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Robust Average Formation Tracking for Multi-Agent Systems With Multiple Leaders ⋆

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Abstract: In this paper, the formation tracking problem of the multi-agent system under disturbances and unmodeled uncertainties has been studied. An identifier-based robust control algorithm using the neighboring relative information has been proposed to ensure the followers to maintain a given, and time-varying formation and track the average state of the leaders at the same time. Some sufficient conditions for the second-order multi-agent system with multiple leaders in the presence of disturbances and unmodeled uncertainties have been proposed based on the graph theory and the Lyapunov method. Numerical simulations are provided to testify the validity of the algorithm.

Keywords: Formation control, Robust formation tracking, Multi-agent systems, Lyapunov analysis.

1. INTRODUCTION

Formation control is a fundamental topic in cooperative control, whose control objective is to make multiple agents maintain some predefined, potentially time-varying, formation under a coordinated control scheme, when the agents carry out a given task. The applications of formation control range from military field to civil field including exploration of unknown environments (Das et al., 2002), search and rescue operation (Chen and Wang, 2005), performance of uninhabited combat air vehicles (UCAVs) (Duan et al., 2008).

The formation problem for multi-agent systems has been studied far and wide in the existing literature, and the past decades have witnessed significant advances in this field. For instance, Balch and Arkin (1998) presents a behavior-based approach to robotic line formation-keeping, which enables three behaviors, i.e., moving to the destination, avoiding obstacles, and formation keeping, to perform simultaneously and cooperatively. In Ren and Beard (2004, 2002), the virtual structure approach is proposed, where the whole formation is treated as a single structure. Shao et al. (2007) provides a leader-following approach to achieve predetermined formations for controlling groups of autonomous mobile vehicles, just like Ren and Sorensen (2008), Mylvaganam and Astolfi (2015), Wang and Ding (2014), where only formation control problems are considered. In Dong et al. (2015a), tracking problems are taken into account, where the followers form a given, and time-varying formation while tracking the state of the leader, but the problem are limited to the case of one leader. The formation containment problem for swarm systems is considered in Ferrari-Trecate et al. (2006), Liu et al. (2014) and Dimarogonas et al. (2006). Dong et al. (2015b) investigates the formation containment problem in the multiple leaders case, presenting protocols to make the leaders to realize the expected formation and make the followers to converge to the convex hull of the states of leaders. In Lewis and Tan (1997), virtual agents, determined by the desired formation, are brought in for each agent correspondingly, which transforms the formation problem into a consensus problem (Hu, 2012; Liu et al., 2012, 2016). Dong et al. (2016) considers the average formation tracking problem for second-order multi-agent systems with multiple leaders and directed interaction topologies. Despite the considerable achievements on formation control, one critical issue arises when the robustness of the formation strategy is required.

It is ineluctable that disturbances and unmodeled uncertainties exist in practical situation, while most of the existing work on the formation problem relies on the assumption that we know the exact model of the agent dynamics, without taking into account the disturbances and unmodeled uncertainties. Actuated by this fact, we are going to improve the robustness of the formation strategy by introducing an identifier to compensate for the disturbances and unmodeled uncertainties.

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The objective of this paper is to address the formation tracking problem for the second-order multi-agent system with multiple leaders in the presence of disturbances and unmodeled uncertainties. Only a subset of followers are informed of the time-varying states of the multiple leaders. First, an identifier is designed to identify and compensate for the disturbances and unmodeled uncertainties. Second, an identifier-based formation tracking protocol is proposed such that the states of the followers can maintain a given, and time-varying formation and track the average state of the leaders at the same time. When compared to the previous work on formation tracking control (Lin et al., 2005) (Balch and Arkin, 1998) (Dong et al., 2015a) (Dong et al., 2016), the contribution of this paper is that we propose a robust formation tracking control algorithm based on a nonlinear identifier, where the influence of the disturbances and unmodeled uncertainties can be eliminated.

This paper is organized as follows. In Section II, some basic notations on graph theory and some useful results on nonlinear systems are reviewed, and the problem formulation is described. Section III presents the identifier-based robust control algorithm to address the formation tracking problem, and some sufficient conditions are given to guarantee the multi-agent system to reach the control objective. The numerical simulations are provided in Section IV. Section V concludes the entire paper.

