University of Groningen

Online marketing

Wu, Meng; Teunter, Ruud H.; Zhu, Stuart X.

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Link to publication in University of Groningen/UMCG research database

Full Length Article

Online marketing: When to offer a refund for advanced sales

Meng Wu [a, b], Ruud H. Teunter [b], Stuart X. Zhu [b,*]

[a] Business School, Sichuan University, Chengdu 610064, China
[b] Department of Operations, University of Groningen, P.O. Box 800, 9700AV Groningen, The Netherlands

A R T I C L E   I N F O

A B S T R A C T

Advance selling is a marketing strategy commonly used by online retailers to increase sales by exploiting consumer valuation uncertainty. Recently, some online retailers have started to allow refunds on products sold in advance. On the one hand this reduces the net advance sales, but on the other hand it allows a higher advance sales price. This research is the first to explore the overall effect of allowing a refund on profits from advance sales, identifying conditions where advance selling with or without refunds (or no advance selling at all) is best. We analytically compare the profits of three advance selling strategies: none, without refund, and with refund. We show that selling in advance and allowing a refund is optimal for products with a relatively small profit margin and small strategic market size, and that the added profit can be considerable. Our results guide managers in selecting the right advance selling strategy. To facilitate this, we graphically display, based on the two dimensions of regular profit margin and strategic market size, under what conditions the different strategies are optimal.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

Driven by the "customer is king" attitude, consumer return and refund policies are ubiquitous in regular sales today. Annually, U.S. customers return $264 billion worth of products (Kerr, 2013). Consumer return rates range from 5% to 9% for most brick-and-mortar retailers and can be up to 35% for fashion apparel (Ferguson, Guide, & Souza, 2006). For mail order/online retailers, return rates are usually larger than 18% and can be as high as 74% (Mostard, de Koster, & Teunter, 2005). Most returned products have no functional or cosmetic defects and can be resold again (Lawton, 2008). These so-called false failure returns result mainly from inventory overstock and from consumers dissatisfaction if a product does not live up to expectations.

Although a liberal return policy may increase sales, handling returns comes at a significant expense to retailers. Return losses can be up to 30% of product value for short life-cycle, time insensitive products (Guide, Souza, van Wassenhove, & Blackburn, 2006), and are around 25% for computer manufacturers (Ferguson et al., 2006). To maintain a profitable business, online retailers often attempt to reduce return costs either by cutting down the unit return handling costs and/or by reducing the quantity of consumer returns. The latter is obviously preferred as it avoids all handling costs and increases sales revenue. One way to achieve fewer returns is to charge customers for returns, either directly through a return (shipment) cost or indirectly by e.g., a restocking fee in B2B situations (Shulman, Coughlan, & Savaskan, 2009). Another way is to make consumers better informed, reducing valuation uncertainty and thereby the risk of dissatisfaction. The latter way is especially crucial for online retailers.

---

* Corresponding author.
  E-mail addresses: wumeng@scu.edu.cn (M. Wu), r.h.teunter@rug.nl (R.H. Teunter), x.zhu@rug.nl (S.X. Zhu).

However, consumer valuation uncertainty is not always bad. Indeed, many companies exploit it by offering consumers an advance selling opportunity before a new product is available. In recent years, as a widely used marketing strategy by online businesses, advance selling has been shown to be profitable for many industries by increasing sales, and has become common for a wide variety of products (e.g., consumer electronics, fashion products, software, videos, games, and books). Moreover, advance selling could benefit the retailer by reducing demand uncertainty (e.g., Tang, Rajaram, Alptekinoglu, & Ou, 2004; Li & Zhang, 2013), and by updating/improving the demand forecast (e.g., Prasad, Stecke, & Zhao, 2011; Wu, Zhu, & Teunter, 2017).

Although quite a few authors have considered the profitability of advanced selling, to the best of our knowledge, none have considered offering a refund for advanced sales. This research is the first to explore the overall effect of allowing a refund on profits from advance sales, identifying conditions where advance selling with or without refunds (or no advance selling at all) is best. Most firms that sell in advance offer a substantial discount, e.g., Amazon offered a 20% pre-order discount on all newly released video games in December 2016. Large discounts are needed to entice customers to buy in advance, despite their often considerable valuation uncertainties. Improving product information can help in this respect, but obviously not eliminate all valuation uncertainties completely. Allowing returns is an alternative or additional way to reduce the consumer's risk, thereby enabling firms to increase the advance selling price. Some online retailers have indeed started to allow returns for advanced sales. For example, as one of the two largest B2C online retailers in China, JD accepts returns without any reasons within 7 days for all advanced sales since September 2015. Moreover, as the largest digital distribution platform for PC gaming, Steam offers a full refund policy for all pre-ordered games if the refund request is submitted within two weeks of the game's release and the game has been played for less than 2 hours. We remark that although consumers have a statutory right to return unsatisfied products and get a full refund for regular sales in many countries, few countries offer this protection for advance sales. So, whether or not to offer a refund option for advance sales is optional in most countries.

Our research reveals many new structural results and insights. We show that the (spot selling) profit margin and the size of the strategic market (i.e., consumers who consider buying in advance) are decisive factors in advance selling strategy selection. Indeed, based on these two dimensions, we graphically show which policy (no advance selling, without refund, with refund) is optimal. In particular, if the strategic market size and profit margin are both relatively small, advance selling with a refund is best and can be considerably more profitable than other strategies. Moreover, our results show that the optimal advance selling strategy is significantly affected by consumer valuation uncertainty and return-related costs. To select the right advance selling strategy, our results suggest that online retailers should precisely estimate the strategic market size, accurately predict the uncertainty in consumer valuation, and control/reduce the return-related costs.

The remainder of this paper is organized as follows. In the following section, we review the related literature. In Section 3, we introduce the model settings. In Section 4, we propose a model for the newsvendor problem without advance selling as a benchmark. In Section 5, we study the newsvendor's advance selling decisions without refund. In Section 6, we study the newsvendor's advance selling decisions with refund. In Section 7, we provide numerical studies to examine how valuation uncertainty and return-related costs affect the retailer's optimal strategy and profit. In Section 8, we conclude the paper.

2. Literature review

In line with the existing literature and our modeling in the next section, we mainly review contributions on advance selling in the newsvendor framework. The literature is surveyed from two aspects: advance selling and product returns in Sections 2.1 and 2.2, respectively. In Section 2.3, we summarize and discuss our core contributions.

2.1. The newsvendor problem with advance selling

Existing papers that study advance selling typically consider consumers who have an uncertain value about the product before it is on the market. Despite this uncertainty, these customers may be enticed to buy in advance when offered a discount, thereby exploiting valuation uncertainty. Assuming independent consumer valuations, Shugan and Xie (2005) and Xie and Shugan (2001) show that advance selling can indeed improve a retailer's profits. Further, by assuming that both the market size and the amount of returned products are deterministic and modeling consumer returns in an "all or nothing" setting, Xie and Shugan (2001) study the optimal refunding strategy and show that offering a partial refund can increase the profit from advance selling. Fay and Xie (2010) compare an advance selling strategy with a probabilistic selling strategy and show that uncertainty and heterogeneity of consumer valuation can benefit the retailer under both strategies in different ways. In cases with limited supply, products can sometimes even be sold at a price premium in advance (e.g., Xie & Shugan, 2001). Yu, Kapuscinski, and Ahn (2015) study the impact of interdependent valuations on a retailer's advance selling strategy, and show that the retailer should use a discount (premium) advance selling strategy if valuations are highly diverse (related). Besides the strategic considerations of valuation uncertainty vs. discount, some researchers have recently considered other types of consumer behavior. Nasiry and Popescu (2012) consider advance selling to strategic and regret-averse consumers, who anticipate regret when deciding whether to advance purchase. Lim and Tang (2013) consider selling in advance to speculators who resell the purchased products rather than consume them. Lee, Choi, and Cheng (2015) and Wu et al. (2017) study the advance selling problem when consumers are both strategic and loss averse.

Besides exploiting consumer valuation uncertainty, some researchers consider the benefit of utilizing the realized advance demand information as a driver for advance selling. When facing regular (myopic) consumers, Tang et al. (2004) and McCardle, Rajaram, and Tang (2004) study the benefits of an advance booking discount program and show that advance selling can

reduce inventory risk for a retailer. However, when facing strategic consumers, Li and Zhang (2013) study the impact of advance demand information and show that more accurate advance demand information does not necessarily benefit the seller. Further, Wu et al. (2017) consider benefits from both valuation uncertainty and improved demand forecasting by advertising. They show that more accurate spot demand information from advance selling and advertising is beneficial only if the advance selling discount is relatively small and strategic market size is relatively large. Zhao and Stecke (2010) and Prasad et al. (2011) consider the benefits from both demand updating and valuation uncertainty to decide when a retailer should sell in advance to risk averse and loss averse consumers. Cho and Tang (2013) and Zhao, Pang, and Stecke (2016) study how the advance selling option affects the interactions between a retailer and a manufacturer in a decentralized supply chain. They show that advance selling can hurt both retailer profit and supply chain performance. Cachon and Feldman (2017) show that advance selling is less beneficial and may be harmful under competition.

In summary, consumer's strategic behavior has been shown to have a significant impact on a retailer's optimal sales strategy. However, to the best of our knowledge, no one has considered the option of allowing product returns with a full refund for items sold in advance. Offering such an option may increase profit from advance selling (further), as consumers may be persuaded to buy at a lower discount. Moreover, there are real-life situations where firms are legally bound to offer a (free) return policy under certain conditions (e.g., within 30 days and undamaged/unused). We next discuss the literature on product returns.

2.2. Product returns

False failure product returns include both consumers unsatisfactory returns and overstocking returns caused by inaccurate stocking decisions. The impact of product returns has received much attention from both the operation and the marketing communities. In the operations literature, there is a body of work on stochastic inventory models with consumers returns. For example, for a single period inventory problem, Mostard et al. (2005) and Mostard and Teunter (2006) study the newsvendor problem with resaleable returns and derive the optimal order quantity by using a net demand approach. Others have developed both push and pull type strategies for controlling the stock of both returned products and (re)manufactured product simultaneously. We refer interested readers to Dekker, Fleischmann, Inderfurth, and van Wassenhove (2004), Pokharel and Mutha (2009) and Govindan, Soleimani, and Kannan (2015) for detailed review of reverse logistics and remanufacturing.

Researchers have also analyzed supply chain performance and relationships between retailers and manufacturers under different returns policies and channel structures. Ferguson et al. (2006) propose a target rebate contract to coordinate supply chain with returns and show that false failure returns can be reduced via supply chain coordination methods. Su (2009) discusses the supply chain performance under both a full refund policy and a partial refund policy, and demonstrates that consumer refund policies may distort incentives under coordination contracts. Xiao, Shi, and Yang (2010) integrate the consumer refund policy and manufacturer buy-back policy and design a buyback/markdown money contract to coordinate the supply chain under a partial refund policy. Chen and Bell (2011) propose a new buy-back contract and set two buy-back prices for overstocking returns and consumer unsatisfactory returns, respectively. They show that this type of contract can achieve perfect supply chain coordination. Shulman, Coughlan, and Savaskan (2010) consider two reverse channel structures (salvaged by the retailer or salvaged by the manufacturer) for returned products. They study the impact of the reverse channel structure on the equilibrium return policy and profit, and show that the manufacturer may earn more profit by accepting returns even if the retailer has a more efficient way for salvaging returned products.

The marketing literature has mostly focused on how retailers can manage product-return policies to maximize future (long term) profits. For a complete review on product returns from the marketing literature, we refer to Minnema, Bijmolt, Petersen, and Shulman (2018) and Petersen and Anderson (2015).

Because many retailers still treat product returns as a cost driver, they aim to limit product returns by setting a strict product return policy which disincentivizes customers to return products. One way to reduce or prevent consumer returns is to impose costs on consumers for returning (e.g., Davis, Hagerty, & Gerstner, 1998). More specifically, a retailer can create a hassle for customers (i.e., effort leniency), impose a deadline on returns (i.e., time leniency), or charge a (restocking) fee (i.e., monetary leniency). Janakiraman, Syrdal, and Freling (2016) review the effect of return policy leniency on a consumer's return decision. Recently, Altug and Aydinliyim (2016) study how the consumer's strategic purchase behavior affects an online retailer's return policy. They show that retailers should not offer a refund option for a high-margin product with a low salvage value. Even for retailers who should optimally permit returns, the optimal refund should not exceed the salvage value.

