

University of Groningen

## Local Operators in the Eternal Black Hole

Papadodimas, Kyriakos; Raju, Suvrat

*Published in:*  
Physical Review Letters

*DOI:*  
[10.1103/PhysRevLett.115.211601](https://doi.org/10.1103/PhysRevLett.115.211601)

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2015

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*  
Papadodimas, K., & Raju, S. (2015). Local Operators in the Eternal Black Hole. *Physical Review Letters*, 115(21), [211601]. <https://doi.org/10.1103/PhysRevLett.115.211601>

### Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

### Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

## Local Operators in the Eternal Black Hole

Kyriakos Papadodimas\*

*Van Swinderen Institute for Particle Physics and Gravity, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands and Theory Group, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*

Suvrat Raju†

*International Centre for Theoretical Sciences, Tata Institute of Fundamental Research, IISc Campus, Bengaluru 560012, India*

(Received 9 May 2015; revised manuscript received 4 October 2015; published 16 November 2015)

In the AdS/CFT correspondence, states obtained by Hamiltonian evolution of the thermofield doubled state are also dual to an eternal black-hole geometry, which is glued to the boundary with a time shift generated by a large diffeomorphism. We describe gauge-invariant relational observables that probe the black hole interior in these states and constrain their properties using effective field theory. By adapting recent versions of the information paradox we show that these observables are necessarily described by state-dependent bulk-boundary maps, which we construct explicitly.

DOI: [10.1103/PhysRevLett.115.211601](https://doi.org/10.1103/PhysRevLett.115.211601)

PACS numbers: 11.25.Tq, 04.70.Dy, 89.70.Cf

*Introduction.*—There has been significant recent debate on whether the AdS/CFT correspondence [1] can meaningfully describe the interior of a black hole. The authors of [2,3] [Almheiri, Marolf, Polchinski, Stanford, and Sully (AMPSS)] argued, following Mathur [4], that the CFT did not contain operators with the right properties to play the role of local perturbative excitations in the interior of a black hole. However, in [5,6], we showed how to construct such operators to leading order in the  $1/N$  expansion. Our construction circumvented the AMPSS arguments by allowing the map between bulk and boundary operators to be state dependent. In this Letter, we show that versions of the AMPSS paradoxes also appear in the eternal black hole and must, once again, be resolved using state-dependent bulk-boundary maps.

We will assume that the eternal black hole has a smooth interior and is dual to a particular entangled state of two decoupled conformal field theories [7],

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_E e^{-(\beta E/2)} |E, E\rangle,$$

where the sum is over all energy eigenstates,  $\beta$  is the inverse temperature of the black hole, and the partition function is  $Z(\beta) = \text{Tr}(e^{-\beta H_R})$ .

We will first argue that if one accepts this duality, then it follows that a much broader class of states is dual to the same “geometry,” but glued differently to the boundary. These states are obtained by evolving  $|\Psi\rangle$  with either of the two boundary Hamiltonians for a time  $T$ ,

$$|\Psi_T\rangle = e^{iH_L T} |\Psi\rangle = e^{iH_R T} |\Psi\rangle. \quad (1)$$

Using a variant of the arguments of [2,3], we show that it is impossible to find a global linear map between bulk and boundary fields that reproduces the predictions of effective field theory behind the horizon for all states in the class (1).

We show this, by expanding on the work of AMPSS, in order to reveal a paradox that can be phrased in terms of a precise question regarding the existence of operators in the CFT with certain properties. In this, our method differs from work [8,9] that also discussed some subtleties of the map between the thermofield doubled state and the eternal black hole.

Our method also allows us to clarify the assumptions that are required to prove the nonexistence of a global bulk-boundary map. In particular, our result depends on intermediate inferences (which we justify but do not prove) that the gravity dual of an unentangled pair of eigenstates does not have any wormhole and that the thermofield state remains smooth even after an exponentially long time.

