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Analysis of gray markets in differentiated duopoly

Hai Li[a], Stuart X. Zhu[b], Nanfang Cui[c] and Jianbin Li[c]*

[a]*School of Business Administration, Zhongnan University of Economics and Law, Wuhan, China;* [b]*Department of Operations, University of Groningen, Groningen, The Netherlands;* [c]*School of Management, Huazhong University of Science and Technology, Wuhan, China*

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In recent years, gray markets have become a significant phenomenon in the business practice. This paper investigates the gray markets issues in differentiated duopoly case by considering quantity competition among firms. We develop a game-theoretic model and provide equilibrium results for three scenarios, i.e. the benchmark scenario 'no gray market', the scenario 'parallel imports act as a buffer against a follower's product' and the scenario 'gray markets stimulate the competition'. By the analysis of the equilibrium results, some important managerial insights are obtained. Finally, by comparison of the equilibrium results among different scenarios, we study the impact of gray markets on manufacturers' optimal strategies and profits in differentiated duopoly.

Keywords: gray markets; parallel imports; manufacturer competition; differentiated duopoly

1. Introduction

Gray markets refer to the trade of genuine-brand products through unauthorised distribution channel. Unlike black markets in which counterfeit goods or even stolen goods are sold illegally, gray markets are generally considered legal since the gray market goods (also known as parallel imports) are genuine-brand products diverted from authorised channels. With the efficient global logistics networks and thriving e-business, gray markets have become so prevalent that a wide range of industries all over the world are greatly affected. For instance, in Malaysia, gray market sales of cell phone account for 70% of the total cell phone sales (Antia, Dutta, and Bergen 2004); in India, sales of gray market flash cards account for 25% of the market (Autrey, Bova, and Soberman 2015); in China, about 1 million iPhones diverted from other countries are unlocked and sold in gray market in 2007 (New York Times 2008). According to a survey by KPMG (2008), the IT products sold in gray markets were worth about 58 billion dollars and account for about 8% of total global IT sales.

Due to the prevalence of gray markets, parallel importation becomes a significant phenomenon challenging the firms. Managing gray markets has already been an important but difficult issue for firms in their decisions-making processes. In reality, many firms argue that the presence of gray markets hurts their profits. For example, in September 2001, Apple, HP and some other IT companies set up a so-called 'Anti-Gray Market Alliance' to lobby against gray markets. Many researchers such as Autrey and Bova (2012), Li and Robles(2007), Maskus and Chen (2004), and Antia, Dutta, and Bergen (2004) also take the position that gray markets are the serious problem faced by the firms. However, the impact of gray markets on firms' profits is still rather vague. Hu, Pavlin, and Shi (2013) find that the impact of gray markets depends on the reseller's inventory holding cost. Ahmadi, Iravani, and Mamani (2015) claim that manufacturer blocks parallel importer when the product is a commodity but ignores parallel importer when the product is a fashion item. Further, there is much literature showing that the gray markets are beneficial for firms. Ahmadi and Yang (2000) argue that the manufacturer can benefit from the presence of gray markets when it incurs higher transaction costs in higher priced market. Other researchers also claim that the presence of gray markets can be beneficial to manufacturers under some conditions (Gerstner and Holthausen 1986; Dutta, Bergen, and John 1994; Yang, Ahmadi, and Monroe 1998; Raff and Schmitt 2007; Xiao, Palekar, and Liu 2011; Ichino 2014). As pointed out by these works, parallel imports regarded as a low-quality substitute of the authorised products can be used as a device of price discrimination to enhance manufacturers' profits. As drivers of gray markets, price differentiation caused by differences in demand between markets, currency exchange rates or segmentation strategy across markets is most commonly considered as the

*Corresponding author. Email: jbli@hust.edu.cn

main driver of gray markets (Ahmadi and Yang 2000; Chen and Maskus 2005; Xiao, Palekar, and Liu 2011; Autrey, Bova, and Soberman 2014; Ahmadi, Iravani, and Mamani 2015). For example, Ahmadi and Yang (2000) analyse a game-theoretic model in which a vertically integrated manufacturer sells to two countries at different prices and an independent agent takes the advantage of price differential between the two markets for arbitrage as a parallel importer. In addition to this stream of work, there is other literature considering demand uncertainty or misaligned incentives in the supply chain as the driver of gray markets from the operational level (Dasu, Ahmadi, and Carr 2012; Su and Mukhopadhyay 2012; Altug and van Ryzin 2013; Hu, Pavlin, and Shi 2013). Recently, Shulman (2014) shows that retailers may divert products to unauthorised sellers in the absence of commonly viewed necessary conditions. Autrey, Bova, and Soberman (2014) use a model of Cournot quantity competition to analyse the firm's organisational structure decision in the presence of gray markets. They find that decentralisation is optimal under some conditions including the condition that gray market is relatively uncompetitive.

In the extant literature, although numerous studies have been successful in investigating the issue of gray markets, most of them analyse it in a monopolistic context. To the best of our knowledge, only few papers consider the problem of gray markets in differentiated duopoly. For instance, Shavandi, Valizadeh Khaki, and Khedmati (2015) consider the competition between manufacturers in a low market. The authors neglect the asymmetry between manufacturers for simplicity and find that the profit of the manufacturer challenged by the gray markets may be better off under some conditions.

However, in practice, when there exists manufacturers' competition in the high-demand market, parallel imports may not only challenge the market share of a leading manufacturer but also the market share of other competing manufacturers. To provide a real-life example of the interactions we seek to model, we choose the example of the competition between Apple Inc. and Huawei Technologies Co. Ltd in the smartphone market to illustrate. Huawei is the top 10 mobile phone manufacturer all over the world and its market mainly focuses on local market, i.e. fast-growing Chinese market.[1] And it is very hard for Huawei to enter the market of developed countries, such as the US market, due to many factors (e.g. political barriers). In addition, the majority of customers in developed countries prefer to use the premium brand products such as iPhone, there is limit demand for Huawei. As it is well-known, iPhone is very popular in the Chinese gray market and a large number of unauthorised iPhones sourced from the US floods into China due to price differentiation (As aforementioned, about 1 million iPhones are unlocked and diverted to China in 2007). In fact, the unauthorised iPhone (i.e. parallel imports) becomes a big threat to Huawei in Chinese market, because the unauthorised iPhone can play the role of inferior version in Apple's product line (similar to the role of iPhone 5c in Apple's product line) and attract the consumers of Huawei at a low price. Under this situation, besides the cannibalisation effect, gray market also helps the leading manufacturer to intensify the competition towards the follower and squeeze the follower's market share.

To close the gap in the literature, we investigate the competition between manufacturers in a high market and consider quality difference among manufacturers' products and parallel imports. We analyse a supply chain model consisting of two manufacturers and one parallel importer to answer the following interesting research questions. Given that the quality of manufacturer 1's authorised product is the highest, when the quality of parallel imports is higher than that of manufacturer 2's product, parallel imports actually become a buffer between manufacturers 1 and 2 and help manufacturer 1 to seize market share from manufacturer 2, could manufacturer 1 earn a greater profit under this scenario? When the quality of parallel imports is lower than that of manufacturer 2's product, parallel imports only compete with manufacturer 2 in the low market directly and the cannibalisation effect is restricted. How would the profit change of manufacturer 1 be? By comparison of the cases with no gray markets setting, how would the profits of two manufacturers change in these two different scenarios?

To answer the above questions, we adopt the vertical differentiation model (Shaked and Sutton 1982; Motta 1993; Mussa and Rosen 1978) commonly used in economics and marketing, and allow consumers to have three options for quality, i.e. buying authorised products from manufacturer 1 or manufacturer 2 and buying the parallel imports produced by manufacturer 1 (hereafter manufacturer 1 represents the leading manufacturer who produces products with the best quality, while manufacturer 2 is the follower producing inferior products). We develop a game-theoretic model and provide equilibrium results for three scenarios, i.e. the benchmark scenario 'no gray market', the scenario 'parallel imports act as a buffer against a follower's product' and the scenario 'gray markets stimulate the competition'. By the analysis of the equilibrium results, some important managerial insights are obtained. Finally, by comparison of the equilibrium results among different scenarios, we study the impact of gray markets on manufacturers' optimal strategies and profits in differentiated duopoly

The contributions of this paper are threefold. First, we study the gray markets issues under manufacturer competition in the high market. This paper complements the literature and shows that under various possible ranks of perceived values of authorised products and parallel imports, both manufacturers are hurt by the presence of gray markets. Second, we consider the asymmetry between manufacturers and adopt the vertical differentiation model and shows that both manufacturers will prefer the situation ‘Stimulating the competition’ when the gray markets emerge. Third, our results indicate that the leading manufacturer copes with the challenges from the following manufacturer and parallel importer only by adjusting the sale quantity in low market but never changes the sale quantity decision in high market. In response to the threat from parallel imports, the following manufacturer should improve the quality of its product.

The rest of the paper is organised as follows. Section 2 describes the basic setting of the model. Section 3 presents equilibrium outcomes in three scenarios, including the benchmark scenario ‘no gray markets’, the scenario ‘parallel imports act as the buffer against follower’s product’ and the scenario ‘gray markets stimulate the competition’. Section 4 compares the equilibrium outcomes of different scenarios and gets some valuable managerial insights. Section 5 provides the conclusions of this paper and directions for future research.

2. The model

Our model considers two manufacturers, indexed 1 and 2. Each of them produces differentiated product (i.e. manufacturer i produces product i, $i = 1, 2$). Manufacturer 1 sells product 1 in two different markets indexed by j ($j = 1, 2$), while manufacturer 2 only sells product 2 in market 2 and engages in quantity competition with manufacturer 1. These two different markets can be also regarded as two different countries (See Ahmadi and Yang 2000; Autrey, Bova, and Soberman 2014, 2015). In addition, when gray market is active, parallel imports might be treated as the third type of product (i.e. product 3 denoted as $i = 3$). Similar to Ahmadi and Yang (2000), we assume consumers are differentiated according to their willingness to pay for the products in each market, and define market 1 as a ‘low’ market with a low consumer willingness to pay and market 2 as a ‘high’ market with a high consumer willingness to pay. Here, we can take market 1 (the US market) as the low market and market 2 (Chinese market) as the high market. Note that the lower willingness to pay in the US may reflect the fierce competition of the market (Szymanski and Valletti 2005). In China, many consumers, especially young people, are enthusiasts of Apple products. Thus, the willingness to pay for iPhone in China is still much larger than that in the US. As a result, iPhone is priced much higher in China than in the US.

