



University of Groningen

Gain-Scheduled Fault Detection Filter For Discrete-time LPV Systems

Carvalho, L. P.; Palma, J. M.; Rosa, T. E.; Jayawardhana, B.; Costa, O. L.V.

Published in: **IEEE Access**

DOI: 10.1109/ACCESS.2021.3121188

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version Publisher's PDF, also known as Version of record

Publication date: 2021

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA): Carvalho, L. P., Palma, J. M., Rosa, T. E., Jayawardhana, B., & Costa, O. L. V. (2021). Gain-Scheduled Fault Detection Filter For Discrete-time LPV Systems. *IEEE Access, 9*, 143349-143365. https://doi.org/10.1109/ACCESS.2021.3121188

Copyright Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: https://www.rug.nl/library/open-access/self-archiving-pure/taverneamendment.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.



Received August 16, 2021, accepted October 7, 2021, date of publication October 18, 2021, date of current version October 27, 2021. Digital Object Identifier 10.1109/ACCESS.2021.3121188

Gain-Scheduled Fault Detection Filter for Discrete-Time LPV Systems

LEONARDO DE PAULA CARVALHO^{®1,2}, JONATHAN M. PALMA^{®3}, TABITHA E. ROSA^{®2}, BAYU JAYAWARDHANA^{®2}, (Senior Member, IEEE),

AND OSWALDO LUIZ DO VALLE COSTA^{®1}, (Senior Member, IEEE)

¹Departamento de Engenharia de Telecomunicações e Controle, Escola Politécnica na Universidade de São Paulo, São Paulo 05508-010, Brazil
²Discrete Technology and Production Automation (DTPA), Rijksuniversiteit Groningen, 9712 CP Groningen, The Netherlands
³Facultad de Ingeniería, Universidad de Talca-Curico, Talca, Maule 430050, Chile

Corresponding author: Leonardo de Paula Carvalho (carvalho.lp@usp.br)

The work of Leonardo de Paula Carvalho was supported in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-Brazil (CAPES) under Grant 88882.333365/2019-01. The work of Leonardo de Paula Carvalho and Jonathan M. Palma was supported by the Chilean Agency through the Agencia Nacional de Investigación y Desarrollo-Fondo Nacional de Desarrollo Científico y Tecnológico under Grant 11201049. The work of Tabitha E. Rosa and Bayu Jayawardhana was supported by the Stichting voor de Technische Wetenschappen Smart Industry 2016 Program through the STW Project under Grant 15472. The Oswaldo Luiz Do Valle Costa was supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under Grant 304149/2019 – 5, in part by the Fundação de Amparo à Pesquisa do Estado de São Paulo/Shell Research Center for Gas Innovation (RCGI) Process FAPESP under Grant 2020/15230 – 5, and in part by the Instituto Nacional de Ciência e Tecnologia para Sistemas Autônomos Cooperativos (INSAC), under Grant CNPq/INCT –465755/2014 – 3 and Grant FAPESP/INCT-2014/50851 – 0.

ABSTRACT The present work investigates a fault detection problem using a gain-scheduled filter for discrete-time Linear Parameter Varying systems. We assume that we cannot directly measure the scheduling parameter but, instead, it is estimated. On the one hand, this assumption imposes the challenge that the fault detection filter should perform properly even when using an inexact parameter. On the other, it avoids the burden associated with designing a complex estimation process for this parameter. We propose three design approaches: the H_2 , H_{∞} , and mixed H_2 / H_{∞} gain-scheduled Fault Detection Filters designed via Linear Matrix Inequalities. We also provide numerical simulations to illustrate the applicability and performance of the proposed novel methods.

INDEX TERMS Fault detection and isolation, LPV systems, \mathcal{H}_2 gain-scheduled filter, \mathcal{H}_{∞} gain-scheduled filter.

I. INTRODUCTION

Faults are inherent in any complex engineering system, such as, for instance, in a multitude of sensor and actuator systems used to drive and operate high degree-of-freedom electromechanical mechanisms. The presence of these faults may, among others, lead to significant performance degradation and can yield to unsafe operations [1]. Therefore it is of utmost importance for the optimal and safe operation of such complex engineering systems that the faults are promptly detected and isolated so that they can be subsequently compensated.

In the literature, there are two main approaches for dealing with fault occurrence: the data-driven [2] and the modelbased [3] methods. For a particular application of wind turbines, a direct comparison of both approaches is presented

The associate editor coordinating the review of this manuscript and approving it for publication was Binit Lukose^D.

in [4]. In [5], the authors present a comparison between the model-based and data-driven approaches considering an Unmanned Aerial Vehicle. In general, while it is not trivial to develop or to identify an accurate dynamical model of a given complex system, the model-based approach is often preferable in practice as it provides a rigorous design framework and is applicable in many practical cases. For instance, in cases where the system is only partially observable or where it requires further implementation of sensors, controllers, or security measurements, it is difficult or impossible to gather enough data to implement a data-driven approach. However, it is still possible to obtain a model to describe the system.

Fault Detection (FD) is a well-established model-based approach where we can integrate the models on the design of fault detection filters (FDF) [3], [6]–[9]. The main idea of an FD is based on the use of residue generator filters, where the model-based filters provide a residue signal that corresponds

to the occurrence of faults. This signal is close to zero when the system is in its nominal state, and it changes markedly when a fault occurs [6], [9].

When the systems are nonlinear, the application of FD using a set of linear approximations of the nonlinear models may lead to practical problems. For instance, if a nonlinear system is simply represented by a linear time-invariant model, then it can generate false alarms or return false negative signals due to the lack of dynamical information in the model [10]. In this case, the use of a Linear Parameter Varying (LPV) modelling framework to model nonlinear systems in the FD context has been favoured in the literature, as advocated in [11]–[14].

The effectiveness of the LPV framework for FD problems has been demonstrated in many real applications, such as the FD for a fixed-wing using a set-valued observer for LPV systems [15], the FD for a winding machine using a polytopic LPV approach [10], the FD for a glucose-insulin system [16] and the FD for a Grid-Connected Hybrid Power Plant using LPV systems associated with a search algorithm [17]. There has been significant progress in the design methods and analysis of LPV systems for the past decades. Some of the recent results are, among many others, the design of FD for LPV systems with bounded uncertainties as proposed in [18], the unknown input observer (UIO) with LPV framework in [19], the FD method for Lur'e systems in [20], the FD approach for polytopic sliding mode observers in [21], and the robust fault detection approach and the design of a set-theoretic unknown input observer for LPV systems in [22]. In [23], the authors present an FDF design under a discrete-time Takagi-Sugeno Fuzzy Markovian jumping system (MJS) framework with the presence of time-varying delay, missing measurement, and partly unknown transition probability. The authors in [24] study the dissipative-based finite-time FDF for a discrete-time complex system in the presence of random delays and channel fading. In [25], the authors propose an event-triggered observer-based FDF design using switched non-linear network control systems. Shen et al. [26] present the design of fuzzy fault-tolerant control for MJS, where the faults are reconstructed using proportional integral observers. In [27], the authors study the design of a simultaneous FDF and state-feedback control for MJS under the premise that the transition matrix is partially unknown. The paper [28] provides the design of a Fault Accommodation Control for hidden-MJS, assuming that the transition matrix in the Markov chain is partially unknown. In [29] the filtering problem for the class of fuzzy singular MJS is analyzed, considering that the system is subjected to time-varying delays. An important aspect that must be considered in the design of an FD scheme is that FD filters need to be resilient against noise to minimize the occurrence of false alarms [6], [9]. Another desirable aspect is that a fault must be detected as soon as possible. To fulfil both tasks, using \mathcal{H}_2 and \mathcal{H}_{∞} norms can be useful as performance indexes. In this case, the papers [30]-[32] present parameter-dependent Linear Matrix Inequalities (LMI) constraints to obtain \mathcal{H}_2 and

 \mathcal{H}_{∞} guaranteed cost values for the LPV systems. In [33], the \mathcal{H}_2 guaranteed cost values are obtained through a gain scheduled filter using LMI with Pólya's relaxations, based on a polynomial structure within the context of uncertain scheduled parameters.

The LPV framework allows us to design control and filter solutions that depend on time-varying parameters, where these parameters may be associated with any part of the system. Regarding the structure of the FDF in the LPV framework, we can distinguish them by the dependency of the time-varying parameter: the parameter-independent approach, which is also known as the robust structure [34], [35]; and the parameter-dependent one, which is typically referred to as the gain-scheduled structure [36], [37]. Comparing the conservative levels between the two different strategies, the gain-scheduled one has a noticeable advantage since it can be reconfigured on-line using the measurements of the time-varying parameter. We consider in this paper that the time-varying parameter is not directly accessible but, instead, it is measured subject to the uncertainties associated with this measurement. Hence, this imprecision is an additional challenge that needs to be taken into account within the FD context, as it cannot be mistakenly considered as a fault, which could lead to the occurrence of false alarms. We can refer to [37] for works dealing with inexact measurements of the time-varying parameter. These design methods are based on the formulation of optimization problems as semi-definite programming using LMI constraints. This formulation allows us to draw FD solutions in the robust or gain-scheduled structures. The distinction between such structures is made during the design process, where the polynomial degree associated with each decision variable changes according to the desired formulation. For a detailed explanation and motivation on this matter, we refer the interested readers to [30], [38]-[41].

A central distinguishing assumption in this paper is that we consider discrete-time LPV systems in which the time-varying parameters are estimated, and that these estimated parameters are contaminated by additive noise. Typically, in the existing methods in the literature, this imprecision is ignored for design simplification. However, depending on the system dynamics and the applied estimation process, this discrepancy may lead the LPV system to lose performance or to become unstable since it may be working outside its designed operational range. An option within this scenario is to implement a more sophisticated estimation process, which will increase the computational burden in an on-line fashion. Adding the assumption that the time-varying parameters are imprecise in the design process of the FDF is particularly useful since it allows to deal with the inexact estimated parameter, presuming that the imprecision is inherent to the estimation process. From the practical standpoint, the proposed Gain-Scheduled Fault Detection Filter design, which is inspired by the results in [33], [42]-[44], tackles this problem without overburdening the estimation process.

While the motivation and applicability of using performance indexes in the context of FD are conceivable, to the best of the authors' knowledge, they have not yet been discussed thoroughly in the literature under the assumption that the scheduling parameter is imprecisely known. Bearing this in mind, the main contributions of the present paper, under the imprecisely known scenario for the scheduling parameter, are the design and analysis of:

- \mathcal{H}_2 FDF for discrete-time LPV systems,
- \mathcal{H}_{∞} FDF for discrete-time LPV systems,
- mixed \mathcal{H}_2 / \mathcal{H}_∞ FDF for discrete-time LPV systems.

Furthermore, illustrative simulations to exemplify the proposed approaches and to explore the implementation aspects for all aforementioned FD filters are provided. The assumption that the scheduling parameter is imprecise is considered during the design process using the more recent LMI parses ROLMIP [38], associated with YALMIP [45]. The conditions for solving such problems are obtained via LMI and are based on the results in [30], [31].

This paper is organized as follows: Sections II and III present the necessary theoretical background and the problem formulation, Section IV presents the main theoretical results, Section V illustrates the results with an illustrative example, and Section VI concludes the paper with some final comments.

II. PRELIMINARIES

A. NOTATIONS

N and N⁺ denote respectively, the set of real numbers and the set of positive real numbers. The *n*-th dimensional Euclidian space with norm $\|\cdot\|$ is denoted by \mathbb{R}^n . The symbol ' denotes the transpose matrix, and • represents blocks induced by symmetry in a square matrix. The operator Her(·) denotes the symmetric sum, e.g. Her(X) = X' + X. The expected value operator is represented by $\mathbb{E}(\cdot)$. The set \mathcal{L}_2 is the class of square-summable sequences, and for $w = \{w(0), w(1), \ldots\} \in \mathcal{L}_2$ we write $\|w\|_2^2 = \sum_{k=0}^{\infty} \|w(k)\|^2$.

