Aggregate Energy Production in Wind Farms via Dynamic Robust Coalitional Games

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***Abstract*—This letter investigates the benefits of aggregating independent wind power producers. First, we quantify the expected joint profit in the case where such producers act as a single entity in a one-stage contract interval, and in a multi-stage contract interval with recourse. Second, we provide a constructive method to design stable allocation mechanisms in the case where the expected profit is changing with time. Third, we prove that the allocation policy stabilizes the coalition by correcting allocations in real-time in the case where the realized power output deviates from the contract size of the ahead market.**

***Index Terms*—Coalitional games, game theory, robust control, wind energy aggregation.**

I. Introduction

AGGREGATION of independent wind farms leads to a reduction of the aggregate volatility [1], [23], which implies less conservative bidding [29]. Producers contract with one aggregator which schedules the power outputs and manages the financial flows to each producer [23]. The aggregator bids on a bus which is common to all producers, see, e.g., [1, Assumption A1]. To deal with the increasing amount of renewables new business models are developed, based on the idea of market recourse, in which the producer uses improved forecasts to adjust its initial bid in the day-ahead (DA) forward market [4]. New business models, including real time pricing [24], can be addressed within the framework of coalitional games with transferable utility (TU games). In [26] the authors use coalitional games to study novel cooperative strategies between the micro-grids. Game theory in energy trading is studied in [28]. Coalitional games appear in [22] where the Shapley value is used to incentivize demand response. In [13], similar concepts are used to model the direct trading between suppliers and end-users. Results on the Shapley value and Harsanyi power solutions in coalitional control are also in [18], [19]. TU games in multi-agent systems are discussed in [25]. Coalitional control under cooperation cost is studied in [11], [16].

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The main contribution of this letter is a quantitative analysis of the profits generated by aggregating producers, who act as a single entity in a repeated two-settlement market. We show that when the producers offer joint bids they increase their profits due to a reduction of statistical dispersion, smaller deviations from the sizes of contracts and consequently smaller penalties. We assume that the producers act as price takers and the price is assumed known, see, e.g., [1, Assumption A2]. We conduct our study in the framework of dynamic coalitional games [12] and robust control invariance [5]–[7]. At each time, the coalitions' values are obtained from maximizing the one-stage expected profit over a joint contract size. While the expected profit is known, the realized value is uncertain and the resulting game is a robust dynamic coalitional game [2], [3]. Such a robust dynamic setting makes this letter substantially different from [1]. The control design is based on a decomposition method. First a feedforward control is designed based on the Shapley value. The feedforward allocation is based on the expected profit. Second a feedback control is obtained as function of the excesses. The feedback allocation is used to correct allocation errors due to wind volatility. The Shapley value guarantees allocation in the core in the long run if the core is nonempty, and can be linked to the ϵ-core in all other cases [15].

A. Preliminaries on TU Games

Let a set $N = \{1, \dots, n\}$ of players be given and let $\mathcal{S} := 2^N \setminus \emptyset$ be the set of all nonempty coalitions. Let $m = |\mathcal{S}| = 2^n - 1$ be the number of possible nonempty coalitions, where we denote by $|\mathcal{S}|$ the cardinality of set $\mathcal{S}$. Consider a function $\eta : S \mapsto \mathbb{R}$ defined for each nonempty coalition $S \in \mathcal{S}$. We denote by $\langle N, \eta \rangle$ the TU game with players set N and characteristic function η, which quantifies the gain of coalition S. Let η^j be the value of the characteristic function η associated with a nonempty coalition $S_j \in \mathcal{S}$. Here j is a generic element of $M := \{1, \dots, m\}$ according to some arbitrary mapping of $\mathcal{S}$ into M. Let the grand coalition be denoted by N.

The core set $C(\eta)$ is the set of allocations that are

- *efficient*, that is, the sum of the components of the allocation vector is equal to the value of the grand coalition, and

- *individually rational*, namely the players do not benefit from splitting from the grand coalition and playing alone,
- *stable with respect to subcoalitions*, the players do not benefit from creating any sub-coalition.

More formally, the core set is defined as:

$$C(\eta) = \{\tilde{u} \in \mathbb{R}^n | \sum_{i \in N} \tilde{u}_i = \eta^m, \ \tilde{u}_i \geq \eta^i, \ \forall S_i \in \mathcal{S}', \sum_{i \in S_j} \tilde{u}_i \geq \eta^j, \quad \forall S_j \in \mathcal{S} \setminus N\},$$

where $\tilde{u}_i$ is the reward allocated to player i, η^m is the value of the grand coalition N, and $\mathcal{S}'$ is the set of all coalitions which consist of a single player.

A common allocation mechanism is based on the Shapley value.

Definition 1: Let $S_r := S_j \cup \{i\}$. The Shapley value of player i is:

$$\phi_i(\eta) = \sum_{S_j \subset N \setminus \{i\}} \frac{|S_j|!(|N| - |S_j| - 1)!}{|N|!} (\eta^r - \eta^j).$$

The structure of the letter is as follows. In Section II, we formulate the problem. In Section III, we present a decomposition approach. The main results are presented in Section IV. In Section V, a constructed numerical example is given to illustrate the proposed method. Finally, in Section VI, we provide conclusions.

