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Abstract

When parents divorce and have common children, the parents have to agree on how much each parent should contribute to cover the expenses of common children. We call this the *divorced-parents problem*. When parents cannot reach an agreement, they can start a law case. In many cases the situation can be easily settled by a judge, but finding a solution for complicated situations with parents having multiple children from different partners is considerably more difficult. In fact, it is observed that judges lack methods to find a good and consistent solutions. As a result, it occurs that in similar situations, the outcomes of the court cases are different, thereby leading to inequalities in law. In yet other cases, outcomes are even in direct conflict with the decisions of the Dutch supreme court.

In this note we develop an algorithm to find the unique proportionally fair distribution for the divorced-parents problem. Such a proportionally fair distribution has at least three advantages. The existence of a unique solution may prevent parents to resettle the distribution via court procedures, which are (very) costly for parents and society. Second, it can be computed efficiently so it can easily cope with changes in income, schooling costs, and so on. Third, the solution generalizes the proportional rule that is currently applied to simple two-parents-one-child networks to larger networks.

1 Introduction

When two Dutch parents divorce and have common children, they both have a financial responsibility to cover the monthly expenses of the children, for housing, schooling, and so on. The legal process to determine the financial contribution of each parent to each child works roughly as follows. First, a mediator, or a judge, establishes a network that formalizes which parent is financially responsible for which child; this is not always easy, for instance, in the presence of step parents. Next, the mediator uses rules to determine the financial need of each child and the financial capacity of each parent. These rules are partly based on (case) law and partly on specific circumstances such as income, schooling costs, and so on. Once the network of responsibilities, capacities and needs is specified, it remains to determine a distribution of contributions of the parents to the children. Henceforth we refer to this problem as the *divorced-parents* problem.

Based on an analysis of multiple law cases, Jonker et al. [2020] establish the following rules that Dutch judges strive to use to solve the divorced-parents problem:

1. Parental capacity is meant to be 'used' for its purpose, in other words, a child cannot have a shortage unless both its parents already spent their full capacity.

2. When two parents of one (or multiple common) child have an overage, distribute the overage of the parents relative to the capacity that each parent has available for the child.

3. Children of one parent should be treated equally, for instance, children born in later marriages should have the same rights as children born in earlier marriages.

4. The capacities of all parents should be taken into account, to the extent possible. In other words, if a parent has obligations towards multiple children, the old and new partners should also take responsibility for the children of the parent.

In the sequel we say that a distribution, or a solution, is *proportionally fair* when it satisfies these rules.

In the simple case of two parents and one child (or multiple 'equal' and common children), the law cases directly apply the above rules to distribute parental overages, and it is easy to see that this distribution is unique. However, most situations that are brought to court are considerably more complicated. For instance, one case mentions one woman having five children from four different partners, and the partners' jobs and incomes vary on a nearly monthly basis. In these more difficult cases, Jonker et al. [2020] show that judges attempt to find a proportionally fair solution, but have to settle on approximations due to the complexities of computations. These approximations, however, have significant drawbacks. First, for more or less similar situations, the distributions can differ significantly, thereby leading to legal inequalities between cases. Second, as the manual computation of even approximately fair distributions is (very) time-consuming, the contributions of the parents are not updated even when there are significant changes in income, newly born children, and so on. These problems give rise to additional conflicts between ex-spouses when they perceive the settlements as 'unfair' or 'arbitrary'; these conflicts sometimes lead again to new (costly and lengthy) court cases.

In this paper we prove that a unique proportionally fair solution exists for the divorced-parent problem for networks of arbitrary size, and we provide an algorithm to compute this distribution. The existence of a solution was earlier proved by Moulin and Sethuraman [2013] but they do not provide an algorithm to actually compute the solution.

2 Model and Proof

Parents and children are represented as nodes in a directed bi-partite graph. Parents have (financial) capacities $d = (d_1, \dots, d_M)$ to cover the (financial) needs $b = (b_1, \dots, b_N)$ of the children. The children for whom a parent is (financially) responsible are represented by directed arcs from the parent to the children. We use the δ function to represent the parent-child relations; $\delta_{ij} = 1$ when parent i is responsible for child j, otherwise $\delta_{ij} = 0$. Note that these relations can also be enforced by setting the transportation cost $c_{ij} = (1 - \delta_{ij})/\delta_{ij}$ from node i to node j.

