Attractor Dynamics of Dyadic Interaction

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Attractor Dynamics of Dyadic Interaction: A Recurrence Based Analysis

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Abstract: *The aim of this study was to investigate interpersonal coordination in young children during dyadic problem solving, by using Cross-Recurrence Quantification Analysis (CRQA). We examined the interactions of seven dyads of children (M_{age}= 5.1 years) in a longitudinal design (6 sessions) with a sequence of problem-solving tasks increasing in difficulty. An innovative implementation of CRQA is presented in order to study the attractor dynamics of dyadic coordination. The analysis consisted of distinguishing two recurrent states in the relationship between children and the task. In other words, the analysis is focused on how the dyadic interaction oscillates between two stable states that for their recurrent presence are considered to be attractors. The distributed dyadic interaction (DDI) state indicates that both children contribute equally to the solution of the task. The unequal dyadic interaction (UDI) state indicating that only one of the children contributes actively to the solution of the task. Results showed that the DDI was more frequent than the UDI but that the dynamics of these two attractor states were quite similar. The behaviors within these states increased in complexity over time, although they did so in DDI more strongly than UDI. The overall recurrence, which indicates the global level of coordination between the individuals in the dyad across all time points, was moderately correlated with the performance of the children.*

Key Words: cross-recurrence quantification analysis (CRQA), dyadic interaction, interpersonal coordination, problem-solving, reasoning

INTRODUCTION

From the level of the dyad to the large scale of society, individuals are able to engage with each other in a flexible and adaptive way (Fusaroli, Konvalinka & Wallot, 2014). In order to understand these interactions between individuals, they must be studied as a process of self-organization (Dale, Fusaroli & Richardson, 2013). Self-organization is the process in which a system evolves towards a more organized level (Haken, 2006), and in which the global state of organization of the system emerges from interactions between the

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components of the system (Camazine et al., 2001). Looking at social interactions from the perspective of complex systems dynamics, self-organization can be understood as the process by which interactional structure emerges from the multimodal and multifaceted exchange between the individuals at many timescales. From this ongoing exchange, patterns of coupled behaviors arise and are sustained over time, and become more organized, that is, are of higher order and stability than the states of the individuals separately. In general, the study of interpersonal coordination requires paying attention to the structure of variability of human behavior in real time. This relation of the individual and the context is known as a process of "soft-assembly" (Kloos & Van Orden, 2009) in which self-organization is based on the interaction between the components of the behavior interact with the characteristics of the given context.

A growing number of interdisciplinary studies have explored the temporal structure of interpersonal coordination by using nonlinear time-series techniques, such as cross-recurrence quantification analysis (CRQA). Recurrence methods characterize the dynamics of processes by describing their recurrent state behaviors, which can be represented in recurrence plots (Marwan & Webber, 2015). CRQA quantifies patterns of interpersonal coordination, basing their analyses on the sequence of behaviors (single time series), performed in real-time activities (Shockley & Riley, 2015). CRQA (Shockley, Butwill & Webber, 2002) is a variation of RQA that has been used to analyze the coupling structure of two separate time series of continuous or categorical data (see Coco & Dale, 2014). Basically, CRQA identifies to what degree a behavioral state in some system (e.g. child 1) is matched by a particular behavioral state in another system (e.g. child 2), at some time earlier or later during the interaction or at the same time (Reuzel et al., 2013; Webber & Zbilut, 2005). CRQA has been used previously to study the interpersonal coordination, for instance, in studies on eye movement coordination (Richardson & Dale, 2005), attachment and infant sleep (De Graag, Cox, Hasselman, Jansen & de Weerth, 2012), early language development (Cox & van Dijk, 2013; Richardson, Dale & Kirkham, 2007; Spivey & Dale, 2006), aggression (Lichtwarck-Aschoff, Hasselman, Cox, Pepler & Granic, 2012), client-staff effective communication (Reuzel et al., 2013, 2014), and movements (Schmidt & Richardson, 2008; Shockley, 2005).

Interpersonal coordination is a broad topic that can be described into two aspects, such as behavioral matching and synchrony of interactions (Hove & Risen, 2009). Delaherche et al., (2012) offer an overview of how synchrony has been used in different ways to describe the interdependence of dyadic patterns behaviors during social interaction. As a result, they try to unify criteria by suggesting the following definition: "synchrony is the dynamic and reciprocal adaptation of the temporal structure of behaviors between interactive partners (…) that can occur in *all interactive* contexts: cooperative (playing a piece of music in duo) or not cooperative activities (fighting), linguistic (telephone conversations) or not linguistic interaction (catching a ball;" Delaherche et al, 2012, p. 3).

About the methodology, the study interpersonal coordination in dyads has been typically studied in the controlled context of the laboratory and often with motion capturing technology (Shockley & Riley, 2015). For instance, capturing the dyadic synchrony through the combination of perceptual modalities while performing a finger oscillation task (e.g. Gibson & Gorman, 2016), measuring through galvanic skin responses (GSRs) the coordinated performance of dyads and long work teams (Guastello et al., 2016); the language synchronization of dyads (Orsucci, et al., 2016), or measuring the coordination of speech trough electroencephalogram (EEGs; Kawasaki, Yamada, Ushiku, Miyauchi & Yamaguchi, 2016).

Synchrony of dyadic interactions in more natural settings of learning has been explored much less. It is well known that higher cognitive processes are not only the result of individual cognitive skills, but also the result of social interaction (Melander, 2012). In everyday activities, peer interaction is important to foster learning processes (Anderson & Soden, 2001). For this reason, the study of this type of dyadic interaction in the context of problem-solving is indispensable for a better understanding of cognitive development. There is consensus on the relevance of collaborative work as an ideal condition in which learners can reach complex levels of reasoning in comparison to working individually (Kerr & Tindale, 2004). Nevertheless, collaboration is not always more efficient than individual work (Bahrami et al., 2010; Koriat, 2012; Storch, 2012). In addition, collaborative and individual work are not the only types of interaction that can occur. Dyadic interaction can be characterized by the levels of *engagement* and *mutuality* that are displayed in a task or activity. In a previous study (Guevara, van Dijk, & van Geert, 2016), we described the occurrence of five types of interaction in young children (i.e. no work, passive work, imitative work, parallel work and collaborative work), and distinguished two types of dyadic interactions: distributed and unequal interactions. In the *distributed dyadic interactions (DDI)*, both children contribute equally to the solution of the task, whereas, in contrast, the *unequal dyadic interactions (UDI)* indicate that only one of the children contributes actively to the solution of the task. The current study explores the dyadic coupling of preschoolers in the context of problem-solving, based on these two attractor states of interaction.

As has been mentioned before, interpersonal coordination emerges in relation to specific contextual constraints. For instance, it has been suggested that interpersonal coordination is more sensitive to context than intrapersonal coordination (e.g. Paxton & Dale, 2013). In relation to this, a previous study on joint action in a cooperative precision task, observed in postural coordination (Ramenzoni, Davis, Riley, Shockley & Baker, 2011), revealed that interpersonal coordination is affected by the demands of the task. As the task increased in difficulty, the interpersonal coordination also increased in degree and stability. These results suggest that joint action and cooperation emerge in relation to the constraints and the cooperative demands of the task.

The current study is part of the Dutch Program called “Curious Minds” (“Talentenkracht” in Dutch, www.talentenkracht.nl), in which researchers of seven universities in the Netherlands and Belgium examine and promote the talents of children in the areas of science and technology. The project studies various forms of reasoning, mainly in young children, and the conditions that foster interest and performance in science activities. Based on the findings of a previous study within this program (Guevara et al., 2016) we aim to analyze the temporal structure of the dyadic interactions of preschool children, who are working together on a series of problem-solving tasks. In the current study, we aim to go beyond the mere *occurrence* of types of interactions (e.g. collaborative or not) and specifically focus on the *interpersonal coordination* of children's interaction. We therefore study the coupling of interaction in terms of distributed and unequal interactions, over multiple time scales, exploring how the structure of the interactions changes in relation to the tasks and their increasing difficulty.

CRQA of Interpersonal Coordination

Interaction between people can be conceived of as a complex dynamic system in which every individual continuously affects and is affected by the others (Dale, Fusaroli, Duran & Richardson, 2013). Interpersonal coordination not only consists of the components of the individuals' behavior, such as the elements of speech or movement, but also forms larger patterns of coupled responses of the individuals involved in the interaction. This way, the structure of the interactions can be conceptualized in terms of attractor dynamics. An attractor refers to the stable state of a system that recurs over time. This regularity on the dynamics of the system generates a relative stability (Hollenstein, 2013). If an attractor state is resistant to the perturbations, it is considered to be stronger (deeper). In contrast, an attractor is considered weaker (shallower) if it is more sensitive to the perturbations. This characterization applies to fixed point and limit cycle attractors. However, deeper attractors are not necessary "better" or "worse" than more shallow attractors. In fact, a certain degree of variability has been found to be optimal in complex behaviors (Guastello, 2015).