There is a gray market in which the parallel importer is an independent agent engaged in arbitrage by buying authorised products of manufacturer 1 in market 1 and reimporting them into market 2. Furthermore, we assume only the leading manufacturer’s product (i.e. product 1) might be the target of parallel importation, while product 2 does not suffer from parallel importation problems. In reality, the fact is that the leading manufacturer’s product is much more likely to be the target of parallel importation than other products. For example, a huge number of iPhones are sold by gray market firms while other brands of mobile phones (like Huawei mobile phone) rarely flood into gray markets. Therefore, there are three players in market 2, including manufacturer 1, manufacturer 2 and parallel importer selling unauthorised product 1. In market 1, there is only one manufacturer (manufacturer 1) acting as monopolist. There can be many reasons that manufacturer 2 can only sell its products in one market, such as production cost and governmental or political factors. Figure 1 illustrates the market structure presumed in the presence of gray markets.

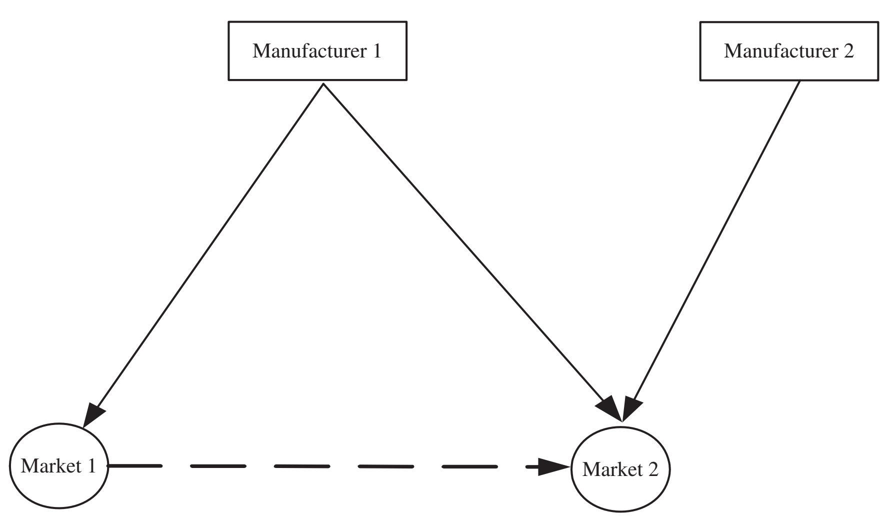

Figure 1. Markets structures with parallel importation in differentiated duopoly (Parallel importation is represented by a dashed arrow from market 1 to market 2).

Next, we specify the sequence of stages of the game in our model. To consider the relationship between manufactures, we model manufacturer 1 dominating the market (e.g. Apple Inc.) is the leader in this Stackelberg game and manufacturer 2 (e.g. Huawei Technologies Co. Ltd) is the follower. When the gray market is inactive, in stage 1, manufacturer 1 decides on the sale quantity of product 1 in both the market 1 and market 2. In stage 2, given the quantities chosen by manufacturer 1, manufacturer 2 decides on the sale quantity of product 2 in market 2. When the gray market is active, the timeline is shown as follows: in stage 1, manufacturer 1 decides on the quantity sold in both market 1 and market 2. In stage 2, given the quantities chosen by manufacturer 1, manufacturer 2 and parallel importer decide on the quantity sold in market 2 simultaneously. Taking the launch of iPhone, for example, Apple obtains the first-mover advantage in the market and Huawei is the follower. After Apple launches the new version of iPhone, Huawei will launch the new version of mobile phone correspondingly to compete with newest iPhone and the parallel importer reacts to the launch of new iPhone immediately in a short time. In this short period, the following manufacturer and parallel importer do not know each other's decision information and decide on the quantity sold simultaneously.

Further, we assume that the quality of parallel imports is perceived lower than the quality of the authorised products in consumer valuation. That is because parallel imports may not include after-sales or warranty service and may have incompatibility problems. Actually, there are three kinds of products sold by manufacturer 1, manufacturer 2 and parallel importer in market 2. For notation convenience, we use product 3 to denote parallel imports and these two terms are used interchangeably in our model.

Next, we set up the environment defined by some assumptions in order to make the analysis tractable and enable us to focus on the issues we want to explore. These assumptions are given as below:

(1) We assume that manufacturer 1 is the leader of the market, whose products have the highest quality. Normalising the quality of product 1 to 1, we use $\delta < 1$ to represent the quality of product 2 and $\theta < 1$ to represent the discounted quality of parallel imports (Hereafter, for simplicity, we use the term 'quality' to denote the perceived valuation of products). In short, we allow for the vertical differentiation among these three kinds of products by setting the quality of product 1, product 2 and parallel imports to be 1, δ and θ, respectively (i.e. $0 < \delta, \theta < 1$). Thus, product 1 is the best product in quality and the quality of parallel imports could be higher or lower than that of product 2.

(2) In each market, consumers are differentiated according to their willingness to pay for the product and buy at most one unit of product offered in up to three quality levels. Let v_{ij} and p_{ij} denote the quality and the retail price of product i ($i = 1, 2, 3$) sold in market j, respectively, and let t_j represent in market $j (j = 1, 2)$, the consumer's marginal willingness to pay for perceived quality v_{ij}. Then a consumer in market j derives the following utility from purchasing product i: $U_{ij} = v_{ij}t_j - p_{ij}$.

(3) For simplicity, we normalise both the production costs, selling costs and other transaction costs such as taxes and shipping cost to zero as in Xiao, Palekar, and Liu (2011) and Shavandi, Valizadeh Khaki, and Khedmati (2015). In market 1, there is a unit mass of consumers whose willingness to pay for quality t_1 is uniformly distributed on the interval $[0, a_1]$. In market 2, there is also a unit mass of consumers whose willingness to pay for quality t_2 is uniformly distributed on the interval $[0, a_2]$. And we assume that $a_2 > a_1$ in order to define market 1 as the low market and market 2 as the high market.

We follow the typical assumption that each consumer purchases either one unit of product with the highest net utility or nothing if consumption utility is negative or zero. In market 1, a consumer's utility from purchasing product 1 is $t_1 - p_{11}$. In market 2, a consumer's utility from purchasing product 1 is $t_2 - p_{12}$, and the utility derived from the product 2 and product 3 is $\delta t_2 - p_{22}$ and $\theta t_2 - p_{32}$, respectively. Then we determine the rank order of the three types of products in terms of quality, and there are two different cases considered in our study, which are the case ($\delta < \theta < 1$) and the case ($\theta < \delta < 1$). In the first case, the quality of parallel imports is higher than that of product 2 while in the second case the quality of parallel imports is lower than that of product 2.

We derive the demand functions by identifying the marginal consumers who are indifferent between a pair of alternatives (including the no purchase option). The marginal indifferent consumer in market 1 is located at $t_1^* = p_{11}$, and then we derived the demand of product 1 in market 1: $q_{11} = \frac{1}{a_1}(a_1 - t_1^*) = 1 - \frac{p_{11}}{a_1}$.To demonstrate consumer segmentation in market 2, we plot the utility functions in the case ($\delta < \theta < 1$) as Figure 2 and that in the case ($\theta < \delta < 1$) as Figure 3, respectively.

In the case ($\delta < \theta < 1$), the consumers are segmented into four groups in market 2: consumers with the highest willingness to pay purchase product 1; consumers with the second highest willingness to pay purchase product 3 (i.e. parallel imports); consumers with the third highest willingness to pay purchase product 2; consumers with the lowest willingness to pay choose to purchase nothing. The consumer who is indifferent between product 1 and product 3 is located at

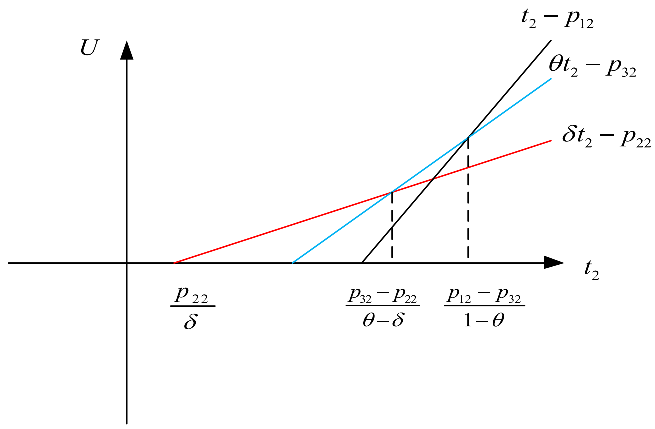

Figure 2. Consumer segmentation at equilibrium in the case ($\delta < \theta < 1$).

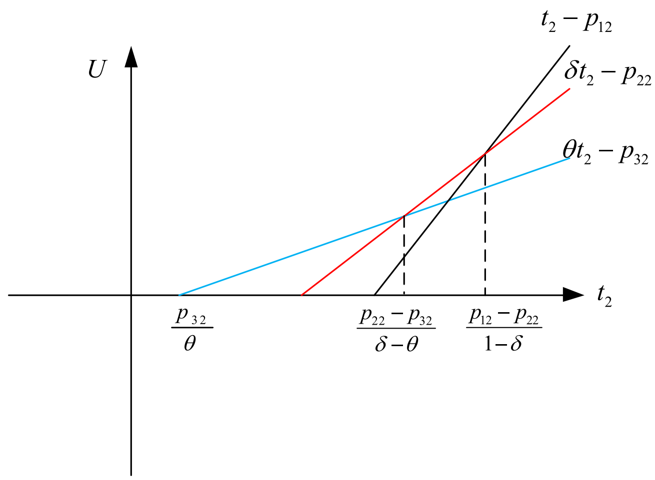

Figure 3. Consumer segmentation at equilibrium in the case ($\theta < \delta < 1$).

$\frac{p_{12}-p_{32}}{1-\theta}$, the consumer who is indifferent between product 3 and product 2 is located at $\frac{p_{32}-p_{22}}{\theta-\delta}$ and the consumer who is indifferent between product 3 and nothing to buy is located at $\frac{p_{22}}{\delta}$. Thus, the demands for product i ($i = 1, 2, 3$) in market 2 are given by $q_{12} = 1 - \frac{p_{12}-p_{32}}{a_2(1-\theta)}$, $q_{32} = \frac{1}{a_2}\left(\frac{p_{12}-p_{32}}{1-\theta} - \frac{p_{32}-p_{22}}{\theta-\delta}\right)$, $q_{22} = \frac{1}{a_2}\left(\frac{p_{32}-p_{22}}{\theta-\delta} - \frac{p_{22}}{\delta}\right)$.

