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Optimal generation in structure-preserving power networks with second-order turbine-governor dynamics*

S. Trip¹ and C. De Persis¹

Abstract— Recently we have been exploring the role of passivity and the internal model principle in power network control in the presence of uncertainties due to unmeasured demand and supply. In this work we continue this line of research and extend our results to include more complex dynamics at the generation side. Namely, we study frequency stabilization by primary control and frequency regulation by optimal generation control, where we additionally incorporate second-order turbine-governor dynamics. The power network is represented by the structure-preserving Bergen-Hill model [1]. Distributed controllers that require local frequency measurements are proposed and are shown to minimize the generation costs. Asymptotic convergence is proven when the generators satisfy a local matrix condition. The effectiveness of proposed controllers is demonstrated in a case study.

I. INTRODUCTION

To guarantee reliable operation of the power network it is important to keep the frequency close to its nominal value of e.g. 50 Hz. This is traditionally achieved by primary proportional control (droop-control) and a secondary PI-control at the different generators in the network. An increased penetration of uncertain and volatile renewable energy sources created renewed interest in this control issue, where computer-based control and communication networks offer new possibilities to improve efficiency and reliability [2]. Especially, economically efficient frequency regulation has gained much attention over the recent years, where different approaches to minimize generation costs are proposed.

Literature review. We briefly discuss some existing results that are relevant to the presented work and focus on work that studies frequency regulation rather than frequency stabilization by primary control [3]. For a linearized model of the power network distributed and centralized controllers that require the knowledge of frequency deviations at their own bus and its neighboring buses are

proposed in [4]. In [5] optimal frequency regulation is investigated within a game-theoretical framework. Optimal frequency regulation based on primal-dual gradient dynamics is proposed in [6] and [7], where a suitable optimal power flow problem is solved using primal-dual decomposition. This approach has been also applied in [8], where second-order turbine-governor dynamics are incorporated and in [9] where it is formulated within a port-Hamiltonian framework. Despite their capability of handling various operational constraints, primal-dual based algorithms (as well as the approach in [5]) generally suffer from required measurements of loads or power flows. This issue is avoided in [10], [11], [12] where optimal frequency regulation is treated within a framework of incremental passivity and output regulation on networks. Besides frequency regulation in high voltage networks, there is related work on microgrids, where research shifted from stability analysis of the frequency [13], towards optimal regulation, solving similar optimization problems as the aforementioned papers, [14], [15], [16].

Main contributions. Detailed models of power networks often include a second-order model of the turbine-governor dynamics [17], [18]. In the aforementioned works, these dynamics are however neglected in the stability analysis (with a notable exception of [3] and [8]). The presented results in this paper are noteworthy as it is, to best of the authors knowledge, the first time where second-order turbine-governor dynamics are explicitly taken into account in the stability analysis of the nonlinear structure-preserving ‘Bergen-Hill’ model, where optimal frequency regulation is (practically) obtained without direct measurements of the loads or the power flows. This paper can therefore be regarded as a continuation of our previous efforts ([11], [12]) to design distributed controllers obtaining economically efficient frequency regulation, exploring the role of incremental passivity and the internal model principle [19], [20]. The way we obtain our result appears to be relevant to related research [3], [8], and can help to relax needed assumptions on the system parameters. To prove asymptotic stability of the power network we study the eigenvalues of a Hamiltonian matrix depending on (local) system parameters. Doing so, we connect (optimal) frequency regulation in power networks to the study of Hamiltonian matrices. Although we do not aim at an extensive analysis of the obtained Hamiltonian matrices in the present

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work, we do believe that results on the (perturbation of the) eigenvalues of Hamiltonian matrices ([21], [22]) are helpful to provide guidelines on how to stabilize a potentially unstable network.

The remainder of this paper is organized as follows. In Section II, we introduce the dynamic model of the power network, including turbine-governor dynamics and load buses. In Section III, we discuss a passivity property of the power network. In Section IV, we discuss primary frequency control and show that it leads to a nonzero frequency deviation. In Section V, we propose a distributed dynamic controller which ensures frequency regulation and achieves economic optimality. In Section VI, we test our controllers for an academic case study using simulations. In Section VII, conclusions are given.

II. SYSTEM MODEL

We consider a nonlinear structure-preserving model of the power network, which is commonly referred to as the Bergen-Hill model [1]. The network consists of n_G generator and n_L load buses. Each bus is assumed, without loss of generality, to be either a generator or a load bus, such that the total number of buses in the network is $n = n_G + n_L$. The network is represented by a connected and undirected graph $G = (\mathcal{V}_G \cup \mathcal{V}_L, \mathcal{E})$, where $\mathcal{V}_G = \{1, \dots, n_G\}$ is the set of generator buses, $\mathcal{V}_L = \{1, \dots, n_L\}$ is the set of load buses and $\mathcal{E} \subset (\mathcal{V}_G \cup \mathcal{V}_L) \times (\mathcal{V}_G \cup \mathcal{V}_L) = \{1, \dots, e\}$ is the set of transmission lines connecting the buses. The network structure can be represented by its corresponding incidence matrix $D \in \mathbb{R}^{n \times e}$. The ends of edge k are arbitrary labeled with a ‘+’ and a ‘-’. Then