Although many retailers dislike product returns, stricter product return policies are not always more profitable. A too strict policy can negatively affect total demand by reducing a customer's option value to purchase (Anderson, Hansen, & Simester, 2009), and has a negative effect on long-term customer value (Petersen & Kumar, 2009). Indeed, a satisfactory product return experience may lead to future benefits for the retailer. More specifically, allowing returns can lead to a positive effect on the number of customer future purchases (Petersen & Kumar, 2009), referral behavior (Petersen & Kumar, 2010), customer perceived purchase risk (Petersen & Kumar, 2015), and customer information value (Minnema, Bijmolt, Gensler, & Wiesel, 2016). Moreover, Petersen and Kumar (2015) point out that product returns play a important role in the firm-customer exchange process, noting that over 60% of surveyed retailers still do not consider product return behavior when allocating marketing resources.

In summary, the existing operation literature mainly focuses on channel relationships under some contract or refund policy, and does not incorporate consumer behavior as is done in the marketing literature.

2.3. Contribution

This paper differs from the existing literature in three ways. First, our study is the first to explore whether online retailers should offer a full refund option for advanced sales in an uncertain environment. Second, although some studies (e.g., Su, 2009) have modeled consumer returns caused by valuation uncertainty, we explicitly model the consumer returns of advanced sales by incorporating both consumer strategic behavior and valuation uncertainty. As our analysis will show, this provides important new insights on the profitability of advanced selling in relation to the strategic market size. Third, whereas most existing literature (e.g., Shulman et al., 2010) on consumer returns considers the situation that all returned products have to be salvaged, we consider the case that returned products can be resold (after some reprocessing activities). This leads to important insights on the profitability of offering a refund option for advance sales dependent on the reselling cost.

3. Problem settings

Our general setting is that of a retailer who faces a single period newsvendor problem. The retailer can allow consumers to place advance orders and also has the option to allow consumers to return items bought in advance. We explore the profitability of both options by comparing to the benchmark situation without advance selling. Table 1 lists the notation used in this paper.

3.1. Retailer settings

In the advance selling period, which ends before the start of the spot selling period, the retailer can allow consumers to make purchases at a price p_A per item and commits to fulfilling advance purchase orders in the spot selling period, guaranteeing delivery to these consumers (i.e., guarding them against a potential stock out).

The retailer uses a price commitment strategy, i.e. announces the advance price and the spot selling price p simultaneously at the beginning of the advance selling period. Since strategic customers in the spot selling market are less brand-loyal, the spot selling market is more competitive and the demand is more sensitive to the spot selling price. The spot selling price p is assumed to be exogenous and its value is determined by using the reference pricing strategy, i.e., the selling price just equals (below) the main price of its competitor. Note that we will discuss the impacts of spot selling profit margin on the optimal policy and on the profit changes from advance selling vs. spot selling in a numerical study.

To reduce the valuation risk for consumers who buy in advance, not having seen the actual product, the retailer can offer consumers the option to return items bought in advance for a full refund. Even if the full purchase price is refunded, a retailer or a third-party logistics company may charge for return shipping and/or the consumer may face the time and effort related cost of returning an item. For example, Amazon charges a convenience fee of $6 per order for picking up customer returns at their preferred addresses by UPS, Tmall asks customers to return the products by any third-party logistics company and to pay all shipping costs for the return.

We will refer to the sum of all return costs simply as the return cost, denoted by r_b. We remark that the return cost is not a decision variable of the retailer. If the item is returned then, after testing/repackaging, the returned product can be resold. We denote the cost involved with a return by r_c and refer to it as the reselling cost in order to avoid confusion with the consumer return cost.

At the beginning of the spot selling period, the retailer delivers the preordered products and decides on the order quantity. This quantity can be split as $Q + d_A$, where d_A products are used to fulfill orders from the advance selling period. Note that the observed demand in the advance selling period can be used to update the demand forecast for the spot selling season. Further, returned products can be resold and so the uncertain number of returns should be taken into account when deciding on Q.

Table 1
Model notation.

p_A	Advance selling price per unit;
p	Spot selling price per unit;
Q	Order quantity for the spot selling season
c	Cost per unit, $c < p$;
r_b	Return cost per unit, the total cost of sending product back to the retailer;
r_c	Reselling cost per unit, $r_c < c$;
s	Salvage value per unit, $s \leq c - r_c$;
m	Spot selling profit margin per unit, $m = p - c$;
V	Consumer valuation for a product which has mean μ_v, standard deviation σ_v, CDF $F(\cdot)$ and PDF $f(\cdot)$ with support on $[s, U]$;
D_A	Demand (number of consumers who buy) in the advance selling period;
D_S	Demand (number of consumers who buy) in the spot selling season with mean μ_S and standard deviation σ_S;
$\mathcal{U_A}$	Expected utilities for buying in the advance selling period
$\mathcal{U_W}$	Expected utilities for waiting until the spot selling period
N_s	Number of (informed) strategic consumers, a normally random variable, i.e., $N_s \sim N(\mu_s, \sigma_s^2)$;
N_m	Number of regular (myopic) consumers, a normally random variable, i.e., $N_m \sim N(\mu_m, \sigma_m^2)$;
R	Number of returned product;
ϵ	Return risk, a normally random variable, i.e., $\epsilon \sim N(0, \sigma_\epsilon^2)$;
k	A critical fractile for spot selling demand, i.e., $k = \Phi^{-1}((p-c)/(p-s))$.

3.2. Consumer settings

3.2.1. Consumer valuation

Consumer valuation is the maximum value a consumer is willing to pay. Consumers are uncertain about their own valuation during the advance selling period. Therefore, we assume that consumer valuation V during this period is a random variable, which has mean μ_v, standard deviation σ_v, a PDF $f(\cdot)$ and a CDF $F(\cdot)$ with a finite support on $[s, U]$ where s is the salvage value per unit. The realized valuation in the spot selling period is denoted by v. Note that the valuation uncertainty can be affected by many factors and valuations can differ across consumers. To keep the problem tractable, however, we assume that all consumers have the same valuation distribution.

3.2.2. Consumer surplus

Consumer surplus is the difference between the consumer valuation and the actual price he/she pays, i.e., $V - p$. If a consumer buys in advance at an advance selling price p_A and decides not to return, the uncertain surplus is $V - p_A$. If the item is returned, then there is a negative surplus $-r_b$. Buying in the spot selling period implies a consumer surplus of $v - p$. Fig. 1 summarizes the consumer surplus options with advance selling.

3.2.3. Consumer behavior, classification and surplus

Although consumers have the same valuation distribution, we introduce consumer heterogeneity by grouping consumers into two types: myopic and (informed) strategic. Informed strategic customers potentially consider buying in advance, but myopic customers (including uninformed strategic customers) do not. Note that the definition of myopic consumer is consistent with most of the literature (e.g., Shen & Su, 2007; Prasad, Venkatesh, & Mahajan, 2015), i.e., myopic customers make a buy-now-or-leave-forever purchase decision when products are available. Let N_m and N_s denote the number of myopic and strategic consumers, respectively. We assume that N_m and N_s follow a bivariate normal distribution with means μ_m and μ_s, standard deviations σ_m and σ_s and correlation coefficient $\rho \in (-1, 1)$, i.e., $(N_m, N_s) \sim N(\mu_m, \mu_s, \sigma_m^2, \sigma_s^2, \rho)$.

A myopic or uninformed strategic customer buys if his surplus $v - p$ is non-negative. The corresponding consumers' expected surplus is

$$\mathcal{U}_W = E(V - p)^+ = \mu_v - p + \int_s^p F(x)dx. \tag{1}$$

Since consumer's valuations are realized in the spot selling period, there are no valuation risk-related returns in the spot selling period. We remark that in real life, some uncertainty may remain and so returns of items purchased in the spot selling period could also be considered. However, as our focus is on the profitability of advance sales and associated possible returns,
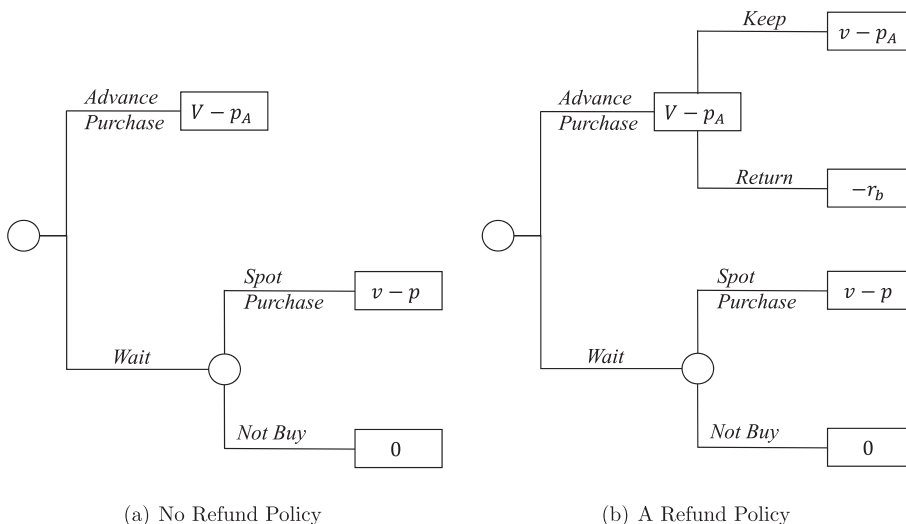

(a) No Refund Policy   (b) A Refund Policy

Fig. 1. Consumer surplus for different decisions.

we focus purely on those returns. Moreover, our model can easily be extended to include spot selling returns by letting D_S be the net (i.e., not returned) spot period demand and modifying the spot selling margin.

Informed strategic customers first need to decide between buying in advance or in the spot selling period (or not at all). If they buy in advance, then they need to decide whether or not to return after the order is delivered and their consumer surplus is realized.

We remark that the availability risk is not considered in our model, i.e., informed strategic consumers can get the product for sure if they choose to buy in the spot selling period. Our reasons are as follows. First, both the consumer's estimation of availability risk and the retailer's order quantity are private information. There is no evidence to support that these are exchanged. Second, for informed strategic consumers who choose to buy in the spot selling period, it is rational to buy the product at the start of the spot selling period rather than wait until the end. Further, we discuss the impact of the availability risk on the main results in Section 8.2.

Under the no-refund policy, the consumer's expected surplus is

$$\mathcal{U}_A^{NR} = E(V - p_A) = \mu_v - p_A. \tag{2}$$

Under a refund policy, a strategic customer returns the item and bears the return cost r_b if his realized surplus $v - p_A$ is lower than the negative surplus $-r_b$. Otherwise, strategic consumers should keep the product and obtain a surplus of $v - p_A$. In summary, the expected surplus of buying early is

$$\mathcal{U}_A^R = -\int_s^{p_A - r_b} r_b dF(x) + \int_{p_A - r_b}^U (x - p_A) dF(x) = \mu_v - p_A + \int_s^{p_A - r_b} F(x) dx. \tag{3}$$

Finally, we model uncertainty in the total number of returns. It is easy to see that a consumer (say i) chooses to return at price p_A if and only if his/her consumer utility is less than $p_A - r_b$, i.e., $v_i - p_A \leq -r_b$. So, aggregating over realized advance demand d_A, the expected total number of returns equals $d_A F(p_A - r_b)$. To include uncertainty in this number, which is realistic, we model the total number of returns as

$$R|_{d_A} = d_A(F(p_A - r_b) + \epsilon) \tag{4}$$

where ϵ is independent with both N_s and N_m and follows a normal distribution, i.e., $\epsilon \sim N(0, \sigma_\epsilon^2)$. Note that without return risk ϵ, the number of returns is known exactly after advance demand is realized, and makes the problem become trivial. Retailer's decisions are summarized and depicted in Fig. 2.

4. A benchmark: the classic newsvendor

We first determine the optimal order quantity and associated profit without advance selling (and so without refund), which serves as a benchmark for assessing the profitability of introducing the advance ordering opportunity.

