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Contents lists available at ScienceDirect

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A loss averse competitive newsvendor problem with anchoring

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ARTICLE INFO

ABSTRACT

We study a loss averse competitive newsvendor problem with anchoring under prospect theory. We consider two demand-splitting rules for quantity competition, including proportional demand allocation and demand reallocation. We characterize the optimal order quantity decisions under both demand rules. We find that the newsvendor's order quantity is decreasing with the degree of loss aversion and the value of the anchor. Compared with an integrated risk-neutral supply chain, a positive anchor always leads to inventory understocking, whereas a negative anchor may result in a serious overstocking. Under competition with homogeneous newsvendors, competition always makes newsvendors order more, which does not necessarily lead to a loss of profit. For newsvendors with a high anchor, competition helps to prevent understocking caused by the anchoring effect, which leads to an increase in profit. For newsvendors with a low anchor, competition exacerbates overstocking, which results in a loss of profit. Under competition with heterogeneous newsvendors, a newsvendor with a higher degree of loss aversion or with a higher anchor adopts a more conservative strategy (i.e. choose a lower order quantity), which results in a smaller market share.

1. Introduction

The newsvendor problem is a classic model in stochastic inventory management that has been widely used and analysed in operations management since the work of [1]. We refer interested readers to [2] and [3] for a detailed review of the newsvendor problem and its extensions.

In recent years, empirical investigations have shown that actual orders often deviate from the optimal order quantity of the risk-neutral newsvendor. Schweitzer & Cachon [4] explain the over-ordering/under-ordering pattern by relying on the risk attitudes towards gains and losses. They point out that prospect theory can explain the ordering bias because it shows that newsvendors are risk averse (seeking) when facing gains (loss) and, thus, should always under order (over order). According to the expected utility theory, individuals only care about absolute wealth, rather than relative wealth in any given situation. In contrast, prospect theory states that people are more sensitive to changes to an anchor (reference point) than they are to absolute changes, where the notion of an anchor was first introduced by Slovic [5]. According to Tver-sky & Kahneman [6], the anchoring effect, as a cognitive bias, is the disproportionate influence on decision-makers to make judgements that are biased towards an initially presented value. Many studies have illustrated that the anchoring effect is prevalent in human decision-making in a variety of fields. We refer interested readers to Furnham & Boo [7] for detailed review of the anchoring effect.

Recently, Nagarajan & Shechter [8] confirmed that the newsvendor always under orders in the low-profit case and over orders in the high-profit case, under a certain prospect theory utility, which contradicts existing experimental results. Therefore, they claim that prospect theory cannot explain the ordering bias. However, Zhao & Geng [9] point out that the reason why prospect theory cannot explain this bias is that the utility function in Nagarajan & Shechter [8] misses a key feature, namely, an anchor. If an anchor is appropriately determined, then prospect theory can be used to explain the ordering bias. Furthermore, Ren & Croson [10] and Ren et al. [11] use experiments to show that the decision bias may be caused by overconfidence/over-precision when estimating the demand risk (with an inappropriate anchor). They also find that if the anchoring effect is considered, then prospect theory can explain the behavioural deviations in the newsvendor problem, without relying on risk preferences. However, most existing studies on the loss averse newsvendor problem under prospect theory are

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based on a zero anchor (zero payoff) and, thus, ignore the anchoring effect. How an anchor affects the optimal order quantity of a loss averse newsvendor, and whether such a newsvendor with an anchor benefits the supply chain remain unclear.

To fill these research gaps, we study the loss averse competitive newsvendor problem with anchoring under a piecewise loss averse utility function. Here, we use a target unit profit as an anchor, and consider quantity competition under both the demand-reallocation rule and the proportional demand-allocation rule. We prove that there exists a unique Nash equilibrium under both demand-splitting rules. Our results show that both loss aversion and anchoring decrease the newsvendor's order quantity. In particular, compared with an integrated risk-neutral supply chain, a positive anchor always leads to understocking for a loss averse monopoly newsvendor, while a negative anchor may lead to overstocking. For loss averse competitive newsvendors, a relatively high anchor always leads to inventory understocking. Both competition and a lower anchor can help counter this effect by having the newsvendor order more stock, which leads to an increase in profit and benefits the supply chain. However, as newsvendors lower their anchors furthermore, a relatively low anchor may lead to inventory overstocking, which results in a (significant) loss of profit. Since the anchor determines whether an outcome is perceived as a loss or a gain, our results stress that anchoring dominates loss aversion in reducing order quantities. Furthermore, we show that demand-splitting rules can affect the profits of competitive newsvendors. The profit gain in the demand reallocation is higher than that in the proportional demand allocation because only part of the demand is reallocated in the former case. Moreover, for heterogeneous newsvendors, a newsvendor that is more loss averse or that has a higher anchor is more conservative when ordering, which results in a smaller market share.

In summary, the contributions of this study to the existing literature on loss averse newsvendor models are threefold. First, although some studies (e.g. [12,13]) have considered the loss averse competitive newsvendor problem under a loss averse utility with a zero anchor, they ignore the anchoring effect, which may lead to incomplete and less rigorous conclusions. Therefore, we consider the loss averse competitive newsvendor problem with the anchoring effect, which has not yet been studied. Second, the related literature (e.g. [12]) has only explored quantity competition under a certain demand-splitting rule, namely, proportional demand allocation. How different splitting rules (proportional demand allocation vs demand reallocation) influence the equilibrium order quantity and the total profits of newsvendors has not yet been studied. Third, the prior studies on the loss averse competitive newsvendor problem assume that newsvendors are homogeneous (e.g. [12]). Therefore, the effect of heterogeneity on this problem is still unknown.

The remainder of this paper is organized as follows. In the next section, we review the literature on the loss averse newsvendor problem with anchoring, as well as the loss/risk-averse competitive newsvendor problems. Section 3 introduces the proposed model. Section 4 considers the competitive model under both the demand-reallocation rule and the proportional demand-allocation rule. Section 5 discusses the effect of loss aversion, anchoring, and competition, and presents our numerical results. Section 6 discusses the heterogeneous competitive newsvendor problem by means of numerical examples. Finally, Section 7 concludes the paper. All proofs are available in the appendix.

2. Literature review

We survey existing studies on the newsvendor problem, which follow two streams of research: anchoring and competition.

2.1. The loss averse newsvendor problem with anchoring

Based on the prospect theory established by Kahneman & Tversky [14], the loss averse newsvendor problem has attracted much attention in recent years. Schweitzer & Cachon [4] were the first to study this problem under prospect theory and to use an experiment to verify their results. Wang & Webster [15] study the newsvendor problem under loss averse utility with a zero anchor. Wang [12] and Liu et al. [13] extend their work to game settings under the proportional demand-allocation rule and by including production substitution, respectively. Ma et al. [16] study a loss averse newsvendor problem with uncertain supply under the same utility criterion used by Wang & Webster [15]. Using the same loss averse utility with a zero anchor, these studies all show that loss aversion always leads to a decrease in order quantity.

As a zero anchor is a special case in prospect theory, and the anchoring effect as a cognitive bias may significantly affect people's decisions, some researchers study how an exogenous (nonzero) anchor affects the order quantity of a loss averse newsvendor. Herweg [17] points out that a newsvendor's order quantity depends heavily on the selected anchors. He shows that loss averse newsvendor with a given exogenous anchor always orders less than the risk-neutral newsvendor does. If the value of the anchor is extremely high or low, then loss aversion plays no role. Long & Nasiry [18] study a similar loss averse newsvendor problem with a nonzero anchor. They also find that, for certain anchors, the anchoring effect can explain the newsvendor's ordering behaviour, without needing to incorporate the newsvendor's attitude to risk or loss. Furthermore, they show that a newsvendor with a sufficiently low anchor may order more stock.

2.2. The competitive newsvendor problem

The competitive newsvendor problem has been studied under different risk criteria. For risk-neutral newsvendors, Parlar [19] first studies the risk-neutral newsvendor problem under quantity competition, in which two substitutable products are sold to two identical newsvendors. [20] study a competitive newsvendor problem with a single product, in which random demand is allocated among competing newsvendors with certain demand-splitting rules. Cachon [21] considers the same problem with a proportional demand-allocation rule; that is, the supplier allocates demand among the newsvendors in proportion to their orders. These studies all find that quantity competition always leads to overstocking and a loss of profit. For risk-averse newsvendor, Wu et al. [22] investigate the risk-averse newsvendor problem with quantity competition and price competition under the CVaR criterion. By considering both the proportional demand-allocation rule and the demand-reallocation rule, they show that quantity competition does not necessarily lead to a loss of profit in certain competitive environments when newsvendors are risk averse. For loss averse newsvendors, based on prospect theory and under the proportional demand-allocation rule, Wang [12] extends the classic competitive newsvendor problem to a game setting in which newsvendors are loss averse. Using the same utility criterion as in [12] and under the demand-reallocation rule, Liu et al. [13] study the loss averse competitive newsvendor problem with production substitution. Both studies show that loss aversion always leads to a decrease in total order quantities of all newsvendors, and may lead to supply chain understocking.

To the best of our knowledge, existing studies on the anchoring effect are based on the loss averse newsvendor problem in which there is only a monopoly vendor in the market. Furthermore, studies on loss averse competitive newsvendor problems are based on a certain demand-splitting rule. Some interesting and unexplored questions are as follows. *How does the joint effect of loss aversion,*

Table 1
Model notation.

