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Passivity-Based Control



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Abstract

Stabilization of physical systems by shaping their energy function is a well-established technique whose roots date back to the work of Lagrange and Legendre. Potential energy shaping for fully actuated mechanical systems was first introduced in Takegaki and Arimoto (Trans ASME J Dyn Syst Meas Control 12:119–125, 1981) more than 30 years ago. In Ortega and Spong (Automatica 25(6):877–888, 1989) it was proved that passivity was the key property underlying the stabilization mechanism of these designs, and the, now widely popular, term of passivity-based control was coined. In this chapter we summarize the basic principles and some of the main developments of this controller design technique.

Keywords

Stabilization · Lyapunov function · Energy shaping · Storage function · Dissipation

Introduction

Energy is one of the fundamental concepts in science and engineering practice, where it is common to view dynamical systems as energy transformation devices. This perspective is particularly useful in studying *complex nonlinear* systems by decomposing them into simpler subsystems which, upon interconnection, add up their energies to determine the full system's behavior. The action of a controller may be also understood in energy terms as another dynamical system – typically implemented in a computer – interconnected with the process to modify its behavior. Then, the control problem can be recast as finding a dynamical system and an interconnection pattern such that the overall energy and dissipation functions take the desired form. This “energy-shaping plus dissipation” approach is the essence of the controller design technique – known as passivity-based control (PBC) – that is reviewed in this chapter.

We consider dynamical systems represented as multiports with port variables $(u, y) \in \mathbb{R}^m \times \mathbb{R}^m$. In physical systems, the port variables are conjugated, in the sense that their product represents power. In this case, passivity captures the well-known property that the energy that can be extracted from the system is bounded, that is,

there exists $\beta \in \mathbb{R}$ such that

$$-\int_0^t u^\top(s)y(s)ds \leq \beta.$$

Motivated by practical applications, we assume that the system admits a state-space representation given in the input-state-output form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x), \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, and we view the system as a mapping $\Sigma : u \mapsto y$. It is said to be passive if there exists a function $H(x) \geq 0$ – called the storage function – such that the power-balance inequality

$$\dot{H} \leq u^\top y \quad (2)$$

holds. In physical systems a reasonable candidate for $H(x)$ is the energy of its energy-storing elements, e.g., capacitors, inductors, dampers, and masses.

An algebraic description of passive systems of the form (1) is given in Hill and Moylan (1980), where it is shown that a necessary condition for a system to be passive is that

$$h(x) = g^\top(x)\nabla H(x). \quad (3)$$

A geometric characterization of systems that can be rendered passive via state feedback is reported in Byrnes et al. (1991).

The objective of PBC, in the simplest static state-feedback formulation, is to find a function $\hat{u} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that the system (1) in closed-loop with the control law $u = \hat{u}(x) + v$ satisfies the new power balance inequality

$$\dot{H}_d \leq v^\top y_d, \quad (4)$$

where $(v, y_d) \in \mathbb{R}^m \times \mathbb{R}^m$ are the new port variables and $H_d(x) \geq 0$ is the *desired* storage function. This first step of PBC is known as *energy shaping*. The second step in PBC, known

as *damping injection*, is to set $v = -K_{\text{DI}}y_d$, with $K_{\text{DI}} > 0$, then $\dot{H}_d \leq -y_d^\top K_{\text{DI}}y_d$.

Chapter notation All mappings are supposed smooth. Consider the mapping $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$. We define the ij -th element of the $n \times m$ matrix $\nabla_x F(x)$ as $(\nabla_x F)_{ij} := \frac{\partial F_j}{\partial x_i}$. When clear from the context, the subindex in ∇ is omitted. For any F and the distinguished element $x^* \in \mathbb{R}^n$, we define the constant matrix $F^* := F(x^*)$. Consider the case $n \geq m$ and $\text{rank}\{F\} = m$. Then, the pseudoinverse of F is denoted by F^\dagger , that is, $F^\dagger := (F^\top F)^{-1}F^\top$ and $F^\dagger F = I_m$. The left annihilator of F is represented as F^\perp , where $\text{rank}\{F^\perp\} = n - m$, and $F^\perp F = \mathbf{0}_{(n-m) \times m}$.

