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# Robust Passivity-Based Control of Boost Converters in DC Microgrids<sup>\*</sup>

Michele Cucuzzella<sup>1</sup>, Riccardo Lazzari<sup>2</sup>, Yu Kawano<sup>3</sup>, Krishna C. Kosaraju<sup>1</sup> and Jacquelin M. A. Scherpen<sup>1</sup>

**Abstract**—This paper deals with the design of a robust and decentralized passivity-based control scheme for regulating the voltage of a DC microgrid through boost converters. A Krasovskii-type storage function is proposed and a (local) passivity property for DC microgrids comprising unknown ZIP (constant impedance ‘Z’, constant current ‘I’ and constant power ‘P’) loads is established. More precisely, the input port-variable of the corresponding passive map is equal to the first-time derivative of the control input. Then, the integrated input port-variable is used to shape the closed loop storage function such that it has a minimum at the desired equilibrium point. Convergence to the desired equilibrium is theoretically analyzed and the proposed control scheme is validated through experiments on a real DC microgrid.

## I. INTRODUCTION

Distributed Generation (DG) requires fundamental transformations of the conventional power generation, transmission and distribution systems [1]. DG represents a conceptual solution to *i*) enhance the integration of Renewable Energy Sources (RES) in order to reduce CO<sub>2</sub> emissions and the dependency on fossil fuels, *ii*) increase the energy efficiency by reducing the transmission power losses, *iii*) improve the service quality by enabling the operation of portions of the network disconnected from the main grid and *iv*) minimize the costs for electrifying remote areas or re-powering the existing power networks due to the ever increasing electric demand. A set of multiple DG Units (DGUs), loads and energy storage devices is identified as a *microgrid* [2].

In the last decades, due to the prevalence of Alternating Current (AC) networks, the literature on microgrids mainly considered AC systems (see for instance [3]–[6] and the references therein). However, the recent widespread use of RES as DGUs is motivating the design and operation of Direct Current (DC) microgrids [7]. Several devices (e.g. electric vehicles, electronic appliances, batteries and photovoltaic panels) can indeed be directly connected to a DC network avoiding lossy DC-AC conversion stages and the

issues related to the frequency and reactive power control [8]. Besides the development of industrial, commercial and residential DC distribution networks, some examples of existing or promising DC microgrid applications are ships, mobile military bases, trains, aircrafts and charging stations for electric vehicles. For all these reasons, control of DC microgrids and, consequently, DC-DC power converters is gaining growing interest.

In DC microgrids, control schemes are usually designed to achieve voltage stabilization and current (or power) sharing (see for instance [9]–[14] and the references therein). However, the dynamics of the power converters are often neglected or described by linear models (e.g., buck converters). Differently from [9]–[14], in this paper we design a robust and decentralized passivity-based control scheme for regulating the voltage of a DC microgrid through *boost* converters, the dynamics of which are *nonlinear*. Regulating the voltage towards the nominal value is required to ensure a proper operation of the connected loads and guarantee the network stability.

## A. Literature Review and Main Contributions

We now provide a brief comparison with some existing theoretical results dealing with the design of voltage controllers for boost converters. Simple tuning rules of passivity-preserving controllers are provided in [15], while stability in presence of a bounded control input is analyzed in [16]. However, only constant impedance loads are considered, the network dynamics are neglected and in [16] the load resistance is assumed to be known. In [17] and [18], Plug-and-Play voltage controllers are proposed. More precisely, the controller designed in [17] is robust with respect to load uncertainties. However, the line dynamics are neglected and the model linearized around the equilibrium point is studied. In [18] the microgrid stability is proved considering a bounded control input. However, only constant current loads are considered and the controller requires local information (including the load) and the value of the resistance of the lines interconnecting neighboring nodes. Under the assumption that the equilibrium point is known, a novel nonlinear control law that takes into account the constraints of the control action is proposed in [19].

We can now list the main contributions of this work:

1) *Robustness*: Differently from [16], [18] and [19], the proposed decentralized control scheme is robust with respect to unknown loads and other parameter uncertainty (e.g., line and filter impedances). Specifically, robustness is obtained by exploiting a *new* passivity property.

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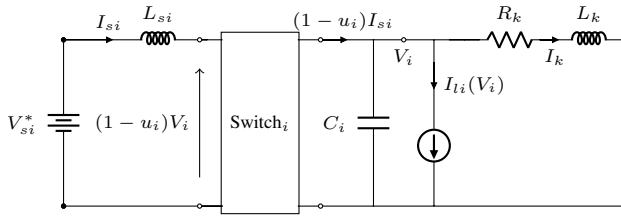


Fig. 1. Electrical scheme of DGU  $i$  and line  $k$ .