CRQA is a technique that allows indexing the attractor strength (Richardson, Schmidt & Kay, 2007). It is becoming increasingly clear that the study of interpersonal coordination can be approached from the general framework of complex dynamical systems (Kelso, 2009; Schmidt, & Richardson, 2008). A main characteristic of this framework is that it exploits the temporal structure of a system's behavior to gain information about the self-organization processes underlying that behavior. CRQA offers an accessible and powerful way to analyze the temporal structure of dyadic interaction at different time scales ranging from micro-interactions to the duration of the entire observation. In the present study we analyze how the behavior of dyads of children is coupled in the context of problem-solving tasks, by implementing CRQA in an innovative way. Our implementation goes beyond the traditional dichotomy categorization two states (recurrent vs. not recurrent). Instead, for this study, the CRQA has been designed to illustrate two different types of recurrent behaviors as part of the natural dynamic observed in the children's performance (i.e. DDI and UDI) to a non-recurrent state (i.e. non-dyadic interaction). Although previously others have explored alternative conceptualizations of recurrence (e.g. Reuzel et al., 2013; 2014), we extended this idea by employing a theory-based and context-appropriate "behavioral matching" of the children's individuals behaviors (see Cox, van der Steen, De Jonge-Hoekstra, Guevara, & van Dijk, 2016). For instance, usually, an implementation of CRQA will show whether children's interaction recurs or not with each other. However, in our study the novelty is that we define two types of recurrent behaviors. In addition, the application to the field of collaboration during scientific reasoning is rather new.

The CRQA method not only compares the behaviors of the children at each individual time point, but also does so with all possible temporal shifts (i.e. delays) between the individual behaviors of the children. These progressively delayed comparisons provide information about coupling and attunement of children's behaviors in the "here and now" as well as earlier and later during the interaction. Specifically, CRQA (see Fusaroli, Konvalinka & Wallot, 2014; Marwan, Zou, Donner & Kurths, 2009; Marwan, Romano, Thiel & Kurths, 2007; Shockley, 2005; Weber & Zbilut, 2005) is a nonlinear time-series analysis tool that enables researchers to quantify the coupled dynamics of interpersonal behavior, by providing several measures of dynamic organization.

For instance, the *recurrence rate on the line of synchrony* (RR_{LOS}) quantifies the proportion in which two individuals present a matching behavior at the same point in time. Also, the *recurrence rate overall* ($RR_{OVERALL}$) quantifies the proportion of recurrent states in the entire recurrence plot, and expresses the global level of coordination between the individuals in the dyad across all time points. Other measures aim at larger structures of recurrent points, and quantify the amount, length and distribution of diagonal and vertical line structures in the recurrence plot (see Fig. 1).

Figure 1 shows a recurrence plot of peer interaction. The *upper part* shows how different areas of the recurrence plot (RP) relate to the temporal coordination of the dyads: The two grey triangular areas indicate coupling of the children on different time scales, where behavior is matched either earlier or later in time. The black diagonal line (line of synchrony) indicates moments in which the dyadic interaction matches at the same point on time. The *bottom part* illustrates two empirical RPs of dyadic interaction during a problem-solving session. The different colors depict recurrent stable states of the system (attractors): Blue areas represent DDI and red areas UDI. Additionally, the white areas represent NDI.

The recurrence domains revealed by the characteristic "checkerboard texture" (block structures) in the recurrence plots are typically indicating metastability (beim Graben & Hutt, 2013; Eckmann, Kamphorst & Ruelle, 1987), and are evidently more pronounced in recurrence analysis of categorical time series. In order to quantify the temporal patterns (i.e. attractor states) of interpersonal coordination we focused on the CRQA-measures of the vertical line structures (De Graag, Cox, Hasselman, Jansen & de Weerth, 2012) for each of the states DDI, UDI, and NDI: RR_{LOS}, $RR_{OVERALL}$, laminarity (LAM), maximal vertical line (MaxVL), and entropy of vertical lines (EntVL).

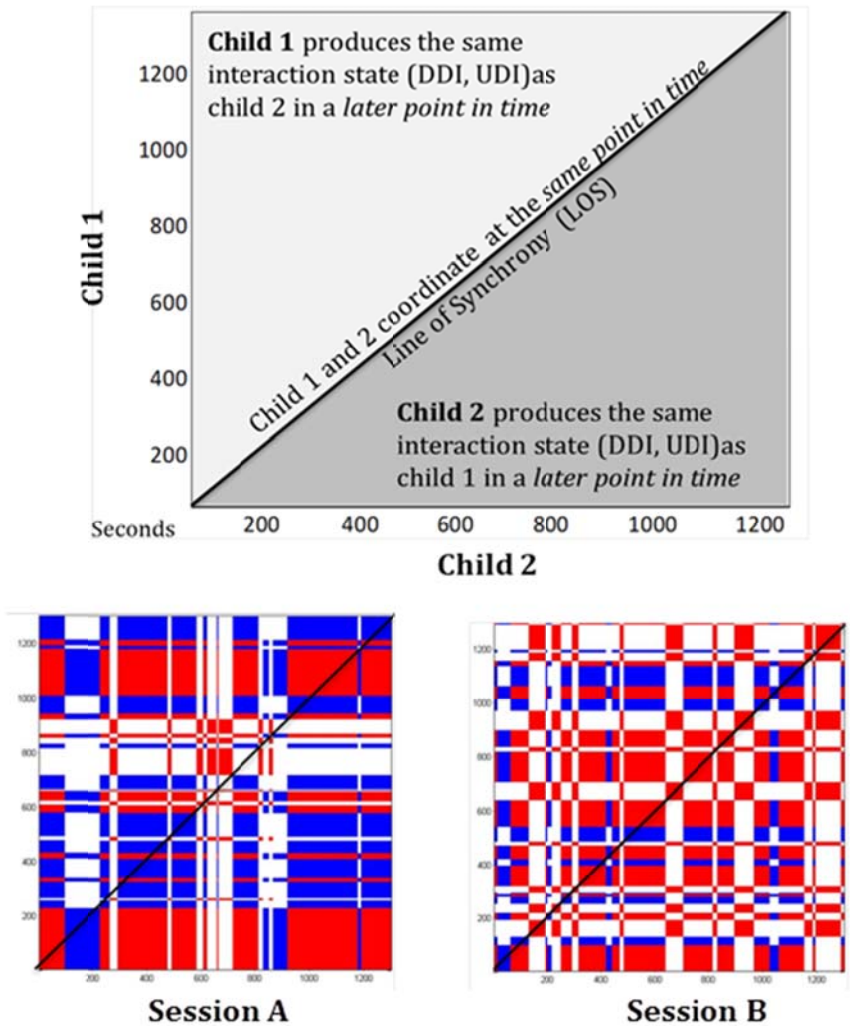


Fig. 1. Recurrence Plots (RPs) of Dyadic Interaction. The upper part depicts a schematic recurrence plot of the temporal dyadic coordination. The bottom part, illustrates the empirical RPs of a dyad during two sessions (A, B) of problem-solving.

The CRQA-measures were conceptualized as follows: *Trapping time* (TT) indicates how stable a particular attractor state is, that is as a recurrent behavioral pattern to which the matching behavior tends to return over and over. *Maximum vertical lines* (MaxVL) indicates the strength of a particular attractor state by quantifying the maximal duration of the matching behavior. Complementary to this, *laminarity* (LAM) is a measure of flexibility of the coupled behaviors and *entropy of vertical lines* (EntVL) is a measure that captures the complexity of the deterministic structure of the system (Marwan, Romano, Thiel & Kurths, 2007). High entropy indicates both high levels of uncertainty and disorder in a system or high levels of complexity, whereas low entropy reflects regularity. Therefore, entropy increases when the system evolves toward a more chaotic dynamics, but also when the system is driven by a more random dynamics (Letellier, 2006). In our study, EntVL was used as an indicator of the complexity of dyadic interaction. Thus, the higher the level of entropy, the higher the complexity of the dyadic system is.

Research Questions

In this study, we were interested in the attractor dynamics underlying dyadic interaction based on the temporal states we described above: "distributed" and "unequal" dyadic interactions, and a third state of "no work" in which the child is neither involved with the task nor with the peer (see Fig. 2). The tasks we have employed resemble what is common in everyday preschool activities in the sense that they are cognitively challenging, they increase in difficulty over time and some sort of peer interaction is asked for. The exploratory questions we addressed were whether in these conditions the coordination of dyadic interaction changes in the course of repeated task exposure, how similar the two types of coordination (DDI and UDI) are with respect to their dynamic properties and whether coordination may be related to

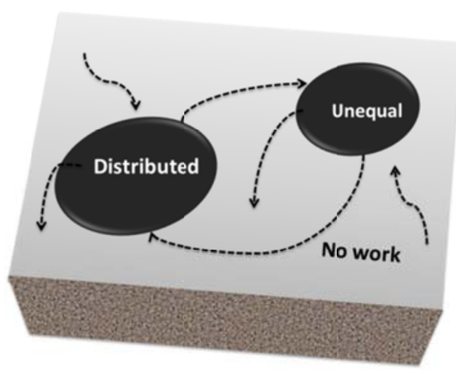


Fig. 2. The hypothetical state space in this study, with two recurrent stable states: the attractors of Distributed Dyadic Interaction (DDI) and Unequal Dyadic Interaction (UDI).

global reasoning complexity. We hypothesized that dyadic coordination may become more stable over time, because of the increasing cognitive demands of the tasks and the fact that the dyadic partners become more acquainted with each other over time. In addition, we speculated that one of the interaction types (DDI or UDI) might become more robust over time, whereas the other may become less frequent and more sensitive to perturbations. And finally, we explored whether there is a relation between dyadic coordination on the one hand and problem solving behavior on the other hand.

