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Published in:
Computers & Industrial Engineering

DOI:
10.1016/j.cie.2016.02.011

Computers & Industrial Engineering 95 (2016) 175–186

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

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ARTICLE INFO

Article history:
Received 10 August 2015
Received in revised form 14 February 2016
Accepted 15 February 2016
Available online 3 March 2016

Keywords:
School timetabling problem
Class-teacher model
Rescheduling
Disturbances

ABSTRACT

School timetables are one or multiple times per year generated to assign class-teacher combinations to class rooms and timeslots. Post-publication disturbances such as absence of teachers typically pose a need for schedulers to rapidly implement some minor changes to avoid empty periods in the timetable. In this paper our aim is to define methods to efficiently solve the school timetabling problem under disturbances. We present three types of solution methods, namely a simple rule-of-thumb, a heuristic and an optimization approach. Exhaustive numerical experiments have been performed with data from five high schools in The Netherlands, each with their unique characteristics in number of classes, number of teachers and number of daily meetings. For each of the three methods we show advantages and disadvantages as well as the effects of resulting changes in the schedules.

1. Introduction

Main decisions in school timetable problems are to make class-teacher-room assignments and allocate meetings to empty slots in a schedule. Schedulers need to take a variety of constraints into account, such as teachers' availability and preferences, room capacity, lesson spread for classes and load balancing for teachers. Observations in practice learn that the generation of timetables is a time consuming process which is executed by software in combination with the manual interaction of a scheduler. The general school timetabling problem is proven to be NP-complete (Even, Itai, & Shamir, 1976). Post et al. (2012) concluded that the field of educational timetabling is nowhere near solving all possible instances of high-school timetabling. Initially, mathematical programming approaches were used in deriving feasible timetables (e.g., Papoutsis, Valouxis, & Housos, 2003; Tripathy, 1984). Later mainly heuristics have been designed (e.g., Fonseca & Santos, 2014; Zhang, Liu, M'Hallah, & Leung, 2010). Typically those methods are intended to be used to design new school timetables from scratch for a (part of) a year. However, timetable users must be able to make minor changes rapidly and easily after publication due to disturbances such as teachers' illness or extracurricular activities. In practice, this rescheduling process is mainly arranged manually. There is a need for new methods to efficiently reschedule parts of school timetables that can be applied at different types of schools (Pillay, 2014). Our aim is to present a model and a variety of solution approaches to define and solve the school timetabling problem under disturbances. Exhaustive experiments are performed to show the outcomes of the different methods, and the ability to generalize outcomes. To this end we use data of five high schools in The Netherlands each with different characteristics instead of as commonly seen in literature only of a single high school (Pillay, 2014).

In the school timetabling problem under disturbances, typically an initial timetable is rescheduled and the altered timetable is compared to the initial timetable. Commonly, in The Netherlands, meetings between absent teachers and their classes will be removed from the schedule and will not be rescheduled in another time period. Consequently, the number of empty periods for classes in the timetable of that day increase. Empty periods are perceived negatively and the overall aim is to keep the number of empty periods as low as possible. Schedulers focus on reducing those resulting additional periods by making short term changes in the schedule. The reduction of empty periods is obtained by temporarily shifting meetings of other teachers to other time periods in the new timetable. However, the reduction of the number of empty periods comes at a cost. Shifting meetings force classes and teachers to adapt to sudden changes of the schedule, which can be experienced as something negative. Therefore, the scheduler has to create a balance between reducing the number of empty periods, keeping the schedule stable, i.e., not deviating too much from the old timetable and being alert on the amount of shifts on a specific day and over days. The latter kind of shifts is typically less valued than shifts on a specific day. Consequently, the quality of the new timetable is determined by its compactness expressed by the number of empty periods, the stability of the

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http://dx.doi.org/10.1016/j.cie.2016.02.011
0360-8352/

schedule and the type of shifts. Typically in literature the quality of the schedule is expressed in soft constraints (e.g., Pillay, 2014). In our newly designed rescheduling policies we explicitly incorporate those performance measures in the objective.