2) *Passivity framework*: Differently from [15], [16], [18] and [19], where a shifted storage function (or shifted Hamiltonian) is adopted, we propose a Krasovskii-type storage function (see for instance [20] and [21]) and establish a *new* (local) passivity property for the considered DC microgrid. More precisely, the input port-variable of the corresponding passive map is equal to the first-time derivative of the control input. Then, the integrated input port-variable is used to shape (*input shaping* methodology [22]) the closed loop storage function such that it has a minimum at the desired equilibrium point. Convergence to the desired equilibrium is established together with extremely simple tuning rules.

3) *Nonlinear load model*: Besides considering the nonlinear dynamics of the boost converter and resistive-inductive lines, differently from [15]–[19], we adopt a general nonlinear load model (called ZIP) including constant impedance ‘Z’, constant current ‘I’ and constant power ‘P’.

4) *Validation*: The proposed control strategy is verified through experiments on a real DC microgrid test facility at Ricerca sul Sistema Energetico (RSE), Milan, Italy, showing excellent closed-loop performance (see [23] for more information about the experimental setup where we have performed our tests).

## B. Outline

The present paper is organized as follows. The microgrid model is described in Section II, while the control objective is formulated in Section III. In Section IV, the proposed control scheme is designed and the stability of the controlled microgrid analyzed. In Section V, the proposed control scheme is validated through experiments on a real DC microgrid and, finally, conclusions are gathered in Section VI.

## C. Notation

Let  $\mathbf{0}$  be the vector of all zeros of suitable dimension and let  $\mathbf{1}_n \in \mathbb{R}^n$  be the vector containing all ones. The  $i$ -th element of vector  $x$  is denoted by  $x_i$ . A steady state solution to system  $\dot{x} = \zeta(x)$ , is denoted by  $\bar{x}$ , i.e.,  $\mathbf{0} = \zeta(\bar{x})$ . A constant signal is denoted by  $x^*$ . Given a vector  $x \in \mathbb{R}^n$ ,  $[x] \in \mathbb{R}^{n \times n}$  indicates the diagonal matrix whose diagonal entries are the components of  $x$ . Let ‘ $\circ$ ’ denote the Hadamard product, i.e., given vectors  $x, y \in \mathbb{R}^n$ ,  $(x \circ y) \in \mathbb{R}^n$  is a vector with elements  $(x \circ y)_i := x_i y_i$  for all  $i = 1, \dots, n$ . Let  $A \in \mathbb{R}^{n \times n}$  be a matrix. In case  $A$  is a positive definite (positive semi-definite) matrix, we write  $A \succ 0$  ( $A \succeq 0$ ). The  $n \times n$  identity matrix is denoted by  $\mathbb{I}_n$ . Given a set  $\Omega$ ,  $|\Omega|$  represents the cardinality of  $\Omega$ .

TABLE I  
DESCRIPTION OF THE USED SYMBOLS

State variables		ZIP loads	
$I_s$	Generated current	$G_l^*$	Load conductance
$V$	Load voltage	$I_l^*$	Load current
$I$	Line current	$P_l^*$	Load power
Inputs		Filters and lines	
$u$	Control input	$L_s, C$	Filter inductance, capacitance
$V_s^*$	Voltage source	$R, L$	Line resistance, inductance

## II. DC MICROGRID MODEL

The DC microgrid is represented by a connected and undirected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V} = \{1, \dots, n\}$  is the set of nodes and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  represents the set of the resistive-inductive lines interconnecting the nodes. Each node, which we call *Distributed Generation Unit*<sup>1</sup> (DGU), includes a DC-DC boost converter supplying an unknown load. A schematic electrical diagram of the considered DC network including a DGU and a power line is illustrated in Fig. 1 (see also Table I for the description of the used symbols).

