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# Is it really possible to build a bridge between cost-benefit analysis and cost-effectiveness analysis?[1]

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**Abstract**

Economists have attempted to build a welfare economic bridge between cost-benefit analysis and cost effectiveness analysis in order to give the latter a basis in welfare economics. This paper develops these attempts and finds that highly unrealistic assumptions are necessary to facilitate the constant willingness-to-pay per unit of health outcome such a bridge requires. We develop an impossibility theorem that shows it is not possible to link CBA and CEA if: (i) the axioms of expected utility theory hold; (ii) the QALY model is valid in a welfare economic sense; and (iii) illness affects the ability to enjoy consumption. We conclude that, within a welfare economic framework, it would be unwise to rely on a link between CBA and CEA in economic evaluations.

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## 1. Introduction

Welfare economists typically advocate the use of cost-benefit analysis (CBA) when evaluating public sector resource allocation decisions (see Mishan 1988). Under CBA, the costs and benefits from any given programme are expressed in monetary units, and the sign of the net benefit across all affected individuals is used as the decision criterion. CBA aims to maximise aggregate welfare and is the only methodology that, at least in theory, provides information on the absolute benefit of different programmes. However, potential ethical and methodological problems in attaching a monetary value to non-market benefits (see Hausman 1993) have led to the development of alternative methods for measuring benefits. In health economics, this has led to the development of cost-effectiveness analysis (CEA), in which health-related benefits are expressed in a single measure, such as gains in life years or quality-adjusted life-years (QALYs).[2] Indeed, CEA has been used in most economic evaluations of health care interventions (see Elixhauser *et al* 1993).

In essence, CEA considers only health-related measures of benefit to be relevant. This has led Kenkel (1997) to conclude, “when we accept the methodology of welfare economics, we should use cost-benefit analysis, not cost-effectiveness analysis”. Nonetheless, this has not prevented economists from attempting to link CEA with CBA. Such a link would be appealing to many economists since the results from the ever-increasing number of CEAs could be interpreted within a standard welfare economic framework. Johannesson (1995) has argued that where CEA counts all societal costs and uses a cost-per-QALY threshold, it can be interpreted as a CBA since the threshold value can be used to translate the non-monetary benefits in CEA into

[2] We use the term CEA to represent analyses that express benefits in any health-related units, although the term cost-utility analysis is often used when information on quality of life is combined with information on length of life.

monetary terms for CBA. To do this, there must be a constant willingness to pay (WTP) per QALY. There have been two main attempts to set out the conditions under which this will hold.

First, Johannesson and Meltzer (1998) have claimed that an article by Pratt and Zeckhauser (1996, hereafter PZ) “provides the strongest theoretical evidence to date” for the use of a constant WTP-per-QALY figure. PZ’s model uses a veil of ignorance based on perfectly comparable utility functions. Here, linking CBA and CEA requires that the benefit (in utility terms) from a given health improvement is constant across all individuals, so that maximising expected benefits behind the veil necessarily maximises aggregate health. Section 2 considers the prospects for a CBA-CEA link within PZ’s framework and shows that highly restrictive and counter-intuitive assumptions are required. Second, Bleichrodt and Quiggin (1999) show the conditions under which life-cycle preferences are consistent with QALY maximisation. By arguing that individuals will consume the same amount in each period, they set out the conditions under which all individuals weight their own QALYs equally, and so form a basis for CEA in welfare theoretic terms. Section 3 discusses the results obtained by Bleichrodt and Quiggin (1999) and argues that they do not in fact link the analyses, even when the conditions they set down are met.

In Section 4, we propose a general impossibility theorem for links between CBA and CEA and argue that, as things stand, the link must be based on unrealistic assumptions which either arbitrarily set key variables to be constant (as in Johannesson and Meltzer 1998), or which rely on special cases that do not exist (as in Bleichrodt and Quiggin 1999). The impossibility theorem shows the conditions that *any* link between CBA and CEA must satisfy under expected utility theory, and sets out the extremely stringent restrictions on the utility function that are required. Whilst we acknowledge a potential benefit of linking CBA and CEA, we conclude that it is impossible at present.

## 2. Finding a societal WTP per QALY

PZ consider how to determine the socially optimal level of expenditure on reducing mortality risks, and argue that each individual's WTP for his risk reduction must be corrected for his own risk and his own wealth. To do this, PZ place individuals behind a veil of ignorance that prevents them from perceiving their risk type and wealth level. Behind the veil, individuals are assumed to have an equal subjective probability of being each person, and so, when they maximise expected utility (EU), they also maximise the average cardinal utility of those in society behind the veil.[3] PZ show that EU maximisation requires that the cost of a marginal decrease in risk is set equal to societal WTP for that same marginal decrease. The first order conditions imply that each marginal risk reduction is valued equally where wealth is constant across society. Where wealth differs, the optimal reduction varies negatively with wealth, such that society places a greater weight on risk reductions for wealthy individuals. Behind PZ's veil, society makes decisions by maximising the average of cardinal utilities so, if good health improves the ability to enjoy wealth, then individuals with higher wealth gain a greater increase in utility from a given health improvement. Other factors being equal, this suggests that society is willing to pay a higher amount to save the life of a wealthy individual.

Of course, this violates the assumption of a constant WTP-per-QALY, as programmes focusing on the rich generate greater benefits to society for an equal number of QALYs than those focused on the poor. Johannesson and Meltzer seek to avoid this by assuming that incomes are constant across society. As income is the only non-health factor in PZ's model, there can be no interaction between health and non-health factors, and so the model is consistent with the health focus of CEA. However, the reliance on a point distribution for income necessarily places the status of any link between CBA and CEA in doubt. We begin by developing PZ's model to incorporate life expectancy and quality of life. Following PZ, we assume that there are no interpersonal aspects to utility, so that social WTP corresponds to individual WTP because individuals are unconcerned with the health of others.[4] We then find the conditions under which a constant societal WTP-per-QALY holds when the unrealistic constant income assumption is relaxed.

