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Uncertainty shocks of Trump election in an interval model of stock market

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This paper proposes a new class of nonlinear interval models for interval-valued time series. By matching the interval model with interval observations, we develop a nonlinear minimum-distance estimation method for the proposed models, and establish the asymptotic theory for the proposed estimators. Superior to traditional point-based methods, the proposed interval modelling approach can assess the change in both the trend and volatility simultaneously. Within the proposed interval framework, this paper examines the impact of the 2016 US presidential election (henceforth Trump election) on the US stock market as a case study. Considering the validity of daily high-low range as a proxy of market efficiency, we employ an interval-valued return to jointly measure the fundamental value movement and market efficiency simultaneously. Empirical results suggest a strong evidence that the Trump election has increased the level/trend and lowered the volatility of the S&P 500 index in both ex ante and ex post analysis. Furthermore, a longer half-life period for the impact on fundamental value (62.4 days) than high-low range (15.9 days) has shown that the impact of Trump's victory on fundamental value is more persistent than its impact on market efficiency.

Keywords: Interval dummy variables; Interval time series; Nonlinear minimum-distance estimator; Range volatility; Trump election

JEL Classification: C2, C13

1. Introduction

Political uncertainty has drawn increasing attention in economics and finance over the last few decades; see Bloom (2009), Baker *et al.* (2014, 2016), Brogaard and Detzel (2015), and Basu and Bundick (2017). In financial markets, political risks broadly refer to the uncertainty of return on investment caused by the instability of political policies and arrangements; see Bernanke (1983) and Pastor and Veronesi (2005). It is widely acknowledged that political risks are important sources of systematic risks, which cannot be hedged by investment diversification. As a typical example, the relationship between the Trump election and financial performance in US stock market is analyzed in this paper. In September 2016, the *Financial Times* columnist Wolfgang Münchau pointed out that the victory of Donald Trump might occur and it could be the most cataclysmic global political event of that year. Therefore, this paper attempts to understand the impacts of Trump's election on financial variables regarding assets' fundamental values and market efficiency.

Different from most other political risks, Trump's election can be regarded as an exogenous shock to the stock market. In many cases, the strong co-movement between economic variables and political policies strongly motivates us to examine the impact of political uncertainty. An appropriate approach should enable us to assess political risks' impacts under the control of other correlated macro-economic variables. It is an intractable problem that different choices of control variables may lead to different results. However, one convenient method to study the impact of Trump's election is that the co-movement between election and macro-economic variables is expected to be very weak. Unlike many events which affect stock markets (e.g. rights issues, take-overs, wars, etc.), the date of an election (once announced) is known and only its outcome is uncertain (e.g. Gemmill 1992). Actually, this outcome is quite contrary to many people's expectation. For instance, the probability of Clinton winning on Betfair was 83%, while on the day of the election, the FiveThirtyEight

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forecast,† which gave Trump the highest probability of winning still puts the Clinton odds at almost 71%; see Wagner *et al.* (2017). Hence, Trump's victory can be regarded as an exogenous shock which is barely reflected by any macroeconomic variables in the short-term.

Recently, there is a growing body of literature that attempts to assess Trump election's impacts on financial markets. Wolfers and Zitzewitz (2016) compare the financial performance before and after Clinton and Trump's presidential debate event on September 26, 2016. Their results reveal that Trump's victory will reduce assets' prices and increase volatility in international stock markets. Bouoiyour and Selmi (2017) examine the fluctuation of various stock indices in US stock market surrounding the election day, and find that Trump's victory divides the investors into losing and winning sectors. Wagner *et al.* (2017) utilize individual firms' data to investigate the Trump election's effect on US financial markets, and conclude High-Beta firms and High-Tax firms are boosted by this unexpected outcome. Born *et al.* (2017) analyze the relation between Trump's tweets and financial market's reaction, and confirm that common stocks' abnormal returns indeed respond to Trump's tweets positively. Hoe and Nippani (2017) attempt to study Trump election's impact on China's stock market, but they do not attain any conclusive results.

However, the aforementioned literature, which mainly focuses on Trump election's impact on assets' prices, does not give enough consideration to market efficiency. Market efficiency is commonly defined as financial market's ability to reflect assets' intrinsic values. The traditional efficient market hypothesis (EMH) states that assets' prices fully reflect all available information. This implies that observed assets' prices should be equal to fundamental values. A potential issue is that the extreme version of the efficient market hypothesis rarely holds. Imperfect market potentially drives prices to deviate from fundamental values, e.g. asymmetric information, illiquidity, micro-noise and investors' sentiment; see DeLong *et al.* (1991), Barberis *et al.* (1998), Easley *et al.* (2002), and Lewellen and Shanken (2002).

Most existing tests for efficient market hypothesis lead to powerful results only if the equilibrium pricing models (e.g. Fama-French three-factor model) are correctly specified. However, the result of market anomaly may be ambiguous, since those tests are joint tests for both efficient market hypothesis and model setting. It is possible that the anomaly is caused by market inefficiency or a misspecified pricing model, as Fama (1991) points out. Different from these traditional tests, intra-day high-low range is firstly considered as a proxy of market efficiency in this paper. One advantage of this method is that little restriction is imposed on the model setting. This idea is in the light of two findings from existing literature. On one hand, overnight returns reflect more information about assets' fundamental values, while intra-day price fluctuation reflects more information about micro-noise and investors' sentiment. Apparently, most of the micro-noises are merely observable in intra-day price fluctuations, e.g. bid-ask bounce and order imbalance. Undoubtedly, high-low range is the ideal choice to capture the market inefficiency caused by these micro-noises. Additionally, the daytime returns reflect more information about investors' sentiment than overnight returns, and overnight returns are mainly driven by assets' fundamental value movements. For more discussions on investor's sentiment, see Tetlock (2007). On the other hand, it is widely acknowledged that assets' prices should fluctuate around their fundamental values. The fundamental value of an asset on a given day is expected to lie in the interval between the lowest price and highest price of the day. Then, the high-low range sets an upper bound for maximal deviation of asset's prices from its fundamental value. Therefore, it is highly desirable to utilize daily high-low range to gauge market efficiency.

However, it has been frequently noticed that the aforementioned literature generally concerns the event impacts using the point-valued closing price data, which might suffer from the loss of volatility information (Yang *et al.* 2012, 2016, Lin 2013, Lin and González-Rivera 2016, 2019), not to mention not examining market efficiency. Instead, a main advantage of the interval time series (ITS) modelling approach over traditional point-based approaches is that interval-valued inputs can capture the evolution in both the level and range of an asset price process simultaneously (Yang *et al.* 2013, Han *et al.* 2016, Sun *et al.* 2018, 2019, Qiao *et al.* 2019). For discussions on the range/volatility of an ITS and the associated tools for modelling, see Chou (2005), Engle and Gallo (1982) and Martens and Van Dijk (2007). ITS-based approaches can be roughly grouped into two categories: bivariate models and set models. The idea of bivariate models is to consider an ITS as two point-valued processes, see Neto and de Carvalho (2008, 2010), Brito and Duarte Silva (2012), González-Rivera and Lin (2013), and Teles and Brito (2015). To some extent, the bivariate-based method is not very well-suited for utilizing information efficiently because it is unable to relate various interval processes within a unified model (see Blanco-Fernández *et al.* 2011, Yang *et al.* 2016). Thus, this paper uses set models to analyze ITS. Set models are proposed by Han *et al.* (2016) to model an interval dataset as a random set, which is an inseparable unity generated from the sample space. Subsequently, Sun *et al.* (2018) propose a parsimonious threshold autoregressive interval (TARI) model to capture the nonlinear feature within an ITS system. Thus, we expect more powerful statistical inference by using the ITS returns rather than using the point-valued closing prices to consider the impact of Trump's election.

In this paper, we firstly propose a class of general nonlinear models for an ITS, which is essentially an interval generalization of the classical nonlinear model for a point-valued time series. A nonlinear minimum-distance estimation method is developed and the asymptotic theory of the proposed estimators is established. In empirical application, an interval-valued return is constructed for capturing the movements of assets' fundamental values and market efficiency. Its left bound is the difference between logarithmic lowest price of a trading day and logarithmic closing price of the previous trading day, and its right bound is calculated as the difference between the logarithmic highest price of a trading day and logarithmic closing price of the previous trading day. Intuitively, the level of this interval-valued return reflects the asset's return

† https://projects.fivethirtyeight.com/2016-election-forecast

on fundamental values, and its range can proxy market efficiency. Instead of traditional point-valued data regression, the ITS modelling approach is firstly proposed in this paper to simultaneously capture the evolution of assets' fundamental values and market efficiency.

The proposed interval model in this paper and linear interval regression proposed by Yang *et al.* (2012, 2016) are employed in a case study which analyzes the Trump election's impacts from two aspects: ex post analysis and ex ante analysis, respectively. For ex ante analysis, a linear interval regression model is used to examine the Trump election's impacts on the S&P500 index's expected return and high-low range before the election. Meanwhile, for ex post analysis, a nonlinear interval regression model is developed for investigating the Trump victory's impacts on the S&P500 index's expected return and high-low range simultaneously after the election. Furthermore, these impacts' duration is estimated as well. Our findings reveal that interval dummy variables can measure the shifts in both the level and range of interval-valued returns of the S&P 500 index. Specifically, ex ante analysis shows that the Trump election has an insignificantly negative impact on the S&P500's expected return before the election, while its range is significantly reduced. This result can be explained by the gradually eliminated uncertainty of an election's outcome as it approaches the election day. Besides, ex post analysis indicates that Trump's victory had a significantly negative impact on the S&P 500's range after the election, while it had a positive impact, which is relatively persistent from ex ante, on the S&P500's expected return.