By applying the Kirchhoff’s laws, the *average*<sup>2</sup> governing dynamic equations of the node  $i \in \mathcal{V}$  are the following:

$$\begin{aligned} L_{si} \dot{I}_{si} &= -(1 - u_i) V_i + V_{si}^* \\ C_i \dot{V}_i &= (1 - u_i) I_{si} - I_{li}(V_i) - \sum_{k \in \mathcal{E}_i} I_k, \end{aligned} \quad (1)$$

where  $I_{si} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ ,  $I_k : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ ,  $V_i : \mathbb{R}_{> 0} \rightarrow \mathbb{R}_{> 0}$ ,  $I_{li}(V_i) : \mathbb{R}_{> 0} \rightarrow \mathbb{R}_{\geq 0}$ ,  $u_i : \mathbb{R}_{\geq 0} \rightarrow [0, 1)$  and  $V_{si}^*, L_{si}, C_i \in \mathbb{R}_{> 0}$ . Moreover,  $\mathcal{E}_i$  is the set of power lines connected to the DGU  $i$  and  $I_k$  is the current flowing on the line  $k \in \mathcal{E}_i$ . Let  $k$  be the power line interconnecting DGUs  $i, j \in \mathcal{V}$ . Then, the dynamic of  $I_k$  in (1) is given by

$$L_k \dot{I}_k = (V_i - V_j) - R_k I_k, \quad (2)$$

with  $L_k, R_k \in \mathbb{R}_{> 0}$ . Moreover, the term  $I_{li}(V_i)$  in (1) represents the current demand<sup>3</sup> of load  $i \in \mathcal{V}$  and (generally) depends on the node voltage  $V_i$ . In this work, we consider a general nonlinear load model including the parallel combination of the following load components:

- 1) constant impedance:  $I_{li} = G_l^* V_i$ , with  $G_l^* \in \mathbb{R}_{> 0}$ ,
- 2) constant current:  $I_{li} = I_l^*$ , with  $I_l^* \in \mathbb{R}_{\geq 0}$ , and
- 3) constant power:  $I_{li} = V_i^{-1} P_l^*$ , with  $P_l^* \in \mathbb{R}_{\geq 0}$ .

To refer to the load types above, the letters ‘Z’, ‘I’ and ‘P’, respectively, are often used in the literature [9]. Therefore, in presence of the so-called ZIP loads,  $I_{li}(V_i)$  in (1) is given by

$$I_{li}(V_i) = G_l^* V_i + I_l^* + V_i^{-1} P_l^*. \quad (3)$$

<sup>1</sup>Note that, we consider generation units only for the sake of simplicity and without loss of generality. In the experiments (see Section V), the controlled nodes are indeed energy storage units, i.e., batteries.

<sup>2</sup>Under the condition that the Pulse Width Modulation (PWM) frequency is sufficiently high, the state of the system can be replaced by the average state representing the average inductor currents and capacitor voltages. Consequently, the switching input is replaced by the so-called duty cycle.

<sup>3</sup>The results presented in this work hold also in case of the so-called *net generating* loads, i.e.,  $I_{li}(V_i) < 0$ .

The symbols used in (1)–(3) are described in Table I.

We represent the microgrid topology by using its corresponding incidence matrix  $D \in \mathbb{R}^{n \times |\mathcal{E}|}$ . The ends of edge  $k \in \mathcal{E}$  are arbitrarily labeled with a + and a -. More precisely, the entries of  $D$  are given by  $D_{ik} = 1$  if  $i$  is the positive end of  $k$ ,  $D_{ik} = -1$  if  $i$  is the negative end of  $k$ , and  $D_{ik} = 0$  otherwise. The overall microgrid system (1), (2) in presence of ZIP loads (3) can now be written compactly for all nodes  $i \in \mathcal{V}$  as follows:

$$\begin{aligned} L_s \dot{I}_s &= -(\mathbb{1}_n - u) \circ V + V_s^* \\ C \dot{V} &= (\mathbb{1}_n - u) \circ I_s - G_l^* V - I_l^* - [V]^{-1} P_l^* + DI \\ L \dot{I} &= -D^\top V - RI, \end{aligned} \quad (4)$$

where  $I_s : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ ,  $V : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{> 0}^n$ ,  $I : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{|\mathcal{E}|}$ ,  $u : \mathbb{R}_{\geq 0} \rightarrow [0, 1]^n$ ,  $V_s^* \in \mathbb{R}_{> 0}^n$  and  $I_l^*, P_l^* \in \mathbb{R}_{> 0}^n$ . Moreover, the matrices  $L_s, C, G_l^*, L$  and  $R$  have appropriate dimensions and are constant, positive definite and diagonal.