II. Model and Problem Set-Up

A. One-Stage Contract Interval Optimization

Consider a market with a *two-settlement structure* [27] in which producers compete by submitting bids for energy. The structure involves two ex-ante markets, namely a day-ahead (DA) forward market and a real-time (RT) spot market, and an ex-post mechanism to penalize uninstructed deviations from ex-ante bids. Let $\mathcal{N} = \{1, \dots, n\}$ be a set of producers. For each coalition $S_j \in \mathcal{S}$, let y (MWh) be a random variable which represents the aggregate wind power produced by that coalition. Let us assume that y has continuous probability density function f and cumulative distribution function (CDF) F. Let C (MWh) be the contract that a coalition offers in a single ex-ante DA forward market. Let p in $\mathbb{R}_+$ be the price of electricity in a DA forward market (£/MWh).

From [29] we know that there is an optimal trade-off in terms of coalition's size which guarantees uncertainty reduction but no significant market power. In such a scenario the producers capacity is small in comparison to the entire market and the producers are therefore price-takers. In this letter, we assume that such forward price is known but changes with time.

In addition, let s in $\mathbb{R}_+$ be the penalty price which settles uninstructed negative deviations from a contract offered ex-ante (£/MWh) in the RT sport market. Henceforth, we refer to a negative deviation as production *shortage*. Also, let h in $\mathbb{R}_+$ be the penalty price which settles uninstructed positive deviations from a contract offered ex-ante (£/MWh). In the following we refer to a positive deviation as production *excess*. Analogously to [1], the penalty prices for uninstructed deviations are revealed after the RT spot market is cleared. Thus, both prices s and h are unknown at the close of the DA forward market. In addition to this, imbalance prices are volatile and difficult to forecast, and therefore we model them as random variables independent of the wind.

Let the expected shortage and excess be

$$\mathbf{E}_s = \mathbb{E}[\max(0, C - y)], \quad \mathbf{E}_h = \mathbb{E}[\max(0, y - C)], \quad (1)$$

which satisfy the condition $\mathbf{E}_s = C - \mu + \mathbf{E}_h$, where μ is the mean and is given by $\int_0^\infty f(y) y dy$. The problem of each coalition is the one of maximizing the expected profit with respect to the size of the constant power contract C, which is the decision variable:

$$\max_C \mathbb{E}J_{S_j}(y, C) := \max_C \{pC - s\mathbf{E}_s - h\mathbf{E}_h\},$$

where $\mathbb{E}J_{S_j}(y, C)$ is the expected profit for the generic coalition $S_j \in \mathcal{S}$ under the constant power contract C. Assuming concavity of $\mathbb{E}J_{S_j}(y, C)$, from [4] the optimal constant power contract C^* is obtained as

$$F(C^*) = \frac{p + h}{s + h}, \quad C^* = F^{-1}\left(\frac{p + h}{s + h}\right), \quad (2)$$

where F^{-1} is the inverse function of the CDF F. In the above it is reasonable to assume that uninstructed negative deviations from a contract offered ex-ante are penalized more than positive deviations namely $s > h$. It is also reasonable to assume that the penalty for negative deviations exceeds the price of electricity itself, namely $s > p$. Indeed, note that if on the contrary we had $s < p$ the producers would tend to bid at maximal capacity. Then, the optimal expected profit is

$$\mathbb{E}J_{S_j}(y, C^*) = pC^* - s(C^* - \mu + \mathbf{E}_h^*) - h\mathbf{E}_h^*, \quad (3)$$

where we denote by $\mathbf{E}_h^*$ the expected excess under the optimal size of the constant power contract C^*. We refer the reader to the proof in [4, Th. IV.1] for a detailed analysis of concavity conditions.

B. Multi-Stage Recursion

Consider a sequence of contract intervals with recursion, for instance one contract per each day k. Assume that the wind power produced at day k has a generic distribution CDF which we denote by F_k. From (2) we have

$$F_k(C^*) = \frac{p_k + h_k}{s_k + h_k}, \quad C^* = F_k^{-1}\left(\frac{p_k + h_k}{s_k + h_k}\right), \quad (4)$$

where F_k^{-1} is the inverse function of F_k. Note that all prices p_k, h_k and s_k now depend on time k, This is true also for the optimal contract size C^* and the optimal expected profit in (3) though for simplicity of notation the dependence on index k is omitted.

In the spirit of [2], [3], [21], the dynamic TU game is denoted by $\langle \mathcal{N}, \eta_k \rangle$, where the jth component of the m-dimensional vector η_k, denoted by η_k^j is the value of the corresponding nonempty coalition S_j at time k in the game $\langle \mathcal{N}, \eta_k \rangle$.