D_T	Total demand with CDF $F_T(\cdot)$ and PDF $f_T(\cdot)$;
D_i	Newsvendor i's (initial) demand with CDF $F_i(\cdot)$ and PDF $f_i(\cdot)$;
R_i	Newsvendor i's realized demand after demand reallocation;
p_i	Newsvendor i's selling price per unit;
c_i	Newsvendor i's purchasing cost per unit, $c_i < p_i$;
s_i	newsvendor i's salvage value per unit, $0 < s_i < c_i$;
w_0^i	Newsvendor i's target profit per unit (anchor);
λ_i	Newsvendor i's loss-aversion degree, i.e. $\lambda_i \geq 1$;
Q_i	Newsvendor i's order quantity;
Q_{LC}^{*}	Equilibrium order quantity of two identical loss averse competitive newsvendors;
Q_C^{*}	Equilibrium order quantity of two identical risk-neutral competitive newsvendors;
Q_0^{*}	Optimal order quantity of the classic newsvendor;
Q_L^{*}	Optimal order quantity of a loss averse monopoly newsvendor;
$\pi(Q_i, Q_j)$	Newsvendor i's profit for a given Q_j;
$U(\pi(Q_i, Q_j))$	Newsvendor i's utility for a given Q_j;
$D_0(Q_i)$	Newsvendor i's break-even demand (such that he has zero utility).

anchoring, and competition influence newsvendors' total order quantity? Compared with the non-competitive case, does quantity competition still hurt loss averse newsvendors? What happens in different competitive environments with the anchoring effect? These questions form the main focus of this research. Here, we consider the anchoring effect and competition under different demand-splitting rules to gain insights into the effects of both anchoring and competition on loss averse newsvendors' optimal strategies and profit performance.

3. Model description

We consider two competing newsvendors who sell a perishable product to the same market. Newsvendor i faces a random demand D_i with a cumulative distribution function (CDF) $F_i(\cdot)$ and a probability density function (PDF) $f_i(\cdot)$. Let $f_T(\cdot)$ and $F_T(\cdot)$ be the PDF and CDF of the total demand D_T of both newsvendors. Without loss of generality, we assume that $F(0) = F_i(0) = 0$. Throughout this paper, we drop the subscript for identical newsvendors, and suppose that all information on each newsvendor's demand distribution and cost structure are common knowledge. The notation used in the paper is given in Table 1.

We consider a loss averse competitive newsvendor problem by employing a loss averse utility function. This approach was first proposed by [14], who use experiments to show that the decision-maker is risk averse in choices involving sure gains, and is risk-seeking in choices involving sure losses. In addition, they show that there is a greater impact of losses than gains, given the same variation. More specifically, we focus on a piecewise-linear form of the loss averse utility function, defined as follows:

$$U(W) = \begin{cases} W - W_0, & \text{if } W \geq W_0, \\ \lambda(W - W_0), & \text{if } W < W_0, \end{cases} \tag{1}$$

where W is a realized profit, W_0 is an anchor that makes the decision-maker change his attitude to loss, and $\lambda > 1$ is a loss-aversion coefficient. A higher value of λ implies a higher degree of loss aversion. When $\lambda = 1$, the loss averse utility function (1) reduces to a risk-neutral utility function. Furthermore, λ can be thought of as the coefficient of the penalty for failing to reach the anchor. This piecewise-linear form of the loss averse utility function (1) is a special case of a prospect theory function, and has been widely used in economics and operation management literature (e.g. see [15,18]). Note that the determination of the anchor in prospect theory is an important research topic (e.g. [23]).

From the classic newsvendor problem, the profit of a newsvendor (say i) is given by

$$\pi(Q_i) = \begin{cases} \pi_-(Q_i) = (p-s)D_i - (c-s)Q_i, & \text{if } D_i \leq Q_i; \\ \pi_+(Q_i) = (p-c)Q_i, & \text{if } D_i > Q_i. \end{cases}$$

Note that newsvendor's profit depends heavily on the order quantity. Although there is no effective mechanism for setting an anchor in prospect theory, it may be more realistic for a manager to set an anchor as a target unit profit, rather than a target gross profit. For example, for new entrants to enter a competitive market, the average profit per unit sets the standard for the rest of the decision-making. In this sense, we introduce a target unit profit or aspiration level of unit profit $w_0 := W_0/Q$ as an anchor for further analysis[18]. also assume that the reference point is a function of the order quantity.

Under the loss averse criterion with a piece-wise linear utility function (1), newsvendor i's utility is given by

$$U(\pi(Q_i)) = \begin{cases} \pi(Q_i) - w_0^i Q_i, & \text{if } w_0^i \leq \dfrac{\pi(Q_i)}{Q_i}, \\ \lambda_i\big(\pi(Q_i) - w_0^i Q_i\big), & \text{if } w_0^i > \dfrac{\pi(Q_i)}{Q_i}, \end{cases}$$

where $w_0^i \in [-(c-s), p-c]$. Furthermore, newsvendor i's expected utility is given by

$$E[U(\pi(Q_i)] = E[\pi(Q_i) - w_0^i Q_i] + (\lambda_i - 1) \\ \times \int_0^{D_0(Q_i)} [\pi_-(Q_i) - w_0^i Q_i] dF_i(x),$$

where $D_0(Q_i)$ is newsvendor i's break-even demand when he orders Q_i items. The aim of a loss averse decision-maker is to maximize his expected utility (i.e. $\max_{Q_i} E[U(\pi(Q_i))]$).

4. Equilibrium and quantity competition

In this section, we consider quantity competition with two loss averse competing newsvendors, where competition occurs by allocating the initial/excess demand among the newsvendors. We derive the equilibrium order quantities under the demand-reallocation rule and the proportional demand-allocation rule.

4.1. Demand-reallocation model

For demand reallocation, there are two aspects of demand: the initial allocation and reallocation. First, the initial demand allocation does not depend upon the inventory levels chosen by the

newsvendors. However, the rule by which demand is initially allocated can be quite complicated. A general initial allocation rule is the randomized splitting rule. Here, the initial demand D_i and D_j are independent random variables, satisfying $D_T = D_i + D_j$, where D_T is total demand. Second, if applicable, a portion of the excess demand is reallocated to other newsvendors. Demand reallocation is considered to be the most general demand allocation scheme for the competitive newsvendors (e.g. [20]).

In this model, the total demand is initially allocated to two newsvendors via randomized splitting rules. If there is unsupplied demand at newsvendor j (i.e. $D_j > Q_j$), then some portion of unsupplied consumers attempt to make purchases at newsvendor i. This transfer of excess demand represents the reallocation. Let R_i denote the realized or effective demand at newsvendor i, including its reallocation, so that

$$R_i = D_i + \alpha_i (D_j - Q_j)^+,$$

where $\alpha_i \in [0, 1]$ is the proportion of j's excess demand allocated to newsvendor i, and α_i reflects the intensity of quantity competition, because it measures the substitutability between the two competing newsvendors if one of them is out of stock.

Under the loss averse criterion, for a given Q_j, newsvendor i's utility $U(\pi(Q_i, Q_j))$ is

$$U(\pi(Q_i, Q_j)) = \begin{cases} \pi(Q_i, Q_j) - w_0^i Q_i, & \text{if } w_0^i \leq \dfrac{\pi(Q_i, Q_j)}{Q_i}, \\ \lambda_i \left(\pi(Q_i, Q_j) - w_0^i Q_i \right), & \text{if } w_0^i > \dfrac{\pi(Q_i, Q_j)}{Q_i}. \end{cases}$$

The break-even demand $D_0^r(Q_i) = \frac{c_i - s_i + w_0^i}{p_i - s_i} Q_i$ is such that $\pi_-(Q_i, Q_j) = w_0^i Q_i$. Thus, newsvendor i's expected utility $E(U(\pi(Q_i, Q_j)))$ is given by

$$E[U(\pi(Q_i, Q_j))] = (p_i - c_i - w_0^i) Q_i$$
$$- (p_i - s_i) \left\{ \int_0^{Q_j} \int_0^{Q_i} (Q_i - x) dF_i(x) F_j(y) \right.$$
$$+ \int_{Q_j}^{Q_j + \frac{Q_i}{\alpha_i}} \int_0^{Q_i - \alpha_i (y - Q_j)}$$
$$\times (Q_i - x - \alpha_i (y - Q_j)) dF_i(x) F_j(y) \Big\}$$
$$- (\lambda_i - 1)(p_i - s_i) \left\{ \int_0^{Q_j} \int_0^{D_0^r(Q_i)} \right.$$
$$\times (D_0^r(Q_i) - x) dF_i(x) dF_j(y)$$
$$+ \int_{Q_j}^{Q_j + \frac{D_0^r(Q_i)}{\alpha_i}} \int_0^{D_0^r(Q_i) - \alpha_i (y - Q_j)}$$
$$\times (D_0^r(Q_i) - x - \alpha_i (y - Q_j)) dF_i(x) dF_j(y) \Big\}.$$

Under the loss averse criterion, Proposition 4.1 characterizes newsvendor i's best responses to the other newsvendor's order quantities.

Proposition 4.1. *Under the demand-reallocation rule, there exists a unique optimal order quantity $Q_i^*(Q_j)$ for a loss averse newsvendor that satisfies the following first order condition:*

$$\frac{p_i - c_i - w_0^i}{p_i - s_i} = F_i(Q_i^*(Q_j)) F_j(Q_j)$$
$$+ \int_{Q_j}^{Q_j + \frac{Q_i^*(Q_j)}{\alpha_i}} F_i(Q_i^*(Q_j) - \alpha_i (y - Q_j)) dF_j(y)$$

$$+ \frac{(\lambda_i - 1)(c_i - s_i + w_0^i)}{p_i - s_i} \left\{ F_i\left(D_0^r(Q_i^*(Q_j))\right) F_j(Q_j) \right.$$
$$+ \int_{Q_j}^{Q_j + \frac{D_0^r(Q_i^*(Q_j))}{\alpha_i}} F_i(D_0^r(Q_i^*(Q_j)) - \alpha_i (y - Q_j)) dF_j(y) \Big\},$$

where $D_0^r(Q_i^*(Q_j)) = \frac{c_i - s_i + w_0^i}{p_i - s_i} Q_i^*(Q_j)$.

Proof. See Appendix A. □

Next, we consider a special case with two identical loss averse newsvendors in order to obtain closed-form results for the equilibrium order quantities.