### III. PROBLEM FORMULATION: VOLTAGE REGULATION

In this section, we formulate the control objective aiming at regulating the voltage of a boost-based DC microgrid. First, we notice that for given  $u^*, V_s^*, G_l^*, I_l^*$  and  $P_l^*$ , a steady state solution  $(\bar{I}_s, \bar{V}, \bar{I})$  to system (4) satisfies

$$\bar{V} = (\mathbb{1}_n - [u^*])^{-1} V_s^* \quad (5a)$$

$$(\mathbb{1}_n - [u^*]) \bar{I}_s = G_l^* \bar{V} + I_l^* + [\bar{V}]^{-1} P_l^* - D \bar{I} \quad (5b)$$

$$\bar{I} = -R^{-1} D^\top \bar{V}, \quad (5c)$$

where from (5a) it follows that the  $i$ -th boost output voltage  $\bar{V}_i$  is higher than the voltage source  $V_{si}^*$ ,  $i \in \mathcal{V}$ , while (5b) implies<sup>4</sup> that *current balance* is achieved at the steady state, i.e., the total current  $\mathbb{1}^\top (\mathbb{1}_n - [u^*]) \bar{I}_s$  injected by the boost converters is equal to the total current  $\mathbb{1}^\top (G_l^* \bar{V} + I_l^* + [\bar{V}]^{-1} P_l^*)$  demand of the ZIP loads. Moreover, in order to guarantee a proper functioning of the connected loads, it is required that the current balance is achieved at the desired voltage value. Consequently, before formulating the control objective, we introduce the following assumption on the existence of a desired reference voltage for each DGU:

**Assumption 1 (Desired voltage)** *There exists a constant desired reference voltage  $V_{di}^*$  satisfying<sup>5</sup>  $V_{di}^* \geq V_{si}^*$  and  $V_{di}^* > \sqrt{P_{li}^*/G_{li}^*}$ , for all  $i \in \mathcal{V}$ .*

Given  $V_d^* = [V_{d1}^*, \dots, V_{dn}^*]^\top$ , the control objective is then formulated as follows:

#### Objective 1 (Voltage regulation)

$$\lim_{t \rightarrow \infty} V(t) = \bar{V} = V_d^*.$$

<sup>4</sup>The incidence matrix  $D$ , satisfies  $\mathbb{1}_n^\top D = \mathbf{0}$ .

<sup>5</sup>The condition  $V_{di}^* \geq V_{si}^*$  is a requirement for any boost converter, the aim of which is indeed to increase the output voltage  $V_i$  with respect to the voltage source  $V_{si}^*$ . The condition  $V_{di}^* > \sqrt{P_{li}^*/G_{li}^*}$  will be clear after defining the set  $\mathcal{Z}_{ZIP}$  in the next section. We simply notice that in absence of constant power load, i.e.,  $P_{li}^* = 0$ , it becomes  $V_{di}^* > 0$ .

Moreover, in order to permit the controller design in the next section, the following assumption is introduced on the available information:

**Assumption 2 (Available information)** *The state variables  $I_{si}$ ,  $V_i$  and the voltage source  $V_{si}^*$  are locally available at the DGU  $i$ .*

Consequently, the control scheme we design in Section IV to achieve Objective 1 needs to be fully *decentralized*, increasing the practical applicability of the proposed approach.

**Remark 1 (Microgrid uncertainty)** *Note that, according to Assumption 2, the parameters  $I_l^*, P_l^*, G_l^*, L_s, L, C, R$  of the ZIP loads, lines and boost converters are not known. As a consequence, we need to design a control scheme that achieves Objective 1 independently of the system parameters. This is in contrast to [16], [18] and [19], where the controller requires some information about the system parameters.*

### IV. THE PROPOSED SOLUTION

In this section, we introduce the key aspects of the proposed decentralized passivity-based control scheme aiming at achieving Objective 1. More precisely, we first augment system (4) with additional dynamics. Secondly, we propose a Krasovskii-type storage function (see for instance [20] and [21]) and establish a (local) passivity property for the augmented system. The input port-variable of the corresponding passive map is equal to the first-time derivative of the control input. Then, we use the integrated input port-variable to shape the closed loop storage function such that it has a minimum at the desired equilibrium point.

Consider the following *auxiliary*<sup>6</sup> system:

$$L_s \dot{I}_s = -(\mathbb{1}_n - u) \circ V + V_s^* \quad (6a)$$

$$C \dot{V} = (\mathbb{1}_n - u) \circ I_s - G_l^* V - I_l^* - [V]^{-1} P_l^* + DI \quad (6b)$$

$$L \dot{I} = -D^\top V - RI \quad (6c)$$

$$L_s \ddot{I}_s = -(\mathbb{1}_n - u) \circ \dot{V} + v_c \circ V \quad (6d)$$

$$C \ddot{V} = (\mathbb{1}_n - u) \circ \dot{I}_s - v_c \circ I_s - (G_l^* - [V]^{-2} [P_l^*]) \dot{V} + D \dot{I} \quad (6e)$$

$$L \ddot{I} = -D^\top \dot{V} - R \dot{I} \quad (6f)$$

$$\dot{u} = v_c, \quad (6g)$$

which includes also the dynamics of the first-time derivative of the state and input of system (4).