We take the value of coalition S_j equal to the profit obtained for generic power output y under a contract size C^* as in (4) detracted by the cost of cooperation. More specifically we have $\eta_k := [J_{S_j}(y, C^*)]_{S_j \in \mathcal{S}} - [\psi_{S_j}]_{S_j \in \mathcal{S}} \in \mathbb{R}^m$, where ψ_{S_j} is the cost of forming coalition S_j. It is sensible to assume that this cost is monotonically increasing in the size of the coalition. We can also assume that it is deterministic and independent on the wind output y. Denote by $B_{\mathcal{H}}$ the matrix whose rows are the characteristic vectors $c^{S_j} \in \mathbb{R}^n$. The characteristic vectors are in turn binary vectors where $c_i^{S_j} = 1$ if $i \in S_j$ and $c_i^{S_j} = 0$ if $i \notin S_j$. Allocations ξ_k in the *core* of the game $C(\eta_k)$ satisfy

$$\xi_k \in C(\eta_k) \Leftrightarrow B_{\mathcal{H}} \xi_k \geq \eta_k, \tag{5}$$

where the inequality is to be interpreted componentwise. Inequality (5) can be rewritten as an equality by using an augmented allocation vector given by $u_k := \begin{bmatrix} \xi_k \\ \lambda_k \end{bmatrix} \in \mathbb{R}^{n+m}$, where λ_k is a vector of m nonnegative surplus variables. Then, we have

$$\Big[B_{\mathcal{H}} \Big| - I\Big] u_k = \eta_k. \tag{6}$$

Note that each surplus variable λ_k^j describes the difference between the allocation and the coalitional value of coalition S_j, i.e., $\lambda_k^j = \sum_{i \in S_j} \xi_k^i - \eta_k^j$. Efficiency is guaranteed by bounding the surplus variable for the grand coalition around zero.

C. Dynamic Coalitional Game With Uncertainty

Let us now formulate our problem as a stabilizability problem of an uncertain dynamical system. To do this, note that equation (6) in compact form can be written as

$$B u_k = -w_k, \quad w_k := -\eta_k. \tag{7}$$

If we knew w_k (this would be possible only if we knew the power output with certainty in each day k) we could select a fair and stable allocation vector u_k to satisfy the above equation and consequently there would be no allocation error. However, in reality w_k is uncertain, and therefore we need to introduce an error dynamics in x_k, also referred to as *cumulative allocation* error, which evolves in accordance to

$$x_{k+1} = x_k + B u_k + w_k, \quad x_0 > 0, \tag{8}$$

and we refer to x_k as the *state* of the system. If the state of the system x_k is zero then it means that the allocation is in the core of the game and the allocation error is zero. To see this, note that $x_k = 0 \Rightarrow \sum_{\tau=0}^{k-1} B u_\tau = -\sum_{\tau=0}^{k-1} w_\tau$. Under uncertain w_τ to keep the state null is not possible. However, one can try to bound the error x_k in a neighborhood of zero. Our goal is then to solve the problem of steering x_k to the neighborhood of a prescribed level.

Then, consider the target set $X(0, \chi) := \{x \in \mathbb{R}^m | 0 \leq x \leq \chi\}$. The condition we wish to satisfy is $x_k \to X$ if x_k is not in X and $x_k \in X$ for all $k \geq \hat{k}$, for some finite $\hat{k} > 0$. Driving x_k to X means that the cumulative allocation error is bounded and therefore the coalitions of wind farm producers are stable. In other words, we have $0 \leq \sum_{\tau=0}^{k-1} B u_\tau + \sum_{\tau=0}^{k-1} w_\tau \leq \chi$. Note that lower bounds can always be taken equal to 0 by a proper shift of the state variables. In the following we assume that u_k and w_k lie in preassigned bounding sets, denoted by U and W, respectively. Sets U and W model bounded allocation budgets and coalitions' values. We also denote by $\mathbb{Z}_+$ the set of nonnegative integers.

Definition 2 (Robust Control Invariance): A set $X(0, \chi)$ is robust control invariant if there exists a state-feedback control strategy $\Phi : X(0, \chi) \to U \subset \mathbb{R}^{n+m}$ such that if $x_k \in X(0, \chi)$ then $x_{k+1} = x_k + B\Phi(x_k) + w_k \in X(0, \chi)$ for any disturbance $w_k \in W$.

Definition 3 (Robust Global Attraction): A set $X(0, \chi)$ is robustly globally attractive if there exists a control law $\Phi : \mathbb{R}^m \setminus X(0, \chi) \to U$ such that for every initial condition $x_0 \in \mathbb{R}^m \setminus X(0, \chi)$ and disturbance $w : \mathbb{Z}_+ \to W$, the corresponding state trajectory satisfies $x_\tau \in X(0, \chi)$ for some $\tau \in \mathbb{Z}_+$.

We wish to find the smallest control invariant and robustly globally attractive set $X^{opt}(0, \chi^{opt}) := \{x \in \mathbb{R}^m | 0 \leq x \leq \chi^{opt}\}$ and the corresponding control strategy $u_k = \Phi(x_k) \in U$, such that, for any initial condition $x_0 \in \mathbb{R}^m \setminus X(0, \chi)$ and disturbance $w : \mathbb{Z}_+ \to W$ the state is kept in the control invariant set if the initial condition belongs to it, namely $x_k \in X(0, \chi)$, for any $k > 0$ if $x_0 \in X(0, \chi)$, and the state is driven in a finite time to it if the initial condition is not in the set, namely $x_k \to X(0, \chi)$, in finite time τ if $x_0 \notin X(0, \chi)$.