After calculations and simplifying, the corresponding inverse demand functions are given by

$$p_{11} = a_1 - a_1 q_{11}, \tag{1}$$

$$p_{12} = a_2(1 - q_{12} - \delta q_{22} - \theta q_{32}), \tag{2}$$

$$p_{22} = a_2\delta(1 - q_{12} - q_{22} - q_{32}), \tag{3}$$

$$p_{32} = a_2(\theta - \theta q_{12} - \delta q_{22} - \theta q_{32}). \tag{4}$$

In the case ($\theta < \delta < 1$), by following the similar argument, we can derive the demand functions given by $q_{11} = 1 - \frac{p_{11}}{a_1}$, $q_{12} = 1 - \frac{p_{12}-p_{22}}{a_2(1-\delta)}$, $q_{22} = \frac{1}{a_2}\left(\frac{p_{12}-p_{22}}{1-\delta} - \frac{p_{22}-p_{32}}{\delta-\theta}\right)$, and $q_{32} = \frac{1}{a_2}\left(\frac{p_{22}-p_{32}}{\delta-\theta} - \frac{p_{32}}{\theta}\right)$. Then, the inverse demand functions are given by:

$$p_{11} = a_1 - a_1 q_{11}, \tag{5}$$

$$p_{12} = a_2(1 - q_{12} - \delta q_{22} - \theta q_{32}), \tag{6}$$

$$p_{22} = a_2(\delta - \delta q_{12} - \delta q_{22} - \theta q_{32}), \tag{7}$$

$$p_{32} = a_2\theta(1 - q_{12} - q_{22} - q_{32}). \tag{8}$$

In order to focus on the impact of gray markets, we first set up a benchmark model in which parallel importation does not occur, and we refer to it as the 'no gray markets' scenario. Next, we assume that authorised product 1, product 2 and parallel imports of product 1 coexist in the high market. There are two scenarios: the first scenario is referred as 'Buffer against follower's product', and the second scenario is referred as 'Stimulating the competition'. In the first scenario, $\delta < \theta < 1$, the parallel imports enter the market 2 as a buffer between the competing manufacturers' products (i.e. product 1 and product 2). In the second scenario, the parallel imports only attack product 2 directly. However, product 1 is still impacted by the parallel imports entry because the attack from parallel imports may cause manufacturer 2 to react by adjusting sale quantity of product 2 and this reaction would directly affects product 1. Under this scenario, parallel imports can 'stimulate' the competition between manufacturers by a chain of reactions. For each scenario mentioned above, we set up different reverse demand functions to examine the effect of gray markets in the context of quantity competition.

3. Equilibrium outcomes

In this section, we characterise the equilibrium for the following three cases: 'No gray markets', 'Buffer against follower's product' and 'Stimulating the competition'.

3.1 *Case 1: no gray markets*

In the absence of gray markets, it is straightforward to derive the demand functions for the two markets, i.e. $q_{11} = 1 - \frac{p_{11}}{a_1}$, $q_{12} = 1 - \frac{p_{12} - p_{22}}{a_2(1-\delta)}$, $q_{22} = \frac{\delta p_{12} - p_{22}}{a_2(1-\delta)\delta}$. The corresponding reverse demand functions are given by $p_{11} = a_1 - a_1 q_{11}$, $p_{12} = a_2(1 - q_{12} - \delta q_{22})$, $p_{22} = a_2\delta(1 - q_{12} - q_{22})$, respectively.

Given the sequence of stages of the game, we solve the model by backward induction. In stage 2, manufacturer 2 chooses the value of q_{22} to maximise his profit (π_{M2}).

$$\max_{q_{22}} \pi_{M2} = p_{22} q_{22}. \tag{9}$$

Solving the first-order condition from (9) yields $q_{22} = \frac{1}{2} - \frac{q_{12}}{2}$. Given the quantity chosen by manufacturer 2, manufacturer 1 decides on the optimal sale quantities to maximise the total profit in the two markets (π_{M1}).

$$\max_{q_{11}, q_{12}} \pi_{M1} = p_{12} q_{12} + p_{11} q_{11}. \tag{10}$$

Solving the first-order condition from (10) yields the optimal quantities, i.e. $q_{11} = \frac{1}{2}$ and $q_{12} = \frac{1}{2}$.

Further, we obtain $q_{22} = \frac{1}{4}$, $p_{11} = \frac{a_1}{2}$, $p_{12} = \left(\frac{1}{2} - \frac{\delta}{4}\right)a_2$, $p_{22} = \frac{a_2\delta}{4}$, $\pi_{M1} = \left(\frac{1}{4} - \frac{\delta}{8}\right)a_2 + \frac{a_1}{4}$ and $\pi_{M2} = \frac{a_2\delta}{16}$. To ensure $p_{11} < p_{12}$, the condition $\frac{a_2}{a_1} > \frac{2}{2-\delta}$ must be satisfied, which indicates that to ensure the existence of price gap between markets, the demand difference between markets should be big enough and the demand difference should be bigger when the quality of product 2 becomes higher.

3.2 *Case 2: buffer against follower's product* ($\delta < \theta < 1$)

In the presence of gray market, parallel imports flood into market 2 due to arbitrage opportunity. For $\delta < \theta < 1$, parallel imports may provide a buffer for manufacturer 1 against his competitor. In this scenario, parallel imports not only have cannibalisation effect on product 1 but also seize market share from product 2.

As shown in Section 2, the inverse demand functions for this case are given by $p_{11} = a_1 - a_1 q_{11}$, $p_{12} = a_2(1 - q_{12} - \delta q_{22} - \theta q_{32})$, $p_{22} = a_2\delta(1 - q_{12} - q_{22} - q_{32})$, $p_{32} = a_2(\theta - \theta q_{12} - \delta q_{22} - \theta q_{32})$.

Solving by backward induction, parallel importer and manufacturer 2 simultaneously choose the quantities that maximise their own profits (π_{P3} and π_{M2}) as follows.

$$\max_{q_{32}} \pi_{P3} = (p_{32} - p_{11})q_{32}, \tag{11}$$

$$\max_{q_{22}} \pi_{M2} = p_{22}q_{22}. \tag{12}$$

Jointly solving the first-order conditions for Equations (11) and (12), we obtain the equilibrium as follows.

$$q_{22} = \frac{a_1(1 - q_{11}) + a_2\theta(1 - q_{12})}{a_2(4\theta - \delta)}, \tag{13}$$

$$q_{32} = \frac{2a_1(q_{11} - 1) + (2a_2\theta - a_2\delta)(1 - q_{12})}{a_2(4\theta - \delta)}. \tag{14}$$

In contrast to Case 1, it is interesting to find that the quantity of product 2 sold in market 2 decreases with respect to the quantity of product 1 sold in market 1. This is because that the more product 1 sold in market 1, the lower the price of product 1 in market 1 becomes. The low price of product 1 results in a low cost of parallel importation. Consequently, the low cost of parallel importation leads to fiercer competition between product 2 and parallel imports, which reduces the quantity of product 2 sold in market 2.

Anticipating quantity responses from manufacturer 2 and parallel importer, manufacturer 1 chooses optimal quantities to maximise the total profit (π_{M1}) in both market 1 and market 2.

$$\max_{q_{11},q_{12}} \pi_{M1} = p_{12}q_{12} + p_{11}(q_{11} + q_{32}) \tag{15}$$

Solving the first-order condition yields the optimal quantities:

$$q_{11} = \frac{2a_1 + a_2\theta}{2a_1 - a_2\delta + 4a_2\theta}, \tag{16}$$

$$q_{12} = \frac{1}{2}. \tag{17}$$

Substituting $q_{11} = \frac{2a_1 + a_2\theta}{2a_1 - a_2\delta + 4a_2\theta}$ and $q_{12} = \frac{1}{2}$ into (13) and (14) yields

$$q_{22} = \frac{2a_1 + a_2\theta}{4a_1 - 2a_2\delta + 8a_2\theta}, \tag{18}$$

$$q_{32} = \frac{1}{2} - \frac{2a_1 + a_2\theta}{2a_1 - a_2\delta + 4a_2\theta}. \tag{19}$$

From Equation (19), we obtain $\frac{a_2}{a_1} > \frac{2}{2\theta - \delta}$ to satisfy $q_{32} > 0$. That means the demand difference should be big enough to allow the existence of gray markets. The lower the perceived value of parallel imports is, or the higher the quality of product 2 is, the higher the demand difference should be.

Further, we derive other equilibrium as follow:

$$p_{11} = \frac{a_1 a_2(3\theta - \delta)}{2a_1 - a_2\delta + 4a_2\theta}, \quad p_{12} = \frac{a_2}{2} + a_1 - \frac{(2a_1 + a_2\theta)(a_1 + a_2\theta)}{2a_1 - a_2\delta + 4a_2\theta},$$

$$p_{22} = \frac{a_2\delta(2a_1 + a_2\theta)}{4a_1 - 2a_2\delta + 8a_2\theta}, \quad p_{32} = \frac{a_2(2a_1 + a_2\theta)(2\theta - \delta)}{4a_1 - 2a_2\delta + 8a_2\theta}.$$

The profits of manufacturers and parallel importer are given as follows: $\pi_{M1} = \frac{a_2}{4} + a_1 - \frac{(2a_1+a_2\theta)^2}{2(2a_1-a_2\delta+4a_2\theta)}$, $\pi_{M2} = \frac{a_2\delta(2a_1+a_2\theta)^2}{4(2a_1-a_2\delta+4a_2\theta)^2}$ and $\pi_{P3} = \frac{a_2\theta(2a_2\theta-2a_1-a_2\delta)^2}{4(2a_1-a_2\delta+4a_2\theta)^2}$.

3.3 *Case 3: stimulating the competition* (θ < δ < *1*)

Here, we consider the case that the quality of parallel imports is lower than that of product 2. In this case, two versions of product 1 compete with product 2 directly in the market 2. In other words, authorised product 1 competes with product 2 for the consumers with the high willingness to pay and parallel imports compete with product 2 for the consumers with the low willingness to pay. The intuition for this case is that manufacturer 1 might seize a larger share of market 2 than that of Case 1 and Case 2, which is very beneficial to manufacturer 1 in this case. However, could the profit of manufacturer 1 be really improved? To answer the question, we have to derive the equilibrium for this case.