Let the vector  $z := (I_s^\top, V^\top, I^\top, \dot{I}_s^\top, \dot{V}^\top, \dot{I}^\top, u^\top)^\top \in \mathcal{Z} := \{z \in \mathbb{R}^{5n+2|\mathcal{E}|} : V \in \mathbb{R}_{> 0}^n, u \in [0, 1]^n\}$  denote the state of the auxiliary system (6). In order to establish a passivity property for system (6), we first introduce the following set:

$$\mathcal{Z}_{ZIP} := \{z \in \mathcal{Z} : G_l^* - [V]^{-2} [P_l^*] > 0\}.$$

<sup>6</sup>The state variables and the control input of the *auxiliary* system are  $I_s, V, I, \dot{I}_s, \dot{V}, \dot{I}, u$  and  $v_c$ , respectively.

Then, the following result can be proved.

**Lemma 1 (Passivity property)** *System (6) is passive with respect to the supply rate  $v_c^\top (\dot{I}_s \circ V - \dot{V} \circ I_s)$  and the storage function*

$$S(z) = \frac{1}{2} \dot{I}_s^\top L_s \dot{I}_s + \frac{1}{2} \dot{V}^\top C \dot{V} + \frac{1}{2} \dot{I}^\top L \dot{I}, \quad (7)$$

for all the trajectories  $z \in \mathcal{Z}_{\text{ZIP}}$ .

*Proof:* The storage function  $S$  in (7) satisfies

$$\begin{aligned} \dot{S} &= -\dot{V}^\top (G_l^* - [V]^{-2} [P_l^*]) \dot{V} - \dot{I}^\top R \dot{I} \\ &\quad + v_c^\top (\dot{I}_s \circ V - \dot{V} \circ I_s) \\ &\leq v_c^\top (\dot{I}_s \circ V - \dot{V} \circ I_s), \end{aligned}$$

along the solutions  $z \in \mathcal{Z}_{\text{ZIP}}$  to system (6), which concludes the proof.  $\blacksquare$

**Remark 2 (Insights on the storage function  $S$ )** *The storage function  $S$  in (7) depends on  $z$ , i.e., the entire state of the auxiliary system (6). This is evident from replacing  $\dot{I}_s, \dot{V}, \dot{I}$  by the corresponding dynamics (6a)–(6c), or rewriting  $S$  as follows:*

$$\begin{aligned} S(z) &= \frac{1}{4} \left( \dot{I}_s^\top L_s \dot{I}_s + \dot{V}^\top C \dot{V} + \dot{I}^\top L \dot{I} \right) \\ &\quad + \frac{1}{4} \left( f_{I_s}^\top L_s^{-1} f_{I_s} + f_V^\top C^{-1} f_V + f_I^\top L^{-1} f_I \right), \end{aligned}$$

where  $f_{I_s} : \mathbb{R}_{>0}^n \times [0, 1]^n \rightarrow \mathbb{R}^n$ ,  $f_V : \mathbb{R}^n \times \mathbb{R}_{>0}^n \times \mathbb{R}^{|\mathcal{E}|} \times [0, 1]^n \rightarrow \mathbb{R}^n$ , and  $f_I : \mathbb{R}_{>0}^n \times \mathbb{R}^{|\mathcal{E}|} \rightarrow \mathbb{R}^{|\mathcal{E}|}$  represent the right-hand sides of (6a)–(6c), respectively. Moreover, it will be shown in Theorem 1 that using (7) to design the controller permits, differently from [16] and [18], the achievement of Objective 1 despite the system uncertainty (see Remark 1). However, the cost of guaranteeing robustness is the need of information about the first-time derivative of the signals  $\dot{I}_s$  and  $\dot{V}$  (see Remark 3).

Before designing the controller and introducing the main result of this work, we show that a unique steady state solution to system (6) exists.