The problem of school timetabling under disturbances can be classified in the field of school timetabling. The school timetabling problem, also referred to as the class-teacher model, consists of assigning meetings to periods for a specific class-teacher combination such that no teacher or class is involved in more than one meeting at a time (e.g., Carter & Laporte, 1998; De Werra, 1985). We can roughly divide literature on school timetabling into two categories, namely (1) class-teacher assignment (Asratian & De Werra, 2002; Azmi Al-Betar & Khader, 2012; De Werra, Asratian, & Durand, 2002); and (2) class scheduling to assign meetings for a specific subject for each class to timeslots and rooms (Burke, Mareček, Parkes, & Rudová, 2012; Sampson, Freeland, & Weiss, 1995; Sampson & Weiss, 1995). Some papers (Al-Yakoob & Sherali, 2007; Alvarez-Valdes, Martin, & Tamarit, 1996) address those decision problems in a sequential way. To our knowledge no methods are specifically designed to perform limited alterations to already published school timetables.

The structure of this paper is as follows. We define the school timetabling problem under disturbances in Section 2. Section 3 presents the different solution approaches, namely a simple rule-of-thumb, an optimal approach and a heuristic. Section 4 shows for a specific setting the outcomes of the different methods. Experiments and data collection at 5 different schools are defined in Section 5. Results and numerical insights are shown in Section 6. Finally, we present conclusions in Section 7.

2. Problem definition

In this section, we formally define and formulate the school timetabling problem under disturbances. Two types of school timetabling problems can be considered, namely one where all pupils in a class follow exactly the same meetings and one where pupils in a class may attend different meetings (Post et al., 2012). In the lower classes in the educational system in The Netherlands all pupils in a class follow exactly the same meetings, which is also the case for some other countries (e.g., the 11- to 14- year-olds in English Secondary Schools). For an overview of the timetabling problem in different countries we refer to Post et al. (2012). In this paper, we focus on the lower classes of high schools where all pupils in a class follow exactly the same meetings. Specifically for those pupils it is the general understanding that the amount of empty periods should be as low as possible.

A specific set of disturbances can be represented by changes in teachers' availability. A published timetable is available showing an assignment of teachers to classes and time periods. This assignment will be input in the school timetabling problem. If a teacher is unavailable, no other teacher will take over to teach this subject to a class. Given that the related meeting is not scheduled at the start or end of a day, we define the resulting time slot as an empty period. If the canceled meeting was scheduled at the start of the end of the day, the students will start/end their day later/earlier. In a feasible schedule sufficient room capacity is available to match meetings to rooms. Given that, we do not consider subject-room assignment decisions in the school timetabling problem under disturbances. Consequently, the model aims to re-allocate *meetings* for each teacher-class combination to a timeslot given the new information on teachers' availability. The goal is, as explained in Section 1, to minimize a weighted sum of the number of empty periods and the number of shifts made between the old and the new schedule. As mentioned in the introduction, we make a distinction between the number of shifts on a specific day and over days.

In defining the parameters and variables we follow where applicable the notation as presented by Santos, Uchoa, Ochi, and Maculan, 2010. The following set of parameters is defined:

C: Set of classes;
T: Set of teachers;
D: Set of days;
P: Set of time periods on a day, where for each day the time periods are numbered from 1 to $|P|$;
$\tilde{R}$: Requirement matrix, where $\tilde{r}_{tc}$ specifies the number of meetings involving teacher t and class c, excluding the disturbed meetings;
$\tilde{T}$: Availability matrix, where $\tilde{t}_{tdp} = 1$ if teacher t is available at time period p of day $d, \tilde{t}_{tdp} = 0$ otherwise;

$$\bar{x}_{tcdp} = \begin{cases} 1 & \begin{array}{l}\text{if teacher } t \text{ and class } c \text{ meet at time period} \\ \ p \text{ of day } d \text{ in the old schedule} \\ \text{and teacher } t \text{ is not disturbed at time period} \\ \ p \text{ of day } d\end{array} \\ 0 & \text{otherwise;} \end{cases}$$

w_1: Penalty for each empty period;
w_2: Penalty for the shift of a meeting to another time period;
w_3: Penalty for the shift of a meeting to another day;

The decision variables and auxiliary variables are defined as follows:

$$x_{tcdp} = \begin{cases} 1 & \text{if teacher } t \text{ and class } c \text{ meet at time period } p \\ & \text{ of day } d, \\ 0 & \text{otherwise;} \end{cases}$$

$h_{cd} \in \mathbb{Z}_+$: Number of empty periods for class c at day d;
$\overline{a}_{cd} \in \mathbb{Z}_+$: Time period of the first meeting of class c at day d;
$\underline{a}_{cd} \in \mathbb{Z}_+$: Time period of the last meeting of class c at day d;
$g_{tcd} \in \mathbb{Z}_+$: Number of meetings between teacher t and class c shifted to day $d$from another day;
s_{tcdp}: Binary variable equal to one if a meeting between teacher t and class c is shifted to time period p at day d.