In sum, the following research questions were addressed: (a) How does the temporal structure of the dyadic coupling of interactions develop over a series of repeated problem solving tasks that increase in difficulty? (b) Is there a difference in the change of the strength of the two dyadic attractor states (i.e. distributed vs. unequal interaction)? (c) How are the measures of dynamic organization (derived from CRQA) related to task performance (i.e. skill levels of children's actions) of the dyads working in two sets of problem-solving (i.e. air pressure and inclined plane)?

METHOD

Participants

For the current study, we used the longitudinal data of the interaction behaviors of seven dyads of children (M_{age}= 5.1 years) observed during their performance on a series of problem-solving tasks. Seven dyads were randomly assigned to working on one of two sets of hands-on tasks related to physics notions. Four of them (Dyads 1 to 4) were working on a task set about air pressure and the other three dyads (Dyads 5 to 7) on a task set about inclined planes. All children were students from an international primary school in The Netherlands and were proficient in English as a first or second language. The dyads were selected by the teacher, based on the children's mutually positive relationships. The participation of the children in the study was based on the informed consent of their parents and on the respective approval of the Ethical Committee of Psychology (ECP) of the University of Groningen.

Data Collection

All the dyads were video recorded solving a particular set of tasks (air pressure or inclined plane) during six sessions, each one with a duration of 20-25 minutes; see Tables 1 and 2. This database of dyadic interaction of verbal and non-verbal behaviors is part of a larger study of dyadic interaction and reasoning (Guevara, 2015). In a previous study, we analyzed the dyadic interaction during verbal reasoning through a nonlinear analysis (Guevara et al., 2016). The data were collected during one school year, with a period of two months between each session. The tasks were increasing in difficulty along sessions. Each task was presented to the dyads of children, under the instruction of working together. During the administration of the task, children were able to manipulate with the material and performed several attempts to solve the tasks.

Five categories of interaction were used to code children's behaviors (for detailed information see Guevara et al., 2016): 1= *no work*: no engagement with the task; 2= *passive work*: engagement with the task without participating actively; 3= *imitative work*: engagement with the other member of the dyad without considering the task; 4= *parallel work*: engagement with the task without considering the other member of the dyad; and 5= *collaborative work*: active engagement with the task considering the other member of the dyad. The children's interaction behaviors were coded by second for each one of the six sessions of data collection. As a result, for each session, we obtained two individual time series, one for each child of the dyad (i.e. child 1 and child 2).

In order to design the CRQA (see Coco & Dale, 2014; Dale & Spivey, 2005; Reuzel et al., 2014, as examples of CRQA with categorical data), we characterized the dyadic interaction in terms of recurrent and non-recurrent states by combining the five *individual categories of interaction* (i.e. no work, passive work, imitative work, parallel work, collaborative work). It is important to note that the CRQA was carried out on the observed behaviors of the dyad (interaction codes), and depending on the nature of the interactions, the recurrence measure was defined either as UDI or DDI. Figure 3a depicts the resulting hypothetical state space of the dyadic interaction as the intersection areas of the categorical values (from 1 to 5) of child 1 and child 2 interactions along the X- and Y- coordinates. Looking at the combination of children's behaviors, we characterized the dyadic system into two possible dyadic attractor states of interaction (*distributed* or DDI and *unequal* or UDI) and one *non-dyadic interaction* state (NDI).

DDI (in Fig. 3) indicates that both children are actively engaged with the task and both contribute to it solution. For instance, both children would work in parallel (value 4) or in collaboration (value 5). UDI (in Fig. 3) indicates that only one child is contributing to solve the task while the other child of the dyad is engaged in the task but not active. It means that one child can work in parallel (value 4) while the other child is not focusing on the task (no work, value 1), remains passive (value 2), or imitates their partner (value 3). NDI (see also in Fig. 3), occurs when the behaviors of the two children that do not lead to the dyadic interaction (i.e. no work and passive work). Finally, the dashed areas indicate that these combinations of individual behaviors are not possible to co-occur. Figure 3b shows the way CRQA characterized the individual time series data into the dyadic interaction states: UDI, DDI and NDI

The performance of the dyads solving the task sets (non-verbal skills) was scored according to Fischer's skill theory (1980). This is a dynamic theory that claims that cognitive development does not follow a linear sequence of stages, but different pathways as a result of the self-organized process of skill levels over time. Therefore, Fischer's skill theory provides specific criteria to operationalize children's performance according to their cognitive complexity. This measure is used in various studies to explore reasoning skills in young ages during problem-solving (Meindertsma, van Dijk, Steenbeek. & van Geert, 2012; van der Steen, Steenbeek, van Dijk & van Geert, 2012). Using this framework, the actions of the children were ranked from the simplest to the most complex as follows: 1= the children centered their actions on isolated elements of the task,

Table 1. Description of the Task Set "Air Pressure."

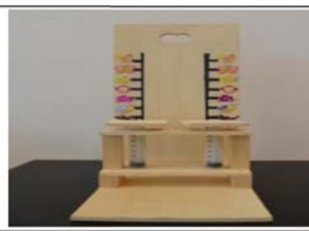
N.	*Figure*	*Task's name*	*Goal*	*Key Elements*
0		Familiarization	To press the plunger in and out, covering the tip of the syringe with the finger	---
1		Task Syringes and tubs	To push the plunge out until the red mark	Medium syringe, Straight tube (short/long), Target syringe *Connection*: Target syringe + tube + medium syringe
2		The platform lift	To lift the weigh until the edge of the wooden wall	Small syringe, Straight tube (short/long), Target syringe *Connection:* Target syringe + tube + small syringe
3		Two platforms	To push the platform up until the bee mark	Big syringe, Straight tube (short/long), Target syringe *Connection*: Target syringe + tube + Big syringe

Table 1 (Continued).

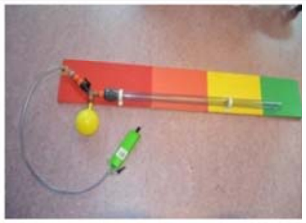
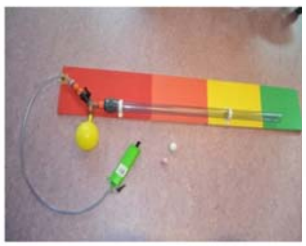
N.	*Figure*	*Task's name*	*Goal*	*Key Elements*
4		Machine with balloon	To inflating a balloon	Pump, Balloon, Slim tube Valves of the balloon and the big tube *Connection:* Open→ slim tube. Close valve→ big tube Pump→ slim tube Balloon→ open access
5		Machine with trail	To roll a ball inside the big tube until the end of the trail (green)	Light ball (pin-pon), Pump, Big tube, Main valve, valves of the balloon and the big tube. *Connection:* Open valves→ big tube, main valve. Close valve→ slim tube Pump→ slim tube Pump→ slim tube Balloon→ open access
6		Black Box	To draw the mechanism that should be inside the box, and to make a frog – attached to the box- jump	Pump, Balloon, Syringe, Frog Valves inside the box *Connection:* Opened valve→ placed on the tube connected to the frog. Closed valves placed on the tube connected to the syringe/Balloon Balloon → open access

Table 2. Description of the Task Set "Inclined Plane."

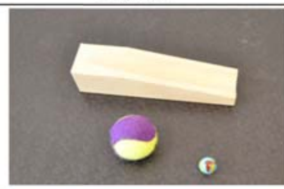
N.	Figure	Task's name	Goal	Key Elements
0		Familiarization	Exploring the function of the ramp and the characteristics of the marble/texture ball	---
1		Texture ramps	To make a marble roll the furthest	Smooth ramp Big marble
2		Marble Container (Drum)	To make a marble roll down and stay inside of a container (drum)	Medium column Smooth ramp Connection of the elements
3		Marble Basket	To make a marble roll into the basket	Columns higher that the basket Inclined ramps Connection of the elements

Table 2 (Continued).