We first adopt the inverse demand functions derived for this scenario from Section 2: $p_{11} = a_1 - a_1q_{11}$, $p_{12} = a_2(1 - q_{12} - \delta q_{22} - \theta q_{32})$, $p_{22} = a_2(\delta - \delta q_{12} - \delta q_{22} - \theta q_{32})$, $p_{32} = a_2\theta(1 - q_{12} - q_{22} - q_{32})$.

Using backward induction, manufacturer 2 and parallel importer simultaneously choose optimal quantities that maximise their own profits (π_{P3} and π_{M2}), respectively, i.e.

$$\max_{q_{32}} \pi_{P3} = (p_{32} - p_{11})q_{32} \tag{20}$$

$$\max_{q_{22}} \pi_{M2} = p_{22}q_{22}. \tag{21}$$

Jointly solving the first-order conditions for (20) and (21), we obtain the equilibrium given by

$$q_{22} = \frac{a_1(1 - q_{11}) + (2a_2\delta - a_2\theta)(1 - q_{12})}{a_2(4\delta - \theta)}, \tag{22}$$

$$q_{32} = \frac{\delta[a_2\theta(1 - q_{12}) - 2a_1(1 - q_{11})]}{a_2\theta(4\delta - \theta)}. \tag{23}$$

From the Equations (22) and (23), we develop some managerial insights as follows. It is interesting to note that the quantity of product 1 sold in market 1 has a negative impact on the sale quantity of product 2 in market 2. This is because of the pro-competitive effect of parallel imports. When the quantity of product 1 in market 1 increases, the quantity of parallel imports also rises due to a low cost of parallel importation. As a result, more and more parallel imports flood into market 2 leading to fiercer competition against product 2, resulting in the sale decrease of product 2. Besides, the quantity of product 1 sold in market 1 or market 2 also influences the quantity of parallel imports. When the sale quantity of product 1 in market 1 increases or the sale quantity of product 1 in market 2 decreases, parallel importation quantity becomes higher.

Anticipating quantity responses from manufacturer 2 and parallel importer, manufacturer 1 chooses optimal quantities to maximise the total profit in both market 1 and market 2.

$$\max_{q_{11},q_{12}} \pi_{M1} = p_{12}q_{12} + p_{11}(q_{11} + q_{32}) \tag{24}$$

Solving the first-order condition yields the optimal quantities, i.e.

$$q_{11} = \frac{1}{2} - \frac{\delta(a_2\theta - 2a_1)}{8a_2\delta\theta + 4a_1\delta - 2a_2\theta^2}, \tag{25}$$

$$q_{12} = \frac{1}{2}. \tag{26}$$

Substituting (25) and (26) into (22) and (23), we obtain $q_{22} = \frac{1}{2} - \frac{a_1\delta+\theta(2a_2\delta-a_1)}{8a_2\delta\theta+4a_1\delta-2a_2\theta^2}$ and $q_{32} = \frac{\delta(a_2\theta-2a_1)}{8a_2\delta\theta+4a_1\delta-2a_2\theta^2}$.

By ensuring $q_{32} > 0$, we have $a_2\theta > 2a_1$ or $\frac{a_2}{a_1} > \frac{2}{\theta}$. It implies that the quality of parallel imports should not be too low and the difference between markets should not be too small such that parallel importer quits the gray markets. It is intriguing to find that whether the gray markets emerge only depends on the discounted value of parallel imports, which also means the emergence of gray markets has no relationship with the quality of competing manufacturer's products in Case 3. However, when the parallel imports act as a buffer in Case 2, the emergence of gray markets not only depends on the discounted value of parallel imports but also the quality of competing manufacturer's products.

Furthermore, by substitution and simplification, other equilibrium outcomes are given as follows.

$$p_{11} = \frac{a_1a_2\theta(5\delta-\theta)}{2\left(4a_2\delta\theta + 2a_1\delta - a_2\theta^2\right)}, \quad p_{12} = \frac{a_2}{2} - \frac{\theta\left(2a_2^2\delta^2 - a_1a_2\delta\right) + a_1a_2\delta^2}{8a_2\delta\theta + 4a_1\delta - 2a_2\theta^2},$$

$$p_{22} = \frac{a_2\delta\left(a_1\delta + a_1\theta - a_2\theta^2 + 2a_2\delta\theta\right)}{8a_2\delta\theta + 4a_1\delta - 2a_2\theta^2}, \quad p_{32} = \frac{a_2\theta(3a_1\delta - a_1\theta + a_2\delta\theta)}{8a_2\delta\theta + 4a_1\delta - 2a_2\theta^2}$$

And the profits of manufacturers and parallel importer are given as follows:

$$\pi_{M1} = \frac{6a_2\delta\theta + 2a_2\delta - a_2\theta^2 - a_2\delta^2}{8\delta} - \frac{a_2^2\theta^2(5\delta-\theta)^2}{8\delta\left(-a_2\theta^2 + 4a_2\delta\theta + 2a_1\delta\right)},$$

$$\pi_{M2} = \frac{a_2\delta\left(a_1\delta + a_1\theta - a_2\theta^2 + 2a_2\delta\theta\right)^2}{4\left(-a_2\theta^2 + 4a_2\delta\theta + 2a_1\delta\right)^2}, \quad \pi_{P3} = \frac{a_2\delta^2\theta(2a_1 - a_2\theta)^2}{4\left(4a_2\delta\theta + 2a_1\delta - a_2\theta^2\right)^2}.$$

4. Comparison and managerial implications

We first compare the equilibrium results among the three cases: 'No gray markets', 'Buffer against follower's product' and 'Stimulating the competition'. All the comparison results are summarised in Tables 1–3. Note that superscript i stands for Case $i (i = 1, 2, 3)$.

From Table 1, it is interesting to find that manufacturer 1 always keeps sale quantity in high market unchanged as $\frac{1}{2}$. Moreover, the ranking of prices is $p_{12}^1 > p_{12}^3 > p_{12}^2$, which implies that although the sale quantity is kept unchanged, the price is mainly influenced by the competition from the parallel imports. The competition from parallel imports reduces the price of product 1 in high market (p_{12}). That is, when the quality of product 2 is given, the higher the perceived value of parallel imports is, the lower the price of product 1 in high market (p_{12}) is. The sales of product 2 in high market are also severely impacted by the competition from parallel imports, both the sale quantity and the price of product 2 decrease in the presence of gray markets. When the quality of product 2 is given, the higher the perceived value of parallel imports is, the lower the price and sale quantity of product 2 in high market become. Hence, we have $q_{22}^1 > q_{22}^3 > q_{22}^2$ and $p_{22}^1 > p_{22}^3 > p_{22}^2$. We can conclude that in the presence of gray markets, both the price of product 1 and the price of product 2 in high market are reduced. Finally, the price ranking of product 1 sold in low market is $p_{11}^2 > p_{11}^3 > p_{11}^1$, which implies that manufacturer 1 raises the price in market 1 to prevent parallel importation according to the competiveness of parallel imports.

In the differentiated duopoly, although the demand for parallel imports can improve the leading manufacturer's profit in low market and the sales of parallel imports can take part of market share from the competing manufacturer in high market, the presence of gray markets still hurts both the competing manufacturer's profit and the leading manufacturer's

Table 1. Comparison of quantities.

Comparison	Case 1 vs. Case 2	Case 1 vs. Case 3	Case 2 vs. Case 3
q_{11}	$q_{11}^1 > q_{11}^2$	$q_{11}^1 > q_{11}^3$	$q_{11}^2 < q_{11}^3$
q_{12}	$q_{12}^1 = q_{12}^2 = 1/2$	$q_{12}^1 = q_{12}^3 = 1/2$	$q_{12}^2 = q_{12}^3 = 1/2$
q_{22}	$q_{22}^1 > q_{22}^2$	$q_{22}^1 > q_{22}^3$	$q_{22}^2 < q_{22}^3$
q_{32}	n.a.	n.a.	$q_{32}^2 > q_{32}^3$

Note: Henceforth, 'n.a.' is short for 'not applicable'.

Table 2. Comparison of prices.

Comparison	Case 1 vs. Case 2	Case 1 vs. Case 3	Case 2 vs. Case 3
p_{11}	$p_{11}^1 < p_{11}^2$	$p_{11}^1 < p_{11}^3$	$p_{11}^2 > p_{11}^3$
p_{12}	$p_{12}^1 > p_{12}^2$	$p_{12}^1 > p_{12}^3$	$p_{12}^2 < p_{12}^3$
p_{22}	$p_{22}^1 > p_{22}^2$	$p_{22}^1 > p_{22}^3$	$p_{22}^2 < p_{22}^3$
p_{32}	n.a.	n.a.	$p_{32}^2 > p_{32}^3$

Table 3. Comparison of profits.

Comparison	Case 1 vs. Case 2	Case 1 vs. Case 3	Case 2 vs. Case 3
π_{M1}	$\pi_{M1}^1 > \pi_{M1}^2$	$\pi_{M1}^1 > \pi_{M1}^3$	$\pi_{M1}^2 < \pi_{M1}^3$
π_{M2}	$\pi_{M2}^1 > \pi_{M2}^2$	$\pi_{M2}^1 > \pi_{M2}^3$	$\pi_{M2}^2 < \pi_{M2}^3$
π_{P3}	n.a.	n.a.	$\pi_{P3}^2 > \pi_{P3}^3$

profit. The higher the perceived value of parallel imports is, the lower the manufacturers' profits are. It implies that the competing effect from parallel imports can reduce both manufacturers' profits in the differentiated duopoly setting.

Next, we summarise some important management insight in the following propositions. The proofs of these propositions are given in the Appendix 1.

Proposition 1. In the case of 'Buffer against follower's product', the profits of both manufacturer 1 and manufacturer 2 are worse off compared with the case without gray markets, i.e. $\Delta\pi_{M1} = \pi_{M1}(Case1) - \pi_{M1}(Case2) = \frac{(2a_2\theta - 2a_1 - a_2\delta)^2}{8(2a_1 - a_2\delta + 4a_2\theta)} > 0$ and $\Delta\pi_{M2} = \pi_{M2}(Case1) - \pi_{M2}(Case2) = \frac{1}{16}a_2\delta\left(1 - \frac{4(2a_1 + a_2\theta)^2}{(2a_1 - a_2\delta + 4a_2\theta)^2}\right) > 0$. In the presence of gray markets, manufacturer 1 reduces the quantity of product 1 sold in low market (the price of product 1 would rise) while keeping the sale quantity in high market unchanged, and manufacturer 2 reduces the sale quantity in high market.