3. Solution approaches

In this section we define three different solution approaches to solve the timetabling problem under disturbances. First, we define a simple rule-of-thumb that can be performed manually without the need of a computer. Secondly, we construct an integer linear programming model (ILP) that solves the problem to optimality. Given the complexity of the problem, we finally define a heuristic procedure that can generate results efficiently. In Section 6 we will compare the different methods to analyze the changes in the schedules obtained.

3.1. Simple rule-of-thumb

A simple rule-of-thumb to solve the timetabling problem under disturbances can be described as follows: pick the first of the empty periods caused by a disturbance and try to shift the last or first meeting of the day to the empty period. Whenever this is not possible, try to find another time period at this day whose scheduled meeting can be shifted to the empty period and where the last or first meeting of the day can be shifted to. Whenever this is not possible either, check whether a meeting at the end or start of another day can be shifted to the empty period. In Appendix A, the pseudocode for this rule-of-thumb is given. The description of the parameters and variables not described in the pseudocode, can be found in Section 2.

Note that if we run, for example, the heuristic on Monday and there is an empty period at Thursday, the heuristic only considers the meetings on Thursday and Friday to be shifted to the empty period. However, it might be possible to shift a meeting from one of the other days to the empty period of Thursday. Therefore, in our experiments, we both run the heuristic described above, and the heuristic with the slight alteration that the meetings of all future days are allowed to be shifted. We report the results of the best found schedule.

3.2. Integer linear programming model

We introduce an ILP model to solve the problem as presented in Section 2. This model is partly based on the formulation for the class-teacher timetabling problem with compactness constraints of Santos et al. (2010). Specific to our problem are: (a) a class perspective instead of a teacher perspective; (b) the formulation of the objective where we focus on minimizing penalties for shifts in the schedule; (c) need for specific constraint (7) and (d) need for specific constraint (8).

$$\text{minimize} \sum_{c\in C}\sum_{d\in D} w_1 h_{cd} + \sum_{t\in T}\sum_{c\in C}\sum_{d\in D}\sum_{p\in P} w_2 s_{tcdp} + \sum_{t\in T}\sum_{c\in C}\sum_{d\in D} w_3 g_{tcd}$$

s.t.

$$\sum_{d\in D}\sum_{p\in P} x_{tcdp} = \tilde{r}_{tc} \qquad \forall t\in T, \quad c\in C \tag{1}$$

$$\sum_{t\in T} x_{tcdp} \leqslant 1 \qquad \forall c\in C, \quad d\in D, \quad p\in P \tag{2}$$

$$\sum_{c\in C} x_{tcdp} \leqslant \tilde{t}_{tdp} \qquad \forall t\in T, \quad d\in D, \quad p\in P \tag{3}$$

$$\bar{a}_{cd} \leqslant (|P|+1) - (|P|+1-p)\sum_{t\in T} x_{tcdp} \qquad \forall c\in C, \quad d\in D, \quad p\in P \tag{4}$$

$$\underline{a}_{cd} \geqslant p\sum_{t\in T} x_{tcdp} \qquad \forall c\in C, \quad d\in D, \quad p\in P \tag{5}$$

$$h_{cd} \geqslant \underline{a}_{cd} - \bar{a}_{cd} + 1 - \sum_{t\in T}\sum_{p\in P} x_{tcdp} \qquad \forall c\in C, \quad d\in D \tag{6}$$

$$\bar{x}_{tcdp} - x_{tcdp} + s_{tcdp} \geqslant 0 \qquad \forall t\in T, \quad c\in C, \quad d\in D, \quad p\in P \tag{7}$$

$$g_{tcd} \geqslant \sum_{p\in P} (\bar{x}_{tcdp} - x_{tcdp}) \qquad \forall t\in T, \quad c\in C, \quad d\in D \tag{8}$$

$$h_{cd}, \quad \underline{a}_{cd}, \quad \bar{a}_{cd} \in \mathbb{Z}_+ \qquad \forall c\in C, \quad d\in D \tag{9}$$

$$g_{tcd} \in \mathbb{Z}_+ \qquad \forall t\in T, \quad c\in C, \quad d\in D \tag{10}$$

$$x_{tcdp}, \quad s_{tcdp} \in \{0,1\} \qquad \forall t\in T, \quad c\in C, \quad d\in D, \quad p\in P \tag{11}$$