2. PRELIMINARIES

2.1 Notions and useful results

Some notions from algebraic graph theory are presented following (Godsil and Royle, 2013). A directed graph is denoted by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where $\mathcal{V} = \{1, 2, \dots N\}$ is the set of agents, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{W} = [w_{ij}], i, j = 1, 2, ..., N$ is the weighted adjacency matrix with elements w_{ij}. If $(j, i) \in \mathcal{E}$, then $w_{ij} > 0$, otherwise $w_{ij} = 0$ and the diagonal entries of $\mathcal{W}$ are zero, i.e., $w_{ii} = 0$. Let $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. $\mathcal{G}$ is said to be undirected if,$(i, j) \in \mathcal{E}$ implies that $(j, i) \in \mathcal{E}$. A directed path from agent i to agent j is a sequence of edges $(i, s_1), (s_1, s_2), ..., (s_k, j)$, where $(i, s_1), (s_1, s_2), ..., (s_k, j) \in \mathcal{E}$. $\mathcal{G}$ is said to be strongly connected if, any pair of agents in the graph are linked by a path. The Laplacian matrix of $\mathcal{G}$ is denoted as: $L = [l_{ij}] \in \mathbb{R}^{n\times n}, l_{ii} = \sum_{j=1}^{n} w_{ij}$, and $l_{ij} = -w_{ij}$, for $i \neq j$. It is clear that matrix L is symmetric if the graph is undirected.

Lemma 1. (Khalil and Grizzle, 1996) Consider the system $\dot{x} = f(t, x)$, where $f(t, x)$ is piecewise continuous in t, and locally Lipschitz in x on $[0, \infty) \times D$, where $D \in \mathbb{R}$ is a domain containing the origin $x = 0$. And the origin $x = 0$ satisfies that, for all $t \geq 0$, $f(t, x)$ is uniformly bounded. Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continuously differentiable function such that the following inequality satisfied

$$W_1(x) \leq V(t, x) \leq W_2(x),$$

$$\dot{V}(t, x) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -W_3(x),$$

for all $t \geq 0$ and $x \in \mathbb{D}$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions and $W_3(x)$ is a continuous positive semidefinite function on D. Select $r > 0$ and $s > 0$ so that $B_r \subset D$ and $s < \min_{\|x\|=r} W_1(x)$. Therefor, all solutions of $\dot{x} = f(t, x)$ with $x(0) \in \{x \in B_r | W_2(x) \leq s\}$ are bounded and when $t \to \infty$, it has

$$W_3(x(t)) \to 0.$$

What's more, the argument holds for $x(0) \in \mathbb{R}^n$ if the following two conditions are satisfied: (i) all the assumptions hold globally, (ii) $W_1(x)$ is radially unbounded.

2.2 Problem formulation

Consider a second-order multi-agent system consisting of N agents distributed on graph $\mathcal{G}$. It is assumed that there exist M $(M < N)$ followers and $N - M$ leaders. Let $F = \{1, 2, \dots M\}$ and $E = \{M+1, M+2, \dots N\}$ denote the set of followers and the set of leaders, respectively. A leader is an agent who does not receive any information from others. Otherwise, it is called a follower. The dynamics of the follower i is given by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = \alpha_x x_i(t) + \alpha_v v_i(t) + u_i(t) + \delta_i(t), i \in F, \\ \dot{x}_l(t) = v_l(t), \\ \dot{v}_l(t) = \alpha_x x_l(t) + \alpha_v v_l(t), l \in E, \end{cases} \tag{2.1}$$