Proposition 2. In the scenario of 'Stimulating the competition', the profits of both manufacturer 1 and manufacturer 2 are hurt by the presence of gray markets, i.e. $\Delta\pi_{M1} = \pi_{M1}(Case1) - \pi_{M1}(Case3) = \frac{\delta(a_2\theta - 2a_1)^2}{8\left(4a_2\delta\theta - a_2\theta^2 + 2a_1\delta\right)} > 0$ and $\Delta\pi_{M2} = \pi_{M2}(Case1) - \pi_{M2}(Case3) = \frac{1}{16}a_2\delta\left(1 - \frac{4(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))^2}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2}\right) > 0$. In the presence of gray markets, manufacturer 1 reduces the sale quantity in low market and manufacturer 2 reduces the sale quantity in high market.

Combining Proposition 2 with Proposition 1, we conjecture that no matter whether the quality of parallel imports is higher or lower than that of product 2, both manufacturers are always worse off in the presence of gray markets. We can also infer that the consumer surplus in market 1 is reduced while the consumer surplus of buying product 1 in market 2 increases when gray markets emerge.

To further investigate the manufacturers' profit loss caused by gray markets, we perform sensitivity analysis about the impact of parameters on the profit gap as shown in Table 4. Note that the proof of Table 4 is available in Appendix 1. From Table 4, it is interesting to find that the profit gaps of both manufacturer 1 and manufacturer 2 will be reduced when the quality of manufacturer 2's product increases. The explanation is as follows. When the quality of manufacturer 2 is improved, parallel imports from gray markets become less attractive for consumers. Consequently, the

Table 4. Impact of parameters on profit gap (↗: increasing, ↘: decreasing, ⊥: constant, n.a.: not applicable).

Case	Profit Gap	a_1	a_2	δ	θ
Case 1 vs. Case 2	$\Delta\pi_{M1} = \pi_{M1}(\text{Case1}) - \pi_{M1}(\text{Case2})$	↘	↗	↘	↗
	$\Delta\pi_{M2} = \pi_{M2}(\text{Case1}) - \pi_{M2}(\text{Case2})$	↘	n.a.	n.a.	↗
Case 1 vs. Case 3	$\Delta\pi_{M1} = \pi_{M1}(\text{Case1}) - \pi_{M1}(\text{Case3})$	↘	↗	↘	↗
	$\Delta\pi_{M2} = \pi_{M2}(\text{Case1}) - \pi_{M2}(\text{Case3})$	↘	↗	↘	↗

market share is mainly dominated by manufacturers 1 and 2, which increases the profits for both manufacturers. Although we cannot get analytical results at some places in Table 4, we can conjecture that in most cases when either a_1 or δ increases (either a_2 or θ decreases), the parallel importation is restricted so that the profit loss of manufacturers becomes smaller.

Finally, we compare the equilibrium results between the scenarios 'Buffer against follower's product' and 'Stimulating the competition' and we have the Propositions 3 and 4 as follows.

Proposition 3. Both manufacturers are worse off in the scenario 'Buffer against follower's product' compared with the scenario 'Stimulating the competition'. Both manufacturers will prefer the situation 'Stimulating the competition' when the gray markets emerge. In these two scenarios, both manufacturers' profits decrease when the perceived value of parallel imports becomes higher.

Proposition 4. Comparing the scenarios under gray markets setting, parallel importer is better off in the scenario 'Buffer against follower's product'. The profit of parallel importer always increases as perceived value of parallel imports becomes higher and decreases as the quality of product 2 increases.

According to Proposition 3, it is beneficial for both manufacturers if manufacture 2 could improve the quality of its products. To gain more managerial insights, we examine the impact of δ on the profit change ratios of both manufacturers 1 and 2. We consider two settings of parameters: $a_1 = 1$, $a_2 = 5$ and $\theta = 0.8$ for Table 5(i) and $a_1 = 1$, $a_2 = 5$ and $\theta = 0.6$ Table 5(ii). For each setting, we test multiple values of δ and compute the corresponding profit change ratios. For instance, we test seven values ($k = 7$) of δ for Table 5(i) with $\delta^1 = 0.6, \ldots, \delta^7 = 0.9$. π^k_{M1} represents the profit of manufacturer 1 with δ^k.

From Tables 5(i) and 5(ii) we can find that the values of profit change ratios for both manufacturers decrease as δ increases. As for manufacturer 2, the profit change ratio is the largest when the value of δ is lowest. When δ is above the quality of parallel imports θ, the profit change ratio decreases significantly. Based on this observation, manufacturer 2 should invest more on the quality improvement when the quality of product 2 is relatively low compared with the quality of parallel imports. The reason is that such an improvement of quality can result in a sufficient increase of market share for manufacture 2, which yields a high profit.

Comparing Table 5(i) with Table 5(ii), we can find that when the quality of parallel imports θ becomes higher, the impact of parameter δ on the profit change ratios becomes weaker. Therefore, the presence of gray markets discourages manufacturer 2's motivation on the improvement of product quality. In particular, when the quality of parallel imports is relatively high, manufacturer 2 has less incentive to invest in the quality improvement of its product due to the marginal increase of the profit. In short, from the view point of production research, our findings provide a valuable guideline for manufacturer 2 to make a wise investment for product design.

Finally, we perform sensitivity analysis for all three cases in Table 6. Note that the proof of Table 6 is in Appendix 1. We observe that without gray markets, the sale quantities are constants. With gray markets, the sale quantity of product 1 in market 1 and sale quantity of product 2 in market 2 are affected by all these parameters listed in Table 6.

Moreover, for Cases 2 and 3, we find that an increase in the discounted value of parallel imports always leads to an increase of price of product 1 in market 2 but a decrease of price (sale quantity) of product 2 in market 2. And the price and sale quantity of parallel imports become higher when the discounted value of parallel imports increases. In the presence of gray markets, although manufacturer 2 only sells products in the high market, manufacturer 2 may still benefit from the increase of demand in the low market.

Furthermore, we observe that with the development of quality (the increase of δ) in the high market, manufacturer 2 increases the sale quantity of product 2 and the price of product 2 also increases. In response to the quality improvement of competitor's product, manufacturer 1 increases the sale quantity in the low market and keeps the sale quantity

Table 5(i). The impact of parameter δ on the profit change ratios ($a_1 = 1$, $a_2 = 5$, $\theta = 0.8$).

δ^k	0.6	0.65	0.7	0.75	0.8	0.85	0.9
π^k_{M1}	1.05	1.03	1.01	0.99	0.96	0.93	0.90
π^k_{M2}	0.12	0.13	0.15	0.17	0.18	0.20	0.22
$\frac{\pi^{k+1}_{M1} - \pi^k_{M1}}{\pi^k_{M1}} \times 100$		−1.94	−2.04	−2.16	−2.29	−3.18	−3.29
$\frac{\pi^{k+1}_{M2} - \pi^k_{M2}}{\pi^k_{M2}} \times 100$		12.04	11.44	10.94	10.51	8.90	8.13

Table 5(ii). The impact of parameter δ on the profit change ratios ($a_1 = 1$, $a_2 = 5$, $\theta = 0.6$).

δ^k	0.5	0.55	0.6	0.65	0.7	0.75	0.8
π^k_{M1}	1.16	1.14	1.11	1.08	1.05	1.02	0.99
π^k_{M2}	0.12	0.14	0.16	0.17	0.19	0.20	0.22
$\frac{\pi^{k+1}_{M1}-\pi^k_{M1}}{\pi^k_{M1}} \times 100$		−2.08	−2.22	−2.79	−2.87	−2.96	−3.05
$\frac{\pi^{k+1}_{M2}-\pi^k_{M2}}{\pi^k_{M2}} \times 100$		14.94	14.11	10.42	9.39	8.55	7.85

Table 6. Impact of parameters on optimal solution ($\nearrow$: increasing, $\searrow$: decreasing, $\perp$: constant, n.a.: not applicable).

Case		a_1	a_2	δ	θ
Case 1	q_{11}	$\perp$	$\perp$	$\perp$	n.a.
	q_{12}	$\perp$	$\perp$	$\perp$	n.a.
	q_{22}	$\perp$	$\perp$	$\perp$	n.a.
	p_{11}	$\nearrow$	$\perp$	$\perp$	n.a.
	p_{12}	$\perp$	$\nearrow$	$\searrow$	n.a.
	p_{22}	$\perp$	$\nearrow$	$\nearrow$	n.a.
	π_{M1}	$\nearrow$	$\nearrow$	$\searrow$	n.a.
	π_{M2}	$\perp$	$\nearrow$	$\nearrow$	n.a.
Case 2	q_{11}	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
	q_{12}	$\perp$	$\perp$	$\perp$	$\perp$
	q_{22}	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
	q_{32}	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{11}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{12}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{22}	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$
	p_{32}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	π_{M1}	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
	π_{M2}	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$
	π_{P3}	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$
Case 3	q_{11}	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
	q_{12}	$\perp$	$\perp$	$\perp$	$\perp$
	q_{22}	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
	q_{32}	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{11}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{12}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	p_{22}	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$
	p_{32}	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
	π_{M1}	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
	π_{M2}	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$
	π_{P3}	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

in the high market unchanged and the prices of product 1 in both markets decrease. In addition, parallel importer reduces the sale quantity of parallel imports. This can be explained as follows. Because parallel imports are affected directly by the quality improvement of product 2, the sale quantity of parallel imports is reduced by the fiercer competition from product 2. Regarding parallel imports as one version of product 1, to maintain the market share, manufacturer 1 has to encourage more parallel imports sold in the high market to compete against manufacturer 2's product. Namely, the manufacturer increases the sale quantity in the low market and the price is correspondingly reduced, which leads to a lower cost for parallel importation. It is interesting to find that the sale quantity of product 1 in the high market is constant for all cases and is not affected by any parameters. In another word, the leading manufacturer copes with the challenges from the following manufacturer and parallel importer only by adjusting the sale quantity in the low market but never changes the sale quantity in the high market.

5. Conclusion

The paper studies the impact of gray markets on manufacturers' optimal strategies and profits in differentiated duopoly by introducing manufacturer competition in the high market. Using 'no gray markets' as benchmark scenario, we address the gray marketing issues in differentiated duopoly setting characterised by two scenarios (i.e. the case of 'buffer against follower's product' and the case of 'stimulating the competition'). By analysing these two scenarios, we obtain equilibrium results for manufacturers and parallel importer in differentiated duopoly.

