Short-run inter-country price formation

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Short-run inter-country price formation: An analysis of international market dependence by price-endogenized input-output model*

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ABSTRACT

The assumption of fixed input coefficients in the traditional input-output analysis is relaxed by introducing the possibility of factor substitution, and the simultaneous repercussion of prices and outputs in the world market is analyzed based on the 1990 international input-output table. Taking factor substitution into account, it is found that the repercussion of prices is not negligible and the effect of an increase in the final demand on output is more dispersed from the original country to the other economic regions than the fixed coefficient case. The findings indicate the possibility of the over-estimation of the domestic multiplier effect and the under-estimation of an extent of the inter-regional dependence in the fixed coefficient case.

Keywords: factor substitution, excess supply function, international market dependence

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1. Introduction

This study presents an attempt to analyze the market dependence between four major economic regions in the world on the basis of the input-output model. The basic viewpoint in this study, however, is somewhat different from the traditional input-output analysis that assumes fixed input coefficients. The assumption of the fixed input coefficient has been widely accepted by the scholars in this field and applied to several fields of interest in the analysis of the inter-industry dependence. As far as the production structure within a closed region is concerned, the change of input coefficients matters little at least in the short-run because input coefficients can be regarded as a technical or an engineering relationship between output and factors of production.[1] It only mattes in the long-run where a technical progress matters.

On the contrary, once we take into account the choice between domestic and imported products, the actually observed input coefficient should be regarded as the result of the economic behavior rather than the technical relationship. In other words, the products should be distinguished depending on where they are produced even if they are technologically equivalent in the production process. In this case the price matters and input coefficients may be sensitive to the price change even in the short-run with given technology level. Accordingly, in an international setting of the input-output analysis, the assumption of the fixed input coefficient should be relaxed and a more general framework may be desired. In this paper, we try to relax the assumption of fixed input coefficient on the basis of the neoclassical production function that allows for the substitution between factors of production corresponding to the change of prices.[2]

The rest of the paper is organized as follows. In section 2 the basic accounting scheme and the analytical framework of the study will be explained. In section 3 the mechanism of the simultaneous determination of output and price will be discussed based on the Jacobian matrix of the excess supply function. Also, a comparative static on price change will be performed on the basis of the estimated Jacobian matrix. In section 4, in order to clarify the economic implication of the model in this study, the impact of the increase in the final demand on the level of output is analyzed in comparison with that of the traditional Leontief model that assumes fixed input coefficients. Finally, section 5 is devoted to concluding remarks and the summary of remaining problems for future improvement.

2. Basic accounting scheme of the model

The basic accounting scheme of the study is an international input-output table, which divides the world market into the five economic regions, Japan (J), the United States (U), the European Union (E), the Asian countries (A), and the Rest of the World(R). The flow of goods and services among these five economic regions are expressed in terms of the international input-output table like Table 1.

In Table 1, $\mathbf{X}_{kl}$'s ($k = J, U, E, A$; $l = J, U, E, A, R$) are ($n\times n$) square transaction matrices, where n is the number of goods and services transacted, and subscript k and l indicate the origin and the destination of inter-regional transactions, respectively.[3] For example, $\mathbf{X}_{EE}$ presents the transaction within the European Union, while $\mathbf{X}_{UE}$ shows the imports of intermediate inputs from the United States to the European Union. Although this table is basically a nominal table, we take a conventional approach to regard this table as a real one by assuming all the prices, including the wage rates and the cost of capital, are normalized to unity in 1990.[4]

Each matrix $\mathbf{X}_{kl}$ has the following structure:

$$\mathbf{X}_{kl} = \begin{bmatrix} X_{11}^{kl} & \cdots & X_{1n}^{kl} \\ \vdots & & \vdots \\ X_{n1}^{kl} & \cdots & X_{nn}^{kl} \end{bmatrix}, \qquad (1)$$

where X_{ij}^{kl} ($i, j = 1,\ldots, n$) is the input of industry j of country l from industry i of country k.

Vectors $\mathbf{x}_k$, $\mathbf{f}_k$ and $\mathbf{v}_k$ are n-dimensional column vectors, which respectively indicate the output, final demand, and value added of country k. These vectors also take the form:

$$\mathbf{x}_k = \begin{bmatrix} X_1^k \\ \vdots \\ X_n^k \end{bmatrix}, \ \mathbf{f}_k = \begin{bmatrix} F_1^k \\ \vdots \\ F_n^k \end{bmatrix}, \ \mathbf{v}_k = \begin{bmatrix} V_1^k \\ \vdots \\ V_n^k \end{bmatrix}, \qquad (2)$$

where X_i^k, F_i^k and V_i^k are, respectively, output, final demand and value added of industry i of country k.[5]

The market equilibrium of all the products, including those of the rest of the world can be expressed by the following identities:

$$X_i^k = \sum_{l=J}^{A} \sum_{j=1}^{n} X_{ij}^{kl} + F_i^k, \qquad (k = J, U, E, A, R;\ i=1,\ldots,n) \qquad (3)$$

