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# Rabin's calibration theorem revisited

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## ABSTRACT

We simplify and refine the theoretical results behind Rabin's famous calibration theorem for expected utility preferences and present the resulting tightened versions of his numerical illustrations.

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## 1. Introduction

The calibration theorem of Rabin (2000) is one of the most cited theoretical results in microeconomics of the past 25 years. In his famous paper, Rabin considers a risk-averse expected utility maximizer who refuses a 50–50 “gain  $g$  or lose  $l$ ” gamble for a range of initial wealth levels. From this prerequisite, he derives conditions on numbers  $m$  and  $k$  such that the same agent also refuses a 50–50 “gain  $m \cdot g$  or lose  $k \cdot l$ ” gamble. In a nutshell, the message of Rabin's paper is that  $m$  can grow far more quickly than  $k$  so that the agent will end up refusing gambles whose potential gains outweigh the losses by far. For instance, by consistency considerations, an agent who refuses a 50–50 “gain \$11 or lose \$10” gamble for all initial wealth levels must also refuse a 50–50 gamble with a possible loss of \$88 regardless of how high the possible gain is. In Rabin's paper, these results are interpreted primarily as an argument against expected utility as a behavioral theory – and from this perspective they have mostly been cited. Yet, the general question which conclusions about an agent's utility function can be drawn from limited information is interesting for other reasons as well, e.g. when dealing with preference uncertainty (Armbruster and Delage, 2015) or when evaluating empirical data on choices between lotteries (Moscati, 2019).

While the main body of Rabin (2000) is extremely well-written, a closer look at the paper's mathematical appendix reveals that it was probably not written with the idea that

generations of students would carefully study it line by line – as turned out to be the case, with the paper being a standard reference that is taught in many educational programs around the world.<sup>1</sup> This is the starting point of this short paper which revisits the theoretical results of Rabin (2000), streamlines the arguments, simplifies the statements and exploits the standing assumptions more fully to make the results both simpler and stronger.<sup>2</sup> Here are some examples:

- While the geometric series is applied once in the proof of Rabin's Corollary, the paper contains various further expressions that can be simplified using the same trick, leading to mathematical expressions which are easier to interpret and evaluate.
- While Rabin's theorem uses concavity of the utility function to extrapolate beyond the range where preference information is available, interpolation within that range is based only on monotonicity. We exploit concavity more fully to tighten the bounds.
- Extrapolating results about “gain  $g$  or lose  $l$ ” gambles to “gain  $m \cdot g$  or lose  $k \cdot g$ ” gambles is actually easier than proving the results about “gain  $m \cdot g$  or lose  $k \cdot l$ ” gambles Rabin presents. In order to move from “lose  $k \cdot g$ ” to “lose

<sup>1</sup> Indeed, several rounds of teaching (Rabin, 2000) were a major motivation for writing this note.

<sup>2</sup> In particular, while there are a few minor typos in the original results, we do not think that anything is “wrong” with them. The typos include, e.g., a missing factor 2 in the Corollary, and a flip of  $\leq$  and  $\geq$  in the first displayed equation in the proof of part (ii) of the Theorem, the counterpart of our Lemma 1.

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$k \cdot l$ ", Rabin needs to introduce an extra assumption,  $g < 2l$ , and take several additional steps. The results about "gain  $m \cdot g$  or lose  $k \cdot g$ " gambles are never mentioned even though the resulting bounds are sharper and do not need the extra assumption.

Since our theoretical results are tighter, we can also improve the tables with quantitative results in Rabin (2000). Some later articles such as Safra and Segal (2008), Freeman (2015) and Safra and Segal (2020) have extended Rabin's results from expected utility to even broader classes of preferences. In these later works, Safra and Segal (2008)'s Theorem 1 has replaced Rabin's Theorem, providing a closed-form expression that is easy to evaluate. While we stay within the more narrow scope of Rabin's original expected utility setting, we find that within that scope our quantitative bounds are considerably tighter than Safra and Segal (2008)'s theorem. We illustrate and discuss this further at the end of this paper.