The key contribution of this study is to provide a comprehensive understanding of the impact of gray markets in differentiate duopoly. In comparison of gray markets in the monopoly case, gray markets would play a completely different role in this differentiated duopoly case and the situation is much more complicated. On the one hand, parallel imports can help the leading manufacturer to enlarge the market share by competing with the follower's products. On the another hand, the parallel importer enters the market by selling unauthorised products as the competitor of the leading manufacturer, which cannibalises the sale of authorised products of the leading manufacturer. Hence, the impact of gray markets on the leading manufacturer's profit is a trade-off between the cannibalisation effect and the demand enlargement effect caused by the gray markets. Our study explicitly characterises such a trade-off. Further, in face of competition from the leading manufacturer and parallel importers, our study gives a guideline to the following manufacturer about how it should strategically design its own product with an appropriate quality in order to maintain competiveness.

Next, we highlight our findings that are different from the related literature. For the monopoly case, both Ahmadi and Yang (2000) and Xiao, Palekar, and Liu (2011) show that the manufacturer can benefit from gray markets under certain channel structures. Different from their findings, our results show that no matter whether the quality of parallel imports is higher or lower than follower's products, the leading manufacturer's profit is always worse off by the presence of gray markets. Although both manufacturers' profits are hurt by the presence of gray markets, the case 'stimulating the competition' is relatively better for them between these two kinds of gray market settings. Parallel imports with a high perceived value will decrease both manufacturers' profits compared with parallel imports with a low perceived value. And the profit of parallel importer always increases as the perceived value of parallel imports becomes high and decreases as the quality of product 2 increases.

For the duopoly case, Shavandi, Valizadeh Khaki, and Khedmati (2015) show that the profit of the manufacturer challenged by gray markets may be better off in the low market with a symmetric demand. Different from their findings, our results indicate that manufacturers suffer from a profit loss under competition in the high market with asymmetric demand. Moreover, our paper shows that the leading manufacturer can deal with the fierce competition from the following manufacturer and parallel importer only by adjusting sale quantity decision in low market. To increase the profit, the following manufacturer should make great efforts on the improving the quality of its products when the quality of parallel imports is relatively low compared with the authorised products.

As a direction of future research, one limitation of the current model is that we ignore the exogenous transaction costs. Since the exogenous transaction costs may be beneficial to manufacturers in the presence of gray markets, it may be worthy to take the transaction costs into account in the analysis of gray markets. The other limitation is our assumption of the deterministic demand. It is interesting to investigate the impact of gray market under a stochastic demand environment.

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Note

1. http://www.reuters.com/article/2014/01/15/us-huawei-results-idUSBREA0E02P20140115

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Appendix 1

Proof of proposition 1

Compared with benchmark scenario (i.e. no gray markets), we have $q_{11} < \frac{1}{2}$, $q_{22} < \frac{1}{4}$, $q_{12} = \frac{1}{2}$, $p_{11} > \frac{a_1}{2}$, $p_{12} < \left(\frac{1}{2} - \frac{\delta}{4}\right)a_2$, $p_{22} > \frac{a_2\delta}{4}$. And we obtain that $\Delta\pi_{M1} = -\frac{(2a_2\theta - 2a_1 - a_2\delta)^2}{8(2a_1 - a_2\delta + 4a_2\theta)} < 0$ and $\Delta\pi_{M2} = \frac{1}{16}a_2\delta\left(-1 + \frac{4(2a_1 + a_2\theta)^2}{(2a_1 - a_2\delta + 4a_2\theta)^2}\right) < 0$ when the constraints of $\theta > \delta$ and $(2\theta - \delta)a_2 > 2a_1$ are binding.
Here, $p_{12} - \left(\frac{1}{2} - \frac{\delta}{4}\right)a_2 = \frac{(a_2\delta - 2a_2\theta)(2a_2\theta - a_2\delta - 2a_1)}{2a_1 - a_2\delta + 4a_2\theta} < 0$ and other expressions can be obtained easily and hence the detailed derivation is omitted.

Proof of proposition 2

Compared with no gray market scenario, it is clear that $q_{11} < \frac{1}{2}$, $q_{22} < \frac{1}{4}$, $p_{11} > \frac{a_1}{2}$, $p_{22} < \frac{a_2\delta}{4}$, $\pi_{M2} < \frac{a_2\delta}{16}$.

The expressions $\Delta\pi_{M1} = -\frac{\delta(a_2\theta - 2a_1)^2}{8\left(4a_2\delta\theta - a_2\theta^2 + 2a_1\delta\right)} < 0$ and $\Delta\pi_{M2} = \frac{1}{16}a_2\delta\left(-1 + \frac{4(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))^2}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2}\right) < 0$ holds under the condition $a_2\theta > 2a_1$. Further, we obtain $p_{12} > \left(\frac{1}{2} - \frac{\delta}{4}\right)a_2$ when $\frac{a_2}{a_1} > \frac{\delta(2-\theta)}{4(1-\delta^2)}$. Then, Proposition 2 is directly followed from the above results.

Proof of proposition 3

We have $\pi_{M1} = \frac{a_2}{4} + a_1 - \frac{(2a_1 + a_2\theta)^2}{2(2a_1 - a_2\delta + 4a_2\theta)}$ $(\delta < \theta < 1)$,

$$\pi_{M1} = \frac{6a_2\delta\theta + 2a_2\delta - a_2\theta^2 - a_2\delta^2}{8\delta} - \frac{a_2^2\theta^2(5\delta - \theta)^2}{8\delta\left(-a_2\theta^2 + 4a_2\delta\theta + 2a_1\delta\right)} \quad (\theta < \delta < 1),$$

$$\pi_{M2} = \frac{a_2\delta(2a_1 + a_2\theta)^2}{4(2a_1 - a_2\delta + 4a_2\theta)^2} \quad (\delta < \theta < 1),$$

$$\pi_{M2} = \frac{a_2\delta\left(a_1\delta + a_1\theta - a_2\theta^2 + 2a_2\delta\theta\right)^2}{4\left(-a_2\theta^2 + 4a_2\delta\theta + 2a_1\delta\right)^2} \quad (\theta < \delta < 1).$$

Under the scenario 'Buffer against follower's product' $(\delta < \theta < 1)$, it requires $\frac{a_2}{a_1} > \frac{2}{2\theta - \delta}$ to satisfy $q_{32} > 0$. Under the scenario 'Stimulating the competition' $(\theta < \delta < 1)$, it requires $\frac{a_2}{a_1} > \frac{2}{\theta}$ to satisfy $q_{32} > 0$.

Hence, under the scenario 'Buffer against follower's product' $(\delta < \theta < 1)$, we have: $\frac{\partial\pi_{M1}}{\partial\theta} = \frac{a_2\left(4a_1^2 + 2a_1a_2\delta - 2a_1a_2\theta + a_2^2\delta\theta - 2a_2^2\theta^2\right)}{(2a_1 - a_2\delta + 4a_2\theta)^2} = \frac{(2a_1 + a_2\delta - 2a_2\theta)(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$ and $\frac{\partial\pi_{M2}}{\partial\theta} = -\frac{a_2^2\delta(6a_1 + a_2\delta)(2a_1 + a_2\theta)}{2(2a_1 - a_2\delta + 4a_2\theta)^3} < 0$. Under the scenario 'Stimulating the competition' $(\theta < \delta < 1)$, we have $\frac{\partial\pi_{M1}}{\partial\theta} = \frac{a_2\delta(2a_1 - a_2\theta)(3a_1\delta - a_1\theta + a_2\delta\theta)}{2\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^2} < 0$ and $\frac{\partial\pi_{M2}}{\partial\theta} = \frac{a_2\delta\left(a_1\delta + a_1\theta + 2a_2\delta\theta - a_2\theta^2\right)\left(2a_1^2\delta - 2a_1a_2\delta\theta + a_1a_2\theta^2 - 2a_2^2\delta\theta^2\right)}{2\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^3} < 0$.

When $\theta = \delta$, we can get $\pi_{M1}(\delta < \theta < 1) = \pi_{M1}(\theta < \delta < 1)$ and $\pi_{M2}(\delta < \theta < 1) = \pi_{M2}(\theta < \delta < 1)$. Then, we conclude that given the value of δ, the profits of manufacturers decrease as the value of θ increases. This implies that the parallel imports with a high perceived value will decrease both manufacturers' profits compared with parallel imports with a low perceived value. Furthermore, we can infer that both manufacturers are worse off in the scenario 'Buffer against follower's product' compared with the scenario 'Stimulating the competition'.

Proof of proposition 4

Under the scenario 'Buffer against follower's product' $(\delta < \theta < 1)$, we have:

$$\frac{\partial\pi_{P3}}{\partial\theta} = \frac{a_2(2a_2\theta - 2a_1 - a_2\delta)\left(a_2^2\delta^2 + 8a_2^2\theta^2 + 20a_1a_2\theta - 2a_2^2\delta\theta - 4a_1^2\right)}{4(2a_1 - a_2\delta + 4a_2\theta)^3} > 0 \quad \text{and}$$

$$\frac{\partial\pi_{P3}}{\partial\delta} = \frac{a_2^2\theta(2a_1 + a_2\delta - 2a_2\theta)(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^3} < 0.$$

Under the scenario 'Stimulating the competition' $(\theta < \delta < 1)$, we have: $\frac{\partial\pi_{P3}}{\partial\theta} = \frac{a_2\delta^2(a_2\theta - 2a_1)\left(14a_1a_2\delta\theta + 4a_2^2\delta\theta^2 + a_2^2\theta^3 - 4a_1^2\delta - 6a_1a_2\theta^2\right)}{4\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^3} > 0$ and $\frac{\partial\pi_{P3}}{\partial\delta} = -\frac{a_2^2\delta\theta^3(2a_1 - a_2\theta)^2}{2\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^3} < 0$.