We write $P_X[x]$ to denote the projection of a vector x on a set X, and we write $|x|_X$ for the distance from x to X, i.e., $P_X[x] = \arg\min_{y \in X} \|x - y\|$ and $|x|_X = \|x - P_X[x]\|$, respectively. The allocation strategy can be formulated as a robust optimization problem with finite horizon $[0, T]$:

$$\min_{u_0} \max_{w_0} \ldots \min_{u_{T-1}} \max_{w_{T-1}} |x_T|_X^2$$
$$\left. \begin{array}{l} x_{k+1} = x_k + B u_k + w_k \\ u_k \in U \\ w_k = -[J_{S_j}(y, C^*)]_{S_j \in \mathcal{S}} \in W \end{array} \right\} \quad k = 0, \ldots T-1. \tag{9}$$

Then, designing a stable allocation turns into a zero-sum game, in which player 1 allocates the revenues to drive the state x_k to $X(0, \chi)$, while player 2 (the disturbance) pushes the state far from it.

III. Decomposition Method

The main idea of this section is to isolate the uncertainty in the discrete-time dynamics (8). Let us start by decomposing (8) as

$$x_{k+1} = x_k + B u_k + w_k^{(1)} + w_k^{(2)}. \tag{10}$$

The uncertainty can be decomposed in two terms as $w_k = w_k^{(1)} + w_k^{(2)}$. The first term is the expected loss:

$$w_k^{(1)} = -[\mathbb{E} J_{S_j}(y, C^*)]_{S_j \in \mathcal{S}} + [\psi_{S_j}]_{S_j \in \mathcal{S}}. \tag{11}$$

The second term is an uncertain but bounded deviation from the expected loss, given by

$$\begin{aligned} w_k^{(2)} &= [-J_{S_j}(y, C^*) + \mathbb{E} J_{S_j}(y, C^*)]_{S_j \in \mathcal{S}} \\ &\in W^{(2)} := \{w \in \mathbb{R}^m | \underline{\delta} \leq w \leq \overline{\delta}\}, \end{aligned} \tag{12}$$

where upper and lower bounds are obtained as

$$\underline{\delta}^j := -J_{S_j}(\overline{y}, C^*) + \mathbb{E} J_{S_j}(y, C^*), \tag{13}$$
$$\overline{\delta}^j := -J_{S_j}(\underline{y}, C^*) + \mathbb{E} J_{S_j}(y, C^*). \tag{14}$$

In the above, $\underline{\delta}^j$ is the jth component of the lower bound $\underline{\delta}$ which is obtained by maximizing the profit of S_j with respect to y: $\bar{y} := \arg\max_y J_{S_j}(y, C^*) = C^*$. Then, the maximal profit for coalition S_j is

$$\max_y J_{S_j}(y, C^*) = J_{S_j}(\bar{y}, C^*) = J_{S_j}(C^*, C^*) = pC^*.$$

Substituting the above in (13), we have $\underline{\delta}^j := -pC^* + \mathbb{E}J_{S_j}(y, C^*)$. Similarly, in (14), the term $\bar{\delta}^j$ is the jth component of the upper bound $\bar{\delta}$ which is obtained by minimizing the profit of S_j with respect to y : $\underline{y} := \arg\min_y J_{S_j}(y, C^*)$. Note that as the penalty for shortage exceeds the penalty for excess, $\underline{y}$ is usually the minimum power output. Given $\underline{y}$, the minimum profit is

$$\min_y J_{S_j}(y, C^*) = J_{S_j}(\underline{y}, C^*) = pC^* - s(C^* - \underline{y}).$$

Substituting the above in (14), we have $\bar{\delta}^j := s(C^* - \underline{y}) - pC^* + \mathbb{E}J_{S_j}(y, C^*)$. We can conclude that

$$\begin{aligned} w_k^{(2)} \in W^{(2)} := \{&w \in \mathbb{R}^m | [-pC^* + \mathbb{E}J_{S_j}(y, C^*)]_{S_j\in\mathcal{S}} \\ &\leq w \leq [s(C^* - \underline{y}) - pC^* + \mathbb{E}J_{S_j}(y, C^*)]_{S_j\in\mathcal{S}}\}. \end{aligned}$$

The above defines the size of the uncertainty and allows us to relate it to the bound χ of the target set X.

IV. Design of Stable Allocations

We provide a constructive procedure to design stable allocation schemes. Let us decompose the allocation vector as $u_k = u_k^{(1)} + u_k^{(2)}$. Let us design $u_k^{(1)}$ based on the Shapley value obtained from the optimal expected profit:

$$Bu_k^{(1)} = -w_k^{(1)} = [\mathbb{E}J_{S_j}(y, C^*)]_{S_j\in\mathcal{S}} - [\psi_{S_j}]_{S_j\in\mathcal{S}}. \quad (15)$$

Under (15), dynamics (8) can be rewritten as

$$x_{k+1} = x_k + Bu_k^{(2)} + w_k^{(2)}, \quad (16)$$

along with bounding sets $U^{(2)} = \{u \in \mathbb{R}^{n+m} | u + u_k^{(1)} \in U\}$ and $W^{(2)} = \{w \in \mathbb{R}^m | w + w_k^{(1)} \in W\}$. Given matrix $B \in \{-1, 0, 1\}^{m\times(n+m)}$ and set $U^{(2)}$ we define $BU^{(2)} := \{\tilde{u} \in \mathbb{R}^m : \tilde{u} = Bu^{(2)}, \forall u^{(2)} \in U^{(2)}\}$.

Lemma 1: If $-BU^{(2)} \supseteq W^{(2)}$ the smallest invariant set is

$$X(0, \chi^{opt}) = \{x \in \mathbb{R}^m | 0 \leq x \leq \bar{\delta} - \underline{\delta}\}.$$

Proof: We need to prove that $\chi^{opt} = \bar{\delta} - \underline{\delta}$. To this purpose, note that

$$X(0, \chi) := \{x \in \mathbb{R}^m | 0 \leq x + Bu \leq \chi\}$$

is controlled invariant if and only if $\exists u \in U$ such that

$$X(0, \chi)_{D^{(2)}} := \{x \in X(0, \chi) | -\underline{\delta} \leq x + BU \leq \chi - \bar{\delta}\} \neq \emptyset.$$

The minimum value for χ for which $X(0, \chi)_{D^{(2)}} \neq \emptyset$ is then $\chi^{opt} = \bar{\delta} - \underline{\delta}$ in which case we have

$$X(0, \chi^{opt})_{D^{(2)}} := \{x \in X(0, \chi^{opt}) | -\underline{\delta} \leq x + Bu \leq -\underline{\delta}\}.$$

The above means that there must exists $u = \Phi(x) \in U$ such that $x + Bu = -\underline{\delta}$.

Now, we show that this is true under the assumption that $-BU^{(2)} \supset W^{(2)}$. To see this, let us choose a target point $x_{ref} \in \mathbb{R}^m$ as $x_{ref} = -\underline{\delta}$. If $x \in X(0, \chi^{opt})$ then $\underline{\delta} \leq x - x_{ref} \leq \bar{\delta}$ and consequently $\exists u \in U^{(2)}$ such that $Bu = x_{ref} - x$. The resulting dynamics is then $0 \leq x_{k+1} = x_{ref} + w_k^{(2)} = -\underline{\delta} + w_k^{(2)} \leq -\underline{\delta} + \bar{\delta}$. ∎

Then, the problem with finite horizon $[0, T]$ to be solved by the game designer is as follows: Find $u_k^{(2)} = \Phi(x_k)$ such that

$$\min_{u_0^{(2)}} \max_{w_0^{(2)}} \dots \min_{u_{T-1}^{(2)}} \max_{w_{T-1}^{(2)}} |x_T|^2_{X(0,\chi^{opt})}$$
$$\left.\begin{aligned} &x_{k+1} = x_k + Bu_k^{(2)} + w_k^{(2)} \\ &u_k^{(2)} \in U^{(2)}, \quad w_k^{(2)} \in W^{(2)} \end{aligned}\right\} \quad k = 0, \dots T-1. \quad (17)$$

Note that if $X(0, \chi^{opt})$ is robustly globally attractive in accordance to Definition 3, then there exists a time T for which the distance of x_T from the set $X(0, \chi^{opt})$ is zero, namely $|x_T|^2_{X(0,\chi^{opt})} = 0$. In other words the optimum value of the objective function for the minimax game in (17) is zero.

A. On Robust Global Attraction

To solve (17), let us consider the discrete-time dynamics

$$x_{k+1} = x_k + Bu_k^{(2)} + w_k^{(2)},$$

and define the new variable $\tilde{x}_{k+1} = x_k + Bu_k^{(2)}$. The proof of the following theorem is constructive and provides a solution algorithm for (17).

Theorem 1: If $-BU^{(2)} \supset W^{(2)}$ the set $X^{opt}(0, \chi^{opt})$ is robustly globally attractive.

Proof: We introduce the function $V(\tilde{x}) = \max_j |\tilde{x}^j - x^j_{ref}|$. Note that it holds $V(\tilde{x}) \geq 0$ for all $\tilde{x}$ in $\mathbb{R}^m$ and $V(\tilde{x}) = 0$ if and only if $\tilde{x} = x_{ref}$, where x_{ref} is as in Lemma 1. Furthermore, if $-BU^{(2)} \supset W^{(2)}$ we also have

$$V(\tilde{x}_{k+1}) - V(\tilde{x}_k) < 0$$

under the control strategy $u^*(\tilde{x})$ obtained from solving the following linear program (LP)

$$\min_u\{\lambda | x_{ref} \leq \tilde{x} + Bu \leq x_{ref} + \lambda\epsilon\theta, \ u \in U\}, \quad (18)$$