N.	*Figure*	*Task's name*	*Goal*	*Key Elements*
4		Rolling down	To roll a right marble down so that it jumps into a container from both the top and middle mark	Big column Round tip column Big heavy marble Connection of the elements
5		Rolling up	To roll a marble up until it reaches a certain distance	Medium column Round tip column Small marble (light) Connection of the elements
6		The Funnel	To make a marble roll until the funnel	Propeller, two ramps and two targets (funnel and basket net)

2 = children used functional actions, relating elements of the task, 3 = children coordinate different elements of the task, performing partial solutions, and 4 = children coordinate the key elements of the task, building the complete mechanism (air pressure/inclined plane) to solve the task. The demand of both task sets– despite the differences in their appearance and goal – was the comprehension of the underlying mechanism of air pressure and inclined plane. The dyads of children were asked to solve the tasks by selecting the materials they thought would enable to solve the tasks. In all cases the dyad needed to build systems of air through using syringes and tubes or inclined plane systems by using blocks of wood and ramps.

The interaction behavior and the cognitive performance of each member of the dyad were coded in “The Observer XT 10” (Noldus, 2010). For each dyad, six time-series of observations were obtained, one for each session, resulting in a total of 42 time series. The inter-rater reliability of interaction was calculated for 71% of the videos (30 out of 42 videos). In addition, the inter-reliability of the cognitive performance of the dyads was calculated for the air pressure task set with 75% of videos (18 out of 24 videos) and for the inclined plane with 66% of the videos (12 out of 18 videos). The percentage of agreement for the coded interactions was 95.8%, and for the coded performance was 90%. In both cases resulting in an “almost perfect kappa” (Viera & Garrett, 2005). For the interaction the kappa was .956 and for the performance of the air pressure and inclined plane, was .985 and 962, respectively.

Because the length of the observations (data points) varied for each session, we used the shortest time series for each dyad as a cutting point for the rest of their trajectories (see note in Table 3) in order to carry out CRQA. As a result, within each dyad we analyzed the same range of data points for all the six time series resulting of the six sessions.

Table 3. Length of the Number of Data Points for the Six Trajectories of Interaction of Each Dyad.

Time series (Trajectories)	*Number of data points per time series*						
	Dyad 1	*Dyad 2*	*Dyad 3*	*Dyad 4*	*Dyad 5*	*Dyad 6*	*Dyad 7*
1	1875	705*	1395	1681	858	823*	853
2	1301*	908	1025*	1078	876	1046	980
3	1361	974	1234	1093	1303	1154	696*
4	1736	1101	1063	1443	1058	1690	1354
5	1489	1383	1173	913*	857*	1160	1043
6	2667	779	1242	1841	960	1194	1136

Note. * indicates the cutting point of data used in each dyad for CRQA. Each measurement point of the time series corresponds to one second of observation.

Table 4. Description of Measurements Used to Examine Two states of Dyadic Interaction: Distributed and Unequal.

Measure	*Description*	*Equation*
%Synchrony	Quantifies the proportion in which two individuals present the same behavior (synchrony) as matched or coupled states of the dyadic system. ***CR*** is the cross-recurrence matrix. *N* is the length of the time series.	$\%Sync = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{CR}_{i,i}$
%Recurrence	Quantifies the proportion of sharing a particular behavior. The dyads can present at least two types of recurrence states (distributed or unequal) which are the result of the combination of particular dyadic behaviors.	$RR(color) = \frac{1}{N^2}\sum_{i,j=1}^{N} \boldsymbol{CR}_{i,j\ (color)}$
LAM	Quantifies the recurrence of a matched behavior of the dyad along their history of interactions at any point of their time series. The histogram $P(v)$ represents the number of vertical lines of length v in the cross-recurrence plot. The number v_{min} is the minimum length of a vertical line; here $v_{min} = 2$.	$LAM = \frac{\sum_{v=vmin}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$
TT	Quantifies how long a behavior is trapped in a particular state. It is a synchrony measure that a show how rigid is the dyadic interaction in terms of the variation between the different dyadic interaction patterns.	$TT = \frac{\sum_{v=vmin}^{N} vP(v)}{\sum_{v=vmin}^{N} P(v)}$
MaxVL	Quantifies the duration of the longest interaction pattern that children share with each other. N_v is the total number of vertical lines (of different length).	$MaxVL = \max(\{v_l\}_{l=1}^{Nv})$

Note: computationally the CRQA measures were carried out for each of the two recurrent states of dyadic interactions (DDI, UDI).

Data Analysis

Three analyses were performed in this study. First, CRQA was performed twice, in order to quantify, separately, the temporal structure of each of the two attractors of dyadic interaction, DDI and UDI. Second, a Monte Carlo analysis was carried out to test the significance of the possible change of the CRQA outcomes over time, and finally a product-moment correlation was carried out to test the relation between the CRQA measures and the dyadic skill levels of performance (non-verbal behavior) to solve the tasks. These procedures are described below.

CRQA

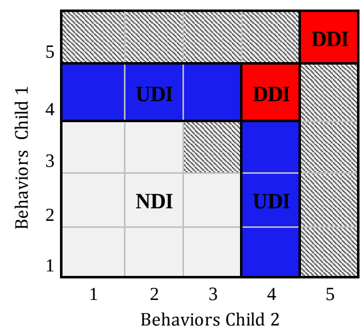
Although others have previously explored alternative conceptualizations of recurrence (e.g. Reuzel et al., 2013, 2014), we extended this idea by employing a theory-based and context-appropriate "behavioral matching" of the children's individuals behaviors (Cox, Van der Steen, De Jonge-Hoekstra, Guevara, & van Dijk, 2016). (*Note*: There was a missing data point for the dyad 1 in session 1). Figure 3 depicts the three types of dyadic interaction states (see the shadow areas inside the square: DDI, UDI and NDI) defined in this study. For a detailed description of the measures used on our analysis of interaction, see Table 4. To our knowledge, this is the first CRQA study that explicitly distinguishes between several types of recurrences throughout the analysis.

The DDI area in Fig. 3a indicates that both children contribute to solve the task (dyadic interactions: collaborative- collaborative and parallel-parallel). The UDI area in Fig. 3a indicates that only one child is contributing to solve the task while the other child of the dyad is not active (dyadic interactions: parallel-no work, parallel-passive and parallel-imitative). The NDI area in Fig. 3a, refers to the combination of behaviors that do not result in the two dyadic attractor states.

Data resulting from the observations constitute categorical time series. The various kinds of recurrences can be depicted in a recurrence plot (RP) as colored dots (red and blue for DDI, UDI, respectively, and white for NDI; see Fig. 1.

Table 5 presents the baseline of possible occurrences of the three states of interaction. These baselines represent the a-priori theoretical probability based on the possible combinations of the five interaction states (i.e. no work, passive work, imitative work, parallel work, collaborative work), and apply to the global CRQA measures of RR_{LOS} and $RR_{OVERALL}$. The baselines of RR_{LOS} and $RR_{OVERALL}$ are estimated from the two coupled dyadic interactions of DDI and UDI states as a total and separately. For the $RR_{OVERALL}$, a baseline for the non-recurrent state, NDI, also was included. If the two types of the dyadic states (DDI, UDI) significantly deviated from their baseline, this means that the underlying dyadic attractor states of interaction are not uniformly distributed across the state space. In other words, the deviation of the baseline indicates the extent to which a state of the dyadic interaction is more likely to be coupled to each other than on the basis of chance. In contrast, when the CRQA measures are at their baseline level this indicates the absence of coupling, because there is an equal chance for each individual behavior to occur.

a.



b.

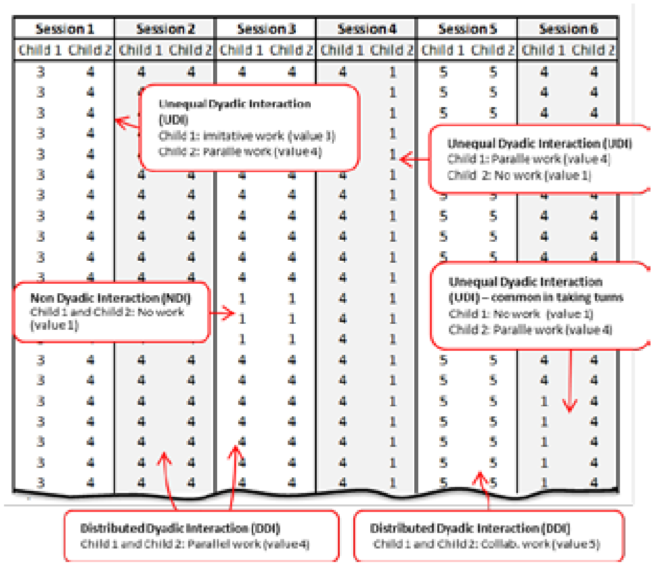


Fig. 3. (a) Hypothetical dyadic state space (child 1- child 2) with categorical dimensions represented with values from 1 to 5 in the axes x and y as follows: No work (1), Passive (2), Imitative (3), Parallel (4), and Collaboration (5). “DDI” correspond to the Distributed Dyadic Interaction and the “UDI” refers to the Unequal Dyadic Interactions respectively. The area with code “NDI” indicates the non-Dyadic Interaction or non-recurrence of distributed or unequal interactions. The dashed areas indicate behaviors that are not possible to co-occur. (b) Example of the CRQA characterization of the time series of a dyad in terms of the dyadic interaction states: UDI, DDI and NDI. Note that in session 6, the time series of the children change of code. It means that a change in the type of interaction was observed. In this case, children shift from working both of them in parallel (code 4) to a parallel-passive dynamic of interaction (codes 4 and 1 respectively).