Under this scenario, demand in the advance selling period is obviously zero, i.e., $D_A = 0$. During the spot selling period, consumer valuations have realized and consumer i makes a purchase at price p if and only if his/her consumer surplus is non-negative, i.e., $v_i - p \geq 0$. The fraction of all consumers who buy at price p is $E(1(v_i \geq p)) = \bar{F}(p)$. Therefore, the spot selling demand is given by

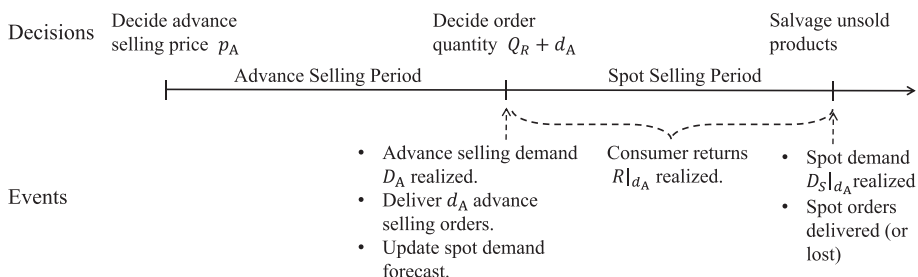$$D_S = \sum_{i=1}^{N_s + N_m} E(1(v_i \geq p)) = (N_s + N_m)\bar{F}(p),$$

Fig. 2. Timeline of decisions and events under the advance selling strategy with refund.

where $1(k)$ is an indicator function, i.e., $1(k) = 1$ if k is true, otherwise $1(k) = 0$. Since the numbers of myopic and strategic consumers are bivariate normally distributed, we have that the spot selling demand is normally distributed with a mean and a variance as follows,

$$\mu_S = E(D_S) = (\mu_s + \mu_m)\bar{F}(p),$$
$$\sigma_S^2 = Var(D_S) = (\sigma_s^2 + \sigma_m^2 + 2\rho\sigma_s\sigma_m)\bar{F}^2(p).$$

The retailer's expected profit is

$$\pi_{SS}(Q) = E_{D_S}\{p\min\{Q, D_S\} + s(Q - D_S)^+ - cQ\}.$$

Deciding on the optimal order quantity is a classic newsvendor problem with normally distributed demand. Following the standard solution method (e.g., Silver, Pyke, & Peterson, 1998), the optimal order quantity and the optimal expected profit are

$$Q_{SS} = (\mu_s + \mu_m)\bar{F}(p) + k\sqrt{\sigma_s^2 + \sigma_m^2 + 2\rho\sigma_s\sigma_m}\bar{F}(p),$$
$$\pi_{SS} = (p - c)(\mu_s + \mu_m)\bar{F}(p) - (p - s)\phi(k)\bar{F}(p)\sqrt{\sigma_s^2 + \sigma_m^2} + 2\rho\sigma_s\sigma_m, \tag{5}$$

where $k = \Phi^{-1}((p - c)/(p - s))$, $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and the PDF of the standard normal distribution respectively.

5. Advance selling without refund

Different from the classical newsvendor problem of the previous section, the advance selling price p_A^{NR} needs to be optimized besides the spot period order quantity Q^{NR}. An important starting observation is that introducing the advanced selling opportunity only makes sense if the price discount is large enough to make strategic customers buy in advance, so that a two-period selling problem indeed materializes.

We will first optimize the optimal order quantity Q^{NR} for the spot selling period, given p_A^{NR}, and then optimize p_A^{NR}. As for the case without advance sales (analyzed in Section 4), the retailer faces a newsvendor problem. However, the retailer can update its forecast for demand in the spot selling season based on the advance demand realization d_A (which does not depend on p_A^{NR} as long as strategic consumers buy in advance). Using standard statistics, we get that the number of myopic consumers N_m follows a normal distribution with updated mean and standard deviation as given by,

$$\mu_m|_{d_A} = \mu_m + \rho(d_A - \mu_s)\frac{\sigma_m}{\sigma_s},$$
$$\sigma_m^2|_{d_A} = \sigma_m^2(1 - \rho^2),$$

respectively. The demand during the spot selling season given d_A, $D_S|_{d_A}$, has an updated mean and standard deviation as follows,

$$\mu_S|_{d_A} = \left(\mu_m + \rho(d_A - \mu_s)\frac{\sigma_m}{\sigma_s}\right)\bar{F}(p),$$
$$\sigma_S^2|_{d_A} = \sigma_m^2(1 - \rho^2)\bar{F}^2(p).$$

Note that the updated demand variance is decreasing in the correlation coefficient ρ. The larger the correlation, the larger the reduction of demand variance in the spot selling period, i.e., the larger the forecast accuracy improvement. So the optimal order quantity $Q^{NR}|_{d_A}$ is given by

$$Q^{NR}|_{d_A} = \mu_S|_{d_A} + k\sigma_S|_{d_A} = \left(\mu_m + \rho(d_A - \mu_s)\frac{\sigma_m}{\sigma_s} + \sigma_m\sqrt{1 - \rho^2}\right)\bar{F}(p). \tag{6}$$

Note from Eq. (6) that $Q^{NR}|_{d_A}$ is not affected by p_A^{NR}. Therefore, the optimal value for p_A^{NR} must be the smallest value that makes strategic consumers buy in advance, i.e., for which $\mathcal{U}_A^{NR} \geq \mathcal{U}_W$. Obviously, the newsvendor will set $\mathcal{U}_A^{NR} = \mathcal{U}_W$ to maximize his profit. Combining Eqs. (1) and (2), we have

$$p_A^{NR} = p - \int_s^p F(x)dx. \tag{7}$$

Based on Eqs. (6) and (7), the maximum expected profit is

$$\pi_{AS}^{NR} = \left(p - c - \int_s^p F(x)dx\right)\mu_s + (p - c)\mu_m\bar{F}(p) - (p - s)\phi(k)\sigma_m\sqrt{1 - \rho^2}\bar{F}(p). \tag{8}$$

The difference between the optimal profit of the advance selling newsvendor without refund and the classic newsvendor problem is given by

$$
\begin{aligned}
\Delta\pi_{AS}^{NR} =& \pi_{AS}^{NR} - \pi_{SS} \\
=& (p-c)\mu_m\bar{F}(p) + \left(p - c - \int_s^p F(x)dx\right)\mu_s - (p-s)\phi(k)\sigma_m\sqrt{1-\rho^2}\bar{F}(p) \\
& - (p-c)(\mu_s + \mu_m)\bar{F}(p) + (p-s)\phi(k)\bar{F}(p)\sqrt{\sigma_s^2 + \sigma_m^2 + 2\rho\sigma_s\sigma_m} \\
=& (p_A^{NR} - c)F(p)\mu_s - \left(p - p_A^{NR}\right)\bar{F}(p)\mu_s + (p-s)\phi(k)\Delta\sigma_0^{NR},
\end{aligned}
\tag{9}
$$

where $\Delta\sigma_0^{NR} = \bar{F}(p)\left(\sqrt{\sigma_s^2 + \sigma_m^2 + 2\rho\sigma_s\sigma_m} - \sigma_m\sqrt{1-\rho^2}\right) \geq 0$.

Note that the three terms of the above profit difference expressions have clear interpretations. The first term represents the added profit from selling $F(p)\mu_s$ more items at price p_A^{NR}, by exploiting consumers' valuation uncertainty. The second term is the loss from selling at a discount to strategic customers who would otherwise have bought the item at the spot selling price. The final term is the safety stock reduction that results from better forecasting of spot selling demand after observing the advanced sales.

The profit difference expression and its interpretation also lead to the identification of parameter settings where advance selling is profitable. If the profit margin $p - c$ is large enough, then the profit from additional sales $(p_A^{NR} - c)F(p)\mu_s$ is larger than the discount loss $(p - p_A^{NR})\bar{F}(p)\mu_s$ for all values of μ_s as both terms are linear in μ_s. For small profit margins, the combination effect of terms $(p_A^{NR} - c)F(p)\mu_s$ and $(p - p_A^{NR})\bar{F}(p)\mu_s$ on the profit is negative, and the corresponding profit deduction is increasing with μ_s. The safety stock reduction $(p-s)\phi(k)\Delta\sigma_0^{NR}$ is not affected by μ_s and therefore only outweighs the profit reduction from offering a discount if μ_s is sufficiently small. This is formalized in the following proposition:

Proposition 5.1. *(AS without refund vs. no AS)*

- *If the spot selling profit margin is higher than some threshold, i.e., $m \geq m^{NR} = \frac{p - p_A^{NR}}{F(p)}$ where $m := p - c$, then a retailer should always sell in advance.*
- *Otherwise, the retailer should sell in advance only if the average number μ_s of informed strategic consumers is less than a threshold, i.e., $\mu_s \leq \mu_s^{NR}(m)$, where*

$$
\mu_s^{NR}(m) = \frac{(p-s)\phi(\Phi^{-1}(\frac{m}{p-s}))\Delta\sigma_0^{NR}}{p - p_A^{NR} - mF(p)}.
$$

Note that $\mu_s^{NR}(0) = 0$ and $\lim_{m \to m^{NR}}\mu_s^{NR}(m) = +\infty$.

Proof. See Appendix. □

Fig. 3 provides a graphical illustration of when advance selling is profitable. Recall that we only consider cases where $c > s$, to avoid situations with an infinite optimal ordering quantity. Therefore, the maximum profit margin considered is $p - s$. Proposition 5.1 implies that whether or not advance selling is profitable depends on both the spot selling profit margin and the market size of informed strategic consumers. If the advance selling discount is relatively small, then advance selling always benefits the retailer. Otherwise, advance selling is profitable only if relatively few potential consumers consider buying in the advance selling period.

Our results on whether or not to sell in advance mirror and build on those of Xie and Shugan (2001) and Prasad et al. (2011). Under the assumption of constant market sizes, Xie and Shugan (2001) found that retailers should sell in advance when marginal costs are below some lower threshold of consumer valuation. If serving the low valuation segment is unprofitable due to a high marginal cost, then spot selling to the high valuation segment is preferred. If market sizes are uncertain, then Prasad et al. (2011) found that no retailer should sell in advance if the difference between consumer expected valuation and consumer expected surplus of waiting is higher than a threshold. Our results further show that, besides the marginal valuation/cost, the market size of potential (informed strategic) consumers in the advance selling period affects the optimal strategy on advance selling.

6. Advance selling with refund

We now consider advance selling with a refund option. Analogously to Section 5, we start by deriving the optimal ordering quantity Q^R given p_A^R. Again, the retailer faces a newsvendor problem with demand forecast updates based on the advance
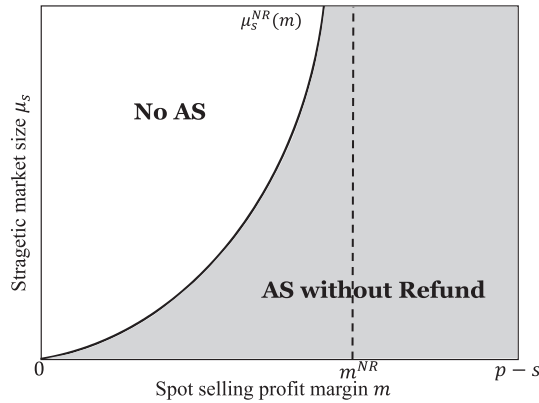
Fig. 3. Optimal advanced selling strategy — no AS vs. AS without refund.

demand realization d_A, where the retailer can resell returned products from the advanced sales period. Recall from Section 3 that the number of returns is given by

$$R|_{d_A} = d_A(F(p_A^R - r_b) + \epsilon)$$

where ϵ is independent with both N_s and N_m and follows a normal distribution, i.e., $\epsilon \sim N(0, \sigma_\epsilon^2)$. Therefore, the return of advanced sales $R|_{d_A}$ follows a normal distribution with mean $d_A F(p_A^R - r_b)$ and variance $d_A^2 \sigma_\epsilon^2$.