Theorem 1. *Under the demand-reallocation rule and with identical loss averse newsvendors, there exists a unique equilibrium order quantity Q_{LC}^* that satisfies the following equation:*

$$\frac{p - c - w_0}{p - s} = F^2(Q_{LC}^*) + \int_{Q_{LC}^*}^{Q_{LC}^*(1 + \frac{1}{\alpha})} F\left(Q_{LC}^* - \alpha(y - Q_{LC}^*)\right) dF(y)$$
$$+ \frac{(\lambda - 1)(c - s + w_0)}{p - s} \left\{ F(Q_{LC}^*) F(D_0^r(Q_{LC}^*)) \right.$$
$$+ \int_{Q_{LC}^*}^{Q_{LC}^* + \frac{D_0^r(Q_{LC}^*)}{\alpha}} F\left(D_0^r(Q_{LC}^*) - \alpha(y - Q_{LC}^*)\right) dF(y) \Big\}, \quad (2)$$

where $D_0^r(Q_{LC}^*) = \frac{c - s + w_0}{p - s} Q_{LC}^*$.

Proof. See Appendix B. □

4.2. Proportional demand-allocation model

In contrast to the demand-reallocation model, in this model, competition occurs only in initial demand by allocating demand among the newsvendors in proportion to their inventory. More specifically, the total demand D_T is assumed to be divided among the newsvendors in proportion to their order quantities:

$$D_i = \frac{Q_i}{Q_i + Q_j} D_T \text{ and } D_j = \frac{Q_j}{Q_i + Q_j} D_T.$$

This rule is known as the proportional demand-allocation rule. Compared with the demand-reallocation rule, there is no reallocation under this rule (i.e. $R_i = D_i$), and the initial allocation is split by the newsvendors' market share of the total demand. As pointed out by [21], the proportional demand-allocation rule is a reasonable model when customers have a relatively low search cost (e.g. online shopping).

Under this rule, newsvendor i's profit is

$$\pi(Q_i, Q_j)$$
$$= \begin{cases} \pi_-(Q_i, Q_j) = (p_i - s_i) \frac{Q_i}{Q_i + Q_j} D_T - (c_i - s_i) Q_i, & \text{if } D_T \leq Q_i + Q_j, \\ \pi_+(Q_i, Q_j) = (p_i - c_i) Q_i, & \text{if } D_T > Q_i + Q_j. \end{cases}$$

Under the loss averse criterion, newsvendor i's expected utility $E(U(\pi(Q_i, Q_j)))$ is

$$E[U(\pi(Q_i, Q_j))] = E[\pi(Q_i, Q_j) - w_0^i Q_i]$$
$$+ (\lambda_i - 1) \int_0^{D_0^p(Q_i)} [\pi_-(Q_i, Q_j) - w_0^i Q_i] dF_T(x)$$
$$= (p_i - c_i - w_0^i) Q_i - (p_i - s_i) \frac{Q_i}{Q_i + Q_j} \left\{ \int_0^{Q_i + Q_j} \right.$$
$$\times F_T(x) dx + (\lambda_i - 1) \int_0^{D_0^p(Q_i)} F_T(x) dx \Big\},$$

where the break-even demand $D_0^p(Q_i) = \frac{c_i - s_i + w_0^i}{p_i - s_i} (Q_i + Q_j)$ is such that $\pi_-(Q_i, Q_j) = w_0^i Q_i$.

Under the loss averse criterion, Proposition 4.2 characterizes newsvendor i's best responses to the other newsvendor's order quantity.

Proposition 4.2. *Under the proportional demand-allocation rule, there exists a unique optimal order quantity $Q_i^*(Q_j)$ for a loss averse newsvendor that satisfies the following first-order condition:*

$$\frac{p_i - c_i - w_0^i}{p_i - s_i} = \left\{ F_T(Q_i^*(Q_j) + Q_j) \right.$$
$$- \frac{Q_j}{(Q_i^*(Q_j) + Q_j)^2} \int_0^{Q_i^*(Q_j)+Q_j} x dF_T(x) \right\} + (\lambda_i - 1)$$
$$\times \left\{ \frac{c_i - s_i + w_0^i}{p_i - s_i} F_T(D_0^p(Q_i^*(Q_j))) \right.$$
$$\left. - \frac{Q_j}{(Q_i^*(Q_j) + Q_j)^2} \int_0^{D_0^p(Q_i^*(Q_j))} x dF_T(x) \right\},$$

where $D_0^p(Q_i^*(Q_j)) = \frac{c_i - s_i + w_0^i}{p_i - s_i}(Q_i^*(Q_j) + Q_j)$.

Proof. See Appendix C. □

Next, we consider a special case with two identical loss averse newsvendors in order to obtain closed-form results for the equilibrium order quantities.

Theorem 2. *Under the proportional demand-allocation rule and with identical loss averse newsvendors, there exists a unique equilibrium order quantity Q_{LC}^* that satisfies*

$$\frac{p - c - w_0}{p - s} = \left\{ F_T(2Q_{LC}^*) - \frac{1}{4Q_{LC}^*} \int_0^{2Q_{LC}^*} x dF_T(x) \right\}$$
$$+ (\lambda - 1) \left\{ \frac{c - s + w_0}{p - s} F_T(D_0^p(Q_{LC}^*)) \right.$$
$$\left. - \frac{1}{4Q_{LC}^*} \int_0^{D_0^p(Q_{LC}^*)} x dF_T(x) \right\}, \qquad (3)$$

where $D_0^p(Q_{LC}^*) = \frac{2(c - s + w_0)}{p - s} Q_{LC}^*$.

Proof. See Appendix D. □

5. Discussion

In this section, based on derived equilibrium order quantities for competitive loss averse newsvendors, we discuss the loss-aversion effect and the anchoring effect on order quantities. We also examine how competition affects the newsvendors' equilibrium ordering quantities in two different competitive settings.

For the effects of loss aversion and anchoring, we have the following theorem.

Theorem 3. *Consider two identical loss averse newsvendors. Under both the proportional demand-allocation and demand-reallocation rules, the following results hold:*

(a) *Loss-Aversion Effect: for any given anchor w_0, the order quantity Q_{LC}^* decreases with the degree of loss aversion λ;*
(b) *Anchoring Effect: for any given loss aversion of degree λ, the order quantity Q_{LC}^* decreases with the value of anchor w_0.*

Proof. See Appendix E. □

Remark 1. If $w_0 = 0$, then Theorem 3(a) is consistent with the conclusion of [12]. Furthermore, our results generalize his results to the nonzero anchor situation. We show that loss aversion and anchoring have similar effects on order quantities; that is, a higher (lower) anchor leads to a lower (higher) order quantity.

Theorem 3 shows that loss aversion and anchoring lead to a decrease in the order quantities of competitive newsvendors. This can

be explained by the fact that loss averse newsvendors are more sensitive to losses than they are to gains. They may receive a low utility by ordering less, but they suffer from utility loss by ordering more. The anchor determines whether an outcome is perceived as a loss or a gain. As the value of an anchor increases, an outcome is more likely to be perceived as a loss. Therefore, based on a similar explanation to that of loss aversion, the order quantity decreases as the anchor increases.

According to traditional wisdom, a negative effect of quantity competition is inventory overstocking, which may lead to a loss of profit. However, Theorem 3 shows that loss aversion and anchoring lead to a decrease in the order quantities of competitive newsvendors. Therefore, they can mitigate the consequence of overstocking caused by quantity competition. To further discuss the joint effect of loss aversion, anchoring, and competition, we next compare the total optimal order quantity and the corresponding optimal profit of loss averse competitive newsvendors with that of an integrated supply chain (a monopoly newsvendor).

For a loss averse monopoly newsvendor with anchoring, the optimal order quantity satisfies

$$\frac{p - c - w_0}{p - s} = F_T(Q_L^*) + (\lambda - 1) \frac{c - s + w_0}{p - s} F_T(\frac{c - s + w_0}{p - s} Q_L^*).$$

Note that if $\lambda = 1$ and $w_0 = 0$, then the loss averse newsvendor problem reduces to the classic risk-neutral newsvendor problem. The optimal order quantity of an integrated risk-neutral supply chain (risk-neutral newsvendor) Q_0^* is equal to $F_T^{-1}(\frac{p-c}{p-s})$.

To facilitate our analysis, we denote

$$\lambda_p(w_0) = 1 + \frac{\int_0^{Q_0^*} x dF_T(x) - \frac{2w_0 Q_0^*}{p - s}}{2D_0^p(\frac{1}{2}Q_0^*) F_T(D_0^p(\frac{1}{2}Q_0^*)) - \int_0^{D_0^p(\frac{1}{2}Q_0^*)} x dF_T(x)} \qquad (4)$$

and

$$w_p = \frac{p - s}{2Q_0^*} \int_0^{Q_0^*} x dF_T(x) \qquad (5)$$

as the threshold of the loss aversion degree and the threshold of the anchor, respectively, under the proportional demand-allocation rule. Furthermore, we denote

$$\lambda_r(w_0) = 1$$
$$+ \frac{\frac{p - c - w_0}{p - s} - \left(F(\frac{Q_0^*}{2})^2 + \int_{\frac{Q_0^*}{2}}^{\frac{Q_0^*}{2}(1 + \frac{1}{\alpha})} F(\frac{Q_0^*}{2} - \alpha(y - \frac{Q_0^*}{2})) dF(y) \right)}{\frac{c - s + w_0}{p - s} \left(F(\frac{Q_0^*}{2}) F(D_0^r(\frac{Q_0^*}{2})) + \int_{\frac{1}{2}Q_0^*}^{\frac{1}{2}Q_0^* + \frac{D_0^r(\frac{1}{2}Q_0^*)}{\alpha}} F(D_0^r(\frac{Q_0^*}{2}) - \alpha(y - \frac{Q_0^*}{2})) dF(y) \right)}$$
$$(6)$$

and

$$w_r = p - c - (p - s) \left(F\left(\frac{Q_0^*}{2}\right)^2 \right.$$
$$\left. + \int_{\frac{Q_0^*}{2}}^{\frac{Q_0^*}{2}(1 + \frac{1}{\alpha})} F(\frac{Q_0^*}{2} - \alpha(y - \frac{Q_0^*}{2})) dF(y) \right) \qquad (7)$$

as the threshold of the loss aversion degree and the threshold of the anchor, respectively, under the demand-reallocation rule. Compared with an integrated risk-neutral supply chain, we obtain the following result for the joint effect of loss aversion, anchoring, and competition on newsvendors' total order quantity.