Monte Carlo Analysis

The Monte Carlo analysis was a permutation test that consisted of comparing the repeated randomized data created by a resampling procedure (10000 permutations) against the empirical data, to estimate the probability that a particular observation deviates from a random result. If the chance that the observed result is based on the simulated data is very small, the test will indicate a significant *p*-value. For the two dyadic attractor states (DDI and UDI), we tested the different CRQA measures (RR_{LOS}, $RR_{OVERALL}$, TT, MaxVL and EntVL) against the null hypothesis that there was no increase or decrease compared to randomized data.

Table 5. Baseline of Recurrence Rate of Line of Synchrony (RR_{LOS}) and Recurrence Rate Overall ($RR_{OVERALL}$).

Measure Type	*Baseline*	*State Space Possibilities*
Total RR_{LOS} *	50%	8 out of 16
RR_{LOS} _DDI	12.5%	2 out of 16
RR_{LOS} _UDI	37.5%	6 out of 16
Total $RR_{OVERALL}$ *	32%	8 out of 25
$RR_{OVERALL}$ _DDI	8%	2 out of 25
$RR_{OVERALL}$ _UDI	24%	6 out of 25
$RR_{OVERALL}$ _NDI	68%	17 out of 25

Note. From the total of the 25 possible combinations of interactions, RR_{LOS} as coupling at the same point in time can only take place in 16 combinations. (*) Total RR_{LOS} and Total $RR_{OVERALL}$ refer to the respective total recurrence rate of adding distributed (DDI) and unequal dyadic interaction (UDI).

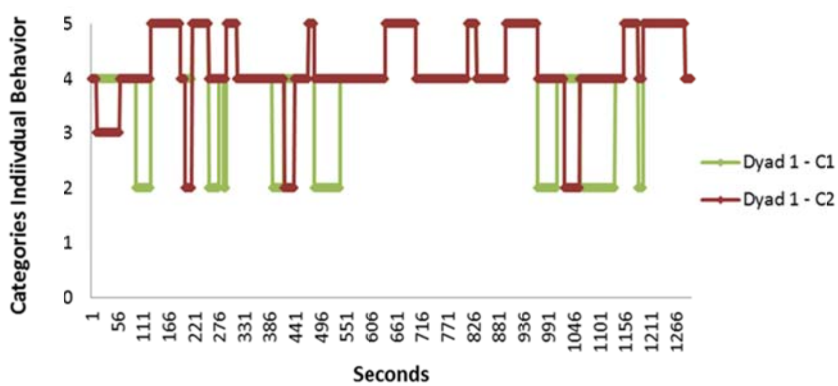
We carried out the following steps: (a) We created random data by randomizing (i.e. shuffling) the empirical time series of each individual dyad in order to simulate the null hypothesis that there is no particular structure in the pattern of variation over time, except for the overall frequencies of the categories over time. (b) CRQA was carried out using both the empirical and randomized time series. An illustration of these time-series is presented in Fig. 4. As a result, we obtained two outcomes of CRQA measures (RR_{LOS}, $RR_{OVERALL}$, LAM, TT, MaxVL, EntVL), one for the empirical and one for the randomized data. (c) We calculated the linear slope of all CRQA measures – for empirical and randomized data – for DDI and UDI based on the group average of their trajectories and compared the respective slopes by using 10000 permutations. As a result we obtained the p-values for each CRQA measure and the attractor states (DDI and UDI).

Product-moment Correlation

A product-moment correlation analysis was carried out in order to explore the relation between the CRQA measures (TT, MaxVL, $RR_{OVERALL}$, and

EntVL) and the performance on the task. To do this, we used the average optimal performance of the dyad as a whole per session. The correlation is based on all values of all dyads and does not take the nested structure (i.e. six waves of data collection with tasks changing in difficulty) of the data into account. It therefore only provides a very global measure of association between nonverbal problem solving and dyadic coordination.

Empirical time-series



Randomized time-series

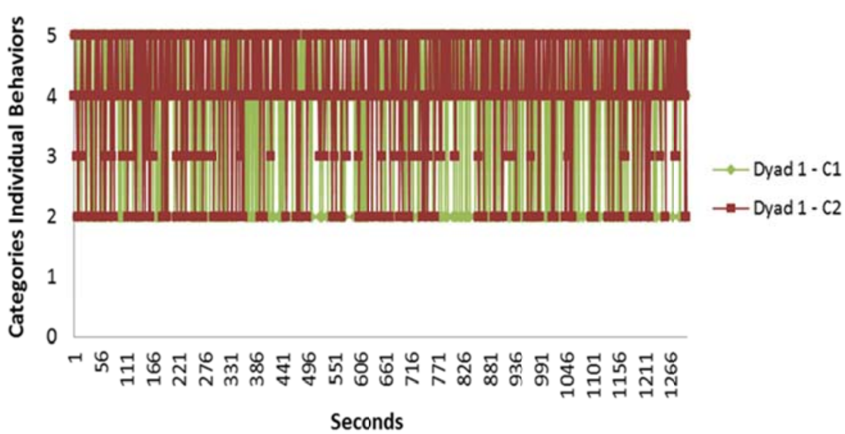


Fig. 4. Example of the empirical and randomized time-series of a dyad corresponding to a single session of problem-solving. C1= trajectory of child 1, C2= trajectory of child 2.

RESULTS

In order to examine the coupling of the dyadic interaction and its relation with cognitive processes, we first present the CRQAs of the different types of dyadic interaction. After this, we show the global performance of the dyads over time and explore the relation of the two attractor states of dyadic interaction (DDI and UDI) with the performance level reached by the dyads in the problem-solving tasks.

CRQA of Dyadic Interaction

Based on the categorical time series of dyadic interactions, the results of the CRQA reveals the hidden structure of the attractor dynamics. Figure 5 shows the changes in RR_{LOS} and $RR_{OVERALL}$ over the six sessions of observation, for both DDI and UDI. A visual inspection reveals that DDI is clearly the predominant attractor state. Across all six sessions, the values for DDI in both recurrent measures (RR_{LOS}, $RR_{OVERALL}$) are higher and exceed the baseline in every observation. In contrast, the values for UDI were below (RR_{LOS}) or around ($RR_{OVERALL}$) the baseline.

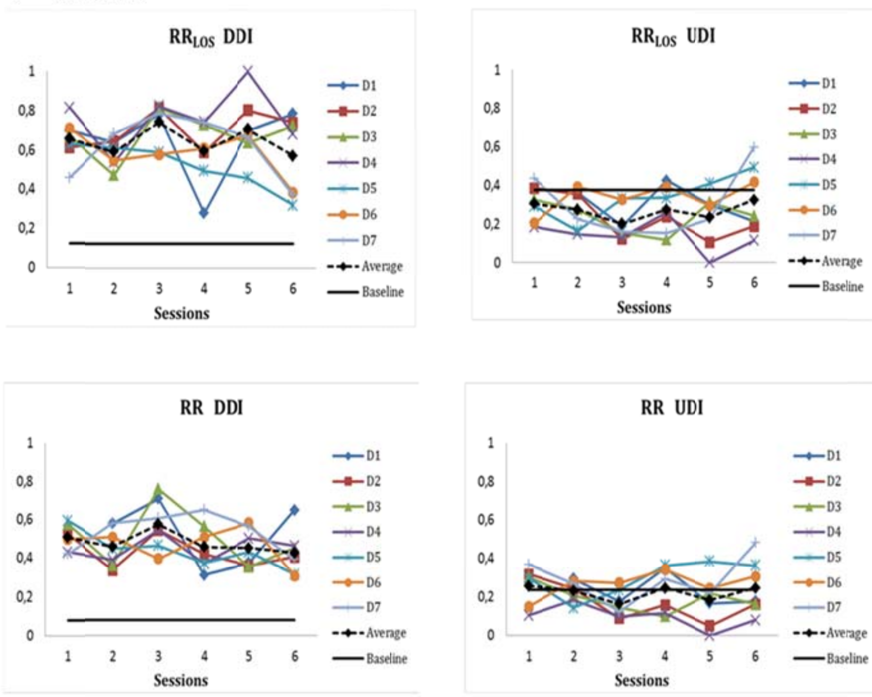


Fig. 5. Recurrence Rate of the Line of Synchrony (RR_{LOS}) and Recurrence Rate Overall ($RR_{OVERALL}$) for distributed and unequal dyadic interactions (DDI, UDI) along sessions increasing in difficulty.