The net demand during the spot selling season given d_A, $D_{SN}|_{d_A}$ is defined as

$$D_{SN}|_{d_A} = (D_S - R)|_{d_A} = D_S|_{d_A} - R|_{d_A}.$$

Since the covariance between $D_S|_{d_A}$ and $R|_{d_A}$ is

$$Cov(D_S|_{d_A}, R|_{d_A}) = Cov(N_m|_{d_A}\bar{F}(p), d_A(F(p_A^R - r_b) + \epsilon)) = d_A\bar{F}(p)Cov(N_m|_{d_A}, \epsilon) = 0,$$

we have that $D_S|_{d_A}$ are $R|_{d_A}$ independent. Therefore, the net demand during the spot selling season given d_A, $D_{SN}|_{d_A}$, has a mean and a standard deviation as follows,

$$\mu_{SN}|_{d_A} = \mu_S|_{d_A} - d_A F(p_A^R - r_b) = \left(\mu_m + \rho(d_A - \mu_s)\frac{\sigma_m}{\sigma_s}\right)\bar{F}(p) - d_A F(p_A^R - r_b),$$

$$\sigma_{SN}^2|_{d_A} = \sigma_S^2|_{d_A} + d_A^2\sigma_\epsilon^2 = \sigma_m^2(1 - \rho^2)\bar{F}^2(p) + d_A^2\sigma_\epsilon^2.$$

So the optimal order quantity Q^R is given by

$$
\begin{aligned}
Q^R|_{d_A} &= \mu_{SN}|_{d_A} + k\sigma_{SN}|_{d_A} \\
&= \left(\mu_m + \rho(d_A - \mu_s)\frac{\sigma_m}{\sigma_s}\right)\bar{F}(p) - d_A F(p_A^R - r_b) + k\sqrt{\sigma_m^2(1 - \rho^2)\bar{F}^2(p) + d_A^2\sigma_\epsilon^2}.
\end{aligned}
\tag{10}
$$

Further, the maximum expected profit is as follows:

$$
\begin{aligned}
\pi_{AS}^R &= (p - c)\mu_m\bar{F}(p) + (p_A^R - c)\mu_s\bar{F}(p_A^R - r_b) \\
&\quad - r_c\mu_s F(p_A^R - r_b) - (p - s)\phi(k)E_{D_A}(\sigma_{SN}|_{d_A})
\end{aligned}
\tag{11}
$$

where $E_{D_A}(\sigma_{SN}|_{d_A}) = E_{D_A}\sqrt{\sigma_{SN}^2|_{d_A}} = E_{D_A}\sqrt{\sigma_m^2(1 - \rho^2)\bar{F}^2(p) + D_A^2\sigma_\epsilon^2}$.

Note from the third term of Eq. (11) that a higher advance selling price is not always better, as it will increase returns and associated costs. So, different from the case without refund, it may be optimal to set p_A^R below the maximum advance selling price at which strategic consumers buy in advance. Ignoring this bound on the advance selling price for now, Eq. (11) leads to the following first-order derivative with respect to p_A^R:

$$\frac{d\pi_{AS}^R}{dp_A^R} = \mu_s\bar{F}\left(p_A^R - r_b\right)\left(1 - (p_A^R - c + r_c)\frac{f\left(p_A^R - r_b\right)}{\bar{F}\left(p_A^R - r_b\right)}\right).$$

This derivative is decreasing in p_A^R for all valuation functions has an increasing failure rate (IFR), i.e., $\left(\frac{f(x)}{F(x)}\right)' \geq 0$ for all x in its domain. Most widely used valuation distributions (e.g., the uniform, normal, and negative exponential) in the operations management literature are of this type (see Lariviere, 2006), and so we will restrict our attention to them. Let p_A^0 denote the unique advance selling price for which the marginal profit of advanced sales is zero, i.e.,

$$\bar{F}\left(p_A^0 - r_b\right) = \left(p_A^0 - c + r_c\right)f\left(p_A^0 - r_b\right). \tag{12}$$

Note that it can be easily proved that p_A^0 is increasing in c.

Of course, we also need to ensure that p_A^R is small enough to make strategic consumers buy in advance, as mentioned before similar to the analysis in Section 5, $\mathcal{U}_A^R \geq 0$ and $\mathcal{U}_A^R \geq \mathcal{U}_W$ must hold. Using Eqs. (1) and (3), we then get $p_A^R \leq \bar{p}_A$ where $\bar{p}_A$ satisfies

$$p - \bar{p}_A = \int_{\bar{p}_A - r_b}^{p} F(x)dx. \tag{13}$$

So, the optimal advance selling price is

$$p_A^R = \begin{cases} p_A^0, & \text{if} \quad c \in [s, c^0], \\ \bar{p}_A, & \text{if} \quad c \in [c^0, p], \end{cases}$$

where c^0 satisfies $p_A^0(c^0) = \bar{p}_A$. We further obtain the following analytical sensitivity results:

Proposition 6.1.

(1) *The optimal advance selling price decreases as the spot selling profit margin increases, i.e., $dp_A^R/dm \leq 0$.*
(2) *The optimal advance selling price decreases as the reselling cost increases, i.e., $dp_A^R/dr_c \leq 0$.*
(3) *The optimal advance selling price first rises and then drops as the return cost increases, i.e., $dp_A^R/dr_b \geq 0$ if $r_b \leq r_b^0$ and $dp_A^R/dr_b \leq 0$ otherwise, where r_b^0 satisfies $p_A^0\left(r_b^0\right) = \bar{p}_A\left(r_b^0\right)$.*

Proof. See Appendix. □

The first result in Proposition 6.1 is in line with previous arguments. If the spot selling margin is higher, then advance selling remains profitable for a larger price discount range. Result (2) is also intuitive. The effect of the return cost on the optimal advance selling price is two-fold. For small enough return costs, the advance sales discount is larger than what is needed to make strategic consumers buy in advance. In this return cost range, a higher return cost reduces the return probability, even if the advance selling price is somewhat higher. This explains why the advance selling price is increasing in the return cost if that cost remains relatively small. For a relatively high return cost, however, the discount is set just high enough to make strategic consumers buy in advance. A (further) increase in the return cost reduces the utility of buying early below than the utility of waiting. To ensure that consumers still buy in advance, the retailer should then lower the advance selling price.

Proposition 6.2. *The expected profit of advance selling with refund decreases as the variance of return risk increases, i.e., $d\pi_{AS}^R/d\sigma_\epsilon^2 \leq 0$.*

The proof is straightforward and omitted. The result in Proposition 6.2 is intuitive. If returns become more uncertain, then more products are returned which results in a profit loss. Therefore, uncertainty in consumer return always hurts retailers.

The difference between the optimal profit of the advance selling newsvendor with refund and that of classic newsvendor is

$$\begin{aligned} \Delta\pi_{AS}^R &= \pi_{AS}^R - \pi_{SS} \\ &= (p-c)\mu_m\bar{F}(p) + (p_A^R - c)\mu_s\bar{F}(p_A^R - r_b) - r_c\mu_s F\left(p_A^R - r_b\right) - (p-s)\phi(k)E_{D_A}(\sigma_{SN}') \\ &\quad - (p-c)(\mu_m + \mu_s)\bar{F}(p) + (p-s)\phi(k)\bar{F}(p)\sqrt{\sigma_s^2 + \sigma_m^2 + 2\rho\sigma_s\sigma_m} \\ &= (p_A^R - c)F(p)\mu_s - \left(p - p_A^R\right)\bar{F}(p)\mu_s - \left(p_A^R - c + r_c\right)F\left(p_A^R - r_b\right)\mu_s \\ &\quad + (p-s)\phi(k)\Delta\sigma_0^{RN}(\mu_s) \end{aligned} \tag{14}$$

where $\Delta\sigma_0^{RN}(\mu_s) = \bar{F}(p)\sqrt{\sigma_s^2 + \sigma_m^2} + 2\rho\sigma_s\sigma_m - E_{D_A}(\sigma_{SN}|_{d_A})$.

All terms of the profit difference expression again have clear interpretations. The first term represents the added profit from selling $F(p)\mu_s$ more items at price p_A^R. The second term is the loss from selling at a discount to strategic customers who would

otherwise have bought the item at the spot selling price. The third term is the reselling loss from consumers returns. The final term corresponds to the safety stock reduction/increment that results from the trade-off between better forecasting of spot selling demand and increased uncertainty from returns.

This profit difference expression and interpretation also lead to the identification of parameter settings where advance selling with refund is profitable. If the profit margin $p - c$ is relative large, then the profit from additional sales $(p_A^R - c) F(p)\mu_s$ is larger than the discount loss $(p - p_A^R) \bar{F}(p)\mu_s$ and the return loss $(p_A^R - c + r_c) F (p_A^R - r_b) \mu_s$ for all values of μ_s. The safety stock reduction $(p-s)\phi(k)\Delta\sigma_0^{NR}(\mu_s)$ is decreasing in μ_s and therefore the retailer should sell in advance and offering a refund option if μ_s is relative small. For small profit margins, the combination effect of terms $(p_A^R - c) F(p)\mu_s$, $(p - p_A^R) \bar{F}(p)\mu_s$ and $(p_A^R - c + r_c) F (p_A^R - r_b) \mu_s$ on the profit is negative, and the corresponding profit deduction is increasing with μ_s. Therefore, the retailer should sell in advance only if μ_s is sufficiently small. This is formalized in the following theorem:

Theorem 1. *(AS with refund vs. no AS) If the average number of informed strategic consumers is less than a threshold, i.e., $\mu_s \leq \mu_s^R(m)$, then the retailer should sell in advance with a refund option; otherwise, the retailer should sell in the spot period only, where $\mu_s^R(m)$ satisfies*

$$\mu_s^R(m) = \frac{(p - s)\phi\left(\Phi^{-1}\left(\frac{m}{p-s}\right)\right)\Delta\sigma_0^{RN}\left(\mu_s^R(m)\right)}{(p - p_A^R) \bar{F}(p) - (p_A^R - c) F(p) + (p_A^R - c + r_c) F (p_A^R - r_b)}.$$

Proof. See Appendix.   □

Fig. 4 provides a graphical illustration of Theorem 1. Note that there are two cases. Fig. 4 (a) is similar to Fig. 3, and arguably most intuitive. Advance selling always increases total net demand, even if some products are returned, which becomes more attractive for larger profit margins. However, if the return fraction is large and managing returns is costly, then the negative effect of increased returns handling cost can almost cancel out the increase in sales revenue. Under such conditions, the safety stock effect becomes dominant for strategy selection. For very small profit margins, the safety stock is very small and so there is not much safety stock cost reduction potential from demand forecast improvements through advance selling. For very large profit margins and so, relatively, a very small cost c, there is again a low benefit of improved forecasting as stocks are not expensive. For cases in between, where the profit margin is neither very high nor very low, the cost savings potential from improved forecasting are highest. This then leads to the policy trade-off as illustrated in Fig. 4 (b).

Combining Proposition 5.1 and Theorem 1, we find the advantages of advance selling with or without refund stem from improved demand forecast accuracy as well as increased total sales. However, advance selling does imply a smaller profit margin and increased costs for handling consumer returns.

Having analyzed no advance selling vs. first advance selling without and then with refund, we next compare the profits of advance selling with or without refund. Combined with Proposition 5.1, this will then allow us to paint a complete picture of when no advance sales, advance sales without refund or advance sales with refund is optimal.
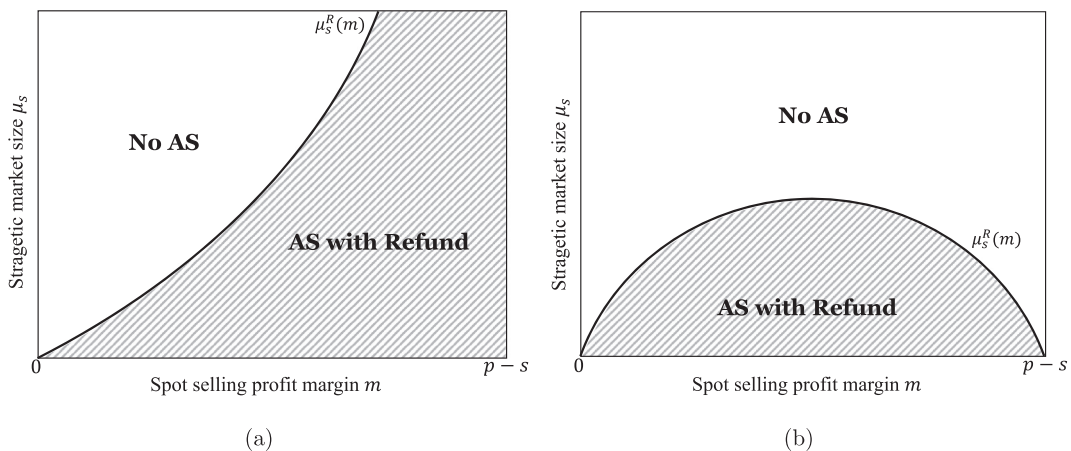

Fig. 4. Optimal advanced selling strategy — no AS vs. AS with refund.