Theorem 4. *For two identical competitive loss averse newsvendors, under both the demand-reallocation rule and the proportional demand-allocation rule, the following results hold:*

(1) *If $w_0 \geq w_p$ (or w_r), then competition leads to understocking (i.e. $2Q_{LC}^* \leq Q_0^*$);*

Table 2
Percentage of profit gain from competition for the proportional demand-allocation model.

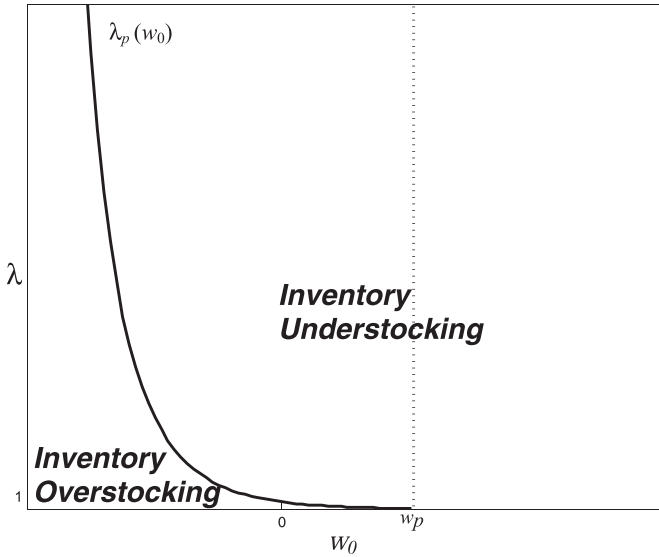| λ	w_0								
	-1.5	-1	-0.5	0	0.5	1.057	1.5	2	2.5
1.0	−350.9%	−100.0%	−32.7%	−11.7%	−1.9%	4.6%	8.3%	11.6%	14.8%
1.5	−338.8%	−94.6%	−30.0%	−9.9%	−0.4%	5.9%	9.5%	12.8%	16.3%
2.0	−328.5%	−90.0%	−27.8%	−8.5%	0.8%	7.0%	10.5%	13.8%	17.7%
2.25	−323.8%	−87.9%	−26.8%	−7.8%	1.3%	7.4%	10.9%	14.3%	18.3%
2.5	−319.4%	−86.0%	−25.9%	−7.2%	1.8%	7.8%	11.3%	14.7%	18.9%
3.0	−311.3%	−82.4%	−24.3%	−6.2%	2.6%	8.6%	12.0%	15.5%	20.2%
3.5	−304.1%	−79.2%	−22.9%	−5.3%	3.3%	9.2%	12.7%	16.2%	21.5%

Note. $w_p = 1.057$.

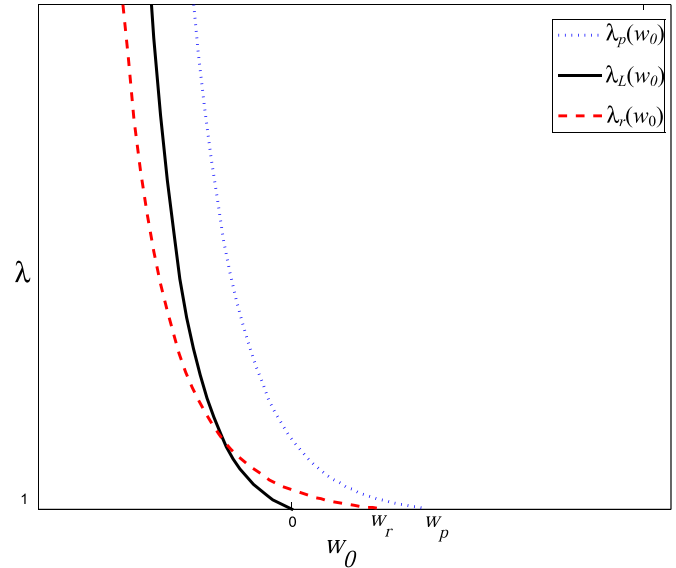**Fig. 1.** The joint effect of loss aversion and anchoring.

Fig. 2. The impact of competition on the thresholds of both loss aversion and the anchor.

(2) *If $w_0 \leq w_p$ (or w_r), then competition leads to overstocking when $\lambda \leq \lambda_p(w_0)$ (or $\lambda_r(w_0)$), or understocking when $\lambda \geq \lambda_p(w_0)$ (or $\lambda_r(w_0)$).*

Proof. See Appendix F. □

The results of Theorem 4 are displayed graphically in Fig. 1. If each newsvendor's anchor is higher than a threshold w_p (or w_r), then the anchoring effect dominates the loss-aversion effect in decreasing the supply chain's total inventory level. Consequently, inventory understocking occurs. On the other hand, if each newsvendor's anchor is lower than the threshold, then the anchoring effect leads to an increase in the order quantity, which counterbalances the effect of the loss aversion in decreasing the supply chain's total inventory level. Furthermore, although loss averse newsvendors are less inclined to increase their order quantities, a lower degree of loss aversion (i.e. $\lambda \leq \lambda_p$ (or λ_r)) could lead to inventory overstocking.

Theorem 4 indicates that whether competition leads to inventory overstocking depends on both a critical anchor and a critical degree of loss aversion. To investigate how these two factors are affected by competition, we need to derive critical values for both for a loss averse monopoly newsvendor. After comparing the difference in order quantity between a loss averse monopoly newsvendor and a risk-neutral monopoly newsvendor, we have the following result.

Corollary 1. *For a loss averse monopoly newsvendor, both the anchoring effect and the loss-aversion effect still hold; that is Q_L^* is*

decreasing in λ and w_0, respectively. Moreover, the following results hold:

(1) *If $w_0 \geq 0$, then loss aversion leads to understocking (i.e. $Q_L^* \leq Q_0^*$);*
(2) *If $w_0 < 0$, then loss aversion leads to overstocking when $\lambda \leq \lambda_L$, or understocking when $\lambda \geq \lambda_L(w_0)$, where*

$$\lambda_L(w_0) := 1 - \frac{w_0}{(c - s + w_0)F_T(\frac{c-s+w_0}{p-s}Q_0^*)}.$$

Proof. See Appendix G. □

A loss averse newsvendor with a zero anchor always understocks, no matter how loss averse he is. However, Corollary 1 shows that this is not always true for non-zero anchors. More specifically, newsvendors always understock for a positive anchor, whereas they may overstock for a negative anchor. The explanation is that decision-makers may be overconfident in estimating demand risk, and display risk-seeking behaviour by setting a relatively low anchor, which leads to an increase in their order quantities. This finding is consistent with those of existing experimental studies on newsvendors' ordering biases. For example, [10] have shown that almost one-third of the observed ordering biases in newsvendor problems are caused by overconfidence.

Comparing Theorem 4 with Corollary 1, Fig. 2 shows that the threshold of the anchor can be enlarged by competition (i.e. $w_p \geq 0$ and $w_r \geq 0$). This can be explained by the fact that competition leads to an increase in the order quantity. In order to keep the total order quantity unchanged, loss averse newsvendors should raise

Table 3
Percentage of profit gain from competition for the demand-reallocation model.

	w_0								
λ	−1.5	−1	−0.5	0	0.5	0.685	1.5	2	2.5
1.0	−3.5%	1.1%	4%	6.4%	8.4%	9.2%	12.2%	14.3%	17.8%
1.5	−3.1%	1.6%	4.7%	6.9%	8.7%	9.3%	12.0%	14.3%	19.1%
2.0	−2.6%	2.1%	5.1%	7.2%	8.8%	9.3%	11.8%	14.4%	20.5%
2.25	−2.4%	2.4%	5.3%	7.3%	8.8%	9.3%	11.7%	14.5%	21.3%
2.5	−2.2%	2.6%	5.5%	7.4%	8.8%	9.2%	11.6%	14.6%	22.1%
3.0	−1.8%	3.1%	5.9%	7.6%	8.7%	9.0%	11.5%	15.0%	23.9%
3.5	−1.5%	3.5%	6.2%	7.7%	8.5%	8.8%	11.4%	15.4%	25.6%

Note. $w_r = 0.685$.

Table 4
Order quantity for the proportional demand-allocation model.

	w_0				
(λ_i, λ_j)	−1	0	0.5	1.057	2.5
(1,1)	(125.04,125.04)	(71.57,71.57)	(62.59,62.59)	(51.32,51.32)	(31.70,31.70)
(1,1.5)	(125.02,121.99)	(72.60,69.06)	(64.42,59.06)	(52.98,47.51)	(38.01,22.29)
(1,2)	(124.98,119.24)	(73.55,66.77)	(66.08,55.91)	(54.58,44.26)	(42.52,15.77)
(1,2.5)	(124.91,116.76)	(74.44,64.67)	(67.40,53.06)	(56.12,41.42)	(45.92,10.95)
(1,3)	(124.82,114.48)	(75.28,62.72)	(69.01,50.46)	(57.60,38.89)	(48.58,7.24)
(1,3.5)	(124.72,112.38)	(76.07,69.90)	(70.33,48.07)	(59.03,36.62)	(50.72,4.29)

their anchors, which can counter the competition effect by ordering less. However, the changes to the critical degree of loss aversion (e.g. $\lambda_r(w_0)$ and $\lambda_p(w_0)$) depend on both competitive patterns (demand-splitting rules) and other market settings. Furthermore, for newsvendors who have relatively high anchors, Theorem 4 implies that the anchoring effect dominates the loss aversion in decreasing the order quantity.

Next, we use numerical examples to further analyse the effects of loss aversion and anchoring on newsvendors' order quantities in two different competitive settings. Following the settings of [24], we assume the median degree of loss aversion is 2.25. Accordingly, the parameters used are as follows: $p = 6$, $c = 3$, $s = 1$, $\alpha = 1$, $\lambda \in [1, 3.5]$, and $D_T \sim N(100, 36^2)$. Correspondingly, $D_i, D_j \sim N(50, \frac{36^2}{2})$, $w_p = 1.057$, and $w_r = 0.685$. Under the risk-neutral criterion, the optimal order quantity of a monopoly newsvendor Q_0^* and the total equilibrium order quantity of two competitive newsvendors $2Q_C^*$ are 109.121 and 143.146, respectively.