Our conclusion is also different from Refs. [8,9], which argued that the thermofield-eternal black-hole duality is either incomplete or incorrect. Instead, we show that if we consider a given state from (1) and small fluctuations about this state, then, as we show in the following, we can explicitly write down boundary operators that are dual to local bulk operators.

In recent work, Marolf and Polchinski [10] have argued that if generic pure states in a single CFT are to have smooth horizons in a state-dependent description, then states with near-unit product in the Hilbert space must be physically distinct. While we will address this in forthcoming work, we note that the discussion of the eternal black hole in this Letter is logically independent of the validity of these arguments. Moreover, the state-dependent

---

*Published by the American Physical Society under the terms of the Creative Commons Attribution 3.0 License. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.*

interior operators that we write down explicitly for the class of states (1) do not suffer from the Born-rule violations that are discussed in [10].

Therefore, an important result of this Letter is that state-dependent bulk-boundary maps—which were also explored in [11,12], and which are discussed in greater detail in [13]—are necessary to preserve quantum effective field theory for the infalling observer and also are sufficient to resolve extensions of the AMPSS paradoxes. This suggests that state dependence is a broader feature of local operators in quantum gravity.

*Review of the eternal black hole.*—The metric of the eternal AdS<sub>d+1</sub>-Schwarzschild black hole is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_{d-1}^2,$$

where  $f(r) = r^2 + 1 - c_d GM r^{2-d}$ , and  $c_d = 8(d-1)^{-1} \pi^{(2-d)/2} \Gamma(d/2)$ .

We introduce the tortoise coordinate  $dr_*/dr = f(r)^{-1}$  so that the right boundary is at  $r_* = 0$  and the future horizon is at  $r_* \rightarrow -\infty$ ,  $t \rightarrow \infty$ . This metric can be smoothly extended past the horizon by defining the Kruskal variables  $U = -e^{(2\pi/\beta)(r_*-t)}$  and  $V = e^{(2\pi/\beta)(r_*+t)}$ . The future horizon is then at  $U = 0$ , with  $V$  finite. The past horizon is at  $V = 0$ , with  $U$  finite.

The quadrant connected to the right boundary is called region I. We can also introduce Schwarzschild coordinates in the other quadrants of the extended geometry. In region II, inside the black hole, we write  $U = e^{(2\pi/\beta)(r_*-t)}$ ,  $V = e^{(2\pi/\beta)(r_*+t)}$ , and in region III, connected to the left asymptotic region, we have  $U = e^{(2\pi/\beta)(r_*-t)}$ ,  $V = -e^{(2\pi/\beta)(r_*+t)}$ . The two boundaries are at  $UV = -1$ .

It is important to note that the geometry is “glued” to the left CFT with a flip in the time coordinate in region III. Therefore, while the time in CFT<sub>R</sub> is identified as  $t_R = t$ , the time in CFT<sub>L</sub> is identified as  $t_L = -t$ . Correspondingly, the isometry of the geometry, generated by  $t \rightarrow t + T$  in all regions of spacetime, is dual to the identity  $e^{i(H_R - H_L)T} |\Psi\rangle = |\Psi\rangle$ .

*Time-shifted thermofield states.*—We now consider the time-shifted thermofield states defined in (1). In any theory of quantum gravity, evolution with a boundary Hamiltonian simply corresponds to a large diffeomorphism in the bulk that does not die off at the boundary [14].

The precise action of the CFT Hamiltonian in the bulk is a gauge-dependent quantity, because the Dirac brackets between the boundary Hamiltonian and bulk operators depend on the gauge-fixing conditions. However, on the boundary, its action is gauge invariant: the action of  $e^{iH_L T}$  corresponds to a large diffeomorphism that induces a flow on the left boundary,  $t_L \rightarrow t_L + T$ ; correspondingly,  $e^{iH_R T}$  includes a flow on the right boundary,  $t_R \rightarrow t_R + T$ .

