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Essays on Monetary Economics and Central Banking

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Essays on Monetary Economics and Central Banking

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DISSERTATION

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THE UNIVERSITY OF TEXAS AT AUSTIN August 2011 Dedicated to my parents Derya Ibrahimagaoglu and Omer Lutfi Ikizler.

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Essays on Monetary Economics and Central Banking

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In the first chapter, I analyze the US banking industry in order to explain two facts. First, larger banks have lower but less volatile returns on loans compared to smaller banks over the years. Second, larger borrowers have better financial records, i.e. verifiable "hard" information, and they are more likely to match with larger banks, as documented by Berger et al.(2005). I show that these two facts can be explained using a segmented loan markets model with loan contracts between banks and borrowers. Moreover, I show that the difference between the banks returns is not due to diversification advantage of larger banks. Instead, it is because of the fact that larger banks can operate in both large and small loan markets, whereas small banks can only operate in small loans market. Therefore large banks are able to match with larger and less risky borrowers more frequently, which are less likely to default. Moreover, I take the model to infinite horizon allowing bank size to be endogenous to answer multiple policy questions about the future of small business finance and consolidation. I use the data set from the Consolidated Reports of Condition and Income provided by FDIC for 1984-2010 to motivate our research question and to estimate the model.

My second chapter revisits the welfare cost of anticipated inflation in an incomplete markets environment where agents can substitute time for money by increasing their shopping frequency. Shopping activity provides an insurance channel to individuals against changes in the return on nominal balances through inflation as documented by Aguiar and Hurst (2007) and McKenzie and Schargrodsky (2011). In my model economy, a higher level of inflation affects people through two channels. First, it distorts the portfolio decision between real and nominal balances, second it redistributes wealth from those who hold more money to those who hold less. People, on average, respond to a higher level of inflation by increasing their price search activity, as they relative return on nominal balances goes down. I find that a 5% increase in inflation causes the welfare level go down by 2% if people are allowed to substitute time for money, and by 10% if we take this channel away from the model.

Finally, in the third chapter, I compare the indirect measure of inflation expectations derived by Ireland (1996b) to the direct measures obtained from expectations surveys in multiple countries. Our results show that the inflation bounds calculated for US and UK data are more volatile than survey results, and are too narrow to contain them due to low standard errors in consumption growth series stemming from high persistence. For Chilean and Turkish cases, however, computed bound for inflation expectations seems to fit the survey results better. Out of three different surveys on inflation expectations in Turkey compared with the bounds computed using Turkish data, expectations obtained by the Consumer Tendency Survey fall within these bounds throughout the whole sample period. The success in the Turkish and Chilean cases can be attributed to the fact that volatility in the consumption series, whereas the failure in US and UK cases are most probably stemming from the fact that the current theoretical model is missing a risk-premium component.

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Chapter 1

US Banking Industry Dynamics: A Matching Model with Segmented Loan Markets

1.1 Introduction

This papers explores the following questions: Why do larger banks have lower and less volatile return on loans compared to smaller banks? What are the bank and borrower characteristics that determine the match between banks and borrowers, and the interest rates on loan contracts? In order to understand these phenomena, my paper is motivated to use the following facts from the empirical finance literature. First, larger borrowers have better financial records, i.e. verifiable "hard" information, and are more likely to work with larger banks, as documented by Berger et al. (2005) using the National Survey of Small Business Finance. Their results show that if the size of the firm and the size of the loan both double, the size of the bank that provides the loan increase by about 40%. Second, Jimenez and Saurina (2004), using information on more than three million loans entries (1988-2000) collected by the Bank of Spain, shows that larger sized loans have lower probability of default, controlling for the collateral. Moreover, Degryse and Ongena (2005), using 15,000 loan entries comprising the entire loan portfolio of a large Belgian bank, documents that larger loans get charged lower interest rates compared to smaller loans. Together, I show that these facts can explain the observed average return-volatility difference between large vs. small banks using a matching model with loan contracts.

Uncovering the channels behind these two facts is very important for two reasons. First of all, these facts are closely related to numerous policy issues. Small business finance is a very big concern due to consolidation trends in the industry. US banking industry has been going through a dramatic restructuring over the last 30 years. Bank exits during the savings and loans crisis during 80s, and interstate branching and merger waves caused by the banking regulations during 90s are the two big reasons why US banking industry experienced an enormous consolidation over the last few decades¹. Second, how the bank size distribution affects systematic risk is a very hot research topic especially after the financial crisis. Our model can be used to conduct policy experiments to answer these questions.

The fact that we use the stable matchings with contracts as our notion of static loan market equilibrium is a unique aspect of our analysis. The quote by Crawford (1991) explains the reason very effectively: "Perhaps the most important advantage of the matching approach is its robustness to heterogeneity. A traditional competitive equilibrium cannot exist in general unless the goods traded in each market are homogeneous, because all goods in the same market must sell at the same price. A traditional model of labor markets with

 $^{^1 \}rm See$ Berger, Kashyap, and Scalise (1995) and Jones and Critchfield (2005) for a detailed analysis of the transformation of US banking industry

the degree of heterogeneity normally encountered therefore has the structure of a multi-market general equilibrium model. The theory of matching markets replaces this collection of markets with a single market game, in which the terms of partnerships are determined endogenously, along with the matching, via negotiations between prospective partners. The notion of stability, suitably generalized, formalizes the idea of competition, and thereby makes it possible to evaluate the robustness of traditional competitive analysis to heterogeneity." A matching approach is a lot more suitable for our model of segmented loan markets. Since each loan size has it's own supply and demand by different groups of banks.

Our paper builds on the paper by Corbae and Derasmo (2011), which uses a very rich model of banking to explain how the US banking industry structure interacts with the business cycles. The focus of this study is instead on the difference in loan returns between large and small banks. To understand this fact, we analyze the effect of bank size on how banks and borrowers match in the loan market. The study by Emmons et al (2004) finds evidence that expanding in asset size in a single location provides better default risk-reduction than expanding to different locations for community banks. A diversification argument in favor of large banks cannot explain these findings altogether. Our mechanism explores this channel as follows: large banks more frequently match with larger borrowers because of their size advantage, and larger borrowers are more likely to be verified borrowers who have lower probability of default; therefore large banks have lower and less volatile returns. Our paper is also closely related to work by Hatfield and Milgrom (2005) which provides a general theory of two sided matching with contracts. I use their equilibrium concept and extend their model to an infinite number of agents (with still finite types and therefore finite types of contracts) on one side of the market and apply it to the US banking industry.

Contribution of our paper to the existing literature is twofold: First, we consider heterogeneity among banks as well as borrowers to explain the differences among small and large banks. Moreover to our knowledge we are the first paper letting borrowers to send a signal to banks in terms of their project productivity (i.e. getting their financial documents audited), but among those who can't afford to send signal, we still observe credit rationing in equilibrium as in Stiglitz and Weiss (1981).

Literature in finance on bank size and relationship banking is very large. Understanding banks' side of the story, Stein (2002) shows theoretically how large organizations face inefficiencies in processing soft, unverifiable information (i.e. difficulty in transferring soft information). Applying this mechanism to banking industry, Berger et al. (2005) empirically analyzes the existence of this theory in US banking data. They document large banks have comparative advantages in lending technologies such as credit scoring that are based on "hard" quantitative data. Small banks, in contrast, may have comparative advantages in lending technologies such as relationship lending that are based on "soft" information that is difficult to quantify and transmit through the communication channels of large banking organizations. In particular, large banks are less willing to lend to informationally difficult credits, such as firms with no financial records. Moreover, after controlling for the endogeneity of bank-firm matching, they find that large banks interact more impersonally with their borrowers, have shorter and less exclusive relationships, and do not alleviate credit constraints as effectively. Moreover, controlling for firm size, firms that have financial records borrow from banks that are roughly 24% larger. This is consistent with the idea that, all else equal, larger banks are at a comparative advantage in lending to firms for which hard information is more readily available. Consistent with these arguments, Berger, Kashyap, and Scalise (1995) show that large banks relative to small banks in the U.S. have been found to lend proportionately less of their assets to Small and Medium Enterprises (from now on SMEs), to lend to larger, older, more financially secure SMEs when they do so (e.g., Haynes, Ou, and Berey (1999)), to charge lower rates, earn lower yields, and require collateral less often on their SME loans (e.g., Berger and Udell, (1996), Carter, McNulty, and Verbrugge, (2004)). DeYoung, Hunter, and Udell (2004) shows that relative to large banks, small banks tend to serve smaller and local customers.

Rest of the paper is organized as follows: Next we will look at the empirical facts in detail, then introduce our model and discuss our equilibrium further, present the results and conclude.

1.2 Data and Motivation

We use data from the Consolidated Reports of Condition and Income provided by FDIC for 1984-2010. Data is publicly available through Federal bank of Chicago. Our data set is quarterly available for most variables and yearly available for some. Table 1 shows some statistics concerning our main motivation for this study. For each quarter, we ranked banks according to total assets, and calculated average return on loans for each group of banks and, finally calculated the averages and standard deviation over the sample period².

Table 1.1. All Ioans, 1964-2010				
Group of banks	Avg Returns	St Deviation		
Top 100	5.07	1.08		
All but top 100	6.2	1.34		

Table 1.1: All loans, 1984-2010

According to data, those banks that are larger in size, have on average lower but less volatile returns on loans. In the table below, we repeat the same exercise focusing on the commercial and industrial(C&I)loans only³. This is indeed the statistic that is more accurate for our study. Commercial and industrial loans are those that are given out to businesses and enterprises, as opposed to real estate or agricultural loans. Each category requires a different approach since the risks and borrower characteristics associated with each type

 $^{^{2}}$ For total assets we use rcfd2170, for income from loans we use riad4010 and riad4059 and for total loans we used rcfd2122. Calculating the return on loans we divided total income from loans by total loans.

 $^{^3 \}rm We$ use the variable riad 4012 for the returns, and rcon1766 for the total C&I loans. Note that rcon1766 is reported on FFIEC041 form unlike other variables

of loan is going to be different. Our focus is going to be on the differences on C&I loans because these are the loans where bank size really matters in terms of how they match with different types of borrowers. Figure 1 depicts the differences in charge off rates⁴ for all loans and for C&I loans. To be more clear, each point on the graph refers to the percentage of loans not received by small banks minus percentage of loans not received by large banks. Throughout the whole sample period, the charge off rate difference for C&I loans is always above the charge off rate difference for all loans (except for one quarter for which the economy is in serious recession). This means that small banks more often match with riskier borrowers when they give out C&I loans, therefore a smaller portion of them are likely to pay back. For this reason, we will focus on C&I loans.