Definition 1 (Unit-Simplex): The unit-simplex Λ_N of dimension $N \in \mathbb{N}$, with $N \ge 2$ is defined as

$$\Lambda_N = \left\{ \zeta \in \mathbb{R}^N : \sum_{i=1}^N \zeta_i = 1, \, \zeta_i \ge 0, \, i = 1, \dots, N \right\}.$$
 (1)

Definition 2 (Multi-Simplex): The multi-simplex $\Lambda_{m,N}$ is defined as the Cartesian product of *m* simplexes (as in (1)) with dimension of *N*, that is, $\Lambda_{m,N} = \Lambda_N \times \cdots \times \lambda_N$ with the Cartesian product containing *m* terms. Thus any $\theta \in \Lambda_{m,N}$ can be decomposed as $\theta = (\theta_1, \theta_2, \dots, \theta_m)$, with $\theta_i = (\theta_{i1}, \theta_{i2}, \dots, \theta_{iN}) \in \Lambda_N, i \in \{1, \dots, m\}.$

Definition 3 (Homogeneous Polynomial): For a unitsimplex Λ_N of dimension $N \in \mathbb{N}$, a polynomial $g(\theta), \theta \in \Lambda_N$ is named a homogeneous polynomial of degree $l \in \mathbb{N}$ if all its monomials have the same total degree l. As an example, assuming $\theta = [\theta_1, \theta_2] \in \Lambda_2$, and $g(\theta) = \theta_1^3 + \theta_1^2 \theta_2 + \theta_1 \theta_2^2 + \theta_2^3, g(\theta)$ is said to be a homogeneous polynomial with a degree l = 3. Set $\mathbb{K}_N^{(l)}$ as the set of *N*-tuples obtained from all possible combinations of *N* nonnegative integers $k_j, j = 1, \dots, N$, with sum $k_1 + k_2 + \dots + k_N = l$. A homogeneous polynomial with l degree is defined as

$$A(\theta) = \sum_{k \in \mathbb{K}_N^{(l)}} \theta^k A_k, \tag{2}$$

where $\theta^k = \theta_1^{k_1} \cdot \theta_2^{k_2} \cdot \dots \cdot \theta_N^{k_N} = \prod_{j=1}^N \theta_j^{k_j}$.

B. LPV SYSTEMS

Consider the following discrete-time LPV system

$$\mathcal{G} := \begin{cases} x(k+1) = A_{\theta(k)}x(k) + J_{\theta(k)}w(k) \\ z(k) = C_{\theta(k)}x(k) + D_{\theta(k)}w(k), \end{cases}$$
(3)

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector, $w(k) \in \mathbb{R}^{n_w}$ represents the exogenous input, and the $z(k) \in \mathbb{R}^{n_z}$ denotes the output signal. We assume that the matrices $A_{\theta(k)}$, $J_{\theta(k)}$, $C_{\theta(k)}$, $D_{\theta(k)}$ in (3) depend on the parameter $\theta(k)$ in the affine form as

$$A_{\theta(k)} = A_0 + \sum_{i=1}^{m} \theta_i(k) A_i,$$
(4)

where A_0, \ldots, A_m are given matrices and $\theta(k) = (\theta_1(k), \ldots, \theta_m(k))$ are bounded time-varying parameters satisfying $|\theta_i(k)| \le t_i, t_i \in \mathbb{R}^+, i = 1, \ldots, m, \forall k \ge 0$. Similarly for $J_{\theta(k)}, C_{\theta(k)}, D_{\theta(k)}$. Observe that the affine form is a particular case of the parameterized form in (2) with degree equal to 1. Note that if we describe the matrices in (3) as polynomials with a degree equal to 0, system (3) becomes parameter-independent.

The procedure to choose each matrix A_0 and A_i , i = 1, 2, ..., m that compose $A_{\theta(k)}$ in (4) is as follows. The matrix A_0 is related to the portion of the system's dynamic that is fixed while the matrices A_i are related to each time-varying parameter, for example, A_1 denotes how the time-varying parameter $\theta_1(k)$ influences the system's dynamic, etc. As an illustrative example, we may have

$$\underbrace{\begin{bmatrix} 1 & 0.1+\theta_1(k) \\ 0 & -0.75+\theta_2(k) \end{bmatrix}}_{A_{\theta(k)}} = \underbrace{\begin{bmatrix} 1 & 0.1 \\ 0 & -0.75 \end{bmatrix}}_{A_0} + \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_1} \theta_1(k) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{A_2} \theta_2(k) \,.$$
(5)

C. \mathcal{H}_2 GUARANTEED COST ANALYSIS

The \mathcal{H}_2 norm is a performance criterion that is associated with the energy of the impulse response of the system or, in other words,

$$\|\mathcal{G}\|_2 = \limsup_{T \to \infty} \mathbb{E} \left\{ \frac{1}{T} \sum_{k=0}^T z(k)' z(k) \right\},\tag{6}$$

where *T* is a positive integer that represents the time horizon and w(k) is a standard white noise (Gaussian zero-mean in which the covariance matrix is equal to the identity matrix) as defined in [46].

Considering an asymptotically stable system in the form (3), an upper bound for its \mathcal{H}_2 norm can be obtained by a set of parameter-dependent LMI constraints, as introduced

in [30] and shown in the following lemma. For the sake of simplicity in (7)-(9) below we set $\theta = \theta(k)$, and $\beta = \theta(k+1)$.

Lemma 1: If there exist symmetric positive definite matrices P_{θ} , and W_{θ} , such that

$$\begin{bmatrix} P_{\beta} - A_{\theta} P_{\theta} A'_{\theta} & \bullet \\ J'_{\theta} & I \end{bmatrix} > 0,$$
(7)

$$\begin{bmatrix} W_{\theta} - D_{\theta} D'_{\theta} & \bullet \\ P_{\theta} C'_{\theta} & P_{\theta} \end{bmatrix} > 0, \tag{8}$$

and

$$\operatorname{Tr}(W_{\theta}) < \mu^2, \tag{9}$$

hold for all $\theta(k)$, $k \ge 0$, then μ is an upper bound for the \mathcal{H}_2 norm of system (3), that is, $\|\mathcal{G}\|_2 < \mu$.

Lemma 1 and its proof are presented in [30, Theorem 2].

D. \mathcal{H}_{∞} GUARANTEED COST ANALYSIS

In this subsection we introduce a few concepts that will be important later on regarding the \mathcal{H}_{∞} norm. The \mathcal{H}_{∞} norm is a classical performance criterion which can be computed using the Bounded Real Lemma (BRL), as proposed in [31] for LPV systems. For the system as in (3), its \mathcal{H}_{∞} norm is defined by

$$\|\mathcal{G}\|_{\infty} = \sup_{\|w\|_{2} \neq 0} \frac{\|z\|_{2}}{\|w\|_{2}}, \quad w \in \mathcal{L}_{2}.$$
 (10)

In the following lemma, based on the conditions from [30], we present the BRL for LPV systems where an upper bound for the \mathcal{H}_{∞} norm is computed via parameter-dependent LMI.

Lemma 2: If there exists a symmetric positive definite matrix P_{θ} , such that

$$\begin{bmatrix} P_{\beta} & \bullet & \bullet & \bullet \\ P_{\theta}A'_{\theta} & P_{\theta} & \bullet & \bullet \\ J'_{\theta} & 0 & \gamma I & \bullet \\ 0 & C_{\theta}P_{\theta} & D_{\theta} & \gamma I \end{bmatrix} > 0,$$
(11)

holds for all $\theta(k)$, $k \ge 0$, then γ is an upper bound for the \mathcal{H}_{∞} norm of system (3), that is, $\|\mathcal{G}\|_{\infty} < \gamma$.

The proof for Lemma 2 can be found in [47, Lemma 3].

III. GAIN SCHEDULED RESIDUE GENERATION PROBLEM FORMULATION

A. PROBLEM FORMULATION

Consider the following LPV discrete-time system

$$\mathcal{G}_{f} := \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + J_{\theta(k)}w(k) \\ \cdots + F_{\theta(k)}f(k) \\ y(k) = C_{\theta(k)}x(k) + D_{\theta(k)}w(k) \\ \cdots + D_{f\theta(k)}f(k), \end{cases}$$
(12)

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector, $u(k) \in \mathbb{R}^{n_u}$ denotes the control input, $w(k) \in \mathbb{R}^{n_w}$ is the exogenous input, $y(k) \in \mathbb{R}^{n_y}$ is the measurement signal and $f(k) \in \mathbb{R}^{n_f}$ is the fault signal. We also consider that the signals $w, f \in \mathcal{L}_2$ and recall that the time-varying parameter $\theta(k)$ is bounded as $|\theta_i(k)| \le t_i, t_i \in \mathbb{R}^+, i = 1, ..., m, \forall k \ge 0.$



FIGURE 1. The gray region represents the feasible region generated by each pair of the parameters ($\theta_i(k), \sigma_i(k)$), borrowed from [33].

Remark 1: Notice that, as in [48, Chapter 11.3], it is assumed that the faults can be model as an unknown and not accessible additive signal f(k).

The FDF is defined as follows

$$\mathcal{F} := \begin{cases} \eta(k+1) = \mathfrak{A}_{\eta\hat{\theta}(k)}\eta(k) + \mathfrak{M}_{\eta\hat{\theta}(k)}u(k) \\ \cdots + \mathfrak{B}_{\eta\hat{\theta}(k)}y(k) \\ r(k) = \mathfrak{C}_{\eta\hat{\theta}(k)}\eta(k) + \mathfrak{D}_{\eta\hat{\theta}(k)}y(k), \end{cases}$$
(13)

where $\eta(k) \in \mathbb{R}^{n_{\eta}}$ denote the filter state, $u(k) \in \mathbb{R}^{n_{u}}$ denotes the control input, $y(k) \in \mathbb{R}^{n_{y}}$ is the measurement signal and $r(k) \in \mathbb{R}^{n_{r}}$ is the residue signal. Note that the FDF (13) depends only on the estimated parameter $\hat{\theta}$. We assume that the FDF in (13) can be written in the affine form similarly to (4), so that the matrices in (13) are defined as

$$\mathfrak{A}_{\eta\hat{\theta}(k)} = \mathfrak{A}_{\eta 0} + \sum_{i=1}^{m} \hat{\theta}_{i}(k)\mathfrak{A}_{\eta i}, \qquad (14)$$

and similarly for $\mathfrak{M}_{\eta\hat{\theta}(k)}$, $\mathfrak{B}_{\eta\hat{\theta}(k)}$, $\mathfrak{C}_{\eta\hat{\theta}(k)}$, $\mathfrak{D}_{\eta\hat{\theta}(k)}$. Hence, the main focus of this paper is to design all the matrices $\mathfrak{A}_{\eta i}$, $\mathfrak{M}_{\eta i}$, $\mathfrak{B}_{\eta i}$, $\mathfrak{C}_{\eta i}$, $\mathfrak{D}_{\eta i}$, $i \in \{1, \ldots, m\}$.

B. PARAMETER UNDER ADDITIVE UNCERTAINTY

One of the major premises of the present paper is that the time-varying parameters $\theta(k)$ are not directly accessible. Instead, we implement estimation procedures to gather an estimation $\hat{\theta}(k)$ of the time-varying parameter $\theta(k)$, which are not completely precise, meaning that we must assume that $\hat{\theta}(k)$ is an inexact measurement of $\theta(k)$. The design under the assumption of inexact measurements is dealt with a general model described in [33], [49], in which we assume that the estimated parameters $\hat{\theta}(k)$ is a sum of the actual parameter $\theta(k)$ with an orthogonal additive uncertainty $\sigma(k)$, that is

$$\hat{\theta}_i(k) = \theta_i(k) + \sigma_i(k), \quad i = 1, \dots, m$$
(15)

where $|\sigma_i(k)| \leq d_i$, $d_i \in \mathbb{R}^+$, i = 1, ..., m. Thus, the domain of $(\theta(k), \sigma(k))$ is as displayed in Fig.1.