The total equilibrium order quantities with different degrees of loss aversion and anchors for the proportional demand-allocation model and the demand-reallocation model are shown in Fig. 3(a) and (b), respectively. Depending on the choices of λ and w_0, the total order quantities of loss averse competitive newsvendors may be higher or lower than those of the risk-neutral competitive newsvendors and the classic monopoly newsvendor. It appears from both figures that the order quantity is decreasing in both the degree of loss aversion and the anchor. This is explained by the fact that a lower anchor and being less loss averse motivate newsvendors to order more products. Furthermore, raising the anchor has a greater effect on reducing the order quantity than increasing the newsvendor's loss aversion does. For instance, considering that $w_0 = 0.5$ in the demand-reallocation model, the total order quantity of two loss averse newsvendors with $\lambda = 1$ is 112.91. If the degree of loss aversion is increased by 250% (i.e. $\lambda = 3.5$), the total order quantity is reduced to 99.01. Instead, an increase of 200% in anchor ($w_0 = 1.5$) makes the loss averse newsvendor order even less (only 90.72). This implies that loss averse newsvendors are more sensitive to changes with respect to the anchor. As a result, anchoring dominates loss aversion in reducing the order quantity. Moreover, since extreme anchors perceive all outcomes as losses or gains, the curves of the order quantity become steeper when the target unit profit approaches its maximum or minimum allowable value.

Although competition always leads newsvendors to order more, it is not clear whether competition benefits a supply chain that includes loss averse newsvendors. To examine the benefit of competition, we compare the total profits of loss averse competitive newsvendors under two demand-splitting rules with that of a loss averse monopoly newsvendor (i.e. $2E(\pi(Q_{LC}^*))$ vs. $E(\pi(Q_L^*))$). Note that the percentage profit gain from competition is measured by

$$\Delta \pi = \frac{2E(\pi(Q_{LC}^*)) - E(\pi(Q_L^*))}{E(\pi(Q_L^*))} \times 100\%.$$

Tables 2 and 3 illustrate that the profit gain and loss from competition in two competition models can be quite significant and are highly sensitive to both the degree of loss aversion and the anchor. On the one hand, loss averse newsvendors and newsvendors with a higher anchor are inclined to order less, which may lead to inventory understocking. Competition can compensate for the negative effect of anchoring and loss aversion by ordering more, which can result in an increase in profit. On the other hand, newsvendors with a lower anchor are inclined to order more, and competition can further lead to inventory overstocking. Therefore, the anchoring effect and competition may jointly lead to a significant loss (even more than 300%). In contrast to the proportional demand model in Table 2, the profit gain from having loss averse newsvendors is much bigger in the demand-reallocation model. The reason is that only a portion of the demand can be reallocated.

6. Heterogeneous newsvendors

In this section, we consider the case of two heterogeneous loss averse newsvendors with different degrees of loss aversion and anchors. In order to obtain insights into the effects of anchoring and loss aversion on the competitive outcome between heterogeneous newsvendors, we consider quantity competition with proportional demand allocation and demand reallocation, respectively. For both problems, we first derive a pair of optimality conditions. Then, we perform a numerical study on the equilibrium order quantities and corresponding profits.

Based on a similar analysis to that in Sections 4 and 5, we can prove that the optimal equilibrium order quantities for the demand-reallocation problem and proportional demand-allocation problem are uniquely given by the following first-order conditions. For the demand-reallocation model, the equilibrium order quantity
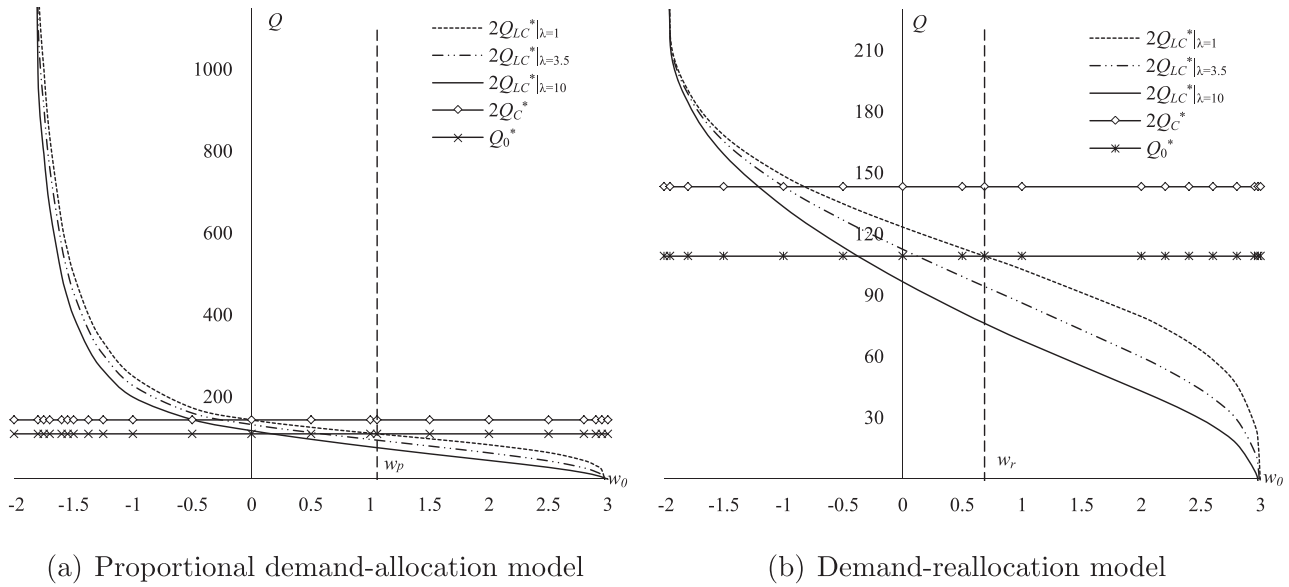
(a) Proportional demand-allocation model

(b) Demand-reallocation model

Fig. 3. Total order quantities of loss averse competitive newsvendors.

Table 5
Profit for the proportional demand-allocation model.

(λ_i, λ_j)	w_0				
	−1	0	0.5	1.057	2.5
(1,1)	(0.01,0.01)	(101.86,101.86)	(112.02,112.02)	(115.30,115.30)	(87.91,87.91)
(1,1.5)	(3.10,3.02)	(105.48,100.33)	(117.56,107.76)	(124.00,106.49)	(106.37,62.39)
(1,2)	(5.99,5.71)	(108.83,98.80)	(122.61,103.73)	(131.66,98.54)	(119.65,44.37)
(1,2.5)	(8.69,8.13)	(111.97,97.27)	(127.27,99.89)	(138.52,91.31)	(129.70,30.93)
(1,3)	(11.24,10.31)	(114.92,95.75)	(131.60,96.22)	(144.72,84.67)	(137.57,20.50)
(1,3.5)	(13.64,12.29)	(117.71,94.25)	(135.64,92.72)	(150.39,78.55)	(143.90,12.17)

Table 6
Order quantity for the demand-reallocation model.

(λ_i, λ_j)	w_0				
	−1	0	0.685	1	2.5
(1,1)	(74.28,74.28)	(61.68,61.68)	(54.56,54.56)	(51.32,51.32)	(31.42,31.42)
(1,1.5)	(74.44,73.54)	(62.34,59.73)	(55.84,51.39)	(52.98,47.51)	(35.58,24.42)
(1,2)	(74.60,72.84)	(62.99,57.94)	(57.09,48.63)	(54.58,44.26)	(38.93,19.48)
(1,2.5)	(74.76,72.15)	(63.64,56.29)	(58.31,46.16)	(56.12,41.42)	(41.66,15.77)
(1,3)	(74.92,71.50)	(64.28,54.75)	(59.50,43.94)	(57.60,38.89)	(43.95,12.85)
(1,3.5)	(75.08,70.86)	(64.92,53.31)	(60.66,41.90)	(59.03,36.62)	(45.88,10.48)

Table 7
Profit for the demand-reallocation model.

(λ_i, λ_j)	w_0				
	−1	0	0.685	1	2.5
(1,1)	(110.88,110.88)	(122.68,122.68)	(123.54,123.54)	(122.19,122.19)	(90.24,90.24)
(1,1.5)	(111.16,111.36)	(124.18,121.74)	(126.80,119.73)	(126.56,116.41)	(102.45,70.96)
(1,2)	(111.44,111.79)	(125.68,120.68)	(129.98,115.97)	(130.78,110.93)	(112.33,57.05)
(1,2.5)	(111.72,112.17)	(127.16,119.53)	(133.10,112.28)	(134.87,105.75)	(120.45,46.43)
(1,3)	(112.00,112.52)	(128.64,118.30)	(136.15,108.68)	(138.82,100.85)	(127.24,37.98)
(1,3.5)	(112.28,112.84)	(130.11,117.03)	(139.15,105.18)	(142.64,96.22)	(133.01,31.07)

Table 8
Equilibrium order quantity for the proportional demand-allocation problem.

(w_0^i, w_0^j)	λ				
	1	1.5	2.25	3	3.5
(0, −1.5)	(39.59, 161.42)	(36.10, 157.23)	(32.23, 153.16)	(29.21, 150.50)	(27.49, 149.20)
(0, −0.5)	(64.17, 91.15)	(61.75, 90.53)	(58.77, 89.78)	(56.30, 89.19)	(54.85, 88.84)
(0, 0)	(71.57, 71.57)	(70.13, 70.13)	(68.32, 68.32)	(66.81, 66.81)	(65.92, 65.92)
(0, 0.5)	(78.93, 54.55)	(78.58, 51.79)	(78.12, 48.44)	(77.71, 45.70)	(77.46, 44.13)
(0, 1.5)	(95.86, 21.85)	(97.97, 16.24)	(100.10, 10.24)	(101.46, 5.93)	(102.09, 3.65)
(0, 1.6)	(97.83, 18.45)	(100.15, 12.65)	(102.43, 6.56)	(103.85, 2.26)	(104.48, 0.03)

Table 9
Profit for the proportional demand-allocation problem.