From Eqs. (8) and (11), we get

$$
\begin{aligned}
\Delta\pi^R &= \pi_{AS}^R - \pi_{AS}^{NR} \\
&= (p-c)\mu_m\bar{F}(p) + \left(p_A^R - c\right)\mu_s\bar{F}\left(p_A^R - r_b\right) - r_c\mu_s F\left(p_A^R - r_b\right) - (p-s)\phi(k)E_{D_A}(\sigma'_{SN}) \\
&\quad - (p-c)\mu_m\bar{F}(p) - \left(p - c - \int_s^p F(x)dx\right)\mu_s + (p-s)\phi(k)\sigma_m\sqrt{1-\rho^2}\bar{F}(p) \\
&= \left(p_A^R - p_A^{NR}\right)\mu_s - \left(p_A^R - c + r_c\right)F\left(p_A^R - r_b\right)\mu_s - (p-s)\phi(k)\Delta\sigma_0^R(\mu_s),
\end{aligned}
$$

where $\Delta\sigma_0^R(\mu_s) = E_{D_A}(\sigma_{SN}|_{d_A}) - \sigma_m\sqrt{1-\rho^2}\bar{F}(p) \geq 0$.

The first term of the above profit difference expression represents the added profit from selling μ_s items at a higher advance selling price p_A^R, by offering consumers a refund option. The second term is the loss from consumers returns and the remanufacturing of returned products. The final term is the loss from safety stock increases that results from uncertain returns. We obtain the following structural result:

Theorem 2. *(AS with refund vs. AS without refund) A retailer who sells a product in advance should offer a refund option if the average number of informed strategic consumers μ_s is below $\mu_s^0(m)$, where $\mu_s^0(m)$ is decreasing in spot selling profit margin m and satisfies the following equation:*

$$
\left.\frac{dE_{D_A}\left(\sigma_{SN}|_{d_A}\right)}{d\mu_s}\right|_{\mu_s = \mu_s^0(m)} = \frac{(p - p_A^R(m) - m - r_c)F\left(p_A^R(m) - r_b\right) + p_A^R(m) - p_A^{NR}}{(p-s)\phi\left(\Phi^{-1}\left(\frac{m}{p-s}\right)\right)}.
$$

Proof. See Appendix. □

Fig. 5 provides a graphical illustration of Theorem 2. As the only advantage, offering a refund option reduces the advance selling discount, thereby increasing the added profit per product sold in advance that is not returned. This is particularly an important benefit if the spot selling profit margin is small to start with. However, allowing returns does lead to an increase in net demand uncertainty and thereby in the safety stock, more so if many consumers buy in advance. Combined, these effects explain why (in Fig. 5) offering a refund option is profitable for small enough profit margins and small enough strategic market sizes.

By combining Proposition 5.1 and Theorems 1–2, we can identify settings where either of the three considered strategies is preferable. This is depicted in Fig. 6. Please note that this figure is again only an illustration, but the next section will present numerical examples that confirm its shape. We also remark that the $\mu_s^{NR}(m)$, $\mu_s^R(m)$ and $\mu_s^0(m)$ lines may not intersect at (approximately) the same point, but our numerical results will show that they typically do.

Fig. 6 has clear and relevant interpretations. If the profit margin is (very) large, then advance selling is best as it leads to considerable extra revenue and avoids costly returns. For small profit margins and large strategic market sizes, advance selling is not profitable at all since the discount loss offered to many customers outweighs the benefits from increased total sales and more accurate forecasting. For small profit margins and small strategic market sizes, advance selling with refund is optimal. By
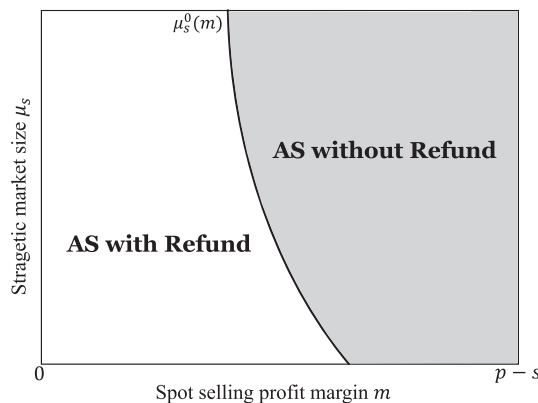

Fig. 5. Optimal advanced selling strategy — AS without refund vs. AS with refund.
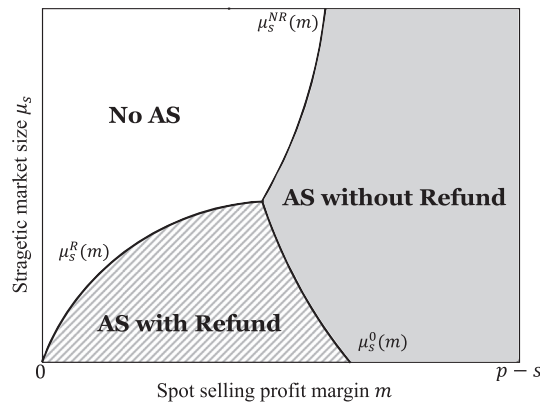
Fig. 6. Optimal advanced selling strategy − no AS vs. AS without refund vs. AS with refund.

offering the refund option, firms can sell in advance at a lower discount, making advance selling profitable even at lower profit margins, as long as the strategic market size is not too large.

The combination of a low margin (say below 30%) and a relatively low strategic market size (of brand enthusiasts) is realistic, and so there may indeed be many real-life cases where advance selling with refund is the most profitable strategy. The potential profit increase from allowing returns will be explored numerically in the next section.

7. Numerical study

So far, we have obtained analytical insights into the regions (combinations of strategic market size and profit margin) where a certain type of strategy is optimal. Recalling that our key contribution is to include the advance selling with refund strategy, this section provides further numerical insights into (a) the size of the region where this is the best strategy, and (b) the profit increase from allowing a refund.

We do so in relation to the return costs for consumers, the returns handling cost for the retailer, and the valuation uncertainty. These three model elements are closely linked to allowing a refund. The first two obviously affect return-related costs. The third is related to the benefit from returns, since more valuation uncertainty implies a higher advance selling discount that can be reduced by allowing a refund. Parameters r_c, r_b and σ_v will therefore be varied for the examples considered. The other parameters are fixed as follows: $p = 5$, $s = 1$, $\mu_v = 5$, $\rho = 0.5$, $\sigma_\epsilon = 0.1$, $\mu_s = 150$, $\sigma_m = 40$ and $\sigma_s = 10$. Other examples produced similar results. Note also that σ_s is set considerably lower than σ_m as the strategic market (of brand enthusiasts) typically has a smaller size than the myopic market (and may also be relatively less uncertain due to frequent interaction with those consumers).

7.1. The impact of return-related market parameters on strategy selection

We start by varying the standard deviations of consumer valuation $\sigma_v \in \{1, 2.5, 4\}$, fixing r_c at 0.4 and r_b at 0.1. Fig. 7 presents the critical thresholds of the strategic market size, i.e., $\mu_s^{NR}(m)$, $\mu_s^{R}(m)$ and $\mu_s^{0}(m)$. Note from this figure that more valuation uncertainty increases the area where offering a refund option for advance sales is beneficial. A more uncertain valuation implies a larger advance selling discount deduction without a refund option, and correspondingly a larger benefit from reducing that discount by offering a refund option.

We next vary the unit reselling cost $r_c \in \{0.4, 1, 1.6\}$, fixing σ_v at 2.5 and r_b at 0.1; and vary the unit return cost $r_b \in \{0.1, 1.3, 2.5\}$, fixing σ_v at 2.5 and r_c at 0.4. Figs. 8 and 9 present the critical thresholds of the strategic market size (i.e., $\mu_s^{NR}(m)$, $\mu_s^{R}(m)$ and $\mu_s^{0}(m)$) with varying unit reselling cost and varying unit return cost, respectively. Note that varying the unit return cost has a very similar effect on the optimal strategy as varying the unit reselling cost. More specifically, a higher reselling (return) cost reduces the size of the area where offering a refund option for advance sales is beneficial. Since fewer customers (retailers) choose to return (offer a refund option) if the return (reselling) cost is relatively high, this result is intuitive.

7.2. The impact of return-related market parameters on the profitability of allowing a refund

Rather than showing the profit increase from advance sales with or without refund for all combinations of strategic market size and spot selling profit margin (which would require a 3D picture), we consider a relatively small ($\mu_s = 10$) and large strategic market size ($\mu_s = 70$). Let $\sigma_v = 2.5$, $r_c = 0.4$, $r_b = 0.1$. In Fig. 10 we consider these situations side by side, and show the profit increase from advance selling with (dashed line) and without refund (solid line) vs. no advance selling.

As Fig. 10 (a) shows, advance selling with refund is the most profitable strategy for 'medium' profit margins between 5% and 30%. For even bigger spot selling period profit margins, reducing the discount through allowing a refund plays less of a role
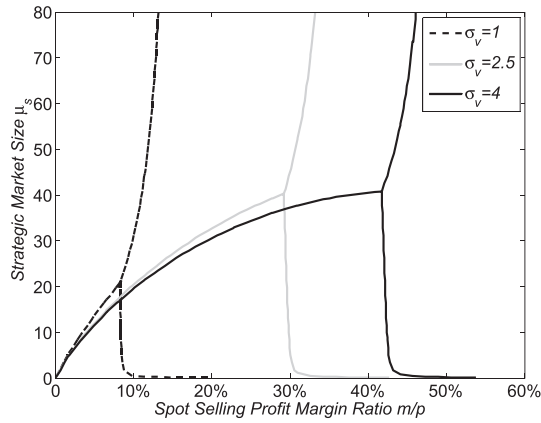
Fig. 7. Optimal policy with varying standard deviations of consumer's valuation.

and advance selling without refund is more profitable. For very low-profit margins, the required discount is too high even with refunds allowed, and spot selling only is best. An interesting result is that for most of the 5%–30% profit margin range where advance selling with refund is the best strategy, it outperforms the other strategies by more than 5% and sometimes even more than 10%. However, for a relative big strategic market considered in Fig. 10 (b), offering a refund option is never profitable. In this situation, the loss from returns is too large compared to the other strategies.

We next discuss the profit increase from advance sales with or without refund for combinations of spot profit margin and unit return cost. We consider a relative small ($m/p = 10\%$, i.e., $c = 4.5$) and big profit margin ratio ($m/p = 40\%$, i.e., $c = 3$). Let $\sigma_v = 2.5, r_c = 0.4, \mu_s = 10$. In Fig. 11 we consider these situations side by side, and show the profit increase from advance selling with (dashed line) and without refund (solid line) vs. no advance selling.

As Fig. 11 (a) shows, for a relative big profit margin product, offering a refund for advanced sales is never optimal although it is always profitable. Since increasing the return cost reduce the return probability which may result in a decrease in advance selling discount, the loss from offering a refund can be reduced by increasing the return cost. However, for a relatively small profit margin product considered in Fig. 11 (b), advance selling with refund is the most profitable strategy for the 'low' return cost between 0% and 25%. For an even higher return cost, further increasing the return cost reduces the utility of buying early below than the utility of waiting. To ensure that consumers still buy in advance, the retailer should offer a bigger advance selling discount. Since the required discount is too high even without a refund, spot selling only is best. An encouraging result is that for a large and arguably the most realistic return cost range (0%–25%), advance selling with refund is the best strategy.

7.3. Extension: endogenousness of spot selling price

In this section, we study how an endogenous spot selling price affects the retailer's advance selling strategy. This is done by numerically solving the first-order conditions of π_{SS}, π_{AS}^{NR} and π_{AS}^{NR} given in Eqs. (5), (8) and (11), respectively. We remark that,
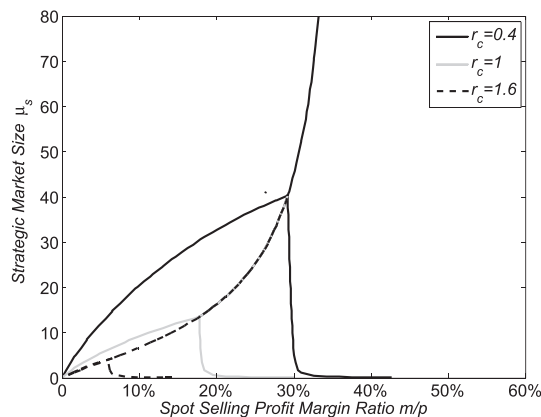
Fig. 8. Optimal policy with varying unit reselling costs.
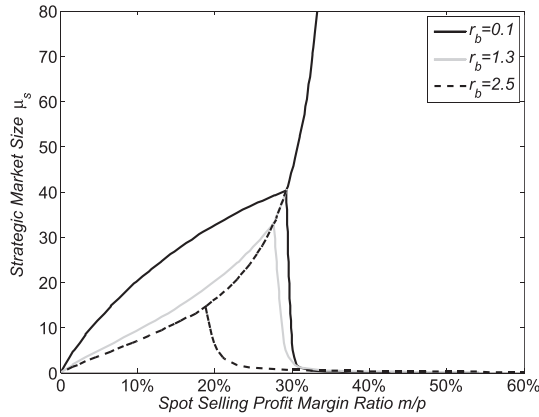
Fig. 9. Optimal policy with varying unit return costs.

under the advance selling with refund strategy, the optimal advance selling price are in two different cases, thereby the optimal spot selling price is the price that gives the highest profit in both cases.