C. THE AUGMENTED SYSTEM

From the aforementioned discussion we may define the augmented system which depends on both time-varying

parameters $\theta(k)$, $\hat{\theta}(k)$, by taking e(k) = r(k) - f(k), as

$$\mathcal{G}_{aug} := \begin{cases} \check{x}(k+1) = \check{A}_{\hat{\theta}(k)\theta(k)}\check{x}(k) + \check{J}_{\hat{\theta}(k)\theta(k)}\check{w}(k) \\ e(k) = \check{C}_{\hat{\theta}(k)\theta(k)}\check{x}(k) + \check{D}_{\hat{\theta}(k)\theta(k)}\check{w}(k), \end{cases}$$
(16)

where we consider the augmented vectors $\check{x} = [x'(k) \eta'(k)]'$, $\check{w} = [u'(k) d'(k) f'(k)]'$. In order to simplify the visualization of the resulting LMI, we consider hereafter $\theta = \theta(k)$, and $\hat{\theta} = \hat{\theta}(k)$. The following augmented matrices can be obtained:

$$\begin{split} \check{A}_{\hat{\theta}\theta} &= \begin{bmatrix} B_{\theta} & 0\\ \mathfrak{B}_{\eta\hat{\theta}}C_{\theta} & \mathfrak{A}_{\eta\hat{\theta}} \end{bmatrix}, \quad \check{J}_{\hat{\theta}\theta} &= \begin{bmatrix} B_{\theta} & J_{\theta} & \mathfrak{B}_{\eta\hat{\theta}}D_{\theta} & \mathfrak{B}_{\eta\hat{\theta}}D_{f\theta} \end{bmatrix}, \\ \check{C}_{\hat{\theta}\theta} &= \begin{bmatrix} \mathfrak{D}_{\eta\hat{\theta}}C_{\theta} & \mathfrak{C}_{\eta\hat{\theta}} \end{bmatrix}, \quad \check{D}_{\hat{\theta}\theta} &= \begin{bmatrix} 0 & \mathfrak{D}_{\eta\hat{\theta}}D_{\theta} & \mathfrak{D}_{\eta\hat{\theta}}D_{f\theta}-I \end{bmatrix}. \end{split}$$

Based on the augmented system as above, we can define the \mathcal{H}_2 Fault Detection problem as follows.

 \mathcal{H}_2 Fault Detection problem: Given a desired \mathcal{H}_2 -gain $\mu > 0$, design the FDF as in (13) such that the \mathcal{H}_2 norm of the augmented system (16) satisfies

$$\|\mathcal{G}_{aug}\|_2 = \limsup_{T \to \infty} \mathbb{E}\left\{\frac{1}{T} \sum_{k=0}^T e(k)' e(k)\right\} < \mu.$$
(17)

Similarly, we can define the \mathcal{H}_{∞} Fault Detection problem as follows.

 \mathcal{H}_{∞} Fault Detection problem: Given a desired \mathcal{H}_{∞} -gain $\gamma > 0$, design the FDF as in (13) such that the \mathcal{H}_{∞} norm of the augmented system (16) satisfies

$$\|\mathcal{G}_{aug}\|_{\infty} = \sup_{\|\check{w}\|_{2} \neq 0, \check{w} \in \mathcal{L}_{2}} \frac{\|e\|_{2}}{\|w\|_{2}} < \gamma.$$
(18)

To guarantee that the FDF distinguishes properly the fault signal from the disturbance one, we define the FD problem by considering the norms as in (17) and (18). On the one hand, the \mathcal{H}_2 norm is used as a performance index to make the residue signal r(k) sensitive to the abnormal energy surge in the system, which characterizes a fault. On the other hand, the \mathcal{H}_{∞} norm is defined to guarantee that the residue signal r(k) can be made resilient against the disturbance. These two definitions provide a distinction on the sensitivity of the residue signal to the fault f(k) or to the disturbance w(k). Furthermore, by using both definitions simultaneously, a mixed $\mathcal{H}_2/\mathcal{H}_{\infty}$ can be designed to combine the characteristic of each approach to the FDF, aiming at obtaining a balanced design of FDF.

D. CHANGE OF VARIABLES

From the discussion presented in the previous subsections, a major assumption in this paper is that the parameter used by the filter is an estimation of the real one affecting the system. To deal with this assumption, it is necessary to employ some procedures to design the fault detection filter (13). Using, for instance, the procedures given in [49], [50], we can perform a variable transformation to deal with this type of parameters subjected to additive uncertainty. These variable transformations, applied to our context can be seen as

$$\alpha_{i1}(k) = \frac{\theta_i(k) + t_i}{2t_i}, \quad \hat{\alpha}_{i1}(k) = \frac{\sigma_i(k) + d_i}{2d_i}.$$

VOLUME 9, 2021

Recalling that $|\theta_i(k)| \le t_i$, $|\sigma_i(k)| \le d_i$, it follows that $0 \le \alpha_{i1}(k) \le 1$, $0 \le \hat{\alpha}_{i1}(k) \le 1$. The original parameters are retrieved, for i = 1, ..., m, as

$$\theta_i(k) = 2 t_i \alpha_{i1}(k) - t_i, \quad \sigma_i(k) = 2 d_i \hat{\alpha}_{i1}(k) - d_i.$$

Thus, we have that $\alpha_i(k) = (\alpha_{i1}(k), \alpha_{i2}(k))$ and $\hat{\alpha}_i(k) =$ $(\hat{\alpha}_{i1}(k), \hat{\alpha}_{i2}(k)),$ where $\alpha_{i2}(k) = 1 - \alpha_{i1}(k), \hat{\alpha}_{i2}(k) = 1 - \alpha_{i1}(k)$ $\hat{\alpha}_{i1}(k)$, belong to the unit-simplex as in (1) with N = 2, so that $\alpha(k) = (\alpha_1(k), \dots, \alpha_m(k))$ and $\hat{\alpha}(k) = (\hat{\alpha}_1(k), \dots, \hat{\alpha}_m(k))$ belong to the multi-simplex $\Lambda_{m,2} = \Lambda_2 \times \cdots \times \Lambda_2$ with m terms, according to the Definition II in Section II. We set $\tilde{\alpha}(k) = (\alpha(k), \hat{\alpha}(k)) \in \Lambda_{m,2} \times \Lambda_{m,2}$, where $\alpha(k)$ is related to $\theta(k)$, and $\hat{\alpha}(k)$ to $\sigma(k)$ (the additive noise time-varying parameter). Notice that the matrices in system (3) and in the FDF in (13) can be rewritten using the new multi-simplex $\tilde{\alpha}(k)$, following the procedure explained in [49], which uses the polynomial homogenisation process presented in [51]. The use of the parse ROLMIP [38], combined with YALMIP [45], yields to a procedure as simple as setting the degrees of the multi-simplex polynomials and the parameter boundaries. Thus for the numerical procedure this change of variable will be applied to derive the FDF in (13).

Another assumption made for the numerical procedure is that the parameters are arbitrarily fast in time, so that, by consequence, $\theta(k + 1)$ is independent from $\theta(k)$. However, the proposed conditions could be extended to deal with the case with bounded rates of variation, as presented in [30], [31].

IV. MAIN RESULTS

In this section, we describe the main contributions of this paper on the design of fault detection filters for solving the previously defined \mathcal{H}_2 and \mathcal{H}_∞ fault detection problems. The main advantage of the FDF designed by using the \mathcal{H}_2 approach is that it yields a fast system's response whenever there is an unexpected additive input. On the other hand, the main purpose of designing an FDF using the \mathcal{H}_{∞} setup is the ability to mitigate the influence of the disturbance on the residue signal, preventing the occurrence of false alarms. Combining the characteristics of both approaches through a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem, makes it possible to increase the performance of the resulting FDF. It is important to stress that the results will be presented in terms of the original parameters $\theta(k)$ and $\hat{\theta}(k)$ to highlight that the derived filter only depends on the measurable parameter $\hat{\theta}(k)$. For the numerical procedure, the change of variables presented in sub-section III-D should be applied so that we end up with multi-simplex polynomials with the new multi-simplex parameter $\tilde{\alpha} \in \Lambda_{m,2} \times \Lambda_{m,2}$. In what follows, by feasible θ , β, θ , we mean that the constraints imposed in Section III are satisfied.

A. \mathcal{H}_2 FILTER DESIGN

The following theorem presents the LPV FDF design with an upper bound for the guaranteed cost for the \mathcal{H}_2 norm of system (16).

Theorem 1: If there exist a scalar $\mu > 0$ and symmetric positive definite matrices $Y_{11\theta}$, $Y_{22\theta}$, M_{θ} , and matrices $Y_{12\theta}$, $X_{1\hat{\theta}}$, $X_{2\hat{\theta}}$, $\bar{X}_{\hat{\theta}}$, $\Omega_{\hat{\theta}}$, $\nabla_{\hat{\theta}}$, $\Gamma_{\hat{\theta}}$, $\mathfrak{C}_{\eta\hat{\theta}}$, $\mathfrak{D}_{\eta\hat{\theta}}$ with compatible dimensions, and a given scalar parameter ξ such that the inequalities (19), (20), (21), as shown at the bottom of the page, hold for all feasible θ , β , $\hat{\theta}$, then the LPV FDF (13) with $\mathfrak{A}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1} \nabla_{\hat{\theta}}$, $\mathfrak{B}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1} \Omega_{\hat{\theta}}$, $\mathfrak{M}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1} \Gamma_{\hat{\theta}}$, $\mathfrak{C}_{\eta\hat{\theta}} = \mathfrak{C}_{\eta\hat{\theta}}$, and $\mathfrak{D}_{\eta\hat{\theta}} = \mathfrak{D}_{\eta\hat{\theta}}$ solves the \mathcal{H}_2 fault detection problem (17).

Proof: First, apply the variable substitution $\nabla_{\hat{\theta}} = \bar{X}_{\hat{\theta}} \mathfrak{A}_{\eta\hat{\theta}}$, $\Omega_{\hat{\theta}} = \bar{X}_{\hat{\theta}} \mathfrak{B}_{\eta\hat{\theta}}, \Gamma_{\hat{\theta}} = \bar{X}_{\hat{\theta}} \mathfrak{M}_{\eta\hat{\theta}}, \mathfrak{C}_{\eta\hat{\theta}} = \mathfrak{C}_{\eta\hat{\theta}}, \text{ and } \mathfrak{D}_{\eta\hat{\theta}} = \mathfrak{D}_{\eta\hat{\theta}} \text{ in }$ (20). Consider the following structures for $X_{\hat{\theta}}, Y_{\theta}, Y_{\theta}$:

$$X_{\hat{\theta}} = \begin{bmatrix} X_{1\hat{\theta}} & \bar{X}_{\hat{\theta}} \\ X_{2\hat{\theta}} & \bar{X}_{\hat{\theta}} \end{bmatrix}, \quad Y_{\theta} = \begin{bmatrix} Y_{11\theta} & \bullet \\ Y_{21\theta} & Y_{22\theta} \end{bmatrix}, \quad Y_{\beta} = \begin{bmatrix} Y_{11\beta} & \bullet \\ Y_{21\beta} & Y_{22\beta} \end{bmatrix}.$$
(22)

From the augmented matrices given in (16) and (22) it follows that

$$\begin{split} X_{\hat{\theta}}\check{A}_{\theta\hat{\theta}} &= \begin{bmatrix} X_{1\hat{\theta}} & X_{\hat{\theta}} \\ X_{2\hat{\theta}} & \bar{X}_{\hat{\theta}} \end{bmatrix} \begin{bmatrix} \mathfrak{B}_{\eta\hat{\theta}} C_{\theta} & \mathfrak{A}_{\eta\hat{\theta}} \\ \mathfrak{B}_{\eta\hat{\theta}} C_{\theta} & \mathfrak{A}_{\eta\hat{\theta}} \end{bmatrix} \\ &= \begin{bmatrix} X_{1\hat{\theta}} A_{\theta} + \bar{X}_{\hat{\theta}} \mathfrak{B}_{\eta\hat{\theta}} C_{\theta} & \bar{X}_{\hat{\theta}} \mathfrak{A}_{\eta\hat{\theta}} \\ X_{2\hat{\theta}} A_{\theta} + \bar{X}_{\hat{\theta}} \mathfrak{B}_{\eta\hat{\theta}} C_{\theta} & \bar{X}_{\hat{\theta}} \mathfrak{A}_{\eta\hat{\theta}} \end{bmatrix}, \\ \check{J}_{\theta\hat{\theta}}'\check{X}_{\hat{\theta}}' &= \begin{bmatrix} B'_{\theta} & \mathfrak{M}'_{\eta\hat{\theta}} \\ J'_{\theta} & D'_{\theta} \mathfrak{B}'_{\eta\hat{\theta}} \\ F'_{\theta} & D'_{\theta} \mathfrak{B}'_{\eta\hat{\theta}} \end{bmatrix} \begin{bmatrix} X'_{1\hat{\theta}} & X'_{2\hat{\theta}} \\ \bar{X}'_{\hat{\theta}} & \bar{X}'_{\hat{\theta}} \end{bmatrix} \\ &= \begin{bmatrix} B'_{\theta} X'_{1\hat{\theta}} + \mathfrak{M}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} & B'_{\theta} X'_{2\hat{\theta}} + \mathfrak{M}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} \\ J'_{\theta} X'_{1\hat{\theta}} + D'_{\theta} \mathfrak{B}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} & J'_{\theta} X'_{2\hat{\theta}} + D'_{\theta} \mathfrak{B}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} \\ F'_{\theta} X'_{1\hat{\theta}} + D'_{f\theta} \mathfrak{B}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} & F'_{\theta} X'_{2\hat{\theta}} + D'_{f\theta} \mathfrak{B}'_{\eta\hat{\theta}} \bar{X}'_{\hat{\theta}} \end{bmatrix}, \end{split}$$

so that we can rewrite the constraint (20) as

$$\begin{bmatrix} -Y_{\theta} + \xi(\operatorname{Her}(X_{\hat{\theta}}\check{A}_{\theta\hat{\theta}})) & \bullet & \bullet \\ X_{\hat{\theta}}\check{A}_{\theta\hat{\theta}} - \xi X'_{\hat{\theta}} & Y_{\beta} - \operatorname{Her}(X_{\hat{\theta}}) & \bullet \\ \xi \check{J}'_{\theta\hat{\theta}}X'_{\hat{\theta}} & \check{J}'_{\theta\hat{\theta}}X'_{\hat{\theta}} & -I \end{bmatrix} < 0.$$
(23)