	λ				
(w_0^i, w_0^j)	1	1.5	2.25	3	3.5
(0, −1.5)	(19.30,78.69)	(21.14,92.07)	(22.40,106.43)	(22.74,117.14)	(22.67,123.03)
(0, −0.5)	(76.29,108.37)	(76.93,112.78)	(77.41,118.25)	(77.52,122.81)	(77.47,125.47)
(0, 0)	(101.86,101.86)	(103.86,103.86)	(106.19,106.19)	(107.97,107.97)	(108.94,108.94)
(0, 0.5)	(127.78,88.31)	(132.23,87.15)	(137.60,85.32)	(141.91,83.46)	(144.36,82.24)
(0, 1.5)	(186.23,42.45)	(197.22,32.70)	(209.16,21.40)	(217.80,12.72)	(222.35,7.94)
(0, 1.6)	(192.89,36.37)	(204.43,25.81)	(216.73,13.88)	(225.45,4.91)	(229.97,0.06)

Table 10
Equilibrium order quantity for the demand-reallocation problem.

	λ				
(w_0^i, w_0^j)	1	1.5	2.25	3	3.5
(0, −1.5)	(56.89, 93.60)	(55.10, 94.40)	(52.72, 95.55)	(50.59, 96.66)	(49.29, 97.37)
(0, −0.5)	(59.49, 70.02)	(57.85, 69.53)	(55.65, 68.90)	(53.70, 68.38)	(52.51, 68.08)
(0, 0)	(61.68, 61.68)	(60.37, 60.37)	(58.64, 58.64)	(57.14, 57.14)	(56.24, 56.24)
(0, 0.5)	(64.68, 53.96)	(63.91, 51.72)	(63.04, 48.85)	(62.36, 46.39)	(61.99, 44.91)
(0, 1.5)	(73.71, 38.46)	(75.05, 34.44)	(76.74, 29.76)	(78.17, 26.07)	(79.01, 24.00)
(0, 2.5)	(91.65, 17.74)	(95.06, 12.90)	(98.39, 8.02)	(100.63, 4.60)	(101.78, 2.79)

Table 11
Profit for the demand-reallocation problem.

	λ				
(w_0^i, w_0^j)	1	1.5	2.25	3	3.5
(0, −1.5)	(111.57,107.62)	(111.04,109.48)	(109.96,112.18)	(108.66,114.80)	(107.70,116.51)
(0, −0.5)	(117.64,124.04)	(117.42,125.72)	(116.79,127.99)	(115.88,130.04)	(115.17,131.31)
(0, 0)	(122.68,122.68)	(123.21,123.21)	(123.67,123.67)	(123.82,123.82)	(123.80,123.80)
(0, 0.5)	(129.44,117.62)	(131.37,116.19)	(133.81,113.74)	(135.91,111.09)	(137.16,109.25)
(0, 1.5)	(150.69, 96.89)	(157.61, 89.57)	(166.34, 79.93)	(173.67, 71.59)	(177.97, 66.62)
(0, 2.5)	(196.59, 50.57)	(209.33, 37.44)	(222.60, 23.63)	(232.07, 13.67)	(237.09, 8.33)

(Q_i^*, Q_j^*) satisfies the following equations:

$$
\begin{cases}
\frac{p-c-w_0^i}{p-s} = \frac{(\lambda_i-1)(c-s+w_0^i)}{p-s}\left\{ F\left(D_0^r(Q_i^*)\right)F(Q_j^*)\right. \\
\quad + \int_{Q_j^*}^{Q_j^*+\frac{D_0^r(Q_i^*)}{\alpha}} F(D_0^r(Q_i^*)-\alpha(y-Q_j^*))dF(y)\Big\} \\
\quad + F(Q_i^*)F(Q_j^*) + \int_{Q_j^*}^{Q_j^*+\frac{Q_i^*}{\alpha}} F(Q_i^*-\alpha(y-Q_j^*))dF(y), \\
\frac{p-c-w_0^j}{p-s} = \frac{(\lambda_j-1)(c-s+w_0^j)}{p-s}\left\{ F\left(D_0^r(Q_j^*)\right)F(Q_i^*)\right. \\
\quad + \int_{Q_i^*}^{Q_i^*+\frac{D_0^r(Q_j^*)}{\alpha}} F(D_0^r(Q_j^*)-\alpha(y-Q_i^*))dF(y)\Big\} \\
\quad + F(Q_i^*)F(Q_j^*) + \int_{Q_i^*}^{Q_i^*+\frac{Q_j^*}{\alpha_j}} F(Q_j^*-\alpha_j(y-Q_i^*))dF(y),
\end{cases}
$$

where $D_0^r(Q_k^*) = \frac{c-s+w_0^k}{p-s}Q_k^*$, $k = i, j$. For the proportional demand-allocation model, the equilibrium order quantity (Q_i^*, Q_j^*) satisfies the following equations:

$$
\begin{cases}
\frac{p-c-w_0^i}{p-s} = F_T(Q_i^*+Q_j^*) - \frac{Q_j^*}{(Q_i^*+Q_j^*)^2}\int_0^{Q_i^*+Q_j^*} xdF_T(x) \\
\quad + (\lambda_i - 1)\left(\frac{c-s+w_0^i}{p-s}F_T(D_0^p(Q_i^*))\right. \\
\quad - \frac{Q_j^*}{(Q_i^*+Q_j^*)^2}\int_0^{D_0^p(Q_i^*)} xdF_T(x)\Big), \\
\frac{p-c-w_0^j}{p-s} = F_T(Q_i^*+Q_j^*) - \frac{Q_i^*}{(Q_i^*+Q_j^*)^2}\int_0^{Q_i^*+Q_j^*} xdF_T(x) \\
\quad + (\lambda_j - 1)\left(\frac{c-s+w_0^j}{p-s}F_T(D_0^p(Q_j^*))\right. \\
\quad - \frac{Q_i^*}{(Q_i^*+Q_j^*)^2}\int_0^{D_0^p(Q_j^*)} xdF_T(x)\Big),
\end{cases}
$$

where $D_0^p(Q_k^*) = \frac{c-s+w_0^k}{p-s}(Q_i^*+Q_j^*)$, $k = i, j$.
The following numerical analysis is based on the same settings as in Section 5. For heterogeneous competitive newsvendors with

different degrees of loss aversion and identical anchors, the order quantities and the corresponding profits under the two different types of competition are given in Tables 4 to 7. We find that most of the observations for homogeneous newsvendors still apply. Furthermore, we observe that the differences in both order quantities and profits between heterogeneous newsvendors are increasing with respect to the difference in loss aversion. Compared with his or her competitor, a newsvendor who has a lower degree of loss aversion can gain more in both competitive environments. A highly loss averse newsvendor with a relatively high anchor can be pushed out of the market. Compared with the homogeneous case, it further appears from Tables 4 and 6 that the total order quantity of heterogeneous newsvendors is lower than that of loss averse newsvendors with an identical degree of loss aversion. Then, as shown in Tables 5 and 7, the corresponding total profits of heterogeneous newsvendors may be higher than those of loss averse newsvendors with identical degrees of loss aversion. These results are related to the observation in Section 5 that newsvendors with a relatively low anchor overstock in both types of competitive environment, and that loss aversion can help counter this effect.

For heterogeneous competitive newsvendors with different anchors and identical degrees of loss aversion, the order quantities and the corresponding profits under the two different types of competition are given in Tables 8 to 11. We find that a newsvendor with a lower anchor could benefits from competition, and loss aversion can further increase the benefits from competition. A loss averse newsvendor with a relatively high anchor can be pushed out of the market if the competitor has a relatively low anchor. Compared with the homogeneous case, it further appears from Tables 8 and 10 that the total order quantity of heterogeneous newsvendors with relatively high (low) anchors is lower (higher) than that of loss averse newsvendors with identical an-

chors. As shown in Tables 9 and 11, the corresponding total profits of heterogeneous newsvendors with relatively high anchors may be higher than those of loss averse newsvendors with identical anchors. However, heterogeneous newsvendors with relatively low anchors always have a lower profit. This also relates to the observation in Section 5 that, for a relatively high anchor, the anchoring effect can help counter the overstocking caused by competition. This results in a decrease in the total order quantity and an increase in the total profit. For a relatively low anchor, the anchoring effect can exacerbate overstocking, which results in a loss of profit.

7. Conclusion

In this study, we examine the effect of anchoring and loss aversion on competitive newsvendors' ordering decisions by introducing a target unit profit as an anchor, and considering quantitative competition in two different demand-splitting rules: proportional demand allocation and demand reallocation.

In contrast to [15], who study a loss averse monopoly newsvendor problem based on a zero anchor, and ignore the anchoring effect, our results indicate that anchoring has a significant effect on newsvendors' ordering decisions. This is because the anchor determines whether an outcome (a realized profit) is perceived as a loss or a gain. More specifically, for a positive anchor, the anchoring effect dominates the loss aversion in reducing the inventory level. However, in contrast to loss aversion, which always leads to a decrease in the order quantity, a negative anchor may lead to inventory overstocking compared with the case of an integrated risk-neutral supply chain.

Next, we discuss the interaction effect of anchoring and competition on the order quantities and corresponding profits of loss averse newsvendors. Most previous studies on loss averse competitive newsvendor problems (e.g. [12,13]) ignore the anchoring effect. Our results show that loss aversion and increasing the anchor always lead to understocking, whereas competition and decreasing the anchor can help counterbalance this effect by ordering more, which leads to an increase in profit. However, a relatively low anchor and competition may lead to inventory overstocking, which results in a loss of profit.

Finally, to the best of our knowledge, our study is the first to discuss the effects of different demand-splitting rules and heterogeneity in the context of the loss averse competitive newsvendor problem. We considered quantity competition in two different games: proportional demand allocation and demand reallocation. For proportional demand allocation, the demand for the two competing newsvendors are actually dependent (i.e. $D_i = \frac{Q_i}{Q_i+Q_j} D_T$ and $D_j = \frac{Q_j}{Q_i+Q_j} D_T$, where D_T represents the total demand). For demand reallocation, although we assume that the initial demand of the two competing newsvendors are independent, the effective demand are dependent (i.e. $R_i = D_i + \alpha_i(D_j - Q_j)^+$ and $R_j = D_j + \alpha_j(D_i - Q_i)^+$). After comparing the profits in two different rules, since only part of the demand can be reallocated in the demand-reallocation problem, our results show that the profit gain in the demand reallocation is larger than that in the proportional demand allocation. Numerical investigations for heterogeneous newsvendors show that a newsvendor with a higher degree of loss aversion or with a higher anchor orders less, which results in a smaller market share and a lower profit. Under the proportional demand-allocation game, competition among newsvendors with less loss aversion and lower anchors may lead to serious overstocking, which results in a lose–lose situation.