where $x_i(t), v_i(t) \in \mathbb{R}^n$ are the position and the velocity of the follower i, $x_l(t), v_l(t) \in \mathbb{R}^n$ are the position and the velocity of the leader l. $u_i(t) \in \mathbb{R}^n$ is the control input. $\delta_i(t)$ is given by $\delta_i(t) = \delta_{1i}(t) + \delta_{2i}(v_i)$, where $\delta_{1i}(t) \in \mathbb{R}^n$ is the disturbance, and $\delta_{2i}(v_i) \in \mathbb{R}^n$ is the unmodeled dynamics. The disturbance $\delta_{1i}(t)$ and its first order and second-order time derivatives are bounded, i.e., $\delta_{1i}(t)$, $\dot{\delta}_{1i}(t), \ddot{\delta}_{1i}(t) \in \mathcal{L}_\infty, i = 1, ..., M$. The unmodeled dynamics $\delta_{2i}(t)$ and its first order and second-order derivatives with regard to x_i are bounded, i.e., if $x_i \in \mathcal{L}_\infty$, then $\delta_{2i}(t)$, $\frac{\partial \delta_{2i}(t)}{\partial x_i}$, $\frac{\partial^2 \delta_{2i}(t)}{\partial^2 x_i} \in \mathcal{L}_\infty, i = 1, ..., M$. $\alpha_x \in \mathbb{R}$ and $\alpha_v \in \mathbb{R}$ are known constant damping gains. Let $n = 1$ for briefness. It can be proved that all the results still hold when $n > 1$ using the property of Kronecker product.

The state of the followers are expected to maintain a time-varying formation. A piecewise continuously differentiable vector $d_i(t) = [d_{xi}(t), d_{vi}(t)]^T$ $(i \in F)$ represents the the expected time-varying formation for followers, and $D_F(t) = [d^T{}_1(t), ..., d^T{}_M(t)]^T$. Let $\xi_i(t) = [x_i(t), v_i(t)]^T, i = 1, ..., N$, and $\xi_F(t) = [\xi^T{}_1(t), ..., \xi^T{}_M(t)]^T$.

Definition 1. (Dong et al., 2016) The multi-agent system (2.1) is said to achieve average formation tracking if for any given bounded initial states and any $i \in F$,

$$\lim_{t\to\infty} \left(\xi_i(t) - d_i(t) - \frac{1}{N-M} \sum_{l=M+1}^{N} \xi_l(t) \right) = 0.$$

The interaction topology of the multi-agent system (2.1) is described by the directed graph $\mathcal{G}$. From the definitions of the leader and the follower, the Laplacian matrix of the directed graph $\mathcal{G}$ is written as

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)} \end{bmatrix},$$

where $L_1 \in \mathbb{R}^{M\times M}$ is the interaction among the followers, and $L_2 \in \mathbb{R}^{M\times(N-M)}$ denotes the connections from the leaders to the followers.

Assumption 1. For any follower, its neighbor set contains either all the leaders or no leaders. For each follower whose neighbor set contains no leaders, there exists at least one directed path from each leader to it.

Lemma 2. (Dong et al., 2016) If the directed graph $\mathcal{G}$ satisfies Assumption 1, then all the eigenvalues of L_1 have positive real parts; all the entries of $-L_1^{-1}L_2$ are identical and equal to $1/(N-M)$.

Remark 1. If the undirected graph $\mathcal{G}_1$ is connected, and satisfies Assumption 1, then the symmetric matrix L_1 is positive definite.

3. MAIN RESULTS

In this section, we consider the multi-agent system (2.1) in the presence of disturbance and unmodeled dynamics. The average formation tracking analysis and design problems are studied. Some sufficient conditions are proposed to achieve the control objective.

Define $\zeta_i(t) = \xi_i(t) - d_i(t)$, $i \in F$, and let $\zeta_i(t) = [\zeta_{xi}(t), \zeta_{vi}(t)]^T$ and $\zeta_F(t) = [\zeta^T{}_1(t), ..., \zeta^T{}_M(t)]^T$.

Let

$$\eta_{xi}(t) = \sum_{j=1}^{M} w_{ij}(t)(\zeta_{xi}(t) - \zeta_{xj}(t)) + \sum_{l=M+1}^{N} w_{il}(t)(\zeta_{xi}(t) - x_l(t)),$$

$$\eta_{vi}(t) = \sum_{j=1}^{M} w_{ij}(t)(\zeta_{vi}(t) - \zeta_{vj}(t)) + \sum_{l=M+1}^{N} w_{il}(t)(\zeta_{vi}(t) - v_l(t)),$$

$$\eta_i(t) = \beta\eta_{xi}(t) + \eta_{vi}(t),$$

where $i \in F$, $\beta \in \mathbb{R}$ is a positive constant scalar to be designed.