Group of banks	Avg Return	St Deviation
Top 100	3.79	0.7
Top 1000	4.31	0.81
Bottom 1000	4.85	0.88

Table 1.2: Commercial and Industrial(C&I)Loans, 2001-2010

As you can see in Figure 2, the difference in bank returns between large and small banks is not only a statistic true on average but hold at every point in our sample period. 5

Figure 3 shows the data from individual banks. We look at the banks that were in business through the sample period (excluding new entrants and

 $^{{}^{4}}$ Ratio of loan payments not received at least 60 days after the due date.

⁵This is true even when we look at all loans for different groups of banks and time periods.

those that exit at some point). There are 6931 banks that continuously operated for 38 quarters in 2001-2010 and for each individual banks, we calculated the mean and standard deviation of returns on C&I loans over the sample period. This figure shows the tradeoff between returns and risk.

1.3 Static Model

We start with introducing the model. There are finite number of banks, each of which identified by their sizes $d_i \in D = \{d_1, d_2...d_N\}$, and there are infinitely many borrowers, characterized by their project size and productivity parameters $(\ell, z) \in L \times Z$. We assume that both the project size and the productivity have finite supports $Z = \{z_1, z_2...z_{M_z}\}$ and $L = \{\ell_1, \ell_2...\ell_{M_\ell}\}$. We also assume that project size is observable to everyone but productivity is private information and only observed by the borrower herself. Timing of events in our model is as follows:

- i. Borrowers draw their project size (observable) and productivity (unobservable) (z, ℓ) with according to cdf $G(z, \ell)$, (and pdf $g(z, \ell)$))
- ii. Borrowers choose if they want to get audited $(\theta = 1)$ or $not(\theta = 0)$, if audited they can reveal(and verify) their productivity to others
- iii. Applications submitted, banks choose how many contracts they want to offer to each type of borrower, borrowers choose the contract with lowest interest rate, and matching process continues until a stable set of contracts is reached

iv. Both sides match, borrowers who received a loan choose the level of risk and projects undertaken and profits realize.

A bank and a borrower are matched through a loan contract, which specifies the project size, the borrower productivity (if the borrower is audited) and the interest rate on the loan. Note that only the interest rate is going to be an endogenous term of the contract. As banks have different deposit capacities, each bank might give out different number of loans in equilibrium. Our solution technique for the equilibrium matching problem between the borrowers and the banks is stability, and it is similar in essence to Hatfield and Milgrom (2005). We solve for the a set of contracts⁶ from which there are no incentives for any agent to deviate by forming other matches, and use that as our equilibrium concept. First, we will define the set of contracts, banks' and the borrowers' problem, and then return to equilibrium in greater detail.

1.3.1 Contracts

Let banks and borrowers indexed by i and j, respectively. Let set of interest rates that can be specified in a contract be the set of real numbers. The set of contracts is defined as:

$$X = \{(i, j, r) | i \in D, j \in Z \cup \{u\} \times L, r \in R\}$$
(1.3.1)

Each contract $x \in X$ specifies a bank by i, a borrower by j and an interest rate $r \in R$ on the loan. Note that we assume that there are finite

 $^{^{6}}$ We will later discuss in detail why we think this is the right choice.

number of banks(N) and finite types of borrowers(however infinite number of borrowers). Here, it is important to be careful about what is on the contract and what is not. Project size is part of the contract as it is observable by everyone. However, project productivity can only be revealed if the borrower pays the verification cost of being audited as we will describe below. Therefore j can either specify a productivity level $z \in Z$ or denote a u that stands for unaudited. Both banks and borrowers will choose their optimal contracts from the available set of contracts in each round of our matching algorithm, and we will find the equilibrium subset of contracts. That's why the set of contracts is the most important aspect of our analysis.

1.3.2 Banks

The problem of a bank is to choose how many contracts to offer to which type of borrowers so as to maximize the expected profits given the capacity constraint. Our bank profit function is very similar to the one by Boyd and De Nicolo (2005). The only difference is the fact that our bank is capable of given out multiple loan contracts to different borrowers since each bank have different amount of deposits available. Banks enter the period knowing the current state of the banking industry. Static profit function of a bank with deposit level d is:

$$\pi_{i}(d,\mu) = \max_{\eta_{ij}} \sum_{j=1}^{J} \eta_{ij} \{ p(R_{j},\theta_{j}z_{j})(r_{ij}^{\ell} - r^{d}(\mu)) \} \ell_{j}$$

$$s.t. \quad \sum_{j} \eta_{ij} l_{j} \leq d$$

$$s.t. \quad 0 \leq \eta_{ij} \leq g(z_{j},\ell_{j}), \quad \forall j$$
(1.3.2)

where η_{ij} is the number of type *j* borrowers bank *i* would like to sign a contract with, R_j is the level of risk the borrower j is going to choose after the contract is signed, d is the deposit capacity for bank i, and μ is the state of the banking industry, which defines how many banks are currently there at each deposit capacity level d. Moreover, $\theta = 0$ means that the borrower chose not to be audited so that $\theta_i z_i = 0$ as well, and that's why the contract for that borrower do not depend on his productivity. However, $\theta = 1$ and $\theta_i z_i = 1$ simply imply that the borrower got audited and revealed his productivity type so that contract terms will be determined accordingly. Profits of the bank are summed over the individual loan contracts, i.e. $\{\forall j \mid \eta_{ij} \neq 0\}$, she signs in equilibrium. Therefore the optimization problem of our banks is choosing the most profitable set of contracts among the available ones. Through this optimization problem we obtain an induced preference ordering over the sets of contracts available to each bank. We will be more specific about the preferences over the sets of borrowers and how this procedure relates to our equilibrium below. The function $p(R_j, \theta_j z_j)$ is the probability of success of the contract with borrower type j. More specifically, we assume that the probability of success depends on the type of the borrower (if audited), i.e. z, and the level of risk the borrower will choose. Note that the level of risk chosen by the borrower will also depend on the interest rate on the loan contract as we will see in the borrowers problem.

Note that the first constraint implies a segmented markets structure since we are only allowing banks to work with borrowers up to their sizes. One obvious consequence of this market structure is the fact that larger banks will have the advantage of choosing from a larger pool of borrowers, therefore in equilibrium they will end up working with more productive borrowers.

1.3.3 Borrowers

There is an infinite number of borrowers in our model economy and Γ is the distribution of borrowers over the possible productivity and project sizes, the set $Z \times L$. Therefore, Γ tells us how many borrowers there are at each state. We assume that the project productivity of a borrower is unobservable unless the borrower gets audited. We see each borrower as a firm or entrepreneur looking for funding for their investment projects.

All the borrowers live only for one period. Therefore we assume limited liability on the borrower side. In case of failure on the project undertaken, the borrower defaults.⁷ Another implication of the one period lived borrower assumption is that there is multiple period relationship banking in our model. Every period newly born borrowers with no history apply for loans.

 $^{^7\}mathrm{Note}$ that it is either failure or success on the project, there is no partial success in our model.

Our borrowers make three decisions: First, each borrower decide whether they want to be audited or not prior to loan applications, then once the applications are submitted, borrowers look for the bank that offers the lowest interest rate loan contract (since each borrower will accept only one loan contract). Once banks and borrowers match and the equilibrium set of contracts are reached, then borrowers with a loan contract get to choose the risk level they want to undertake.

First we will describe the auditing decision:

$$U^{A}(z,\ell;\mu,\sigma) = \max\{\Gamma(\ell,\mu,\sigma)p(R^{*}(r^{\ell}(\mu,\sigma)),z)[R^{*}(r^{\ell}(\mu,\sigma)) - r^{\ell}(\mu,\sigma)]\ell, - c(\theta,\ell) + \Gamma(z,\ell,\mu,\sigma)p(R^{*}(r^{\ell}(z,\mu,\sigma)),z)[R^{*}(r^{\ell}(z,\mu,\sigma)) - r^{\ell}(z,\mu,\sigma)]\ell, 0\}$$
(1.3.3)

Here, μ is the state of the banking industry, σ is the strategy vector for all borrowers, and R^* is the optimal choice of risk by the borrower once the loan contract is signed as a function of the interest rate. $\Gamma(z, \ell, \mu, \sigma)$ denotes the probability of getting a loan. Obviously, being able to get a loan depends, on the loan supply, i.e. μ^8 . In this stage, each borrower chooses to get audited or not, comparing the expected interest rate gain of being audited to the cost being audited. Below is the equation solving for the cutoff value for getting audited:

$$\frac{c(\theta,\ell)}{\ell} = \Gamma(z,\ell,\mu,\sigma)p(R^*,z_{\ell}^*)[R^* - r^{\ell}(z_{\ell}^*,\mu,\sigma)] - \Gamma(\ell,\mu,\sigma)p(R^*,z_{\ell}^*)[R^* - r^{\ell}(\mu,\sigma)]$$
(1.3.4)

⁸In the simply case where loan supply is equal to loan demand, $\Gamma(z, \ell, \mu, \sigma) = 1, \forall z, l$

where z_{ℓ}^* is the cutoff for choosing $\theta = 1$, i.e. for getting audited, that solves the equation above for project size ℓ . Left hand side is the cost of getting audited per unit of loans and the right hand side describes the benefit from getting audited per unit of loan. We assume that the cost function satisfies:

$$\frac{\partial c(\theta,\ell)}{\partial \ell} \ge 0, \quad c(\theta,\ell=0) > 0 \tag{1.3.5}$$

Therefore, the cost of auditing is increasing in borrowers size and has a fixed component. This is consistent with the data, as presented in O'Keefe et al.(1994). This assumption has implications for the cutoff level of productivity and how it is affected by the loan size.

Only those borrowers that are more productive than z_{ℓ}^* will find it profitable to get audited and reveal their project productivity to banks. Once the auditing decisions are made, next step is finding a stable matching between banks and borrowers. All borrowers will look for the lowest interest rate available among the set of contracts offered. However, there is still an important aspect about the borrower choice that needs to be clarified. If there are more than one bank offering the same interest rate to a borrower, which one is she going to choose? Here, we solve the model using the following tie-breaking rule: for any two banks with sizes d_1 and d_2 that are offering the same interest rate, the borrower will choose bank 1 with probability $\frac{d_1}{d_1+d_2}$, i.e. probability being proportional to the size of the bank⁹.