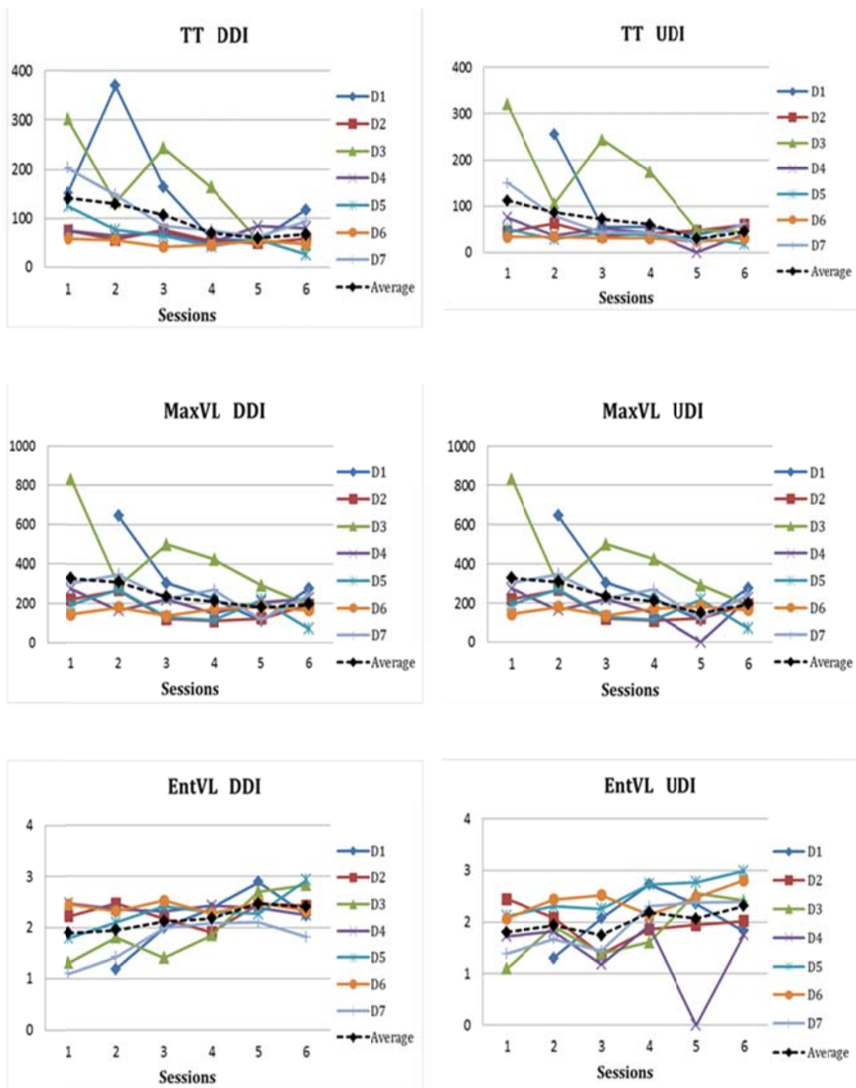


Fig. 6. Trapping time (TT), Maximal vertical lines (MaxVL) and Entropy of vertical lines (EntVL) for the distributed and unequal dyadic interactions (DDI, UDI) along six sessions with tasks increasing in difficulty.

What also stands out is the difference in variability of the DDI over time for the recurrence measures of each of the two attractors of dyadic interaction. The variability between dyads appears slightly larger for RR_{LOS} (bandwidth 46% - 82% to 32% - 78%) than for $RR_{OVERALL}$ (bandwidth: 42%-60% to

31% - 65%). In addition, the overall recurrence of DDI is more similar across dyads.

The Monte Carlo analysis revealed no significant increase or decrease in the global recurrence rate (in terms of the linear slope) for any of the two attractor states (RR_{LOS} *DDI*, p=.302; RR_{LOS} *UDI*, p=.421; $RR_{OVERALL}$ *DDI*, p=.107; $RR_{OVERALL}$ *UDI*, p=.334). The results of the CRQA measures pertaining to the vertical line structure are presented in Fig. 6. Values of Laminarity (LAM) were at maximum value (100%) across all the sessions for DDI and UDI. This means that all recurrent points (matching behaviors) constitute vertical lines, which is reflecting a high degree of patternedness in the interaction.

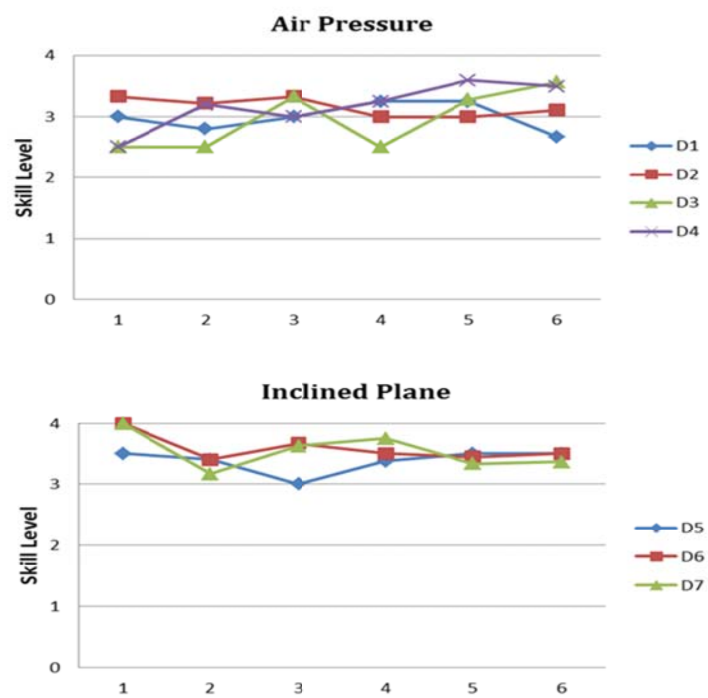


Fig. 7. Average of the optimal skill levels of the dyadic performance on the two task sets along six sessions with tasks increasing in difficulty.

In contrast, the measures of TT and MaxVL showed a global decrease from the first session to the final session (TT *DDI*= 119.6 to 66.5; TT *UDI*= 96.2 to 45; MaxVL *DDI*= 280.6 to 194.9; MaxVL *UDI*= 280.6 to 194.9). Since TT indicates the rigidity of the dyadic system and MaxVL the strength of a particular attractor state, their decrease over time is interpreted as increase of flexibility in the interpersonal coordination of the dyads. TT and MaxVL show a similar pattern over time for DDI and UDI. In addition, the patterns of both DDI and UDI trajectories were similar for the three laminar state measures of LAM, TT and MaxVL.The measure of EntVL as indicator of complexity of the coupling of dyadic interaction, showed a global increase for the DDI and UDI, where the trajectories of UDI were slightly more variable. The Monte Carlo analysis revealed that the probability that the observed decrease (linear slope) of the measures of TT and MaxVL (indicators of the attractor states), was produced by chance was very low for both attractors of dyadic interaction (TT *DDI*, p =.001; TT *UDI*, p = .001; MaxVL *DDI*, p = .001; MaxVL *UDI*, p = .001). This was also the case for the increase (slope) of EntVL or complexity of the dyadic system, in both attractors of dyadic interaction (EntVL *DDI*, p = .016; EntVL *UDI*, p = .060).

Dyadic Skill Level of Performance

Figure 7 presents the average complexity of the performance of the dyads solving the two tasks sets during six sessions of observation. Dyads working on the air pressure set showed a medium-high range of performances (Min average = 2.7; Max average = 3.7), which seems to increase only slightly across time. Most of the children focused their performance in the exploration of the materials (SL 2, use of functional actions), at the same time that other children presented partial and global solutions (SL 3 and 4, coordination of functional actions). Dyads working on the inclined pressure task seem to show a slightly higher performance (Min average = 3.6, Max average = 3.3), by mainly using partial and global solutions (SL 3 and 4, coordination of functional actions).

Interpersonal Coordination and Dyadic Skill Level of Performance

Table 6 presents the product-moment correlations and shows that the performance levels are moderately to strongly related with some CRQA measures, such as $RR_{OVERALL}$ (DDI: r =.381, p = .012; UDI: r = .370, p = .015), EntVL (UDI: r =.277, p = . 076).

Table 6. Correlation of the average skill levels of the dyadic performance (Skill levels) with the average of the CRQA measures. (*df* = 40).

CRQA Measures	*DDI*	*UDI*
TT	-.046	-.135
MaxVL	.212	.212
$RR_{OVERALL}$	.381**	.370**
EntVL	.212	.277*

*p < .10, **p < .05

DISCUSSION

In this study, we examined the dynamics of two recurrent stable states (i.e. attractors) of dyadic interaction (DDI and UDI). These attractor states reflect the different levels of involvement of the children with each other and the task, as well as its relation with the performance that was attained on the solution of the tasks with increasing difficulty.

CRQA revealed the coupled dynamics between children within dyadic interactions along a sequence of tasks. The respective CRQA results not only revealed the coupling between the children at different timescales (based on recurrence states), but also differentiated the types of coupling behavior according to the level of engagement with the task and their pair (i.e. distributed- and unequal- dyadic interactions). First, the results showed maximum levels of LAM across all sessions for DDI and UDI. Since LAM quantifies how much recurrent points are on a vertical line structure (with varying duration), this indicates a very high degree of patternedness or regularity in the interactions.