In summary, our results provide a comprehensive understanding of quantity competition under a loss averse environment. In particular, when newsvendors have their own original markets, they can normally benefit from both loss aversion and anchoring

by avoiding overstocking. However, in a perfectly competitive market, loss averse newsvendors with lower anchors may suffer significant losses. Furthermore, our results show that the anchoring effect dominates loss aversion in decreasing the order quantity for a relatively high anchor. Since loss aversion depends heavily on personal characteristics and there is no effective way to measure the degree of loss aversion accurately, our results show that the loss-aversion effect can be achieved by setting appropriate anchors.

Our model and analyses can be extended in several directions. First, we can examine the effect of backordering. More specifically, suppose that newsvendor i can backorder β_i percent of its excess demand at cost c_i^b. For the proportional demand-allocation model, newsvendor i's profit is

$$\pi(Q_i, Q_j) = (p_i - c_i)Q_i + \beta_i(p_i - c_i^b)(\frac{Q_i}{Q_i + Q_j}D_T - Q_i)$$

$$-[(p_i - s_i) - \beta_i(p_i - c_i^b)](Q_i - \frac{Q_i}{Q_i + Q_j}D_T)^+.$$

With identical loss averse newsvendors, there exists a unique equilibrium order quantity Q_{LC}^β that satisfies

$$\frac{p - c - w_0}{p - s} + \frac{\beta(p - c^b)}{p - s}[\frac{E(D_T)}{4Q_{LC}^\beta} - 1] = \frac{[p - s - \beta(p - c^b)]}{p - s}$$

$$\times \left\{ F_T(2Q_{LC}^\beta) - \frac{1}{4Q_{LC}^\beta} \int_0^{2Q_{LC}^\beta} x dF_T(x) \right\}$$

$$+ (\lambda - 1)\left\{ \frac{c - s + w_0}{p - s} F_T(D_0^p(Q_{LC}^\beta)) \right.$$

$$\left. - \frac{1}{4Q_{LC}^\beta} \int_0^{D_0^p(Q_{LC}^\beta)} x dF_T(x) \right\},$$

where $D_0^p(Q_{LC}^\beta) = \frac{2(c - s + w_0)}{p - s}Q_{LC}^\beta$.

After comparing Q_{LC}^β and Q_{LC}^*, where Q_{LC}^* is given by (3), we find that backordering does not necessarily lead to a decrease in order quantity in a competitive market, although it always decreases the order quantity in a monopoly market. Since backordering can be used to hedge against (reduce) demand uncertainty, it can counter the effect of loss aversion and anchoring in decreasing the order quantity. Thus, the equilibrium order quantity could be increased by backordering.

Next, a critical assumption of our study is that w_0 (the newsvendor's anchor value) is given exogenously. The value of an anchor is usually based on a decision-maker's self-comparison or social comparison. If decision-makers exhibit a tendency to compare themselves to their peers (social comparison), then they may set the anchor as a ranking or the performance of those who perform better or worse (e.g. [25]). If a decision-maker's decision can be affected by past performance (self-comparison), then the value of the anchor depends on his or her aspirations. In this sense, the anchor can be understood as a target profit (e.g. [26]). In our study, the value of the anchor is based on the self-comparison effect; that is, the anchor's value is set by a target unit profit. Since demand information, cost structure, and selling price change dynamically at different selling periods, it would be interesting to further examine how the anchor reacts to changes in a multi-period problem, and how time-varying anchors affect newsvendors' decisions. However, the endogenous relations between the anchor and demand, cost, and price remain unclear. Therefore, we suggest that this could be studied in future when more support from laboratory experiments and natural experiments are available.

Appendix A. Proof of Proposition 4.1

Proof. The first-order and second-order partial derivatives of newsvendor i's expected utility with respect to Q_i are as follows:

$$\frac{\partial E[U(\pi(Q_i, Q_j))]}{\partial Q_i}$$

$$= (p_i - c_i) - w_0^i - (p_i - s_i)\left\{ \int_0^{Q_j} \int_0^{Q_i} dF_i(x)dF_j(y) \right.$$

$$\left. + \int_{Q_j}^{Q_j + \frac{Q_i}{\alpha_i}} \int_0^{Q_i - \alpha_i(y - Q_j)} dF_i(x)dF_j(y) \right\}$$

$$- (\lambda_i - 1)(c_i - s_i + w_0^i)\left\{ \int_0^{Q_j} \int_0^{D_0^r(Q_i)} dF_i(x)dF_j(y) \right.$$

$$\left. + \int_{Q_j}^{Q_j + \frac{D_0^r(Q_i)}{\alpha_i}} \int_0^{D_0^r(Q_i) - \alpha_i(y - Q_j)} dF_i(x)dF_j(y) \right\} \tag{A.1}$$

and

$$\frac{\partial^2 E[U(\pi(Q_i, Q_j))]}{\partial Q_i^2}$$

$$= -(p_i - s_i)\left\{ \int_0^{Q_j} f_i(Q_i)dF_j(y) \right.$$

$$\left. + \int_{Q_j}^{Q_j + \frac{Q_i}{\alpha_i}} f_i\big(Q_i - \alpha_i(y - Q_j)\big)dF_j(y) \right\}$$

$$- (\lambda_i - 1)(c_i - s_i + w_0^i)\left\{ \int_0^{Q_j} f_i(D_0^r(Q_i))dF_j(y) \right.$$

$$\left. + \int_{Q_j}^{Q_j + \frac{D_0^r(Q_i)}{\alpha_i}} f_i\big(D_0^r(Q_i) - \alpha_i(y - Q_j)\big)dF_j(y) \right\} < 0.$$

This implies $E[U(\pi(Q_i(Q_j), Q_j))]$ is concave. Therefore, for any given Q_j, setting (A.1) equal to 0 gives a unique optimal order quantity $Q_i^*(Q_j)$, that is, newsvendor i's best response to Q_j. □

Appendix B. Proof of Theorem 1

Proof. It is not difficult to verify that there exists at least one Nash equilibrium order quantity that satisfies Eq. (2) in this model. To verify whether Q_{LC}^* is unique, we introduce a function defined as follows:

$$e^r(Q) := p - c - w_0 - (p - s)\left\{ F^2(Q) \right.$$

$$\left. + \int_Q^{Q(1 + \frac{1}{\alpha})} F\big(Q - \alpha(y - Q)\big)dF(y) \right\}$$

$$- (\lambda - 1)(c - s + w_0)\left\{ F(Q)F(D_0^r(Q)) \right.$$

$$\left. + \int_Q^{Q + \frac{D_0^r(Q)}{\alpha}} F\big(D_0^r(Q) - \alpha(y - Q)\big)dF(y) \right\}.$$

Differentiating with respect to Q, we have

$$\frac{de^r(Q)}{dQ} = -(p - s)\left\{ f(Q)F(Q) \right.$$

$$\left. + (1 + \alpha)\int_Q^{Q(1 + \frac{1}{\alpha})} f\big(Q - \alpha(y - Q)\big)dF(y) \right\}$$

$$- (\lambda - 1)(c - s + w_0)\left\{ \frac{c - s + w_0}{p - s}F(Q)f(D_0^r(Q)) \right.$$

$$+ \left(\frac{c - s + w_0}{p - s} + \alpha \right)\int_Q^{Q + \frac{D_0^r(Q)}{\alpha}}$$

$$\left. \times f\big(D_0^r(Q) - \alpha(y - Q)\big)dF(y) \right\} < 0,$$

which implies $e^r(Q)$ is decreasing in Q. Since $\lim_{Q \to 0} e^r(Q) > 0$ and $\lim_{Q \to +\infty} e^r(Q) < 0$, there exists a symmetric unique equilibrium order quantity Q_{LC}^* that satisfies (2). □

Appendix C. Proof of Proposition 4.2

Proof. The first-order and second-order partial derivatives of newsvendor i's expected utility with respect to Q_i are as follows:

$$\frac{\partial E[U(\pi(Q_i, Q_j))]}{\partial Q_i}$$

$$= (p_i - c_i - w_0^i) - (p_i - s_i)\left\{ F_T(Q_i + Q_j) \right.$$

$$\left. - \frac{Q_j}{(Q_i + Q_j)^2}\int_0^{Q_i + Q_j} xdF_T(x) \right\}$$

$$- \frac{(\lambda_i - 1)(p_i - s_i)}{Q_i + Q_j}\left\{ D_0^p(Q_i)F_T(D_0^p(Q_i)) \right.$$

$$\left. - \frac{Q_j}{Q_i + Q_j}\int_0^{D_0^p(Q_i)} xdF_T(x) \right\} \tag{C.1}$$

and

$$\frac{\partial^2 E[U(\pi(Q_i, Q_j))]}{\partial Q_i^2}$$

$$= -\frac{2(p_i - s_i)Q_j}{(Q_i + Q_j)^3}\left\{ \int_0^{Q_i + Q_j} xdF_T(x) + (\lambda_i - 1)\int_0^{D_0^p(Q_i)} xdF_T(x) \right\}$$

$$- \frac{Q_i}{Q_i + Q_j}\left\{ (p_i - s_i)f_T(Q_i + Q_j) \right.$$

$$\left. + \frac{\lambda_i - 1}{p_i - s_i}f_T(D_0^p(Q_i))(c_i - s_i + w_0^i)^2 \right\} < 0.$$

For any given Q_j, setting (C.1) equal to 0 gives a unique optimal order quantity $Q_i^*(Q_j)$ (i.e. newsvendor i's best response to Q_j). □

Appendix D. Proof of Theorem 2

Proof. It is not difficult to verify that there exists at least one Nash equilibrium order quantity which satisfies Eq. (3) in this model. To verify whether Q_{LC}^* is unique, we introduce a function defined as follows:

$$e^p(Q) := p - c - w_0 - (p - s)\left\{ F_T(2Q) - \frac{1}{4Q}\int_0^{2Q} xdF_T(x) \right\}$$

$$- (p - s)\frac{\lambda - 1}{2Q}\left\{ D_0^p(Q)F_T(D_0^p(Q)) - \frac{1}{2}\int_0^{D_0^p(Q)} xdF_T(x) \right\}.$$

Differentiating with respect to Q, we have

$$\frac{de^p(Q)}{dQ} = -\frac{p - s}{4Q^2}\left\{ \int_0^{2Q} xdF_T(x) + (\lambda - 1)\int_0^{D_0^p(Q)} xdF_T(x) \right\}$$

$$- \left\{ (p - s)f_T(2Q) + \frac{\lambda - 1}{p - s}(c - s + w_0)^2 f_T(D_0^p(Q)) \right\} < 0,$$

which implies $e^p(Q)$ is decreasing in Q. Since $\lim_{Q \to 0} e^p(Q) > 0$ and $\lim_{Q \to +\infty} e^p(Q) < 0$, there must exist a unique Q_{LC}^* that satisfies $e^p(Q) = 0$ (i.e. Q_{LC}^* is unique). □

Appendix E. Proof of Theorem 3

Proof.