for sufficiently small scalar $\epsilon > 0$ and vector $\theta \in \mathbb{R}^m$, $\theta > 0$. The latter depends on the sizes of $-BU^{(2)}$ and $W^{(2)}$, see, e.g., [7, eq. (20)]. Rewrite equivalently for $\tilde{x}_k$ as follows

$$\tilde{x}_{k+1} = \tilde{x}_k + w_{k-1}^{(2)} + Bu_k^{(2)} + \epsilon\theta_k^* - \epsilon\theta_k^*,$$

where the jth component of θ_k^* is

$$\theta_k^{j*} = \begin{cases} \theta^j, & \text{if } \tilde{x}_k^j - \epsilon\theta^j > -\underline{\delta}^j, \\ \frac{\tilde{x}_k^j + \underline{\delta}^j}{\epsilon}, & \text{otherwise.} \end{cases}$$

From $-BU^{(2)} \supset W^{(2)}$ we have that there exists u which is feasible for (18) and such that $Bu = -w_{k-1}^{(2)} - \epsilon\theta^*$ therefore $\tilde{x}_{k+1}^j = \tilde{x}_k^j - \epsilon\theta^{j*}$. In light of this, by solving

$$\min\{\lambda_k | x_{ref} \leq \tilde{x}_k + Bu_k \leq x_{ref} + \lambda_k\epsilon\theta, \ u \in U\}, \quad (19)$$

we obtain $Bu_k \leq -w_{k-1}^{(2)} - \epsilon\theta^*$ which leads to $\tilde{x}_{k+1}^j \leq \tilde{x}_k^j - \epsilon\theta^{j*}$. The above implies, for all k

$$|\tilde{x}_{k+1}^j - x_{ref}^j| \leq |\tilde{x}_k^j - \epsilon\theta^{j*} - x_{ref}^j| < |\tilde{x}_k^j - x_{ref}^j|.$$

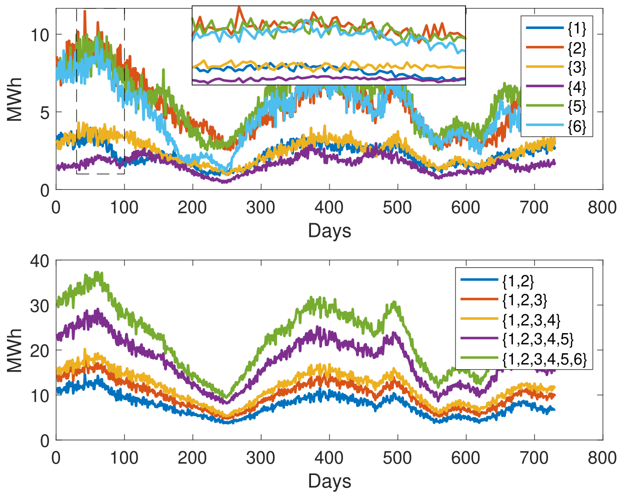

Fig. 1. Simulations of daily production for each coalition of wind farms. Each plot refers to a coalition (see legend).

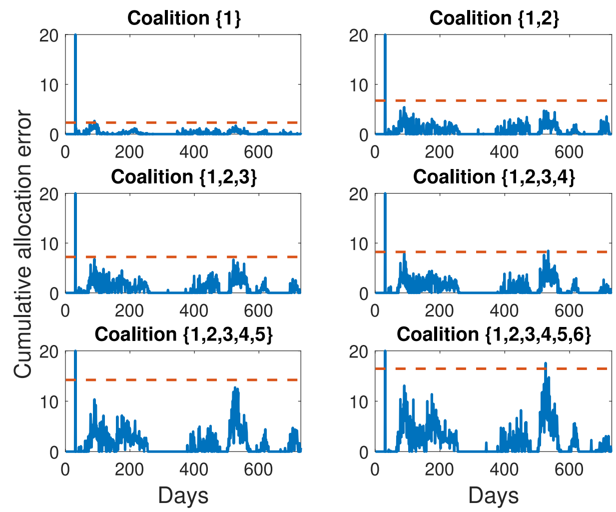

Fig. 2. Time plot of the cumulative allocation error for six coalitions of different size. The error is bounded within a neighborhood of the origin (dashed line).

As a consequence we also have

$$V(\tilde{x}_{k+1}) < \max_j |\tilde{x}_k^j - x_{ref}^j| = V(\tilde{x}_k).$$

Then $\tilde{x}_k \to x_{ref}$. Now, from $\tilde{x}_{k+1} = x_k + Bu_k^{(2)}$, we have $x_{k+1} = \tilde{x}_{k+1} + w_k^{(2)} \to x_{ref} + w_k^{(2)} = -\underline{\delta} + w_k^{(2)} \in X^{opt}$. This concludes the proof. ■

Note that (19) can be solved offline. Also it is a linear program and therefore it has bounded complexity. We refer the reader to [7, Corollary 5.1] for details on the convergence time.