It is easy to find explicit examples of such diffeomorphisms:  $e^{iH_L T}$  can be implemented by

$$\begin{aligned} U &\rightarrow U[\gamma\hat{\theta}(-X) + \hat{\theta}(X)]; \\ V &\rightarrow \frac{V}{\gamma}[\hat{\theta}(-X) + \gamma\hat{\theta}(X)], \end{aligned} \quad (2)$$

where  $X = V - U$ ,  $\gamma = e^{2\pi T/\beta}$ , and  $\hat{\theta}$  is a smooth version of the theta function. We set  $\hat{\theta}(x) = \theta(x)$  for  $|x| > \epsilon$ , and the function makes a smooth transition between its values in the range  $[-\epsilon, \epsilon]$  where  $\epsilon \ll 1$ .

In general, such a diffeomorphism changes the state. In defining the theory we do not mod out by diffeomorphisms that act nontrivially on the boundary. However, we do mod out by trivial diffeomorphisms: diffeomorphisms with the same boundary action—even if they differ in the bulk—must be identified. The reader may find it useful to recall the Brown-Henneaux analysis [15], where the global AdS<sub>3</sub> vacuum was excited by large diffeomorphisms corresponding to the Virasoro algebra. Therefore, the diffeomorphism in (2) is not unique; it is a representative of an equivalence class of diffeomorphisms which all have the common property that they “slide” the left boundary by  $T$ .

Since  $e^{iH_L T}$  is just a diffeomorphism, it evidently leaves all quantities that are intrinsic to the bulk geometry invariant. The specific statement that we will be interested in below is that an infalling observer from the right perceives a smooth horizon in all states  $|\Psi_T\rangle$ . This point was also made in the recent paper [16], and was discussed in [17].

Because we did not use the classical equations of motion anywhere in reaching this conclusion, and our only input was the interpretation of Hamiltonian evolution as a large diffeomorphism, we conjecture that this statement is *exact*. The states  $|\Psi_T\rangle$  are smooth even for  $T = O(e^S)$ , where  $S$  is the entropy of the black hole. Note that these exponentially long times are still parameterically smaller than the Poincaré recurrence time [18], which does not play a role in our discussion.

To support this strong claim, we provide two other perspectives. Using the isometry (1) of the thermofield state, the experience of the right infalling observer jumping into the state  $|\Psi_T\rangle$  at  $t_R = 0$  is the same as the experience of the right observer jumping into the state  $|\Psi\rangle$  at  $t_R = -T$ . From the geometry, it is clear that the right observer’s experience is independent of the time at which he jumps in. If we accept this as an exact statement, then it follows that  $|\Psi_T\rangle$  is smooth for all  $T$ .

Second, from the point of view of the CFT, we would like to treat the state  $|\Psi_T\rangle$  on the same footing as  $|\Psi\rangle$  because there is no natural common origin of time in the two CFTs; the new states correspond to using a shifted time origin on the left. We explore this in more detail in [13].

It is also useful to recognize that the states  $|\Psi_T\rangle$  are phase-shifted states in the CFT,

$$|\Psi_T\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_E e^{(-\beta E/2)} e^{i\phi[E]} |E, E\rangle, \quad (3)$$

where  $\phi[E] = ET \bmod 2\pi$ . In fact, since the spectrum of the CFT is chaotic at high energies [19], we can choose  $T$  to approximate any desired phase to arbitrary accuracy for a conformal primary. However, since the energy of conformal descendants is integrally quantized, we require that, within an irreducible representation,

$$\phi[E] - \phi[E + 1] = \phi[E + 1] - \phi[E + 2] \bmod 2\pi.$$

Supersymmetric states also have integrally quantized energies, but they are exponentially unimportant in (3). The advantage of this perspective is that the phases corresponding to  $T \sim O(1)$  are not qualitatively different from the phases generated by exponentially long  $T$ . This makes it natural that  $|\Psi_T\rangle$  represents a smooth geometry even for long times.

*Relational observables.*—In the presence of gravity, we cannot assume that the coordinates are fixed as one changes the state. Instead we must work with gauge-invariant relational observables. These are not strictly local, but they behave like local operators for many purposes [20]. Intuitively, we want to start from a point on the boundary and follow a null geodesic for a given affine parameter to relationally specify a point in the bulk. To normalize the affine parameter, we must be more careful.