Our discussions are centered around two different (small vs. large) strategic market sizes ($\mu_s = 10$ and $\mu_s = 70$). Let $\sigma_v = 2.5$, $r_c = 0.1$ and $r_b = 0.1$. In Tables 2 and 3, we consider the small and large market size settings side by side, and show the optimal spot selling prices and their corresponding optimal profits under the three different strategies (no advance selling, advance selling without refund, and advance selling with refund).

From Tables 2 and 3, it is clear that no advance selling (advance selling without refund) strategy is optimal only if the retailer has a relatively high (low) cost. Offering a refund for advanced sales is the best strategy only if the informed strategic consumers has a relatively small population and the retailer has a medium purchasing cost. Otherwise, offering a refund is never optimal. Recall that the same effects were observed analytically for a fixed price comparison in Fig. 6.

However, some changes can be noticed that relate to the exogenousness of spot selling price. Since the mean and the standard deviation of consumer valuation are assumed to be 5 and 2.5, i.e., $\mu_v = 5$ and $\sigma_v = 2.5$, from Table 2, the probability that no advance selling strategy is optimal is very small (less than 6%). Since the (expected) consumer valuation is normally higher than the cost, advance selling with or without refund is always optimal if the strategic market size is relatively big.

Moreover, from Tables 2 and 3, it is clear that the optimal spot prices (spot profit margins, i.e., $p - c$) are increasing (decreasing) with respect to the cost. The exact range of margins achieved depends on the selected strategy. Under the no advance selling strategy, the spot profit margins range [2.796, 4.933] for $\mu_s = 10$, and [2.606, 4.923] for $\mu_s = 100$. If retailers employ advance selling with refund strategy, the spot profit margin range is narrowed to [3.121, 4.737] for $\mu_s = 10$ and [3.273, 4.74] for $\mu_s = 100$. However, not offering a refund for advanced sales leads to larger ranges [2.866, 5.15] for $\mu_s = 10$ and [4.245, 6.657] for $\mu_s = 100$. This is explained as follows. To offer discounts for advanced sales, the retailer has to ensure that advance selling strategy has a higher spot profit margin than no advance selling strategy. As a result, the lower bounds of

(a) Small strategic market $\mu_s = 10$

(b) Big strategic market $\mu_s = 70$
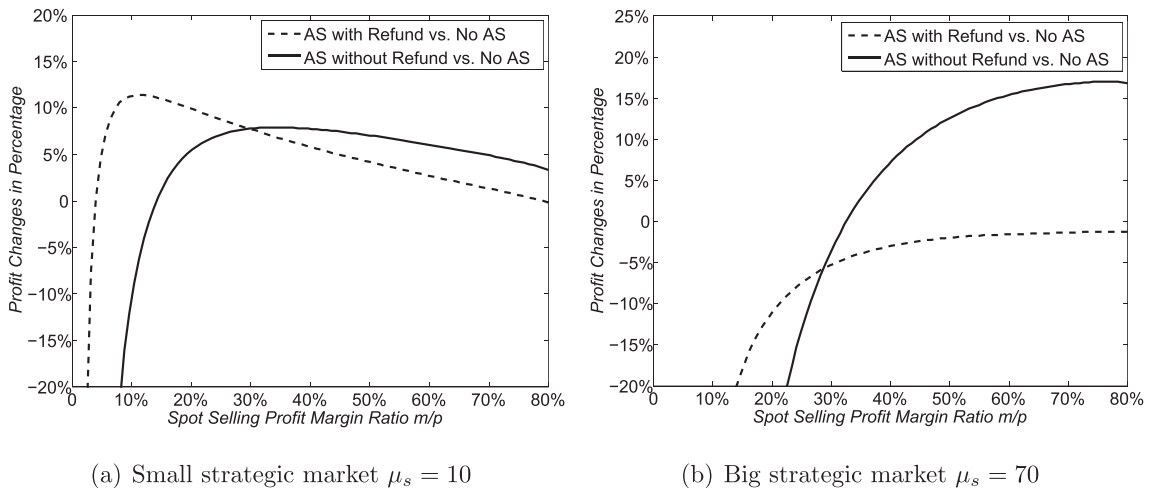
Fig. 10. Profit gap changes in profit margin and strategic market size.

(a) Big profit margin ratio $m/p = 40\%$ (b) Small profit margin ratio $m/p = 10\%$
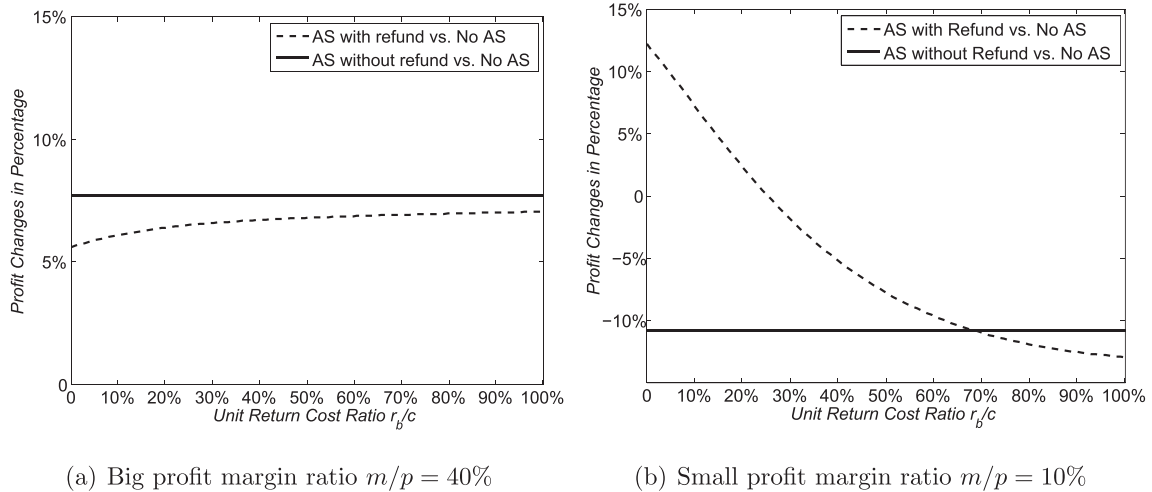
Fig. 11. Profit gap changes in return cost and profit margin.

the spot profit margin are increased by advance selling. Further, when offering refunds for advanced sales, the retailer needs to ensure that the spot profit margin is small enough to make strategic consumers buy in advance and not return. Therefore, the upper bounds of the spot profit margin under the advance selling with refund strategy are decreased. However, without offering a refund, the retailer only needs to ensure that the advance selling discount is large enough. This results in an increase in the spot profit margin. Therefore, the upper bound of the spot profit margin is increased under the advance selling without refund strategy.

8. Conclusion

Our study is the first to explore whether online retailers should offer a full refund option for advanced sales. Our contribution to the literature is two-fold. First, our results reveal when allowing a refund is profitable for retailers. Most advance selling literature (e.g., Xie & Shugan, 2001; Prasad et al., 2011) considers advance selling for its ability to exploit valuation uncertainty of consumers. From this perspective, allowing returns is not intuitive, as dissatisfied consumers can return these products. However, we show that allowing returns leads to a higher advance sales price and can thereby increase the profit margin from unreturned products. This is particularly important if the regular spot selling profit margin is not large to start with. We further show that, besides the profit margin, the market size of potential (informed strategic) consumers in the advance selling period also plays a crucial role in determining whether or not allowing a refund is optimal.

Second, our results contribute to the product returns literature from both operations and marketing perspectives. Most operations literature (Mostard & Teunter, 2006; Shulman et al., 2010; Ferguson et al., 2006; Su, 2009) considers the situation that the product return rate is exogenously given and/or all returned products are not resalable (but be salvaged or returned to the manufacturer). Our study characterizes the consumer's product return behavior and considers the case that returned products can be resold (after reprocessing). This leads to important insights on the profitability of offering a refund option for advance sales in relation to the reselling cost. As the reselling cost increases, allowing returns is less likely to be profitable. The marketing literature (Petersen & Kumar, 2009, 2010, 2015; Minnema et al., 2016) shows that allowing returns and using a

Table 2
Optimal price and maximal profit with a small strategic market size of $\mu_s = 10$.

cost c	No AS		AS without refund		AS with refund	
	Optimal price p_{SS}	Maximal profit π_{SS}	Optimal price p^{NR}	Maximal profit π_{AS}^{NR}	Optimal price p^R	Maximal profit π_{AS}^R
11	13.796	**10.777**	13.866	−43.438	14.121	9.086
10	12.975	**16.728**	13.050	−27.727	13.268	16.218
9	12.171	25.295	12.251	−9.576	12.421	**26.112**
8	11.384	37.308	11.470	11.754	11.570	**39.592**
7	10.615	53.750	10.708	37.121	10.718	**57.590**
6	9.864	75.767	9.966	67.501	9.868	**81.124**
5	9.131	104.694	9.242	104.002	9.027	**111.299**
4	8.412	142.134	8.535	147.912	8.213	**149.377**
3	7.704	190.160	7.841	**200.846**	7.435	197.027
2	6.993	251.991	7.150	**265.226**	6.737	257.020

Table 3
Optimal price and maximal profit with a big strategic market size of $\mu_s = 100$.

Cost	No AS		AS without refund		AS with refund	
	Optimal price	Maximal profit	Optimal price	Maximal profit	Optimal price	Maximal profit
c	p_{SS}	π_{SS}	p^{NR}	π_{AS}^{NR}	p^R	π_{AS}^R
11	13.606	**20.730**	15.245	−534.194	14.273	−24.271
10	12.787	**31.646**	14.454	−430.921	13.118	−7.148
9	11.985	**47.041**	13.679	−326.129	12.100	13.777
8	11.203	**68.173**	12.919	−219.294	11.175	40.171
7	10.440	**96.461**	12.175	−109.776	10.313	73.636
6	9.699	**133.461**	11.447	3.188	9.492	115.709
5	8.980	**180.874**	10.735	120.503	8.702	167.900
4	8.280	240.596	10.036	**243.257**	7.940	231.813
3	7.597	314.909	9.347	**372.830**	7.372	309.669
2	6.923	407.139	8.657	**511.243**	6.740	404.557

lenient return policy can have long-run benefits for retailers. Our study indicates that allowing returns is profitable even in a short term as it allows a reduction of the advance sales discount and an increase of the spot selling price, thereby increases the profit margin from both unreturned products and regular sales.

8.1. Managerial implications

Allowing returns for advanced sales can be effective for increasing the retailer's short-run profit. It allows a reduction of the advanced sales discount and an increase of the spot selling price. This improves the profitability of advanced sales especially when profit margins are slim (less than 30%). However, if the fraction of strategic (informed) customers is too large, then it is still better to not sell in advance at all, since otherwise too many products would be sold at a discount. Therefore, advance selling with a refund option works best for products with a small profit margin and a relatively small fraction of strategic consumers. In practice, it may be difficult to estimate how many potential consumers are informed. In such a situation, pre-selling a limited quantity is a good alternative which has been widely used by JD in China.

For a relatively high-margin product category, not offering a refund option for advanced sales maximizes short-term profit as there is relatively less to gain and more to lose through costly returns. Even if the cost for handling returns is low, offering a refund is no longer optimal for a sufficiently high profit margin. However, we remark that it is unclear whether no-refund policy is then also the best from a long-run perspective, since unsatisfied purchase experiences with return options may prevent future purchases.