## 2. Calibration theorems

We now derive utility comparisons in the spirit of Rabin's. To this end, we first present two results, a theorem and corollary, which carry the same message as their counterparts in his paper but are more explicit and easier to prove. We then modify these results to get closer to his original versions. Throughout, the goal is to assess the consequences of the following assumption on the preferences of an expected utility maximizer with utility function  $u$ .

**Assumption 1.** The utility function  $u : \mathbb{R} \rightarrow [-\infty, \infty)$  is weakly increasing and weakly concave. Moreover, there exists an interval  $W \subseteq \mathbb{R}$  and real numbers  $g$  and  $l$ ,  $g > l > 0$ , such that  $u(w) > -\infty$  for all  $w \in W$  and such that an agent with utility function  $u$  rejects a 50–50 gain  $g$  or lose  $l$  gamble for all wealth levels  $w \in W$ ,

$$\frac{1}{2}u(w - l) + \frac{1}{2}u(w + g) \leq u(w)$$

or, equivalently,

$$u(w + g) - u(w) \leq u(w) - u(w - l). \tag{1}$$

Condition (1) is a strengthening of concavity which only implies a ranking of the increments of  $u$  for adjacent intervals of equal length,

$$u(w + g) - u(w) \leq u(w) - u(w - g).$$

Condition (1) states that this ranking holds even when the interval on the left is shorter, comparing an increment over length  $l$  to an increment over length  $g > l$ . Intuitively, an increment over length  $l$  should be about a factor  $l/g$  smaller than an increment over length  $g$ . With this intuition and concavity, we can turn (1) into a quantitative bound on the growth behavior of  $u$  over intervals of length  $g$ .

**Lemma 1.** Under Assumption 1, we have for all  $w \in W$

$$u(w + g) - u(w) \leq \frac{l}{g}(u(w) - u(w - g)). \tag{2}$$

**Proof.** Since we can write  $\frac{l}{g}(w - g) + \left(1 - \frac{l}{g}\right)w = w - l$ , concavity of  $u$  implies

$$\frac{l}{g}u(w - g) + \left(1 - \frac{l}{g}\right)u(w) \leq u(w - l)$$

<sup>3</sup> By allowing  $u$  to take the value  $-\infty$  but assuming  $u(w) > -\infty$  for  $w \in W$ , our setting is flexible enough to include, e.g., the possibility that  $u$  is a logarithmic utility function which takes the value  $-\infty$  for  $w \leq 0$ .

$$\Rightarrow u(w) - u(w - l) \leq \frac{l}{g}(u(w) - u(w - g)). \tag{3}$$

Combining (1) and (3) yields (2).  $\square$

We next prove a version of Rabin's Theorem, bounding the growth of  $u$  in terms of  $l/g$ .

**Theorem 1.** Under Assumption 1, consider two integers  $m$  and  $k$  and a wealth level  $w$  such that  $[w - kg, w + mg] \subseteq W$  and define  $r(w) = u(w + g) - u(w)$ . Then the utility function  $u$  satisfies the bounds

$$u(w + mg) - u(w) \leq \frac{1 - \left(\frac{l}{g}\right)^m}{1 - \frac{l}{g}} r(w) \quad \text{and}$$

$$u(w) - u(w - kg) \geq \frac{\left(\frac{g}{l}\right)^k - 1}{1 - \frac{l}{g}} r(w).$$

**Proof.** Iterating condition (2), we find that for non-negative integers  $i \leq m$

$$u(w + (i + 1)g) - u(w + ig) \leq \left(\frac{l}{g}\right)^i (u(w + g) - u(w)). \tag{4}$$