**Lemma 2 (Existence of a unique steady state solution)** *Given  $v_c = \mathbf{0}$  and  $\bar{u} \in [0, 1]^n$ , there exists a unique steady state solution  $\bar{z} = (\bar{I}_s, \bar{V}, \bar{I}, \mathbf{0}, \mathbf{0}, \bar{u}) \in \mathcal{Z}$  to system (6), satisfying*

$$\begin{aligned} \bar{V} &= (\mathbb{I}_n - [\bar{u}])^{-1} V_s^* \\ \bar{I}_s &= (\mathbb{I}_n - [\bar{u}])^{-1} (G_l^* \bar{V} + I_l^* + [\bar{V}]^{-1} P_l^* - D \bar{I}) \\ \bar{I} &= -R^{-1} D^\top \bar{V} \\ \mathbf{0} &= \dot{V} \\ \mathbf{0} &= \dot{I}_s \\ \mathbf{0} &= \dot{I} \\ \mathbf{0} &= v_c. \end{aligned} \quad (8)$$

*Proof:* The proof follows from setting the left-hand-side of system (6) to zero.  $\blacksquare$

We can now show the main result of this paper concerning the design of a controller that (provably) stabilizes system (6) achieving Objective 1.

**Theorem 1 (Stability)** *Let Assumptions 1-2 hold. Consider system (6) controlled by*

$$T_c v_c = -K_c (u - u_d^*) - (\dot{I}_s \circ V - \dot{V} \circ I_s), \quad (9)$$

where  $u_{di}^* = 1 - V_{si}^*/V_{di}^*$  is the desired value of the duty cycle of the boost converter  $i \in \mathcal{V}$ ,  $T_c = \text{diag}(T_{c1}, \dots, T_{cn})$ ,  $K_c = \text{diag}(K_{c1}, \dots, K_{cn})$  and  $T_{ci}, K_{ci} \in \mathbb{R}_{>0}$  are the gains of the controller  $i \in \mathcal{V}$ . Then, the equilibrium  $\bar{z} = (\bar{I}_s, V_d^*, \bar{I}, \mathbf{0}, \mathbf{0}, u_d^*) \in \mathcal{Z}_{\text{ZIP}}$  is asymptotically stable in  $\mathcal{Z}_{\text{ZIP}}$ .

*Proof:* Consider the desired closed-loop storage function

$$S_d(z) = S(z) + \frac{1}{2} (u - u_d^*)^\top K_c (u - u_d^*), \quad (10)$$

where  $S$  is given by (7). Then, it is immediate to see that  $S_d$  attains a minimum at the equilibrium  $(\bar{I}_s, V_d^*, \bar{I}, \mathbf{0}, \mathbf{0}, u_d^*)$ , where  $\bar{V} = V_d^*$  follows from the first line of (8) with  $\bar{u} = u_d^* = \mathbf{1}_n - [V_d^*]^{-1} V_s^*$ . Furthermore,  $S_d$  satisfies

$$\begin{aligned} \dot{S}_d &= -\dot{V}^\top (G_l^* - [V]^{-2} [P_l^*]) \dot{V} - \dot{I}^\top R \dot{I} \\ &\quad + v_c^\top (K_c (u - u_d^*) + \dot{I}_s \circ V - \dot{V} \circ I_s) \\ &= -\dot{V}^\top (G_l^* - [V]^{-2} [P_l^*]) \dot{V} - \dot{I}^\top R \dot{I} - v_c^\top T_c v_c, \end{aligned} \quad (11)$$

along the solutions to system (6). From the last line of (11) it follows that  $S_d$  satisfies  $\dot{S}_d \leq 0$  for all  $z \in \mathcal{Z}_{\text{ZIP}}$ . Then, given  $\varepsilon > 0$ , choose  $r \in (0, \varepsilon]$  such that there exists a ball  $\mathcal{B}_r(\bar{z}) \subset \mathcal{Z}_{\text{ZIP}}$  centred in  $\bar{z} = (\bar{I}_s, V_d^*, \bar{I}, \mathbf{0}, \mathbf{0}, u_d^*) \in \mathcal{Z}_{\text{ZIP}}$ , i.e.,

$$\mathcal{B}_r(\bar{z}) := \{z \in \mathcal{Z}_{\text{ZIP}} : \|z - \bar{z}\| \leq r\} \subset \mathcal{Z}_{\text{ZIP}}.$$

Moreover, let  $\alpha$  denote the minimum value of  $S_d$  on the boundary of  $\mathcal{B}_r(\bar{z})$ , i.e.,  $\alpha = \min_{\|z - \bar{z}\| = r} S_d(z)$ . Since  $S_d$  is positive definite, then  $\alpha > 0$ . Take  $\beta \in (0, \alpha)$ , then the set

$$\Omega_\beta := \{z \in \mathcal{B}_r(\bar{z}) : S_d(z) \leq \beta\}$$

is compact, positively invariant and in the interior of  $\mathcal{B}_r(\bar{z})$  (see [24, Theorem 4.1]). Let now  $E$  denote the set of all points in  $\Omega_\beta$  where  $\dot{S}_d = 0$ , i.e.,

$$E := \left\{ z \in \Omega_\beta : \dot{V} = \mathbf{0}, \dot{I} = \mathbf{0}, v_c = \mathbf{0} \right\}.$$