$$d_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases}$$

Generator bus $i \in \mathcal{V}_G$ is modelled as

$$\begin{aligned} \dot{\delta}_i &= \omega_{Gi} \\ M_i \dot{\omega}_{Gi} &= -A_{Gi} \omega_{Gi} \\ &\quad - \sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin(\delta_i - \delta_j) + P_{mi}, \end{aligned}$$

where \mathcal{N}_i is the set of buses connected to bus i . The generated power P_{mi} is modeled as the output of a second-order model, describing the turbine and governor dynamics [8], [17], [18], and is given by

$$\begin{aligned} T_{gi} \dot{P}_{gi} &= -P_{gi} - K_i^{-1} \omega_{Gi} + u_{Gi} \\ T_{mi} \dot{P}_{mi} &= -P_{mi} + P_{gi}, \end{aligned} \quad (1)$$

where u_{Gi} is an additional control input. The loads P_{li} are assumed to be frequency dependent and we model a load bus $i \in \mathcal{V}_L$ as

$$\begin{aligned} \dot{\delta}_i &= \omega_{Li} \\ \mathbf{0} &= -A_{Li} \omega_{Li} - \sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin(\delta_i - \delta_j) - P_{li}. \end{aligned}$$

An overview of the used symbols is provided in Table 1. For all nodes the power system including turbine-

State variables	
δ_i	Voltage angle
ω_{Gi}	Frequency deviation at the generator bus
ω_{Li}	Frequency deviation at the load bus
P_{gi}	Steam power
P_{mi}	Mechanical power
Parameters	
M_i	Moment of inertia
A_{Gi}	Damping constant of the generator
A_{Li}	Damping constant of the load
B_{ij}	Susceptance of the transmission line
V_i	Voltage
T_{gi}	Governor time constant
T_{mi}	Steam chest time constant
K_i	Permanent droop constant
Controllable input	
u_{Gi}	Power generation control
Uncontrollable input	
P_{li}	Unknown power demand

Table 1: Description of main variables and parameters appearing in the system model.

governor dynamics is written as

$$\begin{aligned} \dot{\eta} &= D^T \omega \\ M \dot{\omega}_G &= -A_G \omega_G - D_G \Gamma \sin(\eta) + P_m \\ \mathbf{0} &= -A_L \omega_L - D_L \Gamma \sin(\eta) - P_l \\ T_g \dot{P}_g &= -P_g - K^{-1} \omega_G + u_g \\ T_m \dot{P}_m &= -P_m + P_g, \end{aligned} \quad (2)$$

where $\omega = (\omega_G^T, \omega_L^T)^T$, $\eta = D^T \delta$ and $\Gamma = \text{diag}\{\gamma_1, \dots, \gamma_m\}$, with $\gamma_k = V_i V_j B_{ij} = V_j V_i B_{ji}$ and the index k denoting the line $\{i, j\}$. The matrices $D_G \in \mathbb{R}^{n_G \times e}$ and $D_L \in \mathbb{R}^{n_L \times e}$ are obtained by collecting from D the rows indexed by \mathcal{V}_G and \mathcal{V}_L respectively.

III. INCREMENTAL CYCLO-PASSIVITY OF THE POWER NETWORK

We recall in this section a notable result from [12], which is fundamental to the subsequent analysis in this paper. Namely, the power network, without the turbine-governor dynamics, is an incrementally cyclo-passive system with respect to its steady state solution, if we consider ω_G as the output and P_m as the input.

Remark 1 *In our previous work [11], [12] we refer to the system as being incrementally passive, instead of incrementally cyclo-passive. The latter property is more appropriate in this section, since we do not yet assume the existence of a (local) minimum of the considered incremental storage function. For the stability analysis a local minimum is however required and is ensured by Assumption 1 in the next section. A more in depth treatment on incremental passivity and cyclo-passivity is e.g. provided in respectively [23] and [24].*

To establish this incremental cyclo-passivity property, we eliminate ω_L in (2) by exploiting the identity $\omega_L = A_L^{-1}(-D_L \Gamma \sin(\eta) - P_l)$ and realizing that $D^T \omega =$

$D_{\mathcal{G}}^T \omega_{\mathcal{G}} + D_{\mathcal{L}}^T \omega_{\mathcal{L}}$ [25]. As a result we rewrite (2) as

$$\begin{aligned} \dot{\eta} &= D_{\mathcal{G}}^T \omega_{\mathcal{G}} + D_{\mathcal{L}}^T A_{\mathcal{L}}^{-1} (-D_{\mathcal{L}} \Gamma \sin(\eta) - P_l) \\ M \dot{\omega}_{\mathcal{G}} &= -A_{\mathcal{G}} \omega_{\mathcal{G}} - D_{\mathcal{G}} \Gamma \sin(\eta) + P_m. \\ T_g \dot{P}_g &= -P_g - K^{-1} \omega_{\mathcal{G}} + u_{\mathcal{G}} \\ T_m \dot{P}_m &= -P_m + P_g, \end{aligned} \quad (3)$$