Current strategies at Amazon and JD are the most compelling examples to support these implications. Amazon only presells digital products (video, digital games, softwares and kindle books), which have relatively high profit margins and low returns handling costs. Consistent with our findings, Amazon never allow returns after the release date. Different from Amazon, JD presells in almost all product categories (e.g., consumer electronics, computer, food, and clothing), including many low margin products. JD does allow returns of products purchased in advance, and in fact uses the same product return policy (7-day unconditional return) as for regular selling products. Moreover, JD limits the amount of products sold in advance. This is consistent with our results that allowing refunds for advance sales is indeed profitable for (mainly) low margin products, but only if not too many products are sold in advance at a discount.

8.2. Research limitations and future research

Some limitations of our work and corresponding future research opportunities should be noted. We omitted the availability risk. Availability risk could result in a decrease of consumer's expected utility for waiting. As a result, the advance selling price (e.g., p_A^{NR} and $p_A^R = \bar{p}_A$) could be lowered further. This implies that the spot selling only strategy becomes more profitable compared to the other strategies, i.e., the area where no advance selling is optimal could grow in Fig. 6. However, when offering a refund for a relatively high-profit margin product, the advance selling price is unaffected by the availability risk, i.e., $p_A^R = p_A^0$, where p_A^0 is determined by the marginal profit of advanced sales. Since the advance selling price under the no-refund strategy is lowered by the availability risk, offering a refund option for advance sales is more beneficial, i.e., the area where advance selling without refund is optimal could shrink in Fig. 6. For high-profit margin products, allowing refund for advanced sales may still be optimal if the availability risk is relatively high.

In line with previous research, our model assumes that all consumers are independent and have the same product valuation. In practice, consumers valuations are different (i.e., a part is insensitive to price) and may be dependent (i.e., through group buying). It is worthwhile to extend our model with heterogeneous and/or dependent consumers. Another future research avenue is to extend our analysis into a competitive environment. Although most companies choose to avoid head-to-head competition during the advance selling period in practice, it is interesting to study how competition in the spot selling period (or in both periods) affects the advance selling strategy as well as the refund strategy. Besides, it is interesting to consider a newsvendor with limited liquidity (capital constraint). Advance selling not only is a marketing strategy, but can also be a

financial strategy. A retailer can gain financial capital and improve his cash flow by selling in advance. As a result, a retailer can procure a sufficient amount of stock to buffer against variations for spot selling demand. However, selling in advance may increase the bankruptcy risk because of the advance selling discount and increased net demand uncertainty. Therefore, it is interesting to study this operations-finance interface problem, and investigate the impact of limited liquidity on the advance selling strategy.

Finally, to strengthen of the credibility of our results, it is valuable to test them empirically. Case studies at firms such as Amazon or JD can explore through interviews with marketing and operations managers how retailers decide whether or not to offer a refund for advanced sales. Lab experiments can verify whether decision makers act in line with the theoretical results and in particular whether they are more likely to opt for a refund on advanced sales for lower profit margins and a smaller strategic market size. Hypothesis based on our findings, such as "the size of the strategic market is negatively associated with the number of product returns", can be tested in a survey approach.

Acknowledgments

Appendix

Proof of Proposition 5.1. Let $\gamma(m) = mF(p) - \int_s^p F(x)dx$. It is clear that $\gamma(m)$ is increasing in m, $\gamma(0) \leq 0$ and $\gamma(p) \geq 0$. Therefore, there exist a critical

$$m^{NR} = \frac{\int_s^p F(x)dx}{F(p)} = \frac{p - p_A^{NR}}{F(p)},$$

such that $\gamma(m^{NR}) = 0$. If $m \geq m^{NR}$, then $\Delta\pi_{AS}^{NR} \geq 0$ always holds. Otherwise, $\Delta\pi_{AS}^{NR}$ is decreasing in μ_s and there exists a critical μ_s^{NR} such that $\Delta\pi_{AS}^{NR}(\mu_s^{NR}) = 0$. Note that the critical market size μ_s^{NR} is increasing in m and $\mu_s^{NR}(0) = 0$. If $\mu_s \leq \mu_s^{NR}(m)$, then $\Delta\pi_{AS}^{NR} \geq 0$; otherwise, $\Delta\pi_{AS}^{NR} \leq 0$. □

Proof of Proposition 6.1. (1) Recall that p_A^0 satisfies $\frac{d\pi_{AS}^R}{dp_A^R} = 0$. Using Eq. (11) and differentiating $\frac{d\pi_{AS}^R}{dp_A^R}$ with respect to c gives,

$\frac{d^2\pi_{AS}^R}{dp_A^R dc} = \mu_s f(p_A^0 - r_b) \geq 0$ which implies p_A^0 is increasing in c. Since $\bar{p}_A$ is independent from c and the profit margin m is decreasing in c, we have that p_A^R is increasing in c and decreasing in m.

(2) The proof is very similar to that of part (1) and omitted here.

(3) There exists a critical r_b^0 such that $p_A^0(r_b^0) = \bar{p}_A(r_b^0)$. If $r_b \geq r_b^0$ or $p_A^R = \bar{p}_A$, then differentiating both sides of Eq. (13) with respect to r_b gives $\frac{d\bar{p}_A}{dr_b} = -\frac{F(\bar{p}_A - r_b)}{\bar{F}(\bar{p}_A - r_b)} \leq 0$, which implies that the optimal advance selling price is decreasing in r_b.

If $r_b \leq r_b^0$ or $p_A^R = p_A^0$, then using Eq. (11) and differentiating $\frac{d\pi_{AS}^R}{dp_A^R}$ with respect to r_b gives

$$\frac{d^2\pi_{AS}^R}{dp_A^R dr_b} = \mu_s \left(f(p_A^0 - r_b) + (p_A^0 - c + r_c)f'(p_A^0 - r_b) \right).$$

Since the valuation function has an IFR, by the definition of IFR, we have that $f'(x)\bar{F}(x) + f^2(x) \geq 0$ or $f'(x) \geq -\frac{f^2(x)}{\bar{F}(x)}$ for all x in its domain. Therefore,

$$\frac{d^2\pi_{AS}^R}{dp_A^R dr_b} \geq \mu_s \left(f(p_A^0 - r_b) - (p_A^0 - c + r_c)\frac{f^2(p_A^0 - r_b)}{\bar{F}(p_A^0 - r_b)} \right)$$

$$= \frac{\mu_s f(p_A^0 - r_b)}{\bar{F}(p_A^0 - r_b)} \cdot \left(\bar{F}(p_A^0 - r_b) - (p_A^0 - c + r_c)f(p_A^0 - r_b) \right) = 0,$$

which implies that the optimal advance selling price is increasing in r_b. □

Proof of Theorem 1. Let $\Gamma(c) := (p_A^R - c)F(p) - (p - p_A^R)\bar{F}(p) - (p_A^R - c + r_c)F(p_A^R - r_b)$. Differentiating $\Delta\pi_{AS}^R$ with respect to μ_s gives

$$\frac{d\Delta\pi_{AS}^R}{d\mu_s} = \Gamma(c) - (p - s)\phi(k)\frac{dE_{D_A}(\sigma_{SN}|d_A)}{d\mu_s}.$$

Note that $\lim\limits_{\mu_s \to 0} \Delta\pi_{AS}^R = \lim\limits_{\mu_s \to 0}(p-s)\phi(k)\Delta\sigma_0^{RN}(\mu_s) \geq 0$. Therefore, if $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \geq 0$, then $\Delta\pi_{AS}^R \geq 0$ which implies the retailer always benefit from offering a refund option. If $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \leq 0$, then there exist a critical μ_s^R such that $\Delta\pi_{AS}^R(\mu_s^R) = 0$. If $\mu_s \leq \mu_s^R$, then $\Delta\pi_{AS}^R \geq 0$ which indicates that the retailer should choose a refund policy; otherwise, the retailer should choose spot selling only. Therefore, to prove this theorem, we need to verify the sign of $\frac{d\Delta\pi_{AS}^R}{d\mu_s}$.

Differentiating $\Gamma(c)$ with respect to c gives

$$\Gamma'(c) = F\left(p_A^R - r_b\right) - F(p) + \left(\bar{F}\left(p_A^R - r_b\right) - \left(p_A^R - c + r_c\right)f\left(p_A^R - r_b\right)\right)\frac{dp_A^R}{dc}.$$

Note that if $p_A^R = p_A^0$, i.e., $p_A^0 \leq \bar{p}_A$, then $\bar{F}\left(p_A^R - r_b\right) - \left(p_A^R - c + r_c\right)f\left(p_A^R - r_b\right) = 0$, which implies that $\Gamma'(c) = F\left(p_A^R - r_b\right) - F(p) \leq 0$. If $p_A^R = \bar{p}_A$, i.e., $p_A^0 \geq \bar{p}_A$, then we have $\bar{F}\left(p_A^R - r_b\right) - \left(p_A^R - c + r_c\right)f\left(p_A^R - r_b\right) \geq 0$ and $\frac{dp_A^R}{dc} = 0$. In summary, we have that

$$\Gamma'(c) = F\left(p_A^R - r_b\right) - F(p) \leq 0.$$

Further, we have that if $\Gamma(s) \leq 0$, then $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \leq 0$ holds for all c; otherwise, then there exist a critical cost c^0 such that $\Gamma(c^0) = 0$. If $c \geq c^0$, then $\Gamma(c) \leq 0$ which results $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \leq 0$, otherwise $\Gamma(c) \geq 0$, but the sign of $\frac{d\Delta\pi_{AS}^R}{d\mu_s}$ is uncertain.

Next, we verify the sign of $\frac{d\Delta\pi_{AS}^R}{d\mu_s}$ when $c \leq c^0$. Differentiating $\frac{d\Delta\pi_{AS}^R}{d\mu_s}$ with respect to c gives

$$\frac{d^2\Delta\pi_{AS}^R}{d\mu_s dc} = \Gamma'(c) + \frac{\phi'(k)}{\phi(k)}\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s}$$

and

$$\frac{d^3\Delta\pi_{AS}^R}{d\mu_s dc^2} = f\left(p_A^R - r_b\right)\frac{dp_A^R}{dc} - \frac{\phi''(k)\phi(k) - (\phi'(k))^2}{(p-s)\phi^3(k)}\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s}.$$

Since the PDF of the standard normal distribution is a concave function, i.e., $\phi''(k) \leq 0$, and p_A^R is increasing in c, we have that $\frac{d^3\Delta\pi_{AS}^R}{d\mu_s dc^2} \geq 0$ indicating that $\frac{d\Delta\pi_{AS}^R}{d\mu_s}$ is convex in c. Note that $\frac{d\Delta\pi_{AS}^R}{d\mu_s}|_{c=s} = \Gamma(s) \geq 0$ and $\frac{d\Delta\pi_{AS}^R}{d\mu_s}|_{c=c^0} \leq 0$. So, there exists a unique c^R such that $\frac{d^2\Delta\pi_{AS}^R}{d\mu_s dc}|_{c=c^R} = 0$. If $c \in [s, c^R]$, then $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \geq 0$. If $c \in [c^R, c^0]$, then $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \leq 0$.

In summary, when $\Gamma(s) \geq 0$, we have that if $c \leq c^R$ (or $m \geq m^R$), then $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \geq 0$, which implies $\Delta\pi_{AS}^R \geq 0$; if $c \geq c^R$ (or $m \leq m^R$), then $\frac{d\Delta\pi_{AS}^R}{d\mu_s} \leq 0$. Then, if $\mu_s \leq \mu_s^R$, $\Delta\pi_{AS}^R \geq 0$; otherwise $\Delta\pi_{AS}^R \leq 0$. When $\Gamma(s) \leq 0$, $\Delta\pi_{AS}^R \geq 0$ if $\mu_s \leq \mu_s^R$, otherwise $\Delta\pi_{AS}^R \leq 0$.

Proof of Theorem 2. Let $\tilde{\Gamma}(c) := \Gamma(c) - (p-c)F(p) + (p - p_A^{NR}) = (p_A^R - p_A^{NR}) - (p_A^R - c + r_c)F(p_A^R - r_b)$. Differentiating $\Delta\pi^R$ with respect to μ_s gives

$$\frac{d\Delta\pi^R}{d\mu_s} = \tilde{\Gamma}(c) - (p-s)\phi(k)\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s}.$$

Note that $\lim\limits_{\mu_s \to 0} \Delta\pi^R = 0$. Thus, we have that if $\frac{d\Delta\pi^R}{d\mu_s} \geq 0$, then $\Delta\pi^R \geq 0$ holds for all μ_s; otherwise $\Delta\pi^R \leq 0$.