Moreover, let  $M$  be the largest invariant set in  $E$ . Then, by LaSalle's invariance principle [24, Theorem 4.4], every solution starting in  $\Omega_\beta$  approaches  $M$  as  $t$  approaches infinity. Consequently, from (6e) we obtain  $\dot{I}_s = \mathbf{0}$  in  $M$ , and from (9) we can conclude that, in the largest invariant set  $M$ ,  $\bar{u} = u_d^*$ , implying from (6a) that  $V$  asymptotically converges to  $V_d^*$ . We finally conclude the proof observing

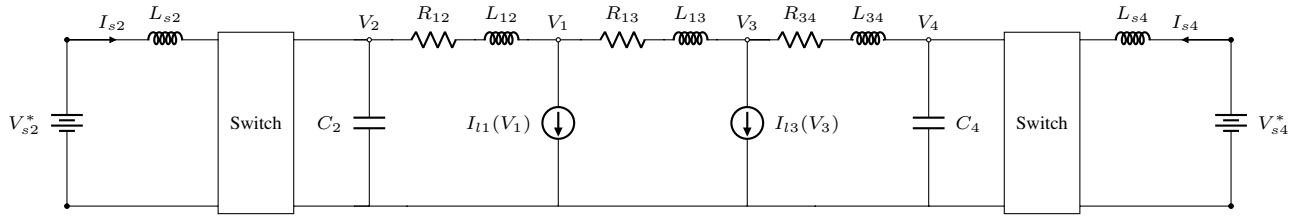


Fig. 2. Electrical scheme of the RSE's DC microgrid.

from (6b) and (6c) that also  $I_s$  and  $I$  converge to a constant value satisfying (8). ■

**Remark 3 (Robustness)** Note that controller (9) requires the first-time derivative of the current  $I_s$  and voltage  $V$ . This makes the proposed controller independent from the load, line and boost parameters. Controller (9) requires indeed only the knowledge of  $u_d^*$ , which depends on  $V_s^*$ . This is in contrast to [16], [18] and [19], where the controller requires some information about the system parameters.

**Remark 4 (ZI loads)** We observe that in case of only ZI loads, i.e.,  $P_l^* = \mathbf{0}$ , the results developed in this section can be strengthened. The absence of constant power loads implies indeed that the passivity property of system (6) and the result of Theorem 1 hold in the whole set  $\mathcal{Z}$ . This immediately follows by noticing that  $P_l^* = \mathbf{0}$  implies  $\mathcal{Z}_{ZIP} \equiv \mathcal{Z}$ .

## V. EXPERIMENTAL RESULTS

In order to validate the proposed control scheme, experimental tests have been performed on the DC microgrid test facility at RSE. The electrical scheme of the setup is shown in Fig. 2. The RSE's DC microgrid is unipolar with a nominal voltage of 380 V and includes a ZIP load, a DC generator (which emulates a PV plant) and two storage devices. The batteries are connected to the DC network through bidirectional boost converters (see [23] and [25] for more information about the RSE's DC microgrid and its parameters). In order to regulate the voltages  $V_2$  and  $V_4$  at the nodes 2 and 4 towards the corresponding desired value  $V_d^* = 380$  V, the control strategy proposed in Section IV (with  $T_c = 1 \times 10^7$  and  $K_c = 1 \times 10^9$ ) is implemented through dSpace controllers. The currents  $I_{l1}(V_1)$  and  $I_{l3}(V_3)$  demanded by the load and generated by the PV emulator are treated as disturbances. In the following, we arbitrarily assume the passive sign convention<sup>7</sup>.

In the first scenario the system is in a steady state condition with zero power absorbed by the load or provided by the generator. Each battery converter regulates its output voltage at the desired value  $V_d^* = 380$  V. At the time instant  $t = 5$  s the load (see Fig. 3) or the PV emulator (see Fig. 4) absorbs/generates 20 kW until the time instant  $t = 45$  s. From Fig. 3 and in Fig. 4, one can observe that, after a transient

<sup>7</sup> $I_{l1}(V_1) \geq 0$ ,  $I_{l3}(V_3) \leq 0$ ,  $I_{s1}, I_{s2} > 0$  ( $I_{s1}, I_{s2} < 0$ ) if the batteries charge (discharge).

due to the load/generator variations, the system exhibits a stable performance. This clearly shows the robustness of the proposed controller with respect to unknown loads.