Parent i pays a (care) contribution $x_{ij} \geq 0$ to child j; of course, $x_{ij} = 0$ when $\delta_{ij} = 0$. For a given set of contributions $x = \{x_{ij}\}$ we define the *overage*, or surplus, of parent i as

$$y_i = d_i - \sum_{j=1}^{N} \delta_{ij} x_{ij}, \tag{1}$$

and the *budget* that parent i has available for child j after meeting all its obligations to all other children for whom s/he is responsible as

$$y_i + x_{ij} = d_i - \sum_{k \neq j} \delta_{ik} x_{ik}. \tag{2}$$

Analogously, define the *shortage* of child j as

$$z_j = b_j - \sum_{i=1}^{M} \delta_{ij} x_{ij}.$$

We write $y = (y_1, \ldots, y_M)$ and $z = (z_1, \ldots, z_N)$. Note that y and z depend on the payments matrix $x = \{x_{ij}\}$.

The max-flow problem is equivalent to finding a solution for Rule 1 of the Introduction, and can be written as the linear program (LP)

$$\min_x \left\{ \sum_j z_j \, ; x \geq 0, y \geq 0, z \geq 0 \right\}. \tag{3}$$

The constraints are evident: the parental contributions x cannot be negative; parents pay at most their capacity, hence $y \geq 0$; and children receive at most their need, hence $z \geq 0$. (The inequality $y \geq 0$ means $y_i \geq 0$ for each term.)

In case the solution lies on a corner of the feasible set, this LP suffices to find the optimal solution. However, when multiple solutions exist, we can Rule 2 of the Introduction to distribute any overages and shortages in a fair way. For this, we first assume that we deal with a network in which all parents have an overages, hence $y > 0$ and $z = 0$. Then we discuss general networks.

2.1 A network with overages

In a proportional distribution, the overages for parents i and k that are both responsible for child j should be such that the relative overages of both parents are the same. Recalling (1) and (2), we mean by this that x and y must be such that

$$\frac{y_i}{y_i + x_{ij}} = \frac{y_k}{y_k + x_{kj}}, \tag{4}$$

where $y_i/(y_i + x_{ij})$ has the interpretation of the overage of parent i relative to the total budget that parent i has at its disposal to meet the needs of child j. From (1) and (2) we see that $y_i = y_i + x_{ij} - x_{ij}$, so that the above relation can be rewritten to

$$\frac{x_{ij}}{y_i + x_{ij}} = \frac{x_{kj}}{y_k + x_{kj}}. \tag{5}$$

In words, instead of proportionally distributing the relative overages we can just as well distribute the relative payments of the parents.

Now observe that (5) together with the assumption $y > 0$ imply that $x_{ij}/y_i = x_{kj}/y_k$, which in turn implies that there exists a proportionality factor $\beta_j > 0$ such that $x_{ij} = \beta_j y_i$. Observe that $x_{ij} > 0$ since $y_i > 0$ by assumption. When parent y_i is not responsible for child x_j we have that $\delta_{ij} = 0$. Thus, we have established that

$$x_{ij} = \delta_{ij} y_i \beta_j. \tag{6}$$

Next, as we are dealing with a network with overages, we can impose the condition that $z = 0$, i.e., all needs are satisfied. In particular, for child j this means that

$$z_j = 0 \iff \sum_i \delta_{ij} x_{ij} = b_j. \tag{7}$$

Substitute the expression (6) for x_{ij} into this equation to see that β_j must satisfy $\beta_j \sum_k \delta_{kj} y_k = b_j$. From $y_k > 0$ it follows that $\sum_k \delta_{kj} y_k > 0$ for each child j, thereby allowing us to write $\beta_j = b_j / \sum_k \delta_{kj} y_k$. Let us substitute this into (6) to obtain

$$x_{ij} = \delta_{ij} \frac{y_i}{\sum_k \delta_{kj} y_k} b_j. \tag{8}$$

Finally, using this in (2), it follows that y has to satisfy the equality

$$y_i \left(1 + \sum_j \delta_{ij} \frac{b_j}{\sum_k \delta_{kj} y_k} \right) = d_i. \tag{9}$$

Thus, suppose we can find a vector of overages $y > 0$ that solves (9). Then, with (8), we can find a set of parental contributions $x \geq 0$. It is clear from the construction that x satisfies Rule 1, namely by (7). Moreover, Rule 2 is simultaneously satisfied via (5) and (4).