Further, we have

$$\frac{d^2\Delta\pi^R}{d\mu_s dc} = F\left(p_A^R - r_b\right) + \frac{\phi'(k)}{\phi(k)}\cdot\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s} = F\left(p_A^R - r_b\right) - k\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s}$$

and

$$\frac{d^3\Delta\pi^R}{d\mu_s dc^2} = f\left(p_A^R - r_b\right)\frac{dp_A^R}{dc} + \frac{1}{(p-s)\phi(k)}\cdot\frac{dE_{D_A}(\sigma_{SN}|_{d_A})}{d\mu_s} \geq 0,$$

which indicates that $d\Delta\pi^R/d\mu_s$ is convex in c. Note that

$$\frac{d\Delta\pi^R}{d\mu_s}\bigg|_{c=s} = \tilde{\Gamma}(s) \quad \text{and} \quad \frac{d\Delta\pi^R}{d\mu_s}\bigg|_{c=p} = \tilde{\Gamma}(p) \geq \tilde{\Gamma}(s).$$

It follows from Proposition 5.1 and Theorem 1 that offering a refund option at a very high profit margin is not always optimal. Thus, we only consider the case that $\tilde{\Gamma}(s) < 0$. If $\tilde{\Gamma}(p) \leq 0$ then $\frac{d\Delta\pi^R}{d\mu_s} \leq 0$ always holds, which implies $\Delta\pi^R \leq 0$. If $\tilde{\Gamma}(s) \leq 0$ and $\tilde{\Gamma}(p) \geq 0$ then there exist a unique c^0 or $m^0 = p - c^0$ such that $\frac{d\Delta\pi^R}{d\mu_s}|_{c=c^0} = 0$ and $\frac{d^2\Delta\pi^R}{d\mu_s dc}|_{c=c^0} \geq 0$, where c^0 is a function of μ_s. So c^0 satisfies

$$\tilde{\Gamma}(c^0) = (p - s)\phi(k(c^0))\frac{dE_{D_A}(\sigma_{SN|d_A})}{d\mu_s}. \tag{15}$$

If $c \leq c^0$ or $m \geq m^0$ then $\frac{d\Delta\pi^R}{d\mu_s} \leq 0$, which implies $\Delta\pi^R \leq 0$. Otherwise $\frac{d\Delta\pi^R}{d\mu_s} \geq 0$, which implies $\Delta\pi^R \geq 0$.

Note that

$$\frac{dE_{D_A}(\sigma_{SN|d_A})}{d\mu_s} = \int_0^\infty \frac{x - \mu_s}{\sqrt{2\pi}\sigma_s^3} e^{-\frac{(x-\mu_s)^2}{2\sigma_s^2}} \cdot \sqrt{\sigma_m^2(1-\rho^2)\bar{F}^2(p) + x^2\sigma_\epsilon^2} dx \geq 0,$$

$$\frac{d^2E_{D_A}(\sigma_{SN|d_A})}{d\mu_s^2} = -\int_0^\infty \frac{(x - \mu_s)^2 + \sigma_s^2}{\sqrt{2\pi}\sigma_s^5} e^{-\frac{(x-\mu_s)^2}{2\sigma_s^2}} \cdot \sqrt{\sigma_m^2(1-\rho^2)\bar{F}^2(p) + x^2\sigma_\epsilon^2} dx \leq 0.$$

Differentiating both sides of Eq. (15) with respect to μ_s gives

$$\frac{d\tilde{\Gamma}(c^0)}{dc^0}\frac{dc^0}{d\mu_s} = (p - s)\left(\phi(k(c^0))\frac{d^2E_{D_A}(\sigma_{SN|d_A})}{d\mu_s^2} + \frac{d\phi(k(c^0))}{dc^0}\frac{dc^0}{d\mu_s}\frac{dE_{D_A}(\sigma_{SN|d_A})}{d\mu_s}\right),$$

which gives

$$\frac{dc^0}{d\mu_s} = \frac{(p - s)\phi(k(c^0))}{\frac{d^2\Delta\pi^R}{d\mu_s dc}|_{c=c^0}} \cdot \frac{d^2E_{D_A}(\sigma_{SN|d_A})}{d\mu_s^2} \geq 0.$$

Therefore $c^0(\mu_s)$ (or $m^0(\mu_s)$) is increasing (decreasing) in μ_s.

From Eq. (15), $c^0(\mu_s)$ also can be rewritten as a function μ_s with respect to c or m, i.e., $\mu_s^0(c)$ or $\mu_s^0(m)$ where $\mu_s^0(m)$ is decreasing in m and satisfies

$$\frac{dE_{D_A}(\sigma_{SN|d_A})}{d\mu_s}\bigg|_{\mu_s=\mu_s^0(m)} = \frac{(p - p_A^R(m) - m - r_c)F(p_A^R(m) - r_b) + p_A^R(m) - p_A^{NR}}{(p - s)\phi\left(\Phi^{-1}(\frac{m}{p-s})\right)}.$$

If $\mu_s \leq \mu_s^0(m)$, then the retailer should choose offer a refund option. Otherwise, the retailer should not do so.

References

Altug, M. S., & Aydinliyim, T. (2016). Counteracting strategic purchase deferrals: The impact of online retailers' return policy decisions. *Manufacturing & Service Operations Management, 18,* 376–392.

Anderson, E. T., Hansen, K., & Simester, D. (2009). The option value of returns: Theory and empirical evidence. *Marketing Science, 28,* 405–423.

Cachon, G. P., & Feldman, P. (2017). Is advance selling desirable with competition? *Marketing Science, 36,* 214–231.

Chen, J., & Bell, P. C. (2011). Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy. *European Journal of Operational Research, 212,* 293–300.

Cho, S.-H., & Tang, C. S. (2013). Advance selling in a supply chain under uncertain supply and demand. *Manufacturing & Service Operations Management, 15,* 305–319.

Davis, S., Hagerty, M., & Gerstner, E. (1998). Return policies and the optimal level of "hassle". *Journal of Economics and Business, 50,* 445–460.

Dekker, R., Fleischmann, M., Inderfurth, K., & van Wassenhove, L. N. (2004). *Reverse logistics: Quantitative models for closed-loop supply chains.* Springer, Berlin.

Fay, S., & Xie, J. (2010). The economics of buyer uncertainty: Advance selling vs. probabilistic selling. *Marketing Science, 29,* 1040–1057.

Ferguson, M., Guide, V. D. R., Jr, & Souza, G. C. (2006). Supply chain coordination for false failure returns. *Manufacturing & Service Operations Management, 8,* 376–393.

Govindan, K., Soleimani, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research, 240,* 603–626.

Guide, V. D. R., Jr, Souza, G. C., Van Wassenhove, L. N., & Blackburn, J. D. (2006). Time value of commercial product returns. *Management Science, 52,* 1200–1214.

Janakiraman, N., Syrdal, H. A., & Freling, R. (2016). The effect of return policy leniency on consumer purchase and return decisions: A meta-analytic review. *Journal of Retailing, 92,* 226–235.

Kerr, J. C. (2013). Serial returners, beware: Retailers are tracking you. NBC Today, August 12.

Lariviere, M. A. (2006). A note on probability distributions with increasing generalized failure rates. *Operations Research, 54,* 602–604.

Lawton, C. (2008). The war on returns. The Wall Steet Journal, May 8.

Lee, C. H., Choi, T.-M., & Cheng, T. (2015). Selling to strategic and loss-averse consumers: Stocking, procurement, and product design policies. *Naval Research Logistics, 62,* 435–453.

Li, C., & Zhang, F. (2013). Advance demand information, price discrimination, and preorder strategies. *Manufacturing & Service Operations Management, 15,* 57–71.

Lim, W. S., & Tang, C. S. (2013). Advance selling in the presence of speculators and forward-looking consumers. *Production and Operations Management, 22,* 571–587.

McCardle, K., Rajaram, K., & Tang, C. S. (2004). Advance booking discount programs under retail competition. *Management Science, 50,* 701–708.

Minnema, A., Bijmolt, T. H. A., Gensler, S., & Wiesel, T. (2016). To keep or not to keep: Effects of online customer reviews on product returns. *Journal of Retailing, 92,* 253–267.

Minnema, A., Bijmolt, T. H. A., Petersen, J. A., & Shulman, J. D. (2018). Managing product returns within the customer value framework. In R. W. Palmatier, V. Kumar, & C. M. Harmeling (Eds.), *Customer engagement marketing.* (pp. 95–118). Cham: Springer.

Mostard, J., de Koster, R., & Teunter, R. (2005). The distribution-free newsboy problem with resalable returns. *International Journal of Production Economics, 97,* 329–342.

Mostard, J., & Teunter, R. (2006). The newsboy problem with resalable returns: A single period model and case study. *European Journal of Operational Research, 169,* 81–96.

Nasiry, J., & Popescu, I. (2012). Advance selling when consumers regret. *Management Science, 58,* 1160–1177.

Petersen, J. A., & Anderson, E. T. (2015). Leveraging product returns to maximize customer equity. In V. Kumar, & D. Shah (Eds.), *Handbook of research on customer equity in marketing.* (pp. 160–177). Cheltenham: Edward Elgar.

Petersen, J. A., & Kumar, V. (2009). Are product returns a necessary evil? Antecedents and consequences. *Journal of Marketing, 73,* 35–51.

Petersen, J. A., & Kumar, V. (2010). Can product returns make you money? *MIT Sloan Management Review, 51,* 85–89.

Petersen, J. A., & Kumar, V. (2015). Perceived risk, product returns, and optimal resource allocation: Evidence from a field experiment. *Journal of Marketing Research, 52,* 268–285.

Pokharel, S., & Mutha, A. (2009). Perspectives in reverse logistics: A review. *Resources, Conservation and Recycling, 53,* 175–182.

Prasad, A., Stecke, K. E., & Zhao, X. (2011). Advance selling by a newsvendor retailer. *Production and Operations Management, 20,* 129–142.

Prasad, A., Venkatesh, R., & Mahajan, V. (2015). Product bundling or reserved product pricing? Price discrimination with myopic and strategic consumers. *International Journal of Research in Marketing, 32,* 1–8.

Shen, Z.-J. M., & Su, X. (2007). Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and Operations Management, 16,* 713–728.

Shugan, S. M., & Xie, J. (2005). Advance-selling as a competitive marketing tool. *International Journal of Research in Marketing, 22,* 351–373.

Shulman, J. D., Coughlan, A. T., & Savaskan, R. C. (2009). Optimal restocking fees and information provision in an integrated demand-supply model of product returns. *Manufacturing & Service Operations Management, 11,* 577–594.

Shulman, J. D., Coughlan, A. T., & Savaskan, R. C. (2010). Optimal reverse channel structure for consumer product returns. *Marketing Science, 29,* 1071–1085.

Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling.* John Wiley & Sons, Inc., New York.

Su, X. (2009). Consumer returns policies and supply chain performance. *Manufacturing & Service Operations Management, 11,* 595–612.

Tang, C. S., Rajaram, K., Alptekinoglu, A., & Ou, J. (2004). The benefits of advance booking discount programs: Model and analysis. *Management Science, 50,* 465–478.

Wu, M., Zhu, S. X., & Teunter, R. H. (2017). Advance selling and advertising: A newsvendor framework. *Working paper.* Groningen: University of Groningen.

Xiao, T., Shi, K., & Yang, D. (2010). Coordination of a supply chain with consumer return under demand uncertainty. *International Journal of Production Economics, 124,* 171–180.

Xie, J., & Shugan, S. M. (2001). Electronic tickets, smart cards, and online prepayments: When and how to advance sell. *Marketing Science, 20,* 219–243.

Yu, M., Kapuscinski, R., & Ahn, H.-S. (2015). Advance selling: Effects of interdependent consumer valuations and seller's capacity. *Management Science, 61,* 2100–2117.

Zhao, X., Pang, Z., & Stecke, K. E. (2016). When does a retailer's advance selling capability benefit manufacturer, retailer, or both? *Production and Operations Management, 25,* 1073–1087.

Zhao, X., & Stecke, K. E. (2010). Pre-orders for new to-be-released products considering consumer loss aversion. *Production and Operations Management, 19,* 198–215.