First, starting with a given boundary point  $(t_R, \Omega)$ , we consider a null geodesic parameterized by ordinary asymptotic Schwarzschild time  $t$ , and with no initial velocity along  $S^{d-1}$ :  $\dot{\Omega} = 0$ . We can then specify all points in front of the horizon as intersection points of this geodesic with another geodesic that hits the boundary at a later time  $(t'_R, \Omega')$  with no final velocity  $\dot{\Omega}' = 0$ . The value of  $\Omega'$  must be chosen to ensure these geodesics intersect.

We can now use these points to normalize the affine parameter [13] of a null geodesic, and follow it into the horizon for some affine time. All points in regions I and II can be reached in this manner.

Now, let us consider the field  $\phi$  as measured at one of these points, which we will denote by  $\phi^{\text{rel},R}(t_R, \lambda)$  suppressing the dependence on the  $S^{d-1}$  which is interesting [13] but not relevant for our discussion. The qualifier “rel, R” indicates that this is a relational observable defined with respect to the right boundary.

We note that  $\phi^{\text{rel},R}(t_R, \lambda)$  is invariant under any diffeomorphism that leaves the right boundary invariant, including diffeomorphisms that do not vanish at the left boundary. We can see this in several ways. By means of a trivial diffeomorphism—one that leaves both boundaries invariant—we can transform any diffeomorphism that induces a flow on the left boundary into one that vanishes everywhere except for a small region localized infinitesimally close to the left boundary. Thus, it leaves the experience of the right observer unchanged. More formally, consider a diffeomorphism taking bulk points  $x \rightarrow g(x)$ . We can equivalently represent this as an action on the fields  $\phi(x) \rightarrow \phi[g^{-1}(x)]$  and as corresponding actions on the metric and higher spin fields. On the other hand, the solution to the geodesic equation for a given affine parameter  $\lambda$  (determined as above) transforms under the new connection coefficients as  $x(\lambda) \rightarrow g[x(\lambda)]$ . The relational observable  $\phi^{\text{rel},R}(t_R, \lambda)$  is thus left invariant.

Furthermore, diffeomorphisms that induce a flow along the right boundary just shift the value of  $t_R$ . Together, this implies that

$$\begin{aligned} e^{iH_L T} \phi^{\text{rel},R}(t_R, \lambda) e^{-iH_L T} &= \phi^{\text{rel},R}(t_R, \lambda), \\ e^{iH_R T} \phi^{\text{rel},R}(t_R, \lambda) e^{-iH_R T} &= \phi^{\text{rel},R}(t_R + T, \lambda). \end{aligned} \quad (4)$$

Now, by solving the geodesic equation in the metric given by the eternal black hole, we can trade the parameters  $t_R, \lambda$  for the usual Kruskal coordinates,  $U(t_R, \lambda), V(t_R, \lambda)$ . Near the horizon,  $U = 0$ , the field can be expanded in creation and annihilation operators

$$\begin{aligned} \lim_{U \rightarrow 0^-} \phi^{\text{rel},R}(U, V) &= \sum_{\omega_n} \omega_n^{-1/2} [a_{R\omega_n}^{\text{rel}} (e^{i\delta_n} U^{i\beta\omega_n/2\pi} + e^{-i\delta_n} V^{-i\beta\omega_n/2\pi}) + \text{H.c.}], \\ \lim_{U \rightarrow 0^+} \phi^{\text{rel},R}(U, V) &= \sum_{\omega_n} \omega_n^{-1/2} [\tilde{a}_{R\omega_n}^{\text{rel}} e^{-i\delta_n} U^{-i\beta\omega_n/2\pi} + a_{R\omega_n}^{\text{rel}} e^{-i\delta_n} V^{-i\beta\omega_n/2\pi} + \text{H.c.}], \end{aligned} \quad (5)$$

where  $\delta_n$  depends on details of scattering in the black-hole geometry [5]. The relations (4) now translate into

$$\begin{aligned} [H_R, a_{R\omega_n}^{\text{rel}}] &= -\omega_n a_{R\omega_n}^{\text{rel}}, & [H_L, a_{R\omega_n}^{\text{rel}}] &= 0, \\ [H_R, \tilde{a}_{R\omega_n}^{\text{rel}}] &= \omega_n \tilde{a}_{R\omega_n}^{\text{rel}}, & [H_L, \tilde{a}_{R\omega_n}^{\text{rel}}] &= 0. \end{aligned}$$