⁹We can rationalize this assumption by assuming that number of branches a bank has is proportional to its size and those branches are evenly distributed over where borrowers are located, and borrowers choose the bank that is the closest to them.

Now we turn to the last step of the borrower's problem. Below we state the investment problem for borrowers, i.e. choice of riskiness R. Once the borrower signs a loan contract with r^{ℓ} ,:

$$U^{b}(z,\ell,r^{\ell}) = \max_{R} \{ p(R,z)(R-r^{\ell}) \} \ell$$
(1.3.6)

Therefore, depending on how high the interest rate on the loan contract, borrower will choose the level of riskiness accordingly. To be more clear, FOC implies:

$$\frac{\partial U^b}{\partial R} = \{ \frac{\partial p(R^*, z)}{\partial R} (R^* - r^\ell) + p(R^*, z) \} \ell = 0$$
(1.3.7)

where $R^* = R(r^{\ell}, z)$ is the optimum level of risk chosen by the borrower. Note here that the level of risk chosen is increasing in the interest rate on the loan and decreasing both in the borrower productivity and the project size:

$$\frac{\partial R^*}{\partial r^\ell} = -\frac{\frac{\partial p(R^*,z)}{\partial R}}{\frac{\partial^2 p(R^*,z)}{\partial R^2}(R^* - r^\ell) + 2\frac{\partial p(R^*,z)}{\partial R}} \ge 0$$
(1.3.8)

This property of the borrowers' problem will induce an optimum loan interest rate for the bank. An interest rate that is too high will cause the borrower to take too much risk, therefore reducing the expected profit for the bank, and any lower interest rate will reduce the risk along with the return. So, banks profit function will be hump shaped¹⁰.

1.3.4 Static Loan Market Equilibrium

We now turn to definition of our static market equilibrium. Since we will use the notion of stability as our equilibrium concept, we need to precisely define stability. First we need to introduce some notation:

- **Definition 1.** *i.* A matching between banks and borrowers is stable if it is not blocked by any individual bank, borrower or bank-borrower pair.
 - *ii.* A group stable matching is one that is not blocked by any coalition of banks and borrowers.

Now we state the concept of stability in terms of our value functions taking advantage of the preference assumptions in our model.

Definition 2. *i.* Let $V_{ij}(r) = \{p(R_j, \theta_j z_j)(r_{ij}^{\ell} - r^d(\mu))\}\ell_j$ be the value of a contract with borrower type j for bank i at the interest rate r.

 $^{^{10}{\}rm This}$ property has been introduced by Stiglitz and Weiss (81), and used by Corbae and Derasmo (2010) in their problem for borrowers as well

ii. Let $U_j^b(r^\ell) = \{p(R^*, z_j)(R^* - r^{\ell_j})\}\ell_j$ be the value of a loan contract at the interest rate r with borrower j^{11} .

Definition 3. An equilibrium in this market consists of a vector of cutoff values $\{z_{\ell}^*\}_{\ell \in L}$, a vector of interest rates $\{r_{ij}^{\ell}\}_{I \times J}$, a vector of banks portfolio choices $\{\eta_{ij}\}_{I \times J}$ s.t. a subset of contracts $X' \subset X$ is a stable allocation if:

- i. Banks and borrowers problems, i.e. (2), (3) and (6) are satisfied.
- ii. $\forall \ell \in L, z_{\ell}^* \text{ satisfies (4).}$
- iii. Individual Rationality is satisfied:

$$\min\{V_{ij}(r'), U_j(r')\} \ge 0, \forall (i, j, r') \in X'$$
(1.3.9)

iv. No blocking coalitions condition is satisfied:

$$\min_{j|(i^0,j,r')\in X'} \{V_{i^0j}(r')\} \ge V_{i^0j^0(r^0)} \text{ or } U_{j^0}(r') \le U_{j^0}(r^0), \forall (i^0,j^0,r^0) \in X \setminus X'$$
(1.3.10)

Condition (1) implies that given the interest rates, banks portfolio choices solve the bank's profit maximization problem, and borrowers makes their risk/return choices optimally. Condition (2) asserts that the productivity cutoff rule for being audited is satisfied. Therefore, none of the borrowers will have incentive to change their auditing decision, i.e. given the interest rates, it still not profitable for unaudited borrowers to pay the cost and verify

¹¹Note that a borrower's value only depend on the interest rate but not on the bank type, since we assume that all banks are the same for a borrower

their productivity to get a lower interest rate, and vice versa. Condition (3) is the usual participation constraint for signing any contract and condition (4) is the stability condition, which makes sure that no bank or borrower can mutually agree to switch their partner and sign a more profitable contract.

Our equilibrium is unique in terms of the interest rates. However, since banks earn the same expected profits over each type of contract, the different allocations of borrowers among banks (still satisfying the size constraint) will yield other stable matching allocations. However, under our assumption of borrower tie break rule, our equilibrium is completely unique. A stable matching allocation always exists given our preference assumption following from Hatfield and Milgrom (2005).

Figure 4 gives a graphical representation for the determination of equilibrium interest rates. Interest rates are such that they are at bank-profit maximizing level for the least productive types, i.e for the unaudited pool of applicants. Moreover, interest rates on the audited borrowers must be such that those contracts yield the same expected return with the other contracts. This is because, once banks start competing for the most productive types, the interest rates on those contracts starts falling down, until the expected return equals the maximum profit that can be earned with an unaudited type. In this example, we completely abstracted from different loan sizes and kept it simple and give the intuition.

1.3.5Characterization of Equilibrium

Now we present the main result of our paper:

Proposition 4. Let d_i be the deposit size and r_i^{ℓ} be the set of loan interest rates signed in equilibrium by bank i. For any two banks i, j s.t. $d_i < d_j$:

i.
$$\mu_i(r_i^\ell) \ge \mu_j(r_j^\ell)$$

ii. $\sigma_i(r_i^\ell) \ge \sigma_j(r_j^\ell)$

where μ and σ denotes the set average and standard deviation operations over of the set r^{ℓ} .

Proof. See Appendix.

Corollary 5. Take any two different loan markets. The equilibrium profits made on the least valuable contract for a bank in a larger loan market is more(weakly) valuable than the profits made of the least valuable contract in a smaller loan market.

Proof: Follows from the stability condition of our equilibrium definition.

Proposition 6. Take any two loan markets where initial loan supplies are equal. If there is no overlap between the markets, then we have: $\ell_1 < \ell_2 \Rightarrow z^*_{\ell_1} > z^*_{\ell_2}$

Proof. See Appendix.

One important issue is to compare our equilibrium set of contracts to competitive equilibrium. To be able to do that, all we need is to define offthe-equilibrium interest rates for those contracts that have not been actually signed. We need to assign the interest rates on those contracts such that our group stability condition is satisfied, i.e. there are no incentives for individuals or groups to deviate from the equilibrium set of contracts. Then, the set of interest rates in our equilibrium set of contracts combined with those off the equilibrium set of interest rates altogether defines the price vector in our economy that supports the competitive equilibrium counterpart for our solution.

A special case of our equilibrium can be interpreted as Bertrand competition. As long as the total supply for loans are not more than the demand for loans, we could interpret our equilibrium as the banks competing for the most productive types. From this perspective, it is easier to understand why each contract earns the same expected profits in a stable equilibrium, similar to Bertrand competition. In the special case where there is more loan supply then demand, then the borrowers expected utility would have been drawn down their outside option, as then they would be bertrand competing for the loans, not the banks.

As in Kelso and Crawford (1982), we can do comparative statics for our equilibrium. Expectedly, adding more borrowers to one side of the market makes all banks weakly better off and all borrowers weakly worse off. Moreover, reverse is true for the banks. More banks imply more competition on the banks side, implying that borrowers will get better interest rates (weakly) and banks are weakly worse off. Moreover, one can show that interest rates on contracts are completely symmetric over types:

Corollary 7. Any two audited borrowers with the same characteristics sign (if any) contracts with the same interest rate in equilibrium.

Proof. See Appendix.

The corollary above simple follows from the fact that a bank that is currently offering the lower rate would find it beneficial to offer a slightly better rate to the borrower who is currently paying the higher rate, and would exploit this profitable contract. The no blocking condition defined in (10) implies this result directly.

Proposition 8. Assume that no bank has monopoly power in any of the segmented markets¹², then for any two audited borrowers, we have: $z_1 > z_2 \implies$ $r_1^{\ell} < r_2^{\ell}$.

Corollary 9. For any two borrowers with equal project size we have: $z_1 > z_2 \implies r_1^{\ell} \le r_2^{\ell}$.

Proof. See Appendix.

 $^{^{12}}$ Therefore there are at least 2 active banks operating in each of the segmented loan markets.

Proposition (8) suggests that the more productive a borrower, the lower the interest rate in its loan contract. Unlike in many other models, this result does not stem from a zero profit condition on the banks' side. However, in a similar fashion, the stability of our equilibrium requires the banks to earn the exact same profits on each contract. Therefore in essence, this result follow from the same underlying competition for the productive types. Figure (4) summarizes this result in a nutshell. Starting any initial level of interest rates, competition for the productive types will bring the interest rates such that each contract earn the same expected profits.

Note that we have multiple equilibria in our model. Core is always non-empty and contains the set of all stable sets of contracts. However, we restrict our attention to the bank optimal stable set of contracts, which is a unique equilibrium point.¹³ There are couple of reasons to focus on the bank optimal stable outcome. First of all, unlike borrowers, banks have stronger incentives to choose the risk level on their portfolio, therefore it matters a lot for banks to whom they are matched in equilibrium. Moreover, for borrowers, what matters is being able to get the loan with the best possible interest rate independent of the identity of the bank. Secondly, except for the very large borrowers with really good information about their project quality, mostly banks make the ultimate offers, i.e. have the bargaining power.

¹³Also, note that every bank signs the same number of contracts at every point in the core, thus our selection of the bank optimal equilibrium only affects the equilibrium interest rates and bank profitability.

1.3.6 Computational Algorithm

Mechanism we use to find the equilibrium in our environment is in essence similar to that of Hatfiled and Milgrom (2005). We can't apply their fixed point operator simply because we have infinitely many borrowers. However, we find the equilibrium iterating on each side's decisions similar to deferred acceptance algorithm in Roth and Sotomayor (82). We start with the autarky case where all the borrowers choose not get audited, and see what interest rate the bank offers to a pool of applicants all of which unaudited. Given that all the borrowers are identical in this setup, they are all equally likely to get a loan and the probability of getting the loan is derived from the total amount of loans supply available in each market. Given the interest rate offered and the probability of getting a loan, next we check again if there are any incentives to get audited starting from the most productive type in each market¹⁴ and continue until equation (4) is satisfied. Now that we have the best response from the borrowers, we go back to banks decision and post the new interest rates for the updated unaudited pool, and the audited types, which implies new probabilities for being able get a loan for each type. This process continues until each party' best responses do not change.