V. Empirical and Numerical Studies

The analysis exploits wind power time series from the Renewable Energy Foundation (http://www.ref.org.uk). The data set contains average monthly power outputs in the period from Nov. 2017 to Oct. 2019. The wind farms are in the U.K. and are in order: the Humber Gateway (Lincolnshire), the London Array (Kent), the Thanet (off Foreness Point), the Whitelee (East Renfrewshire), the Race Bank (Grimsby) and the Gwynt y Mor (Clwyd).

The wind farms are indexed by $i \in \mathcal{N} := \{1, \dots, 6\}$. From the average monthly output we simulate the daily output for $T = 720$ days (23 months). We first interpolate linearly the data and divide by 30 to obtain the daily average samples $\tilde{y}_k$, $k = 1, \dots 720$ and then we use the recursion $y_k - \tilde{y}_k \sim N(0, \sigma^2)$. Similarly to [29], we assume Gaussian distribution and a standard deviation equal to 8%. We plot the daily power output of each wind farm in Fig. 1 (top) and the aggregate output for five possible coalitions of different size (bottom). The expected clearing price is normalized to one, i.e., $\mathbb{E}p = 1$, and is uniformly distributed in [0.5, 1.5]. The penalty price s is on average twice as much as the clearing price, that is $\mathbb{E}s = 2$. The current price is uniformly distributed in [1.5, 2.5]. Following [1] the penalty price h is null as we assume curtailment of excess energy, thus $h = 0$.

For every day after the first month, $k = 31, \dots, 720$ and every coalition we first extract the CDF F_k from the last

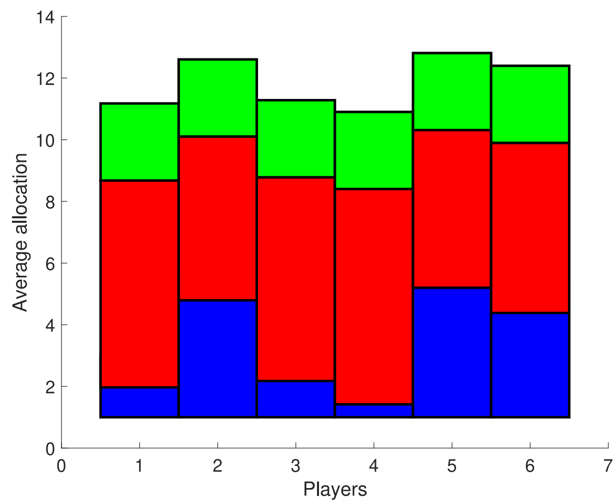

Fig. 3. Expected profit of the single wind farm $i = 1, \dots, 6$ (blue bin); extra expected allocated profit to each wind farm obtained from the Shapley vector when the single wind farm joins the grand coalition with cooperation costs (red bin), and without cooperation costs (green bin).

30 samples (1-month moving horizon) and then calculate the optimal contract size as in (4) and the expected profit as in (3). The cost of forming a coalition is assumed to be equal to three times the size of the coalition, namely $\psi_{S_j} := 3(|S_j| - 1)$. From the last 30 samples we obtain the maximum $\overline{y}$ and the minimum $\underline{y}$ wind output and use these values to obtain the bounds $\underline{\delta}^j$ and $\overline{\delta}^j$ in (13)-(14) for each coalition $j = 1, \dots, 63$.

At every iteration we then solve the optimisation in (19) and simulate the dynamics in (17). Figure 2, displays the cumulative allocation error x_k, for $k = 31, \dots 720$ for six coalitions of wind farms of different sizes. The error is initialized at 20 at time $k = 31$, which explains the initial peak outside the converging set. The plot shows that the error is bounded within a neighborhood of the origin (dashed line) in accordance to Theorem 1.

Finally to quantify the benefit of aggregation, Fig. 3 plots the expected profit of the six wind farms (blue bins) and the extra expected profit obtained from the Shapley vector when the wind farms join the grand coalition with (red bin) and without (green bin) cooperation costs.

VI. Conclusion

We have developed a method to design stable allocation mechanisms based on TU games. This is possible by correcting allocations on the real time market to respond to the deviations from the ex-ante bids. The method is characterized by exponential growth. However, in [29] it has been proven that the optimal size of coalitions is usually bounded. In other words, while aggregating players into a coalition is beneficial, such benefit decreases when the size of the coalition exceeds certain bounds. This is also in accordance with the electricity market regulation which does not admit large coalitions to avoid oligopolistic markets. Furthermore computation of allocations can be performed offline and therefore even exponential calculation can be carried out for moderately large instances. Scalability is related to one of the main properties of Banzhaf value as first discussed in [17]. This relation was first discussed in [14]. Scalability can also be obtained following the approach in [10]. In addition, sampling-based methods can provide accurate estimates of the Shapley value in polynomial time [8], [9], [20].

References

[1] E. Baeyens, E. Y. Bitar, P. P. Khargonekar, and K. Poolla, "Coalitional aggregation of wind power," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3774–3784, Nov. 2013.

[2] D. Bauso and J. Timmer, "Robust dynamic cooperative games," *Int. J. Game Theory*, vol. 38, no. 1, pp. 23–36, 2009.