These surprising commutation relations show that the naive construction of local bulk operators in terms of boundary operators is incorrect. In particular, we may still identify  $a_{R\omega_n}^{\text{rel}}$  with the Fourier modes of an appropriate single trace operator  $\mathcal{O}$  in the right CFT [21],  $a_{R\omega_n}^{\text{rel}} = C_\beta^{-1/2} \mathcal{O}_{R\omega_n}$ , with

$$\mathcal{O}_{R\omega_n} = T_b^{-1/2} \int_{-T_b}^{T_b} \mathcal{O}_R(t_R) e^{i\omega_n t_R} dt_R,$$

$$\mathcal{C}_\beta = \frac{1}{Z(\beta)} \text{Tr}(e^{-\beta H_R} [\mathcal{O}_{R\omega_n}, \mathcal{O}_{R\omega_n}^\dagger]),$$

where  $T_b$  specifies a time band that we can use to define slightly “smeared” Fourier modes [13].

However, we cannot use  $a_{L\omega_n} = \mathcal{C}_\beta^{-1/2} \mathcal{O}_{L\omega_n}$  in place of  $\tilde{a}_{R\omega_n}^{\text{rel}}$  in (5). This has the wrong commutator with the Hamiltonian and the wrong two-point function,

$$\langle \Psi_T | a_{L\omega_n} a_{R\omega_n}^{\text{rel}} | \Psi_T \rangle = (1 - e^{-\beta\omega_n})^{-1} e^{i\omega_n T - (\beta\omega/2)} \delta_{\omega_n, \omega_n'}.$$

A short calculation shows that  $\tilde{a}_{R\omega_n}^{\text{rel}}$  must have a similar two-point function, but without the factor of  $e^{i\omega_n T}$ ; this is essential to reproduce the two-point function of a field propagating about a smooth horizon.

We now proceed to show that *no* state-independent operator in the CFT can play the role of  $\tilde{a}_{R\omega_n}^{\text{rel}}$ .

*Paradoxes in the eternal black hole.*—Consider the number operator as measured by the right infalling observer. With  $c \equiv a_{R\omega_n}^{\text{rel}} - e^{-\beta\omega_n/2} (\tilde{a}_{R\omega_n}^{\text{rel}})^\dagger$  and  $d \equiv \tilde{a}_{R\omega_n}^{\text{rel}} - e^{-\beta\omega_n/2} (a_{R\omega_n}^{\text{rel}})^\dagger$ , we define

$$N_a = (1 - e^{-\beta\omega_n})^{-1} (c^\dagger c + d^\dagger d).$$

At a smooth horizon the infalling observer expects to encounter no particles,

$$\langle \Psi_T | N_a | \Psi_T \rangle = O(S^{-1}),$$

except for small quantum fluctuations, proportional to a power of the entropy  $S$  that are also independent of time.

If we take the long time average

$$\frac{1}{2T_{\text{av}}} \int_{-T_{\text{av}}}^{T_{\text{av}}} \langle \Psi_T | N_a | \Psi_T \rangle dT = \sum_E \frac{e^{-\beta E}}{Z(\beta)} \langle E, E | N_a | E, E \rangle$$

$$+ \sum_{E \neq E'} \frac{e^{-(\beta/2)(E+E')}}{Z(\beta)} \frac{\sin[(E' - E)T_{\text{av}}]}{(E' - E)T_{\text{av}}} \langle E, E | N_a | E', E' \rangle$$

with the averaging time,  $T_{\text{av}}$ , large enough, we see that this is only possible if

$$\sum_E \frac{e^{-\beta E}}{Z(\beta)} \langle E, E | N_a | E, E \rangle = O(S^{-1}).$$

However, since this is true for all  $\beta$  above the Hawking-Page transition temperature, we can do a Legendre transform, and we conclude

$$\langle E, E | N_a | E, E \rangle = O(S^{-1}),$$

for typical energy eigenstates  $E$  relevant to the black hole.