¹⁴The returns to getting audited is highest for the most productive type. However, that is not a imposition on the algorithm, it is a conjecture for the equilibrium and it does not rule out the other cases.

1.4 Conclusions

This paper explores why large banks have lower but less volatile returns on their loans compared to small banks. We bring together evidences from empirical finance literature with a detailed matching model between banks and borrowers to understand the channels through which this phenomenon occurs. We see that size advantage of large banks allows them to be able to match with more productive borrowers overall, i.e. to choose their customers from a larger pool of applicants compared to smaller banks. Moreover, everything else being constant, large borrowers are more advantageous in terms of verifying their project productiveness, because of the fixed costs associated with being audited. In equilibrium, large banks' portfolio of loans includes more large and verified borrowers compared to small banks' portfolios. Therefore, they have lower and less volatile returns on their loans.

Our results are very preliminary but promising. There are couple of assumptions we would like to make endogenous to the model. The most important one is the loan size. Currently, we assume that project size is given and loan size is not endogenously determined in equilibrium. Whereas in real life, both the interest rate and the loan size are endogenously determined, and we abstract from that. In the current model, we only stated and solved the static loan market competition but we are currently working on the dynamic version of the model. In our dynamic version banks make investment, entry and exit decisions, so that bank size is an endogenous variable and the size distribution changes overtime. Our next step is to solve for the long-run indus-
try equilibrium using the notion of oblivious equilibrium by Weintraub et al. (2008). With the addition of dynamic dimension to our model, we can answer a couple of questions: Recent trends in the US banking industry, stability of the banking system against aggregate shocks, i.e. possibly analyze banking crises and the future of small business finance.



Figure 1.1: Charge-off Rate Differences by Loan Type



Figure 1.2: Return on C&I Loans by Bank Size



Figure 1.3: Returns vs Volatility, All banks, C&I loans



Figure 1.4: Stability in the Loan Market

Chapter 2

Shopping time, price search and Optimal Monetary Policy

2.1 Introduction

Welfare costs associated with inflation has been explored for many decades, under different environments¹. Researchers concluded that one way or other, inflation distorts household's decision for nominal assets, and therefore is costly. Recent studies emphasized another channel of interest the previous literature ignored calculating the welfare costs of inflation. Aguiar and Hurst (2007) documented that doubling shopping frequency lowers a goods price by 7 to 10 percent. They use scanner data and time diaries to document how households substitute time for money through shopping and home production. Moreover, McKenzie and Schargrodsky(2011) showed that during the high inflationary financial crisis in Argentina in 2002, people increased their shopping frequency significantly. The devaluation resulted in a significant increase of 41% in the overall consumer price index in 2002, and along with an increase in price dispersion in the economy. They documented that people responded to this phenomenon by increasing their shopping activity both in intensive and

¹See Lucas(2000) for a comparison of basic approaches.

extensive margin. Mean shopping days increased from 5.02 in 2001 to 5.21 in 2002. This increase translates into almost two-thirds of households shopping an extra day each month.

In the light of these observations, we realize that one should take into account the fact that people do substitute time for money, when calculating the welfare implications of inflation. In this paper, we allow people to use time and money together to increase their consumption. When faced with a higher inflation, returns from holding nominal assets go down, and people respond by increasing their shopping activity instead. We show that the actual welfare cost of inflation is lower under the existence of this additional channel.

In his very recent work, Wang (2011) uses a general equilibrium search model with endogenous price dispersion to analyze the welfare cost of inflation. Their model is built on the work by Lagos(2005), where there are search frictions, sellers post prices and buyers decide how much of their real resources to allocate for searching lower prices(linear and discrete). In their model, they abstain from time substitution however. They find that search channel increases the welfare cost of inflation at a very ignorable amount. Our model however shows that the existence of the shopping activity reduces the welfare loss associated with inflation.

Rocheteau and Wright(2005) analyzes the optimality of the Friedman rule under three different environments with search frictions. They find that when prices are determined competitively, or through bargaining between buyers and sellers the Friedman rule is unable to correct the inefficiencies associated with the market structure. Even if it is the optimal rule, there are welfare costs associated with inflation. However, when market structure allows for competitive search through price posting mechanism, then the Friedman rule is not only optimal but also corrects the inefficiencies, creating no welfare loss.

In their early work, Cooley(1991) analyzes the interaction between the welfare cost of inflation and the distortion due to existing fiscal tools. They find that welfare cost of inflation almost doubles when there are are additional taxes on capital and labor. Our paper is essentially similar to their because we explore the reverse mechanism, where instead of distortions, if there is an additional insurance mechanism for people to substitute time for money through a shopping technology.

Bhattacharya et al. (2005) shows that under heterogeneity of agents and the lack of fiscal tools to redistribute wealth among agents, the Friedman rule is no longer optimal since inflation is the only way to redistribute the wealth among agents. Even though we are not focusing on this channel in our work, we still want to include wealth heterogeneity in our environment because we want to have our model to match the fact that differences in wealth levels are one of the main determinants of the search behavior.

The idea of substituting away from nominal balances when the inflation is high not new obviously. Bailey(1956)describes how consumers spend real resources in alternative means of exchange to avoid the inflation tax. Moreover, Gillman (1993) calculates the welfare cost of inflation in a cash-in-advance model where people can use cash or costly credit to purchase goods. As inflation rate gets higher, substitute away from cash balances towards using costly credit. He shows that the associated welfare cost is higher in this environment than the standard cash-in-advance type environments. Dotsey and Ireland(1996) studies the very similar environment in a general equilibrium framework and shows that existence of production side amplifies the welfare cost of inflation very seriously.

Among others, Burstein and Hellwig(2008) quantitatively evaluates the welfare effects of inflation in a menu cost model of price adjustment using a money in the utility type model. They use a general equilibrium model and find that the relative contribution of the price setting distortion, i.e. menu cost, is very minimal compared to distortion created by the the opportunity cost of real money balances. Their results encourages our assumption of partial equilibrium since we abstain from price setting side of the story.

Our environment entails similar features to the literature above and connects some of the links to be able to explain the patters we see in the data. We combine the heterogeneous agents environment with a shopping technology that allows agents to substitute time for money, at different levels. We show that the welfare cost of inflation is lower in this environment. The remainder of this paper is organized as follows. The next section introduces our model and defines the steady state equilibrium for a given level of inflation. Section 3 compares how different levels of inflation affects equilibrium decisions and welfare levels. Section 4 concludes the paper.

2.2 Model

In this section we introduce the model economy. Time is discrete and infinite. Agents maximize lifetime utility in an incomplete market environment² where they face idiosyncratic income risk. In this environment, agents have two resources, one is real endowment and the other is time endowment. The use of real endowment is a choice between holding inflation bearing money and interest rate bearing bonds. Agents need to hold money in order to facilitate consumption. Moreover, we assume that the shopping technology is such that people use time endowment as well as money for consumption. The key assumption is that time and money are substitutes in our shopping technology. We motivate this assumption by the empirical findings on price search and shopping frequency literature.

We assume that people have 1 unit of time endowment to allocate between shopping time and leisure ³. The population consists of a continuum of infinitely lived agents who maximize their expected life time utility:

$$E\sum_{t=0}^{\infty}\beta^{t}u(c_{t},l_{t})$$
(2.2.1)

where β indicates time discount factor, c_t and l_t represents consumption and leisure period t. The budget constraint in real terms is as follows:

 $^{^{2}}$ We also solve the same environment with representative agents where the stream of endowments are constant over time, see the results section for the computational exercise

 $^{^3\}mathrm{We}$ also analyze the case with labor supply, where we see endowment process as being employed, and allocate the time accordingly

$$c_t + m_{t+1}(1 + \sigma_t) + \frac{b_{t+1}}{1 + r_t} \le y_t + m_t + b_t + M_t \sigma_t$$
(2.2.2)

At each period, the agent chooses money and bond holdings, m_{t+1} and b_{t+1} for the next period and allocates his time between shopping and leisure, s_t and l_t . We formulate the problem in the recursive form below:

$$V(b, m, y) = \max_{m', l, b, c} \{u(c, l) + \beta E[V(b', m', y')]\}$$

s.t.
$$c + m'(1 + \sigma) + \frac{b'}{1 + r} \le y + m + b + M\sigma$$

$$s + l \le 1$$

(2.2.3)

where the shopping technology is defined 4 as follows:

$$s = g(c, m')$$
 (2.2.4)

Here m' is the amount of money chosen to be used for shopping, and s is the time spent on shopping. Moreover, note here that government redistributes the newly printed money back to people as a lump sum transfer. So, agents start the period by knowing how much total assets they have, i.e. b + m and they receive their endowment shock, $y \in \{y_l, y_h\}$, which follows first order Markov process with probability support π . The amount of consumption goods one can buy is increasing with the amount of money held for shopping and time

 $^{^{4}}$ We use a very similar formulation following Ljungqvist and Sargent(2004)

spent. Our shopping function satisfies:

$$\frac{\partial g}{\partial c} \ge 0, \frac{\partial g}{\partial m'} \le 0, \frac{\partial^2 g}{\partial c^2} \ge 0, \frac{\partial^2 g}{\partial m'^2} \ge 0, \frac{\partial^2 g}{\partial c \partial m'} \le 0$$
(2.2.5)

Let $\Gamma(m, b, y)$ be the distribution of the population over the state space. Then, the goods and the money market clearing conditions are:

$$\sum_{i=h,l} \int \int \Gamma(m,b,y_i) c(m,b,y_i) dm db = \sum_{i=h,l} \int \int \Gamma(m,b,y_i) y_i dm db$$

$$\sum_{i=h,l} \int \int \Gamma(m,b,y_i) m'(m,b,y_i) = M'$$
(2.2.6)

where:
$$M' = (1 + \sigma)M$$
 (2.2.7)

2.3 Equilibrium

In this section we define the equilibrium. Following the literature on dynamic stochastic equilibrium macroeconomics models, we define our steady state equilibrium as:

Definition 10. A steady state monetary competitive equilibrium, given the money growth rate σ , is a set of decision rules m'(m, b, y), b'(m, b, y), l(m, b, y) and c(m, b, y), and an invariant distribution, $\Gamma(m, b, y)$ over the population s.t.:

i. Individual optimization conditions, i.e. (3) and (4) are satisfied.

ii. Markets clear, i.e. (6) are satisfied.