In order to compensate for the disturbance and unmodeled dynamics in the agent dynamics, we develop a nonlinear observer to estimate the disturbance and unmodeled dynamics. Denote $\hat{\delta}_i(t) \in \mathbb{R}$ is the observer to estimate the disturbance and unmodeled dynamics. Inspired by Hu (2012), Hu et al. (2008) and Makkar et al. (2007), the estimation strategies for $\delta_i(t) \in \mathbb{R}$ is designed as

$$\dot{\hat{\delta}}_i(t) = k_1\dot{\eta}_i(t) + k_2\mathrm{sgn}(\eta_i(t)) + k_3\eta_i(t), \tag{3.2}$$

where $k_1, k_2, k_3 \in \mathbb{R}$ are positive constant control gains.

Let $\alpha = [\alpha_x, \alpha_v]$. Based on the nonlinear observer (3.2), the following robust average formation tracking scheme is proposed.

$$u_i(t) = -\hat{\delta}_i(t) + k\eta_i(t) - \alpha d_i(t) + \dot{d}_{vi}(t), \tag{3.3}$$

where $i \in F$, $k \in \mathbb{R}$ is a positive constant control gain.

It's worth mentioning that we use the integral form of $\dot{\hat{\delta}}_i(t)$ instead of using $\dot{\hat{\delta}}_i(t)$ itself in our work, because the term $\dot{\eta}_i(t)$ is unmeasurable, failing to generate $\dot{\hat{\delta}}_i(t)$ in a direct way (Hu, 2012). The integral form of $\dot{\hat{\delta}}_i(t)$ is

$$\hat{\delta}_i(t) = k_1(\eta_i(t) - \eta_i(0)) + \int_0^t (k_2\mathrm{sgn}(\eta_i(\tau)) + k_3\eta_i(\tau))d\tau.$$

Let

$$X(t) = [x_1(t), ..., x_N(t)]^T, V(t) = [v_1(t), ..., v_N(t)]^T,$$

$$U(t) = [u_1(t), ..., u_M(t)]^T, \eta_{Fx}(t) = [\eta_{x1}(t), ..., \eta_{xM}(t)]^T,$$

$$\eta_{Fv}(t) = [\eta_{v1}(t), ..., \eta_{vM}(t)]^T, \eta_F(t) = [\eta_1(t), ..., \eta_M(t)]^T,$$

$$\Delta(t) = [\delta_{11}(t) + \delta_{21}, ..., \delta_{1M}(t) + \delta_{2M}]^T,$$

$$\hat{\Delta}(t) = [\hat{\delta}_1(t), ..., \hat{\delta}_M(t)]^T.$$

Then,

$$\eta_{Fx}(t) = L_1\zeta_x(t) + L_2X_E(t), \tag{3.4}$$

$$\eta_{Fv}(t) = L_1\zeta_v(t) + L_2V_E(t), \tag{3.5}$$

$$\eta_F(t) = \beta(L_1\zeta_x(t) + L_2X_E(t)) + L_1\zeta_v(t) + L_2V_E(t), \tag{3.6}$$

$$\hat{\Delta}(t) = k_1(\eta_F(t) - \eta_F(0)) + \int_0^t (k_2\mathrm{sgn}(\eta_F(t)) + k_3\eta_F(t))d\tau, \tag{3.7}$$

$$U(t) = -\hat{\Delta}(t) + k\eta_F(t) - \alpha D_F(t) + \dot{D}_{Fv}(t). \tag{3.8}$$

From (3.4)-(3.8), we have

$$\dot{\eta}_{Fx}(t) = \eta_{Fv}(t) = L_1\zeta_v(t) + L_2V_E(t), \tag{3.9}$$

$$\dot{\eta}_F(t) = \beta\dot{\eta}_{Fx}(t) + \dot{\eta}_{Fv}(t) = L_1(\alpha\zeta_F(t) + \beta\zeta_v(t) + k\eta_F(t) - \hat{\Delta}(t) + \Delta(t)) + L_2(\beta V_E(t) + \dot{V}_E(t)). \tag{3.10}$$

Lemma 3. The multi-agent system (2.1) achieves robust average formation tracking, if and only if $\eta_F(t) = 0$.