In the following discussion, however, for avoiding unnecessarily complicated expression with many super- and subscripts, we will principally use the sequentially re-ordered subscript without a superscript that indicates the country. For example, output vector is expressed as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^J \\ \vdots \\ \mathbf{x}^R \end{bmatrix} = \begin{bmatrix} X_1 \\ \vdots \\ X_{5n} \end{bmatrix} \tag{4}$$

where, for example, X_1 is the output of industry 1 of Japan (i = 1), and X_{n+1} is the output of industry 1 of the United States ($i = n$ +1), and so on. Similarly, the transaction matrix can also be expressed like:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{JJ} & \cdots & \mathbf{X}_{JA} \\ \vdots & & \vdots \\ \mathbf{X}_{RJ} & \cdots & \mathbf{X}_{RA} \end{bmatrix} = \begin{bmatrix} X_{11} & \cdots & X_{1,4n} \\ \vdots & & \vdots \\ X_{5n,1} & \cdots & X_{5n,4n} \end{bmatrix}, \tag{5}$$

where, for example, $X_{1,\ n+1}$ is the input from industry 1 of Japan (i = 1) to industry 1 of the United States ($j = n$ +1), and so on. In what follows we will principally use the similar notation as in (4) or (5). Accordingly, the identity of the market equilibrium in (3) can be rewritten in a more intelligible fashion as:

$$X_i = \sum_{i=1}^{4n} X_{ij} + F_i, \qquad (i=1,\ldots,5n) \tag{6}$$

In the traditional input-output model, the equilibrium level of output is determined as a solution of the simultaneous linear equation system by assuming the production function with fixed input coefficients.[6] This implies that all the markets are adjusted only by quantities, while the prices are beside the matter in hand. The determination of prices is quite otherwise with that of quantities. On the contrary to this traditional treatment of the market adjustment, this study assumes that the quantities and prices adjust all the markets simultaneously on the basis of the neoclassical production function that allows for the substitution between factors of production.

The production function of industry j is expressed explicitly as follows:

$$X_j = f_j(X_{1j}, \cdots, X_{5n,j}, L_j, K_j) \tag{7}$$

where L_j and K_j are respectively labor and capital input of industry j.

We assume that the production functions (7) are twice differentiable and homogeneous with degree one. Under the assumption of the short-run profit maximization with given level of capital input $\overline{K}_j$ [7], the demand function for all the intermediate inputs and the supply function of the output of industry j are derived respectively as follows:

$$X_{ij} = x_{ij}(p_1, \cdots, p_{5n}, w_j)\overline{K}_j, \qquad (i=1,\ldots,5n) \tag{8}$$

$$X_j = x_j(p_1, \cdots, p_{5n}, w_j)\overline{K}_j. \tag{9}$$

$$L_j = x_j(p_1, \cdots, p_{5n}, w_j)\overline{K}_j. \tag{10}$$

where p_i is the price of the output of industry i and w_j is the wage rate of industry j. It should be noted here that we do not treat the market adjustment of the labor market by the wage rate and regard the wage rate as an exogenous variable in this study. Same treatment is meted to the inputs from the Rest of the World. The product's prices of the Rest of the World are exogenous and if the demand for the products of the Rest of the World increases, it is assumed that the exactly corresponding amount is always provided. Accordingly, in the following discussion the term "price" means relative price with above exogenous prices being fixed and the market clearing mechanism are analyzed only for the product markets of the four endogenous economic regions; Japan, the United States, the European Union, and the Asian countries. The price mechanism for the products of the Rest of the World will not be considered in this study.

Thus, the excess supply of industry i is defined as:

$$E_i = X_i - \sum_{j=1}^{4n} X_{ij} - F_i. \qquad (i = 1,\ldots, 4n) \tag{11}$$

Substituting (8) and (9) into (11), we can express the excess supply of each market as a function of all the factor prices:

$$E_i = e_i(p_1,\ldots, p_{4n}; \overline{p}_{4n+1},\ldots, \overline{p}_{5n}, \overline{w}_1, \cdots, \overline{w}_{5n}, \mu_i), \quad (i = 1,\ldots, 4n) \tag{12}$$

where μ_i is a shift parameter of the excess supply function of industry i, which includes all the factors such as the scale factor of the production function, the level of technology, and the existing capital stock at the beginning of the period as well as the exogenous final demand, which are assumed to be independent from the current price change.

Prices that equate all the excess supply functions to zero are equilibrium prices.

Denoting the equilibrium prices corresponding to the specific value of the shift parameter μ_i^* as $p_1^*,\ldots,p_{4n}^*$, these prices should satisfy the following:

$$E_i = e_i(p_1^*,\ldots,p_{4n}^*;\overline{p}_{4n+1},\ldots,\overline{p}_{5n},\overline{w}_1,\cdots,\overline{w}_{5n}\mu_i^*) = 0 \quad (i = 1,\ldots, 4n) \tag{13}$$