All three versions of our model is subject to the same equilibrium definition and conditions, whereas the endogenous returns to search case market clearing condition involves and equilibrium return parameter as we will define in the very next section. Our results show that people shopping time to substitute leisure for consumption even in the absence of inflation. Moreover, as documented in the data, search activity is negatively correlated with wealth level, and the income level for the current period. We will present two different cases first, and then compare our results for all three cases.

2.3.1 Case 2: Representative Agents Case

We know from Bhattacharya et al.(2005) that having a heterogenous wealth distribution with inflation redistributing wealth among agents creates a welfare effect for positive levels inflation due to concavity. Our benchmark model environment generates a heterogenous wealth distribution in any steady state. Therefore lower cost of inflation can also be attributed to redistribution of wealth among agents. To separate the effect of inflation through time substitution from the redistributive channel we solve our model economy with complete markets, i.e. as a representative agent model. In this environment all individuals receive a constant stream of endowments and therefore make identical decisions in any steady state. We show that even in this environment shopping activity provides reduces the negative effects of inflation. The individual optimization problem changes into:

$$V(b,m) = \max_{m',l,b,c} \{u(c,l) + \beta V(b',m')\}$$

s.t.
$$c + m'(1+\sigma) + \frac{b'}{1+r} \le y + m + b + M\sigma$$

$$s+l \le 1$$

(2.3.1)

The rest of the problem is indeed very similar to the previous case. The market clearing conditions are identical if we drop the endowment variable y from the distribution function Γ . Note that in any steady state, we have a point mass of agents on the same state in this case, since everyone is identical. Results from this case are of critical importance to us as we isolate the price search effect in this case. The comparison of this environment with no search environment is very indicative of the insurance channel the time substitution contributes to people against inflation. We will present the results in section (4). Before we go on to the results we will address another weak aspect of our model.

2.3.2 Case 3: Endogenous Returns on Shopping Activity

Another main source of critique about our environment is that it is a partial equilibrium model. We don't take into account the fact that producers do respond to shopping time spent by consumers. Actually, price posting is a channel for producers to price discriminate between those who are willing to substitute time for money and those who don't. Coupons, online deals, weekly specials, etc. are the ways producers discriminate those "two" different types of customers. In our environment however, we assume a functional form for the relationship between shopping time, consumption level and the money holdings assuming that producers do not respond to changes in aggregate shopping behavior. In reality however, one would expect the producers re-optimize their strategy. For example, if everyone in the society would happen to go for coupon to get deals, start going to grocery shopping once a day to catch those times when a certain good is on sale, then the initial purpose would have failed for them. To address this problem, we introduce a slightly different version of our environment in which we add an endogenous "return to search time" variable that determines how much return is out there from spending time in the market looking for better prices. We formulate the problem as follows⁵:

$$V(m, y; K) = \max_{m', s, m_c} \{u(c, l) + \beta E[V(m', y')]\}$$

s.t.
$$m_c + m'(1 + \sigma) \le y + m + M\sigma$$

$$s + l \le 1$$

 $^{^{5}}$ Note that in this case we abstain from the bond option. The reason is that in this formulation there is no reason to hold money anymore, unless we introduce another friction like cash in advance. Instead we emphasize the effect of K using the very basic model following Imrohoroglu(1992)

where the shopping technology is defined as follows:

$$c = h(m_c, s) = \frac{m_c}{Ks^{-\theta}}$$
(2.3.2)

Here m_c is the amount of money chosen to be used for shopping, and s is the time spent on shopping. The amount of consumption goods one can buy is increasing with the amount of money held for shopping and time spent on shopping. Moreover, K is the return on shopping activity and is determined in equilibrium through the goods market clearing condition. Note that,given the amount of money hold constant in the economy, if everybody increases their search activity, the parameter K has to adjust(go down) such that the goods market clears. In other words, what really matters is relative search. If an agent is spending more shopping time than everybody else in the economy, then there are gains. On the other hand, however, this mechanism puts a burden on the wealthiest people in the economy as they will either be forced to spend more time or more money for the same amount of goods just because all the other people are spending a lot of time shopping and looking for better prices.

Market clearing conditions are :

goods market :
$$\int \int \int \Gamma(m, y)c(m, y) = \int \int \int \Gamma(m, y)y$$

money market :
$$\int \int \int \Gamma(m, y)m'(m, y) = M'$$

where in real terms :
$$M' = \frac{(1+\tau)}{1+\pi}M$$

and :
$$\frac{1}{1+\pi} = \frac{p}{p'}$$

(2.3.3)

2.4 Results

In this section we present our results for various computational experiments. We calibrate our model economy to US economy and computed steady state equilibrium for different levels of inflation. We assume the following functional forms for the utility function and shopping technology following Ljungqvist and Sargent(2004):

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{l^{1-\alpha}}{1-\alpha}$$

$$s = g(c,m') = \frac{c}{(1+m'(1+\tau))^{\theta}}$$
(2.4.1)

Table 1 shows the parameter values we used for calibration. We took most of the values from the literature

In Table 2 we present the results of our model at different inflation levels. This values are calculated comparing the steady state utility levels, and how much does inflation affect the utility level from compared to 0% inflation rate. For example, a 5% increase in yearly inflation reduces the

Table 2.1: Parameter Calibration						
Parameter	Value	Definition				
β	.98	discount factor				
σ	1.5	CRRA param. for c				
θ	2	convexity for shopping tech.				
π_h	0.97	$P(y_{t+1} = y_h y_t = y_h)$				
πl	0.5	$P(y_{t+1} = y_l y_t = y_l)$				
α	1.5	CRRA param. for l				
y_h	1	high endowment				
y_l	.2	low endowment				

welfare level by 10% in the benchmark environment with no time substitution option. One pattern we see looking at our results is that, expectedly, having both heterogeneity and time substitution together makes the effect of inflation very mild on the welfare.

On the other hand, having endogenous returns on the time spent on shopping reduces the effectiveness of the time substitution as everyone in the economy attempts to use shopping time and the return on it goes down significantly. As we can see, 5% yearly inflation still causes an 8% decline in the welfare level in this case, still lower than the no search case.

Table 2.2: Inflation and Welfare							
Yearly Inflation	No search	Heterogenous	Representative	Endogenous			
5%	-%10	-%2	-%4	-%8			
10%	-%14	-%3.5	-%6	-%11			
20%	-%36	-%12	-%18	-%22			

For comparison reasons, we would like to present our welfare comparison in terms of the consumption levels. In other words, in the table we will present that the amount of real consumption good that is necessary to compensate the consumer for the increase in inflation such that the initial level of steady state utility is reached in each case.

rable 2.5. Wenare in Consumption Terms							
Yearly Inflation	No search	Heterogenous	Representative	Endogenous			
5%	%3.7	%1.1	%2.4	%2.8			
10%	%4.4	%1.9	%3.1	%3.8			
20%	%8.8	%4.2	%5.4	% 6.4			

Table 2.3: Welfare in Consumption Terms

We find that higher levels of inflation induces higher levels of search activity in the economy, consistent with the data presented in McKenzie and Schargrodsky(2011). Moreover, we find that people at lower wealth levels increase their search activity more than people with higher wealth levels, in all of our model specifications. Therefore we can see both channels through which price search interact with inflation. It provides people an insurance mechanism against high inflation, and does it more intensely for poorer people. We also see that having an endogenous return on the shopping time activity does reduce the effectiveness of the mechanism. In this case what matters the most is "relative" search.

2.5 Conclusion

In this study, we focus on the fact that existence of a shopping technology reduces the welfare cost of inflation. Opportunity of substituting time for money gives agents a channel to hedge themselves against inflation. Inflation reduces the return on money, and therefore people can either choose to give away from their bond holdings to be able to maintain a given level of consumption, or substitute time for money. Having this option reduces the welfare cost of inflation, in all specification of the model.

Evidently, people do substitute time for money according to empirical evidence. The policymaker should be taking into account the fact that people respond to changes in inflation by adjusting their shopping frequency as well. In this paper, we illustrate this channel with a very simple example. However, our environment assumes a given shopping technology and therefore only looking at the partial equilibrium. We should consider the effects of inflation on producers price setting decisions, and how changes in inflation affect the price dispersion, and therefore returns to shopping activity. Moreover, a second strong assumption of our model is the independence of our returns to shopping activity from the rest of the economy. In real life, one would expect that if everyone else is increasing the shopping frequency, that will generate a most probably decreasing effect on the returns to shopping activity.

A great extension and a direct application of our model should be to calibrate our environment for the Argentinean economy and try to match the exact changes in shopping time activity across different wealth levels. For that matter, one would need to use to calculate the transition dynamics and focus on the very short term effects of the sudden "unanticipated" increase in inflation, as in it happened during the 2002 crisis in Argentine. Obviously, our case is confined to a comparison of steady states with different levels of anticipated inflation. Therefore one should see our results more from a qualitative standpoint than an exact quantitative match of the real world.

Chapter 3

Direct vs. Indirect Measures of Inflation Expectations: A Case Study in 4 Countries

3.1 Introduction

The ultimate goal of any central bank policy is to achieve and maintain price stability. As an unobserved component, inflation expectations are very crucial for determining future inflation, mainly through price and wage setting behaviors. Therefore those expectations need to be measured with sufficient precision. In this study we derive bounds for inflation expectations for 4 countries: the United States, the United Kingdom, Turkey and Chile using the relationship between interest rates and inflation expectations, and compare these bounds to the results of survey data in those countries. Particularly, we recalculate inflation bounds in Ireland (1996) and compare these bounds with a direct measure of inflation expectations, the median responses of the Livingston survey. We apply the same procedure to all countries in our sample in an attempt to seek a plausible comparison¹.

Our results show that, for the US and UK data, the inflation bounds

¹As we will show later in the paper, restricting our attention to a developed country, US, might lead to adverse conclusions and we believe Turkish economy is a good case for developing countries.

suggested by Ireland (1996) are more volatile than survey results, and too narrow to contain them, due to low standard errors in consumption growth series stemming from high persistence. This result seems to be discouraging for the usefulness of bounds, but the Turkish and Chilean cases offer better results in favor of this approach. Calculated real interest rates are very volatile in Turkey and therefore movements in the nominal interest rates themselves cannot be used as an indicator of changes in inflation expectations. Result are somewhat better in Chilean case as well. Taking risk premia into account can address the problem in the UK and US cases.