The objective of the problem is to minimize a weighted sum of the number of empty periods, the number of meetings shifted, and the number of meetings shifted to another day. Constraints (1) ensure that the number of meetings between a class and teacher are as specified in the requirement matrix. Constraints (2) make sure that each class can at most be assigned to one teacher at a given time period. Due to constraints (3), at most one class meets a teacher at a given time period. Moreover, a class only meets a teacher whenever the teacher is available. Constraints (4) and (5) define the time periods corresponding to the first and last meeting for each class at each day, respectively. Constraints (6) define the number of empty periods for each class and day. Constraints (7) define the number of meetings shifted. Constraints (8) define the number of meetings shifted to another day. Constraints (9)–(11) define the nature and the domain of the decision variables.

3.3. Heuristic

The heuristic we implemented to solve this problem iteratively optimizes the schedules of the different classes. The heuristic procedure starts with the initial schedule for all classes, i.e., $x_{tcdp} = \bar{x}_{tcdp}, \forall t\in T, c\in C, d\in D, p\in P$. The heuristic iteratively optimizes the schedule of each class, starting with the first class, i.e., $\bar{c} = 1$, and ending with the last class, i.e., $\bar{c} = |C|$. The optimization of each schedule requires three steps. The first step is to fix the schedules of the other classes and to update the availability matrix such that a teacher is available at the time slots it was scheduled to teach class $\bar{c}$ in the initial schedule, i.e., if $x_{t,\bar{c},d,p} = 1$, then $\tilde{t}_{tdp} = 1$, $\forall t\in T,\ d\in D,\ p\in P$. The second step is the actual optimization of the schedule of class $\bar{c}$, which is done by solving the ILP, as described in Section 3.2, with $C = \{\bar{c}\}$. The last step is to update the availability matrix such that a teacher is not available at the timeslots at which it is assigned to class $\bar{c}$ in the new schedule, i.e., if $x_{t,\bar{c},d,p} = 1$, then $\tilde{t}_{tdp} = 0$, $\forall t\in T,\ d\in D,\ p\in P$. The procedure continues until each class is optimized exactly once. In Appendix A, the pseudocode for this heuristic is given.

4. Example

We present a small example to study the differences in outcomes obtained with the three methods presented in Section 3. We assume that the cost for a shift at the same day is equal to 1, and the cost for a shift to another is equal to 3. The cost for an empty period is equal to 4. Table 1 (appendix B) shows the initial schedule for five sequential days for class 1 and class 2. Assume that it is currently day 1 and that due to disturbances class 1 has two empty time periods at periods 3 and 5 of day 1. Class 2 has a single empty period at period 4 of day 1. Table 2 (appendix B) shows the availability of the teachers involved for classes 1 and 2.

Tables 3a–3c (see appendix B) presents the outcomes for the different methods. In Table 3a we note that with the *rule-of-thumb*, class 1 will be examined first. The first empty period is at period 3 of day 1. The first possible change as found with the procedure as presented in Appendix will be to assign teacher 36 at period 3 of day 1 instead of at period 1 of day 5. Other earlier changes are not possible due to the availability of teachers as presented in Table 2. The next empty period is period 5 of day 1. This empty period can be eliminated by shifting the meeting between teacher 28 and class 1 from period 7 of day 1 to period 5 of that same day. Class 2 has an empty period at period 4 of day 1. The meeting between teacher 4 and class 2 can move from period 3 to period 4 of that day. As a result, three shifts have been made of which one has been moved to another day. The value of the objective equals 5 in this case.