Reorganizing (23) we get that

$$\begin{bmatrix} -Y_{\theta} & \bullet & \bullet \\ 0 & Y_{\beta} & \bullet \\ 0 & 0 & -I \end{bmatrix} + \begin{bmatrix} A'_{\theta\hat{\theta}} \\ -I \\ J'_{\theta\hat{\theta}} \end{bmatrix} X'_{\hat{\theta}} \begin{bmatrix} \xi I & I & 0 \end{bmatrix} + \cdots \\ \times \begin{bmatrix} \xi I \\ I \\ 0 \end{bmatrix} X_{\hat{\theta}} \begin{bmatrix} \check{A}_{\theta\hat{\theta}} & -I & \check{J}_{\theta\hat{\theta}} \end{bmatrix} < 0 \quad (24)$$

so that (24) can be rewritten as

$$Q_{\theta\beta} + U'_{\theta\hat{\theta}}X'_{\hat{\theta}}V + V'X_{\hat{\theta}}U_{\theta\hat{\theta}} < 0$$
⁽²⁵⁾

where

$$Q_{\theta\beta} = \begin{bmatrix} -Y_{\theta} & \bullet & \bullet \\ 0 & Y_{\beta} & \bullet \\ 0 & 0 & -I \end{bmatrix}, \quad U_{\theta\hat{\theta}} = \begin{bmatrix} \check{A}_{\theta\hat{\theta}} & -I & \check{J}_{\theta\hat{\theta}} \end{bmatrix}, \quad V = \begin{bmatrix} \xi I & I & 0 \end{bmatrix}.$$

Let the null spaces for $U_{\theta\hat{\theta}}$ and V be given by

$$\mathcal{N}_{U} = \begin{bmatrix} I & 0\\ \check{A}_{\theta\hat{\theta}} & \check{J}_{\theta\hat{\theta}}\\ 0 & I \end{bmatrix}, \text{ and } \mathcal{N}_{V} = \begin{bmatrix} -I & 0\\ \xi I & 0\\ 0 & I \end{bmatrix}.$$
(26)

Now, if we pre- and post-multiply (23) by \mathcal{N}'_U and \mathcal{N}_U , respectively, and apply twice the Schur complement to the result of this procedure we recover the conditions presented in (7) with $P_{\theta} = Y_{\theta}^{-1}$ and $P_{\beta} = Y_{\beta}^{-1}$. Pre- and post-multiplying (23) by \mathcal{N}_V we get the bounds for the scalar parameter $\xi \in (-1, 1)$. Regarding the constraints (21) we consider the same variable substitutions as at the start of the proof. After that, applying twice the Schur complement, we obtain the constraint (8) with $W_{\theta} = M_{\theta}$.

B. \mathcal{H}_{∞} **FILTER DESIGN**

In the following theorem, we present the design of LPV FDF via LMI in order to obtain a guaranteed \mathcal{H}_{∞} upper bound of the augmented system in (16).

Theorem 2: If there exist a scalar $\gamma > 0$ and symmetric positive definite matrices $W_{11\theta}$, and $W_{22\theta}$ and matrices $W_{12\theta}$, $K_{1\hat{\theta}}$, $K_{2\hat{\theta}}$, $\bar{K}_{\hat{\theta}}$, $\Omega_{\hat{\theta}}$, $\nabla_{\hat{\theta}}$, $\Gamma_{\hat{\theta}}$, $\mathfrak{C}_{\eta\hat{\theta}}$, $\mathfrak{D}_{\eta\hat{\theta}}$ with compatible dimensions and a given scalar parameter ξ such that (27)

holds for all feasible θ , β , $\hat{\theta}$ then the LPV FDF (13) with $\mathfrak{A}_{\eta\hat{\theta}} = \bar{K}_{\hat{\theta}}^{-1} \nabla_{\hat{\theta}}, \mathfrak{B}_{\eta\hat{\theta}} = \bar{K}_{\hat{\theta}}^{-1} \Omega_{\hat{\theta}}, \mathfrak{M}_{\eta\hat{\theta}} = \bar{K}_{\hat{\theta}}^{-1} \Gamma_{\hat{\theta}}, \mathfrak{C}_{\eta\hat{\theta}} = \mathfrak{C}_{\eta\hat{\theta}},$ and $\mathfrak{D}_{\eta\hat{\theta}} = \mathfrak{D}_{\eta\hat{\theta}}$ solves the \mathcal{H}_{∞} fault detection problem (18).

Proof: We apply the variable substitutions $\nabla_{\hat{\theta}} = \bar{K}_{\hat{\theta}}\mathfrak{A}_{\eta\hat{\theta}}$, $\Omega_{\hat{\theta}} = \bar{K}_{\hat{\theta}}\mathfrak{B}_{\eta\hat{\theta}}, \Gamma_{\hat{\theta}} = \bar{K}_{\hat{\theta}}\mathfrak{M}_{\eta\hat{\theta}}, \mathfrak{C}_{\eta\hat{\theta}} = \mathfrak{C}_{\eta\hat{\theta}}$, and $\mathfrak{D}_{\eta\hat{\theta}} = \mathfrak{D}_{\eta\hat{\theta}}$ in (27), as shown at the bottom of the page. Assuming the structure of \mathcal{W}_{θ} , $\mathcal{K}_{\hat{\alpha}}$, as

$$\mathcal{W}_{\theta} = \begin{bmatrix} W_{11\theta} & W_{12\theta} \\ W_{12\theta}' & W_{22\theta} \end{bmatrix}, \quad \mathcal{K}_{\hat{\theta}} = \begin{bmatrix} K_{1\hat{\theta}} & \bar{K}_{\hat{\theta}} \\ K_{2\hat{\theta}} & \bar{K}_{\hat{\theta}} \end{bmatrix}, \quad (28)$$

as well as the augmented matrices in (16), we get the following matrices

$$\begin{split} \mathcal{K}_{\hat{\theta}}\check{A}_{\theta\hat{\theta}} &= \begin{bmatrix} K_{1\hat{\theta}} & \bar{K}_{\hat{\theta}} \\ K_{2\hat{\theta}} & \bar{K}_{\hat{\theta}} \end{bmatrix} \begin{bmatrix} \mathfrak{A}_{\theta} & 0 \\ \mathfrak{B}_{\eta\hat{\theta}}C_{\theta} & \mathfrak{A}_{\eta\hat{\theta}} \end{bmatrix} \\ &= \begin{bmatrix} K_{1\hat{\theta}}A_{\theta} + \bar{K}_{\hat{\theta}}\mathfrak{B}_{\eta\hat{\theta}}C_{\theta} & \bar{K}_{\hat{\theta}}\mathfrak{A}_{\eta\hat{\theta}} \\ K_{2\hat{\theta}}A_{\theta} + \bar{K}_{\hat{\theta}}\mathfrak{B}_{\eta\hat{\theta}}C_{\theta} & \bar{K}_{\hat{\theta}}\mathfrak{A}_{\eta\hat{\theta}} \end{bmatrix}, \\ \check{J}_{\theta\hat{\theta}}'\check{K}_{\hat{\theta}}' &= \begin{bmatrix} B_{\theta}' & \mathfrak{M}'_{\eta\hat{\theta}} \\ J_{\theta}' & D_{\theta}'\mathfrak{B}'_{\eta\hat{\theta}} \\ F_{\theta}' & D_{f\theta}'\mathfrak{B}'_{\eta\hat{\theta}} \end{bmatrix} \begin{bmatrix} K_{1\hat{\theta}}' & K_{2\hat{\theta}}' \\ \bar{K}_{\hat{\theta}}' & \bar{K}_{\hat{\theta}}' \end{bmatrix} \\ &= \begin{bmatrix} B_{\theta}'K_{1\hat{\theta}}' + \mathfrak{M}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' & B_{\theta}'K_{2\hat{\theta}}' + \mathfrak{M}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' \\ J_{\theta}'K_{1\hat{\theta}}' + D_{\theta}'\mathfrak{B}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' & J_{\theta}'K_{2\hat{\theta}}' + D_{\theta}'\mathfrak{B}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' \\ F_{\theta}'K_{1\hat{\theta}}' + D_{f\theta}'\mathfrak{B}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' & F_{\theta}'K_{2\hat{\theta}}' + D_{f\theta}'\mathfrak{B}'_{\eta\hat{\theta}}\bar{K}_{\hat{\theta}}' \end{bmatrix} \end{split}$$

From the above, inequality (27) can be rewritten as

$$\begin{bmatrix} -\mathcal{W}_{\theta} + \xi(\operatorname{Her}(\mathcal{K}_{\hat{\theta}}\hat{A}_{\theta\hat{\theta}})) & \bullet & \bullet & \bullet \\ \mathcal{K}_{\hat{\theta}}\check{A}_{\theta\hat{\theta}} - \xi\mathcal{K}'_{\hat{\theta}} & -\mathcal{W}_{\hat{\theta}} - \mathcal{K}_{\hat{\theta}} - \mathcal{K}'_{\hat{\theta}} & \bullet & \bullet \\ \xi\check{J}'_{\theta\hat{\theta}}\mathcal{K}'_{\hat{\theta}} & \check{J}'_{\theta\hat{\theta}}\mathcal{K}'_{\hat{\theta}} & -\gamma^{2}I & \bullet \\ \check{C}_{\theta\hat{\theta}} & 0 & \check{D}_{\theta\hat{\theta}} & -I \end{bmatrix} < 0.$$
(29)

We can reorganize (29) as

$$\begin{array}{cccc} -\mathcal{W}_{\theta} & \bullet & \bullet & \bullet \\ 0 & -\mathcal{W}_{\beta} & \bullet & \bullet \\ 0 & 0 & -\gamma^{2}I & \bullet \\ \check{C}_{\theta\hat{\theta}} & 0 & \check{D}_{\theta\hat{\theta}} & -I \end{array} \right] + \begin{bmatrix} \check{A}'_{\theta\hat{\theta}} \\ -I \\ \check{J}'_{\theta\hat{\theta}} \\ 0 \end{bmatrix}} \mathcal{K}'_{\hat{\theta}} \begin{bmatrix} \xi I & I & 0 & 0 \end{bmatrix} + \cdots \\ \times \begin{bmatrix} \xi I \\ 0 \\ 0 \end{bmatrix} \mathcal{K}_{\hat{\theta}} \begin{bmatrix} \check{A}_{\theta\hat{\theta}} & -I & \check{J}_{\theta\hat{\theta}} & 0 \end{bmatrix} < 0$$
(30)

so that (30) can be written as

$$Q_{\theta\hat{\theta}\beta} + U'_{\theta\hat{\theta}}\mathcal{K}'_{\hat{\theta}}V + V'\mathcal{K}_{\hat{\theta}}U_{\theta\hat{\theta}} < 0,$$
(31)

where

$$Q_{\theta\hat{\theta}\beta} = \begin{bmatrix} -W_{\theta} & \bullet & \bullet & \bullet \\ 0 & -W_{\beta} & \bullet & \bullet \\ 0 & 0 & -\gamma^{2}I & \bullet \\ \check{C}_{\theta\hat{\theta}} & 0 & \check{D}_{\theta\hat{\theta}} & -I \end{bmatrix}, \\ U'_{\theta\hat{\theta}} = \begin{bmatrix} \check{A}'_{\theta\hat{\theta}} \\ -I \\ J'_{\theta\hat{\theta}} \\ 0 \end{bmatrix}, \quad V' = \begin{bmatrix} \xi I \\ I \\ 0 \\ 0 \end{bmatrix}.$$
(32)