Second, the CRQA shows that the coupling interaction at the same time point (RR_{LOS}) and the overall coupling interactions ($RR_{OVERALL}$) are more frequent for the DDI attractor than for the UDI. Thus, the coupling of the dyadic interaction –along sessions– indicates a high engagement of both children with the task and each other during the solution of the tasks in which both children contribute actively to the tasks (i.e. collaborative-collaborative and parallel-parallel), also when they increase in difficulty. Thus, for these tasks and these young children, the peer interactions are more strongly coordinated towards a *distributed* type of interaction than to an *unequal* type of interaction.

Third, the measures of laminar states (TT and MaxVL) showed that the amount of coupled behaviors for the DDI and UDI decrease over time; the interaction becomes more flexible over time. This is in contrast to our expectations and findings of interpersonal coordination examined in motor behavior (Ramenzoni et al., 2011), where the coupling for dyadic interactions increased over time. In our study, the structure of the dyadic interaction – DDI and UDI – became less coupled over time and task difficulty. Possibly, this context-sensitivity is an adaptive response to the complex demand of the tasks. If children are becoming more adaptive, their behaviors may be becoming more varied – jumping from state to state– and perseverating less often.

Fourth, concerning the underlying complex dynamics of the dyadic system, we found a significant increase of the EntVL for the DDI trajectories, and an increase for UDI over time that approached significance. These results indicate that the dyadic system tends to evolve towards more complex dynamics. This kind of flexibility and variation on the dyadic interaction is an indicator of the increase of entropy.

Regarding the CRQA measures and their relation to task performance, we found that the skill levels were relatively stable despite increasing task difficulty. In particular, there was a moderate positive correlation between the skill levels of performance with the $RR_{OVERALL}$ of both DDI and UDI, which are global measures of recurrence of the interaction. It suggests that the (nonverbal) cognitive performance of the dyads is better with two relatively strong dyadic attractor states of interaction. However, no statistical association between the skill levels and the other CRQA measures were found. Combined, it means that the skill levels of the children's performance may be related with the global amount of coupling between the children in terms of the DDI and UDI attractor states, but not with the strength, stability, and complexity of those attractors.

The structure of the dyadic system shows that although the children move back and forth from one interaction state to another, the coordination changes over a longer time span: initially, the dyadic interaction is relatively rigid, but it becomes more flexible over time. Depicting the structure of the dyadic system as two valleys representing the two attractors of dyadic interaction (DDI and UDI), the dyadic coupling changed from "dwelling" in a particular attractor (DDI and UDI) before moving to another, to "switching" more frequently (perhaps adaptively) between the two attractor states. In addition, similar patterns of the trajectories were observed for the laminar state measures (LAM, TT and MaxVL) for DDI and UDI, revealing a similar temporal dynamics of these two attractors of dyadic interaction. Combined, these results suggest that the coupling of the interaction behavior becomes more complex over time. However, to get a clear understanding of the underlying dynamics, future research needs to be done contrasting how dyadic interaction evolves in a sequence of tasks with similar or increasing difficulty. This would give us more information about how the two dyadic interactions change in various contexts.

In summary, the results of CRQA have shown that the two attractors of dyadic interaction evolved toward a more complex (EntVL; Letellier, 2006) self-organizing system (Camazine et al., 2001; Haken, 2006). It seems that these changes in the dyadic dynamics take place as part of the soft-assembly process of the children's behaviors and the tasks with different levels of difficulty. We found that young children who work together show high degrees of patterning and regularity in which the distributed interactions (DDI, consisting mostly of parallel work) are a stronger attractor. In addition, the dyadic interactions became less coupled and more complex over time. Contrasting to the findings of joint action in the cooperative precision task (Ramenzoni et al., 2011), we found that interpersonal coordination does not increase in degree of stability as the task becomes more difficult, but decrease over time. The observed increased level of entropy, which this study revealed, indicates that interpersonal coordination becomes more complex and less regular. This may partly be due to increase in the task difficulty.

The limitations of the current study were the small sample size, as well as the related global correlations. Another limitation could be the re-sampling with the time series of the same dyad in order to create a randomized data to test the empirical data. Another approach is to carry out the re-sampling with the trajectories of children from different dyads. However, it is important to consider that surrogate procedures, could lead to higher variance in CRQA measures (Shockley et al., 2007).

Methodologically, this study has provided a new way to approach the examination of dyadic interaction as a dyadic system. For instance, this study presented a new way of implementing CRQA in categorical data, defining multiple states of recurrence of a system. Additionally, CRQA has been a useful tool to examine the dyadic behavior by characterizing the level of engagement between the children and how their interactions are coupled in different points in time. This dynamic exploration of the interpersonal coordination could for instance also be useful to understand the dynamics of classroom interactions. For instance, future studies could use the methodological approach suggested in this study to identify the relationship between teacher-child or child-child interactions and the quality of educational practices (e.g. instructional tasks, problem-solving, free play). Also, it would be interesting to explore whether older students solving difficult tasks present similar or different attractors of interaction from the ones reported in this study with young children. In the same line of ideas, a study with small groups of children (3 to 4 children) would provide more insights about the dynamic role of interaction and children's problem-solving behaviors in more "school-like" learning situations. Further studies about the dynamics of interaction and learning situations are needed. As education around the world is currently promoting educative experiences that are more complex and collaborative, teasing out the nuances of task features and opportunities for coordinated problem solving will become even more important. We hope this study will inspire further explorations of naturalistic behaviors of children, to improve our understanding of childhood cognition and education.

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REFERENCES

Anderson, T., & Soden, R. (2001). Peer interaction and the learning of critical thinking skills. *Psychology Learning & Teaching, 1*(1), 37-40.

Bahrami, B., Olsen, K., Latham, P. E., Roepstorff, A., Rees, G., & Frith, C.D. (2010). Optimally interacting minds. *Science, 329*, 1081-1085.

beim Graben, P. & Hutt, A. (2013). Detecting recurrence domains of dynamical systems by symbolic dynamics. *Physical Review Letters, 110*, 154101.

Camazine, S, Deneubourg, J. L., Franks, N., Sneyd, J., Theraulaz, G., & Bonabeau, E. (2001). *Self-organization in biological systems*. Princeton, NJ: Princeton University Press.

Coco, M. I., & Dale, R. (2014). Cross-recurrence quantification analysis of categorical and continuous time series: An R package. *Frontiers in Psychology, 5*(510), 1-14. doi:10.3389/fpsyg.2014.00510

Cox, R. F. A., & van Dijk, M. (2013). Microdevelopment in parent-child conversations: From global changes to flexibility. *Ecological Psychology, 25*, 304-315. doi:10.1080/10407413.2013.810095

Cox, R. F. A., & van Dijk, M. (2014, March). *The dynamics of mother-infant coordinative behaviour*. Paper presented at Society for Chaos Theory in Psychology & Life Sciences 6th International Nonlinear Science Conference, Nijmegen, Netherlands.

Cox, R. F. A., van der Steen, S., Guevara Guerrero, M., Hoekstra, L., & van Dijk, M. (2016). Chromatic and anisotropic cross-recurrence quantification analysis of interpersonal behavior. In C. Webber, C. Ioana, & N. Marwan (Eds.), *Recurrence plots and their quantifications: Expanding horizons* (pp. 209-225). New York: Springer.

Dale, R., Fusaroli, R., Duran, N. D., & Richardson, D. C. (2013). The self-organization of human interaction. In B. H. Ross (Ed.), *The psychology of learning and motivation* (pp. 43–96). New York: Elsevier Inc., Academic Press.

Dale, R., & Spivey, M. J. (2005). Categorical recurrence analysis of child language. *Proceedings of the Annual Meeting of the Cognitive Science Society, 27*, 530-535.

De Graag, J. A., Cox, R. F. A., Hasselman, F., Jansen, J., & de Weerth, C. (2012). Functioning within a relationship: Mother-infant synchrony and infant sleep. *Infant Behavior & Development, 35*, 252-263. doi: 10.1016/j.infbeh.2011.12.006

Delaherche, E., Chetouani, M., Mahdhaoui, A., Saint-Georges, C., Viaux, S., & Cohen, D. (2012). Interpersonal synchrony: A survey of evaluation methods across disciplines. *IEEE Transactions on Affective Computing 3*, 349-365. doi: http://dx.doi.org/10.1109/T-AFFC.2012.12

Eckmann, J. P., Kamphorst, S. O., & Ruelle, D. (1987). Recurrence plots of dynamical Systems. *Europhysics Letters, 4*, 973-977.

Fischer, K. W. (1980). A theory of cognitive development: The control and construction of hierarchies of skills. *Psychological Review, 87*, 477-531.

Fogel, A. (1992). Movement and communication in human infancy: The social dynamics of development. *Human Movement Science 11*, 387-423.