Compared to the existing literature, this paper has some appealing features. First, the proposed model can capture the impacts of an event on both assets' return and market efficiency, which is proxied by the high-low range of interval-valued asset return, due to the informational gain of an ITS over a point-valued observation. Second, ITS might avoid undesirable noises included in high-frequency point-valued observations and capture information hiding in intra-day price fluctuations. Specifically, point-valued time series vary significantly due to certain disturbances, whose variations may be considered to be noise and can affect the tendency of an asset's returns. Third, there is no theoretical guidance to select the optimal event window, since different event windows give different parameter estimates and economic interpretations. This can raise concerns about whether satisfactory results were obtained by data snooping or by coincidence. This issue is addressed in this paper by using the decay rate to analyze the influence of the event over time.

The rest of this paper proceeds as follows: Section 2 proposes a general nonlinear interval model and establishes the asymptotic theory of the proposed nonlinear minimum distance estimators. Data and some preliminary analysis are described in Section 3. The results of empirical analysis are presented in Section 4 and Section 5 concludes.

2. Statistical model

To capture possible nonlinear dynamics of an ITS, we propose a class of nonlinear interval regression models for the interval-valued $\{Y_t\}$:

$$Y_t = h(\mathbf{Z}_t, \boldsymbol{\theta}) + u_t, \tag{1}$$

where $\{Y_t = [Y_{L,t}, Y_{R,t}]\}_{t=1}^{\infty}$ is an ITS process, $Y_{L,t}$ and $Y_{R,t}$ are the left and right bounds of this ITS process, $\mathbf{Z}_t$ is a vector of interval-valued variables, and $\boldsymbol{\theta}$ is a vector of unknown scalar-valued parameters. Suppose that $\{u_t\}$ is an interval martingale difference sequence (IMDS) with respect to the information set I_{t-1}, that is, $\mathbb{E}(u_t \mid I_{t-1}) = [0, 0]$ almost surely.

Note that $\{Y_t\}$, $\{u_t\}$ and $\{\mathbf{Z}_t\}$ are allowed the left bound of an ITS to be larger than the right bound, namely, extended random interval process. It is different from conventional intervals in bivariate models. In the classical or traditional interval analysis, an interval has been considered as a set of ordered numbers, with the lower bound smaller than the upper bound. In this paper, we extend the concept of an interval to the concept of an extended interval, which is a set of ordered numbers. Specifically, an extended random interval Y on a probability space (Ω, F, P) is a measurable mapping $Y : \Omega \to I_R$, where I_R is the space of closed sets of ordered numbers in R, as $Y(w) = [Y_L(w), Y_R(w)]$, where $Y_L(w)$, $Y_R(w) \in R$ for $w \in \Omega$ denote the left and right bounds of $Y(w)$, respectively. This includes regular intervals and extended intervals where the left-bound may not be smaller than the right bound. In fact, the idea of extended interval has been considered in interval algebra by Kaucher (1980), so-called generalized interval.

This extended interval can cover more applications in economics and finance, since the interval-valued economic variables with the reserved order for the boundaries are not uncommon. We allow, for instance, intervals consist of risk-free rates and asset returns, and thus an interval version of the capital asset pricing model (CAPM). One may argue that such an interval CAPM has no economic interpretation. However, from our interval CAPM, one can derive a point-valued model, which is the difference between the right bound and left bound of the interval. This range model is the conventional CAPM (where the dependent variable is the asset return minus risk-free rate), which has meaningful economic interpretations. The main advantage of our interval modelling approach is that we estimate the model using interval data, which contains more information than the range data. Thus, we can obtain more efficient estimates even if our final interest is the range model.

The nonlinear interval-valued regression model is an interval generalization of the popular nonlinear regression for a point-valued time series, which is a nonlinear function of the parameters θ. It can be used to infer the conditional mean dynamics and to forecast intervals of the interval process. It can also be generalized to capture the conditional mean dynamics of a general set-valued time series. For instance, let $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)'$. If $h(Z_t, \boldsymbol{\theta}) = Z_t\theta_1 I(Z_{r,t} < \theta_3) + Z_t\theta_2 I(Z_{r,t} > \theta_3)$, it is an interval-based threshold regression. This model can display asymmetric patterns in the declining and rising stages of the process. If $h(Z_t, \boldsymbol{\theta}) = \theta_1 + \theta_2 \exp(\theta_3) Z_t$, it can capture an exponential form of parameters in the regression model. What differentiates these examples from the linear regression model is that the conditional mean cannot be written as a linear function of the parameter vector $\boldsymbol{\theta}$. Sometimes, nonlinear regression is adopted because the functional form

$h(\mathbf{Z}_t, \boldsymbol{\theta})$ is suggested by an economic theory. See Section 2.1 for more discussion.

2.1. Special case: event study

In this section, we consider a special case of the proposed general interval model, as follows:

$$\begin{aligned} Y_t = {} & \alpha_0 + \beta_0 I_0 + \Sigma_{i=1}^{p} \beta_i Y_{t-i} + \Sigma_{l=1}^{s} \boldsymbol{\delta}_l' \mathbf{X}_{t-l} \\ & + \Sigma_{n=1}^{N} \mathrm{e}^{-\rho_{nm}(t-T_{0,n})} \gamma_{nm} D_{nm,t} \\ & + \Sigma_{n=1}^{N} \mathrm{e}^{-\rho_{nr}(t-T_{0,n})} \gamma_{nr} D_{nr,t} + u_t, \\ & n = 1, \ldots, N, \end{aligned} \tag{2}$$

where $\mathbf{X}_{t-l}$ is $q \times 1$ interval-valued exogenous variables for $l = 1, \ldots, s$; α_0, $\beta_i, i = 0 \cdots, p$, γ_{nm}, γ_{nr} are scalar-valued unknown parameters; $\boldsymbol{\delta}_l = (\delta_l^1, \ldots, \delta_l^q)'$ is a parameter vector of size $q \times 1$ with elements being scalar-valued parameters; ρ_{nm} and ρ_{nr} denote the decay rate of the trend and range when the nth event occurs, respectively; $I_0 = [-\frac{1}{2}, \frac{1}{2}]$ is a constant unit interval; $\alpha_0 + \beta_0 I_0 = [\alpha_0 - \frac{1}{2}\beta_0, \alpha_0 + \frac{1}{2}\beta_0]$ is an interval intercept; $u_t = [u_{L,t}, u_{R,t}]$ is an interval innovation, assumed as an IMDS with respect to the information set I_{t-1}, that is, $\mathbb{E}(u_t \mid I_{t-1}) = [0, 0]$ almost surely; $D_{nm,t} = [1, 1]\mathbf{1}_{\{t \in [T_{0,n}, T]\}}$, $D_{nr,t} = [-\frac{1}{2}, \frac{1}{2}]\mathbf{1}_{\{t \in [T_{0,n}, T]\}}$, $\mathbf{1}_{\{\cdot\}}$ is an indicator function and $t_{0,n}$ is the time point when the nth event occurs. Let $\mathbf{Z}_t = ([1, 1], I_0, Y_{t-1}, \ldots, Y_1, \mathbf{X}_{t-1}', \ldots, \mathbf{X}_1', D_{1m,t}, \ldots, D_{nm,t}, D_{1r,t}, \ldots, D_{nr,t})'$ and $\boldsymbol{\theta} = (\alpha_0, \beta_0, \beta_1, \ldots, \beta_p, \boldsymbol{\delta}_1', \ldots, \boldsymbol{\delta}_s', \rho_{1m}, \ldots, \rho_{Nm}, \rho_{1r}, \ldots, \rho_{Nr}, \gamma_{1m}, \ldots, \gamma_{nm}, \gamma_{1r}, \ldots, \gamma_{nr})'$.

This model is employed to quantify and assess the Trump election on the stock index based on the ITS modelling. A main advantage of the proposed nonlinear interval models is that our model captures the impacts of events on both trend and volatility of an ITS in a given time period by directly modelling an ITS as a random set. Specifically, it is a set process and results in parsimonious models for an ITS, where the derived two boundary processes are generated by just one set of parameters under the proposed model.

Equation (2) is in essence an interval version for the classic nonlinear model in point-based time series analysis. Specifically, by taking the difference between the left and right bounds, we obtain the point-valued range model to capture the range volatility:

$$\begin{aligned} Y_{r,t} = {} & \beta_0 + \beta_1 Y_{r,t-1} + \sum_{l=1}^{s} \boldsymbol{\delta}_l' \mathbf{X}_{r,t-l} \\ & + \Sigma_{n=1}^{N} \mathrm{e}^{-\rho_{nr}(t-T_{0,n})} \gamma_{nr} D_{nr,t} + u_{r,t}, \end{aligned} \tag{3}$$

where $Y_{r,t}$ and $\mathbf{X}_{r,t}$ are the difference between the left bound and right bound of the corresponding interval-valued variables, respectively. This delivers an alternative method for modelling the range dynamics of a time series. One can make use of the interval sample information, rather than the range sample only, to estimate the parameters more efficiently, even if the interest is range modelling.

Similarly, the trend model for $Y_{m,t}$ is obtained as follows:

$$\begin{aligned} Y_{m,t} = {} & \alpha_0 + \beta_1 Y_{m,t-1} + \sum_{l=1}^{s} \boldsymbol{\delta}_l' \mathbf{X}_{m,t-l} \\ & + \Sigma_{n=1}^{N} \mathrm{e}^{-\rho_{nm}(t-T_{0,n})} \gamma_{nm} D_{nm,t} + u_{m,t}, \end{aligned} \tag{4}$$

where $Y_{m,t}$ and $\mathbf{X}_{m,t}$ are the midpoints between the left bound and right bound of the corresponding interval-valued variables, respectively. This can be used to capture some well-known phenomenon in time series, e.g. level effect. For instance, if $Y_{m,t}$ is the stock return, $\beta_1 < 0$ indicates that a level higher than zero at time t is likely to be followed by another level lower than zero in the next time period, and vice versa. This is the so-called mean reversion.