Differentiating (13) with respect to μ_j, we obtain,

$$\mathbf{\Phi}_p\mathbf{\Gamma}_\mu + \mathbf{\Phi}_\mu = \mathbf{0}\ , \tag{14}$$

where

$$\mathbf{\Phi}_p = \begin{bmatrix} \frac{\partial E_1}{\partial p_1} & \cdots & \frac{\partial E_1}{\partial p_{4n}} \\ \vdots & & \vdots \\ \frac{\partial E_{4n}}{\partial p_1} & \cdots & \frac{\partial E_{4n}}{\partial p_{4n}} \end{bmatrix}, \mathbf{\Gamma}_\mu = \begin{bmatrix} \frac{\partial p_1}{\partial \mu_1} & \cdots & \frac{\partial p_1}{\partial \mu_{4n}} \\ \vdots & & \vdots \\ \frac{\partial p_{4n}}{\partial \mu_1} & \cdots & \frac{\partial p_{4n}}{\partial \mu_{4n}} \end{bmatrix}, \text{ and } \mathbf{\Phi}_\mu = \begin{bmatrix} \frac{\partial E_1}{\partial \mu_1} & \cdots & \frac{\partial E_1}{\partial \mu_{4n}} \\ \vdots & & \vdots \\ \frac{\partial E_{4n}}{\partial \mu_1} & \cdots & \frac{\partial E_{4n}}{\partial \mu_{4n}} \end{bmatrix}.$$

Column j in Γ_μ presents the change of the equilibrium prices induced by the change of the shift parameter of industry j. We will consider the case that a unit increase of the shift parameter of industry i only shifts its excess supply E_i downward by the same amount and does not affect the excess supply functions of other industries at all. It is easily understood that a unit increase in the final demand is a typical case of such a change in the shift parameter. In this case -Φ_μ becomes the unit matrix. Taking this property of Φ_μ into account and solving equation (14) for Γ_μ, we obtain,

$$\Gamma_\mu = \Phi^{-1}{}_p \tag{15}$$

3. Estimation of the Jacobian matrix of the excess supply function

As is seen from equation (15), in order to asses empirically the effect of the change of the shift parameters on the equilibrium prices, the Jacobian matrix of the excess supply function with respect to prices, Φ_p, should be first estimated. The (i, j) element of Φ_p can be obtained by differentiating the excess supply of industry i in equation (11) with respect to p_j as follows.

$$\frac{\partial E_i}{\partial p_j} = \frac{\partial X_i}{\partial p_j} - \sum_{k=1}^{4n} \frac{\partial X_{ik}}{\partial p_j}. \qquad (i = 1,\ldots, 4n; j = 1,\ldots, 4n) \tag{16}$$

As for the specification of the production function (7) in this study, mainly due to the data availability, we adopt a simple Cobb-Douglas function with homogeneity of degree one as follows:

$$X_j = A_j \prod_{i=1}^{5n} X_{ij}^{\alpha_{ij}} \cdot L_j^{\alpha_{Lj}} \cdot K_j^{\alpha_{Kj}}, \qquad (j = 1,\ldots, 4n) \tag{17}$$

where A_j is a scale parameter of the production function, including the level of technology, and

$$\sum_{i=1}^{5n} \alpha_{ij} + \alpha_{Lj} + \alpha_{Kj} = 1. \qquad (j = 1,\ldots, 4n) \tag{18}$$

It is well known that the Cobb-Douglas production function is a very restrictive one in the sense that it assumes unity elasticity of substitution between all the factors of production. From the empirical viewpoints, however, the estimation of the marginal effect of the prices on the factor demand in (16) goes well all the more with this restrictive feature.[8]

Under the assumption of the cost minimization with given level of output X^0_j, the price elasticity of factor demand for input X_{ij} with respect to p_k based on the Cobb-Douglas production function can be estimated as;

$$\varepsilon_{ik}^{(j)} = \left.\frac{\partial \log X_{ij}}{\partial \log p_k}\right|_{X_j = X_j^0} = s_k^{(j)} - \delta_{ik}, \tag{19}$$

where δ_{ik} is a kronecker's δ and $s^{(j)}_k$ is the relative share of input k in total cost of industry j. In the framework of the input-output analysis, the marginal effect of price k on the factor demand for input X_{ij} is estimated as:

$$\frac{\partial X_{ij}}{\partial p_k} = \varepsilon_{ik}^{(j)} \cdot \frac{X_{ij}}{p_k} = (a_{ij} - \delta_{ik}) \cdot \frac{X_{ij}}{p_k}, \tag{20}$$

where a_{ij} is the input coefficient in value term. Similar calculation can also be meted to another factors of production, L and K.

It should be noted, however, that these marginal effects (20) are calculated based on the long-run cost minimization principle but not based on the short-run profit maximization which is assumed in the general equilibrium framework. Accordingly, the above price effects should be converted to those based on the short-run profit maximization. This can be made by comparing the Slutsky equations for long-run cost minimization with given output with that for short-run profit maximization with given capital input. The method is provided at the appendix 1 to this paper. In what follows, all the discussions are based on the short-run profit maximization with given capital input.

Once we have obtained the marginal effect of the price on factor demands, the marginal effect of price on the level of the output can also be calculated as:

$$\frac{\partial X_j}{\partial p_k} = \sum_{i=1}^{5n} f_i \frac{\partial X_{ij}}{\partial p_k} + f_L \frac{\partial L_j}{\partial p_k} = \sum_{i=1}^{5n} \frac{p_i}{p_j} \frac{\partial X_{ij}}{\partial p_k} + \frac{w_j}{p_j} \frac{\partial L_j}{\partial p_k} . \tag{21}$$

Thus, substituting (20) and (21) into (16), we can calculate the (i, j) element of Φ_p numerically.