Now, we pre- and post-multiply the inequality (31) by

$$\begin{bmatrix} I & \dot{A}'_{\theta\hat{\theta}} & 0 & 0\\ 0 & \tilde{J}'_{\theta\hat{\theta}} & I & 0\\ 0 & 0 & 0 & I \end{bmatrix},$$
 (33)

and its transpose, respectively, and after that, applying the Schur complement and using arguments similar to those explained at the end of the proof for Theorem 1, we obtain constraints that are equivalent to those of the BRL (11), concluding the proof. \blacksquare

C. MIXED \mathcal{H}_2 / \mathcal{H}_∞ FAULT DETECTION FILTER DESIGN FOR LPV SYSTEMS

This section provides a mixed procedure aiming to improve the FD performance by combining the results for \mathcal{H}_2 and \mathcal{H}_∞ norms introduced earlier in this section. A simple approach to obtain a mixed solution when dealing with LMI constraints is to solve both optimization problems simultaneously, for instance, we can consider the following two optimization statements

(i) For a fixed weighting scalar $0 < \nu < 1$, we solve the constrains assuming an objective function of the form

$$g(\mu, \gamma) = \inf\{\nu \mu + (1 - \nu)\gamma\},\tag{34}$$

where $||G_{\text{aug}}||_2^2 < \mu$ and $||G_{\text{aug}}||_{\infty}^2 < \gamma$.

(ii) Given one of the upper bounds of the \mathcal{H}_2 or \mathcal{H}_∞ norms, $\mu > 0$ or $\gamma > 0$, respectively, we solve the constraints to minimize the other upper bound.

Before we introduce the main result of this section, consider the following set of variables

$$\begin{split} \psi &= \left\{ W_{11\theta} > 0, \ W_{12\theta}, \ W_{22\theta} > 0, \ X_{1\hat{\theta}}, \ X_{2\hat{\theta}}, \ Y_{11\theta}, \right. \\ &\times K_{1\hat{\theta}}, \ Y_{12\theta}, \ Y_{22\theta}, K_{2\hat{\theta}}, \ M_{\theta} > 0, \ \bar{X}_{\hat{\theta}} = \bar{K}_{\hat{\theta}} > 0, \end{split}$$

	$W_{11\theta} + \xi(\operatorname{Her}(K_{1\hat{\theta}}A_{\theta} + \Omega_{\hat{\theta}}C_{\theta}))$	•	•	•	•	•	•	• -	1	
_	$W_{12\theta}' \! + \! \xi (\nabla_{\hat{\theta}}' \! + \! K_{2\hat{\theta}} A_{\theta} \! + \! C_{\theta}' \Omega_{\hat{\theta}}')$	$-W_{22\theta}'\!+\!\xi(\operatorname{Her}(\nabla_{\hat{\theta}}))$	•	•	•	•	•	•		
	$K_{1\hat{ heta}}A_{ heta} + \Omega_{\hat{ heta}}'C_{ heta} + \xi K_{1\hat{ heta}}'$	$ abla_{\hat{ heta}} + \xi K'_{2\hat{ heta}}$	$W'_{11\beta} - \text{Her}(K_{1\hat{\theta}})$	•	•	•	•	•		
	$K_{2\hat{ heta}}A_{ heta}\!+\!\Omega_{\hat{ heta}}'C_{ heta}\!+\!\xiar{K}_{\hat{ heta}}'$	$ abla_{\hat{ heta}}\!+\!\xiar{K}_{\hat{ heta}}'$	$W_{12\beta}^{\prime}\!-\!ar{K}_{\hat{ heta}}^{\prime}\!-\!K_{2\hat{ heta}}$	$W'_{22\beta} - \text{Her}(\bar{K}_{\hat{\theta}})$	•	•	•	•		
	$\xi(K_{1\hat{\theta}}B_{\theta}+\Gamma_{\hat{\theta}})'$	$\xi(K_{2\hat\theta}B_\theta\!+\!\Gamma_{\hat\theta})'$	$B_{\theta}'K_{1\hat{\theta}}'y + \Gamma_{\hat{\theta}}'$	$B_{\theta}'K_{2\hat{\theta}}'\!+\!\Gamma_{\hat{\theta}}'$	$-\gamma^2 I$	•	•	•	< 0	(27)
	$\xi(K_{1\hat{\theta}}J_{\theta} + \Omega_{\hat{\theta}}D_{d\theta})'$	$\xi(K_{2\hat\theta}J_\theta\!+\!\Omega_{\hat\theta}D_{d\theta})'$	$J_{\theta}'K_{1\hat{\theta}}'{+}D_{d\theta}'\Omega_{\hat{\theta}}'$	$J_{\theta}'K_{2\hat{\theta}}'\!+\!D_{d\theta}'\Omega_{\hat{\theta}}'$	0	$-\gamma^2 I$	•	•		
	$\xi(K_{1\hat{\theta}}F_{\theta} + \Omega_{\hat{\theta}}D_{f\theta})'$	$\xi(K_{2\hat{\theta}}F_{\theta}\!+\!\Omega_{\hat{\theta}}D_{f\theta})'$	$F_{\theta}'K_{1\hat{\theta}}'+D_{f\theta}'\Omega_{\hat{\theta}}'$	$F'_{\theta}K'_{2\hat{\theta}} {+} D'_{f\theta}\Omega'_{\hat{\theta}}$	0	0	$-\gamma^2 I$	•		
_	$\mathfrak{D}_{n\hat{ heta}}C_{ heta}$	$\mathfrak{C}_{n\hat{ heta}}$	0	0	0	$\mathfrak{D}_{n\hat{ heta}} D_{d heta}$	$\mathfrak{D}_{n\hat{\theta}}D_{f\theta}-I$	-I]	

$$\times \nabla_{\hat{\theta}}, \ \Omega_{\hat{\theta}}, \ \Gamma_{\hat{\theta}}, \ \mathfrak{C}_{\eta\hat{\theta}}, \ \mathfrak{D}_{\eta\hat{\theta}} \bigg\} \cup \zeta_{1}, \tag{35}$$

where ζ_1 denotes the set containing μ and γ .

The following theorem provides a sufficient condition for the FDF design for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem.

Theorem 3: If, for given upper bounds $\mu > 0$ and $\gamma > 0$, there exists ψ as in (35) such that the inequalities (27), and (19)-(21) hold for all feasible θ , β , $\hat{\theta}$, then a suitable LPV FDF as in (13) which solves simultaneously the \mathcal{H}_{∞} and \mathcal{H}_{2} fault detection problems (17) and (18) is given by $\mathfrak{A}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1}\nabla_{\hat{\theta}}$, $\mathfrak{B}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1}\Omega_{\hat{\theta}}$, $\mathfrak{M}_{\eta\hat{\theta}} = \bar{X}_{\hat{\theta}}^{-1}\Gamma_{\hat{\theta}}$, $\mathfrak{C}_{\eta\hat{\theta}} = \mathfrak{C}_{\eta\hat{\theta}}$, and $\mathfrak{D}_{\eta\hat{\theta}} = \mathfrak{D}_{\eta\hat{\theta}}$. Alternatively, one can consider both or one of the upper bounds μ and γ as variables and solve the optimization problems in ψ according to (i) or (ii) above.

Proof: The proof follows directly from the proofs of Theorems 1 and 2.

Remark 2: Note that Theorems 1, 2 and 3 are LMI conditions that provide the system performance regarding the $\mathcal{H}_{\infty}, \mathcal{H}_{2}, \text{ and } \mathcal{H}_{2}/\mathcal{H}_{\infty}, \text{ norms respectively. Observe that the }$ LMI conditions in (19), (20), (21), and (27), are defined as an infinite dimensional optimization problem that must be solved, meaning that the LMI constraints depends on θ and $\hat{\theta}$. By using the change of variables presented in sub-section III-D and explained at the beginning of this section, we can rewrite the LMI optimization problems in terms of the new multi-simplex parameter $\tilde{\alpha} \in \Lambda_{m,2} \times \Lambda_{m,2}$, meaning that the LMI constraints now depends solely on $\tilde{\alpha}$. This type of polynomial relaxations allows the problem to be rewritten as an analysis of the positivity of homogeneous polynomial matrices (see Definition II-A), which is the procedure made by ROLMIP [38] and YALMIP [45]. In order to solve the problem that is given in terms of positivity of homogeneous polynomial matrices, which is a finite-dimensional problem, we can use a semi-definite programming solver, such as SeDuMi [52] or Mosek [53].

Remark 3: Note that in Theorems 1, 2 and 3, the variables that define whether the FDF is in the robust form or in the affine form, are $\nabla_{\hat{\theta}}$, $\Omega_{\hat{\theta}}$, $\Gamma_{\hat{\theta}}$, $\mathfrak{C}_{\eta\hat{\theta}}$, $\mathfrak{D}_{\eta\hat{\theta}}$, and $\bar{X}_{\hat{\theta}}$. If the degree of those homogeneous polynomial matrices are set to be 0, the FDF designed will be robust, meaning that the FDF obtained will be parameter-independent. For a homogeneous polynomial matrices with degree equal to 1, the FDF obtained will be in the affine form. Observe that a higher degree of the homogeneous polynomial can be set, leading to the design of FDF with higher degree. It is important to discuss that it is also allowed to change the degree of the other variables in Theorems 1, 2 and 3, such as $Y_{11\theta}$, $Y_{12\theta}$, $Y_{22\theta}$, M_{θ} , $W_{11\theta}$, $W_{12\theta}$, and $W_{22\theta}$, with this choice mainly affecting the level of conservatism and the computational effort.

Remark 4: Another point that should be highlighted is that, as the number of LMI lines increases, the computational effort required to solve the optimization problem also increases. For that reason, the optimization problem for Theorem 3 requires more computational resources since it simultaneously deals with the LMI constraints from Theorem 1 and 2, which more than doubles the number of LMI lines.



FIGURE 2. Illustration of the coupled tank model scheme, where h_1 represents the level for the first tank, h_2 represents the level for the second tank, and u_1 denotes the input on the first tank.

TABLE 1. Numerical parameters of the coupled tank model.

Variable Unit		Description	Value
g	m/s ²	Gravitational acceleration	9.8
A_{cs}	m^2	Tank cross section area	0.40
ω	m^2	Interconnection pipe cross section area	0.01
h_0^1	m^2	height initial condition for the first tank	0.25
h_{0}^{2}	m^2	height initial condition for the second tank	0.22

This effect can be seen in the upper bound analysis of the numerical example presented in the next section.

V. NUMERICAL EXAMPLE

In this section, we present a numerical example to illustrate the applicability of the FD techniques presented in Section IV. We first describe the simulation setup, and later, we present the results obtained applying Theorems 1, 2 and 3.

The physical system we use in this example is the coupled tank system as shown in Fig. 2.

We can describe the dynamical equations of this system that give the relationships between the input and output flows of each tank. A complete description of the modeling process is presented in [54]. Considering $h_1(k)$ and $h_2(k)$ as the water levels in tank 1 and 2, respectively, we set x(k) = $[h_1(k) h_2(k)]'$ as the state vector for the state-space representation of the system with the system's dynamic matrix *A* be given by

$$A = \frac{1}{A_{cs}} \begin{bmatrix} \frac{\omega g}{\sqrt{2gh_0^1}} - \frac{\omega g}{\sqrt{2gh_0^1} - h_0^2} & -\frac{\omega g}{\sqrt{2gh_0^1} - h_0^2} \\ \frac{\omega g}{\sqrt{2gh_0^1} - h_0^2} + \theta(k) & \frac{\omega g}{\sqrt{2gh_0^2}} - \frac{\omega g}{\sqrt{2gh_0^1} - h_0^2} \end{bmatrix}.$$
(36)

The parameter values as given in [54] are presented in Table 1.