Literature on the relationship between future inflation and interest rates starts with Fisher's (1907) early work with a postulate that nominal interest rates, in a perfect foresight world, are equal to the real rate of return plus the future rate of inflation. Two views have been raised about the relationship between the real rate and inflation expectations². The first view, following Mundell (1963) and Tobin (1965) claims that the expected real return component of nominal interest rates is negatively related to the expected inflation component. The intuition behind this view is that in an environment with high inflation, agents economize on their nominal asset holdings and hold more real balances which result in a lower marginal product of capital. The second view is contrary to the first one. This view started with Fama (1975), and advocates the constancy of the real rate through time, and hence that nominal interest rates can be used as a signal of future inflation expectations.

²For a very detailed literature survey, see section 3 of Stock and Watson (2003).

Discussion of the relation of inflation expectations to the ex-ante real rate has extended further after the uncertainty was introduced into the Fisher equation by Lucas (1978). He suggested that, in a world with uncertainty, nominal interest rates consist of a risk premium along with the real rate of return and an inflation premium. None of these three components are observable, but Ireland (1996) managed to characterize bounds on inflation expectations using the risk premium. Using ten-year US Treasury bond yields, he showed that real interest rates are quite stable. Therefore, natural limits on risk premia ³ allowed him to draw the bounds on inflation expectations, which are pretty close to each other for US data due to a low risk premium.

Research on inflation expectations in Turkey is relatively new. In one of the earlier works, Sahinbeyoglu and Yalcin (2000) analyzed inflation expectations in Turkey by applying regressions with several explanatory variables, following Mishkin (1981). They found that the term structure of nominal interest rates has valuable information about inflation expectations. Berument and Malatyali (2001) employed GARCH models to identify anticipated and unanticipated inflation. Their findings support the existence of the Mundell-Tobin effect for the case of Turkey, suggesting that the chronically high level of inflation leads to low real rates and stimulates the Turkish economy ⁴. Our study departs from theirs as well as from other relevant studies using Turkish data in a couple of ways. First, we use a forward-looking model for the

³As will be explained later, there are natural limits on risk premia.

⁴In a more recent paper, Gul and Acikalin (2008) rejected Fisher's hypothesis for Turkish data without using risk premia in the regression equation.

inflation expectations while they assume purely adaptive expectations behavior. Expectations should not be modeled by pure time-series models, because individuals have a larger set of information than just the inflation series, and they use this set fully in their decision-making processes. Therefore, similar to Ireland (1996), we use individual consumption decisions to derive information about inflation expectations. Second, our approach doesn't identify inflation components; rather, it presents inflation bounds incorporated with the inflation risk premium. Our main contribution to this literature is comparing the Turkish survey data with these bounds and testing the usefulness of these survey results.

3.2 The Model

In this section, we introduce a version of the theoretical model originally proposed by Lucas (1978). Our model economy is populated by a continuum of infinitely lived households. The representative agent receives a stream of income, y_t . Each period, he chooses how much to consume c_t and how much to invest on two assets: one real asset b_t that costs one unit of consumption good at time t and returns r_t consumption good at time t+1, and one nominal asset B_t ⁵ costs P_t at time t and returns R_t at time t+1 that can be traded with consumption good at the price P_{t+1} .

There is uncertainty about future variables that will help us form the

 $^{^{5}}$ All nominal variables are represented in capital letters throughout the paper.

bounds on inflation expectations following Ireland (1996). The uncertainty is about future prices, income, consumption, interest rates, and bond holdings. That is, our representative agent may not learn the exact values of P_t , y_t , c_t , R_t , r_t , B_t , and b_t until the beginning of period t; before then, he regards these variables as random. The agent faces the following optimization problem:

$$\max E_t \left\{ \sum_{j=0}^{\infty} \beta^j ln(c_{t+j}) \right\}$$
(3.2.1)

subject to the following budget constraint:

$$c_t + b_t + B_t / P_t \le y_t + r_{t-1}b_{t-1} + R_{t-1}B_{t-1} / P_t \tag{3.2.2}$$

Solution to this optimization problem yields the following two conditions:

$$1/r_t = \beta E_t[(1/x_{t+1})] \tag{3.2.3}$$

$$1/R_t = \beta E_t[(1/x_{t+1})(1/\pi_{t+1})]$$
(3.2.4)

where $x_{t+1} = c_{t+1}/c_t$ and $\pi_{t+1} = P_{t+1}/P_t$ are the rate of consumption and inflation, respectively. The first equation is relating the expected consumption ratio to ex-ante real interest rate. The second equation presents this relation in terms of nominal variables, i.e. nominal interest rates and expected inflation rate. Even though the ex-ante real interest rates are unobservable, this equation lets us use consumption data as a way to obtain an estimate for them. One can rewrite equation (3.2.4) as:

$$1/R_t = \beta Cov_t[(1/x_{t+1}), (1/\pi_{t+1})] + \beta E_t[1/x_{t+1}]E_t[1/\pi_{t+1}]$$
(3.2.5)

combining with equation (3.2.3), we get:

$$1/R_t = \beta Cov_t[(1/x_{t+1}), (1/\pi_{t+1})] + (1/r_t)E_t[1/\pi_{t+1}]$$
(3.2.6)

Equation (3.2.6) is a generalized version of the well-known Fisher equation, which relates real interest rates to inflation and nominal interest rates. Sign of the covariance term here determines how the nominal interest rates are affected by the relation between inverse consumption growth and inflation rate. We follow by replacing the risk premium term as:

$$\beta Cov_t[(1/x_{t+1}), (1/\pi_{t+1})] = \beta \rho_t Std_t[1/x_{t+1}]Std_t[1/\pi_{t+1}]$$
(3.2.7)

where ρ_t is the correlation coefficient defined by:

$$\rho_t = Cov_t[(1/x_{t+1}), (1/\pi_{t+1})] / \{Std_t[1/x_{t+1}]Std_t[1/\pi_{t+1}]\}$$
(3.2.8)

Using the fact that the correlation coefficient has to be between -1 and 1, we derive the following inequality:

$$Std_t[1/x_{t+1}]Std_t[1/\pi_{t+1}] \ge Cov_t[(1/x_{t+1}), (1/\pi_{t+1})] \ge -Std_t[1/x_{t+1}]Std_t[1/\pi_{t+1}]$$
(3.2.9)

This inequality puts bounds on the covariance term we are interested in. Following Ireland (1996), we impose the additional assumption on the size of the coefficient of variation for $1/\pi_{t+1}$:

$$Std_t[1/\pi_{t+1}]/E_t[1/\pi_{t+1}] \le 1$$
 (3.2.10)

As Ireland (1996) has done it for US case, we justified this assumption by looking at the Turkish data as well and found that the coefficient of variation never exceeded 0.05 for 1998-2008 period. Hence, similar to Ireland (1996), our bounds are extremely conservative.

In the light of equation (3.2.10), rearranging equation (3.2.9) using (3.2.5) gives us:

$$\beta R_t \{ E_t[1/x_{t+1}] + Std_t[1/x_{t+1}] \} \ge 1/E[1/\pi_{t+1}] \ge \beta R_t \{ E_t[1/x_{t+1}] - Std_t[1/x_{t+1}] \}$$
(3.2.11)

This is almost exactly what one needs to derive the bounds on expected inflation. If we use the approximation:

$$1/E_t[1/\pi_{t+1}] \approx E_t[\pi_{t+1}] \tag{3.2.12}$$

then we have the bounds ready to be estimated. The width of the bounds will be dependent on the size of the risk premium, which in term will be estimated using the consumption ratio using aggregate consumption data.

3.3 Estimation Methodology for the Real Interest Rate and Bounds on Expected Inflation

We use the same estimation technique proposed by Ireland (1996). The relationship between observed variables, nominal interest rate and consumption, and unobserved variables, real interest rate and bounds on expected inflation, are proposed by equations (3.2.3) and (3.2.11). The only two unknowns in these equations are $E_t[1/x_{t+1}]$ and $Std_t[1/x_{t+1}]$, namely expectation and

standard deviation of next period's inverse growth rate of aggregate consumption, and can be estimated through a time series model fit to $1/x_{t+1}$. Now, for convenience, let $g_{t+1} = 1/x_{t+1}$ and assume that g_{t+1} follows an AR(1) process such that

$$g_{t+1} = \gamma + \rho g_t + \epsilon_{t+1} \tag{3.3.1}$$

where γ is a constant and ρ is the AR(1) parameter. ϵ_{t+1} is the random error term and satisfies

$$E_t[\epsilon_{t+1}] = 0, Std_t[\epsilon_{t+1}] = \sigma, E_t[\epsilon_{t+1}\epsilon_{t-j}] = 0, E_t[\epsilon_{t+1}g_{t-j}] = 0 \ \forall j \qquad (3.3.2)$$

where σ is constant through time. Next, we define the data we use for US.

3.3.1**Inverse Consumption Ratio Estimation Results**

0.013

The table below shows the results from our estimation for the inverse consumption ratio. For $E\{\frac{1}{x_{t+1}}\}$ we use the smaple average of the predicted value, which is different at any quarter. For the actual bounds we use the predicted value for the coming quarter, using only the information available up to that quarter. It seems like the standard deviation of inverse consumption

Table 3.1: Inverse Consumption Rates US UK Turkey Chili $\frac{E\left\{\frac{1}{x_{t+1}}\right\}}{std\left\{\frac{1}{x_{t+1}}\right\}}$ 0.981.0020.981.005

0.01

0.06

0.017

ratio differs significantly across countries, which is partly accountable for our results. Relatively higher standard deviation of this ratio drives the bounds to be wider than the lower standard deviation countries.

3.4 US Data

Our model period is one quarter, similar to Ireland (1996), but we use annual bond yields instead of ten-year returns, i.e. 4-period-ahead expectations are used. Since the model period is finer than the interval of bond yields, estimation using ordinary least squares give consistent estimates of AR parameters but biased σ estimates (see Hansen and Hodrick (1980)⁶). Consistent estimates of σ are derived using the method proposed in Hansen and Hodrick (1980), modified as suggested by Newey and West (1987).

We analyzed 1959:1 to 2009:1 period for US data. The nominal interest rate is measured by the market yield on U.S. Treasury bonds at 1-year constant maturity achieved from the Federal Reserve database. Per capita consumption values are found by dividing the seasonally adjusted series of real personal aggregate nondurables and services expenditures ⁷ by the size of the noninstitutional civilian population, ages 16 and over ⁸.