Using the definition of  $r(w)$ , we can thus deduce the first claim

$$u(w + mg) - u(w) = \sum_{i=0}^{m-1} u(w + (i + 1)g) - u(w + ig)$$

$$\leq r(w) \sum_{i=0}^{m-1} \left(\frac{l}{g}\right)^i = r(w) \frac{1 - \left(\frac{l}{g}\right)^m}{1 - \frac{l}{g}}.$$

For the second one, we replace  $w$  by  $w - ig$  in (4), plug in  $i = j + 1$  and rearrange to obtain

$$u(w - jg) - u(w - (j + 1)g) \geq \left(\frac{g}{l}\right)^{j+1} (u(w + g) - u(w))$$

$$= \left(\frac{g}{l}\right)^{j+1} r(w)$$

for non-negative integers  $j < k$ , using that  $[w - kg, w + mg] \subseteq W$ .<sup>4</sup> Consequently,

$$u(w) - u(w - kg) = \sum_{j=0}^{k-1} u(w - jg) - u(w - (j + 1)g)$$

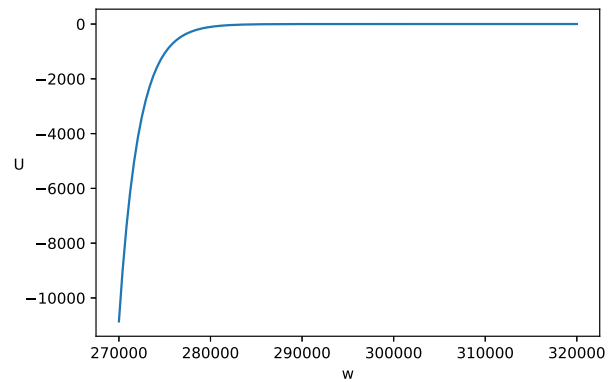
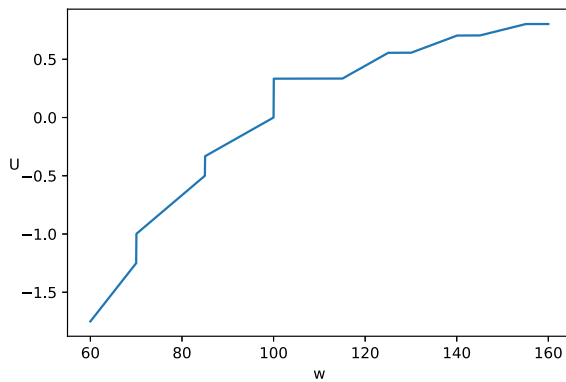
$$\geq r(w) \frac{g}{l} \sum_{j=0}^{k-1} \left(\frac{g}{l}\right)^j = r(w) \frac{\left(\frac{g}{l}\right)^k - 1}{1 - \frac{l}{g}}. \quad \square$$

For  $W = \mathbb{R}$ , the potential gain  $u(w + mg) - u(w)$  is bounded in  $m$  while the potential loss  $u(w) - u(w - kg)$  grows exponentially in  $k$ . This contrast is the essence of Rabin's result which is summarized in the following version of his Corollary.

**Corollary 1.** Suppose that Assumption 1 holds with  $W = \mathbb{R}$ , so that the agent rejects a 50–50 gain  $g$  or lose  $l$  gamble regardless of his wealth and let  $k, m \geq 2$  be two integers.

(i) Suppose that  $\left(\frac{g}{l}\right)^k \geq 2$ , i.e.,  $k \geq \frac{\log(2)}{\log(g) - \log(l)}$ . Then the agent also rejects 50–50 gambles with a loss of  $k \cdot g$  and a gain of  $m \cdot g$  for all  $m \in \mathbb{N}$ .

<sup>4</sup> Carefully inspecting this proof, one can tighten the result a little bit more: Since only the point  $w$  needs to lie in  $W$  in Assumption 1, it suffices to assume  $[w - (k - 1)g, w + (m - 1)g] \subseteq W$  rather than  $[w - kg, w + mg] \subseteq W$ . The same improvement is possible in Propositions 1 and 2.