We also assume that the linearization point is $h_1 = 25$ cm and $h_2 = 10$ cm, where these values are arbitrarily chosen. We consider that the time-varying parameter $\theta(k)$ in the tank model (36) represents the uncertainty in the first tank, modelling a variation on the valve discharge coefficient between tank 1 and tank 2. Therefore, the state-space matrices in the affine form are given by

$$A_1 = \begin{bmatrix} -0.0239 & -0.0127 \\ 0.0127 & -0.0285 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B_{1,2} = \begin{bmatrix} 0.71 \\ 0 \end{bmatrix},$$

$$J_{1,2} = \begin{bmatrix} 0.0071 \\ 0 \end{bmatrix}, \quad F_{1,2} = \begin{bmatrix} 0.71 \\ 0 \end{bmatrix}, \quad C_{1,2} = I^{2 \times 2}, \\ D_{d_{1,2}} = \begin{bmatrix} 0.001 \\ 0.001 \end{bmatrix}, \quad D_{f_{1,2}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad K = \begin{bmatrix} -1.03 & -0.33 \end{bmatrix}, \\ |\theta(k)| \le t_i = 0.03.$$
(37)

Note that, $F_{1,2}$ has the same structure of the control input matrix B, representing an abnormal input in the first tank. Additionally, note that the matrix $D_{f1,2}$ is null since we do not consider sensor fault in the simulation. The matrices presented in (37) represent the model of the coupled tanks in the continuous-time domain. Since we need the system and its polytope described in the discrete-time domain, we use Taylor series expansion, as described in [55]. The procedure allows us to set the number of terms in the Taylor series, which increases the reliability of the discrete representation with the potential of increasing the computational cost. The sampling time was taken as T = 5ms, and the number of terms in the Taylor series was 2.

For the parameter estimation, we need *apriori* information on the range of $\sigma(k)$. Some methods have been presented in the literature to obtain it as, for example, through Monte Carlo simulations, as presented in [33]. The bound value of $\sigma(k)$ can be set using some previous knowledge about the system associated with the estimation process. It is possible to find the value of d_i analytically or via simulation. Since obtaining the range of $\sigma(k)$ is not the main focus of the present paper, for the numerical simulation we set the range of $\sigma(k)$ as $|\sigma(k)| \leq d_i = 0.01$. To obtain the estimated parameter $\hat{\theta}(k)$, we implemented the Recursive Least Square (RLS) algorithm [56], [57]. We note that any other adaptive filter algorithm can also be implemented to obtain $\hat{\theta}$, such as the \mathcal{H}_{∞} adaptive filter algorithm or the Least Mean Square-based algorithm.

Remark 5: Note that the level of reliability in the estimation process is directly connected to the value of $\sigma(k)$. This happens since the value of the obtained $\sigma(k)$ changes depending on the reliability of the estimation process: the less reliable the estimation process is, the higher the value of $\sigma(k)$ will be. Another important discussion is that the increase of bound for $\sigma(k)$ will lead to the increase of conservatism in the optimization problem since the LMI constraints must guarantee the stability and performance within the grey area in Fig. 3(a). On the other hand, assuming that $d_i = 0$, the constraints will guarantee stability and performance solely on the black line in Fig. 3(a). It is important to recall that the only information necessary during the design process are the boundary values of the parameter θ and additive noise σ .

The imprecise parameter $\hat{\theta}(k) = \theta(k) + \sigma(k)$ behavior is presented in Fig. 3(a), which we assume to be the representation of an imprecision in the valve that couples the first tank with the second one. Fig.3(b) shows all three fault signals that the system is subjected to, separately. The dashed blue line in Fig.3(b) represents an abrupt abnormal increase of 1% on the input u_1 that starts at k = 100 (Fault 1). The green line in Fig.3(b) corresponds to an oscillatory fault due to an oscillation on the input u_1 (Fault 2). The magenta line in Fig.3(b) is associated to an incipient fault so that the input



(a) Behavior for the Linear-Parameter variable $\theta(k)$ contaminated by the imprecision $\sigma(k)$.



(b) The blue dashed line represents the abrupt fault, the green curve denotes the oscillatory fault, the magenta dotted line represents an incipient fault.

FIGURE 3. The imprecise scheduled parameter $\hat{\theta}$, and the fault signals.



FIGURE 4. The behavior of the upper bounds μ and γ for Theorems 1 and 2 in function of the scalar ξ .

 u_1 smoothly increases by 1% (Fault 3). The main purpose of the setup here is to cover a wide range of fault types in order to verify whether the proposed approach is able to detect these faults before any severe problem may occur. Lastly, we assume that the system is subjected to a white noise signal.

In the following, we present the simulation results in two distinct parts: the upper bound behavior analysis and temporal analysis. We analyze the obtained values for the upper bounds μ and γ when performing a search in the scalar $\xi \in]-11$ [with 100 steps with the same length.

A. UPPER BOUND ANALYSIS

Fig. 4 presents the upper bound analysis considering two different structures for the FDF, the robust and the affine forms. From Fig. 4 the first plot represents the upper bound



FIGURE 5. The behavior of the upper bound μ for Theorem 3 in function of the scalar ξ , using the fixed value of $\gamma = 0.01$.







FIGURE 6. System states behavior when subjected to the faults separately.

value of μ obtained via Theorem 1, using the ξ scalar term. Notice that there is a visible difference in the values for the robust form, but the difference is not as evident as that for the affine form. The effectiveness of the scalar search is greater for the robust case, which is expected since it has a higher level of conservatism. The same statements can be made for the second plot, which presents the results obtained using Theorem 2. Another information that can be extracted from Fig.4 is that the affine form is indeed a more relaxed solution due to the lower value of the upper bounds throughout the scalar search.

From Fig. 5, it can be observed that the upper bound behavior has a similar behavior to the results shown in Fig. 5 with pronounced curves. The presence of tortuous curves around the origin in Fig. 5 are mainly due to the fact that the optimization problem in Theorem 3 combines the LMI constraints from Theorems 1 and 2. Hence, Theorem 3 has a higher number of LMI lines, consequently increasing the optimization problem's conservatism. As previously discussed, the scalar search has a higher impact on the optimization problem performance as the size of the LMI rises.

Fig. 4 and Fig. 5 present the results for the upper bounds behavior (μ or γ) after performing the scalar search for ξ related to Theorems 1, 2, and 3. In what follows, in order to derive the FDF, the value of ξ was fixed to $\xi = 0.2$.

Regarding the results obtained for the \mathcal{H}_2 norm using Theorem 1 the FDF is given by

$$\begin{split} \mathfrak{A}_{\eta_{\mathrm{aff1}}} &= \begin{bmatrix} -1.02 & -3.75\\ 0.01 & 0.05 \end{bmatrix}, \quad \mathfrak{A}_{\eta_{\mathrm{aff2}}} = \begin{bmatrix} -0.00 & -0.03\\ 0.00 & -0.01 \end{bmatrix}, \\ \mathfrak{B}_{\eta_{\mathrm{aff1}}} &= \begin{bmatrix} -0.99 & -3.99\\ -0.00 & 0.08 \end{bmatrix}, \quad \mathfrak{B}_{\eta_{\mathrm{aff2}}} = \begin{bmatrix} 0.00 & 3.97\\ -0.00 & -0.10 \end{bmatrix}, \\ \mathfrak{M}_{\eta_{\mathrm{aff1}}} &= \begin{bmatrix} -0.71\\ -0.00 \end{bmatrix}, \quad \mathfrak{M}_{\eta_{\mathrm{aff2}}} = \begin{bmatrix} -0.70\\ -0.00 \end{bmatrix}, \\ \mathfrak{C}_{\eta_{\mathrm{aff1}}} &= \begin{bmatrix} 0.49 & 1.82 \end{bmatrix}, \quad \mathfrak{C}_{\eta_{\mathrm{aff2}}} = \begin{bmatrix} 0.00 & 0.04 \end{bmatrix}, \\ \mathfrak{D}_{\eta_{\mathrm{aff1}}} &= \begin{bmatrix} 0.50 & 1.95 \end{bmatrix}, \quad \mathfrak{D}_{\eta_{\mathrm{aff2}}} = \begin{bmatrix} -0.49 & -1.90 \end{bmatrix}. \end{split}$$

From Theorem 2, related to the \mathcal{H}_∞ norm, the FDF is given by

$$\begin{split} \mathfrak{A}_{\eta_{\text{aff1}}} &= \begin{bmatrix} -0.89 & -70.49 \\ 0.01 & 0.87 \end{bmatrix}, \quad \mathfrak{A}_{\eta_{\text{aff2}}} = \begin{bmatrix} 0.12 & 4.89 \\ -0.00 & -0.06 \end{bmatrix}, \\ \mathfrak{B}_{\eta_{\text{aff1}}} &= \begin{bmatrix} -0.93 & -70.16 \\ -0.00 & 0.89 \end{bmatrix}, \quad \mathfrak{B}_{\eta_{\text{aff2}}} = \begin{bmatrix} 0.09 & 71.08 \\ -0.00 & -0.91 \end{bmatrix}, \\ \mathfrak{M}_{\eta_{\text{aff1}}} &= \begin{bmatrix} 0.75 \\ 0.00 \end{bmatrix}, \quad \mathfrak{M}_{\eta_{\text{aff2}}} = \begin{bmatrix} -0.71 \\ 0.00 \end{bmatrix}, \\ \mathfrak{C}_{\eta_{\text{aff1}}} &= \begin{bmatrix} 0.49 & 38.96 \end{bmatrix}, \quad \mathfrak{C}_{\eta_{\text{aff2}}} = \begin{bmatrix} 0.00 & 0.04 \end{bmatrix}, \\ \mathfrak{D}_{\eta_{\text{aff1}}} &= \begin{bmatrix} 0.49 & 38.96 \end{bmatrix}, \quad \mathfrak{D}_{\eta_{\text{aff2}}} = \begin{bmatrix} -0.49 & -38.91 \end{bmatrix}. \end{split}$$

Regarding the mixed $\mathcal{H}_2 / \mathcal{H}_\infty$ results, the affine filter obtained using Theorem 3 is given by

$$\begin{split} \mathfrak{A}_{\eta_{aff1}} &= \begin{bmatrix} -1.02 & -4.23 \\ 0.01 & -0.00 \end{bmatrix}, \quad \mathfrak{A}_{\eta_{aff1}} &= \begin{bmatrix} -0.00 & 1.13 \\ 0.00 & -0.01 \end{bmatrix}, \\ \mathfrak{B}_{\eta_{aff1}} &= \begin{bmatrix} -1.00 & -3.81 \\ 0.00 & 0.01 \end{bmatrix}, \quad \mathfrak{B}_{\eta_{aff2}} &= \begin{bmatrix} -0.00 & 5.04 \\ 0.00 & -0.02 \end{bmatrix}, \\ \mathfrak{M}_{\eta_{aff1}} &= \begin{bmatrix} -0.71 \\ 0.00 \end{bmatrix}, \quad \mathfrak{M}_{\eta_{aff2}} &= \begin{bmatrix} -0.70 \\ -0.00 \end{bmatrix}, \\ \mathfrak{C}_{\eta_{aff1}} &= \begin{bmatrix} 0.49 & 32.34 \end{bmatrix}, \quad \mathfrak{C}_{\eta_{aff2}} &= \begin{bmatrix} 0.00 & -16.76 \end{bmatrix}, \\ \mathfrak{D}_{\eta_{aff1}} &= \begin{bmatrix} 0.49 & 32.25 \end{bmatrix}, \quad \mathfrak{D}_{\eta_{aff2}} &= \begin{bmatrix} -0.50 & -49.21 \end{bmatrix}. \end{split}$$

1) EVALUATION FUNCTION

In a fault detection process, the next step after generating the residue signal is to evaluate the residue signal to properly detect the fault. The evaluation process uses an evaluation function EVAL(k), and a threshold TH. We consider that a fault occurs if the EVAL(k) value surpasses the threshold TH. As in [3], [58], we define EVAL(k) as

$$EVAL(k) \triangleq \sqrt{\sum_{i=k-L}^{k} r(i)'r(i)},$$
(38)

where L denotes the evaluation window. Note that the proper choice of L affects the FD performance, due to the fact that a small L may not detect the fault since the evaluation signal might not have enough time to surpass the threshold. On the opposite side, if L is too large, it may lead to the occurrence of false alarms. For the numerical simulations,



(a) Residue signal for FDF designed via Theorem 1 subjected to Fault 1.