(1) Loss-aversion effect:

For the demand-reallocation game, we have

$$\frac{\partial^2 E[U(\pi(Q_{LC}^*, Q_{LC}^*))]}{\partial Q_{LC}^* \partial \lambda}$$

$$= -(c - s + w_0)\left\{ F(Q_{LC}^*)F(D_0^r(Q_{LC}^*)) \right.$$

$$\left. + \int_{Q_{LC}^*}^{Q_{LC}^* + \frac{D_0^r(Q_{LC}^*)}{\alpha}} F(D_0^r(Q_{LC}^*) - \alpha(y - Q_{LC}^*))dF(y) \right\} \le 0.$$

For the proportional demand-allocation game, we have

$$\frac{\partial^2 E[U(\pi(Q_{LC}^*, Q_{LC}^*))]}{\partial Q_{LC}^* \partial \lambda}$$

$$= -\frac{1}{2}(w_0 + c - s)F_T(D_0^p(Q_{LC}^*)) - \frac{p - s}{4Q_{LC}^*}\int_0^{D_0^p(Q_{LC}^*)} F_T(x)dx \le 0.$$

Therefore, for both games, $E[U(\pi(Q_{LC}^*, Q_{LC}^*))]$ is supermodular in (Q_{LC}^*, λ) which implies that Q_{LC}^* is decreasing in λ.

(2) Anchoring effect:

For the demand-reallocation game, from the first-order condition (2), we have

$$\frac{\partial^2 E[U(\pi(Q_{LC}^*, Q_{LC}^*))]}{\partial Q_{LC}^* \partial w_0}$$

$$= -1 - (\lambda - 1)D_0^r(Q_{LC}^*)\left\{ \int_{Q_{LC}^*}^{Q_{LC}^* + \frac{D_0^r(Q_{LC}^*)}{\alpha}} f(D_0^r(Q_{LC}^*) - \alpha(y - Q_{LC}^*))dF(y) \right.$$

$$\left. + f(D_0^r(Q_{LC}^*))F(Q_{LC}^*) \right\} - (\lambda - 1)\left\{ F(Q_{LC}^*)F(D_0^r(Q_{LC}^*)) \right.$$

$$\left. + \int_{Q_{LC}^*}^{Q_{LC}^* + \frac{D_0^r(Q_{LC}^*)}{\alpha}} F(D_0^r(Q_{LC}^*) - \alpha(y - Q_{LC}^*))dF(y) \right\} < 0.$$

For the proportional demand-allocation game, from the first-order condition (3), we have

$$\frac{\partial^2 E[U(\pi(Q_{LC}^*, Q_{LC}^*))]}{\partial Q_{LC}^* \partial w_0} = -1 - (\lambda - 1)F_T(D_0^p(Q_{LC}^*))$$

$$- (\lambda - 1)D_0^p(\tfrac{1}{2}Q_{LC}^*)f_T(D_0^p(Q_{LC}^*)) < 0.$$

Therefore, for both games, $E[U(\pi(Q_{LC}^*, Q_{LC}^*))]$ is strictly supermodular in (Q_{LC}^*, w_0). Therefore, Q_{LC}^* is strictly decreasing in w_0 (i.e. $\frac{dQ_{LC}^*}{dw_0} < 0$). □

Appendix F. Proof of Theorem 4

Proof. *For the demand-reallocation problem:*

From the proof of Theorem 1, we have that $e^r(Q)$ is decreasing in Q and $e^r(Q_{LC}^*) = 0$. To prove $2Q_{LC}^* \le (or \ge)Q_0^*$, we only need to prove $e^r(Q_0^*/2) \le (or \ge)0$. We then have

$$e^r(\frac{Q_0^*}{2}, \lambda) = p - c - w_0 - (p - s)\left\{ F^2(\frac{Q_0^*}{2}) \right.$$

$$\left. + \int_{\frac{Q_0^*}{2}}^{\frac{Q_0^*}{2}(1 + \frac{1}{\alpha})} F\left(\frac{Q_0^*}{2} - \alpha(y - \frac{Q_0^*}{2})\right)dF(y) \right\}$$

$$- (\lambda - 1)(c - s + w_0)\left\{ F(\frac{Q_0^*}{2})F(D_0^r(\frac{Q_0^*}{2})) \right.$$

$$\left. + \int_{\frac{Q_0^*}{2}}^{\frac{Q_0^*}{2} + \frac{D_0^r(\frac{Q_0^*}{2})}{\alpha}} F\left(D_0^r(\frac{Q_0^*}{2}) - \alpha_i(y - \frac{Q_0^*}{2})\right)dF(y) \right\}.$$

There is a critical w_r defined in (7). If $w_0 \ge w_r$, then $e^r(\frac{Q_0^*}{2}, \lambda) \le 0$, which implies $2Q_{LC}^* \le Q_0^*$. If $w_0 \le w_r$, then there exists a unique critical λ_r, which is defined in (6), such that $e^r(\frac{Q_0^*}{2}, \lambda_r) = 0$. Furthermore, if $\lambda \ge \lambda_r$, then $e^r(\frac{Q_0^*}{2}, \lambda) \le 0$, which implies $2Q_{LC}^* \le Q_0^*$; otherwise, we have $e^r(\frac{Q_0^*}{2}, \lambda) \ge 0$, which implies $2Q_{LC}^* \ge Q_0^*$.

For the proportional-demand allocation problem:

From the proof of Theorem 2, we have that $e^p(Q)$ is decreasing in Q and $e^p(Q_{LC}^*) = 0$. To prove $2Q_{LC}^* \le (or \ge)Q_0^*$, we only need to prove $e^p(Q_0^*/2) \le (or \ge)0$. We then have

$$e^p(\frac{Q_0^*}{2}, \lambda) = \frac{p - s}{2Q_0^*}\int_0^{Q_0^*} xdF_T(x) - w_0$$

$$- (\lambda - 1)\left(\frac{1}{2}(w_0 + c - s)F_T(D_0^p(\tfrac{1}{2}Q_0^*)) \right.$$

$$\left. + \frac{p - s}{2Q_0^*}\int_0^{D_0^p(\frac{1}{2}Q_0^*)} F_T(x)dx \right).$$

There is a critical w_p defined in (5). If $w_0 \ge w_p$, then $e^p(\frac{Q_0^*}{2}, \lambda) \le 0$, which implies $2Q_{LC}^* \le Q_0^*$. If $w_0 \le w_p$, then there exists a unique critical λ_p, which is defined in (4), such that $e^p(\frac{Q_0^*}{2}, \lambda_p) = 0$. Furthermore, if $\lambda \ge \lambda_p$, then $e^p(\frac{Q_0^*}{2}, \lambda) \le 0$, which implies $2Q_{LC}^* \le Q_0^*$; otherwise, we have $e^p(\frac{Q_0^*}{2}, \lambda) \ge 0$, which implies $2Q_{LC}^* \ge Q_0^*$. □

Appendix G. Proof of Corollary 1

Proof. The expected utility function of a loss averse monopoly newsvendor is given by

$$E(\pi(Q)) = (p - c - w_0)Q - (p - s)\int_0^Q F_T(x)dx$$

$$- (p - s)(\lambda - 1)\int_0^{\frac{c - s + w_0}{p - s}Q} F_T(x)dx.$$

It is easy to verify that $E(\pi(Q))$ is concave in Q and $\frac{dE(\pi(Q))}{dQ}|_{Q_L^*} = 0$. Furthermore, we have

$$\frac{dE(\pi(Q))}{dQ}\bigg|_{Q = Q_0^*} = (p - c - w_0) - (p - s)F_T(Q_0^*)$$

$$- (\lambda - 1)(c - s + w_0)F_T\left(\frac{c - s + w_0}{p - s}Q_0^*\right)$$

$$= -w_0 - (\lambda - 1)(c - s + w_0)F_T\left(\frac{c - s + w_0}{p - s}Q_0^*\right),$$

where $Q_0^* = F_T^{-1}(\frac{p - c}{p - s})$. It is clear that, if $w_0 > 0$, then $\frac{dE(\pi(Q))}{dQ}|_{Q = Q_0^*} < 0$ holds, which implies $Q_0^* > Q_L^*$. When $w_0 < 0$, since $\frac{d^2 E(\pi(Q))}{dQdw_0}|_{Q = Q_0^*} < 0$, there exists a critical λ_L, such that $\frac{dE(\pi(Q))}{dQ}|_{Q = Q_0^*, \lambda = \lambda_L} = 0$. If $\lambda \ge \lambda_L$, then $Q_0^* \ge Q_L^*$; otherwise, $Q_0^* \le Q_L^*$.

The proofs of the anchoring effect and the loss-aversion effect are similar to those of Theorem 3. Therefore, we omit the details here. □

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