⁶Hansen and Hodrick (1980) further show that k-step-ahead OLS estimator is dominant to the OLS estimator proposed by the resampling at every kth integer in the sense that (1) the latter exceeds in error variance over the former by a positive definite matrix, and (2) using the former has a higher power in testing the null hypothesis. Therefore, the estimation strategy used in our paper is superior to the natural alternative of adjusting the model period to one year.

⁷Consumption Data Source: Bureau of Economic Analysis.

⁸Population Data Source: Labor Force Statistics from the Current Population Survey.

3.5 Results for US Data

Results for US data is depicted in Figure 3.1. Bounds for inflation expectations are far wider in our model based on one-year returns compared to ten-year returns of Ireland's, bound width is between 1.32% and 1.52% in our model while it is 0.15% to 0.17% in Ireland's. However, they are still too narrow to contain survey results. Particularly, bounds do not contain survey results 53% of the time. The Livingston survey results offer a much smoother path for the inflation expectations than the expectations derived from our model. The main reason behind this result is the excess volatility in nominal interest rates compared to inverse growth in consumption. With a lower variability in the real rate suggested by the observed stable path of inverse consumption growth, inflation expectations capture most of the variability in nominal rates.

3.6 UK Data

For the UK case, we use the NOP Inflation Attitudes Survey that is conducted by the Bank of England from 2001 onwards. The question 2A in the survey asks people the following question: "How much would you expect prices in the shops generally to change over the next 12 months?" We use the median response to this question to form our series for the inflation expectations.

For consumption, we use the seasonally adjusted household final con-

sumption expenditure data⁹ provided by the Office for National Statistics. Also the nominal interest rates, we use the quarterly average yield from British Government Securities, Nominal Par Yield¹⁰ provided by the Bank of England.

3.7 Results for UK data

Figure 3.2 compares the bounds on inflation expectations derived from the model to the survey results. The bounds are unable to contain the survey results for most of the time period, similar to the US case. The only times the survey results are indeed inside the bounded area is when the inflation rate is actually high. As it is in the US case, the survey results are a lot more stable over the time period compared to the bounds.

3.8 Turkish Data

This section analyzes 2000-2009 period for Turkish data. There are reasons for this choice. First, Turkey experienced a disinflation and stabilization process starting from 2000 and a more stable state has been reached by the end of 2003. These two different environments offer a good analysis diversity. Second, and more importantly, data availability on surveys limits our set of possible dates. Only one of the surveys was available before 2001 while the other two surveys we analyzed have starting dates of 2001 and 2003.

Data is gathered as follows. The nominal interest rates are yearly com-

⁹Series ABJR is used in our analysis.

¹⁰Series IUQASNPY is used.

pounded interest rates of treasury discounted auctions ¹¹, available monthly for our sample period. There have been three months where the Turkish Treasury did not auction bills but since no two or more such instances occurred in a certain quarter, we just ignored those dates when we get quarterly averages. Consumption data is obtained by dividing seasonally adjusted private final consumption expenditure figures ¹² by the estimated quarterly population, ages between 15 and 64. Mid-year population estimates and population growth rates are combined with age dependency ratio ¹³ and interpolated to achieve quarterly population figures.

3.8.1 Consumer Tendency Survey

Starting from 2003, TURKSTAT and CBRT have jointly conducted the Consumer Tendency Survey (CTS), which aims at measuring consumer tendencies and expectations. The scope of the survey includes all individuals who are 15 and above and have a job that provide income, in urban or rural areas of Turkey. Survey frequency is one month and the participant size changes between 7100 and 8700 for the 2003-2009 period. Inflation expectations are asked as the direction of changes in prices over the next 12 months and hence point estimates of inflation expectations are unavailable and need to be derived. A recent study by Oral (2009) that quantifies answers about inflation

¹¹Nominal Interest Rates Source: Turkish Undersecretariat of Treasury.

¹²Consumption Data Source: OECD Quarterly National Accounts Dataset, LNBQRSA measure.

¹³Population Data Source: TURKSTAT.

expectations of this survey is used for this purpose.

3.9 Results for Turkish Data

We first derive ex-ante real interest rates from our model and compare them with the ex-post real rates calculated using nominal returns and actual inflation. Figure 3.3 depicts both series ¹⁴. It can be seen as a data fact that ex-post real rates are highly volatile. Even though the induced ex-ante real rates are less volatile than the ex-post rates, they vary within a range of -6.9% to 17.6%, which makes it impossible to infer inflation expectations movements directly from a change in nominal interest rates. Therefore, deriving bounds for Turkish inflation expectations is more important and essential compared to US case.

The bounds for inflation expectations are derived for Turkish data and compared with the survey mentioned above. Actual inflation series is also drawn for comparison purposes. Because of the late availability of the survey, we can make a comparison only for the stable inflation path starting from late 2003. Our results show that the CTS responses are contained within the bounds for the whole sample period.

 $^{^{14}\}mathrm{Note}$ that we covered 1998-2009 period in the figure while our time period for comparison purposes is 2000-2009.

3.10 Chilean Data

Central Bank Of Chile conveys monthly survey of selected academics, consultants, and executives or advisors of financial institutions and corporations. As a part of the Central Bank of Chile Economic Expectations Survey, subjects are asked for their expected inflation for the following 12 month period. Again we use the median responses. This survey is available since 2001. Seasonally adjusted consumption expenditure and nominal average deposit rate at annual percentage are taken from the Central Bank of Chile as well.

3.11 Results for Chilean data

Below is Figure 3.5 that show the bounds on inflation expectations derived from the model and the survey results. The survey results fall within the bounds for most of the time period. As the nominal interest rates are a multiplier for the size of the upper and lower bounds, the higher it is at any time the higher the difference, i.e. the width, of the bounds. That's why in the Chilean case, bounds are a lot wider than the US and UK cases.

3.12 Conclusion

There are direct and indirect measures available to central banks and many different techniques have been proposed in the literature. In this study, we tested the bounds of inflation expectations obtained from Ireland (1996), an indirect measure, using multiple survey results, a direct measure, in 4 countries, US, UK, Chile and Turkey. Our results indicate that, those bounds do a better job containing the survey results in Turkey and Chile compared to US and UK. There are two reasons behind this result. First of all, the sensitivity of the Ireland's methodology to the movements in consumption growth rates. As the estimate for the ex-ante interest rates are calculated using the consumption growth rates in our model, and Turkish consumption data is a lot more volatile than the rest of the countries, inflation expectations bounds for Turkey are a lot wider than the other cases. Also, the nominal interest rate is the coefficient on the in front of the bounds and the higher it is the wider the bounds are.

A secondary result obtained from our analysis is that, unlike in US, real interest rates are extremely volatile in Turkey and movements in nominal interest rates can not be used to predict the changes in inflation expectations. Due to a stable real interest rate and low risk premia in US, Ireland (1996) suggests that movements in the nominal interest rates primarily reflect changes in inflationary expectations. However, Turkish case offers unstable real rates and high risk premia, and therefore computing a good measure of inflation expectations is more essential.

A comprehensive theoretical model that captures a country specific default risk premium component might help explain the difference in volatilities in consumption series. Obviously, in the current formulation we don't account for that. So the bounds for the US and UK case would have been a lot less volatile if our model had accounted for this factor. This would be interesting extension to our work.


Figure 3.1: Inflation Expectations in US: Direct vs Indirect Measures



Figure 3.2: Inflation Expectations in UK: Direct vs Indirect Measures



Figure 3.3: Ex-ante and Ex-post Real Rates in Turkey



Figure 3.4: Inflation Expectations in TR: CTS vs Model



Figure 3.5: Inflation Expectations in Chile: Direct vs Indirect Measures

Appendix

Appendix 1

Proofs for Chapter1

1.1 Proofs

Proof of Proposition 4. Let $d_1 < d_2$ be two banks. Equation (2) defines the banks' portfolio problem and we can see that two banks have the same identical problem except for the capacity constraint. We also know that borrowers have indifferent among different bank sizes. Together, it means that the larger bank, d_2 , has the same optimization problem with a more relaxed capacity constraint. So at any optimum portfolio, large banks optimal portfolio has to be at least as good as the smaller banks portfolio. Hence, the larger bank's portfolio contains proportionately more of the productive borrowers. And we know that more productive borrowers get charged lower interest rate on their loans, and therefore choose a lower risk level on their portfolio. Which yields our main result:

$$\mu_{2}(r_{2}^{\ell}) > \mu_{1}(r_{1}^{\ell})$$

$$\sigma_{1}(r_{1}^{\ell}) > \sigma_{2}(r_{2}^{\ell})$$

$$(1.1.1)$$

Proof of Proposition 6. Take two markets i = 1, 2 s.t. $\ell_1 < \ell_2$. We want to

show that $z_{\ell_1}^* > z_{\ell_2}^*$. Assume for a contradiction that $z_{\ell_1}^* \le z_{\ell_2}^*$. We will define:

$$EU_{i}^{A} = \Gamma(z_{i}, \ell_{i}, \mu, \sigma)p(R^{*}(r^{\ell_{i}}(z_{i}, \mu, \sigma)), z_{i})[R^{*}(r^{\ell_{i}}(z_{i}, \mu, \sigma)) - r^{\ell_{i}}(z_{i}, \mu, \sigma)]$$
$$EU_{i}^{U} = \Gamma(\ell_{i}, \mu, \sigma)p(R^{*}(r^{\ell_{i}}(\mu, \sigma)), z_{i})[R^{*}(r^{\ell_{i}}(\mu, \sigma)) - r^{\ell_{i}}(\mu, \sigma)]$$
$$for \quad i = 1, 2$$
(1.1.2)

Since we assumed that the cost function has a fixed and a proportional component, we know that :

$$\frac{c(\theta,\ell_1)}{\ell_1} > \frac{c(\theta,\ell_2)}{\ell_2} \tag{1.1.3}$$

using equation (4) implies:

$$EU_1^A(z_1^*) - EU_1^U(z_1^*) > EU_2^A(z_2^*) - EU_2^U(z_2^*)$$
(1.1.4)

Next, since we assumed that $z_{\ell_1}^* \leq z_{\ell_2}^*$, it means that in the larger loan market, the borrowers with $z_{\ell_1}^*$ in the second market did not find it profitable to get audited. That implies:

$$EU_2^A(z_2^*) - EU_2^U(z_2^*) > EU_2^A(z_1^*) - EU_2^U(z_1^*)$$
(1.1.5)

i.e. the expected benefit from getting audited is smaller than the cost of getting audited. Equations A.3 and A.4 together implies:

$$EU_1^A(z_1^*) - EU_1^U(z_1^*) > EU_2^A(z_1^*) - EU_2^U(z_1^*)$$
(1.1.6)

Since we assumed that $z_{\ell_1}^* \leq z_{\ell_2}^*$, the pool of applicants that are nonaudited in the larger loan market are of a higher quality, i.e. there are more productive types in the unaudited pool of applicant in the large loan market. In that case, the interest rate that the better pool gets charged is lower, i.e. so $r_2^u \leq r_1^u$. This implies that: $EU_1^U(z_1^*) > EU_2^U(z_1^*)$. Moreover, since expected utility from being audited are the same in both loan markets we have: $EU_1^A(z_1^*) = EU_2^A(z_1^*)$. So we have a contradiction.