Proof. Let $S_i(t) = [\eta_{xi}(t), \eta_{vi}(t)]^T$, and $S_F(t) = [S_1(t), ..., S_M(t)]^T$. If $\eta_F(t) = 0$ holds, it can be obtained from (3.4)-(3.8) that $\eta_{Fx}(t) = 0$ and $\eta_{Fv}(t) = 0$. Furthermore, we have $S_F(t) = 0$. Based on (3.4)-(3.10), we can get

$$S_F(t) = (L_1 \otimes I_2)\zeta_F(t) + (L_2 \otimes I_2)\xi_E(t). \tag{3.11}$$

So, if $\lim_{t\to\infty} S_F(t) = 0$, then $\lim_{t\to\infty}((L_1 \otimes I_2)\zeta_F(t) + (L_2 \otimes I_2)\xi_E(t)) = 0$. Note that L_1 is nonsingular, we can premultiply the both sides of (3.11) by L_1^{-1}, then one gets

$$\lim_{t\to\infty}(\zeta_F(t) + (L_1^{-1}L_2 \otimes I_2)\xi_E(t)) = 0,$$

which can be written as

$$\lim_{t\to\infty}(\xi_F(t) - D_F(t) - (-L_1^{-1}L_2 \otimes I_2)\xi_E(t)) = 0. \quad (3.12)$$

According to Lemma 2, we can see that (3.12) is equivalent to

$$\lim_{t\to\infty}\left(\xi_i(t) - d_i(t) - \frac{1}{N-M}\sum_{l=M+1}^{N}\xi_l(t)\right) = 0,$$

which is the control objective. Thus, $\eta_F(t) = 0$ is the necessary and sufficient conditions for the multi-agent system (2.1) to achieve robust average formation tracking. This is the end of the proof.

Remark 2. Lemma 3 transforms the formation tracking problem into the convergence problem. In the case where $M = N-1$, which means there is only one leader in the multi-agent system, the results can also be applied to tackle the formation tracking problem.

Define

$$\begin{aligned} s(t) &= L_1^{-1}\dot{\eta}_F(t) - k\eta_F(t) \\ &= Z(t) - \hat{\Delta}(t), \end{aligned} \quad (3.13)$$

where
$Z(t) = \alpha\zeta_F(t) + \beta\zeta_v(t) + \Delta(t) + L_1^{-1}L_2(\beta V_E(t) + \dot{V}_E(t))$.

Remark 3. It can be easily proved that $\lim_{t\to\infty} s(t) = 0$ ensures $\lim_{t\to\infty}\eta_F(t) = 0$, by taking Lyapunov stability analysis while choosing a Lyapunov function $V[\eta_F(t)] = \frac{1}{2}\eta_F^T(t)L_1^{-1}\eta_F(t)$.

From (3.13), it can be obtained that

$$\begin{aligned} \dot{s}(t) &= \dot{Z}(t) - \dot{\hat{\Delta}}(t) \\ &= \dot{Z}(t) + \Xi(t) - k_1L_1q(t) - k_2\mathrm{sgn}(\eta_F(t)) - \eta_F(t), \end{aligned}$$

where $\Xi(t) \in \mathbb{R}^n$, and

$$\Xi(t) = \eta_F(t) - k_3\eta_F(t) + kk_1L_1\eta_F(t).$$

Let $k_3 = 1$ for simplicity, then $\Xi(t) = kk_1L_1\eta_F(t)$.

Remark 4. If matric L_1 is positive definite, then $\|L_1\|$ is upper bounded by $\max_i\{\lambda_i(L_1)\}$, where $\lambda_i(L_1)$ is the eigenvalue of matrix L_1. On the basis of Remark 1, it can be proved that

$$\left\|\dot{Z}(t)\right\| \le \theta_1, \left\|L_1^{-1}\ddot{Z}(t)\right\| \le \theta_2,$$

where $\theta_1, \theta_2 \in \mathbb{R}$ denote known positive constants.