(c) Residue signal for FDF designed via Theorem 3 subjected to Fault 1.



(e) Residue signal for FDF designed via Theorem 1 subjected to Fault 2.



(g) Residue signal for FDF designed via Theorem 3 subjected to Fault 2.



(i) Residue signal for FDF designed via Theorem 1 subjected to Fault 3.

FIGURE 7. The residue signal obtained for all FDF designs and all fault types.



(b) Residue signal for FDF designed via Theorem 2 subjected to Fault 1.



(d) Residue signal for FDF designed via [61] subjected to Fault 1.



(f) Residue signal for FDF designed via Theorem 2 subjected to Fault 2.



(h) Residue signal for FDF designed via [61] subjected to Fault 2.



(j) Residue signal for FDF designed via Theorem 2 subjected to Fault 3.



(k) Residue signal for FDF designed via Theorem 3 subjected to Fault 3.



(m) Residue signal for FDF designed via Theorem 1 for the nominal conditions without fault.



(o) Residue signal for FDF designed via Theorem 3 for the nominal conditions without fault.



(l) Residue signal for FDF designed via [61] subjected to Fault 3.



(n) Residue signal for FDF designed via Theorem 2 for the nominal conditions without fault.



(p) Residue signal for FDF designed via [61] for the nominal conditions without fault.



we set TH = 1, and L = 250. There are several ways to define the evaluation function and the threshold. However, since this is not the focus of this work, we refer interested readers to the papers [3], [58] and references therein for a thorough discussion on the topic.

B. TEMPORAL SIMULATION

In this section we provide the residue signal and the evaluation function obtained when the system is subjected to Fault 1, Fault 2, Fault 3, and Faultless. We also compare our results with those by using [59, Theorem 1]. Before that, we present in Fig. 6 the states of the system for all situations.

In Fig.6 the dashed blue line represents the state when subjected to Fault 1, the green line denotes the state behavior when Fault 2 is implied to the system, and the magenta line represents the state when Fault 3 is inflicted in the system.

The dotted black line represents the system in its nominal condition, meaning there is no fault occurrence. Note that all three fault signals cause an error on the state near 10% of the nominal value.

Now the results obtained using the FDF designed with Theorem 1, 2 and 3 are presented, as well as a comparison with the results obtained using [59, Theorem 1]. Firstly, we discuss the results derived from the residue signal and, after that, those from the evaluation function.

From the sub-plots in Fig. 7, it can be seen that the FDF designed using Theorems 1, 2 and 3 respond only when the fault signal was present. Note that the FDF designed using Theorem 3 gives a higher value during the simulation for all three faults. Figs.7 shows that the results obtained using Theorem 3 consistently yield a residue signal with a lower standard deviation and higher peak value. Inspecting Figs. 7 we can state that the residue signal when there are



(a) Evaluation Function for FDF designed via Theorem 1 subjected to Fault 1.



(c) Evaluation Function for FDF designed via Theorem 3 subjected to Fault 1.



(e) Evaluation Function for FDF designed via Theorem 1 subjected to Fault 2.



(g) Evaluation Function for FDF designed via Theorem 3 subjected to Fault 2.



(i) Evaluation Function for FDF designed via Theorem 1 subjected to Fault 3.

FIGURE 8. The evaluation function obtained for all affine FDF designs and all fault types.



(b) Evaluation Function for FDF designed via Theorem 2 subjected to Fault 1.



(d) Evaluation Function for FDF designed via [61] subjected to Fault 1.



(f) Evaluation Function for FDF designed via Theorem 2 subjected to Fault 2.



(h) Evaluation Function for FDF designed via [61] subjected to Fault 2.



(j) Evaluation Function for FDF designed via Theorem 2 subjected to Fault 3.



(k) Evaluation Function for FDF designed via Theorem 3 subjected to Fault 3.



(m) Evaluation Function for FDF designed via Theorem 1 for the nominal conditions without fault.



(o) Evaluation Function for FDF designed via Theorem 3 for the nominal conditions without fault.

FIGURE 8. (Continued.) The evaluation function obtained for all affine FDF designs and all fault types.

TABLE 2. Detection interval obtained for the affine FDF.

Design Method	Fault 1 (k)	Fault 2 (k)	Fault 3 (k)
Theorem 1	$[123\ 133]$	[120 129]	$[279\ 287]$
Theorem 2	$[110\ 116]$	[108 115]	$[245\ 253]$
Theorem 3	$[109\ 116]$	$[105\ 114]$	$[242\ 252]$
[61, Theorem 1]	$[132\ 147]$	$[128\ 141]$	$[291\ 300]$

no faults is close to zero in all simulations, as expected. This characteristic provides a higher value of the evaluation function leading to a lower occurrence of false alarms. Next, we present the evaluation function obtained from the residue signals presented in Fig. 7.

The results for each graphic in Fig. 8 are presented in Table 2. Observe that the FDF designed using Theorem 1, 2 and 3 detected all three faults. Note that [59, Theorem 1] in some cases did not detect Fault 3, which is an incipient



(1) Evaluation Function for FDF designed via [61] subjected to Fault 3.



(n) Evaluation Function for FDF designed via Theorem 2 for the nominal conditions without fault.



(p) Evaluation Function for FDF designed via [61] for the nominal conditions without fault.

fault. The main advantage of our approach is the polynomial relaxation made during the design processes, which allows us to obtain a better solution in the optimization process.

In Fig.9 it is possible to observe the performance for all FDF side-by-side, and it is noticeable that the FDF designed using Theorem 3 presented a higher performance, as intended. It can also be noticed that for Fault 1, after k = 200, the evaluation function changes its behavior by becoming flattered, which is as intended since Fault 1 loses its intensity at this point in the simulation. This same behavior is also apparent for the oscillatory fault. From the obtained results, we can say that the proposed approach seems to be a viable solution for the Fault Detection problem. It is important to point out that the approaches presented herein apply to any system that is possible to be described as an LPV system.



FIGURE 9. Comparison of the evaluation function obtained with the affine FDF for all fault cases.

VI. CONCLUSION

We have presented three design methods of gain-scheduling fault detection filters under the linear parameter varying framework for solving the fault detection and isolation problem. These approaches are developed based on two different performance indexes, the \mathcal{H}_2 -norm and the \mathcal{H}_{∞} -norm, which can be combined to provide a mixed $\mathcal{H}_2/\mathcal{H}_{\infty}$ -norm approach. For all three approaches, we take into account

the availability of real-time estimation of the time-varying parameters that can directly be used in the design of the FDF. A distinguishing feature of our approach is that we assume an unreliable estimation process. The main advantage of implementing such proposed solutions is that it lessens the burden on the on-line estimation process, allowing us to use less sophisticated procedures without losing reliability on the FD process. Simulation results show the efficacy of the proposed approaches for solving the fault detection problem in an illustrative example.

Alongside this line of research, there are some possible ways to improve the results presented here, for instance, by using the sensitivity index H_{-} in the design of an FDF or considering a non-homogeneous Markov chain for the modeling of the noise affecting the measurement of the time-varying parameter.

REFERENCES

- M. Rodrigues, D. Theilliol, S. Aberkane, and D. Sauter, "Fault tolerant control design for polytopic LPV systems," *Int. J. Appl. Math. Comput. Sci.*, vol. 17, no. 1, pp. 27–37, Mar. 2007.
- [2] Z. Chen, "Data-driven fault detection for industrial processes," in *Journal of Process Control*, vol. 1. Wiesbaden, Germany: Springer-Vieweg, 2017.
- [3] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems* (The International Series on Asian Studies in Computer and Information Science), vol. 3. New York, NY, USA: Springer, 2012. [Online]. Available: https://books.google.com.br/books?id=_wvrBwAAQBAJ
- [4] Z. Zhang, "Comparison of data-driven and model based methodologies of wind turbine fault detection with scada data," EWEA, Brussels, Belgium, Tech. Rep., Mar. 2014.
- [5] P. Freeman, R. Pandita, N. Srivastava, and G. J. Balas, "Model-based and data-driven fault detection performance for a small UAV," *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 4, pp. 1300–1309, Aug. 2013.
- [6] R. Isermann, R. Schwarz, and S. Stolzl, "Fault-tolerant drive-by-wire systems," *IEEE Control Syst.*, vol. 22, no. 5, pp. 64–81, Oct. 2002.
- [7] M. Witczak, Fault Diagnosis and Fault-Tolerant Control Strategies for Non-Linear Systems (Lecture Notes in Electrical Engineering), vol. 266. Springer, 2014, pp. 375–392.
- [8] H. Noura, D. Theilliol, J.-C. Ponsart, and A. Chamseddine, Fault-Tolerant Control Systems: Design and Practical Applications (Advances in Industrial Control). London, U.K.: Springer, 2009. [Online]. Available: https://books.google.com.br/books?id=_LDI6-9t6UoC
- [9] R. J. Patton, P. M. Frank, and R. N. Clark, Issues of Fault Diagnosis for Dynamic Systems. Springer, 2013.
- [10] M. Rodrigues, M. Sahnoun, D. Theilliol, and J.-C. Ponsart, "Sensor fault detection and isolation filter for polytopic LPV systems: A winding machine application," *J. Process Control*, vol. 23, no. 6, pp. 805–816, 2013.
- [11] C. Briat, O. Sename, and J.-F. Lafay, "Design of LPV observers for LPV time-delay systems: An algebraic approach," *Int. J. Control*, vol. 84, no. 9, pp. 1533–1542, Sep. 2011.
- [12] C. Hoffmann and H. Werner, "A survey of linear parameter-varying control applications validated by experiments or high-fidelity simulations," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 2, pp. 416–433, Mar. 2015.
- [13] P. N. Kvieska, M. Ait-Ahmed, and G. Lebret, "LPV systems: Theoretical results for gain scheduling," in *Proc. Eur. Control Conf. (ECC)*, Aug. 2009, pp. 3166–3171.
- [14] J. Mohammadpour and C. W. Scherer, Control of Linear Parameter Varying Systems With Applications (SpringerLink: Bücher). New York, NY, USA: Springer, 2012. [Online]. Available: https://books.google.com.br/ books?id=Gvr6yurJI9IC
- [15] P. Rosa and C. Silvestre, "Fault detection and isolation of LPV systems using set-valued observers: An application to a fixed-wing aircraft," *Control Eng. Pract.*, vol. 21, no. 3, pp. 242–252, 2013.
- [16] L. Kovács, "LPV fault detection of glucose-insulin system," in Proc. 14th Medit. Conf. Control Autom., Dec. 2006, pp. 1–5.
- [17] M. M. Morato, D. J. Regner, P. R. C. Mendes, J. E. Normey-Rico, and C. Bordons, "Fault analysis, detection and estimation for a microgrid via H₂/H_∞ LPV observers," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 823–845, Feb. 2019.