Proof of Proposition 8. Note that this proposition does not restrict the borrowers to have the same loan size. That's exactly why we rule out the monopoly case from all segmented loan markets to prove our result. Take any two audited borrowers, a bank's profits over loan contracts per unit of loan is strictly increasing in borrowers productivity, z. Equilibrium requires the contracts to earn the same expected profit per unit of loan, therefore a bank can always improve profits by offering a slightly lower interest rate to a more productive borrower unless the interest rates are such that offering a lower interest rate to a more productive borrower is not profitable anymore. Since the profits per unit is increasing in the loan interest rate (up to the peak point beyond which is never an equilibrium), a more productive borrower will get a lower interest rate in order for the per unit profits to be the same for the banks.

Proof of Corollary 9. Assume that X' is the equilibrium set of contracts. Assume for a contradiction that two borrowers $j_1 = j_2$, with the same productivity and size signed two separate loan contracts, i.e. $\exists x_1, x_2 \in X'$ s.t.

 $x_1 = (i_1, j_1, r_1), x_2 = (i_2, j_2, r_2)$ and $r_1 \neq r_2$. WLOG, assume that $r_1 > r_2$. If there is only one bank that is large enough to offer a contract to these two borrowers, i.e. $i_1 = i_2$, then it would mean that this bank has monopoly power over these borrowers, and since we know that the profit curve is single peaked in the loan interest rate, then we know that at least one of the contracts is not profit maximizing, i.e. does not satisfy banks' problem (2).

Lets assume that $i_1 \neq i_2$. Now lets construct the following contract $x_3 = (i_2, j_1, \frac{r_1+r_2}{2})$. If this contract is already in X', then it means that bank 2 is not optimizing as we stated above. If it is indeed not part if the equilibrium set of contracts, then it violates the no blocking condition, i.e. equation (10). Because this new contract that we constructed is gives better utility to bank 2 and borrower 1, as they are both getting a better offer then before. This completes the proof.

Bibliography

- Aguiar, M. and Hurst, E., 2000. Life-Cycle Prices and Production. The American Economic Review, volume 97-5, pages 1533-1559.
- Allen, F., and D. Gale, 2000. Comparing Financial Systems. Cambridge, MA: MIT Press.
- Bailey, M.J., 1965. The welfare cost of inflationary finance. The Journal of Political Economy, volume 64-2, pp.93-110.
- Berger, A., N. Miller, M. Petersen, R. Rajan, and J. Stein, 2005. Does Function Follow Organizational Form? Evidence from the Lending Practices of Large and Small Banks. Journal of Financial Economics 76 237269.
- Berger, Allen N., Anil K. Kashyap, and Joseph M. Scalise, 1995. The Transformation of the U.S. Banking Industry: What a Long, Strange Trip Its Been. Brookings Papers on Economic Activity 2:54219.
- Berger, Allen N., Asli Demirguc-Kunt, Ross Levine, and Joseph G. Haubrich, 2004. Bank Concentration and Competition: An Evolution in the Making. Journal of Money, Credit, and Banking, Vol. 36, No. 3 (June 2004, Part 2)

- Boyd, J. H. and De Nicoló, G., 2005. The Theory of Bank Risk Taking and Competition Revisited. The Journal of Finance, 60: 13291343. doi: 10.1111/j.1540-6261.2005.00763.x
- Berument, H., Malatyali, K., 2001. Determinants of Interest Rates in Turkey. Russian and East European Finance and Trade 37 (1), 5-16.
- Bhattacharya, J., Haslag, J. H., Martin, A., 2005. Heterogeneity, Redistribution and the Friedman Rule. International Economic Review 46 (2), 437-454.
- Burstein, A. and Hellwig, C., 2008. Welfare costs of inflation in a menu cost model. The American Economic Review, volume 98-2, pp. 438-443
- Cooley, T.F. and Hansen, G.D., 1991. The welfare costs of moderate inflations. Journal of Money, Credit and Banking, volume 23-3, pp. 483-503.
- Corbae, D. and Derasmo, P., 2010. A Quantitative Model of Banking Industry Dynamics. Working Paper.
- **Crawford, Vincent**, 1991. Comparative Statics in Matching Markets. Journal Of Economic Theory 54, 389400.
- Degryse, H. and Ongena, S., 2005. Distance, Lending Relationships, and Competition. The Journal of Finance VOL. LX, NO. 1.
- **Dotsey, M. and Ireland, P.**, 1996. The welfare cost of inflation in general equilibrium. Journal of Monetary Economics, volume 37-1, pp. 29-47.

- Emmons, William R., Gilbert, R. Alton and Timothy J. Yeager, 2004. Reducing the Risk at Small Community Banks: Is it Size or Geographic Diversification that Matters?. Journal Of Financial Services Research. Volume 25, Numbers 2-3, 259-281.
- Fama, E. F., 1975. Short-Term Interest Rates as Predictors of Inflation. American Economic Review 65 (3), 269-282.
- Fisher, I., 1907. The Rate of Interest. New York: MacMillan Company.
- Gillman, M., 1993. The welfare cost of inflation in a cash-in-advance economy with costly credit. Journal of Monetary Economics, volume 31-1, pp. 97-115.
- Gul, E., Acikalin, S., 2008. An Examination of the Fisher Hypothesis: The Case of Turkey. Applied Economics 40 (24), 3227-3231.
- Hansen, L. P., Hodrick, R. J., 1980. Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis. The Journal of Political Economy 88 (5), 829-853.
- Hatfield, John W. and Milgrom, Paul R., 2005. Matching with ContractsThe American Economic Review, Vol. 95, No. 4, pp. 913-935.
- Imrohoroglu, A., 1992. The welfare cost of inflation under imperfect insurance. Journal of Economic Dynamics and Control Volume 16, Issue 1, January 1992, Pages 79-91.

- Ireland, P. N., 1996. Long-Term Interest Rates and Inflation: A Fisherian Approach. Federal Reserve Bank of Richmond Economic Quarterly 82 (1), 21-35.
- Jimenez, G. and Saurina, J., 2004. Collateral, Type of Lender and Relationship Banking as Determinants of Credit Risk. Journal of Banking & Finance 28, 21912212.
- Kenneth D. Jones and Tim Critchfield, 2005. Consolidation in the U.S. Banking Industry: Is the Long, Strange Trip About to End?. FDIC banking review, VOLUME 17, NO. 4.
- Lagos, R. and Wright, R., 2005. A Unified Framework for Monetary Theory and Policy Analysis. Journal of Political Economy, volume 113-3.
- Ljungqvist, L. and Sargent, T.J., 2004. Recursive Macroeconomic Theory. The MIT press.
- Lucas, R. E. Jr., 1978. Asset Prices in an Exchange Economy. Econometrica 46, 14291445.
- Lucas, R. E. Jr., 1978. Inflation and welfare. Econometrica 68-2, 247-274.
- McKenzie, D. and Schargrodsky, E., 2011. Buying Less but Shopping More: The Use of Nonmarket Labor during a Crisis. Economía, volume 11-2, pages 1-35, Brookings Institution Press.

- Mishkin, F. S., 1981. The Real Interest Rate: An Empirical Investigation. NBER Working Paper No. 622.
- Mundell, R., 1963. Inflation and Real Interest. Journal of Political Economy 1, 280-283.
- Newey, W. K., West, K. D., 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. Econometrica 55 (3), 703-708.
- O'Keefe, B. Terrence, Simunic, Dan A. and Stein, Michael T., 1994. The Production of Audit Services: Evidence from a Major Public Accounting Firm. Journal of Accounting Research, Vol. 32, No. 2 (Autumn, 1994), pp. 241-261.
- **Oral, E.**, 2009. Inflation Expectations in Turkey: Consumer Survey Based Results. Unpublished manuscript.
- Sahinbeyoglu, G., Yalcin, C., 2000. The Term Structure of Interest Rates: Does it Tell About Future Inflation? Central Bank of Turkey Research Department Discussion Paper No: 2000/2.
- Petersen, M., and R. Rajan, 1994. The Benefits of Lending Relationships: Evidence from Small Business Data. The Journal of Finance, Vol. 49, No. 1, pp. 3-37.

- Petersen, M., and R. Rajan, 2002. Does Distance Still Matter: The Informational Revolution in Small Business Lending. The Journal of Finance, Vol. 57, No. 6, pp. 2533-2570.
- Rocheteau, G. and Wright, R., 1982. Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. Econometrica, volume 73-1, pp. 175-202.
- Roth, Alvin E. and Sotomayor, Marilda, 1982. Twosided matching: A study in game-theoretic modeling and analysis.Cambridge: Cambridge University Press, 1990.
- Satoru Fujishige and Zaifu Yang, 2003. A Note on Kelso and Crawford's Gross Substitutes Condition. Mathematics of Operations Research, Vol. 28, No. 3 (Aug., 2003), pp. 463-469.
- Stein, J. C., 2002. Information Production and Capital Allocation: Decentralized versus Hierarchical Firms. The Journal of Finance, 57: 18911921. doi: 10.1111/0022-1082.00483.
- Stiglitz, Joseph E. and Weiss, Andrew, 1981. Credit Rationing in Markets with Imperfect Information. The American Economic Review, Vol. 71, No. 3 (Jun., 1981), pp. 393-410.
- Stock, J. H., Watson, M. W., 2003. Forecasting Output and Inflation: The Role of Asset Prices. Journal of Economic Literature 41 (3), 788-829.
- Tobin, J., 1965. Money and Economic Growth. Econometrica 33, 671-684.

- Liang, W., 2011. Inflation and Welfare with Search and Price Dispersion. Working Paper.
- Weintraub, G. Y., Benkard, C. L. and Van Roy, B., 2008. Markov Perfect Industry Dynamics With Many Firms. Econometrica, 76: 13751411. doi: 10.3982/ECTA6158.