Fusaroli, R., Konvalinka, I., & Wallot, S. (2014). Analyzing social interactions: The promises and challenges of using cross recurrence quantification analysis. In N. Marwan (Ed.), *Translational recurrences: Springer proceedings in mathematics & statistics* (pp. 137-152). doi: 10.1007/978-3-319-09531-8_9.

Gibson, C. L., & Gorman, J. C. (2016). Top-down (prior knowledge) and bottom-up (perceptual modality) influences on spontaneous interpersonal synchronization. *Nonlinear Dynamics, Psychology, and Life Sciences, 20*, 193-222.

Guastello, S. (2015). The complexity of the psychological self and the principle of optimum variability. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*, 511-527.

Guastello, S. J., Marra, D. E., Perna, C., Castro, J., Gomez, M., & Peressini, A. F. (2016). Physiological synchronization in emergency response teams: Subjective workload, drivers and empaths. *Nonlinear Dynamics, Psychology, and Life Sciences, 20*, 223-270.

Guevara Guerrero, M. (2015). *Peer interaction and scientific reasoning processes in preschoolers: An intra-individual approach*. [Doctoral dissertation]. Groningen, The Netherlands: University of Groningen.

Guevara, M., van Dijk, M., & van Geert, P. (2016). Microdevelopment of peer interaction and scientific reasoning in young children. *Infancia y Aprendizaje, 39*, 21-41. doi: 10.1080/02103702.2016.1215083

Haken, H. (2006). *Information and self-organization: A macroscopic approach to complex systems*. Heidelberg, Germany: Springer.

Hollenstein, T. (2013). State space grids. In T. Hollenstein (Ed.), *State space grids:*

Depicting dynamics across development (pp. 11-34). New York: Springer.
Hove, M., & Risen, J. L. (2009). It's all in the timing: Interpersonal synchrony increases affiliation. *Social Cognition, 27*, 949–961.
Kawasaki, M. Yamada, Y. Ushiku, Y., Miyauchi, E., & Yamaguchi, Y. (2016). Inter-brain synchronization during coordination of speech rhythm in human-to-human social interaction. *Scientific Reports, 3*, 1692. doi: 10.1038/srep01692
Kelso, J. A. S. (2009). Coordination dynamics. In R. A. Meyers (Ed.), *Encyclopedia of complexity and system science* (pp. 1537–1564). Heidelberg, Germany: Springer.
Kerr, N. L., & Tindale, R. S. (2004). Group performance and decision making. *Annual Review of Psychology, 56*, 623– 655.
Kloos, H., & van Orden, G. C. (2009). Soft-assembled mechanisms for the grand theory. In J. P. Spencer, M. Thomas, & J. McClelland (Eds.), *Toward a new grand theory of development? Connectionism and dynamics systems theory reconsidered* (pp. 253- 267). Oxford: Oxford University Press.
Koriat, A. (2012). When are two heads better than one and why? *Science, 336*(6079), 360–362. doi:10.1126/science.1216549
Letellier, C. (2006). Estimating the Shannon entropy: Recurrence plots versus symbolic dynamics. *Physics Review Letters, 96*, 1-10. doi: 10.1063/1.2435307
Lichtwarck-Aschoff, A., Hasselman, F., Cox, R., Pepler, D., & Granic, I. (2012). A characteristic destabilization profile in parent-child interactions associated with treatment efficacy for aggressive children. *Nonlinear Dynamics, Psychology, and Life Sciences, 16*, 353-379.
Marwan, N. & Webber, C. L., Jr. (2015). Mathematical and computational foundations of recurrence quantifications. In C. L. Webber, Jr. & N. Marwan (Eds.), *Recurrence quantification analysis: Theory and best practices* (pp. 3-43). Heidelberg: Springer. doi:10.1007/978-3-319-07155-8
Marwan, N. (2008). A historical review of recurrence plots. *European Physical Journal ST 164*(1), 3–12. doi:10.1140/epjst/e2008-00829-1
Marwan, N., Donges, J. F., Zou, Y., Donner, R. V. & Kurths, J. (2009). Complex network approach for recurrence analysis of time series. *Physics Letters A, 373*, 4246–4254, doi:10.1016/j.physleta.2009.09
Marwan, N., Romano, M.C., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics Reports, 438*(5-6), 237. doi:10.1016/ j.physrep.2006.11.001
Noldus (2010). *The Observer XT: Reference manual version 10*. Wageningen, The Netherlands: Noldus Information Technology.
Orsucci. F., Musmeci, N., Aas, B., Schiepek, G., Reda, M.A., Canestri, L., Giuliani, A., & de Felice, G. (2016). Synchronization analysis of language and physiology in human dyads. *Nonlinear Dynamics, Psychology, and Life Sciences, 20*, 167-191.
Paxton, A., & Dale, R. (2013). Multimodal networks of interpersonal interaction and conversational contexts. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceedings of the 35th Annual Conference of the Cognitive Science Society* (pp. 1121-1126). Berlin: Cognitive Science Society.
Ramenzoni, V. C., Davis, T. J., Riley, M. A., Shockley, K., & Baker, A. A. (2011). Joint action in a cooperative precision task: Nested processes of intrapersonal and interpersonal coordination. *Experimental Brain Research, 211*, 447-457. doi:10.1007/s00221-011-2653-8.
Reuzel, E., Embregts, P. J. C. M., Bosman, A. M. T., Cox, R. F. A., van Nieuwenhuijzen, M., & Jahoda, A. (2014). Verbal interactional dominance and coordinative structure of speech rhythms of staff and clients with an intellectual disability.

Nonlinear Dynamics Psychology and Life Sciences, 18, 371-396.
Reuzel, E., Embregts, P. J. C. M., Bosman, A. M. T., Cox, R., van Nieuwenhuijzen, M., & Jahoda, A. (2013). Conversational synchronization in naturally occurring settings: A recurrence-based analysis of gaze directions and speech rhythms of staff and clients with intellectual disability. *Journal of Nonverbal Behavior, 37*, 281-305. doi: 10.1007/s10919-013-0158-9.
Richardson, D. C., Dale, R., & Kirkham, N. Z. (2007). The art of conversation is coordination: Common ground and the coupling of eye movements during dialogue. *Psychological Science, 18*, 407–413.
Richardson, D. C, & Dale, R. (2005). Looking to understand: The coupling between speakers' and listeners' eye movements and its relationship to discourse comprehension. *Cognitive Science, 29*, 1045–1060.
Richardson, M. J., Schmidt, R. C. & Kay, B. (2007). Distinguishing the noise and attractor strength of coordinated limb movements using recurrence analysis. *Biological Cybernetics, 96*, 59-78.
Schmidt, R. C., & Richardson, M. J. (2008). Dynamics of interpersonal coordination. In Fuchs & V. Jirsa (Ed.), *Coordination: Neural, behavioral and social dynamics* (pp. 281–308). Heidelberg, Germany: Springer-Verlag. doi: 10.1007/978-3-540-74479-5
Shockley, K. (2005). Cross recurrence quantification of interpersonal postural activity. In M. A. Riley & G. C. van Orden, (Eds.), *Tutorials in contemporary nonlinear methods for the behavioral sciences* (pp. 142–177). Retrieved May 31, 2017 from http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp.
Shockley K., Baker A. A., Richardson M. J., Fowler C. A. (2007). Articulatory constraints on interpersonal postural coordination. *Journal of Experimental Psychology: Human Perception and Performance, 33*, 201-208. doi: 10.1037/0096-1523.33.1.201
Shockley, K., & Riley, M.A. (2015). Interpersonal couplings in human interactions. In C. L. Webber, Jr., & N. Marawn (Eds.), *Recurrence quantification analysis: Theory and best practices* (pp. 399-421). Heidelberg, Germany: Springer.
Shockley, K., Butwill, M., Zbilut, J. P., & Webber, C. L., Jr. (2002). Cross-recurrence quantification of coupled oscillators. *Physics Letters, 305*, 59–69.
Spivey, M. J., & Dale, R. (2006). Continuous dynamics in real-time cognition. *Current Directions in Psychological Science, 15*, 207–211
Storch, N. (2012). *Collaborative language learning: The encyclopedia of applied linguistics.* New York, NY: Wiley-Blackwell Publishing. doi: 10.1002/9781405198431.wbeal0153
van der Steen, S. Steenbeek, H., & van Geert, P. (2012). Using the dynamics of a person-context system to describe children's understanding of air pressure. In H. Kloos, B. J. Morris & J. L. Amaral (Eds.), *Current topics in children's learning and cognition* (pp. 21-44). Rijeka, Croatia: InTech. doi:10.5772/53935
Viera, A. J., & Garrett, J. M. (2005). Understanding interobserver agreement: The kappa statistic. *Family Medicine, 37*, 360-363.
Webber, C. L., Jr., & Zbilut, J. P. (2005). Recurrent quantification analysis of nonlinear dynamical systems. In M. A. Riley & G. C. van Orden, (Eds.), *Tutorials in contemporary nonlinear methods for the behavioral sciences* (pp. 353–400). Retrieved May 31, 2017 from http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp.