Robust state-feedback stabilization of discrete-time linear polytopic positive systems using LMIs

Esteves Rosa, Tábitha; Morais, Cecília F.; Oliveira, Ricardo C.L.F.

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Tábitha E. Rosa*, Cecília F. Morais**, Ricardo C.L.F. Oliveira**
* University of Groningen, 9747 AG, Groningen, The Netherlands
** University of Campinas, 13083-852, Campinas-SP, Brazil
Email: t.esteves.rosa@rug.nl

1 Introduction

A class of systems that properly represents several real phenomena is called *positive systems*, where the state variables, inputs and outputs, can only assume non-negative values. This study field has some open problems since many of the well-establish results for linear systems cannot be directly applied to positive systems. This abstract proproses a new state-feedback design condition for discrete-time linear polytopic positive systems via the extension of the techniques given in [2] and [3], that use linear matrix inequalities (LMIs) combined with scalar searches.

2 Results

Consider the following discrete-time linear polytopic positive system

$$x(k+1) = A(\alpha)x(k) + B(\alpha)u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ represents the state vector, $u(k) \in \mathbb{R}^m$ is the control input. The system matrices belong to a polytopic domain, i.e., they can be written as a convex combination of the N known vertices: $(A,B)(\alpha) = \sum_{i=1}^{N} \alpha_i (A_i, B_i), \alpha \in \Lambda$, and α is a time-invariant parameter belonging to the unit simplex $\Lambda = \{\sum_{i=1}^{N} \alpha_i = 1,\ \ \alpha_i \geq 0,\ \ i = 1,\dots,N\}$. Furthermore, all entries of matrices $(A,B)(\alpha)$ are positive (internally positive system). In order to asymptotically stabilize and guarantee the positiveness of (1) in closed-loop, we consider the following robust state-feedback control law: $u(k) = Kx(k)$. The main result of this work is presented next.

Theorem 1 *For given scalars $\gamma \neq 0$ and ε, if there exist matrices $P(\alpha) = P(\alpha)' > 0 \in \mathbb{R}^n$, $F(\alpha)$ and $G(\alpha) \in \mathbb{R}^{n\times n}$, $L \in \mathbb{R}^{m\times n}$ and a diagonal matrix $S \in \mathbb{R}^{n\times n}$, such that $A(\alpha)S + B(\alpha)L \geq 0$ and*

$$\begin{bmatrix} \begin{pmatrix} A(\alpha)F(\alpha)+ \\ F(\alpha)'A(\alpha)' - P(\alpha) \end{pmatrix} & \star & \star \\ \begin{pmatrix} (A(\alpha)G(\alpha) + \varepsilon B(\alpha)L)' \\ -F(\alpha) \end{pmatrix} & P(\alpha) - G(\alpha) - G(\alpha)' & \star \\ (B(\alpha)L)' - \gamma F(\alpha) & \gamma(\varepsilon S - G(\alpha)) & \gamma(S+S') \end{bmatrix} < 0$$

hold $\forall \alpha \in \Lambda$, then $K = LS^{-1}$ assures the positiveness and the closed-loop asymptotic stability of (1).

The proof follows the same steps of Theorem 1 from [3] as well as the procedure to obtain the finite set of LMIs (programmable).

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To illustrate the effectiveness of the technique, consider the following example which analyses the state-space system whose polytopic matrices have the following vertices

$$\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right] = \left[\begin{array}{ccc|ccc} 7.307 & 3.154 & 5.515 & 9.837 & 4.432 & 7.749 \\ 4.725 & 2.438 & 3.764 & 9.150 & 4.266 & 7.402 \\ 11.18 & 5.754 & 10.028 & 11.158 & 5.246 & 9.410 \\ \hline 9.321 & 4.313 & 7.727 & 11.371 & 5.684 & 9.426 \\ 9.287 & 4.289 & 7.688 & 11.632 & 5.665 & 9.567 \\ 11.47 & 5.554 & 9.581 & 6.928 & 3.268 & 5.698 \end{array}\right]$$

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 4 & 5 \\ 4 & 4 & 5 \\ 5 & 5 & 3 \end{bmatrix}$$

The main goal is to compare Theorem 1 (T1) and other methods from the literature (LMIs (24), (25) and (27) from [1] adapted to handle the stabilization problem). In order to test T1, we consider $\gamma = -1 \times 10^5$ and the scalar search $\varepsilon \in [10^{-5},\ 10]$, containing 7 logarithmically spaced values.

All three conditions from [1] were not able to provide stabilizing gains. Considering T1 with the mentioned scalar search, only $\varepsilon = 10^{-2}$ and 10^{-3} were able to provide feasible solutions, respectively providing the following controllers (truncated with 4 decimal digits)

$$\left[\begin{array}{c} K(\varepsilon = 10^{-2}) \\ \hline K(\varepsilon = 10^{-3}) \end{array}\right] = \left[\begin{array}{ccc} -2.2316 & -1.0491 & -1.8383 \\ \hline -2.2314 & -1.0485 & -1.8378 \end{array}\right]$$

These results validate the advantages of employing slack variables (matrices $F(\alpha)$ and $G(\alpha)$) and scalar searches in order to reduce the conservatism of the synthesis condition at the price of an increase in the computational burden. Finally, note that in Theorem 1 it is not necessary to impose that the Lyapunov matrix is constant or diagonal to obtain robust controllers that guarantee the positiveness and stability of the system.

References

[1] Y. Ebihara, D. Peaucelle, and D. Arzelier. LMI approach to linear positive system analysis and synthesis. *Systems & Control Letters*, 63:50–56, 2014.

[2] T.E. Rosa, C.F. Morais, and R.C.L.F. Oliveira. New robust LMI synthesis conditions for mixed gain-scheduled reduced-order DOF control of discrete-time LPV systems. *Int. J. Robust Nonlinear Control*, 28(18):6122–6145, 2018.

[3] H. S. Vieira, R.C.L.F. Oliveira, and P.L.D. Peres. Robust stabilization and $\mathscr{H}_\infty$ control by means of state-feedback for polytopic linear systems using lmis and scalar searches. In *American Control Conference (ACC), 2015*, pages 5966–5973, Chicago, IL, USA, 2015. IEEE.