### *2.1 Incorporating QALYs into the model*

In the PZ model, the individual simply lives (for an unspecified period of time in an unspecified health state) or dies. Therefore, Johannesson and Meltzer's claim of a 'link' between CBA and CEA depends not only on the assumption that QALYs are a valid cardinal utility function for individuals (as argued by Johannesson 1995) but also on the implications of incorporating life expectancy and health status into PZ's model. In extending PZ's model to include length and quality of life, we allow health status to vary by the risk and income type $x$, where each type occurs with relative frequency $f(x)$. For type $x$, the type-dependent probability of illness is $p(x, e(x))$, where $e(x)$ denotes the present value of the expenditures on each individual of that type. As in PZ's paper, our model deals only with *preventative* interventions as the individual remains in whichever health state they emerge in once the veil of ignorance is lifted.[5]

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[3] This veil of ignorance is analogous to Harsanyi (1955) and is rather 'thinner' than that in Rawls (1971).

[4] Alternative specifications are possible but require certain restrictive assumptions (see Jones-Lee (1992) for an exposition of this).

[5] For curative treatments, we must consider the differential impact of treatment on individuals, and so the type-dependant expenditure of PZ's model is inappropriate unless compound lotteries are introduced. The possibilities for this are not discussed here.

As is common in the health state valuation literature, health status is bounded above by 1 (i.e. full health) and unbounded below, with death denoted by $h = 0$. Further, let utility be a function of income and health, $u(w,h)$, where marginal utility increases in health, so that:

$$0 < \frac{\partial u(w,h_0)}{\partial w} < \frac{\partial u(w,h_1)}{\partial w} \text{ for all } h_0 < h_1.$$

An individual of type $x$ is assumed to live for $L(x)$ years for certain, at which time they receive utility (which is a function of wealth at the time of death) from the legacy they leave their dependants. Each individual is assumed to discount future periods exponentially with a rate of time preference equal to the interest rate[6] and contributes towards societal expenditures with a constant amount $\bar{e}$ over their remaining lifespan[7], where this equals:

$$\bar{e} = \sum_x f(x)e(x) \Big/ \sum_y \tfrac{1}{r} f(y)\left(1 - \mathrm{e}^{-rL(y)}\right)$$

The expected utility of an individual behind a veil of ignorance is the sum of each type's EU weighted by the relative frequency, which acts as the subjective probability of being each type behind the veil. Optimal expenditures on each type were found by solving the first order conditions (see Appendix A for details) of this ex-ante expected utility.

$$(\forall x) \quad -\frac{1\Big/\frac{\partial p(x,e(x))}{\partial e}}{\frac{1}{r}\left(1 - \mathrm{e}^{-rL(x)}\right)\left(1 - h(x)\right)} = \frac{u_\Delta(w(x) - \bar{e}, h(x))}{\left(1 - h(x)\right)} \Bigg/ \frac{\frac{\partial \overline{EU}}{\partial w}}{\sum_y \frac{1}{r} f(y)\left(1 - \mathrm{e}^{-rL(y)}\right)} \qquad (1)$$

where $u_\Delta(w,h) = u(w,1) - u(w,h)$ is the utility gain from a prevented illness.

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[6] Whilst other discount functions (for instance, the hyperbolic one) may represent individual preferences more accurately, we test for the possibility of CBA-CEA link under those conditions most favourable to CEA.

[7] The assumption that PZ used (that costs were borne equally by all) raises equity issues for those with a short life expectancy and is unsuitable for the time-sensitive model presented here. Per-period contributions were rejected because the length of the time period is arbitrary.

The left hand side of (1) represents the marginal cost-per-QALY of treating type *x*, since the numerator is the marginal cost of abating a statistical illness whilst the denominator is the number of QALYs gained when a statistical illness is abated. The numerator on the right hand side is the utility value of a prevented illness per unit of health improvement, while the denominator is the marginal utility of income of an additional unit of expenditure spread across society. The right hand side of (1) is therefore society's marginal WTP-per-QALY. Therefore, EU maximisation requires that society equalise the marginal cost and marginal WTP-per-QALY for each type.

### 2.1.1 The effect of including life expectancy

In (1), differences in life expectancy between individuals affect the marginal cost-per-QALY but not the marginal WTP-per-QALY, other things being equal. Whilst those with longer life expectancies attract greater expenditure, this occurs in a way that is generally consistent with QALY maximisation. Whilst our model uses deterministic life expectancies, we do not consider it worthwhile to add a stochastic element, since this would add nothing to our critique. We note, however, that when people die (as opposed to occupying a health state equivalent to death, which is the case we consider), average cardinal utility falls directly, through their death, and indirectly, through the subsequent need to increase the contribution level for those who remain. This second factor could see treatments that prevent death or treat those with a long life expectancy receiving priority beyond that suggested by their QALY gains alone.

### 2.1.2 The effect of including health status

Health status enters (1) within a utility function that increases in income and decreases in health status. The restriction that QALYs form a valid cardinal utility function defines the way in which health enters the utility function (see Johannesson 1995). Here, the utility from a given health improvement must not be dependent on the type of person benefiting. Therefore, the ratio of utility gain and health must be constant, with the utility function consequently satisfying:

$$(\forall h) \quad u_{\Delta}(w,h) = (1-h)u_{\Delta}(w,0)$$

The first order conditions for EU maximisation become:

$$(\forall x) \quad -\frac{1 \Big/ \frac{\partial p(x,e(x))}{\partial e}}{\frac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)\left(1-h(x)\right)} = u_{\Delta}(w(x)-\bar{e},0) \Bigg/ \frac{\frac{\partial \overline{EU}}{\partial w}}{\sum_{y} \frac{1}{r} f(y)\left(1-\mathrm{e}^{-rL(y)}\right)}$$

Where income is constant, the societal WTP-per-QALY becomes constant with respect to health status. A representation of the type of $u_{\Delta}$ consistent with this assumption is displayed in Figure 1. Note that we require the ‘constant incomes’ assumption since a cursory examination of the diagram reveals that the utility gain from a prevented illness is a function of income. Contrast this with Figure 2, which shows a $u_{\Delta}$ function where the WTP-per-QALY assumption is violated. At any positive level of income, the lines (though equidistant in health status) exhibit decreasing marginal gains in utility.