Lemma 4. If the control gains k and k_2 satisfy the following conditions

$$k_2 - \left\|\dot{Z}(t)\right\| > 0, \quad (3.14)$$

$$a\sqrt{(N-M)}k_2 - b\left(\left\|\dot{Z}(t)\right\| + \frac{1}{k}\left\|L_1^{-1}\ddot{Z}(t)\right\|\right) > 0, \quad (3.15)$$

where $a = \min_i\{\lambda_i(L_1)\}, b = \max_i\{\lambda_i(L_1)\}$ represent the minimum and maximum eigenvalue of matrix L_1. Then

$$h(t) = \eta_F(0)^T(k_2\mathrm{sgn}(\eta_F(0)) - \dot{Z}(0)) - \varphi(t) \ge 0, \quad (3.16)$$

where $\varphi(t)$ is determined by

$$\dot{\varphi}(t) = s(t)^TL_1(\dot{Z}(t) - k_2\mathrm{sgn}(\eta_F(t))).$$

Proof. Based on (3.13), $\varphi(t)$ can be

$$\begin{aligned} \varphi(t) &= (\eta_F(t)^T\dot{Z}(t) - \eta_F(0)^T\dot{Z}(0)) \\ &-(\eta_F(t)^Tk_2\mathrm{sgn}(\eta_F(t)) - \eta_F(0)^Tk_2\mathrm{sgn}(\eta_F(0))) \\ &+\int_0^t kL_1\eta_F(\tau)^T(k_2\mathrm{sgn}(\eta_F(\tau)) - \dot{Z}(\tau) - \frac{1}{k}L_1^{-1}\ddot{Z}(\tau))d\tau. \end{aligned}$$

Define

$$\begin{aligned} \Psi(\tau) = &\|L_1\eta_F(\tau)\|\left(\left\|\dot{Z}(\tau)\right\| + \frac{1}{k}\left\|L_1^{-1}\ddot{Z}(\tau)\right\|\right) \\ &-k_2\eta_F(\tau)^TL_1\mathrm{sgn}(\eta_F(\tau)). \end{aligned}$$

It can be derived on account of $\|L_1\eta_F(\tau)\| \le b\|\eta_F(\tau)\|$ and $\eta_F(\tau)^TL_1\mathrm{sgn}(\eta_F(\tau)) \ge a\sqrt{(N-M)}\|\eta_F(\tau)\|$ that

$$\begin{aligned} \Psi(\tau) \le &b\|\eta_F(\tau)\|\left(\left\|\dot{Z}(\tau)\right\| + \frac{1}{k}\left\|L_1^{-1}\ddot{Z}(\tau)\right\|\right) \\ &-a\sqrt{(N-M)}k_2\|\eta_F(\tau)\|. \end{aligned}$$

Thus, if (3.14)-(3.15) hold, then

$$\varphi(t) \le \eta_F(0)^Tk_2\mathrm{sgn}(\eta_F(0)) - \eta_F(0)^T\dot{Z}(0),$$

which proved the validity of (3.16). This is the end of the proof.

Inspired by the identifier-based robust control in previous work Hu (2012), we can have the following theorem.

Theorem 1. The multi-agent system (2.1) with control (3.3) achieves robust average formation tracking, if the parameters satisfy the following conditions

$$k_3 = 1, \quad (3.17)$$

$$k_2 - \theta_1 > 0, \quad (3.18)$$

$$k_2 - \frac{b(k\theta_1 + \theta_2)}{a\sqrt{(N-M)}k} > 0, \quad (3.19)$$

$$kk_1L_1 - 2I < 0, \quad (3.20)$$

$$2\gamma kL_1 - I > 0, \quad (3.21)$$

where $0 < \gamma < \beta$ can be any constant scalar.