[3] D. Bauso and J. Timmer, "On robustness and dynamics in (un)balanced coalitional games," *Automatica*, vol. 48, no. 10, pp. 2592–2596, 2012.

[4] E. Y. Bitar, R. Rajagopal, P. P. Khargonekar, K. Poolla, and P. Varaiya, "Bringing wind energy to market," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1225–1235, Aug. 2012.

[5] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[6] F. Blanchini, S. Miani, and W. Ukovich, "Control of production-distribution systems with unknown inputs and system failures," *IEEE Trans. Autom. Control*, vol. 45, no. 6, pp. 1072–1081, Jun. 2000.

[7] F. Blanchini, F. Rinaldi, and W. Ukovich, "Least inventory control of multistorage systems with non-stochastic unknown input," *IEEE Trans. Robot. Autom.*, vol. 13, no. 5, pp. 633–645, Oct. 1997.

[8] J. Castro, D. Gómez, E. Molina, and J. Tejada, "Improving polynomial estimation of the shapley value by stratified random sampling with optimum allocation," *Comput. Oper. Res.*, vol. 82, pp. 180–188, Jun. 2017.

[9] J. Castro, D. Gómez, and J. Tejada, "Polynomial calculation of the Shapley value based on sampling," *Comput. Oper. Res.*, vol. 36, no. 5, pp. 1726–1730, 2009.

[10] F. Fele, E. Debada, J. M. Maestre, and E. F. Camacho, "Coalitional control for self-organizing agents," *IEEE Trans. Autom. Control*, vol. 63, no. 9, pp. 2883–2897, Sep. 2018.

[11] F. Fele, J. M. Maestre, and E. F. Camacho, "Coalitional control: Cooperative game theory and control," *IEEE Control Syst. Mag.*, vol. 37, no. 1, pp. 53–69, Feb. 2017.

[12] A. Haurie, "On some properties of the characteristic function and the core of a multistage game of coalitions," *IEEE Trans. Autom. Control*, vol. 20, no. 2, pp. 238–241, Apr. 1975.

[13] W. Lee, L. Xiang, R. Schober, and V. W. S. Wong, "Direct electricity trading in smart grid: A coalitional game analysis," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 7, pp. 1398–1411, Jul. 2014.

[14] E. Lehrer, "An axiomatization of the banzhaf value," *Int. J. Game Theory*, vol. 17, no. 2, pp. 89–99, 1988.

[15] E. Lehrer, "Allocation processes in cooperative games," *Int. J. Game Theory*, vol. 31, pp. 341–351, Jun. 2003.

[16] J. M. Maestre, D. M. de la Peña, A. J. Losada, E. Algaba, and E. F. Camacho, "A coalitional control scheme with applications to cooperative game theory," *Opt. Control Appl. Methods*, vol. 35, no. 5, pp. 592–608, 2014.

[17] F. J. Muros, E. Algaba, J. M. Maestre, and E. F. Camacho, "The banzhaf value as a design tool in coalitional control," *Syst. Control Lett.*, vol. 104, pp. 21–30, Jun. 2017.

[18] F. J. Muros, E. Algaba, J. M. Maestre, and E. F. Camacho, "Harsanyi power solutions in coalitional control systems," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3369–3381, Jul. 2017.

[19] F. J. Muros, J. M. Maestre, E. Algaba, T. Alamo, and E. F. Camacho, "Networked control design for coalitional schemes using game-theoretic methods," *Automatica*, vol. 78, pp. 320–332, Apr. 2017.

[20] F. J. Muros, J. M. Maestre, C. Ocampo-Martinez, E. Algaba, and E. F. Camacho, "A game theoretical randomized method for large-scale systems partitioning," *IEEE Access*, vol. 6, pp. 42245–42263, 2018.

[21] A. Nedić and D. Bauso, "Dynamic coalitional tu games: Distributed bargaining among players' neighbors," *IEEE Trans. Autom. Control.*, vol. 58, no. 6, pp. 1363–1376, Jun. 2013.

[22] G. O'Brien, A. El Gamal, and R. Rajagopal, "Shapley value estimation for compensation of participants in demand response programs," *IEEE Trans. Smart Grid*, vol. 6, no. 6, pp. 2837–2844, Nov. 2015.

[23] C. Opathella and B. Venkatesh, "Managing uncertainty of wind energy with wind generators cooperative," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2918–2928, Aug. 2013.

[24] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1926–1940, Nov. 2012.

[25] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 77–97, Sep. 2009.

[26] W. Saad, Z. Han, and H. V. Poor, "Coalitional game theory for cooperative micro-grid distribution networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kyoto, Japan, 2011, pp. 1–5.

[27] S. Stoft, *Power System Economics: Designing Markets for Electricity*. Piscataway, NJ, USA: IEEE Press/Wiley, 2002.

[28] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Başar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1439–1450, May 2014.

[29] B. Zhang, R. Johari, and R. Rajagopal, "Competition and coalition formation of renewable power producers," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1624–1632, May 2015.