*2.2 Relaxing the constant income assumption*

Johannesson and Meltzer’s claims of a link between CBA and CEA are tenuous, as they rest on a special case of economic unreality where income is constant across society. For a more robust link between CBA and CEA, the utility function must place a constant value on a given health improvement that varies in neither health nor income. The utility gain of a prevented illness must therefore satisfy:

$$(\forall w,h) \quad u_{\Delta}(w,h) = (1-h)u_{\Delta}(0,0)$$

This implies that utility function at a point in time must linearly separable into health and non-health factors with a constant trade-off between wealth and health that is common to all. The intertemporal utility function of a person with wealth *w*, health *h*, and deterministic life expectancy *L* equals:

$$U(w,h,L) = \tfrac{1}{r}u(w,0) + \tfrac{\lambda}{r}\left(1 - e^{-rL(x)}\right)h$$

where $\lambda$ is the common trade-off between the health and non-health portions of utility.[8]

The significance of this restriction is clear in the contrast between Figures 3 and 4. Figure 3 shows a utility function in a world where CBA and CEA are linked when incomes are constant. At any level of income, the value an individual places on preventing an illness is a function of the severity of that illness alone. Figure 4 shows a utility function where the link between CBA and CEA remains but where incomes may vary. Here, the utility that an individual places on a prevented illness holds *across* wealth levels.

Therefore, two additional constraints must be satisfied in addition to the conditions required by PZ's model. First, the utility value of a prevented illness must depend only on the size of the health improvement, and second, the ability of an individual to enjoy wealth cannot be affected by illness. The latter of these conditions is highly counter-intuitive and is not supported by empirical evidence (see Evans and Viscusi 1993 and Sloan *et al* 1998), making any link between CBA and CEA of the sort Johannesson and Meltzer propose untenable.

[8] Note that any positive linear transformation of the utility function must affect the entire function $U(w, h, L)$ and not just $u(w, 0)$. The common trade-off between the health and non-health portions of utility cannot be weakened by transforming only *u*.

In addition, Johannesson and Meltzer's argument requires that all rational economic agents share the same views about society from behind an appropriately specified veil of ignorance. The operational device of the veil of ignorance has been the subject of much controversy in the philosophical literature (see Barry 1989) and so, even if the restrictions outlined here were to be established empirically, the framework may still be a difficult one to accept. Whilst we must use a cardinal utility function in any link between CBA and CEA, we do not have to use a veil of ignorance to achieve it. We now turn our attention to the arguments put forward by Bleichrodt and Quiggin (1999).

## 3. Life cycle preferences over consumption and health

Bleichrodt and Quiggin consider the conditions under which utility maximisation over consumption is consistent with QALY maximisation. Their analysis produces conditions that they suggest link CBA and CEA and which appears to require neither a "constant incomes" assumption nor linear separability under EU. Their analysis notes that any function consistent with QALY maximisation must be a positive linear transformation of the health status enjoyed in each period:

$$U((c_1, h_1),...,(c_T, h_T)) = \sum_{t=1}^{T} \left( v(c_t) h_t + w_t(c_t) \right),$$

where $c_t$ is consumption in period *t* and $h_t$ is the health state in period *t*.

If, as Bleichrodt and Quiggin suggest, the utility of any state equivalent to death is the same then we can ignore $w_t(c_t)$. For the utility function to be a valid positive linear transformation of the number of QALYs it becomes sufficient to ensure $v(c_t)$ is constant. Bleichrodt and Quiggin facilitate this by arguing that consumption is smoothed to ensure the same consumption in each period. Utility functions are therefore of the form:

$$U(\hat{c}; h_1, ..., h_T) = v(\hat{c})\sum_{t=1}^{T} h_t \qquad (2)$$

where $\hat{c}$ is smoothed consumption and $h_t$ is the health state in period $t$

However, very few people are in a position to enjoy such constant consumption. For example, imperfect credit markets prevent consumption smoothing, uncertain life expectancy makes calculation of optimal consumption impossible, and differences in expectations over lifestyle in different life years makes constant consumption unlikely.[9] If the constant consumption assumption is relaxed, then Bleichrodt and Quiggin acknowledge that individual WTP-per-QALY depends on the level of consumption.

Moreover, there are unresolved issues even where consumption smoothing is possible. Firstly, all individuals must agree on a WTP-per-QALY figure to be used in CEA. This is highly unlikely since, as Bleichrodt and Quiggin themselves acknowledge, each individual's WTP-per-QALY will increase in (smoothed) consumption. Therefore, we still require that discounted lifetime income be the same across individuals. Secondly, since health and consumption may be correlated, an individual who is ill may have lower income over his remaining lifespan, which may reduce his per-period consumption. In this case, health improvements improve utility from increased smoothed consumption in addition to its direct effect. Since both are included in individual WTP, we would expect that those interventions that improve earning capacity would be valued more highly under CBA than those that do not.