Note that while we used the number operator,  $N_a$ , in the reasoning above, we could have used some other operator to detect the smoothness of the horizon. The point is that if a state-independent operator predicts that the thermofield state and its time-shifted cousins all have regular interiors, then this operator also predicts that eigenstate pairs of the form  $|E, E\rangle$  are smooth.

The authors of [2,3] argued, however, that state-independent operators cannot describe the black hole interior in individual energy eigenstates  $|E\rangle$  of a single CFT. How can the interior of energy eigenstate pairs  $|E, E\rangle$ —which do not even have any entanglement—then be described by such operators?

To sharpen this question, we additionally assume that the absence of entanglement implies that eigenstate pairs have no wormhole [17,21]. Therefore, no experiment on the left should affect the right infalling observer,

$$\langle E, E | U_L^\dagger N_a U_L | E, E \rangle = \langle E, E | N_a | E, E \rangle, \quad \forall U_L.$$

By suitably selecting  $U_L$ , we infer

$$\langle E', E | N_a | E', E \rangle = O(S^{-1}), \quad (6)$$

where  $E'$  is an *independent* fixed typical energy eigenstate in the left CFT.

We now immediately run into the AMPSS paradoxes. For example, if we denote the orthonormal eigenstates of the number operator  $a_{\omega_n}^\dagger a_{\omega_n}$  by  $|N_j\rangle$ , then we expect

$$\langle E', N_j | N_a | E', N_j \rangle = O(1).$$

This is inconsistent with (6) by a change of basis [3].

To see another paradox, consider a band of energies  $(E - \Delta, E + \Delta)$  that contains  $\mathcal{D}$  eigenstates; let  $P_{E,\Delta,R}$  and  $P_{E,\Delta,L}$  be the projector onto this subspace in the right and left CFTs, respectively. With  $\tilde{N}_n \equiv \tilde{a}_{\omega_n}^\dagger \tilde{a}_{\omega_n}$ ,

$$\text{Tr}(P_{E,\Delta,L} P_{E,\Delta,R} \tilde{N}_n) = \text{Tr}(P_{E,\Delta,L} P_{E+\omega_n,\Delta,R} (\tilde{N}_n + 1)),$$

where we have used the cyclicity of the trace, and the expected commutator of the mirror operator with its adjoint and with the two Hamiltonians. The number of states in the shifted band  $E + \omega_n \pm \Delta$  is  $\mathcal{D}e^{\beta\omega_n}$ . A little algebra leads to

$$\langle E', E | \tilde{N}_n | E', E \rangle = -(1 - e^{-\beta\omega_n})^{-1}.$$

This negative expectation for a manifestly positive operator is absurd and suggests that there is no state-independent operator  $\tilde{a}_{\omega_n}$  with the expected commutation relations.

*State-dependent construction of the interior.*—To correctly construct the interior of the eternal black hole, we must use state-dependent operators, and we drop the requirement that the *same* operators describe the right relational observables in all time-shifted states. These

operators can be obtained as a solution to the linear equations presented in [6]. We present this solution below.