Proof. Define $\Lambda(t) = [\eta_{Fx}(t)^T, \eta_F(t)^T, \mathrm{s}(t)^T, \sqrt{\varphi(t)}]^T$. We use

$$\begin{aligned} V(t,\Lambda) = &\frac{1}{2}\eta_{Fx}(t)^T\eta_{Fx}(t) + \frac{1}{2}\eta_F(t)^T\eta_F(t) \\ &+\frac{1}{2}\mathrm{s}(t)^TL_1\mathrm{s}(t) + \varphi(t) \end{aligned}$$

as a Lyapunov function candidate. We take the time derivative of $V(t,\Lambda)$, it has

$$\dot V(t,\Lambda) = -\beta\eta_{Fx}(t)^T\eta_{Fx}(t) - k\eta_F(t)^T L_1\eta_F(t) - k_1 \mathrm{s}(t)^T L_1^2 \mathrm{s}(t) + kk_1\mathrm{s}(t)^T L_1^2 \eta_F(t) + \gamma\eta_{Fx}(t)^T\eta_{Fx}(t) + \frac{1}{4\gamma}\eta_F(t)^T\eta_F(t)$$
$$\le -(\gamma-\beta)\eta_{Fx}(t)^T\eta_{Fx}(t) - \eta_F(t)^T(\frac{1}{2}kL_1 - \frac{1}{4\gamma}I)\eta_F(t) - \frac{1}{2}k(\eta_F(t) - k_1L_1\mathrm{s}(t))^T L_1(\eta_F(t)-k_1L_1\mathrm{s}(t)) - \mathrm{s}(t)^T k_1 L_1^2(I-\frac{1}{2}kk_1L_1)\mathrm{s}(t). \quad (3.22)$$

If (3.17)-(3.21) are satisfied, subsequently we have $\dot V(t,\Lambda) \le 0$.

Based on Remark 1, it can be easily obtained that

$$\mathrm{a}\|\mathrm{s}(t)\|^2 \le \mathrm{s}(t)^T L_1 \mathrm{s}(t) \le b\|\mathrm{s}(t)\|^2$$

by using Rayleigh-Ritz theorem. Therefore, $W_1(\Lambda)$ and $W_2(\Lambda)$ that are positive definite can be found so that $W_1(\Lambda) \le V(t,\Lambda) \le W_2(\Lambda)$. Furthermore, according to (3.22), there exist a positive semidefinite function $W_3(\Lambda)$ defined as

$$W_3(\Lambda) = (\gamma-\beta)\eta_{Fx}(t)^T\eta_{Fx}(t) + \eta_F(t)^T(\frac{kL_1}{2} - \frac{1}{4\gamma}I)\eta_F(t) + \frac{1}{2}k(\eta_F(t)-k_1L_1\mathrm{s}(t))^T L_1(\eta_F(t)-k_1L_1\mathrm{s}(t)) + \mathrm{s}(t)^T k_1 L_1^2(I - \frac{1}{2}kk_1L_1)\mathrm{s}(t),$$

such that $\dot V(t,\Lambda) \le -W_3(\Lambda)$. Based on Lemma 1, we can arrive at a conclusion that when $t\to\infty$, $\eta_F(t)\to 0$. Thus, it can be concluded from Lemma 3 that robust average formation tracking is achieved for the multi-agent system (2.1) in the presence of disturbance and unmodeled dynamics. This is the end of the proof.

4. NUMERICAL SIMULATIONS

In this section, we consider a multi-agent system (2.1) consisting of four followers and two leaders with $n = 2$. The interaction topology of the agents team is shown in Fig. 1. The damping gains α, disturbance and unmodeled dynamics terms δ_i are given by

$$\alpha = [-5, 0], \delta_i = \begin{bmatrix} \sin(it) + 0.1x_{iX}(t) \\ \sin(it) + 0.1x_{iY}(t)\end{bmatrix}$$

Our control goal is to make the states of the four followers to maintain a square formation while track the average state of the leaders. The formation is defined as

$$d_i(t) = \begin{bmatrix} 0.2\cos(0.8t + \frac{(i-1)\pi}{2}) \\ 0.2\sin(0.8t + \frac{(i-1)\pi}{2}) \\ -0.16\sin(0.8t + \frac{(i-1)\pi}{2}) \\ 0.16\cos(0.8t + \frac{(i-1)\pi}{2}) \end{bmatrix}.$$