Finally, Bleichrodt and Quiggin's model only considers income and health. If a welfare economic bridge is to be built between CBA and CEA then *all* other possible arguments in the

[9] Whilst a perfectly competitive market for annuities can address the problems associated with a stochastic life expectancy, this simply necessitates the swapping of one unrealistic assumption for another.

utility function must be considered. Since there are non-wealth and non-health factors (e.g. autonomy, self-respect, and social standing) that might affect the coefficient on the health-utility index in (2), the following section develops a slightly more general view accommodating such factors, and shows that a link between CBA and CEA is highly unlikely within an EU framework.

## 4. The (im)possibility of a link between CBA and CEA

Following Bleichrodt and Quiggin, we impose the restrictions of *marginality* and *symmetry* on preferences. Marginality allows us to represent preferences as the sum of a series of period-specific utility functions, whilst symmetry guarantees that the same utility function is used in all periods. Where EU maximisation also holds, we can represent an individual's utility function as:

$$U(c_1 ... c_T, \boldsymbol{N}_1 ... \boldsymbol{N}_T, h_1 ... h_T) = \sum_{t=1}^{T} U(c_t, \boldsymbol{N}_t, h_t)$$

where $c_t$ is consumption, $\boldsymbol{N}_t$ is a vector of other non-health factors and $h_t$ is health status at time $t$

We begin by assuming that health does not affect the non-health variables ($c$ and $\boldsymbol{N}$) directly, so that WTP can be found by summing the WTP for each period's improvement in health. We take any link between CBA and CEA to be satisfied where individual WTP is a linear function of the number of individual QALYs gained and where the marginal WTP-per-QALY figure is common to all.

Theorem 1:

A utility function, $U(c_1...c_T, \boldsymbol{N}_1...\boldsymbol{N}_T, h_1...h_T)$, linking CBA and CEA cannot embody all of the following:

1. The axioms of EU theory.
2. Marginality.
3. Symmetry.
4. Weak concavity in consumption ($\frac{\partial U}{\partial c_t}, \frac{\partial^2 U}{\partial c_t^2}$ exist with $\frac{\partial U}{\partial c_t} > 0$ and $\frac{\partial^2 U}{\partial c_t^2} \leq 0$).
5. The property that health improves the ability to enjoy ($\frac{\partial^2 U}{\partial h \partial c} > 0$).

Under a CBA-CEA link, Conditions 1 through 4 require that the marginal utility of consumption does not change in response to a change in health status, so that Condition 5 cannot also hold (for a proof of this, see Appendix B). Where Condition 5 holds then the WTP for a given health improvement will necessarily be greater for those in poor health where utility is less sensitive to consumption. For a QALY to have a monetary value that holds for all individuals across society, we must restrict the utility function so that consumption and health are linearly separable in the utility function. In addition utility must be a linear function of these factors, since any non-linearity will cause the WTP-per-QALY to vary according to the initial levels of health and consumption.

However, Conditions 1 through 4 are not yet sufficient for a link between CEA and CBA, since factors other than health and consumption may affect the utility function even where health and consumption enter separately and linearly. Specifically, these factors may be either valued for their own sake or may affect the marginal utility of consumption. Only where both these possibilities are precluded can a link between CBA and CEA be made.

Theorem 2:

To accommodate a link between CBA and CEA, a utility function, $U(c_1...c_T, \boldsymbol{N}_1...\boldsymbol{N}_T, h_1...h_T)$ satisfying Conditions 1 through 4 must also assume one of the following restrictions:

6(i) Health and consumption are be the only factors in the utility function ($\partial U / \partial \boldsymbol{N}_{ti} = 0$).

6(ii) Any non-health, non-consumption factor(s) must remain fixed in all circumstances ($\boldsymbol{N}$ constant).

Condition 6(i) would explicitly remove any factors other than health and consumption, whilst 6(ii) requires that any such factors must be related to the identity of the individual. In different ways, both these assumptions prevent factors such as autonomy and self-respect from entering meaningfully into an individual's WTP. In both cases, the per-period utility function can be isolated down to a positive linear transformation of the function $U(c, \boldsymbol{N}, h) = c + \lambda h$, where $\lambda$ is the common trade-off between consumption and health and the coefficients of this transformation may include person-specific factors $\boldsymbol{N}$ (under 6(ii)).

For Condition 5 to be accommodated within a link between CEA and CBA, we must relax or replace at least one of Conditions 1-4. However, under EU (Condition 1), both marginality (Condition 2) and symmetry (Condition 3) are required for utility to be the sum of identical per-period utility functions. Condition 4 guarantees a "nice" WTP function, and requires only that the utility function be (strictly) monotonic and concave in consumption. Using a utility function that is constant or decreasing over some levels of consumption (and which is almost certainly non-concave) will also disqualify almost all standard utility functions and so is difficult to justify.

This leaves only Condition 1, which places the link between CBA and CEA within an EU framework. Bleichrodt and Quiggin additionally consider the link under rank-dependent EU but it is unlikely that their analysis could lead to a valid link between CBA and CEA that avoided the problems outlined in Section 4. Generalisations of EU use utility functions in which health can interact with other factors, causing asymmetric weightings and confounding any link between CBA and CEA. Since the normative appeal of CBA may be questionable under theories outside generalised EU, the prospects of a suitable link seem rather dim.

## 5. Conclusion

CEA is increasingly being used to evaluate resource allocation decisions in health care. Most forms of CEA involve the maximisation of an effect variable for a given budget, which typically involves funding all programmes with a cost-per-unit-outcome below a certain threshold level. Economists have considered the extent to which this form of analysis is compatible with a standard welfare economic framework and, in particular, with CBA. Under welfarist models, individuals are the best judges of their own welfare and individual WTP is taken to be the appropriate monetary valuation of any benefit. CBA sums WTP over all those affected and compares this figure to net costs, implementing only those programmes that increase net benefit (defined in monetary terms). CBA is seen as the welfarist ‘gold standard’, whilst CEA can be argued to lack a theoretical foundation (see Johannesson and Karlsson, 1997).