We estimated the Jacobian matrix of excess supply function Φ_p and its inverse $\Phi^{-1}{}_p$ based on the 1990 international input-output table provided by Ministry of International Trade and Industry of the government of Japan. Although the original input-output table consists of 40 sectors in each region, the domestic sectors are aggregated to 10 in this study.[9] Accordingly, the estimated Jacobian matrix becomes a 40×40 matrix.

Estimated Φ_P and $\Phi^{-1}{}_P$ are shown in Table 2 and 3, respectively. Also, the shapes of Φ_P and $\Phi^{-1}{}_P$ are visually shown in Figure 1 and 2, respectively. In spite of the sector aggregation, since Φ_P and $\Phi^{-1}{}_P$ still have 1600 elements, in what follows we will in principle examine the general feature of the estimated Jacobian matrix based on some representative elements.

Table 4 presents the diagonal elements and related statistics that reflects the principal property of the estimated Jacobian matrix, Φ_P. First, let us examine its property from the viewpoint of stability of the equilibrium. The most intuitive but economically most meaningful property of Φ_p that ensures the stability may be "Gross Substitutability" of Φ_p. Negishi (1958) and Hahn (1958) proved that if the Jacobian matrix of excess supply function has a property of "Gross Substitute," the equilibrium is locally stable. If all the diagonals of Φ_p are positive and off-diagonals are negative, Φ_p is called "Gross Substitute" matrix. As is seen from Column (1) of the table, all the diagonal elements of Φ_p are positive, but 668 out of 1560 off-diagonals are not negative. This means that the estimated Φ_p does not have a property of "Gross Substitute." Accordingly, from the viewpoint of "Gross Substitutability," the estimated Φ_p does not ensure the stability of equilibrium. It should be noted, however, that, as column (3) of the table shows, off-diagonals are in general relatively small to diagonals even if they are positive. In addition, the averages of off-diagonals in the corresponding columns are all negative as is shown in Column (4) of the table. From these observed tendency in diagonals and off-diagonals of Φ_p, we can expected that the estimated

Φ_p has an approximate property to "Gross Substitute."

It is well known that "Gross Substitutability" is a sufficient condition for stability, but not a necessary condition. In other word, even if Φ_p is not a "Gross Substitute," there will still be a possibility that our estimated system is stable.

An another important examination of the stability may be the one developed by Mackenzie (1960), who proved that the equilibrium is locally stable if and only if the Jacobian matrix of excess supply function is a dominant diagonal matrix with positive diagonals.

For some $d_1>0, d_2>0,\cdots, d_{4n}>0$, if

$$d_i\left|\frac{\partial E_i}{\partial p_i}\right| > \sum_{\substack{j=1\\ j\neq i}}^{4n} d_j\left|\frac{\partial E_i}{\partial p_i}\right| \quad , (i=1,\ldots,4n) \tag{22}$$

holds, Φ_p is a dominant diagonal matrix. This is equivalent to that the simultaneous equation,

$$\begin{bmatrix} |\partial E_1/\partial p_1| & \cdots & -|\partial E_1/\partial p_{4n}| \\ \vdots & & \vdots \\ -|\partial E_{4n}/\partial p_1| & \cdots & |\partial E_{4n}/\partial p_{4n}| \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_{4n} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_{4n} \end{bmatrix} \tag{23}$$

has the non-negative solutions $d_1\geq 0$, $d_2\geq 0,\ldots$, $d_{4n}\geq 0$, for some constant $c_1>0$, $c_2>0$, ... $c_{4n}>0$. This condition is satisfied if the inverse of the coefficient matrix of (23) is non-negative matrix. As is seen from Table 3 or 4, diagonal elements in the estimated Φ_p are all positive. Also, the inverse of the coefficient matrix of (23) based on the estimated Φ_p is non-negative matrix. Accordingly, the estimated Jacobian matrix of the excess supply function is a dominant diagonal matrix with positive diagonals. This implies that the market adjustment process in our model is stable and gives the following comparative static analysis the economic ground.

Diagonal elements of $\Phi^{-1}{}_p$ and the related statistics are shown in Table 5. Diagonal element of $\Phi^{-1}{}_p$ indicates the effect of a unit increase in the shift parameter on the price of own industry. For example, the (1, 1) element of $\Phi^{-1}{}_p$ indicates that a unit increase of the shift parameter in the industry of agriculture products in Japan increases its product price by 0.000716. The (23, 23) element of $\Phi^{-1}{}_p$, 0.000131 is also a change of the food products' price in the European Union induced by a unit increase in the shift parameter of the same industry, and so on. In what follows, we call this effect "own price effect." In general, we can conceive the relatively large impact of the upper five industries, say agriculture, energy, food products, chemical and petroleum products, and metal products in each country, except for that of trade and transportation of the Asian countries that shows 0.000315, the third largest in the corresponding country.

These impacts on prices, however, depend on the magnitude of the initial equilibrium level of prices and outputs. In this sense the relative importance of a change of the shift parameter that induces a unit excess demand, say 100 thousand dollar, may be different depending on the level of output at the initial equilibrium point. In our case we conduct comparative static by regarding the actual level of prices and outputs in 1990 as the initial equilibrium point. In order to normalize the relative importance of a unit change of the excess demand induced by a change of the shift parameter, Column (2) shows these own price effects in terms of elasticity by multiplying the actual output of corresponding industry in 1990. It can be seen from Column (2) of the table that in terms of elasticity the impacts on prices are less dispersed over industries than in terms of absolute magnitudes. On average, own price effects in terms of elasticity are large for agriculture products, energy and tertiary industries, and small for secondary industries for three developed regions, except for machinery in Japan. On the contrary, own price effects are relatively small for agriculture products and tertiary industries and large for secondary industries for Asian countries. Also it can be seen that, in general they are relatively small for manufacturing in the European Union.

An excess demand in a certain market induced by a change of the shift parameter will first raise its own price. This price change will be dispersed to other markets by affecting the input structure of the corresponding industries through substitution effect and the price change in other markets rebounds upon the price of the market where the initial price change occurs. Diagonals of $\Phi^{-1}{}_{p}$ listed in Column (1) of Table 5 are those which include all the repercussion effects described above. On the contrary, the reciprocal of diagonals of Φ_{P} listed in Column (3) can be regarded as the price change necessary to wipe out the excess demand only by its own price. In this sense, we can call this “intra-effect” of the change of the shift parameter, while the effect in Column (1) is referred to as “total effect.”

It may be of much interest to compare the intra-effect in column (3) with the total effect in column (1). As is seen from the table, the total effects are all larger than the intra-effect in our model. This implies that the effects on the price change are amplified in a positive direction for all the industries. This is easily expected from the fact that the elements in the estimated $\Phi^{-1}{}_{p}$ are all positive. Column (4) shows the ratio of total effect to intra-effect, that is to say, the expansion rate of the intra-effect. The expansion rates are systematically large for agriculture and food products for all the four regions. As is typically perceived in these two industries, if there is a large negative off diagonal in the corresponding column of $\Phi_{p,}$ the expansion rate tends to be large. This is also true for other tertiary industries.

Off diagonals of $\Phi^{-1}{}_{p}$ and the related statistics are presented in Table 6. As is seen from the column (1) of the table, the average of off diagonals are very small both in level and in elasticity for all the industries, indicating that the effects of the shift parameter on other industries are relatively small compared to those on own industry. Actually, the average ratios of off-diagonals to diagonal of the corresponding columns are in general less than one-tenth.

Index of power of dispersion, which measures the relative importance of the corresponding industry in price effect, is also shown in Column (4) of the table.[10] In general, those for primary industry or primary consumption goods industries, like food products, are relatively large. Tertiary industries do no have so strong impact on the prices as expected. Examining the power of dispersion by country, those for Japan and the Asian countries are relatively large, especially the supply and demand balance in the markets of the energy and the chemical products in Asian countries have strong impacts on other industries and regions. This indicates that Japan and the other Asian countries play an important role in the price formation in the world market.

As a final examination of the price effect, it may be of interest to discuss the effect of the change of the shift parameter on the price of the same industry but in the different country. In Table 7 each column shows the impact on the price of the same kind industry in four different economic regions. Remarkable is the correspondence between Japan and the other Asian countries. Especially, the impacts of the supply and demand balance in Japan on the other Asian countries are the largest among other regional correspondences. Same tendency can be perceived in the reverse direction between these two countries. The impacts of the European Union on the same industry in other countries' are relatively small, indicating the relative independence of this economic region from others.

It also can be seen that the effects tend to concentrate on the own country for Japan, the United States, and the European Union, while those of the Asian countries disperse more over other regions.

4. Inter-country and inter-industry dependence in production decision

There is an another detail of much interest on our market adjustment model. The effect of the increase in the final demand on the level of production can be assessed based on the estimated

$\Phi^{-1}{}_p$. As mentioned in section 4, the effect of p_k on the level of the output X_j can be calculated as:

$$\frac{\partial X_j}{\partial p_k} = \sum_{i=1}^{5n} f_i \frac{\partial X_{ij}}{\partial p_k} + f_L \frac{\partial L_j}{\partial p_k} = \sum_{i=1}^{5n} \frac{p_i}{p_j} \frac{\partial X_{ij}}{\partial p_k} + \frac{w_j}{p_j} \frac{\partial L_j}{\partial p_k}. \tag{24}$$

Accordingly, the effect of the change of the shift parameter on the level of output can be expressed based on the estimated $\Phi^{-1}{}_p$ as follows:

$$\mathbf{\Theta}_\mu = \mathbf{\Xi}_P \mathbf{\Phi}_P^{-1} \tag{25}$$

where

$$\mathbf{\Theta}_\mu = \begin{bmatrix} \frac{\partial X_1}{\partial \mu_1} & \cdots & \frac{\partial X_1}{\partial \mu_{4n}} \\ \vdots & & \vdots \\ \frac{\partial X_{4n}}{\partial \mu_1} & \cdots & \frac{\partial X_{4n}}{\partial \mu_{4n}} \end{bmatrix}, \text{ and } \mathbf{\Xi}_P = \begin{bmatrix} \frac{\partial X_1}{\partial p_1} & \cdots & \frac{\partial X_1}{\partial p_{4n}} \\ \vdots & & \vdots \\ \frac{\partial E_{4n}}{\partial p_1} & \cdots & \frac{\partial E_{4n}}{\partial p_{4n}} \end{bmatrix}.$$

As we mentioned before, since we can regard the change of the shift parameter as the increase of the final demand, an analysis of (25) can be discussed in parallel with $(\mathbf{I}\text{-}\mathbf{A})^{-1}$ of the traditional open Leontief model. The comparison of the results by two models will clarify the empirical implication of the different market clearing mechanism assumed in our model from that of the traditional Leontief model.

Diagonals in $\Xi_p\Phi^{-1}{}_p$ and $(\mathbf{I}\text{-}\mathbf{A})^{-1}$ show the "own effect" of a unit increase in the final demand on the level of output of the own industry. The own effects in our model for Japan, the United States, the European Union, and Asian countries are shown respectively in Columns (1) to (4) in the corresponding rows in Table 8. For example, the own effects of Japanese industries are shown in upper 10 rows in Column (1), that is to say, 0.635, 0.690,…, 1.057. Those for the United States are also shown in rows (11) to (20) in Column (2), that is to say, 0.729, 0.719,…, 1.024, and so on. Own effects in Leontief model are also listed in Column (5) to (8) in the same manner.

As is seen from the table, there is a distinct difference in the own effect between our model and Leontief model. In our model own effects are less than unity for 30 industries out of forty, while they are all larger than unity in Leontief model. The difference comes from the shape of the supply curve assumed in each model.

In Leontief model the supply curve is completely flat and the demand curve is vertical to the quantity-axis in price-quantity 2 dimensional plane since those are assumed to be completely independent from the prices. On the contrary, in our model the supply curve is right-upward sloping and the demand curve is right-downward sloping. In this case, the right-upward sloping supply curve has a suppressing effect on the increase of the supply through the rise of the price[11]. Thus, the own effects are in general less in our model than in Leontief model.

The substitution effect also matters in the determination of output in our model. The rise in the output price of the industry will decrease the demand for its product from other industries as an intermediate input. This may suppress the supply of its own output and increase the level of output of other industries. Due to this substitution effect, as is seen in Table 8, the impacts on the level of output of the other industries are in general larger in our model than in Leontief model.[12]

The substitution effect appears more remarkably in the effects on the total output of the industries as a whole. The column sum of $\Xi_p \Phi^{-1}{}_p$ and $(\mathbf{I}\text{-}\mathbf{A})^{-1}$ presents the effect of the increase in the final demand on the level of total output of all four economic regions. Table 9 shows this effect by regions. Columns (1) to (4) are the effects on the total outputs of the corresponding regions estimated by our model, while Columns (5) to (8) are those by Leontief model. In contrast to the own effect, the effects on total output of industries as a whole are larger in our model than Leontief model in most cases. These findings indicate that taking the factor substitution into account, the effects on the output are more dispersed to the other economic regions than the fixed coefficient case. In other words, the domestic multiplier effect would be over-evaluated in the traditional Leontief model.

5. Concluding remarks

We analyzed the market dependence between four economic regions empirically based on the price-endogenized input-output model. The estimated Jacobian matrix of the excess demand function based on the Cobb-Douglas production technology turns out to be a dominant diagonal matrix with positive diagonals, indicating that the market adjustment process in the estimated inter-country market model is at least locally stable.

The obtained results are in general reasonable both theoretically and intuitively. A comparative static on price change showed that all the prices change simultaneously in the same direction when a distortion occurs in supply and demand balance in a certain market. That is to say, occurrence of the excess demand in a certain market increases all the product prices in every country or region. In terms of elasticity, the impacts of Japan on the other Asian countries' markets are relatively large, while the correspondences between other three economic regions are moderate. It should be noted that the impacts of the European Union on the other countries' markets are small in general, indicating the relative independence of this economic region from others.

Also, a comparative static on output showed that taking the factor substitution into account, the effect of a unit increase in the final demand on the output is more dispersed to the other economic regions than the fixed coefficient case. This indicates the possibility of the leakage of the effect to the other regions and the over-estimation of the domestic multiplier effect based on the traditional Leontief model.

There are, however, a couple of important problems, which are still left being unresolved in this study. First, the adoption of the Cobb-Douglas production function due to the data availability may come into question. Although we can estimate all the parameters of this function based on the actual input-output table, the estimated parameters are not approved by the statistical test. This should be improved by applying more general functional form.

The more important problem may be a lump sum treatment of the final demand. Throughout the paper, the empirical implication of the endogenous input coefficient as a function of prices is emphasized from the viewpoint of the generalization of the fixed coefficient model. The model presented in this study, however, also can be considered to be a simple and straightforward description of the multi-region trade based on the neoclassical theory of firm. Although firm's production decision plays a very important role in the recent world trade, the trade as the final goods still occupies a significant share in the total amount of transaction. In this sense, the model should be extended to endogenize , at least, the household consumption demand of each country as a function of prices to make the model more complete.

Appendix 1 Conversion of price elasticity

In most of the multi-sector studies of production function that assumes constant returns to scale, the parameters are often estimated based on the long-run cost minimization principle with given output level. In the short-run analysis in which the level of output should be determined, however, the estimated parameters should be re-evaluated on a basis of profit maximization principle with given capital input. This appendix provides a method of conversion from the parameters on a basis of cost minimization to those on profit maximization in a general but a practical manner.

The Slutsky equation for cost minimization with given output for industry j is expressed in a matrix form as:

$$\begin{bmatrix} \lambda f_{11} & \cdots & \lambda f_{1N} & \lambda f_{1L} & \lambda f_{1K} & f_1 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ \lambda f_{N1} & \cdots & \lambda f_{NN} & \lambda f_{NL} & \lambda f_{NK} & f_N \\ \lambda f_{L1} & \cdots & \lambda f_{LN} & \lambda f_{LL} & \lambda f_{LK} & f_L \\ \lambda f_{K1} & \cdots & \lambda f_{KN} & \lambda f_{KL} & \lambda f_{KK} & f_K \\ f_1 & \cdots & f_N & f_L & f_K & 0 \end{bmatrix} \begin{bmatrix} dX_{1j} \\ \vdots \\ dX_{Nj} \\ dL_j \\ dK_j \\ d\lambda_j \end{bmatrix} = \begin{bmatrix} dp_1 \\ \vdots \\ dp_N \\ dw_j \\ dr_j \\ dX_j \end{bmatrix}, \tag{A.1}$$

where λ is the Lagrangian multiplier in the conditional optimization, f with subscripts are the corresponding first and second partial derivatives of f, and for convenience symbol N is used instead of $5n$.

Let this coefficient matrix be B_C. $\partial X_{ij}/\partial p_k$ with fixed output can be obtained by setting $dp_i=0$ (for $i \neq k$), $dw_j = dr_j = dX_j = 0$ and solving above Slutusky equation for dX_{ij}. That is to say, $\partial X_{ij}/\partial p_k$ ($i = 1, N$; $k = 1, N$) obtained in (19) of the text is the (i, k) element of B_C^{-1}. Thus, if we express B_C^{-1} in more detailed one like,

$$\mathbf{B}_C^{-1} = \left[\begin{array}{ccc|c} b_{11} & \cdots & b_{1,N+2} & b_{1,N+3} \\ \vdots & & \vdots & \vdots \\ b_{N+2,1} & \cdots & b_{N+2,N+2} & b_{N+2,N+3} \\ \hline b_{N+3,1} & \cdots & b_{N+3,N+2} & b_{N+3,N+3} \end{array}\right] = \begin{bmatrix} \mathbf{B}_*^{-1} & \mathbf{b} \\ \mathbf{b}' & b_{N+3,N+3} \end{bmatrix}, \tag{A.2}$$

we have already known empirically $\mathbf{B}_*^{-1}$.

On the other hand, the Slutsky equation for profit maximization with given capital stock is expressed as:

$$\begin{bmatrix} p_i f_{11} & \cdots & p_i f_{1N} & p_i f_{1L} \\ \vdots & & \vdots & \vdots \\ p_i f_{N1} & \cdots & p_i f_{NN} & p_i f_{NL} \\ p_i f_{L1} & \cdots & p_i f_{LN} & p_i f_{LL} \end{bmatrix} \begin{bmatrix} dX_{1j} \\ \vdots \\ dX_{Nj} \\ dL_j \end{bmatrix} = \begin{bmatrix} dp_1 - f_1 dp_i - p_i f_{1K} dK_j \\ \vdots \\ dp_N - f_N dp_i - p_i f_{NK} dK_j \\ dw_j - f_L dp_i - p_i f_{LK} dK_j \end{bmatrix} \tag{A.3}$$

Let the coefficient matrix be B_P. $\partial X_{ij}/\partial p_k$ with given capital input can also be obtained by setting dp_i=0 ($i \neq k$), $dw_j = dK_j = 0$ and solving above Slutusky equation for dX_{ij}. That is to say, $\partial X_{ij}/\partial p_k$ is the (i, k) element of $B^{-1}{}_P$.

At the maximum point of the profit, since each marginal product, f_k, equals to the ratio of the corresponding price to the output price, the right hand side of equation (A.3) becomes,

$$\begin{bmatrix} dp_1 - (p_1/p_j)\, dp_j \\ \vdots \\ dp_N - (p_N/p_j)\, dp_j \\ dw_j - (w_j/p_j)\, dp_j \end{bmatrix}. \tag{A.4}$$

Accordingly, $\partial X_{ij}/\partial p_k$ ($k = i$) with fixed capital input is obtained as an inner product of Row j of $B^{-1}{}_P$ and the vector

$$\begin{bmatrix} -p_1/p_j & -p_2/p_j & \cdots & 0 & \cdots & -p_N/p_j & -w_j/p_j \end{bmatrix}. \tag{A.5}$$

It should be noted that j-th element in vector (A.5) is 0. $\partial X_{ij}/\partial p_k$ for $k \neq i$ is simply the (i, k) element of $B^{-1}{}_P$.

Under the assumption of cost minimization, since at the optimum point Lagrangian multiplier λ is equivalent to the output price p_j, B_P becomes the sub-matrix of B_C as is shown in (A.6).

$$\mathbf{B}_C = \left[\begin{array}{cccc|cc} \lambda f_{11} & \cdots & \lambda f_{1N} & \lambda f_1 & \lambda f_1 & f_1 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ \lambda f_{N1} & \cdots & \lambda f_{NN} & \lambda f_N & \lambda f_N & f_N \\ \lambda f_{1} & \cdots & \lambda f_N & \lambda f & \lambda f & f \\ \hline \lambda f_{1} & \cdots & \lambda f_N & \lambda f & \lambda f & f \\ f_1 & \cdots & f_N & f & f & 0 \end{array}\right] = \begin{bmatrix} \mathbf{B}_P & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} \tag{A.6}$$

If we can have the complete matrix $\mathbf{B}_C$, the marginal effect of prices on factor demand based with fixed capital input $\mathbf{B}_P{}^{-1}$ can be obtained by taking an inverse of the sub-matrix $\mathbf{B}_P$.

As shown in (A.2), we have already derived $\mathbf{B}^{*-1}$ in (A.2) based on equation (17) in the text. Accordingly, if we have the borders of $\mathbf{B}_C^{-1}$, say vector **b** and $b_{N+3,N+3}$, we can have the complete $\mathbf{B}_C^{-1}$. Each element in vector **b** can be expressed as a solution of (A.1) as,

$$b_{i,N+3} = \partial X_{ij} \big/ \partial X_j \ . \tag{A.7}$$

Since we assumes homogeneity of the production function, these are equivalent to

$$b_{i,N+3} = X_{ij} \big/ X_j \ \text{ for } i\text{=1,…}N, \ b_{N+1,N+3} = L_j \big/ X_j \ \text{, and } b_{N+2,N+3} = X_{ij} \big/ X_j$$

The right-end corner of $\mathbf{B}_C^{-1}$, $b_{N+3,N+3}$, is the elasticity of the unit cost with respect to the level of output, which is zero in a homogeneous production function since the unit cost is always equal to the output price in the perfect market.

Thus, we can construct complete $\mathbf{B}_C^{-1}$ numerically. Taking the inverse of $\mathbf{B}_C^{-1}$, we can obtain $\mathbf{B}_C$ and its sub-matrix $\mathbf{B}_P$.

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Footnotes

[1] Shishido et. al. (1999) finds that input coefficients are fairly stable across the countries in the North-East Asian region in spite of their different stage of development. The input coefficients in their study, however, are not those that take into account of the multilateral international transactions.

[2] This is a relatively straightforward extension of the model proposed in Tokutsu (1994) to the international setting.

[3] As can be seen from Table 1, since the Rest of the World does not have the corresponding columns, we can not identify the input structure of this sector.

[4] All the entries in the table are evaluated in 100 thousand dollars on the basis of the average foreign exchange rate in 1990. Accordingly, in this conventional treatment, the “quantity” of all the variables in the table is regarded as being measured in terms of the amount, which can be purchased by 100 thousand dollars in 1990.

[5] It is to be noted that the term “final demand of country *k*” means the final demand for the products of country *k*, but does not mean the final demand from country *k*. It also should be noted that this item includes export to the Rest of the World as intermediate inputs as well as the final demand.

[6] As is stated in footnote 3, the ROW sector has no corresponding columns. Accordingly, the equilibrium levels of only $4n$ outputs for Japan, the United States, the European Union, and the Asian countries can be determined in a system, although the identities of the market equilibrium are defined for all the $5n$ market in equation (6). In this sense, the model based on Table 1 is an “open” model with respect to the outputs of the ROW as well as labor input.

[7] This assumption implies that the industry also adapts its output level corresponding to the change of the output price as well as the change of the demand. This is a definitely different treatment from that of the cost minimization with given output, in which the industry only passively adapts its output level corresponding to the change of the demand. This difference is reflected in the shape of the supply curve. This will be discussed later in section 4.

[8] This method is originally adopted by Saito (1971) based on the substitution theorem in Klein(1952).

[9] Correspondence between the original sector classification and the aggregated sector

classification in this study is presented in Appendix 2 to this paper.

[10] Index of power of dispersion is defined as

$$U_{\cdot j} = \frac{\sum_{i=1, i \neq j}^{4n} \rho_{ij}}{4n-1} \Bigg/ \frac{\sum_{j=1}^{4n} \sum_{i=1, i \neq j}^{4n} \rho_{ij}}{4n(4n-1)} \quad ,$$

where ρ_{ij} is the element (i, j) of the inverse of the Jacobian matrix $\Phi^{-1}{}_{p}$. The index more than unity means that the power of dispersion of the corresponding industry is relatively large compared to the other industries.

[11] In the case of cost minimization with given output, which most studies of the multi-sector production function assume, the supply curve is still flat, while demand curve is right-downward sloping similar to our model.

[12] It should be noted, however, that there are a couple of cases where such an impact is negative.