Proof of Table 4:

By comparing Case 1 and Case 2: we obtain that $\Delta\pi_{M1} = \frac{(2a_2\theta - 2a_1 - a_2\delta)^2}{8(2a_1 - a_2\delta + 4a_2\theta)} > 0$ and $\Delta\pi_{M2} = \frac{1}{16}a_2\delta\left(1 - \frac{4(2a_1 + a_2\theta)^2}{(2a_1 - a_2\delta + 4a_2\theta)^2}\right) > 0$ with the constraints of $\theta > \delta$ and $(2\theta - \delta)a_2 > 2a_1$. And we have:

$$\frac{\partial \Delta\pi_{M1}}{\partial a_1} < 0, \frac{\partial \Delta\pi_{M1}}{\partial a_2} = \frac{(2a_1 + a_2(\delta - 2\theta))(2a_1(3\delta - 8\theta) + a_2(\delta - 4\theta)(2\theta - \delta))}{8(2a_1 - a_2\delta + 4a_2\theta)^2} > 0,$$

$$\frac{\partial \Delta\pi_{M1}}{\partial \theta} = -\frac{a_2(2a_1 + a_2(\delta - 2\theta))(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial \Delta\pi_{M1}}{\partial \delta} = \frac{a_2(2a_1 + a_2(\delta - 2\theta))(6a_1 - a_2\delta + 6a_2\theta)}{8(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial \Delta\pi_{M2}}{\partial a_1} = \frac{a_2^2\delta(\delta - 3\theta)(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^3} < 0$$

$$\frac{\partial \Delta\pi_{M2}}{\partial \theta} = \frac{a_2^2\delta(6a_1 + a_2\delta)(2a_1 + a_2\theta)}{2(2a_1 - a_2\delta + 4a_2\theta)^3} > 0$$

By comparing Case 1 and Case 3: we obtain that $\Delta\pi_{M1} = \frac{\delta(a_2\theta - 2a_1)^2}{8\left(4a_2\delta\theta - a_2\theta^2 + 2a_1\delta\right)} > 0$ and $\Delta\pi_{M2} = \frac{1}{16}a_2\delta\left(1 - \frac{4(a_2(2\delta-\theta)\theta + a_1(\delta+\theta))^2}{(2a_1\delta + a_2(4\delta-\theta)\theta)^2}\right) > 0$ with the constraint $a_2\theta > 2a_1$. And we have:

$$\frac{\partial \Delta\pi_{M1}}{\partial a_1} = \frac{\delta(2a_1 - a_2\theta)(2a_1\delta + a_2(9\delta - 2\theta)\theta)}{4(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0, \quad \frac{\partial \Delta\pi_{M1}}{\partial a_2} = \frac{\delta\theta(2a_1 - a_2\theta)(2a_1(-6\delta + \theta) + a_2\theta(-4\delta + \theta))}{8(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0$$

$$\frac{\partial \Delta\pi_{M1}}{\partial \theta} = -\frac{a_2\delta(2a_1 - a_2\theta)(3a_1\delta - a_1\theta + a_2\delta\theta)}{2\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^2} > 0 \quad \frac{\partial \Delta\pi_{M1}}{\partial \delta} = -\frac{a_2\theta^2(2a_1 - a_2\theta)^2}{8\left(-a_2\theta^2 + \delta(2a_1 + 4a_2\theta)\right)^2} < 0$$

$$\frac{\partial \Delta\pi_{M2}}{\partial a_1} = -\frac{a_2^2\delta(5\delta - \theta)\theta^2(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^3} < 0$$

$$\frac{\partial \Delta\pi_{M2}}{\partial \delta} = \frac{a_2\theta^2(2a_1 - a_2\theta)\left(4a_1^2\delta + a_2^2(4\delta - 3\theta)\theta^2 + 2a_1a_2\theta(5\delta + \theta)\right)}{16(2a_1\delta + a_2(4\delta - \theta)\theta)^3} < 0$$

$$\frac{\partial \Delta\pi_{M2}}{\partial \theta} = -\frac{a_2\delta\left(a_1\delta + a_1\theta + 2a_2\delta\theta - a_2\theta^2\right)\left(2a_1^2\delta - 2a_1a_2\delta\theta + a_1a_2\theta^2 - 2a_2^2\delta\theta^2\right)}{2\left(2a_1\delta + 4a_2\delta\theta - a_2\theta^2\right)^3} > 0$$

Proof of Table 6:

(1) As for Case 1, we have:

$$\frac{\partial p_{11}}{\partial a_1} = \frac{1}{2}, \quad \frac{\partial p_{12}}{\partial a_2} = \frac{1}{2} - \frac{\delta}{4} > 0, \frac{\partial p_{12}}{\partial \delta} = -\frac{a_2}{4}, \frac{\partial p_{22}}{\partial a_2} = \frac{\delta}{4}, \frac{\partial p_{22}}{\partial \delta} = \frac{a2}{4}$$

$$\frac{\partial \pi_{M1}}{\partial a_1} = \frac{1}{4}, \quad \frac{\partial \pi_{M1}}{\partial a_2} = \frac{1}{4} - \frac{\delta}{8}, \quad \frac{\partial \pi_{M1}}{\partial \delta} = -\frac{a_2}{8}, \quad \frac{\partial \pi_{M2}}{\partial a_2} = \frac{\delta}{16}, \quad \frac{\partial \pi_{M2}}{\partial \delta} = \frac{a_2}{16}$$

(2) As for Case 2, $(2\theta - \delta)a_2 > 2a_1$, we have:

$$\frac{\partial q_{11}}{\partial a_1} = -\frac{2a_2(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial q_{11}}{\partial a_2} = \frac{2a_1(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial q_{11}}{\partial \delta} = \frac{a_2(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial q_{11}}{\partial \theta} = -\frac{a_2(6a_1 + a_2\delta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial q_{22}}{\partial a_1} = -\frac{a_2(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial q_{22}}{\partial a_2} = \frac{a_1(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0,$$

$$\frac{\partial q_{22}}{\partial \delta} = \frac{2a_2(2a_1 + a_2\theta)}{(4a_1 - 2a_2\delta + 8a_2\theta)^2} > 0, \quad \frac{\partial q_{22}}{\partial \theta} = -\frac{a_2(6a_1 + a_2\delta)}{2(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial q_{32}}{\partial a_1} = \frac{2a_2(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0, \quad \frac{\partial q_{32}}{\partial a_2} = -\frac{2a_1(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0,$$

$$\frac{\partial q_{32}}{\partial \delta} = -\frac{a_2(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0, \quad \frac{\partial q_{32}}{\partial \theta} = \frac{a_2(6a_1 + a_2\delta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial p_{11}}{\partial a_1} = \frac{a_2^2(\delta - 4\theta)(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial p_{11}}{\partial a_2} = -\frac{2a_1^2(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0,$$

$$\frac{\partial p_{11}}{\partial \delta} = -\frac{a_1a_2(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0, \quad \frac{\partial p_{11}}{\partial \theta} = \frac{a_1a_2(6a_1 + a_2\delta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial p_{12}}{\partial a_1} = \frac{a_2^2(3\theta - \delta)(2\theta - \delta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0,$$

$$\frac{\partial p_{12}}{\partial a_2} = \frac{4a_1^2(1 - \delta + \theta) - 4a_1a_2(\delta + 2(-2 + \theta)\theta) + a_2^2(\delta - 4\theta)(\delta + 2(-2 + \theta)\theta)}{2(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial p_{12}}{\partial \delta} = -\frac{a_2(a_1 + a_2\theta)(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0,$$

$$(2\theta - \delta)a_2 > 2a_1 \Rightarrow \frac{\partial p_{12}}{\partial \theta} = \frac{a_2\left(2a_1^2 + a_1a_2(3\delta - 4\theta) + 2a_2^2(\delta - 2\theta)\theta\right)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial p_{22}}{\partial a_1} = -\frac{a_2^2\delta(\delta - 3\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial p_{22}}{\partial a_2} = \frac{\delta\left(4a_1^2 + 4a_1a_2\theta + a_2^2\theta(-\delta + 4\theta)\right)}{2(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial p_{22}}{\partial \delta} = \frac{a_2(2a_1 + a_2\theta)(a_1 + 2a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial p_{22}}{\partial \theta} = -\frac{a_2^2\delta(6a_1 + a_2\delta)}{2(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial p_{32}}{\partial a_1} = \frac{a_2^2(\delta - 3\theta)(\delta - 2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial p_{32}}{\partial a_2} = \frac{(\delta - 2\theta)\left(-4a_1^2 - 4a_1a_2\theta + a_2^2(\delta - 4\theta)\theta\right)}{2(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial p_{32}}{\partial \delta} = -\frac{a_2(a_1 + a_2\theta)(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0, \quad \frac{\partial p_{32}}{\partial \theta} = \frac{1}{4}a_2\left(1 + \frac{(2a_1 + a_2\delta)(6a_1 + a_2\delta)}{(2a_1 - a_2\delta + 4a_2\theta)^2}\right) > 0$$

$$\frac{\partial \pi_{M1}}{\partial a_1} = \frac{a_2^2(\delta - 3\theta)^2}{(2a_1 - a_2\delta + 4a_2\theta)^2} > 0, \quad \frac{\partial \pi_{M1}}{\partial a_2} = \frac{a_1^2(4 - 8\delta + 16\theta) - 4a_1a_2(\delta + 2(-2 + \theta)\theta) + a_2^2(\delta - 4\theta)(\delta + 2(-2 + \theta)\theta)}{4(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial \pi_{M1}}{\partial \delta} = -\frac{a_2(2a_1 + a_2\theta)^2}{2(2a_1 - a_2\delta + 4a_2\theta)^2} < 0,$$

$$(2\theta - \delta)a_2 > 2a_1 \Rightarrow \frac{\partial \pi_{M1}}{\partial \theta} = \frac{a_2(2a_1 + a_2(\delta - 2\theta))(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^2} < 0$$

$$\frac{\partial \pi_{M2}}{\partial a_1} = \frac{a_2\delta(2a_1 + a_2\theta)^2}{4(2a_1 - a_2\delta + 4a_2\theta)^2} > 0$$

$$\frac{\partial \pi_{M2}}{\partial a_2} = \frac{\delta(2a_1 + a_2\theta)\left(4a_1^2 + 2a_1a_2(\delta - \theta) + a_2^2\theta(-\delta + 4\theta)\right)}{4(2a_1 - a_2\delta + 4a_2\theta)^3} > 0$$

$$\frac{\partial \pi_{M2}}{\partial \delta} = \frac{a_2(2a_1 + a_2\theta)^2(2a_1 + a_2(\delta + 4\theta))}{4(2a_1 - a_2\delta + 4a_2\theta)^3} > 0$$

$$\frac{\partial \pi_{M2}}{\partial \theta} = \frac{a_2^2\delta(6a_1 + a_2\delta)(2a_1 + a_2\theta)}{2(-2a_1 + a_2(\delta - 4\theta))^3} < 0$$

$$\frac{\partial \pi_{P3}}{\partial a_1} = \frac{2a_2^2(2a_1 + a_2(\delta - 2\theta))(\delta - 3\theta)\theta}{(-2a_1 + a_2(\delta - 4\theta))^3} < 0$$

$$\frac{\partial \pi_{P3}}{\partial a_2} = \frac{\theta(-2a_1 - a_2\delta + 2a_2\theta)\left(-4a_1^2 + a_2^2(\delta - 4\theta)(\delta - 2\theta) + 4a_1a_2(-2\delta + 5\theta)\right)}{4(2a_1 - a_2\delta + 4a_2\theta)^3} > 0$$

$$\frac{\partial \pi_{P3}}{\partial \delta} = \frac{a_2^2(2a_1 + a_2(\delta - 2\theta))\theta(2a_1 + a_2\theta)}{(2a_1 - a_2\delta + 4a_2\theta)^3} < 0,$$

$$\frac{\partial \pi_{P3}}{\partial \theta} = -\frac{a_2(2a_1 + a_2(\delta - 2\theta))\left(-4a_1^2 + 20a_1a_2\theta + a_2^2\left(\delta^2 - 2\delta\theta + 8\theta^2\right)\right)}{4(2a_1 - a_2\delta + 4a_2\theta)^3} > 0$$