In this section we focus on the first and second equation of (3), which corresponds to the classic Bergen-Hill model without turbine-governor dynamics

$$\begin{aligned} \dot{\eta} &= D_{\mathcal{G}}^T \omega_{\mathcal{G}} + D_{\mathcal{L}}^T A_{\mathcal{L}}^{-1} (-D_{\mathcal{L}} \Gamma \sin(\eta) - P_l) \\ M \dot{\omega}_{\mathcal{G}} &= -A_{\mathcal{G}} \omega_{\mathcal{G}} - D_{\mathcal{G}} \Gamma \sin(\eta) + P_m. \end{aligned} \quad (4)$$

The incremental (cyclo-)passivity property of (4) has been stated before in [12] and is repeated below for the sake of completeness.

Theorem 1 *System (4) with input P_m and output $\omega_{\mathcal{G}}$ is an output strictly incrementally cyclo-passive system, with respect to the constant equilibrium $(\bar{\eta}, \bar{\omega}_{\mathcal{G}})$ satisfying*

$$\begin{aligned} \mathbf{0} &= D_{\mathcal{G}}^T \bar{\omega}_{\mathcal{G}} + D_{\mathcal{L}}^T A_{\mathcal{L}}^{-1} (-D_{\mathcal{L}} \Gamma \sin(\bar{\eta}) - P_l) \\ \mathbf{0} &= -A_{\mathcal{G}} \bar{\omega}_{\mathcal{G}} - D_{\mathcal{G}} \Gamma \sin(\bar{\eta}) + \bar{P}_m. \end{aligned} \quad (5)$$

Namely, there exists a storage function $U(\omega_{\mathcal{G}}, \bar{\omega}_{\mathcal{G}}, \eta, \bar{\eta})$ which satisfies the following incremental dissipation inequality

$$\begin{aligned} \dot{U}(\omega_{\mathcal{G}}, \bar{\omega}_{\mathcal{G}}, \eta, \bar{\eta}) &\leq -\rho(\omega_{\mathcal{G}} - \bar{\omega}_{\mathcal{G}}) \\ &\quad + (\omega_{\mathcal{G}} - \bar{\omega}_{\mathcal{G}})^T (P_m - \bar{P}_m), \end{aligned}$$

where \dot{U} represents the directional derivative of U along the solutions to (4) and ρ is a positive definite function.

The result of Theorem 1 plays a key role in the remainder of this work, since the considered incremental storage function will be employed in the following sections to establish convergence properties of the power network with additional second-order turbine-governor dynamics.

Remark 2 *There is an increased interest in utilizing load control for frequency regulation [7], [8]. Due to space limitations the incorporation of load control in the present setting is left to a future work. We remark however that an additional control input $u_{\mathcal{L}}$ will lead to the following algebraic relation at the load buses:*

$$\mathbf{0} = -A_{\mathcal{L}} \omega_{\mathcal{L}} - D_{\mathcal{L}} \Gamma \sin(\eta) - P_l - u_{\mathcal{L}}.$$

It can be shown by adapting Theorem 1, that the incremental storage function U then satisfies $\dot{U} \leq -(\omega_{\mathcal{L}} - \bar{\omega}_{\mathcal{L}})^T (u_{\mathcal{L}} - \bar{u}_{\mathcal{L}})$. This incremental passivity property can be exploited in a similar fashion as in [11], [12] to design optimal distributed load controllers.

IV. PRIMARY FREQUENCY CONTROL

In this section we study the stability of the structure preserving power network presented in Section II under a constant control input $u_{\mathcal{G}} = \bar{u}_{\mathcal{G}}$. The steady state

solution $(\bar{\eta}, \bar{\omega} = (\bar{\omega}_{\mathcal{G}}^T, \bar{\omega}_{\mathcal{L}}^T)^T, \bar{P}_g, \bar{P}_m)$ to (2) necessarily satisfies

$$\begin{aligned} \mathbf{0} &= D^T \bar{\omega} \\ \mathbf{0} &= -A_{\mathcal{G}} \bar{\omega}_{\mathcal{G}} - D_{\mathcal{G}} \Gamma \sin(\bar{\eta}) + \bar{P}_m \\ \mathbf{0} &= -A_{\mathcal{L}} \bar{\omega}_{\mathcal{L}} - D_{\mathcal{L}} \Gamma \sin(\bar{\eta}) - P_l \\ \mathbf{0} &= -\bar{P}_g - K^{-1} \bar{\omega}_{\mathcal{G}} + \bar{u}_{\mathcal{G}} \\ \mathbf{0} &= -\bar{P}_m + \bar{P}_g. \end{aligned} \quad (6)$$

Before proceeding we make two assumptions on the solution to (6).

Assumption 1 *The steady state differences in voltage angles $\bar{\eta}$ satisfy $\bar{\eta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^e$.*

The purpose of Assumption 1 is to guarantee that the storage function introduced in Theorem 1 has local minimum at steady state (see [26] for related results on convexity of the storage function). We note that Assumption 1 is generally satisfied in high voltage networks that are studied here. Additionally we make the natural assumption that a solution to (6) exists, which corresponds to assuming that the network is able to transfer the required power at its steady state.

Assumption 2 *For a given $\bar{u}_{\mathcal{G}}$ and P_l , there exist $\bar{\eta} \in \mathcal{R}(D^T)$, $\bar{\omega} \in \mathcal{N}(D^T)$, $\bar{P}_g \in \mathbb{R}^{n_{\mathcal{G}}}$ and $\bar{P}_m \in \mathbb{R}^{n_{\mathcal{G}}}$ such that (6) is satisfied.*

From algebraic manipulations of (6) we can derive the following lemma that makes the frequency deviation at steady state $\bar{\omega}$ explicit.

Lemma 1 *Let Assumption 1 hold, then necessarily $\bar{\omega} = \mathbf{1}_n \omega_*$, with*

$$\omega_* = \frac{\mathbf{1}_{n_{\mathcal{G}}}^T \bar{u}_{\mathcal{G}} - \mathbf{1}_{n_{\mathcal{L}}}^T P_l}{\mathbf{1}_{n_{\mathcal{G}}}^T A_{\mathcal{G}} \mathbf{1}_{n_{\mathcal{G}}} + \mathbf{1}_{n_{\mathcal{L}}}^T A_{\mathcal{L}} \mathbf{1}_{n_{\mathcal{L}}} + \mathbf{1}_{n_{\mathcal{G}}}^T K^{-1} \mathbf{1}_{n_{\mathcal{G}}}},$$

where $\mathbf{1}_n \in \mathbb{R}^n$ is the vector consisting of all ones.

It is clear from Lemma 1 that it is desirable to have a small value of K_i in order to have a small frequency deviation at steady state. Furthermore, we can prove asymptotic stability if the following assumption is satisfied, which relates the parameters of generator i to a Hamiltonian matrix.

Assumption 3 *Let for all $i \in \mathcal{V}_{\mathcal{G}}$ the Hamiltonian matrix*

$$H_i = \begin{pmatrix} A_{Ri} & B_{Ri} B_{Ri}^T \\ -C_{Ri}^T C_{Ri} & -A_{Ri}^T \end{pmatrix}$$

be such that it has no eigenvalues on the imaginary axis, where

$$\begin{aligned} A_{Ri} &= \begin{pmatrix} -\frac{1}{2} T_{gi}^{-1} & -\frac{1}{4} T_{gi}^{-1} K_i^{-1} A_{\mathcal{G}i}^{-1} \\ \frac{1}{2} T_{mi}^{-1} & -\frac{1}{2} T_{mi}^{-1} \end{pmatrix} \\ B_{Ri} B_{Ri}^T &= \begin{pmatrix} \frac{1}{4} A_{\mathcal{G}i}^{-1} T_{gi}^{-2} T_{mi}^{-2} & 0 \\ 0 & 0 \end{pmatrix} \\ C_{Ri}^T C_{Ri} &= \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{4} A_{\mathcal{G}i}^{-1} \end{pmatrix} \end{aligned}$$

Remark 3 Determining the eigenvalues of $H_i \in \mathbb{R}^{4 \times 4}$ in Assumption 3 is straightforward and can be done locally for every generator. It is possible to give an analytic expression of the eigenvalues of matrix H_i , since the associated characteristic polynomial is of fourth order. Obtaining explicit bounds on the generator parameters such that Assumption 3 is satisfied is left for future research. Furthermore, an interesting question is how system parameters should be altered if the Hamiltonian matrix does have eigenvalues on the imaginary axis [21], [22].

We are now ready to present the main result of this section, namely that the solution to (6) is asymptotically stable under the assumptions discussed.

Theorem 2 Consider system (2) with constant power demand P^l and constant control input $u_G = \bar{u}_G$. Let Assumptions 1, 2 and 3 hold. Then, the solutions of system (2) that start in a neighborhood of $(\bar{\eta}, \bar{\omega} = \mathbf{1}_{n_G} \omega^*, \bar{P}_g, \bar{P}_m)$ converge asymptotically to the largest invariant set where $\omega = \mathbf{1}_{n_G} \omega^*$ characterized in Lemma 1, $P_g = \bar{P}_g$ and $P_m = \bar{P}_m$.

The proof is provided in [27].

Remark 4 A related study on primary frequency control is performed in [3] and requires $K_i^{-1} < A_{G_i}$ for the system at hand in order to prove asymptotic stability of the steady state.

A consequence of the analysis in this section is that the power network will generally converge to a steady state frequency deviation ω^* unequal to zero. In the next section we address this issue by designing additional secondary control, which regulates the frequency and minimizes generation costs.

V. ECONOMICALLY EFFICIENT FREQUENCY REGULATION

Before we turn our attention to the design of distributed controllers that regulate the frequency, we discuss the possibility of minimizing generation costs. To this end, we assign to every generator a convex linear-quadratic cost function that relates the generated power P_{mi} to the generation costs $C_i(P_{mi})$, typically expressed in €/MWh, i.e. $C_i(P_{mi}) = \frac{1}{2} q_i P_{mi}^2 + r_i P_{mi} + s_i$. From Lemma 1 we notice that it is required that $\mathbf{1}_{n_G}^T \bar{u}_G - \mathbf{1}_{n_G}^T P^l = 0$ to have a steady state where $\bar{\omega} = \mathbf{1}_{n_G} \omega^* = \mathbf{0}$, i.e. a zero frequency deviation. From (2) it follows furthermore that at this steady state where $\bar{\omega} = \mathbf{0}$, also $\bar{u}_G = \bar{P}_g = \bar{P}_m$. In order to achieve economic optimality we design a controller adjusting u_G such that the steady state $u_G = \bar{u}_G = \bar{P}_g = \bar{P}_m$ is a solution to the following optimization problem:

$$\begin{aligned} \min_{P_m} C(P_m) &= \min_{P_m} \sum_{i \in \mathcal{V}_G} C_i(P_{mi}) \\ \text{s.t. } 0 &= \mathbf{1}_{n_G}^T \bar{P}_m - \mathbf{1}_{n_G}^T P^l. \end{aligned} \quad (7)$$

The total costs can compactly be expressed as $C(P_m) = \frac{1}{2} P_m^T Q P_m + R^T P_m + \mathbf{1}_{n_G}^T S$. Notice that the equality constraint in (7) implies a steady state frequency deviation of zero when $\bar{u}_G = \bar{P}_m$. The solution to (7), indicated by the superscript *opt*, therefore satisfies

$$\begin{aligned} \mathbf{0} &= D^T \mathbf{0} \\ \mathbf{0} &= -A_G \mathbf{0} - D_G \Gamma \sin(\bar{\eta}) + \bar{P}_m^{opt} \\ \mathbf{0} &= -A_L \mathbf{0} - D_L \Gamma \sin(\bar{\eta}) - P^l \\ \mathbf{0} &= -\bar{P}_g^{opt} - K^{-1} \mathbf{0} + \bar{u}_G^{opt} \\ \mathbf{0} &= -\bar{P}_m^{opt} + \bar{P}_g^{opt}. \end{aligned} \quad (8)$$

It is possible to explicitly characterize the solution to (7) and we do so in the following lemma.

Lemma 2 Let $C(P_m) = \frac{1}{2} P_m^T Q P_m + R^T P_m + \mathbf{1}_{n_G}^T S$, with $Q > 0$ and diagonal. The solution \bar{P}_m^{opt} to (7) must satisfy

$$\bar{P}_m^{opt} = Q^{-1}(\bar{\theta} - R), \quad (9)$$

where $\bar{\theta} = \frac{\mathbf{1}_{n_G}(\mathbf{1}_{n_G}^T P^l + \mathbf{1}_{n_G}^T Q^{-1} R)}{\mathbf{1}_{n_G}^T Q^{-1} \mathbf{1}_{n_G}} \in \mathcal{R}(\mathbf{1}_{n_G})$.

The proof is omitted here, but is provided in [12, Lemma 2]. From (9) it is immediate to see that $Q \bar{P}_m^{opt} + R = \bar{\theta} \in \mathcal{R}(\mathbf{1}_{n_G})$, which implies that at the solution to (7) all marginal costs are identical. In order to achieve optimality, the controllers exchange information over a communication network, leading to the following assumption.

Assumption 4 The undirected graph reflecting the topology of information exchange among the controllers is connected.

Before stating the main result of this section, that is, the design of distributed controllers that (practically) regulate the frequency and minimize the generation costs, we state an assumption that play a similar role as Assumption 3 in the previous section.

Assumption 5 Let for all $i \in \mathcal{V}_G$, $v_i \beta_i < 2 + 2T_{gi} T_{mi}^{-1}$, $v_i \in (0, 1]$, $J_i > 0$ and the matrix

$$H_i = \begin{pmatrix} A_{Ri} & B_{Ri} B_{Ri}^T \\ -C_{Ri}^T C_{Ri} & -A_{Ri}^T \end{pmatrix}$$

be such that it has no eigenvalues on the imaginary axis, where

$$A_{Ri} = \begin{pmatrix} -\frac{1}{4} T_{gi}^{-1} (2 - \beta_i v_i) & \frac{1}{4} T_{gi}^{-1} (v_i - \beta_i v_i - K_i^{-1} A_{Gi}^{-1}) \\ \frac{1}{2} T_{mi}^{-1} & -\frac{1}{2} T_{mi}^{-1} \end{pmatrix}$$

$$B_{Ri} B_{Ri}^T = \frac{1}{4} \begin{pmatrix} A_{Gi}^{-1} T_{gi}^{-2} K_i^{-2} + J_i T_{gi}^{-2} v_i^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$C_{Ri}^T C_{Ri} = \frac{1}{4} \begin{pmatrix} J_i \beta_i^2 & J_i \beta_i (1 - \beta_i) \\ J_i \beta_i (1 - \beta_i) & (A_{Gi}^{-1} + J_i (1 - \beta_i)^2) \end{pmatrix}$$

We now provide the distributed controllers that achieve (practical) frequency regulation and economic efficiency.

Theorem 3 Consider system (2) with demand P^l and let Assumptions 1, 2, 4 and 5 hold. Consider the controllers at the nodes $i \in \mathcal{V}_{n_G}$

$$\begin{aligned} T_{\phi_i} \dot{\phi}_i &= -J_i \phi_i + J_i \beta_i P_{g_i} + J_i (1 - \beta_i) P_{m_i} \\ &\quad - q_i \sum_{j \in \mathcal{N}_{G_i}^{comm}} (q_j \phi_j + r_j) \\ u_{G_i} &= v_i \phi_i, \end{aligned} \quad (10)$$

where $\mathcal{N}_{G_i}^{comm}$ denotes the set of neighbours that exchange information with node i . Then the controllers (10) guarantee the solutions of the closed-loop system that start in a neighborhood of $(\bar{\eta}, \bar{\omega}, \bar{P}_g, \bar{P}_m, \bar{\phi})$ to converge asymptotically to the largest invariant set where $\omega = \mathbf{1}_{n_G} \omega^*$ characterized in Lemma 1, $P_g = \bar{P}_g$, $P_m = \bar{P}_m$ and $\phi = \bar{\phi}$

The proof is provided in [27].

Remark 5 We refer to ‘practical’ frequency regulation and economic efficiency, because of the appearance of the tuning variable $v_i \in (0, 1]$, in the controller stated in Theorem 3. As a rule of thumb, the closer the value of v_i is to 1, the better the obtained frequency regulation and economic efficiency is. Choosing $v_i = 1$ for all $i \in \mathcal{V}_G$ allows for a steady state where we have ‘exact’ frequency regulation and economic efficiency, i.e. $\bar{\omega} = \mathbf{0}$ and $\bar{\phi} = \bar{P}_g = \bar{P}_m = \bar{P}_m^{opt}$, where \bar{P}_m^{opt} is characterized in Lemma 4. It appears however that Assumption 5 becomes more difficult to satisfy when v_i approaches 1. The case study in the next section shows nevertheless asymptotic stability with the choice $v_i = 1$. Relaxing Assumption 5 is left to a future research.

Remark 6 The value of β_i can be chosen arbitrarily as long as Assumption 5 is satisfied. The choice $\beta_i \in \{0, 1\}$ is however most relevant as it corresponds to measure either P_{g_i} or P_{m_i} instead of both.

Remark 7 The amount of literature on economically efficient frequency regulation is growing rapidly. A popular approach to solve (7) is based on primal-dual gradient dynamics (see e.g. [6], [7], [8], [9]). These approaches generally require knowledge of the loads or the power flows. A remarkable property of our work is that the distributed controllers solve (7) without such measurements.

VI. CASE STUDY

We test the proposed controllers on a 6-bus system proposed in [28] (see Figure 1). Generator, load and transmission line parameters are based on the values provided in [18] and [28], and are provided in Table 2 and Table 3. Every generator is equipped with the controller presented in Theorem 3. The underlying communication network is also indicated in Figure 1 by the dashed lines. The system is initially at steady state with load $P_l = (1.01, 1.20, 1.18)^T$ pu (assuming a base power of 100 MVA). After 10 seconds the load is increased to $P_l = (1.15, 1.30, 1.21)^T$ pu. From Figure 2 we can see how the frequency deviation is regulated back to zero in such a way that total generation costs are minimized.

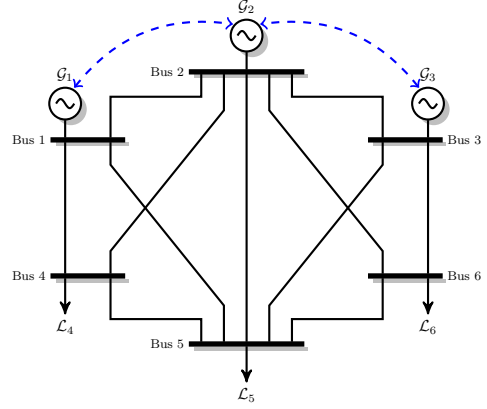


Fig. 1: Diagram for the 6-bus power system model, consisting of 3 generator and 3 load buses. The communication links are represented by the dashed lines.

		Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
M_i	(pu)	4.53	4.27	5.10	–	–	–
A_i	(pu)	1.81	1.48	1.92	0.52	0.58	0.63
V_i	(pu)	1.05	0.98	1.04	1.01	1.03	1.00
T_{g_i}	(pu)	0.08	0.10	0.09	–	–	–
T_{m_i}	(pu)	0.51	0.41	0.35	–	–	–
K_i	(pu)	0.34	0.29	0.41	–	–	–
T_{ϕ_i}	(pu)	0.10	0.10	0.10	–	–	–
J_i	(pu)	1.00	1.00	1.00	–	–	–
β_i	(–)	0.00	0.00	0.00	–	–	–
v_i	(–)	1.00	1.00	1.00	–	–	–
q_i	(10^2 \$/h)	2.40	3.81	3.44	–	–	–
r_i	(10^2 \$/h)	10.5	5.70	8.00	–	–	–
s_i	(10^2 \$/h)	9.10	14.4	13.2	–	–	–

Table 2: Numerical values of the generator and load parameters.

VII. CONCLUSIONS

In this paper we have presented an approach to incorporate second-order turbine-governor dynamics in the design of distributed controllers aiming at economically efficient frequency regulation. Although the considered second-order dynamics are widely used in power network studies, it has been left out in the vast majority of stability studies concerning optimal frequency regulation (see [8] for a notable exception, where however knowledge of loads and power flows is required). This work provides therefore an extension to our previous results and is instrumental to obtain a firm analytical basis of frequency regulation in an interconnected power network (including

B_{ij} (pu)	1	2	3	4	5	6	j
1	–	-4.0	–	-4.7	-3.1	–	
2	-4.0	–	-3.8	-8.0	-3.0	-4.5	
3	–	-3.8	–	–	-3.2	-9.6	
4	-4.7	-8.0	–	–	-2.0	–	
5	-3.1	-3.0	-3.2	-2.0	–	-3.0	
6	–	-4.5	-9.6	–	-3.0	–	
i							

Table 3: Susceptance B_{ij} of the transmission line connecting bus i and bus j . Values are per unit on a base of 100 MVA.

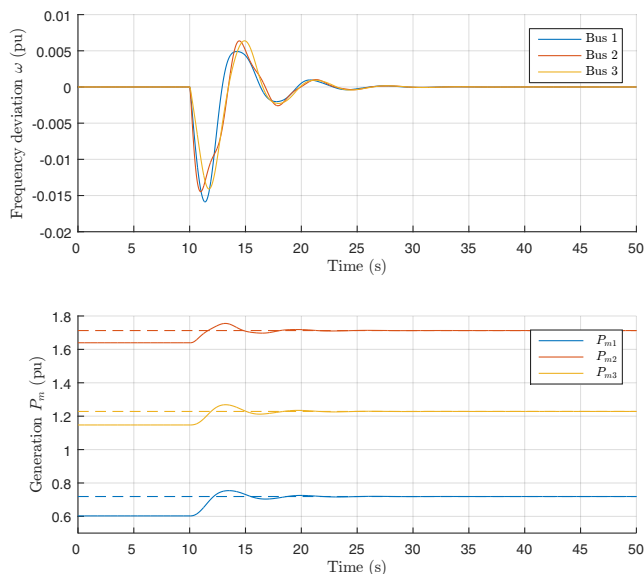


Fig. 2: Frequency response and generated power at the generator buses using the controller of Theorem 3. The constant load is increased at timestep 10, whereafter the frequency deviation is regulated back to zero and generation costs are minimized. The cost minimizing generation \bar{P}_m^{opt} for $t > 10$, characterized in Lemma 2, is given by the dashed lines.

voltage dynamics [11]) and electricity market [17]. In a future work we will focus on incorporating optimal load control, which we now only briefly addressed in Remark 2, and extending the analysis to include convex cost functions.

REFERENCES

- [1] A. Bergen and D. Hill, "A structure preserving model for power system stability analysis," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, pp. 25–35, 1981.
- [2] I. Ibraheem, P. Kumar, and D. Kothari, "Recent philosophies of automatic generation control strategies in power systems," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 346–357, 2005.
- [3] C. Zhao and S. Low, "Optimal decentralized primary frequency control in power networks," in *Proc. of the IEEE 53rd Annual Conference on Decision and Control (CDC)*, 2014.
- [4] M. Andreasson, D. V. Dimarogonas, K. H. Johansson, and H. Sandberg, "Distributed vs. centralized power systems frequency control under unknown load changes," in *Proc. of the 12th European Control Conference (ECC)*, Zurich, Switzerland, 2013, pp. 3524–3529.
- [5] Z. Wang, F. Liu, L. Chen, and S. Mei, "Distributed economic automatic generation control: A game theoretic perspective," in *Proc. of the IEEE Power Energy Society General Meeting*, 2015, pp. 1–5.
- [6] N. Li, C. Zhao, and L. Chen, "Connecting automatic generation control and economic dispatch from an optimization view," *IEEE Transactions on Control of Network Systems*, 2015, to appear.
- [7] X. Zhang and A. Papachristodoulou, "A real-time control framework for smart power networks: Design methodology and stability," *Automatica*, vol. 58, pp. 43–50, 2015.
- [8] C. Zhao, E. Mallada, and S. Low, "Distributed generator and load-side secondary frequency control in power networks," in *Proc. of the Conference on Information Sciences and Systems (CISS)*, Baltimore, MD, USA, 2015, pp. 1–6.
- [9] T. Stegink, C. De Persis, and A. van der Schaft, "A port-Hamiltonian approach to optimal frequency regulation in power grids," *arXiv preprint arXiv:1509.07318 [math.OC]*, 2015.
- [10] M. Bürger, C. De Persis, and S. Trip, "An internal model approach to (optimal) frequency regulation in power grids," in *Proc. of the 21th International Symposium on Mathematical Theory of Networks and Systems (MTNS)*, Groningen, the Netherlands, 2014, pp. 577–583.
- [11] S. Trip, M. Bürger, and C. De Persis, "An internal model approach to (optimal) frequency regulation in power grids with time-varying voltages," *Automatica*, vol. 64, pp. 240–253, 2016.
- [12] S. Trip and C. De Persis, "Optimal frequency regulation in nonlinear structure preserving power networks including turbine dynamics: an incremental passivity approach," *arXiv preprint arXiv:1509.07617 [math.OC]*, 2015.
- [13] J. Schiffer, R. Ortega, A. Astolfi, J. Raisch, and T. Sezi, "Conditions for stability of droop-controlled inverter-based microgrids," *Automatica*, vol. 50, no. 10, pp. 2457–2469, 2014.
- [14] F. Dörfler, J. W. Simpson-Porco, and F. Bullo, "Breaking the hierarchy: Distributed control & economic optimality in microgrids," *arXiv preprint arXiv:1401.1767 [math.OC]*, 2014.
- [15] S. Trip, M. Bürger, and C. De Persis, "An internal model approach to frequency regulation in inverter-based microgrids with time-varying voltages," in *Proc. of the IEEE 53rd Conference on Decision and Control (CDC)*, Los Angeles, CA, USA, 2014, pp. 223–228.
- [16] Q. Shafiee, J. Guerrero, and J. Vasquez, "Distributed secondary control for islanded microgrids; a novel approach," *IEEE Transactions on Power Electronics*, vol. 29, no. 2, pp. 1018–1031, 2014.
- [17] F. Alvarado, J. Meng, C. DeMarco, and W. Mota, "Stability analysis of interconnected power systems coupled with market dynamics," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 695–701, 2001.
- [18] M. Ourari, L.-A. Dessaint, and V.-Q. Do, "Dynamic equivalent modeling of large power systems using structure preservation technique," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1284–1295, 2006.
- [19] M. Bürger and C. De Persis, "Dynamic coupling design for nonlinear output agreement and time-varying flow control," *Automatica*, vol. 51, pp. 210–222, 2015.
- [20] C. De Persis and B. Jayawardhana, "On the internal model principle in the coordination of nonlinear systems," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 3, pp. 272–282, 2014.
- [21] S. Grivet-Talocia, "Passivity enforcement via perturbation of Hamiltonian matrices," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 9, pp. 1755–1769, 2004.
- [22] R. Alam, S. Bora, M. Karow, V. Mehrmann, and J. Moro, "Perturbation theory for Hamiltonian matrices and the distance to bounded-realness," *SIAM Journal on Matrix Analysis and Applications*, vol. 32, no. 2, pp. 484–514, 2011.
- [23] A. Pavlov and L. Marconi, "Incremental passivity and output regulation," *Systems and Control Letters*, vol. 57, pp. 400–409, 2008.
- [24] D. J. Hill and P. J. Moylan, "Dissipative dynamical systems: Basic input-output and state properties," *Journal of the Franklin Institute*, vol. 309, no. 5, pp. 327–357, 1980.
- [25] N. Monshizadeh and C. De Persis, "Output agreement in networks with unmatched disturbances and algebraic constraints," *arXiv preprint arXiv:1504.03609v2 [cs.SY]*, in *Proc. of the IEEE 54th Conference on Decision and Control*, Osaka, Japan.
- [26] K. Dvijotham, S. Low, and M. Chertkov, "Convexity of energy-like functions: Theoretical results and applications to power system operations," *arXiv preprint arXiv:1501.04052 [math.OC]*, 2015.
- [27] S. Trip and C. De Persis, "Optimal generation in structure-preserving power networks with second-order turbine-governor dynamics," *arXiv preprint*, 2016.
- [28] A. Wood and B. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. Wiley, 1996.