- [18] J. Tan, S. Olaru, M. Roman, F. Xu, and B. Liang, "Invariant set-based analysis of minimal detectable fault for discrete-time LPV systems with bounded uncertainties," *IEEE Access*, vol. 7, pp. 152564–152575, 2019.
- [19] A. H. Hassanabadi, M. Shafiee, and V. Puig, "UIO design for singular delayed LPV systems with application to actuator fault detection and isolation," *Int. J. Syst. Sci.*, vol. 47, no. 1, pp. 107–121, 2016.
- [20] A. N. Hanafi, M. M. Seron, and J. A. De Doná, "Fault estimation and controller compensation in lure systems by LPV-embedding," *Int. J. Control*, vol. 92, no. 8, pp. 1914–1927, Aug. 2019.
- [21] H. Hamdi, M. Rodrigues, C. Mechmeche, and N. B. Braiek, "Fault diagnosis based on sliding mode observer for LPV descriptor systems," *Asian J. Control*, vol. 21, no. 1, pp. 89–98, Jan. 2019.
- [22] F. Xu, J. Tan, Y. Wang, X. Wang, B. Liang, and B. Yuan, "Robust fault detection and set-theoretic UIO for discrete-time LPV systems with state and output equations scheduled by inexact scheduling variables," *IEEE Trans. Autom. Control*, vol. 64, no. 12, pp. 4982–4997, Dec. 2019.
- [23] R. Sakthivel, V. T. Suveetha, H. Divya, and R. Sakthivel, "Fault detection finite-time filter design for T–S fuzzy Markovian jump system with missing measurements," *Circuits, Syst., Signal Process.*, vol. 40, no. 4, pp. 1607–1634, Apr. 2021.
- [24] R. Sakthivel, V. T. Suveetha, V. Nithya, and R. Sakthivel, "Finite-time fault detection filter design for complex systems with multiple stochastic communication and distributed delays," *Chaos, Solitons Fractals*, vol. 136, Jul. 2020, Art. no. 109778.
- [25] Y. Liu, A. Arunkumar, R. Sakthivel, V. Nithya, and F. Alsaadi, "Finitetime event-triggered non-fragile control and fault detection for switched networked systems with random packet losses," *J. Franklin Inst.*, vol. 357, no. 16, pp. 11394–11420, Nov. 2020.
- [26] M. Shen, Y. Ma, J. H. Park, and Q.-G. Wang, "Fuzzy tracking control for Markov jump systems with mismatched faults by iterative proportionalintegral observers," *IEEE Trans. Fuzzy Syst.*, early access, Dec. 1, 2020, doi: 10.1109/TFUZZ.2020.3041589.
- [27] L. Carvalho, A. De Oliveira, and O. Costa, "H₂/H_∞ simultaneous fault detection and control for Markov jump linear systems with partial observation," *IEEE Access*, vol. 8, pp. 11979–11990, 2020.
- [28] L. P. Carvalho, T. E. Rosa, B. Jayawardhana, and O. L. V. Costa, "Fault accommodation controller under Markovian jump linear systems with asynchronous modes," *Int. J. Robust Nonlinear Control*, vol. 30, no. 18, pp. 8503–8520, Dec. 2020.
- [29] G. Zhuang, S.-F. Su, J. Xia, and W. Sun, "HMM-based asynchronous H_∞ filtering for fuzzy singular Markovian switching systems with retarded time-varying delays," *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1189–1203, Mar. 2021.
- [30] J. De Caigny, J. F. Camino, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Gain-scheduled H₂ and H_∞ control of discrete-time polytopic time-varying systems," *IET Control Theory Appl.*, vol. 4, no. 3, pp. 362–380, 2010.
- [31] J. De Caigny, J. F. Camino, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Gain-scheduled dynamic output feedback control for discretetime LPV systems," *Int. J. Robust Nonlinear Control*, vol. 22, no. 5, pp. 535–558, 2012.
- [32] T. E. Rosa, C. F. Morais, and R. C. L. F. Oliveira, "New robust LMI synthesis conditions for mixed H2/H_∞ gain-scheduled reduced-order DOF control of discrete-time LPV systems," *Int. J. Robust Nonlinear Control*, vol. 28, no. 18, pp. 6122–6145, Dec. 2018.
- [33] J. M. Palma, C. F. Morais, and R. C. L. F. Oliveira, "*H*₂ gain-scheduled filtering for discrete-time LPV systems using estimated time-varying parameters," in *Proc. Annu. Amer. Control Conf.*, Jun. 2018, pp. 4367–4372.
- [34] L. Frezzatto, M. C. de Oliveira, R. C. L. F. Oliveira, and P. L. D. Peres, "Robust H_∞ filter design with past output measurements for uncertain discrete-time systems," *Automatica*, vol. 71, pp. 151–158, Sep. 2016.
- [35] A. P. Pandey and M. C. de Oliveira, "A new discrete-time stabilizability condition for linear parameter-varying systems," *Automatica*, vol. 79, pp. 214–217, May 2017.
- [36] A. Sadeghzadeh, "Gain-scheduled continuous-time control using polytope-bounded inexact scheduling parameters," *Int. J. Robust Nonlinear Control*, vol. 28, no. 17, pp. 5557–5574, Nov. 2018.
- [37] J. M. Palma, C. F. Morais, and R. C. L. F. Oliveira, "H₂ control and filtering of discrete-time LPV systems exploring statistical information of the timevarying parameters," *J. Franklin Inst.*, vol. 357, no. 6, pp. 3835–3864, Apr. 2020.
- [38] C. M. Agulhari, A. Felipe, R. C. L. F. Oliveira, and P. L. D. Peres, "Algorithm 998: The robust LMI parser—A toolbox to construct LMI conditions for uncertain systems," *ACM Trans. Math. Softw.*, vol. 45, pp. 36:1–36:25, Aug. 2019. [Online]. Available: http://rolmip.github.io

- [39] K. A. Barbosa, C. E. de Souza, and D. Coutinho, "Admissibility analysis of discrete linear time-varying descriptor systems," *Automatica*, vol. 91, pp. 136–143, May 2018.
- [40] B. Zhang, S. Xu, Q. Ma, and Z. Zhang, "Output-feedback stabilization of singular LPV systems subject to inexact scheduling parameters," *Automatica*, vol. 104, pp. 1–7, Jun. 2019.
- [41] M. F. Amorim, A. P. C. Gonçalves, and M. Souza, "Optimal H₂ outputfeedback control of sampled systems," in *Proc. 4th IFAC Symp. Telematics Appl.*, Porto Alegre, RS, Brasil, vol. 49, Nov. 2016, pp. 126–131.
- [42] M. Sato and D. Peaucelle, "Gain-scheduled output-feedback controllers using inexact scheduling parameters for continuous-time LPV systems," *Automatica*, vol. 49, no. 4, pp. 1019–1025, 2013.
- [43] P. Apkarian and P. Gahinet, ⁴A convex characterization of gain-scheduled *H*_∞ controllers," *IEEE Trans. Autom. Control*, vol. 40, no. 5, pp. 853–864, May 1995.
- [44] P. Apkarian and R. J. Adams, "Advanced gain-scheduling techniques for uncertain systems," in *Proc. Amer. Control Conf.*, 1997, pp. 209–228.
- [45] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MAT-LAB," in *Proc. IEEE Int. Conf. Robot. Autom.*, Sep. 2004, pp. 284–289.
- [46] K. A. Barbosa, C. E. de Souza, and A. Trofino, "Robust H_∞/H₂ filtering for discrete-time uncertain linear systems using parameterdependent Lyapunov functions," in *Proc. Amer. Control Conf.*, vol. 4, 2002, pp. 3224–3229.
- [47] C. E. de Souza, K. A. Barbosa, and A. T. Neto, "Robust H_{∞} filtering for discrete-time linear systems with uncertain time-varying parameters," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2110–2118, Jun. 2006.
- [48] R. Isermann, Fault-Diagnosis Systems: An Introduction From Fault Detection to Fault Tolerance. Berlin, Germany: Springer, 2005. [Online]. Available: https://books.google.com.br/books?id=6yUfoZhGMY0C
- [49] M. J. Lacerda, E. S. Tognetti, R. C. L. F. Oliveira, and P. L. D. Peres, "A new approach to handle additive and multiplicative uncertainties in the measurement for LPV filtering," *Int. J. Syst. Sci.*, vol. 47, no. 5, pp. 1042–1053, Apr. 2016.
- [50] C. Briat, *Linear Parameter-Varying and Time-Delay Systems* (Advances in Delays and Dynamics). Berlin, Germany: Springer, 2015.
- [51] R. C. L. F. Oliveira, P.-A. Bliman, and P. L. D. Peres, "Robust LMIs with parameters in multi-simplex: Existence of solutions and applications," in *Proc. 47th IEEE Conf. Decis. Control*, Dec. 2008, pp. 2226–2231.
- [52] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Softw.*, vol. 11, nos. 1–4, pp. 625–653, 1999.
- [53] M. ApS, "MOSEK optimization toolbox for MATLAB," User's Guide Reference Manual, Version, vol. 4, pp. 1–346, Sep. 2019.
- [54] Feedback Instruments Ltd., FeedBack Coupled Tanks Control Experiments 33-041S (For Use With MATLAB), 1 ed. Crowborough, U.K.: Park Road, Jul. 2013, pp. 1–49.
- [55] M. F. Braga, C. F. Morais, E. S. Tognetti, R. C. L. F. Oliveira, and P. L. D. Peres, "A new procedure for discretization and state feedback control of uncertain linear systems," in *Proc. 52nd IEEE Conf. Decis. Control*, Dec. 2013, pp. 6397–6402.
- [56] S. R. D. Paulo, Adaptive Filtering: Algorithms and Practical Implementation. New York, NY, USA: Springer, 2013.
- [57] A. H. Sayed, Adaptive Filters. Hoboken, NJ, USA: Wiley, 2011.
- [58] P. M. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems," *J. Process Control*, vol. 7, no. 6, pp. 403–424, 1997.
- [59] Z.-H. Zhang and G.-H. Yang, "Fault detection for discrete-time uncertain LPV systems using non-minimal order filter," *J. Franklin Inst.*, vol. 355, no. 2, pp. 902–921, Jan. 2018.



LEONARDO DE PAULA CARVALHO received the bachelor's degree in electrical engineering from the Universidade Federal do Mato Grosso do Sul (UFMS), Brazil, in 2012, and the master's degree in electrical engineering from the Universidade Estadual de Campinas (UNICAMP), Brazil, in 2016. He is currently pursuing the Ph.D. degree with the Universidade de São Paulo, Brazil, and the University of Groningen, The Netherlands. His research interests include control and filtering the-

ory, fault detection, fault-tolerant control, convex optimization, Markov jump linear systems, networked control systems, and linear parameter varying.



JONATHAN M. PALMA received the B.Sc. degree in electronic engineering from the University of Bío-Bío, Concepción, Chile, in 2014, and the master's and Ph.D. degrees in electrical engineering from the University of Campinas, in 2016 and 2019, respectively. He currently works as an Academic and a Researcher with the Engineering Faculty, University of Talca, Curico, Chile. His research interests include control theory, mainly investigating linear parameter-varying

(LPV) dynamics, and Markov jump linear systems (MJLS). Other research topics include the development of protocols for energy-efficiency in networked control systems; automatic control of combustion systems using optical instrumentation; and stability analysis, control and mathematical modeling applied to biological systems (also called biomathematics).



BAYU JAYAWARDHANA (Senior Member, IEEE) received the B.Sc. degree in electrical and electronics engineering from the Institut Teknologi Bandung, Bandung, Indonesia, in 2000, the M.Eng. degree in electrical and electronics engineering from Nanyang Technological University, Singapore, in 2003, and the Ph.D. degree in electrical and electronics engineering from Imperial College London, London, U.K., in 2006. He was with Bath University, Bath, U.K., and with

The University of Manchester, Manchester, U.K. He is currently a Professor of mechatronics and control of nonlinear systems with the Faculty of Science and Engineering, University of Groningen, Groningen, The Netherlands. His research interests include the analysis of nonlinear systems, systems with hysteresis, mechatronics, robotics, and systems biology. He is a member of the Conference Editorial Board of the IEEE Control Systems Society. He is a Subject Editor of the *International Journal of Robust and Nonlinear Control* and an Associate Editor of the *European Journal of Control*.



TABITHA E. ROSA received the bachelor's degree in mechatronics engineering from the Centro Federal de Educação Tecnológica de Minas Gerais (CEFET-MG), Brazil, in 2014, and the master's degree in electrical engineering from the Universidade Estadual de Campinas (UNICAMP), Brazil, in 2017. She is currently pursuing the Ph.D. degree with the University of Groningen, The Netherlands. Her master's project focused on robust control, looking more specifically into the problem

of output feedback control for linear parameter varying systems. For her bachelor's thesis, she proposed the automation and modeling of a system of interactive tanks and a discrete-time control project using decoupling techniques for multi-variable systems. For her current research, she investigates the problem of integrating models and real-time data for zero-defect manufacturing control systems.



OSWALDO LUIZ DO VALLE COSTA (Senior Member, IEEE) was born in Rio de Janeiro, Brazil, in 1959. He received the B.Sc. and M.Sc. degrees in electrical engineering from the Catholic University of Rio de Janeiro, in 1981 and 1983, respectively, and the Ph.D. degree in electrical engineering from the Imperial College of Science and Technology, London, U.K., in 1987. From 1987 to 1988, he was a Postdoctoral Research Assistant with the Department of Electrical Engi-

neering, Imperial College. He is currently a Full Professor with the Control Group, Department of Telecommunications and Control Engineering, Polytechnic School of the University of São Paulo, São Paulo, Brazil. His research interests include stochastic control, optimal control, and jump systems.