Some economists have attempted to find conditions under which CBA and CEA produce identical results. Here, a constant cost-per-QALY value must be used (Johannesson, 1995). But this is problematic because the use of one societal WTP-per-QALY figure means that differences in individual valuations of a QALY have to be ignored. Simply overriding individual preferences will do this, but this does not sit easily with the welfarist tradition. Alternatively, conditions can be imposed on individual preferences and this is the approach favoured by many economists.

We have considered two attempts to link CBA and CEA. The first, by Johannesson and Meltzer (1998), requires that incomes be held constant across individuals for WTP to be proportional to the QALY gain. In relaxing this assumption, we find that health must be additively separable to consumption in the utility function, since a relationship between health and income would influence the ability of an individual to enjoy consumption. However, this 'link' does not build a suitable bridge between CBA and CEA in the strict welfarist sense since individual judgements (about the trade-offs between health and income) are overruled in formulating a societal CBA. The second attempt to link CBA and CEA, by Bleichrodt and Quiggin (1999), differs in that individual WTP figures are used. Whilst they find conditions under which individuals would choose to maximise QALYs under a given cost-per-QALY threshold, this threshold will differ across individuals and, without a common threshold, their analysis is not consistent with a single implementation of CEA and so no substantive link exists here either.

We have developed an impossibility theorem that shows that it is not possible to link CBA and CEA if: (i) the axioms of EU theory hold; (ii) the QALY model is valid in a welfare economic sense; and (iii) illness hinders the ability to enjoy consumption. Since (iii) is intuitive and (ii) is essential for CEA, the relaxation of (i) is the only real avenue open for a meaningful welfare economic link between CBA and CEA (at least where costs are assessed from a societal perspective).

In showing that there is currently no meaningful link between CBA and CEA, we have also shown that CEA is not currently justifiable on strictly welfarist grounds. Instead, CEA would seem to be justifiable only on non-welfarist grounds where the output of health care is judged according to its contribution to health itself, rather than according to the extent to which it contributes to overall welfare (as determined by individual preferences). The normative justification for this focus on people's *objective needs* rather than on their *subjective demands* owes much to Sen's (1980) concept of 'basic capabilities'. Culyer (1989) draws heavily on Sen when he argues that health is a crucial characteristic that is important for people's capability to 'flourish' as human beings.

It appears to us that CBA and CEA have such fundamentally different ethical underpinnings, that it would seem futile to further attempt to reconcile them within the welfare economic paradigm. Rather than attempting to find a bridge that is able to reconcile the central conflict between utility and health maximisation, attention should instead be focused on the debate about the appropriateness of CBA vis-à-vis CEA. One way forward might be to consider the extent to which people prefer health care to be distributed according to the principle 'to each according to need' rather than 'to each according to willingness (and ability) to pay' i.e. the extent to which, as citizens, they might be willing to override their preferences as consumers. Whatever the details, future research on the relative merits of CBA and CEA must also consider the relative merits of welfarist and non-welfarist philosophies in the context of allocation decisions in health care (see Brouwer and Koopmanschap 2000).

## APPENDIX A – The Calculation of EU in the updated PZ model

Each individual receives utility from both consumption and the legacy left to her descendants, both of which are discounted exponentially at the prevailing interest rate. Intertemporal utility for an individual with certain income *w*, health *h*, and life expectancy *L* therefore equals:

$$U(w,h,L) = \int_0^L \mathrm{e}^{-rt} u(w,h)\mathrm{d}t + \int_L^\infty \mathrm{e}^{-rt} u(w,0)\mathrm{d}t = \tfrac{1}{r}\left(1-\mathrm{e}^{-rL}\right)u(w,h) + \tfrac{1}{r}\mathrm{e}^{-rL}u(w,0)$$

As the probability of an illness occurring for an individual of type x is p(x, e(x)), EU is:

$$\begin{aligned} EU(w,h(x),L(x)) &= p(x,e(x))U(w,h(x),L(x)) + \left(1 - p(x,e(x))\right)U(w,1,L(x)) \\ &= U(w,1,L(x)) - p(x,e(x))\tfrac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)u_\Delta(w,h(x)) \end{aligned}$$

where $u_\Delta(w,h) = u(w,1) - u(w,h)$

Given constant contribution towards risk-reducing expenditures, type-dependent EU becomes:

$$EU(w(x)-\bar{e},h(x),L(x)) = U(w(x)-\bar{e},1,L(x)) - p(x,e(x))\tfrac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)u_\Delta(w(x)-\bar{e},h(x))$$

EU behind the veil of ignorance is

$$\overline{EU} = \sum_x f(x)U(w(x)-\bar{e},1,L(x)) - \sum_x f(x)p(x,e(x))\tfrac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)u_\Delta(w(x)-\bar{e},h(x))$$

where $\bar{e} = \sum_x f(x)e(x) \Big/ \sum_y \tfrac{1}{r} f(y)\left(1 - \mathrm{e}^{-rL(y)}\right)$.