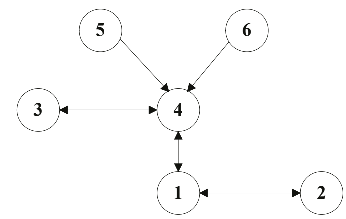

Fig. 1. Undirected interaction topology of the agents

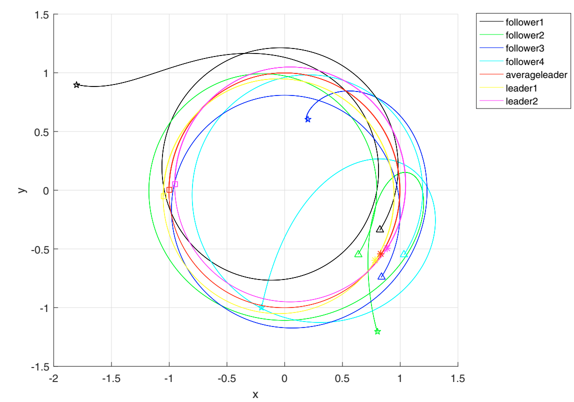

Fig. 2. Trajectories within t=10s

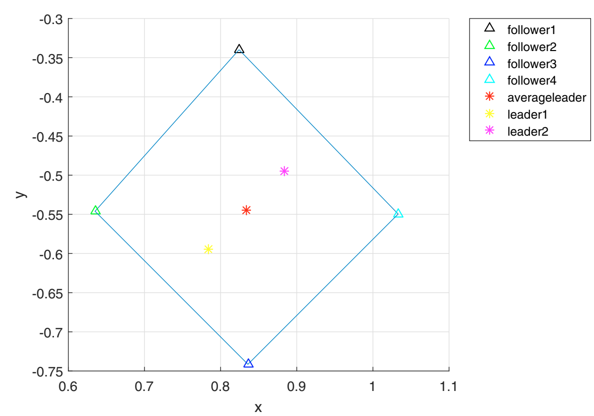

Fig. 3. Snapshots at t=10s

Then, $\lim_{t\to\infty}\sum_{i=1}^{6} d_i(t) = 0$, therefore, the two leaders' average state will be at the centre of the square formation when control objective is achieved.

The control gains are chosen as $k = 4$, $k_1 = 0.1$, $k_2 = 12$, $k_3 = 1$, $\beta = 3$, $\gamma = 1.6$, which satisfy the conditions in Theorem 1.

The initial states of the agents are given by

$$X_{\mathrm{ix}}(0) = [-1.8, 0.8, 0.2, -0.2, -1.1, -0.9]^T,$$
$$X_{\mathrm{iY}}(0) = [0.9, -1.2, 0.6, -1, -0.1, 0.1]^T,$$
$$V_{iX}(0) = [0,0,0,0,0,0,]^T,\ V_{iY}(0) = [0,0,0,0,1,1,]^T.$$

Fig. 2 shows the trajectories of the six agents within $t = 10s$, and Fig. 3 shows a snapshot at $t = 10s$. The asterisk and triangle represent the initial and terminal position of the followers distinguished by color. The initial and terminal position of the leaders are denoted by the square and hexagram. In addition, the red trajectory and the red hexagram represent the average position of the two leaders. From Fig. 2 and Fig. 3, we can see that the states of the four followers maintain a square formation, and two leaders' average state is at the centre of the square formation. Thus, the control objective has been achieved.

5. CONCLUSION

This paper has studied the formation tracking problem of the multi-agent system in the presence of disturbances and unmodeled uncertainties. Only a subset of followers are informed of the time-varying states of the multiple leaders. An identifier-based robust control algorithm using the neighboring relative information has been proposed to ensure the states of the followers to maintain a predefined time-varying formation and track the average state of the leaders at the same time. Some sufficient conditions for the second-order multi-agent system with multiple leaders in the presence of disturbances and unmodeled uncertainties have been proposed based on the graph theory and the Lyapunov method. When the control gains are selected appropriately, the control objective can be achieved. Finally, the numerical simulations are provided to testify the validity of the algorithm.

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