To illustrate how the interval dummy variable represents an event and how to measure the shift of an ITS due to the occurrence of this event, we consider a simple case for an event D_1. Let an ITS be $\{Y_t^* \mid Y_t^* = [Y_{L,t}^*, Y_{R,t}^*]\}$ as the random interval without the effects of events. γ_m and γ_r measure the marginal effects of $D_{m,t}$ and $D_{r,t}$ on Y_t^*, respectively, when the event D_1 happens. Based on the interval operations in Han *et al.* (2016), the occurrences of a set of events induce Y_t^* to become Y_t as

$$\begin{aligned} Y_t &= Y_t^* + \mathrm{e}^{-\rho_{1m}(t-T_{0,1})} \gamma_{1m} D_{1m,t} + \mathrm{e}^{-\rho_{1r}(t-T_{0,1})} \gamma_{1r} D_{1r,t} \\ &= Y_t^* + [\mathrm{e}^{-\rho_{1m}(t-T_{0,1})} \gamma_{1m}, \mathrm{e}^{-\rho_{1r}(t-T_{0,1})} \gamma_{1r}] \\ &= [Y_{m,t}^* + \mathrm{e}^{-\rho_{1m}(t-T_{0,1})} \gamma_{1m}, Y_{r,t}^* + \mathrm{e}^{-\rho_{1r}(t-T_{0,1})} \gamma_{1r}]. \end{aligned} \tag{5}$$

Specifically, when $\gamma_{1m} > 0$, the trend of the ITS moves towards the right with γ_{1m} units at time point $T_{0,1}$. The impact of an event D_1 on the range $Y_{r,t}$ is determined by γ_{1r}, when the event occurs. Besides, when t is larger than $T_{0,1}$, the marginal effects of the event on the trend or the range decays exponentially with a rate ρ_{1m} or ρ_{1r}, respectively. This is consistent with the conventional wisdom that the recent information has a larger impact on today than the remote past information (Engle 1982).

There are various patterns of event's impacts in existing literature, e.g. increasing linearly, exponentially decaying and even exponentially rising. See more discussions in Box and Tiao (1975), Montgomery and Weatherby (1980), and Izenman and Zabell (1981). Here, the exponential decaying is most suitable for this study, since the expected effect of this event would be to produce a gradual reduction in S&P 500. Figure 1 shows the 10 days moving average curve for the ranges of S&P 500 index from Nov 7th 2016 to Feb 28th 2017. In spite of some local peaks and valleys, it displays a general trend that S&P 500 index's ranges sharply decline within the first two weeks after the election, and then slowly move down until the end of our sample. It can be seen that the impact of Trump election on S&P 500 decays gradually over time, with a potential exponentially declining scheme. This pattern provides us with a heuristic guess that exponential function may properly capture the evolution of Trump election's impact due to its satisfactory goodness of fit.

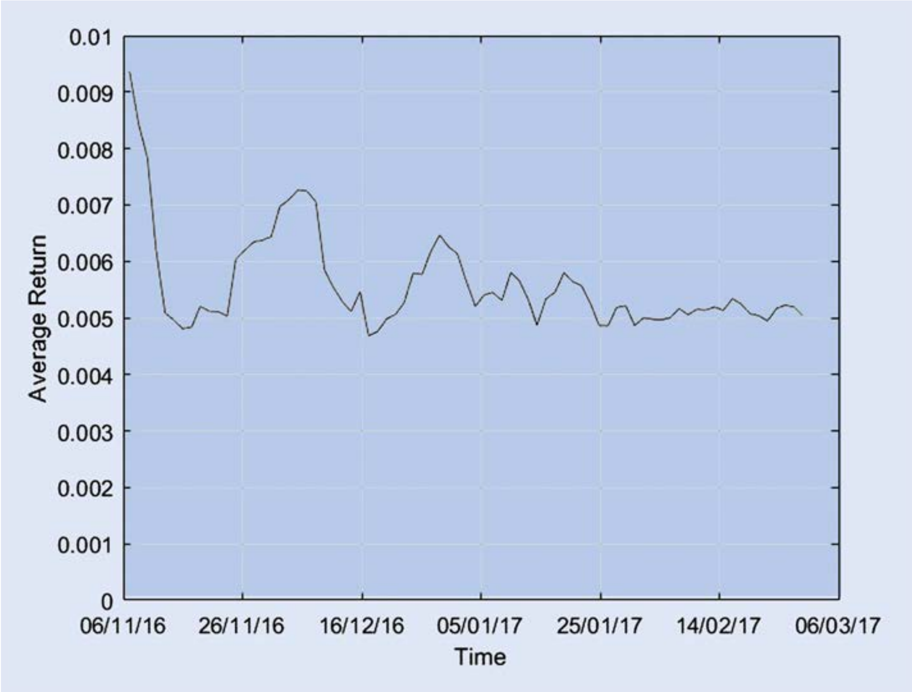

Figure 1. Average return of SP500 over 10 days.
Note: The average stock return is the moving average of returns over every 10 days. The sample is from November 6th, 2016 to March 6th, 2017.

2.2. Half-life period

It is widely acknowledged that the magnitudes of impacts are sensitive to the choice of the event window, pointed out by MacKinlay (1997) and Asgharian *et al.* (2011). For instance, short-horizon methods are powerful only if the abnormal performance is concentrated in the event window, while long-horizon methods are more appropriate when the impact of event has a long duration. Thus, one critical and practical issue with event study is how many observations should be used. However, in the existing literature, the length of event window is often determined by researchers, either in an ad hoc manner or based on empirical experience. For short-period studies, the event window is set to 3-days, 5-days and 7-days in Dahiya *et al.* (2003), while it is set to 1-day in Billett *et al.* (1995). For long-period studies, Zenios and Ziemba (2007) use an event window of one year data, while Yang *et al.* (2016) choose a window length equal to three years. However, there has been no theoretical guidance to choose the optimal event window in event studies, which may raise concerns about whether the reported satisfactory results were obtained simply by chance or by data snooping. The robustness of the empirical results may be subject to suspect because it is likely that only the results from successful windows have been reported.

In this paper, we assume that the impact of the event decays exponentially as t increases. Based on equation (2), the simple derivation is obtained as follows

$$\frac{\mathrm{d}\,\mathrm{e}^{-\rho_{1m}(t-T_{0,1})}}{\mathrm{e}^{-\rho_{1m}(t-T_{0,1})}} = -\rho_{1m}\,\mathrm{d}t, \quad \frac{\mathrm{d}\,\mathrm{e}^{-\rho_{1r}(t-T_{0,1})}}{\mathrm{e}^{-\rho_{1r}(t-T_{0,1})}} = -\rho_{1r}\,\mathrm{d}t, \qquad (6)$$

where ρ_{1m} and ρ_{1r} are decay rates of initial impacts on S&P500 index's expected return and volatility when the event D_1 occurs, respectively. The half-life periods of midpoints or ranges are obtained as $t - T_{0,1} = \ln 2/\rho_{1m}$ or $t - T_{0,1} = \ln 2/\rho_{1r}$, respectively. This implies that it takes $\ln 2/\theta$ days for the impact of the event to decay away to half. These half periods can be considered as event windows, which are hardly sensitive to the sample length. Since the decay rates θ_m and θ_r can be self-adaptively estimated with the associate sample, the problem of event window selection is avoided.

3. Estimation and hypothesis testing

Without losing generality, equation (1) is used to show the nonlinear minimum D_K-distance estimation. Suppose that an ITS sample $\{Y_t\}_{t=1}^{T}$ is generated from equation (2) with the true parameter $\boldsymbol{\theta}^0$ and an interval sample $\{\mathbf{Z}_t\}_{t=1}^{T}$. We derive the minimum D_K-distance estimator $\widehat{\boldsymbol{\theta}}$ by minimizing the sum of squared residuals $\widehat{Q}_T(\boldsymbol{\theta})$ of the interval model, namely

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}\in\Theta} \widehat{Q}_T(\boldsymbol{\theta}), \qquad (7)$$

with associated first order condition

$$g(\widehat{\boldsymbol{\theta}}) = -\frac{2}{T}\sum_{t=1}^{T}\langle s_{Y_t - h(\mathbf{Z}_t,\widehat{\boldsymbol{\theta}})}, s_{\frac{\partial h(\mathbf{Z}_t,\widehat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}}}\rangle_K = \mathbf{0},$$

where $\widehat{Q}_T(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T} q_t(\boldsymbol{\theta})$ and $q_t(\boldsymbol{\theta}) = \langle s_{Y_t-h(\mathbf{Z}_t,\boldsymbol{\theta})}, s_{Y_t-h(\mathbf{Z}_t,\boldsymbol{\theta})}\rangle_K$, i.e. the D_K-distance between the interval-valued residual $\widehat{u}_t$ and the zero interval $[0, 0]$.

Note that our approach is essentially a nonlinear least square approach, based on the distance between interval-valued data, namely, D_K distance, which is developed by Körner (1997) and Körner and Näther (2002). As a special case of random sets, the D_K metric for any pair of intervals $A,\ B \in I_R$ is given by

$$D_K(A,B) = \sqrt{\int_{(u,v)\in S^0} [s_A(u) - s_B(u)][s_A(u) - s_B(v)]\,\mathrm{d}K(u,v)}$$
$$= \sqrt{\langle s_{A-B}, s_{A-B}\rangle_K} = \|A - B\|_K,$$

where the unit space $S^0 = \{u \in R^1, |u| = 1\} = \{-1, 1\}$, $s_A(u)$ is the support function, in the univariate interval context, which is defined as $s_A(u) = A_R$ if $u = 1$, $s_A(u) = -A_L$ if $u = -1$ for the unit space S^0, $\|A - B\|_K$ is the norm for interval $A - B$ with respect to the D_K-distance, $\langle\cdot,\cdot\rangle$ indicates the inner product in S^0 with respect to kernel $K(u, v)$. See Sun *et al.* (2018) for more discussions.

3.1. Asymptotic theory

To establish the consistency and derive the asymptotic distributions of $\widehat{\boldsymbol{\theta}}$, we impose some regularity conditions.

Assumption 1 $\{Y_t\}$ is an interval stochastic process with $\mathbb{E}\|Y_t\|_K^4 < \infty$, and it follows an interval regression in equation (1), where the exogenous variable is stationary ITS.