Therefore, the first order conditions for the EU maximising type-dependent expenditure:

$$\begin{aligned} (\forall x) \quad 0 &= -f(x)\tfrac{\partial p(x,e(x))}{\partial e}\tfrac{1}{r}\left(1 - \mathrm{e}^{-rL(x)}\right)u_\Delta(w(x)-\bar{e},h(x)) + \tfrac{\partial \overline{EU}}{\partial w}\tfrac{\partial w}{\partial \bar{e}}\tfrac{\partial \bar{e}}{\partial e} \\ &= -f(x)\tfrac{\partial p(x,e(x))}{\partial e}\tfrac{1}{r}\left(1 - \mathrm{e}^{-rL(x)}\right)u_\Delta(w(x)-\bar{e},h(x)) - \frac{f(x)\tfrac{\partial \overline{EU}}{\partial w}}{\sum_y \tfrac{1}{r} f(y)\left(1 - \mathrm{e}^{-rL(y)}\right)} \end{aligned}$$

On rearranging, the first order conditions become:

$$(\forall x) \quad -\frac{1 \Big/ \frac{\partial p(x,e(x))}{\partial e}}{\frac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)} = u_\Delta(w(x)-\bar{e}, h(x)) \Bigg/ \frac{\frac{\partial \overline{EU}}{\partial w}}{\sum_y \frac{1}{r} f(y)\left(1-\mathrm{e}^{-rL(y)}\right)}$$

To aid interpretation, this is divided by the benefits from a successful cure, (1 - $h(x)$). Therefore:

$$(\forall x) \quad -\frac{1 \Big/ \frac{\partial p(x,e(x))}{\partial e}}{\frac{1}{r}\left(1-\mathrm{e}^{-rL(x)}\right)\left(1-h(x)\right)} = \frac{u_\Delta(w(x)-\bar{e}, h(x))}{\left(1-h(x)\right)} \Bigg/ \frac{\frac{\partial \overline{EU}}{\partial w}}{\sum_y \frac{1}{r} f(y)\left(1-\mathrm{e}^{-rL(y)}\right)}$$

## APPENDIX B – Proof of Theorems

Theorem 1

Conditions (1)-(3) allow us to represent utility as the sum of identical per-period utility functions (as required by the QALY model). This also allows us to find WTP by summing period-specific WTP figure across periods. For a link between CBA and CEA, we must have a constant WTP-per-QALY. We proceed by showing that this is impossible under all of Conditions (1)-(5).

Let $U(c, \boldsymbol{N}, h)$ be the per-period utility function. Condition (4) guarantees that this function is one-to-one in consumption (holding $h$ and $\boldsymbol{N}$ constant) and guarantees the existence of a consumption-specific inverse that returns the consumption required to achieve a specific level of utility. For such a $C(u, \boldsymbol{N}, h)$, we know that $\frac{\partial C}{\partial u} > 0$ and $\frac{\partial^2 C}{\partial u^2} \geq 0$.

Consider two periods, 0 and 1, in which health differs. WTP is given by the function:

$$\begin{aligned} g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) &= c_0 - C(u, \boldsymbol{N}_1, h_1) \\ &= c_0 - C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1) \end{aligned}$$

This function must satisfy $g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) = \lambda(h_1 - h_0)$ for some common $\lambda \in \Re$ if individual WTP is to be consistent with an implementation of CEA. For the WTP for marginal changes in $h_0$ (holding $h_1$ constant) to be invariant under changes in consumption then:

$$\frac{\partial^2 g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1)}{\partial c_0 \partial h_0} = -\frac{\partial^2 C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1)}{\partial u^2} \frac{\partial^2 U(c_0, \boldsymbol{N}_0, h_0)}{\partial c_0 \partial h_0} = 0$$

Here, either the utility function is additively separable into portions that consider health and consumption (Case I) and/or utility enters linearly into the compensation function (Case II).

*Case I*

Here $\partial^2 U(c_0, \boldsymbol{N}_0, h_0) / \partial c_0 \partial h_0 = 0$ and the utility function is of the form

$$U(c, \boldsymbol{N}, h) = \tfrac{1}{V_1(\boldsymbol{N})} c + \tfrac{1}{V_2(\boldsymbol{N})} h + W(\boldsymbol{N})$$

for some positive functions $V_1(\boldsymbol{N}), V_2(\boldsymbol{N})$. The theorem is established for this case because (5) cannot be accommodated within this function, which uses (1)-(4).

For marginal WTP in $h_1$ (holding $h_0$ constant) to be invariant to changes in $\boldsymbol{N}_1$, we also require:

$$(\forall i) \qquad \frac{\partial^2 g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1)}{\partial (\boldsymbol{N}_1)_i \partial h_1} = -\frac{\partial^2 C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1)}{\partial (\boldsymbol{N}_1)_i \partial h_1} = 0$$

So that $V_1(\boldsymbol{N}) = \lambda V_2(\boldsymbol{N})$,

$$U(c, \boldsymbol{N}, h) = \tfrac{1}{V_1(\boldsymbol{N})}(c + \lambda h) + W(\boldsymbol{N}),$$

And $\quad g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) = c_0 + \lambda h_1 - \tfrac{V_2(\boldsymbol{N}_1)}{V_2(\boldsymbol{N}_0)}(c_0 + \lambda h_0) - \lambda V_2(\boldsymbol{N}_1)(W(\boldsymbol{N}_1) - W(\boldsymbol{N}_0))$

*Case II*

We proceed with ${\partial^2 C}/{\partial u^2} = 0$ so that $C(u, \boldsymbol{N}, h) = \alpha u + u \cdot Y(\boldsymbol{N}, h) + Z(\boldsymbol{N}, h)$ for some $\alpha \in \Re$ and $Y(\boldsymbol{N}, h) > 0$. For marginal WTP for changes in $h_1$ to be invariant to changes in $\boldsymbol{N}_1$, we require:

$$(\forall i) \qquad \frac{\partial^2 g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1)}{\partial(\boldsymbol{N}_1)_i \partial h_1} = -\frac{\partial^2 C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1)}{\partial(\boldsymbol{N}_1)_i \partial h_1} = 0$$

So that $C(u, \boldsymbol{N}_1, h_1) = \alpha u + uY_1(h_1) + uY_2(\boldsymbol{N}_1) + Z_1(h_1) + Z_2(\boldsymbol{N}_1)$.