In the second scenario the system is in a steady state condition with a constant power equal to 20 kW provided by the generator. Each battery converter regulates its output voltage at the desired value  $V_d^* = 380$  V. At the time instant  $t = 5$  s the desired value  $V_d^*$  is changed to 375 V and at the time instant  $t = 45$  s also the desired value  $V_d^*$  is changed to 375 V (dashed line). From Fig. 5, one can observe that the system exhibits a stable performance tracking the new desired voltage value. Tracking capabilities are generally essential to couple primary voltage controllers with higher-level control schemes that modify the voltage reference of each node, in order to achieve power sharing among the nodes of the microgrid. Finally, we notice that in the discussed scenarios, only the voltages  $V_2$  and  $V_4$  are controlled and the deviations from the desired value during the load and generator variations are less than 4%. In the uncontrolled nodes, the deviations of the voltages  $V_1$  and  $V_3$  from the desired value are due to the line impedances between the controlled and uncontrolled nodes.

## VI. CONCLUSIONS

In this paper a decentralized passivity-based control scheme is designed to regulate the voltage of a DC microgrid through *boost* converters. Using a Krasovskii-type storage function, a (local) passivity property for the considered DC microgrid is established. More precisely, the integrated input port-variable is used to shape the closed loop storage function. Convergence to the desired equilibrium is proven in presence of the so-called ZIP (constant impedance 'Z', constant current 'I' and constant power 'P') loads, showing robustness with respect to system parameter uncertainties. The proposed control scheme is validated through experimental tests on a real DC microgrid, showing excellent closed-loop performances.

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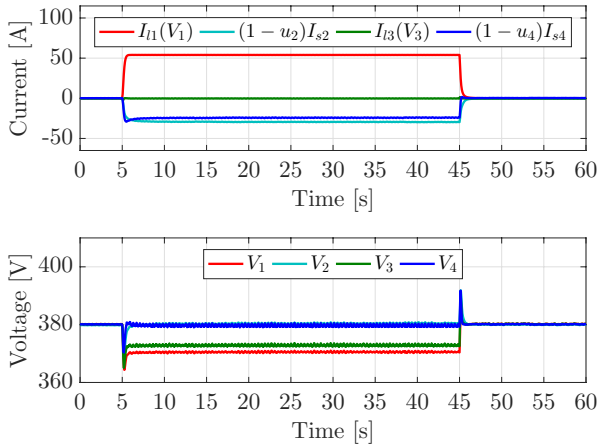


Fig. 3. Scenario 1: closed-loop system performance with a step load variation of 20 kW.

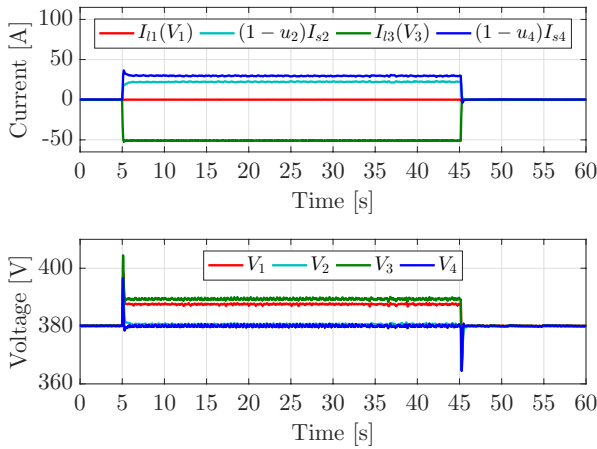


Fig. 4. Scenario 1: closed-loop system performance with a step generator variation of 20 kW.

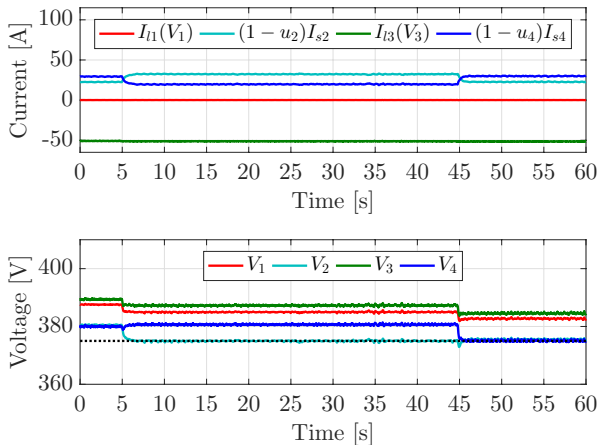


Fig. 5. Scenario 2: closed-loop system performance with a step reference variation of 5 V.

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