(3) As for Case 3, $a_2\theta > 2a_1$, we have:

$$\frac{\partial q_{11}}{\partial a_1} = \frac{a_2\delta(5\delta - \theta)\theta}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0, \quad \frac{\partial q_{11}}{\partial a_2} = \frac{a_1\delta\theta(-5\delta + \theta)}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0,$$

$$\frac{\partial q_{11}}{\partial \delta} = \frac{a_2\theta^2(-2a_1 + a_2\theta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0, \quad \frac{\partial q_{11}}{\partial \theta} = -\frac{a_2\delta\left(10a_1\delta - 4a_1\theta + a_2\theta^2\right)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0$$

$$\frac{\partial q_{22}}{\partial a_1} = \frac{a_2(5\delta - \theta)\theta^2}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0, \quad \frac{\partial q_{22}}{\partial a_2} = \frac{a_1\theta^2(-5\delta + \theta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0,$$

$$\frac{\partial q_{22}}{\partial \delta} = \frac{\theta(-2a_1 + a_2\theta)(a_1 + 2a_2\theta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0, \quad \frac{\partial q_{22}}{\partial \theta} = \frac{2a_1\delta(a_1 - a_2\theta) + a_2\theta^2(a_1 - 2a_2\delta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0$$

$$\frac{\partial q_{32}}{\partial a_1} = \frac{a_2\delta\theta(-5\delta + \theta)}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0, \quad \frac{\partial q_{32}}{\partial a_2} = \frac{a_1\delta(5\delta - \theta)\theta}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0$$

$$\frac{\partial q_{32}}{\partial \delta} = -\frac{a_2\theta^2(-2a_1 + a_2\theta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0, \quad \frac{\partial q_{32}}{\partial \theta} = \frac{a_2\delta\left(10a_1\delta - 4a_1\theta + a_2\theta^2\right)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0$$

$$\frac{\partial p_{11}}{\partial a_1} = \frac{a_2^2\theta^2\left(20\delta^2 - 9\delta\theta + \theta^2\right)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0, \quad \frac{\partial p_{11}}{\partial a_2} = \frac{a_1^2\delta(5\delta - \theta)\theta}{(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0$$

$$\frac{\partial p_{11}}{\partial \delta} = \frac{a_1a_2\theta^2(2a_1 - a_2\theta)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} < 0, \quad \frac{\partial p_{11}}{\partial \theta} = \frac{a_1a_2\delta\left(10a_1\delta - 4a_1\theta + a_2\theta^2\right)}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^2} > 0$$

$$\frac{\partial p_{12}}{\partial a_1}=\frac{a_2^2\delta(5\delta-\theta)\theta^2}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0,$$

$$\frac{\partial p_{12}}{\partial a_2}=\frac{\left(4a_1^2\delta^2-2a_1^2\delta^3+2a_1^2\delta^2\theta+16a_1a_2\delta^2\theta-8a_1a_2\delta^3\theta-4a_1a_2\delta\theta^2+16a_2^2\delta^2\theta^2-8a_2^2\delta^3\theta^2-8a_2^2\delta\theta^3+2a_2^2\delta^2\theta^3+a_2^2\theta^4\right)}{2\left(2a_1\delta+4a_2\delta\theta-a_2\theta^2\right)^2}>0,\frac{\partial p_{12}}{\partial\delta}$$
$$=-\frac{a_2\left(2a_1^2\delta^2+4a_2^2\delta(2\delta-\theta)\theta^2+a_1a_2\theta\left(8\delta^2-2\delta\theta+\theta^2\right)\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0,$$

$$\frac{\partial p_{12}}{\partial\theta}=\frac{a_2\delta\left(2a_1^2\delta-2a_1a_2\delta\theta+a_2(a_1-2a_2\delta)\theta^2\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0$$

$$\frac{\partial p_{22}}{\partial a_1}=\frac{a_2^2\delta(5\delta-\theta)\theta^2}{2\left(2a_1\delta+4a_2\delta\theta-a_2\theta^2\right)^2}>0,$$

$$\frac{\partial p_{22}}{\partial a_2}=\frac{\delta\left(4a_1a_2\delta(2\delta-\theta)\theta+2a_1^2\delta(\delta+\theta)+a_2^2\theta^2\left(8\delta^2-6\delta\theta+\theta^2\right)\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0$$

$$\frac{\partial p_{22}}{\partial\delta}=\frac{a_2\left(2a_1^2\delta^2+a_2^2\theta^2\left(8\delta^2-4\delta\theta+\theta^2\right)-a_1a\theta\left(-8\delta^2+2\delta\theta+\theta^2\right)\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0$$

$$\frac{\partial p_{22}}{\partial\theta}=\frac{a_2\delta\left(2a_1^2\delta-2a_1a_2\delta\theta+a_2(a_1-2a_2\delta)\theta^2\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0$$

$$\frac{\partial p_{32}}{\partial a_1}=\frac{a_2^2\theta^2\left(10\delta^2-7\delta\theta+\theta^2\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0,$$

$$\frac{\partial p_{32}}{\partial a_2}=\frac{\delta\theta\left(a_1^2(6\delta-2\theta)+4a_1a_2\delta\theta+a_2^2(4\delta-\theta)\theta^2\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0$$

$$\frac{\partial p_{32}}{\partial\delta}=\frac{a_2\theta^2(2a_1-a_2\theta)(a_1+a_2\theta)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0,$$

$$\frac{\partial p_{32}}{\partial\theta}=\frac{a_2\delta\left(6a_1^2\delta-4a_1(a_1-a_2\delta)\theta+a_2(-a_1+4a_2\delta)\theta^2\right)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0$$

$$\frac{\partial\pi_{M1}}{\partial a_1}=\frac{a_2^2(5\delta-\theta)^2\theta^2}{4\left(2a_1\delta+4a_2\delta\theta-a_2\theta^2\right)^2}>0,$$

$$\frac{\partial\pi_{M1}}{\partial a_2}=\frac{-\left(4a_1a_2\delta\theta(2(-2+\delta)\delta+\theta)+a_2^2(4\delta-\theta)\theta^2(2(-2+\delta)\delta+\theta)+2a_1^2\delta\left((-2+\delta)\delta-6\delta\theta\theta^2\right)\right)}{4(2a_1\delta+a_2(4\delta-\theta)\theta)^2}>0$$

$$\frac{\partial\pi_{M1}}{\partial\delta}=-\frac{a_2(a_1(\delta-\theta)+2a_2\delta\theta)(a_2(2\delta-\theta)\theta+a_1(\delta+\theta))}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0$$

$$\frac{\partial\pi_{M1}}{\partial\theta}=-\frac{a_2\delta(-2a_1+a_2\theta)(3a_1\delta-a_1\theta+a_2\delta\theta)}{2(2a_1\delta+a_2(4\delta-\theta)\theta)^2}<0$$

$$\frac{\partial \pi_{M2}}{\partial a_1} = \frac{a_2^2 \delta(5\delta - \theta)\theta^2 (a_2(2\delta - \theta)\theta + a_1(\delta + \theta))}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^3} > 0$$

$$\frac{\partial \pi_{M2}}{\partial a_2} = \frac{\left(\delta(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))\left(2a_1^2\delta(\delta + \theta) + a_1 a\theta\left(8\delta^2 - 9\delta\theta + \theta^2\right) + a_2^2\theta^2\left(8\delta^2 - 6\delta\theta + \theta^2\right)\right)\right)}{4(2a_1\delta + a_2(4\delta - \theta)\theta)^3} > 0$$

$$\frac{\partial \pi_{M2}}{\partial \delta} = \frac{\left(a_2(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))\left(2a_1^2\delta(\delta - \theta) + a_2^2\theta^2\left(8\delta^2 - 2\delta\theta + \theta^2\right) - a_1 a_2\theta\left(-8\delta^2 + 5\delta\theta + \theta^2\right)\right)\right)}{4(2a_1\delta + a_2(4\delta - \theta)\theta)^3} > 0$$

$$\frac{\partial \pi_{M2}}{\partial \theta} = -\frac{a_2\delta\left(-2a_1^2\delta + 2a_1 a_2\delta\theta + a_2(-a_1 + 2a_2\delta)\theta^2\right)(a_2(2\delta - \theta)\theta + a_1(\delta + \theta))}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^3} < 0$$

$$\frac{\partial \pi_{P3}}{\partial a_1} = \frac{a_2^2\delta^2\theta^2(-5\delta + \theta)(-2a_1 + a_2\theta)}{(2a_1\delta + a_2(4\delta - \theta)\theta)^3} < 0$$

$$\frac{\partial \pi_{P3}}{\partial a_2} = \frac{\delta^2\theta(2a_1 - a_2\theta)\left(4a_1^2\delta + 2a_1 a_2\theta(-7\delta + \theta) + a_2^2\theta^2(-4\delta + \theta)\right)}{4(2a_1\delta + a_2(4\delta - \theta)\theta)^3} > 0$$

$$\frac{\partial \pi_{P3}}{\partial \delta} = -\frac{a_2^2\delta\theta^3(-2a_1 + a_2\theta)^2}{2(2a_1\delta + a_2(4\delta - \theta)\theta)^3} < 0, \quad \frac{\partial \pi_{P3}}{\partial \theta} = \frac{a_2\delta^2(-2a_1 + a_2\theta)\left(-4a_1^2\delta + 2a_1 a_2(7\delta - 3\theta)\theta + a_2^2\theta^2(4\delta + \theta)\right)}{4(2a_1\delta + a_2(4\delta - \theta)\theta)^3} > 0$$