Equilibrium Stabilization via PBC

PBC is often used for Lyapunov stabilization of equilibria. For this task, the key observation is that if $H_d(x)$ verifies

$$x^* = \arg \min H_d(x), \quad (5)$$

where $f(x^*) + g(x^*)\hat{u}(x^*) = \mathbf{0}_n$, then x^* is a *stable* equilibrium of the system in closed-loop with $u = \hat{u}(x)$, with Lyapunov function $H_d(x)$. Moreover, adding the *damping injection* step, the equilibrium x^* will be *asymptotically* stable if y_d is a detectable output for the closed-loop system – see van der Schaft (2016, Corollary 4.2.2).

Energy-Balancing PBC and the Dissipation Obstacle

The most natural way to carry out the energy shaping is to fix $y_d = y$, to look for a function $H_a : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$, solution of the equation

$$\dot{H}_a = -h^\top(x)\hat{u}(x), \quad (6)$$

and to define $H_d(x) = H(x) + H_a(x)$. This variation of PBC is called energy balancing (EB) (Ortega et al. 2001), because $H_d(x)$ consists of the sum of the systems energy and the energy

provided by the controller, e.g., $y^\top u$. In spite of this appealing interpretation, EB-PBC has two serious shortcomings. On one hand, the Eq. (6), which is a partial differential equation (PDE) of the form

$$[\nabla H_a(x)]^\top [f(x) + g(x)\hat{u}(x)] = -h^\top(x)\hat{u}(x),$$

is not amenable for an easy interpretation – because of its explicit dependence on the feedback signal that we are looking for. On the other hand, it is applicable only to systems that are not constrained by the *dissipation obstacle*, that is, systems where dissipation is absent in steady state and can, therefore, be stabilized extracting a finite amount of energy from the controller. Indeed, it is clear that a necessary condition for the existence of a solution of (6) is that $h^\top(x^*)\hat{u}(x^*) = 0$.

The main domain of application of EB-PBC is in regulation of the position $q \in \mathbb{R}^\ell$ of mechanical systems, where $\ell = \frac{n}{2}$. In this case, the dissipation obstacle is conspicuous by its absence because the passive output is velocity \dot{q} . Moreover, since in this case we are only interested in adding a new function $V_a(q)$ to the potential energy function, the PDE to be solved reduces to $G^\perp(q)\nabla V_a(q) = \mathbf{0}_{\ell-m}$, where the input matrix is of the form $g(q, \dot{q}) = [\mathbf{0}_{\ell \times m}^\top G^\top(q)]^\top$. In the particular case of fully actuated mechanical systems, it is possible to remove the open-loop potential energy and assign a desired one. This idea was introduced in the pioneering work Takegaki and Arimoto (1981).

Generating Alternative Passive Outputs

One way to overcome the dissipation obstacle is to enforce the power balance inequality (4) with outputs $y_d \neq y$. To achieve this end, it is convenient to restrict our attention to *port-Hamiltonian* (pH) systems, that is systems where

$$f(x) = [\mathcal{J}(x) - \mathcal{R}(x)]\nabla H(x), \quad (7)$$

where $\mathcal{J}(x) = \mathcal{J}^\top(x)$ is the interconnection matrix and $\mathcal{R}(x) \geq 0$ is the dissipation matrix. See van der Schaft (2016) for an extensive dis-

cussion, and well-founded practical motivation, of this class of systems. The following result, which characterizes all passive outputs of the pH system with storage function $H(x)$, was recently established in Zhang et al. (2018).

Lemma 1 Consider the pH system (1) and (7). Introduce the factorization $\mathcal{R}(x) = \phi^\top(x)\phi(x)$, where $\phi(x) \in \mathbb{R}^{q \times n}$, with $q \geq \text{rank}\{\mathcal{R}(x)\}$ and define $y_d := h(x) + j(x)u$. The following statements are equivalent.

- (S1) The mapping $u \mapsto y_d$ is passive with storage function $H(x)$.
- (S2) The mappings $h(x)$ and $j(x)$ can be expressed as

$$\begin{aligned} h(x) &= [g(x) + 2\phi^\top(x)w(x)]^\top \nabla H(x) \\ j(x) &= w^\top(x)w(x) + D(x), \end{aligned}$$

for some mappings $w(x) \in \mathbb{R}^{q \times m}$ and $D(x) \in \mathbb{R}^{m \times m}$, with $D(x) = -D^\top(x)$.

One particularly attractive output that allows us to overcome the dissipation obstacle is the so-called *power-shaping* output. With a suitable selection of the mappings $w(x)$ and $D(x)$ in Proposition 1, it is possible to generate the output

$$\begin{aligned} y_d &= -g^\top(x)[\mathcal{J}(x) - \mathcal{R}(x)]^{-\top} \{[\mathcal{J}(x) \\ &\quad - \mathcal{R}(x)]\nabla H(x) + g(x)u\}, \end{aligned}$$

where, for ease of presentation, it is assumed that the matrix $\mathcal{J}(x) - \mathcal{R}(x)$ is full rank. Since $y_d = \mathbf{0}_m$ at the equilibrium, it is clear that the dissipation obstacle is absent for the system $u \mapsto y_d$.

Interconnection and Damping Assignment (IDA)-PBC

A variation of PBC that has been very successful in many practical applications is IDA-PBC. The main result of IDA-PBC is the following proposition, whose proof may be found in Ortega et al. (2002a).

Proposition 1 Consider the system (1). Assume there are matrices $\mathcal{J}_d(x) = -\mathcal{J}_d^\top(x)$, $\mathcal{R}_d(x) =$

$\mathcal{R}_d^\top(x) \geq 0$, and a function $H_d(x)$, that verify the matching equation

$$g^\perp(x)f(x) = g^\perp(x)[\mathcal{J}_d(x) - \mathcal{R}_d(x)]\nabla H_d. \quad (8)$$

Then, the closed-loop system with $u = \hat{u}(x)$, where

$$\hat{u}(x) := g^\dagger(x)\{[\mathcal{J}_d(x) - \mathcal{R}_d(x)]\nabla H_d - f(x)\},$$

takes the pH form $\dot{x} = [\mathcal{J}_d(x) - \mathcal{R}_d(x)]\nabla H_d$. Moreover, if $x^* \in \{x \mid g^\perp(x)f(x) = \mathbf{0}_{n-m}\}$ and (5) holds, it is a stable equilibrium.

Conversely, if there exists $\hat{u}(x)$ that globally asymptotically stabilizes (1), then there exist $\mathcal{J}_d(x)$, $\mathcal{R}_d(x)$, and $H_d(x)$ which satisfy (8).

The key step in the IDA-PBC design is, of course, the solution of (8), and there are several approaches to carry out this task. In the non-parameterized IDA-PBC, we fix $\mathcal{J}_d(x)$ and $\mathcal{R}_d(x)$, and (8) becomes a PDE for $H_d(x)$. In algebraic IDA-PBC (Fujimoto and Sugie 2001), we fix $H_d(x)$, and (8) becomes an algebraic equation in $\mathcal{J}_d(x)$ and $\mathcal{R}_d(x)$. In some applications, it is of interest to fix the structure of the desired energy function, for instance, for mechanical systems, it is reasonable to propose

$$H_d(q, p) = \frac{1}{2}p^\top M_d^{-1}(q)p + V_d(q);$$

whence (8) becomes a PDE in the desired inertia matrix $M_d(q)$ and the desired potential energy function $V_d(q)$ – this approach is called parameterized IDA-PBC (Ortega et al. 2002b), with a Lagrangian version given in Bloch et al. (2000). Clearly, fixing this structure imposes some particular constraints on $\mathcal{J}_d(x)$ and $\mathcal{R}_d(x)$. Finally, for nonlinear systems of the form $\dot{x} = f(x, u)$, Poincaré’s Lemma states that a necessary and sufficient condition for the solution of the matching equation

$$\nabla H_d(x) = [\mathcal{J}_d(x) - \mathcal{R}_d(x)]^{-1}f(x, \hat{u}(x))$$

is that the right-hand side is a gradient vector field. Fixing $\mathcal{J}_d(x)$ and $\mathcal{R}_d(x)$, this condition translates into a PDE directly for $\hat{u}(x)$.

Proportional-Integral-Derivative (PID)-PBC

PID controllers overwhelmingly dominate engineering applications where the control objective is to regulate some measurable signal y around a constant desired value y^* . Commissioning of PIDs reduces to the suitable selection of the controller gains, which is a difficult task for wide-ranging operating systems, where the validity of a linearized approximation is limited, compromising the stability of the closed-loop. Although gain scheduling, auto-tuning, and adaptation provide some help to overcome this problem, they suffer from well-documented drawbacks. In contrast to this scenario, in PID-PBC, where the PID is wrapped around a passive output, the gain tuning step is trivialized. Indeed, PIDs

$$\begin{aligned} \dot{x}_c &= y \\ u &= -K_P y - K_I x_c - K_D \dot{y}, \end{aligned} \quad (9)$$

define, (output-strictly) passive maps $u \mapsto -y$, with storage function $\frac{1}{2}y^\top K_D y + \frac{1}{2}x_c^\top K_I x_c$, for all positive gains. Therefore, convergence of the output to zero and \mathcal{L}_2 -stability of the closed-loop system is always guaranteed – and the designers task is only to select the gains that ensure best transient performance.

Shifted Passivity

It is often the case that the reference output $y^* \neq 0$, that suggests to wrap the PID around the error signal $y - y^*$. Unfortunately, for general nonlinear systems, passivity of the mapping $u \mapsto y$ does not imply passivity of $(u - u^*) \mapsto (y - y^*)$ – a property called “shifted passivity” in van der Schaft (2016). The strongest conditions under which the implication holds for pH systems have been reported in Monshizadeh et al. (2019) where, in particular, the following easily verifiable necessary and sufficient condition is given for systems with quadratic nonlinearities.

Lemma 2 Consider the pH system (1) and (7) with g and \mathcal{R} constants,

$$H(x) = \frac{1}{2}x^\top Qx, \quad Q > 0, \quad \mathcal{J}(x) = \mathcal{J}_0 + \sum_{i=1}^n \mathcal{J}_i x_i.$$

Fix

$$(x^*, u^*) \in \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m \mid [\mathcal{J}(x) - \mathcal{R}]\nabla H(x) + gu = \mathbf{0}_n\}.$$

The system satisfies $\dot{\mathcal{H}} \leq (y - y^*)^\top (u - u^*)$, with $\mathcal{H}(x) := \frac{1}{2}(x - x^*)^\top Q(x - x^*)$, if and only if

$$\left(\sum_{i=1}^n \mathcal{J}_i - \mathcal{R} \right) Q x^* e_i^\top Q^{-1} + \left[\left(\sum_{i=1}^n \mathcal{J}_i - \mathcal{R} \right) Q x^* e_i^\top Q^{-1} \right]^\top - 2\mathcal{R} \leq 0,$$

where $e_i \in \mathbb{R}$ is the i -th element of the orthogonal basis.

Leveraging Lemma 2, we can confidently wrap a PID around the shifted output $y - y^*$. One important observation is that, in the implementation of the PID, there is no need to know u^* . Indeed, by shifting the storage function, we can establish passivity of the map $u \mapsto y - y^*$.

Lyapunov Stabilization via PID-PBC

Another scenario of practical interest is when the control objective cannot be captured by the behavior of an output signal, for instance, when it is desired to drive the full system state to a desired equilibrium x^* . To treat this case, it is necessary to create a Lyapunov function for the closed-loop system, an approach taken for pH systems in Zhang et al. (2018). The main difficulty in this case is how to ensure the positivity (with respect to x^*) of the function. Indeed, although for a passive system (1) in closed-loop with a PID (9), we can prove that the function

$$U(x, x_c) := H(x) + \frac{1}{2}h^\top(x)K_D h(x) + \frac{1}{2}x_c^\top K_I x_c$$

satisfies $\dot{U} \leq -y^\top K_P y$ and the function $U(x, x_c)$ is not positive definite – therefore, it does not qualify as a Lyapunov function. One way to overcome this obstacle is to find a mapping $\gamma(x) \in \mathbb{R}^m$ such that the multilevel set

$$\Gamma := \{(x, x_c) \in \mathbb{R}^n \times \mathbb{R}^m \mid x_c = \gamma(x) + \kappa, \kappa \in \mathbb{R}^m\} \quad (10)$$

is invariant and the function

$$H_d(x) := H(x) + U(x, \gamma(x) + \kappa) \quad (11)$$

verifies (5), for some $\kappa \in \mathbb{R}^m$. As expected, this involves the solution of a PDE, as shown below.

Lemma 3 Consider the pH system (1), (7), and $\dot{x}_c = y_d$, with y_d defined in Lemma 1. Assume there exists mappings $w(x)$ and $D(x)$ such that the PDE

$$\begin{bmatrix} [\nabla H(x)]^\top F^\top(x) \\ g^\top(x) \end{bmatrix} \nabla \gamma(x) = \begin{bmatrix} [\nabla H(x)]^\top [g(x) + 2\phi^\top(x)w(x)] \\ w^\top(x)w(x) - D(x) \end{bmatrix}$$

admits a solution $\gamma(x) \in \mathbb{R}^m$. Then, the set Γ , given in (10), is invariant.

Combining Lemmata 1 and 3, we can complete the design of the PID-PBC as follows.

Proposition 2 Consider the pH system (1) and (7) in closed-loop with the PID-PBC

$$u = -K_P y_d - K_I [\gamma(x) + \kappa] - K_D \dot{y}_d.$$

where $\kappa = -(\gamma^* + K_I^{-1}u^*)$ and y_d is defined as in Lemmata 1 and 3. Assume that $H_d(x)$, given in (11), verifies (5). Then, the closed-loop system has a stable equilibrium at x^* with Lyapunov function (11). Moreover, the equilibrium is asymptotically stable if y_d is a detectable output for the closed-loop system.

Standard PBC of Euler-Lagrange (EL) Systems

Another variation of PBC, particularly suitable for systems described by Euler-Lagrange equations of motion is the so-called standard PBC,

that was thoroughly explored in Ortega et al. (2013).

Mathematical Model and Passivity Property

In this case we deal with dynamical systems with ℓ degrees-of-freedom, generalized coordinates $q \in \mathbb{R}^\ell$, and external forces $Q \in \mathbb{R}^\ell$, which are described by the EL equations

$$\frac{d}{dt} [\nabla_{\dot{q}} \mathcal{L}(q, \dot{q})] - \nabla_q \mathcal{L}(q, \dot{q}) = F, \quad (12)$$

where $\mathcal{L}(q, \dot{q}) := T(q, \dot{q}) - V(q)$ is the Lagrangian function, $T(q, \dot{q}) = \frac{1}{2} \dot{q}^\top M(q) \dot{q}$, is the kinetic (co-energy) function, with $M(q) > 0$ the generalized inertia matrix, and $V(q)$ is the potential energy function. The vector F is the external forces that take the form $F = -\nabla \mathcal{F}(\dot{q}) + G(q)u$, where $u \in \mathbb{R}^m$, $m \leq \ell$ are the control and dissipation forces and $\mathcal{F}(\dot{q})$ is the Rayleigh dissipation function verifying $\dot{q}^\top \nabla \mathcal{F}(\dot{q}) \geq 0$.

It is well-known that the EL system (12) defines a passive operator $u \mapsto G^\top(q) \dot{q}$ with storage function, the system's total energy $H(q, \dot{q}) = T(q, \dot{q}) + V(q)$, see Proposition 2.5 in Ortega et al. (2013). Moreover, it is possible to prove a ‘‘stronger’’ property. Namely, if we define an $\ell \times \ell$ matrix $C(q, \dot{q})$ – called in the robotics literature the ‘‘Coriolis and centrifugal forces’’ matrix – with ik -th entry

$$C_{ik}(q, \dot{q}) = \sum_j^\ell c_{ijk}(q) \dot{q}_j.$$

where

$$c_{ijk}(q) := \frac{1}{2} [\nabla_{q_j} m_{ik}(q) + \nabla_{q_i} m_{jk}(q) - \nabla_{q_k} m_{ij}(q)]$$

are the Christoffel symbols of the first kind (Ortega and Spong 1989). Then, the EL system becomes

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + \nabla \mathcal{F}(\dot{q}) + \nabla V(q) = G(q, \dot{q})u, \quad (13)$$

where

$$\begin{aligned} \dot{M}(q) &= C(q, \dot{q}) + C^\top(q, \dot{q}) \\ &\Leftrightarrow z^\top [\dot{M}(q) - 2C(q, \dot{q})]z = 0, \\ \forall z &\in \mathbb{R}^\ell. \end{aligned} \quad (14)$$

Standard PBC

The skew-symmetry property (14), which identifies the work-less forces, is the key component of standard PBC. The main result, for regulation of q , of this technique is given as follows.

Proposition 3 *Consider the EL system (13) verifying (14). Fix the desired position $q^* \in \{q \in \mathbb{R}^\ell \mid G^\perp(q) \nabla V(q) = \mathbf{0}_{\ell-m}\}$. Assume it is possible to find signals $q_d \in \mathbb{R}^\ell$ satisfying the equation*

$$\begin{aligned} G^\perp(q) [M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + \nabla \mathcal{F}(\dot{q}) \\ + \nabla V(q) - K_D \dot{\tilde{q}} - K_P \tilde{q}] = \mathbf{0}_{\ell-m}, \end{aligned}$$

where $K_D, K_P > 0$, $\tilde{q} := q - q_d$, and that for the system

$$M(q)\ddot{\tilde{q}} + C(q, \dot{q})\dot{\tilde{q}} + K_d \dot{\tilde{q}} + K_p \tilde{q} = \mathbf{0}_\ell, \quad (15)$$

it is possible to prove that

$$\tilde{q}, \dot{\tilde{q}} \in \mathcal{L}_\infty, \quad \dot{\tilde{q}}(t) \rightarrow \mathbf{0}_\ell \Rightarrow q, \dot{q} \in \mathcal{L}_\infty, \quad q(t) \rightarrow q^*. \quad (16)$$

Under these conditions, the system (13) in closed-loop with the control

$$\begin{aligned} u &= G^\dagger(q) [M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + \nabla \mathcal{F}(\dot{q}) \\ &\quad + \nabla V(q) - K_D \dot{\tilde{q}} - K_P \tilde{q}] \end{aligned}$$

ensures $q(t) \rightarrow q^*$ with all internal signals bounded.

The gist of the proof is the observation that the error system is given by (15) and that for this system we have the following

$$\begin{aligned} H_d(\tilde{q}) &:= \frac{1}{2} \dot{\tilde{q}}^\top D \dot{\tilde{q}} + \tilde{q}^\top K_p \tilde{q} \Rightarrow \dot{H}_d \\ &= -\dot{\tilde{q}}^\top K_v \dot{\tilde{q}}. \end{aligned}$$

Then, some easy signal chasing and the implication (16) allow us to complete the proof.

The procedure described above is an extension of the well-known controller for fully actuated robot manipulators of Slotine and Li (1988). In that case, $G(q) = I_\ell$ and stabilization (or tracking) of q^* are ensured by selecting

$$q_d := q^* - \Lambda(q - q^*), \quad \Lambda > 0.$$

For underactuated systems, it is not possible to fix all signals q_d to desired values; whence some of them are defined as states of the controller dynamics. In Ortega et al. (2013) the method is applied to a large variety of physical systems, including underactuated mechanical systems, electrical motors, power converters, and levitated systems. Particularly noteworthy is the proof that applying standard PBC to induction motors yields a controller that contains, as a particular case, the industry standard field-oriented control. One major drawback of the method is that the calculation of the controller involves an implicit inversion of the system dynamics; hence its application is restricted to minimum phase systems. This limitation is conspicuous by its absence in IDA-PBC.

Summary and Future Directions

In physical systems, the concepts of energy and dissipation are well defined. Therefore, passivity theory and PBC techniques emerge as natural options to analyze and control, respectively, this kind of systems. However, the range of applicability of PBC is not constrained to physical systems; as a matter of fact, the possibility of decomposing complex nonlinear systems into simpler subsystems makes this approach appealing to tackle down modern problems in control, such as stabilization of biological systems or smart grids.

Cross-References

- ▶ [Feedback Stabilization of Nonlinear Systems](#)
- ▶ [KYP Lemma and Generalizations/Applications](#)

- ▶ [Nonlinear Adaptive Control](#)
- ▶ [PID Control](#)
- ▶ [Robot Motion Control](#)
- ▶ [Stability: Lyapunov, Linear Systems](#)

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