Consider the space of states formed by exciting a given time-shifted state by operators  $A_\alpha$  that can be written as polynomials comprising  $O(1)$  products of single trace operators. This space is defined more precisely in [6]. We write the projector onto this space as  $\hat{P}_T$ ,

$$\begin{aligned}\hat{P}_T A_\alpha |\Psi_T\rangle &= A_\alpha |\Psi_T\rangle; & \langle v | A_\alpha | \Psi_T \rangle &= 0, \\ \forall \alpha &\Rightarrow \hat{P}_T |v\rangle = 0.\end{aligned}$$

The time-shifted states are mutually almost orthogonal

$$|\langle \Psi_T | \Psi \rangle|^2 = (1 + O(S^{-1})) e^{-T^2 C / \beta^2}, \quad (7)$$

where  $C \propto S$  is the specific heat of the CFT, and the expression is valid for  $T \ll 1$ .

Thus, by means of an  $O(1)$  cutoff,  $T_{\text{cut}}$ , we construct

$$\tilde{a}_{R\omega_n}^{\text{rel}} = \sqrt{\frac{C}{\pi\beta^2}} \int_{T-T_{\text{cut}}}^{T+T_{\text{cut}}} a_{L\omega_n}^{T_i} \hat{P}_{T_i} dT_i, \quad (8)$$

where

$$a_{L\omega_n}^{T_i} = C_\beta^{-1/2} T_b^{-1/2} \int_{-T_b}^{T_b} \mathcal{O}_L(T_i + t_L) e^{i\omega_n t_L} dt_L$$

is a smeared Fourier mode of the left field evaluated about  $T_i$ . When inserted in correlators about the state  $|\Psi_T\rangle$ , and in neighboring states obtained by acting with “reasonable” excitations (including  $e^{iHT_i}$  for  $T_i < T_{\text{cut}}$ ), it is easy to check that  $\tilde{a}_{R\omega_n}^{\text{rel}}$  satisfies the properties that we need up to  $O(S^{-1})$  corrections [13].

On the other hand, for large  $T$ , the inner product (7) does not decrease indefinitely but saturates at  $O(e^{-S/2})$ . If we attempt to take the cutoff  $T_{\text{cut}}$  to be exponentially large, then the “fat tail” from the inner product above implies that interference from distant microstates spoils the good properties of the operator. In particular, therefore, we cannot use the same operator to describe the interior of the black hole in the entire range of states given by (1).

*Conclusions.*—The different time-shifted states are distinct states in the Hilbert space but correspond to the same macroscopic geometry for the right observer; in this sense, they are like “microstates” of the eternal black hole. In this Letter, we have shown that it is impossible to construct state-independent operators that describe a smooth interior in all these microstates. As in our previous paper, given a particular time-shifted state, we can find state-dependent operators with the right properties in reasonable excitations of this state. This construction, obtained by combining (8) and (5), resolves the paradox that we presented here; in a separate work [6], we have discussed how state dependence also resolves other aspects of the information paradox.

Although other possibilities may emerge in the future, at the moment we are able to perceive only two alternatives to

state dependence in the eternal black hole. First, one may declare that the thermofield state itself lacks a smooth horizon [9,21]. However, this contradicts AdS/CFT calculations that appear to explicitly probe the interior [22]. Alternately, one may posit that the state develops a firewall after some long time. We presented several arguments to the contrary, but the strongest is that we can explicitly construct the interior using (8) even for late times.

It is an unusual, and very interesting, aspect of local operators in quantum gravity that one can prove that the “same observable,” like a field at a “given” point in space, is described by different operators in different patches of the Hilbert space. This striking feature deserves further attention.

We are particularly grateful to all members of the string theory groups at ICTS-TIFR, TIFR (Mumbai), IISc (Bangalore), CERN, and the University of Groningen. We are also grateful to the participants in the summer workshop on “Emergent Spacetime in String Theory” at the Aspen Center for Physics, which was supported by NSF Grant No. PHYS-1066293 and the Simons Foundation, and the Santa Barbara Gravity Workshop II. S. R. is partially supported by a Ramanujan Fellowship of the Department of Science and Technology (India). K. P. would like to thank the Royal Netherlands Academy of Sciences (KNAW).