**Fig. 1.** The function  $U$  from Proposition 2 for  $w = 100, g = 15, l = 10$  and  $W = \mathbb{R}$  in the left panel and for  $w = 290,000, g = 105, l = 100$  and  $W = [0, 300,000]$  in the right panel.

**Table 1**

If averse to 50–50 lose \$100 or gain  $g$  bets for all wealth levels, will turn down 50–50 lose  $L$  or gain  $G$  bets;  $G$ 's entered in table.

$L \setminus g$	\$101	\$105	\$110	\$125
\$400	\$415	\$483	\$601	$\infty$
\$600	\$636	\$824	\$1,296	$\infty$
\$800	\$866	\$1,280	\$6,716	$\infty$
\$1,000	\$1,107	\$1,918	$\infty$	$\infty$
\$2,000	\$2,491	$\infty$	$\infty$	$\infty$
\$4,000	\$6,694	$\infty$	$\infty$	$\infty$
\$6,000	\$16,641	$\infty$	$\infty$	$\infty$
\$8,000	$\infty$	$\infty$	$\infty$	$\infty$
\$10,000	$\infty$	$\infty$	$\infty$	$\infty$
\$20,000	$\infty$	$\infty$	$\infty$	$\infty$

(ii) Suppose that  $(\frac{g}{l})^k < 2$ , i.e.,  $k < \frac{\log(2)}{\log(\frac{g}{l}) - \log(l)}$ . Then the agent also rejects 50–50 gambles with a loss of  $k \cdot g$  and a gain of  $m \cdot g$  if  $m$  satisfies

$$m \leq m^* := \frac{\log\left(2 - \left(\frac{g}{l}\right)^k\right)}{\log(l) - \log(g)}.$$

**Proof.** We need to establish sufficient conditions for  $u(w) - u(w - kg) \geq u(w + mg) - u(w)$ . By Theorem 1, one such condition is

$$1 - \left(\frac{l}{g}\right)^m \leq \left(\frac{g}{l}\right)^k - 1. \tag{5}$$

If  $(\frac{g}{l})^k \geq 2$ , i.e., if  $(\frac{g}{l})^k - 1 \geq 1$ , then (5) holds regardless of  $m$  since the left hand side of (5) always lies in  $(0, 1)$ . This settles case (i). If  $(\frac{g}{l})^k - 1 < 1$ , setting  $m$  equal to the (typically non-integer) value  $m^*$  from the proposition turns (5) into an equality. Since the left hand side of (5) is increasing in  $m$ , (5) is satisfied for  $m \leq m^*$ .  $\square$

In the first case of the corollary, the agent rejects a gamble with gains of  $m \cdot g$  and losses of  $k \cdot g$  for any  $m$ . When  $g$  is larger than  $2l$ , we are in this case for all  $k$ . Consider the example from the introduction, an agent who rejects a 50–50 “gain  $g = \$11$  or lose  $l = \$10$ ” gamble for all initial wealth levels. Solving  $(\frac{g}{l})^k = 2$  with these values gives  $k = \log(2)/\log(1.1) \approx 7.3$ . Rounding to  $k = 8$ , we can conclude that this agent rejects a gamble with a possible loss of  $k \cdot g = \$88$  no matter what the possible gain is.

There are two main differences between our results so far and those in Rabin’s original paper. First, we consider lotteries with losses of  $k \cdot g$  while he considers losses of  $k \cdot l$ . Second, his theorem does not just bound the utility function on a grid but also interpolates between the grid points and extrapolates outside the range determined by the set  $W$ . In the remainder of this section, we extend our results into these two directions.

**Multiples of  $l$ .** To extend the results from losses that are multiples of  $g$  to those that are multiples of  $l$ , Rabin assumes that  $g$  lies between  $l$  and  $2l$ . The basic trick is to observe that we do not only have a contraction by a factor  $l/g$  for intervals of length  $g$  – as shown in Lemma 1 – but also for intervals of length  $h > g$  as this only makes the claim weaker.