In Assumption 1, $\{Y_t\}$ is an ITS with finite fourth moments with respect to D_K-distance, which follows a nonlinear regression $h(\mathbf{Z}_t,\cdot)$ with stationary exogenous interval-valued variables. $h(\mathbf{Z}_t,\cdot)$ essentially has various forms, such as $\exp(\mathbf{Z}_t,\cdot)$, $\sin(\mathbf{Z}_t,\cdot)$ and $\mathbf{Z}_t^{\cdot}$, which results in nonlinear behaviors in $\{Y_t\}$.

Assumption 2 The interval innovation $\{u_t\}$ is an IMDS with respect to the information set I_{t-1}, i.e. $\mathbb{E}(u_t\,|\,I_{t-1}) = [0, 0]$, almost surely, and $\mathbb{E}\|u_t\|_K^2 < C$, where C is some positive constant.

In Assumption 2, $\mathbb{E}(u_t\,|\,I_{t-1}) = [0, 0]$ implies that the linearity of the support function s_A satisfying $\mathbb{E}(s_{u_t}\,|\,I_{t-1}) = 0$ a.s., and the cross product satisfies $\mathbb{E}(\langle s_{\mathbf{Z}_t}, s_{u_t}\rangle_K\,|\,I_{t-1}) = 0$. We note that our IMDS assumption includes conditional heteroscedasticity, which can cover more applications in finance. This setting is weaker than i.i.d. condition and a conditional homoscedastic assumption for $\{u_{L,t}, u_{R,t}\}$.

Assumption 3 The parameter space Θ is a finite-dimensional compact space of R^τ, where τ is the size of $\boldsymbol{\theta}^0$. $\boldsymbol{\theta}^0$ is an interior point in Θ, which is the true parameter vector value given in equation (1).

Assumption 4 $h(\mathbf{Z}_t, \boldsymbol{\theta})$ has continuous second partial derivative functions near $\boldsymbol{\theta}^0$.

Assumption 4 is a smoothness condition on $h(\mathbf{Z}_t,\cdot)$ for $\boldsymbol{\theta}$. It requires that the second order derivative of $h(\mathbf{Z}_t,\cdot)$ exists and is continuous, which is necessary for the Taylor expansion of $h(\mathbf{Z}_t,\cdot)$ near $\boldsymbol{\theta}^0$.

Assumption 5 The square matrices

$$\mathbb{E}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}}\rangle_K \quad \text{and} \quad \mathbb{E}[\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}}, s_{u_t(\boldsymbol{\theta})}\rangle_K\,\langle s_{u_t(\boldsymbol{\theta})}, s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}}\rangle_K]$$

are positive definite for all $\boldsymbol{\theta}$ in a small neighborhood of $\boldsymbol{\theta}^0$.

Assumption 5 is necessary to derive the asymptotic distribution of $\widehat{\boldsymbol{\theta}}$ via a Taylor series expansion. Firstly, the consistency of $\widehat{\boldsymbol{\theta}}$ is established.

Theorem 1 *Under Assumptions* 1–4 *and if the model is correctly specified, the nonlinear minimum D_K-distance estimator defined by equation* (1) *is consistent, i.e.*

$$\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0 \xrightarrow{P} \mathbf{0}.$$

Intuitively, from the first order condition of $\widehat{Q}_t(\boldsymbol{\theta})$, it is easy to obtain the difference, i.e. $\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0$. Under these certain regularity conditions, it can be shown that the estimator $\widehat{\boldsymbol{\theta}}$ converges in probability to $\boldsymbol{\theta}^0$ uniformly in Θ as $T \to \infty$. Next, the asymptotic normality of $\widehat{\boldsymbol{\theta}}$ is derived.

Theorem 2 *Suppose Assumptions* 1–5 *and if the model is correctly specified, then*

$$\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0) \sim N(\mathbf{0}, \mathbf{M}^{-1}(\boldsymbol{\theta}^0)\mathbf{V}(\boldsymbol{\theta}^0)\mathbf{M}^{-1}(\boldsymbol{\theta}^0)),$$

where

$$\mathbf{M}(\boldsymbol{\theta}^0) = \mathbb{E}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}\rangle_K, \quad \text{and}$$
$$\mathbf{V}(\boldsymbol{\theta}^0) = \mathbb{E}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}, s_{u_t}\rangle_K \langle s_{u_t}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}\rangle_K.$$

Remark Note that various selections of kernel K will result in different minimum D_K-distance estimators $\widehat{\boldsymbol{\theta}}$, which are consistent for $\boldsymbol{\theta}^0$ in probability. Different kernels imply different weighting functions of all possible pairs of points in intervals. For instance, if $K = (1, 1, 1)$, it measures the distance between the ranges of two intervals, while $K = (\frac{1}{4}, -\frac{1}{4}, \frac{1}{4})$ measures the distance between the midpoints of two intervals. To address this uncertainty of kernel selection, we follow the spirit of Han *et al.* (2015) to use the two-stage minimum kernel K, which is asymptotically most efficient among all symmetric positive definite kernels with the minimum asymptotic variance. Specifically, a two-stage minimum D_K-distance estimation method is proposed as follows: in the first step, a preliminary choice of kernel K is used to estimate parameters and obtain the estimated residuals $\widehat{u}_t(\widehat{\boldsymbol{\theta}})$. In the second step, the optimal kernel $\widehat{K}_{opt}$ is used to estimate the model parameters, where $\widehat{K}_{opt} = (T^{-1}\Sigma_{t=1}^T \widehat{u}_{L,t}^2, T^{-1}\Sigma_{t=1}^T \widehat{u}_{L,t}(\widehat{\boldsymbol{\theta}})\widehat{u}_{R,t}(\widehat{\boldsymbol{\theta}}), T^{-1}\Sigma_{t=1}^T \widehat{u}_{R,t}^2)$. Intuitively, the optimal K^{opt} is proportional to the covariance matrix of the residuals, which discounts the sample squared distance components with larger variance.

A key difference of our nonlinear interval model from a bivariate point-valued modelling is that by treating an interval as a set, we are able to develop a new estimation method for the proposed interval model, through minimizing the distance between the interval model and interval data. Interestingly, our minimum-distance estimator can be interpreted as follows: It employs not only the information on the distances between the bounds of intervals, but also the information on distances between interior points, with appropriate weighting.

3.2. *Inference*

Following the spirit of Han *et al.* (2015), the Wald statistics is used to examine the significance of the coefficients in interval models. We consider the special case of equation (2). The result of the general case is similar. In this case, the null hypothesis $\mathbb{H}_0$ is $\mathbf{R}\boldsymbol{\theta}^0 = \mathbf{r}$, where $\mathbf{R}$ is a $b \times \tau$ nonstochastic matrix of full rank $b \leq \tau$, $\mathbf{r}$ is a $b \times 1$ nonstochastic vector, and $\tau = 2 + p + sq + 4N$ is the dimension of parameter $\boldsymbol{\theta}$ in the interval regression model with interval dummy variables in equation (2). The statistics of Wald testing is:

$$\mathbf{W} = [T(\mathbf{R}\widehat{\boldsymbol{\theta}} - r)'][\mathbf{R}\widehat{\mathbf{M}}_T^{-1}(\widehat{\boldsymbol{\theta}})\widehat{\mathbf{V}}_T(\widehat{\boldsymbol{\theta}})\widehat{\mathbf{M}}_T^{-1}(\widehat{\boldsymbol{\theta}})\mathbf{R}']^{-1}[(\mathbf{R}\widehat{\boldsymbol{\theta}} - \mathbf{r})], \tag{8}$$

where $\widehat{\mathbf{M}}_T(\widehat{\boldsymbol{\theta}})$, $\widehat{\mathbf{V}}_T(\widehat{\boldsymbol{\theta}})$, and $\widehat{\boldsymbol{\theta}}$ are the estimators of $\mathbf{M}_T(\boldsymbol{\theta})$, $\mathbf{V}_T(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$ in Theorem 2.

The Wald statistic used here has good size in finite samples and reasonable power; see more detailed discussions in Han *et al.* (2015) and Yang *et al.* (2016).

Most importantly, the Wald statistics is employed to test the significance of the coefficients of interval-valued dummy variables, which studies the impact of the occurrence of an event on ITS. Here, $\mathbb{H}_0 : \gamma_{nm} = 0$ (or $\gamma_{nr} = 0$), which examines the impact of the nth event on the midpoint (or the range) of the ITS. Specifically, the null hypothesis $\mathbb{H}_0$ suggests that the midpoint (or the range) of the ITS is hardly affected by the nth event. If $\mathbb{H}_0$ is rejected and $\gamma_{nm} > 0$ (or $\gamma_{nr} > 0$), it implies that the nth event increases the midpoint or the range of the ITS.

4. Simulation studies

This section studies the finite sample properties of the proposed two-stage minimum D_K-distance estimators. We consider the nonlinear interval regression model as data generating process:

$$Y_t = \alpha_0 + \beta_0 I_0 + \beta_1 X_{t1} + \lambda \exp^{-\rho t} X_{t2} + \varepsilon_t, \tag{9}$$

where $\boldsymbol{\theta} = (\alpha_0, \beta_0, \beta_1, \rho, \lambda)' = (0.1, 0.2, 0.12, 0.3, -0.5)'$, two boundaries of X_{t_1} and X_{t_2} follow *i.i.d*., bivariate normal distribution $f(\boldsymbol{\mu}, \mathbf{I})$, with $\boldsymbol{\mu} = (2, 2)$ and I is 2×2 identity matrix and ε_t are interval innovations. Following the spirit of Han *et al.* (2016) and Sun *et al.* (2018), the interval innovations ε_t are obtained from ACI model:

$$Y_= \alpha_0 + \beta_0 I_0 + \beta_1 Y_{t-1} + u_t, \tag{10}$$

where parameter values are obtained from a two-stage minimum D_K-distance estimation with a preliminary kernel K_{ab} with $a/b = 2/1$ in the first stage. The data used in ACI model is daily S&P 500 price index from January 1st, 2015 to October 31th, 2017. The interval innovations $\{\varepsilon_t\}$ are obtained via a naive bootstrap from the estimated residuals $\{\widehat{u}_t = Y_t - (\widehat{\alpha}_0 + \widehat{\beta}_0 I_0 + \widehat{\beta}_1 Y_{t-1})\}$, with the sample size $T = 200$, 500, respectively. For each sample size T, we perform 1000 replications.

Table 1. Bias, SD and RMSE of estimators for parameters (10^{-2}).

	$T = 200$			$T = 500$		
	Bias	S.D	RMSE	Bias	S.D	RMSE
α_0	– 0.99	2.35	2.55	– 0.29	1.42	1.45
β_0	– 0.76	1.85	2.00	0.60	0.80	1.00
β_1	3.50	4.22	5.48	1.83	2.32	2.96
ρ	– 3.08	2.41	3.91	– 1.48	1.83	2.36
λ	– 4.92	4.23	6.49	– 1.91	2.74	3.34

Note: Bias, SD and RMSE of each parameter are computed based on 1000 bootstrap replications.

We compute the bias, standard deviation (SD), and root mean square error (RMSE) for each estimator:

$$\begin{aligned} Bias(\widehat{\theta}_i) &= \frac{1}{1000}\sum_{n=1}^{1000}(\widehat{\theta}_i^{(m)} - \theta_i^0), SD(\widehat{\theta}_i) \\ &= \left[\frac{1}{1000}\sum_{n=1}^{1000}(\widehat{\theta}_i^{(m)} - \overline{\theta}_i)^2\right]^{1/2}, RMSE(\widehat{\theta}_i) \\ &= [Bias^2(\widehat{\theta}_i + SD^2(\widehat{\theta}_i))]^{1/2}, \end{aligned}$$

where $\overline{\theta}_i = \frac{1}{1000}\sum_{m=1}^{1000}\widehat{\theta}_i^{(m)}$, and $\widehat{\theta}_i = \widehat{\alpha}_0, \widehat{\beta}_0, \widehat{\beta}_1, \widehat{\rho}, \widehat{\gamma}$, respectively. Table 1 shows that the RMSE for all estimators decrease as T increases, which is consistent with Theorems 1–2 that the convergence rate of estimation depends on the sample length T. Similarly, as T increases, Bias and SD of parameters decrease significantly as expected.

5. Empirical preliminary

5.1. *Data and preliminary analysis*

To investigate the effects of the 2016 election on US equity market, we construct interval-valued daily returns on S&P 500 index as $Y_t = [Y_{L,t}, Y_{R,t}]$, from January 1st, 2015 to October 31th, 2017, where $Y_{L,t}$ and $Y_{R,t}$ are calculated as $Y_{L,t} = \ln(P_t^L/P_{t-1}^C)$, $Y_{R,t} = \ln(P_t^H/P_{t-1}^C)$, respectively. Note that informational gain is obtained in the ITS sample, since an interval data captures both the trend and variation information of a price process in a given day.

The descriptive statistics for the interval-valued S&P 500 index are reported in table 2. The results suggest that the informational gain of the ITS over the induced point-valued data. Firstly, the average of interval-valued daily return is about [-0.0045,0.0043], which captures the S&P500 index's level (i.e. midpoint) and intra-day volatility (i.e. range) simultaneously. Secondly, the standard deviation of two bounds are 0.64% and 0.54%, respectively. Notice that the averages and standard deviations of the two bounds of the interval-valued returns have roughly the same scale, whereas closing price-based daily returns often present a standard deviation more than ten times its average. Hence, interval-valued return is more stable than common daily return based on closing prices. Thirdly, the intra-day volatility (i.e. $Y_{r,t}$) and the two boundaries of returns (i.e. $Y_{L,t}$ and $Y_{R,t}$) appear to have different

Table 2. Summary statistics of interval-valued returns.

	P_t^L	P_t^H	$Y_{L,t}$	$Y_{R,t}$	*Return*	$Y_{r,t}$
Mean	7.6777	7.6865	– 0.0045	0.0043	0.0003	0.0088
Median	7.6519	7.6589	– 0.0032	0.0029	0.0003	0.0070
Maximum	7.8525	7.8567	0.0087	0.0396	0.0383	0.0512
Minimum	7.5011	7.5213	– 0.0541	– 0.0066	– 0.0402	0.0017
Std. Dev.	0.0804	0.0768	0.0064	0.0054	0.0079	0.0060
Skewness	0.4003	0.4817	– 2.2386	1.5504	– 0.3937	2.1357
Kurtosis	2.3107	2.2930	12.0100	6.9932	6.3095	10.2966

Notes: This table reports some basic statistical analysis on the attributes of interval-valued S&P 500 returns. The sample is from January 1st, 2015 to October 31st, 2017. The interval-valued daily returns on S&P 500 index is defined as $Y_t = [Y_{L,t}, Y_{R,t}]$. $Y_{L,t}$ and $Y_{R,t}$ are calculated as $Y_{L,t} = \ln(P_t^L/P_{t-1}^C)$, $Y_{R,t} = \ln(P_t^H/P_{t-1}^C)$ respectively, and $Y_{r,t=} = Y_{R,t} - Y_{L,t}$. *Return* is the logarithm difference of the daily S&P 500 closing prices.

skewness and kurtosis properties. For instance, the intra-day volatility $Y_{r,t}$ of the ITS has a larger skewness and kurtosis than its right bound (i.e. $Y_{R,t}$), which are smaller than its left bound (i.e. $Y_{L,t}$). This implies that only partial information can be explained if just one of these point-valued processes (i.e. $Y_{L,t}$, $Y_{R,t}$ and $Y_{r,t}$). Thus, a parsimonious model for the ITS, in which an interval observation is considered as an inseparable unit, is expected to efficiently utilize the information contained in the ITS and derive more efficient estimation and more powerful inference than point-based models.

5.2. *Control variables*

The choice of explanatory variables (except for the dummy variables) is discussed in our interval regression models. It is widely acknowledged that oil future prices, along with currencies and gold, are the main drivers of world economy. They respond significantly to the economic or political shocks (e.g. Popescu 2016). Thus, these three interval-valued returns are considered in this paper, including West Texas Intermediate (WTI) crude oil future prices, dollar index and New York Commodities Exchange (COMEX) gold future prices. Be aware that these interval-valued control variables are simply constructed by the same method used in S&P 500 returns (i.e. Y_t). For the summary statistics of these variables, see table 3.

Specifically, in terms of WTI returns, there seems to be a significant correlation between the S&P 500 stock market and the oil future market. This is confirmed in Zhang and Wei (2010). Besides, a number of literature in recent years studies the effects of exchange rates on stock markets and suggests a significant relationship between these two markets (e.g. Granger *et al.* 2000, Phylaktis and Ravazzolo 2005, Andersen *et al.* 2007, Ehrmann *et al.* 2011). In this paper, the interval dollar index is used as a proxy to measure the volatility in the foreign exchange markets.

Furthermore, VIX index, a proxy of S&P500 index's implied volatility, is considered as a control variable in this paper. Specifically, VIX index is an integrated measure of the expected volatility implied by S&P500 index options. Previous studies have found that there is a statistically significant relationship between the returns of the stocks and VIX index; see Giot (2005). Different from other volatility indices based on realized values of returns, VIX index reflects investors' expectation of the risk in stock market. Hence, VIX is also known as Fear Index. It is involved in our model as a control variable to capture the evolution of investors' sentiment. We expect that higher VIX is accompanied with lower market efficiency due to more pessimistic investors' sentiment. An interval-valued VIX variable is constructed as $IVIX_t = [-\frac{1}{2}VIX_t, \frac{1}{2}VIX_t]$. Note that the midpoint of the interval is constantly zero. It implies that we potentially make the assumption that investors' sentiment affects market efficiency, but cannot affect assets' fundamental values.

5.3. *Model specification*

Based on the preliminary analysis of the ITS of S&P 500 index and the selected control variables, we use the interval-valued models for the ITS affected by 2016 election.

Table 3. Summary statistics of main control variables.

	$Oil_{L,t}$	$Oil_{R,t}$	$Index_{L,t}$	$Index_{R,t}$	$IVIX_{L,t}$	$IVIX_{R,t}$	$Gold_{L,t}$	$Gold_{R,t}$
Mean	– 0.0178	0.0185	– 0.0037	0.0037	– 1.3296	1.3296	– 0.0066	0.0068
Median	– 0.0143	0.0135	– 0.0028	0.0030	– 1.3107	1.3107	– 0.0054	0.0051
Maximum	0.0870	0.1582	0.0044	0.0381	– 1.1091	1.8536	0.0084	0.0788
Minimum	– 0.0881	– 0.0071	– 0.0329	– 0.0041	– 1.8536	1.1091	– 0.0473	– 0.0067
Std. Dev.	0.0168	0.0177	0.0037	0.0034	0.1281	0.1281	0.0058	0.0066
Skewness	– 0.7142	2.2143	– 2.2427	2.4005	– 0.7906	0.7906	– 1.7483	3.2781
Kurtosis	6.6097	11.0149	12.6869	17.7140	3.4183	3.4183	8.9671	27.5004

Notes: This table reports some basic statistical analysis on the attributes of interval-valued control variables. The sample is from January 1st, 2015 to October 31st, 2017. $Index_t = [Index_{L,t} - Index_{c,t-1}, Index_{R,t} - Index_{c,t-1}]$ is logarithm US dollar index return in an interval format; $IVIX_t = [-\frac{1}{2}VIX_t, \frac{1}{2}VIX_t]$ is the interval-valued volatility index. $Gold_t = [Gold_{L,t} - Gold_{c,t-1}, Gold_{R,t} - Gold_{c,t-1}]$ is the daily interval-valued logarithm Comex gold future return.

Specifically, for ex ante analysis, a linear interval-valued data regression model is proposed as follows,

$$\begin{aligned} Y_t = \alpha_0 + \beta_0 I_0 + \beta_1 Y_{t-1} + \beta_2 Oil_t + \beta_3 Index_t \\ + \beta_4 IVIX_t + \beta_5 Gold_t \\ + \gamma_{1m} D_{t,m} + \gamma_{1r} D_{t,r} + u_t, \end{aligned} \tag{11}$$

where $Y_t = [Y_{L,t}, Y_{R,t}]$ is the interval-valued logarithm daily return on S&P 500 index, namely $Y_{L,t} = \ln(P_{L,t}/P_{c,t-1})$, $Y_{R,t} = \ln(P_{R,t}/P_{c,t-1})$; $Index_t = [Index_{L,t} - Index_{c,t-1}, Index_{R,t} - Index_{c,t-1}]$ is logarithm US dollar index return in an interval format; $IVIX_t = [-\frac{1}{2}VIX_t, \frac{1}{2}VIX_t]$ is the interval-valued volatility index. $Gold_t = [Gold_{L,t} - Gold_{c,t-1}, Gold_{R,t} - Gold_{c,t-1}]$ is the daily interval-valued logarithm Comex gold future return. In addition, interval-valued dummy variable $D_{t,m} = [1, 1]$ (or $D_{t,r} = [-\frac{1}{2}, \frac{1}{2}]$) is included in the one-month event window before the election, otherwise $D_{t,m} = [0, 0]$ (or $D_{t,r} = [0, 0]$); $D_{t,m}$ measures 2016 election's impact on trend of S&P500 index's return, while $D_{t,r}$ reflects its impact on range of S&P500 index's return. We will focus on coefficients γ_m's and γ_r's significance and magnitude.

Different from ex ante analysis, a nonlinear interval-valued data regression model is suggested for ex post analysis as following,

$$\begin{aligned} Y_t = \alpha_0 + \beta_0 I_0 + \beta_1 Y_{t-1} + \beta_2 Oil_t + \beta_3 Index_t \\ + \beta_4 IVIX_t + \beta_5 Gold_t \\ + \gamma_m \,\mathrm{e}^{-\rho_m (t-T_0)} D_{m,t} + \gamma_r \,\mathrm{e}^{-\rho_r (t-T_0)} D_{r,t} + u_t, \end{aligned} \tag{12}$$

where $D_{m,t}$ and $D_{r,t}$ are in the same sense as the above; $D_{m,t} = [1, 1]$ and $D_{r,t} = [-\frac{1}{2}, \frac{1}{2}]$ if trading day t is no earlier than election day, otherwise $D_{t,m} = [0, 0]$ and $D_{t,r} = [0, 0]$; T_0 refers to the election day. It is worth noting that, when $t = T_0$, the trend term, $\gamma_m D_t \,\mathrm{e}^{-\rho_m (t-T_0)}$, equals to γ_m, and volatility term, $\gamma_r D_t \,\mathrm{e}^{-\rho_r (t-T_0)}$, equals to γ_r. Therefore, coefficients γ_m and γ_r can measure the initial impacts of Trump's victory on S&P500's expected return and volatility, respectively.

After that, two hypothesis are used to explore the influence of the event on the trend and the range of the ITS, respectively. One tests $\mathbb{H}_0 : \gamma_m = 0$ and $\mathbb{H}_A : \gamma_m \neq 0$ for the trend of the ITS. The other examines $\mathbb{H}_0 : \gamma_r = 0$ for the marginal effect of the range caused by the Trump election event.

6. Estimation results

6.1. Ex ante analysis

Table 4 reports the two-stage minimum D_K-distance estimators and the P-values for ex ante analysis on 2016 election. Firstly, we focus on the coefficient of the midpoint dummy variable, namely γ_m. The estimator of expected return term γ_m is insignificantly positive (about 0.05%) at 10% level. This implies that we cannot reject the null hypothesis that Trump election has no influence on S&P 500 index in a short period (one month) before the election day. Nevertheless, its sign is consistent with our expectation. Indeed, the uncertainty of the election's outcome is gradually reduced as time approaches the election day. For instance, on September 26, 2016, Clinton and Trump participated in a presidential debate hosted by Hofstra University. After the debate, Clinton's odds of election increased from 63% to 69% in Betfair Prediction Market. To some extent, the enlarged gap between Trump's and Clinton's approval ratings reduces the uncertainty of election's outcome, see Wolfers and Zitzewitz (2016). Thus, the political risk in US stock market is gradually decreased in the event window. Additionally, Referring to Campbell and Hentschel (1992), volatility potentially produces a discount on asset's fundamental value. This is known as volatility feedback effect. Hence, the decrease of market risk is expected to raise S&P500 index's level and produce a positive impact on S&P500 index's return. This is confirmed by the sign of γ_m's estimator, although this effect is not statistically significant.

Secondly, the estimator of volatility's coefficient γ_r is around -0.20 % and significantly negative at 5% level. This suggests a negative impact on S&P500 index range, which implies a rise of market efficiency in the event window. This finding is also consistent with our expectation. As a controversial political event, Trump election may cause investors' to have heterogeneous beliefs. Properly speaking, investors have heterogeneous expectation on the probability distribution of election's result, which leads to lower market efficiency. The heterogeneity is gradually reduced until the election result is released, which then causes the market efficiency to improve.

Furthermore, some other interesting findings are presented as well. Firstly, the coefficients of the interval-valued crude oil prices and dollar index, i.e. β_2 and β_3, are significantly positive, which implies that the volatility in stock market is highly linked to that of crude oil prices and dollar index. Specifically, the higher volatility the gold price has, the higher volatility the S&P500 has. This suggests a significant transmission and

Table 4. Ex ante analysis on S&P500 index's daily return.

		γ_m			γ_r		
Coef (%) (p-value)		0.05 (0.56)			– 0.20** (0.04)		
	β_1	β_2	β_3	β_4	β_5	α_0	β_0
Coef (p-value)	0.00 (0.96)	0.12*** (0.00)	0.18* (0.06)	0.02*** (0.00)	– 0.18*** (0.00)	0.00 (0.43)	– 0.03** (0.00)

Notes: This is the model for equation (4.1). γ_m and γ_r capture the impact of Trump victory on the midpoints and ranges of the interval-valued returns. Results are reported under the titles of 'Coef' and 'P-value' that represent the parameter estimates and the corresponding P-values, respectively. Asterisks ***, **, * denote rejections of the null hypothesis at 1%, 5% and 10% significant levels, respectively.

volatility spillover among the S&P 500 index and commodity markets. Another noticeable result is that VIX has information in explaining S&P 500 index before the Trump election, confirmed by the statistically significant estimates of β_5 at 1% level.

6.2. *Ex post analysis*

Tables 5–7 report the nonlinear minimum D_K-distance estimators and the *P*-values for ex post analysis on Trump's victory's impact on S&P500 index. Tables 5–6 also report the estimation results with a decreasing number of interval-valued exogenous variables for equation (4.2). This gives an insight on the robustness of our results under various interval-valued control variables, which help explain the dynamics of interval-valued S&P 500 returns. The P-values for testing the impact on the trend and range of interval-valued returns, i.e. to test whether γ_m and γ_r are significantly larger than zero, are also collected in tables 5–7.

From table 5, we observe that the initial impact on S&P500 index' expected return, i.e. γ_m, is significantly positive, around 0.24%. This finding indicates that Trump's victory positively affects the fundamental value of aggregated US stock market. This situation is common, since the overall market have a historic tendency to rise after elections. Secondly, its decay rate ρ_m is around 1.11%, which implies that this initial impact will be reduced by 1.11% per trading day. A half-life period, which is calculated as $\tau = \ln 2/\rho_m = 62.4$ days, suggests that the impact on S&P500 index's expected return will be reduced to half of its initial value in one and half week. Thus, Trump's victory has a fairly durable impact on US stock market's expected return.

Besides, Trump's victory's initial impact on S&P500 index's intra-day volatility γ_r is significantly negative (-1.86%). This finding suggests that Trump's victory effectively decreases S&P500 index's intra-day volatility on the election day. Additionally, its decay rate θ_r suggests that this impact will be reduced by 4.35% per trading day. Hence we can conclude that Trump's victory has an extremely durable negative impact on US stock market's intra-day volatility. Its half-life period is $\tau = \ln 2/\rho_r = 15.9$ days, namely, it will take one month to reduce this impact to half of its initial value. One possible reason is that domestic and international society are generally optimistic on the outlook for US economy due to some of the policies claimed by Trump, including positive fiscal expansion and tax cuts. This is consistent with findings in Wagner *et al.* (2017), who point out that both growth prospects and expectations of a major corporate tax cut are viewed positively by the stock market. Indeed, the stock market was up so dramatically when Trump's victory occurred, which implies that many relative losers (e.g. healthcare, medical equipment, textile, etc) actually increased in price, but not nearly as much as relative winners (e.g. heavy

Table 5. Ex post analysis on daily return.

	ρ_m	ρ_r	γ_m	γ_r			
Coef (%) (P-value)	1.11* (0.06)	4.35*** (0.01)	0.24*** (0.00)	– 1.86***** (0.01)			
	β_1	β_2	β_3	β_4	β_5	α_0	β_0
Coef (P-value)	– 0.04*** (0.41)	0.02 (0.14)	0.02 (0.90)	0.02** (0.00)	– 0.04** (0.48)	– 0.00*** (0.14)	– 0.04 (0.00)

Notes: This is the model for equation (11). γ_m and γ_r capture the impact of Trump victory on the midpoints and ranges of the interval-valued returns. Results are reported under the titles of 'Coef' and 'P-value' that represent the parameter estimates and the corresponding P-values, respectively. Asterisks ***, **, * denote rejections of the null hypothesis at 1%, 5% and 10% significant levels, respectively.

Table 6. Ex post analysis on daily return from Jan 2015 to Oct 2017.

	M-2		M-3		M-4		M-5	
	Coef	P-value	Coef	P-value	Coef	P-value	Coef	P-value
α_0	0.00*	0.10	0.00	0.27	0.00	0.45	0.00	0.51
β_0	– 0.02***	0.00	0.01***	0.00	0.01***	0.00	0.01***	0.00
β_1	0.01	0.83	– 0.16***	0.00	0.04	0.32	0.01	0.78
β_2	0.07***	0.00	0.09***	0.00	0.12***	0.00	0.10***	0.00
β_3	– 0.02	0.87	0.18*	0.05	– 0.04	0.64		
β_4	0.01***	0.00						
β_5					– 0.14***	0.00		
ρ_m	0.00	1.00	6.63	0.35	0.73	0.87	0.01	1.00
ρ_r	2.87***	0.00	0.17*	0.06	0.61**	0.01	2.25***	0.00
γ_m	0.00	0.98	0.24	0.19	0.02	0.77	0.00	0.98
γ_r	– 1.65***	0.00	– 0.51***	0.00	– 0.52***	0.00	– 1.19***	0.00

Notes: γ_m and γ_r capture the initial impact of the Trump election on the trends and ranges of the interval-valued stock returns. ρ_m and ρ_r capture the decay rate as t increases. M-2, M-3, M-4 and M-5 represent four nonlinear interval regression models. Results for each model M-i ($i = 2, 3, 4, 5$) are reported in two columns under the titles of 'Coef' and 'P-value' that represent the parameter estimates and the corresponding P-values, respectively. Asterisks ***, **, * denote rejections of the null hypothesis at 1%, 5% and 10% significant levels, respectively.

Table 7. Ex post analysis on daily return from Nov 2015 to Oct 2017.

	M-1		M-2		M-3		M-4		M-5	
	Coef	P-value	Coef	P-value	Coef	P-value	Coef	P-value	Coef	P-value
α_0	0.00*	0.07	0.00**	0.05	0.00**	0.05	0.00	0.54	0.00**	0.03
β_0	– 0.03***	0.00	– 0.03***	0.00	0.01***	0.00	0.01***	0.00	0.01***	0.00
β_1	– 0.02	0.72	0.06	0.19	0.04	0.36	0.00	0.97	0.05	0.25
β_2	0.10***	0.00	– 0.01	0.28	0.13***	0.00	0.13***	0.00	0.09***	0.00
β_3	– 0.01	0.97	0.02	0.79	– 0.03	0.75	0.03	0.77		
β_4	0.01***	0.00	0.02***	0.00						
β_5	– 0.03	0.62					– 0.13***	0.01		
ρ_m	0.01	1.00	0.01	1.00	0.01	1.00	0.48	0.62	0.01	1.00
ρ_r	2.46*	0.09	3.07**	0.03	0.25***	0.01	0.01	0.97	0.21**	0.04
γ_m	0.01	0.85	0.01	0.86	0.01	0.85	0.06	0.33	0.01	0.85
γ_r	– 0.56**	0.04	– 0.63**	0.01	– 0.46***	0.00	– 0.12*	0.06	– 0.39***	0.00

Notes: γ_m and γ_r capture the initial impact of the Trump election on the trends and ranges of the interval-valued stock returns. ρ_m and ρ_r capture the decay rate as t increases. M-1, M-2, M-3, M-4 and M-5 represent four nonlinear interval regression models. Results for each model M-i ($i = 1, 2, 3, 4$) are reported in two columns under the titles of 'Coef' and 'P-value' that represent the parameter estimates and the corresponding P-values, respectively. Asterisks ***, **, * denote rejections of the null hypothesis at 1%, 5% and 10% significant levels, respectively.

industries, financial firms, etc). Although, one may argue that many policies (e.g. tax policy and trade policy) proposed by Trump that would ultimately be implemented were uncertain, since they required Congressional approval. However, it is clear that President Trump wants to cut corporate taxes significantly below its current 35% level, and is very likely to succeed, given that the Republican holds a majority in congress. In addition, many Democratic legislators, have the same preference (Wagner *et al.* 2017).

Next, we discuss the estimates of other interval control variables. From tables 5–7, we observe that the trend of interval-valued returns tend to move towards an equilibrium state, as it is driven by its range in the last period. This confirms mean reversion in Poterba and Summers (1988). Another noticeable result is that the coefficient of $IVIX_t$, i.e. β_4, is significantly negative at 1% level. Volatility index (VIX) shows the market's expectation of 30-day volatility. It implies that higher VIX, more risk, higher range, and further causing inefficiency of stock market. This finding is different from equity volatility puzzle, which is proposed by Shiller (1981). Shiller (1981) concluded that the greater volatility of the stock market could plausibly be explained by any rational view of the future, which is possibly caused by poor market efficiency. Our result supports that the increase in volatility further decreases the market efficiency, which is another supplemental explanation to Shiller's findings.

Finally, we conduct various robustness check to determine whether our results are sensitive to different sample sizes and choices of control variables. This shows that our interval methodology has a robust performance when the sample size is small.

6.3. Alternative explanation

6.3.1. Market efficiency. To gain the insight on the informational advantage of using interval data, we discuss one feature of ITS, a measure of volatility. Volatility analysis has drawn growing attention over the last few decades, see Engle (2001, 2002, 2008). Unlike other volatility measures frequently employed in the existing literature, high-low range is an ideal choice to proxy the market efficiency. It is expected that high-low range performs differently from other classical volatility measures.

We consider the following GARCH-in-mean model

$$\begin{aligned} R_t &= c_0 + \alpha_0\sigma_t^2 + \varepsilon_t, \\ \sigma_t^2 &= c_1 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \beta_2 R_{oil,t} + \beta_2 R_{index,t} \\ &\quad + \beta_4 VIX_t + \beta_5 R_{Gold,t} + \gamma Dummy_t, \end{aligned} \tag{13}$$

where R_t is the daily return on S&P 500; c_0 and c_1 are constants; $R_{oil,t}$, $R_{index,t}$, $R_{glod,t}$ are the daily returns of NYMEX WTI, US dollar index and Comex gold future, respectively; $Dummy_t$ is the dummy variable, which is zero before Trump's victory and one after Trump's victory.

Table 8 reports the estimation results in equation (13). We consider two estimation samples, i.e. January 2015–October 2017, and November 2015–October 2017. The results are quite similar. We then take the latter sample as an example. We observe that the coefficient of the dummy variable $Dummy_t$, i.e. γ, is significantly positive at 5% significance level, and $\widehat{\gamma}$ is around $0.05 * 10^{-4}$ for different sample sizes. This indicates that the Trump election has a significant positive impact on S&P 500 index's volatility. However, this is different from our results in tables 6 and 7. According to our previous result, Trump victory's positive impact on the range of S&P 500 index is significantly negative. That is to say, the high-low range is essentially different from the inter-day volatility.

The aforementioned analysis reveals different aspects of asset prices' evolution. GARCH model attempts to gauge the volatility of assets' fundamental values, while high-low range reflects more information about market efficiency. The fundamental value movements are mostly presented in opening prices of assets, as pointed out by Tetlock (2007). On the other hand, intra-day fluctuation of assets' prices is mainly attributed to micro-noise and investors' sentiment. This suggests that the fundamental values of assets rarely change during the trading periods between the trading periods from

Table 8. S&P500 index's daily return under GARCH models.

	Jan,2015-Oct,2017			Nov,2015-Oct,2017		
	Coefficient	SD	P-value	Coefficient	SD	P-value
α_0	2.01	6.40	0.75	− 1.61	8.68	0.85
c_0(%)	0.06	0.03	0.03	0.07	0.03	0.02
	Variance equation			Variance equation		
c_1(%)	− 0.01	0.00	0.00	− 0.01	0.00	0.00
α_1	0.11	0.04	0.00	0.09	0.04	0.04
β_1	0.44	0.08	0.00	0.51	0.08	0.00
$\beta_2(10^{-4})$	0.09	0.52	0.86	0.08	0.53	0.88
$\beta_3(10^{-4})$	0.64	1.27	0.62	0.46	1.20	0.70
$\beta_4(10^{-4})$	0.60	0.09	0.00	0.48	0.08	0.00
$\beta_5(10^{-2})$	0.06	0.02	0.00	0.05	0.02	0.00
$\gamma(10^{-4})$	0.05	0.03	0.06	0.05	0.02	0.04

Notes: The GARCH model used here is

$$R_t = c_0 + \alpha_0\sigma_t^2 + \varepsilon_t,$$
$$\sigma_t^2 = c_1 + \alpha_1\varepsilon_t^2 + \beta_1\sigma_t^2 + \beta_2 R_{oil,t} + \beta_2 R_{index,t} + \beta_4 VIX_t + \beta_5 R_{Gold,t} + \gamma Dummy_t,$$

where R_t is the daily return on S&P 500; c_0 and c_1 are constants; $R_{oil,t}$, $R_{index,t}$ and $R_{glod,t}$ are the daily returns of NYMEX WTI, US dollar index and Comex gold future, respectively; $Dummy_t$ is a dummy variable, which is zero before the Trump victory and one after the Trump victory.

the market opening to closing. Besides, it is widely acknowledged that observed prices of assets should fluctuate around the fundamental value. This implies that high-low range sets an upper bound for the maximal variation of observed prices' from the fundamental value. Therefore, high-low range is an desirable proxy of market efficiency.