In fact, we can compute a unique solution for (9) with recursion, thereby proving the existence of a unique solution for the divorced parents problem. To this end, define the ith component of the (vector) function f as

$$f_i(v) = 1 + \sum_j \delta_{ij} \frac{b_j}{\sum_k \delta_{kj} v_k},$$

for a set $v > 0$ of overages. Observe that with this, (9) reduces to $y_i f_i(y) = d_i$.

Lemma 2.1. Suppose that there is a vector $v > 0$ such that $v_k f_k(v) \geq d_k$ for all its components v_k, and $v_i f_i(v) > d_i$ for the ith component. Take

$$v'_i = d_i / f_i(v).$$

Then we have that i. $0 < v'_i < v_i$, and ii. $v'_i f(v') > d_i$.

Proof. i.) Observe that $v_i f_i(v) > d_i \implies v_i > d_i / f_i(v) = v'_i$. Next, $v > 0 \implies f(v) > 0$, hence $v' > 0$.

ii.) Use the definition $v'_i = d_i / f_i(v)$ to reduce the inequality $v'_i f_i(v') > d_i$ to the inequality $f_i(v') > f_i(v)$. But this latter inequality directly follows from the definition of f and observing that, by i., $v'_i < v_i$ and $v'_k \leq v_k$.

□

The solution y now follows straightaway computed from recursion. Take $v^0 = d > 0$; the argument being that the overages can never exceed the capacities. From the definition of f, we see that $f(v^0) > 1$, so that $v_i^0 f_i(v^0) > d_i$ for all i. Next, define $v_i^1 = d_i / f_i(v^0)$. By the above lemma, $v^1 < v$ and again $v_i^1 f_i(v^1) > d_i$. Thus, we can apply this lemma again to v^1 to obtain $v_i^2 = d_i / f_i(v^1) < v_i^1$, and so on. Clearly, this recursive procedure yields a monotone decreasing set of vectors v^n that is bounded from below since $v^n > 0$ for all n. Then, by the theorem of Weierstrass, it follows that v^n converges to a unique limit point y. This limit point y must satisfy (9), for if $y_i f_i(y) > 0$ for some i, we can use the lemma to find a smaller vector.

2.2 General networks

In the previous section we assume that there was an overage, i.e., $y > 0$. Let us show how we can apply the same method to a general network. For this we need to split the network into two sub networks, one in which parents have an overage, and the other part in which children have a shortage. Thus, the overage and shortage networks are complementary: each parent (child) must belong to either the overage or the shortage network. The LP (3) proves a highly device to split the network.

In more detail, we associate a Lagrange multiplier λ with the constraint $y \geq 0$, and μ with $z \geq 0$. In the optimal solution, when $y_i > 0$, it is clear that parent i should be assigned to the overage network. Next, by complementary slackness, when $\lambda_i > 0$ for parent i, $y_i = 0$. Hence, any marginal increase in the capacity of parent i can be used to reduce the shortage of a child. Thus, such a parent i must necessarily belong to the shortage network. By analogy, when $z_j > 0$ ($\mu_j > 0$) child j belongs to the shortage (overage) network.

It may happen that the solution is degenerate such that $y_i = \lambda_i = 0$ for parent i. To find out to which sub-network we should assign this parent, we propose to add an extra constraint $y_i > 1$ with multiplier ν_i and solve this augmented LP. With complementary slackness the assignment can follow the same reasoning as earlier, and similar for children with $z_j = \mu_j = 0$.

Another method would be to solve the initial LP (3) into a non-linear optimization problem with objective

$$\sum_i z_i + \epsilon \sum_i y_i^2 + \epsilon \sum_j z_j^2, \tag{10}$$

with $\epsilon \ll 1$. The solution of this satisfies our requirements. To see this, observe that in a degenerate solution of the LP, it is possible to 'move money' from one parent to another without affecting the value of the objective (3) of the LP. However, in the non-linear objective, it is optimal to keep the largest y_i as small as possible so that, as a consequence, the smallest overage will be as large as possible, as long as this does not affect the objective of the LP. Likewise reasoning applies to the shortages z.

With the algorithm of the previous section we can divide overages in an overage network proportionally over the parents. Interestingly, the same algorithm can be used to proportionally distribute the shortages of children in a shortage network. For this, consider the transpose of the shortage network. In other words, we swap the roles of the parents and the children, and we obtain a network in which there is a 'surplus of needs'. Then we apply the algorithm to proportionally distribute these 'surplus of needs'.

Thus, by applying the algorithm first to the overage network and then to the transpose of the shortage network we find a solution for the entire network.

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