**Lemma 2.** Under Assumption 1, we have for all  $w \in W$  and  $h \geq g$

$$u(w) - u(w - h) \geq \frac{l}{g}(u(w + h) - u(w)). \tag{6}$$

**Proof.** Since  $\frac{g}{h}(w + h) + (1 - \frac{g}{h})w = w + g$ , concavity of  $u$  implies

$$\frac{g}{h}u(w + h) + \left(1 - \frac{g}{h}\right)u(w) \leq u(w + g).$$

Rearranging and applying Assumption 1, it follows that

$$\begin{aligned} u(w + h) - u(w) &\leq \frac{h}{g}(u(w + g) - u(w)) \\ &\leq \frac{h}{g}(u(w) - u(w - l)). \end{aligned} \tag{7}$$

Arguing like in (3) but with  $h$  in place of  $g$ , we also know that

$$u(w) - u(w - l) \leq \frac{l}{h}(u(w) - u(w - h)). \tag{8}$$

Plugging (8) into (7) concludes the proof.  $\square$

Assuming  $g \leq 2l$  and setting  $h = 2l$ , we get bounds like in case (i) of Rabin’s theorem:

**Proposition 1.** Under Assumption 1, consider two integers  $m$  and  $k$  and a wealth level  $w$  such that  $[w - kg, w + mg] \subseteq W$ , define  $r(w) = u(w + g) - u(w)$  and suppose that  $g \leq 2l$ . Then the utility function  $u$  satisfies the bounds

$$u(w) - u(w - 2kl) \geq 2 \frac{\left(\frac{g}{l}\right)^k - 1}{\frac{g}{l} - 1} r(w).$$

**Proof.** Arguing like in Theorem 1 but with Lemma 2 in place of Lemma 1, we find that

$$\begin{aligned} u(w) - u(w - kh) &= \sum_{j=0}^{k-1} u(w - jh) - u(w - (j+1)h) \\ &\geq (u(w) - u(w - h)) \frac{\left(\frac{g}{l}\right)^k - 1}{\frac{g}{l} - 1} \end{aligned} \tag{9}$$

**Table 2**  
Table 1 replicated for initial wealth level  $w = 290,000$  and  $W = [0, 300,000]$ .

$L \setminus g$	\$101	\$105	\$110	\$125
\$400	\$415	\$483	\$601	\$1,105,440,200
\$600	\$636	\$824	\$1,296	\$29,432,073,060
\$800	\$866	\$1,280	\$6,716	\$74,987,862,188
\$1,000	\$1,107	\$1,918	\$2,437,547	\$140,096,476,686
\$2,000	\$2,491	\$133,200	\$23,487,753	\$1,185,980,444,937
\$4,000	\$6,694	\$1,015,102	\$192,453,308	\$44,577,197,489,785
\$6,000	\$14,898	\$3,248,647	\$1,146,866,216	\$1,586,142,900,840,091
\$8,000	\$25,644	\$8,905,397	\$6,536,601,709	\$56,353,558,511,463,528
\$10,000	\$38,730	\$23,231,799	\$36,966,107,744	\$2,002,083,024,435,601,152
\$20,000	\$161,526	\$2,466,146,744	\$214,467,260,409,269	\$113,313,887,494,512,423,066,402,816

**Table 3**  
If averse to 50–50 lose \$100 or gain  $g$  bets for all wealth levels in  $[a, b]$ , will turn down 50–50 lose  $L$  or gain  $G$  bets at wealth  $a$ ;  $L$ 's entered in table.

$G$	$b - a$	$g = \$101$	$g = \$105$	$g = \$110$	$g = \$125$
\$1,000,000	\$20,000	\$145,425	\$2,298	\$1,211	\$626
\$5,000,000	\$30,000	\$268,437	\$2,210	\$1,211	\$626
\$10,000,000	\$40,000	\$203,643	\$2,206	\$1,211	\$626

for  $h \geq g$ . Moreover, by concavity and Assumption 1

$$u(w) - u(w - 2l) \geq 2(u(w) - u(w - l)) \geq 2r(w). \tag{10}$$

Plugging (10) into (9) with  $h = 2l$  gives the result.  $\square$

Combining this bound with the bound on  $u(w + mg) - u(w)$  from above, one can prove a version of Rabin's Corollary arguing as in the proof of our Corollary 1. However, the result will not be as tight as Corollary 1 as it relies on replacing Lemma 1 by the weaker Lemma 2. Intuitively, the bounds are weakened first by stretching the  $x$ -axis from multiples of  $g$  to multiples of  $2l$  and then even more when applying (10). To obtain tight bounds, it is preferable to interpolate and extrapolate Theorem 1 directly.

*Interpolation and extrapolation.* With monotonicity and concavity we have two assumptions in place that can help us in extending our bounds from the grid to the real line. Monotonicity implies that bounds on the function can be extended to intermediate values. Concavity implies that bounds on the slope over adjacent intervals give linear bounds on the function. Rabin's theorem only uses monotonicity when interpolating and only concavity when extrapolating. In our proposition, we apply both monotonicity and concavity in the interpolation to achieve a tighter bound on the utility function.

**Proposition 2.** Under Assumption 1, consider two integers  $m$  and  $k, m, k \geq 2$ , and a wealth level  $w$  such that  $[w - kg, w + mg] \subseteq W$ . Assume without loss of generality that  $u(w) = 0$  and  $u(w + g) = 1 - \frac{l}{g}$  and define the sequences  $(x_i), (u_i)$  and  $(s_i), i \in \mathbb{Z}$ , through

$$x_i = w + ig, \quad u_i = 1 - \left(\frac{l}{g}\right)^i \quad \text{and} \quad s_i = \left(\frac{l}{g}\right)^i \left(1 - \frac{l}{g}\right).$$

Then we have for all  $x \in \mathbb{R}$  the bound  $u(x) \leq U(x)$  where the function  $U : \mathbb{R} \rightarrow \mathbb{R}$  is defined piecewise as follows: For  $x > x_m$ ,  $U(x) = u_m + s_m \frac{x-x_m}{g}$ . For  $x \in (x_1, x_m]$ , there exists an integer  $i$  and  $\alpha \in (0, 1]$  such that  $x = x_i + \alpha g$  and we define  $U(x) = \min(u_i + \alpha s_{i-1}, u_{i+1})$ . For  $x \in (x_0, x_1]$ , we define  $U(x) = u_1$ . For  $x \in (x_{-k}, x_0]$ , there exists an integer  $j$  and  $\alpha \in [0, 1)$  such that  $x = x_{-j} - \alpha g$  and we define  $U(x) = u_{-j} - \alpha s_{-j}$ . Finally, for  $x \leq x_{-k}$ , we define  $U(x) = u_{-k} - s_{-k} \frac{x-x_{-k}}{g}$ .

**Proof.** From the proof of Theorem 1, we see that  $(x_i), (u_i)$  and  $(s_i)$  are chosen such that  $u(x_i) \leq u_i$  for  $i = -k, \dots, m$ ,  $u(x_{i+1}) - u(x_i) \leq s_i$  for  $i = 0, \dots, m - 1$  and  $u(x_{-(j-1)}) - u(x_{-j}) \geq s_{-j}$

for  $j = 0, \dots, k$ . The interpolation is based on three ideas. First, by monotonicity, we have for  $i = -k, \dots, m - 1$  and  $\alpha \in [0, 1]$   $u(x_i + \alpha g) \leq u_{i+1}$ . Second, by concavity, the slope anywhere inside the interval  $[x_i, x_{i+1}]$  is less than the slope over  $[x_{i-1}, x_i]$  and thus

$$u(x_i + \alpha g) \leq u(x_i) + (u(x_i) - u(x_{i-1}))\alpha \leq u_i + s_{i-1}\alpha$$

for  $i = 0, \dots, m - 1$  and any  $\alpha > 0$ . Third, by a similar concavity argument,

$$u(x_{-j} - \alpha g) \leq u(x_{-j}) - (u(x_{-(j-1)}) - u(x_{-j}))\alpha \leq u_{-j} - s_{-j}\alpha$$

for  $\alpha > 0$  and  $j = 0, \dots, k$ . The result now follows by suitably combining these bounds.  $\square$

The construction of the function  $U$  is illustrated in the left panel of Fig. 1. We can conclude that in the setting of the proposition, an agent with wealth  $w$  will reject a 50–50 “lose  $L$  or gain  $G$ ” lottery if  $-U(w - L) > U(w + G)$  since this implies

$$u(w) - u(w - L) \geq -U(w - L) > U(w + G) \geq u(w + G) - u(w)$$

where we use  $u(x) \leq U(x)$  and the normalization  $u(w) = 0$ . By solving the indifference condition  $-U(w - L) = U(w + G)$ , we can recreate versions of Rabin's Tables 1 and 2, giving examples of lotteries that would be rejected by an agent whose preferences satisfy Assumption 1.<sup>5</sup> Throughout, our results are sharper, corresponding to larger values in the tables, sometimes remarkably so.<sup>6</sup> The right panel of Fig. 1 shows the function  $U$  for the second column of Table 2. On this scale, the local deviations from concavity in the construction of  $U$  are not visible. To understand the global behavior of  $U$  better, note that it interpolates the concave sequence  $x_i \mapsto u_i$ . Solving  $x_i$  for  $i$  and plugging this into the expression for  $u_i$ , we find that  $x_i \mapsto u_i$  interpolates the function  $x \mapsto \bar{u}(x)$  where

$$\bar{u}(x) = 1 - \left(\frac{l}{g}\right)^{\frac{x-w}{g}} = 1 - C \exp(-\rho x) \quad \text{with} \quad C = \left(\frac{g}{l}\right)^{\frac{w}{g}} \quad \text{and} \quad \rho = \frac{\log(g) - \log(l)}{g}. \tag{11}$$

The function  $\bar{u}$  is a CARA utility function with risk aversion parameter  $\rho$ . On sufficiently coarse scales, the behavior of  $U$  and  $\bar{u}$  is effectively the same inside  $W$ .<sup>7</sup>

Finally, we compare our bounds to Safra and Segal (2008). In Table 3, we present a counterpart to their Table I which illustrates their Theorem 1. Here, all of our bounds are tighter by a factor of

<sup>5</sup> A Python code that creates the tables and figures is available in the supplemental online material.

<sup>6</sup> In particular, our results bind immediately. For an agent who rejects a gain 101 or lose 100 lottery, Rabin can only conclude that he would also reject a gain 400, lose 400 lottery – which follows from risk aversion alone. We find that the agent would also reject a gain 415, lose 400 lottery.

<sup>7</sup> From (11), we have an explicit formula that translates  $l$  and  $g$  into an approximate risk aversion level  $\rho$ . Numerically, this gives approximately the same values as in Rabin's Table 3.

approximately 2, in some cases even more. Recall that, building on our [Lemma 1](#) and thus on the stronger case of Rabin's original theorem, our bounds exploit a contraction by a factor  $l/g$  over intervals of length  $g$ . In contrast, [Safra and Segal \(2008\)](#) only exploit this contraction over intervals of length  $l + g$ . Over an interval of length  $b - a$ , they thus apply the factor  $l/g$  about  $(b - a)/(l + g)$  times, while we apply it  $(b - a)/g$  times. When  $l \approx g$ , it follows that the implied rate of exponential decay behind our results is about twice as large. This has a considerable impact on the quality of the bounds.

#### Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.econlet.2021.110166>.

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