6.3.2. Time-varying coefficient. A time series dynamics usually suffers from abrupt structural breaks or smooth structural changes due to external factors, including policy shifts, technology progress and preference switch. To handle such instability, instead of using all available observations, it is quite common to use only the most recent observations

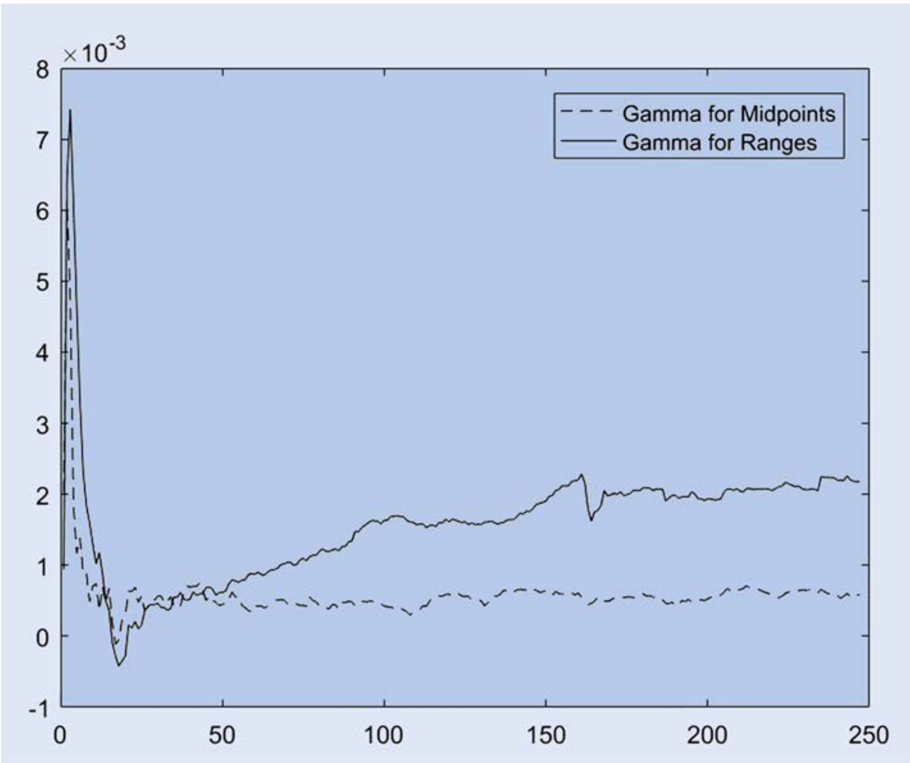

Figure 2. Coefficient estimators for dummy variables.
Note: The rolling estimators of coefficients (i.e. γ_m and γ_r) are listed. The sample starts from 1st January 2015 to 8th November 2016 to estimate parameters at 8th November 2016, and this is so-called fixed rolling window. After that, we estimate parameters increasing a new observation and removing the earliest one. The procedure ends when the sample set is extended to the last day of October 2017.

to estimate the parameters (the so-called rolling estimation method). Rolling estimation always drops the earliest observation as an additional observation is used, which is a simple way to update the sample information. This implicitly assumes that the underlying model parameters are time-varying. So it is a simple tool to capture the evolutionary behavior of economic and financial time series. Thus, we use this estimation method to visually present whether the impact of Trump election on the interval-valued stock returns is exponentially decaying.

We consider the linear interval-valued regression:

$$Y_t = \alpha_0 + \beta_0 I_0 + \beta_1 Y_{t-1} + \beta_2 Oil_t + \beta_3 Index_t + \beta_4 IVIX_t + \beta_5 Gold_t + \gamma_m D_{t,m} + \gamma_r D_{t,r} + u_t, \tag{14}$$

where the interval dummy variables take the nonzero intervals, i.e. $D_{t,m} = [1, 1]$ and $D_{r,t} = [-\frac{1}{2}, \frac{1}{2}]$, during the event window from November 8th, 2016 to October 31st, 2017. Rolling estimation is employed in equation (14). The sample starts from January 1st, 2015 to November 8th, 2016 to estimate the parameters at November 8th, 2016. This is so-called fixed rolling window. After that, we estimate parameters increasing a new observation and removing the earliest one. The procedure ends when the sample set is extended to the last day of October 2017. The movement of coefficients γ_r and γ_m is shown in figure 2.

Figure 2 reports the estimations of γ_m and γ_r after Trump election. From this figure, we can see that the decaying process of these coefficients resembles an exponential function. This is consistent with our model setting. Specifically, we observe parameters estimators of two large peaks (i.e. γ_r and γ_m) are most close to the election day. After that, both of these two curves display a sharp downward pattern. The estimators of γ_m constantly stay in a small neighborhood near zero, after its half-life period of 62.4 days. Besides, the estimators of γ_r declines dramatically relative to γ_m. After almost 15.9 days, which is its half-life period, the estimator γ_r has dropped below zero and stays at a relatively low level, although the subsequent sample shows a slightly upward pattern. These findings support the appropriate model setting (i.e. an exponential function) in this paper.

7. Conclusions

This paper firstly proposes a new class of nonlinear interval models and develops the nonlinear minimum D_K-distance estimator. The asymptotic theory regarding consistency, asymptotic normality and efficiency of the proposed estimators is established. The proposed model is employed to analyze the impacts of the Trump election on stock markets. Furthermore, the Wald statistic is used to quantify the changes in direction and magnitude of both the trend and volatility of the ITS.

In the empirical application, a time series of interval-valued returns on financial assets are constructed as highest, lowest and closing prices of every trading day. We argue that the range of interval-valued return can be considered as a proxy of market efficiency, and its midpoint reflects the return driven by fundamental value movement. Ex ante analysis indicates that the S&P 500 index's fundamental value rises insignificantly in a one-month window prior to the election day, while the market efficiency is improved due to a significantly negative impact on the S&P500 index's interval-valued return in our event window. Ex post analysis finds that the Trump election had a significantly positive impact on the return of the S&P500 index on the election day, but this impact was reduced by 1.11% per trading day. Moreover, the Trump election also improved market efficiency, having a significantly negative impact on the S&P500 index's interval-valued return on election day. However, this impact was also reduced by 4.35% per trading day.

There are several possible extensions for subsequent work. Firstly, the inherent connections between the interval-valued variables should be analyzed, e.g. the Granger causality test can be extended to ITS to examine whether one interval variable is useful in forecasting another. Secondly, it is widely acknowledged that the economy may behave differently if some variable lies in one region rather in another. For instance, the link between financial markets is stronger during the crisis period than during normal periods. It may be suitable to use the nonlinear interval model with different regimes to analyze the linkages between financial markets, which are all affected by Trump's election. Third, model selection is an important aspect of statistical modelling, while most works on interval modelling use traditional tools (Akaike Information Criterion and Bayesian Information Criterion) to select the best bivariate regression model by treating intervals as two points. It is highly desirable to propose a new model selection method for interval-valued data, which efficiently utilizes information contained in intervals. Furthermore, it is possible to combine this approach with other interval predictive models (e.g. Han *et al.* 2015, Sun *et al.* 2018) to improve the forecast accuracy of future volatility in equity, commodity and foreign exchange markets. This could provide some important insight for portfolio hedgers and authorities in making optimal portfolio allocations and engaging in risk management.

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Appendix

Proof of Theorem 1 The first order condition is

$$g(\widehat{\boldsymbol{\theta}}) = -\frac{2}{T}\sum_{t=1}^{T}\langle s_{Y_t-h(\mathbf{Z}_t,\widehat{\boldsymbol{\theta}})}, s_{\frac{\partial h(\mathbf{Z}_t,\widehat{\boldsymbol{\theta}})}{\partial\boldsymbol{\theta}}}\rangle_K = \mathbf{0}. \tag{A1}$$

In the more general cases, this equation is a set of nonlinear equations that do not have an explicit solution.

Applying Taylor expansion of $\widehat{\boldsymbol{\theta}}$ in equation (2.3), we get ($\bar{\boldsymbol{\theta}}$ is between $\widehat{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}^0$)

$$\begin{aligned}\mathbf{0} &= \frac{1}{T}\sum_{t=1}^{T}\left\langle s_{Y_t-h(\mathbf{Z}_t,\boldsymbol{\theta}^0)-\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}, s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}+\frac{\partial^2 h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}\right\rangle_K \\ &= \frac{1}{T}\sum_{t=1}^{T}\left\langle s_{u_t-\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}, s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}\right\rangle_K + (s.o.),\end{aligned} \tag{A2}$$

where $(s.o.)$ denotes smaller order terms.

Thus,

$$\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0 = [T^{-1}\sum_{t=1}^{T}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}\rangle_K]^{-1}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}, s_{u_t}\rangle_K$$

Based on Assumption 2, $\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0 = o_p(1)$ (in fact, $\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0 = O_p(T^{-1/2})$). ■

Proof of Theorem 2 Applying Taylor expansion of $\widehat{\boldsymbol{\theta}}$ in equation (2.3), we get ($\bar{\boldsymbol{\theta}}$ is between $\widehat{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}^0$)

$$\begin{aligned}\mathbf{0} &= \frac{1}{T}\sum_{t=1}^{T}\left\langle s_{Y_t-h(\mathbf{Z}_t,\boldsymbol{\theta}^0)-\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}, s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}+\frac{\partial^2 h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}\right\rangle_K \\ &= \frac{1}{T}\sum_{t=1}^{T}\left\langle s_{u_t-\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}^0)}, s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}\right\rangle_K + (s.o.),\end{aligned} \tag{A3}$$

where $(s.o.)$ denotes smaller order terms. Re-arrange equation (A3) and based on Theorem 3.2 in Han *et al.* (2015), we obtain

$$\begin{aligned}\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^0) &= \left[T^{-1}\sum_{t=1}^{T}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}\rangle_K\right]^{-1} T^{-1/2} \\ &\quad \times \sum_{t=1}^{T} s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}|_{\boldsymbol{\theta}=\bar{\boldsymbol{\theta}}}}, s_{u_t}\rangle_K + (s.o.) \\ &\to N(\mathbf{0}, \mathbb{E}(\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}\rangle_K)^{-1} \\ &\quad \times \mathbb{E}\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}, s_{u_t}\rangle_K \langle s_{u_t}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}\rangle_K \\ &\quad \times \mathbb{E}(\langle s_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}, s'_{\frac{\partial h(\mathbf{Z}_t,\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}}\rangle_K)^{-1} \\ &= N(\mathbf{0}, \mathbf{M}^{-1}(\boldsymbol{\theta}^0)\mathbf{V}(\boldsymbol{\theta}^0)\mathbf{M}^{-1}(\boldsymbol{\theta}^0)). \blacksquare\end{aligned}$$