The analogous restriction for $h_1$ vs. $c0$ is:

$$\frac{\partial^2 g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1)}{\partial c_0 \partial h_1} = -\frac{\partial^2 C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1)}{\partial u \partial h_1} \frac{\partial U(c_0, \boldsymbol{N}_0, h_0)}{\partial c_0} = 0$$

And since $\frac{\partial U}{\partial c} > 0$, $C(u, \boldsymbol{N}_1, h_1) = u \cdot (\delta + Y_2(\boldsymbol{N}_1)) + Z_1(h_1) + Z_2(\boldsymbol{N}_1)$.

Again, since changes in $h_1$ cannot affect marginal WTP for changes in $h_1$, we have.

$$\frac{\partial^2 g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1)}{\partial {h_1}^2} = -\frac{\partial^2 C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_1, h_1)}{\hat{\partial} {h_1}^2} = 0$$

So that $C(u, \boldsymbol{N}_1, h_1) = u \cdot (\delta + Y_2(\boldsymbol{N}_1)) + Z_2(\boldsymbol{N}_1) - \lambda h_1$

Finally, since $c_0 = C(U(c_0, \boldsymbol{N}_0, h_0), \boldsymbol{N}_0, h_0)$.

$$c_0 = U(c_0, \boldsymbol{N}_0, h_0)(\delta + Y_2(\boldsymbol{N}_0)) + Z_2(\boldsymbol{N}_0) - \lambda h_0$$

and $\qquad U(c_0, \boldsymbol{N}_0, h_0) = \frac{1}{\delta + Y_2(\boldsymbol{N}_0)}(c_0 - Z_2(\boldsymbol{N}_0) + \lambda h_0)$

Under a CBA-CEA link, the marginal utility of income is unaffected by the health level, so that (5) cannot hold alongside (1)-(4).

<u>Theorem 2</u>

We can see that the utility function above is trivially different to that required for a CEA-CBA link since we require:

$$g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) = \lambda(h_1 - h_0) \qquad (1)$$

In Theorem 1, Conditions (1)-(4) provide terms in all five variables. For Case I above we have:

$$g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) = c_0 + \lambda h_1 - \tfrac{V_2(\boldsymbol{N}_1)}{V_2(\boldsymbol{N}_0)}(c_0 + \lambda h_0) - \lambda V_2(\boldsymbol{N}_1)(W(\boldsymbol{N}_1) - W(\boldsymbol{N}_0)) \qquad (2)$$

While for Case II:

$$g(c_0, \boldsymbol{N}_0, h_0, \boldsymbol{N}_1, h_1) = c_0 - Z_2(\boldsymbol{N}_1) + \lambda h_1 - \tfrac{\delta + Y_2(\boldsymbol{N}_1)}{\delta + Y_2(\boldsymbol{N}_0)}\left(c_0 - Z_2(\boldsymbol{N}_0) + \lambda h_0\right) \qquad (3)$$

where the functions differ in each case.

In both cases the non-health, non-consumption factors, $\boldsymbol{N}$, confound any CEA-CBA link since they may have a direct affect on utility (through $W(\boldsymbol{N})$ and $Z_2(\boldsymbol{N})$), and can also affect the marginal utility of consumption (through $V_2(\boldsymbol{N})$ and $Y_2(\boldsymbol{N})$).

CEA and CBA are linked only where both these possibilities are precluded. To see this, let us substitute (1) into (2). Solving this, we find that we require both $V_2(\boldsymbol{N}_1) = V_2(\boldsymbol{N}_0)$ and $W(\boldsymbol{N}_1) = W(\boldsymbol{N}_0)$ (since $V_2(\boldsymbol{N}) > 0$). Likewise, substituting (1) into (3), we find that we require $Y_2(\boldsymbol{N}_1) = Y_2(\boldsymbol{N}_0)$ and $Z_2(\boldsymbol{N}_1) = Z_2(\boldsymbol{N}_0)$. Where these restrictions are not met then for some values of the non-health, non-consumption factors then either the marginal utility of consumption changes or a non-health factor must be compensated for directly (that is $\exists \boldsymbol{N}_0, \boldsymbol{N}_1$: $Y_2(\boldsymbol{N}_1) \neq Y_2(\boldsymbol{N}_0)$ or $Z_2(\boldsymbol{N}_1) \neq Z_2(\boldsymbol{N}_0)$).

To accommodate these restrictions, we require either that the functions are constant and/or that the $\boldsymbol{N}$ values are constant. In Case I, where $V_2(\boldsymbol{N}_i) = k$ and $W(\boldsymbol{N}_i) = l$ we have:

$U(c,h) = \frac{1}{k}(c + \lambda h) + l$ and:

$$g(c_0, h_0, h_1) = c_0 + \lambda h_1 - (c_0 + \lambda h_0) - \lambda k(l - l)$$
$$= \lambda(h_1 - h_0)$$

While for Case II, $Y_2(\boldsymbol{N}_i) = \delta_2 - \delta > 0$ and $Z(\boldsymbol{N}_i) = n$ we have:

$$U(c_0, h_0) = \tfrac{1}{\delta_2}(c_0 + \lambda h_0) + \tfrac{n}{\delta_2}$$

and
$$g(c_0, h_0, h_1) = c_0 - n + \lambda h_1 - \left(c_0 - n + \lambda h_0\right)$$
$$= \lambda(h_1 - h_0)$$

In both cases non-health, non-consumption factors are indistinguishable from a linear transformation of the utility function. Where $V_{2,}$ $W$, $Y_2$, and $Z_2$ are constant across all values of ***N*** non-health, non-consumption factors have no meaning and can be said not to exist. Where ***N*** is constant across all possible outcomes then only those non-health, non-consumption factors that are specific to a person can be said to exist, and even here they cannot be said to affect WTP in any meaningful way.

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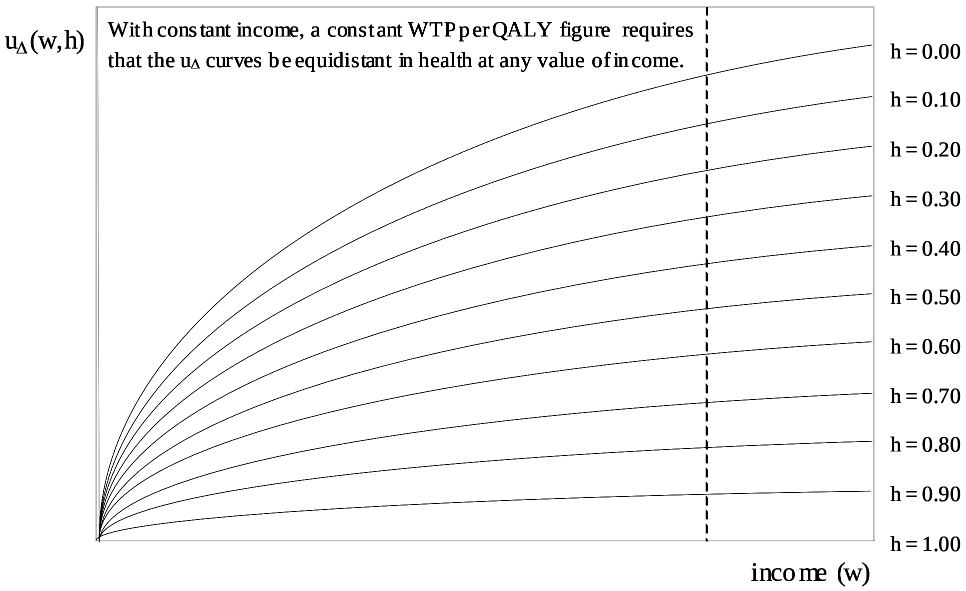
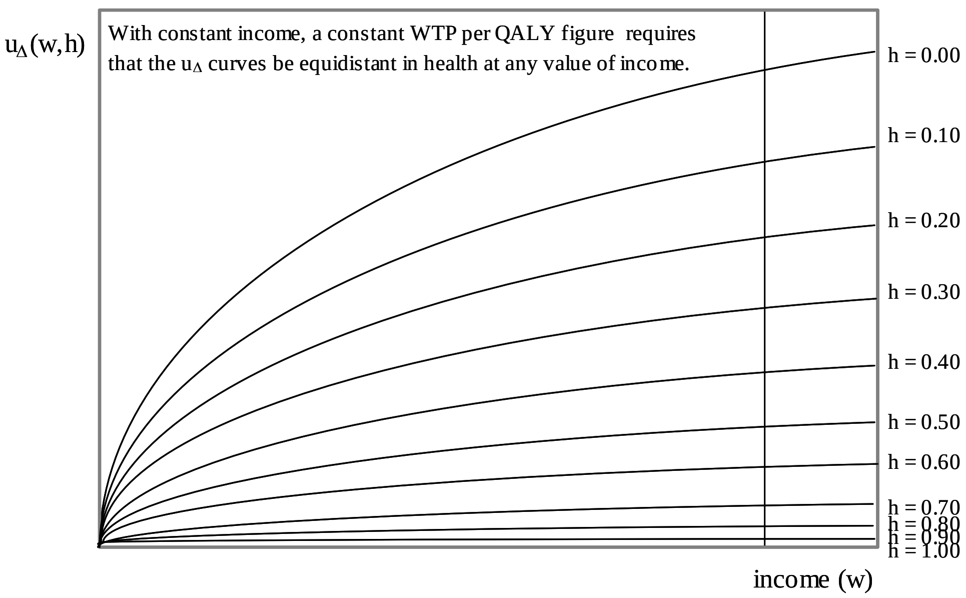
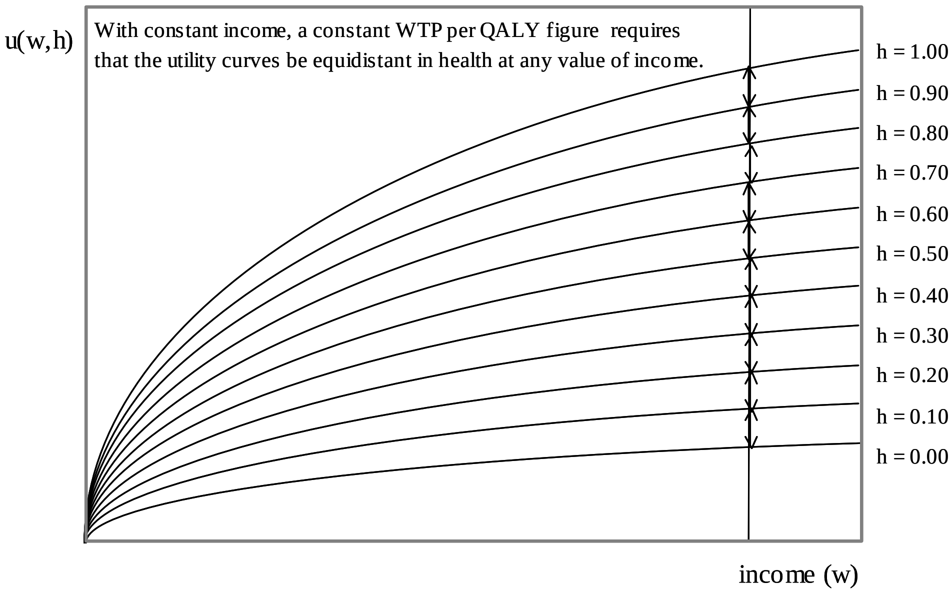
## Figure 1: Constant WTP per QALY at any given income level




## Figure 2: Variable WTP-per-QALY at any positive income level



## Figure 3: Constant WTP per QALY at any given income level




## Figure 4: Constant WTP per QALY at all income levels.