---

\*kyriakos.papadodimas@cern.ch

†suvrat@icts.res.in

- [1] J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998); S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Phys. Lett. B* **428**, 105 (1998).
- [2] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, *J. High Energy Phys.* **02** (2013) 062; **09** (2013) 018.
- [3] D. Marolf and J. Polchinski, *Phys. Rev. Lett.* **111**, 171301 (2013).
- [4] S. D. Mathur, *Classical Quantum Gravity* **26**, 224001 (2009).
- [5] K. Papadodimas and S. Raju, *J. High Energy Phys.* **10** (2013) 212.
- [6] K. Papadodimas and S. Raju, *Phys. Rev. D* **89**, 086010 (2014); *Phys. Rev. Lett.* **112**, 051301 (2014).
- [7] J. M. Maldacena, *J. High Energy Phys.* **04** (2003) 021.
- [8] D. Marolf and A. C. Wall, *Classical Quantum Gravity* **30**, 025001 (2013).
- [9] S. D. Mathur, [arXiv:1402.6378](https://arxiv.org/abs/1402.6378).
- [10] D. Marolf and J. Polchinski, [arXiv:1506.01337](https://arxiv.org/abs/1506.01337).
- [11] E. Verlinde and H. Verlinde, *J. High Energy Phys.* **10** (2013) 107; [arXiv:1306.0515](https://arxiv.org/abs/1306.0515); [arXiv:1306.0516](https://arxiv.org/abs/1306.0516); [arXiv:1311.1137](https://arxiv.org/abs/1311.1137).
- [12] M. Guica and S. F. Ross, *Classical Quantum Gravity* **32**, 055014 (2015).
- [13] K. Papadodimas and S. Raju (to be published).
- [14] T. Regge and C. Teitelboim, *Ann. Phys. (N.Y.)* **88**, 286 (1974); B. S. DeWitt, *Phys. Rev.* **160**, 1113 (1967).

- [15] J. D. Brown and M. Henneaux, *Commun. Math. Phys.* **104**, 207 (1986).
- [16] G. Mandal, R. Sinha, and N. Sorokhaibam, *J. High Energy Phys.* **01** (2015) 036.
- [17] J. Maldacena and L. Susskind, *Fortschr. Phys.* **61**, 781 (2013).
- [18] J. Barbon and E. Rabinovici, *Fortschr. Phys.* **52**, 642 (2004); *J. High Energy Phys.* **11** (2003) 047.
- [19] G. Festuccia and H. Liu, *J. High Energy Phys.* **12** (2007) 027.
- [20] S. B. Giddings, D. Marolf, and J. B. Hartle, *Phys. Rev. D* **74**, 064018 (2006).
- [21] T. Banks, M. R. Douglas, G. T. Horowitz, and E. J. Martinec, [arXiv:hep-th/9808016](https://arxiv.org/abs/hep-th/9808016); I. Bena, *Phys. Rev. D* **62**, 066007 (2000); A. Hamilton, D. N. Kabat, G. Lifschytz, and D. A. Lowe, *Phys. Rev. D* **74**, 066009 (2006); **73**, 086003 (2006); in *Advances in String Theory: The First Sowers Workshop in Theoretical Physics*, AMS/IP Studies in Advanced Mathematics Vol. 44 (American Mathematical Society/International Press, Providence, 2008), pp. 850–100; M. Van Raamsdonk, [arXiv:0907.2939](https://arxiv.org/abs/0907.2939); *Gen. Relativ. Gravit.* **42**, 2323 (2010); *Classical Quantum Gravity* **28**, 065002 (2011); S. H. Shenker and D. Stanford, *J. High Energy Phys.* **03** (2014) 067; **12** (2014) 04; S. G. Avery and B. D. Chowdhury, [arXiv:1312.3346](https://arxiv.org/abs/1312.3346).
- [22] P. Kraus, H. Ooguri, and S. Shenker, *Phys. Rev. D* **67**, 124022 (2003); L. Fidkowski, V. Hubeny, M. Kleban, and S. Shenker, *J. High Energy Phys.* **02** (2004) 014; T. Hartman and J. Maldacena, *J. High Energy Phys.* **05** (2013) 014.