Table 3b shows the results for applying the *heuristic* to the example problem. Also the heuristic obtains a solution in a sequential way. First class 1 and then class 2 is considered. For each class the problem is iteratively solved using a branch-and-bound algorithm applied to the ILP model as shown in Section 3 where the other class is fixed and no changes can be performed for that specific class. For class 1 we note the following changes to reduce the number of empty periods: the meeting of teacher 4 at period 2 of day 1 moves to period 8 at day 1; the meeting of teacher 28 moves from period 7 at day 1 to period 5 at day 1; the meeting of teacher 15 moves from period 4 at day 1 to period 7 at day 1. For class 2 the meeting of teacher 4 moves from period 3 at day 1 to period 4 at day 1. The value of the objective equals 4 in this example. Compared to the rule-of-thumb all shifts can be performed at the same day.

Table 3c shows the results obtained by the *optimal approach* for class 1 and class 2. In this case, the schedules of both classes are optimized simultaneously. For class 2, the meeting of teacher 4 is moved from period 3 at day 1 to period 4 at the same day. For class 1, the meeting of teacher 4 is moved from period 2 of day 1 to period 3 at day 1. Next to that, the meeting of teacher 28 is moved from period 7 to period 5 at day 1. Since the optimal approach considers the schedules of both classes simultaneously, better results are obtained by the optimal approach compared to the outcomes obtained by the rule-of-thumb and the heuristic. By first solving the problem for class 2, the availability of teacher 4 has changed, which directly helps to eliminate one of the empty periods for class 1 with a shift at the same day. For this example the value of the objective equals 3.

5. Experiments

Data was provided by five different high schools in The Netherlands. The schools feature different characteristics concerning for example the number of teachers, the number of classes and the number of time periods during a day. The characteristics of each of the schools can be found in Table 4.

The data made available by the high schools consist of the initial schedules for all lower classes (i.e., classes that each follow the same meetings; refer to Section 1) and teachers and the availability for each of the teachers. The data is slightly adapted to match the characteristics of the problem as specified in Section 2. The average number of meetings scheduled in a week for the lower classes of each of the schools used in our experiments and the number of time periods per day and the duration of a meeting can be found in Table 4.

For each of the five schools, we created two sets of disturbances of each 600 instances. In the first set (experiments 1–600), each disturbance lasts for only one day namely the Monday. In the second set (experiments 601–1200) the teachers are absent for the whole week. The sets are each divided in 6 batches of 100 experiments, where the number of absent teachers in a batch respectively equals 2, 3, 4, 5, 6 and 10. Absent teachers are randomly chosen in such a way that they have at least one meeting scheduled in the initial time table at one of the disturbed time periods. For all experiments, we have only taken into account the classes for which the disturbances do lead to the removal of one or multiple meetings. The average number of classes involved in the experiments for the different schools are given in Table 5.

For the experiments the weights in the objective are set as follows: $w_1 = 4$, $w_2 = 1$, $w_3 = 2$. These values are chosen based on insights of experts in the field. The penalty for an empty period equals 4, the penalty of a shift is set to 1 and the penalty of a shift to another day is set to 2. This means that if a meeting shifts to another day, the total penalty equals $w_1 + w_2 = 3$. We use the GUROBI 5.1 solver in AIMMS 3.13 to solve the model as presented in Section 3. In solving the problem both empty periods resulting from disturbances as well as empty periods already present in the schedule are taken into account.

We provide insights on the differences in performance of the different solution methods under different conditions. To this end we use the following measure to show the gap between the solutions derived by two different solution methods.

$$\text{Gap} = \frac{f(s_1) - f(s_2)}{f(s_2)} \cdot 100\%, \tag{12}$$

where $f(s_i)$ is the objective value of the solution obtained by solution method i, $i = 1, 2$.

Note that it is not our aim to compare different schools on their performance or to test the performance of specific schedulers. The data of the schools are used to compare the outcomes of the three methods as presented in this paper. We aim to derive insights for schedulers to help in their decision making process what types of disturbances to handle in what way, with what method and to get a feeling for the effects and impact of the decisions made.

6. Results

In this section we present the results of the experiments as described in Section 5. First, we discuss the outcomes for the various schools if a small disturbance (set 1) occurs. For each of those experiments the model as introduced in Section 3 could be solved to optimality within reasonable time, i.e., between a few seconds and a couple of hours per instance. For the heuristic and the rule of thumb, the computation time for each instance was within a

Table 4
Characteristics of the different schools.

School	# Classes	# Teachers	# Time periods per day	Duration meeting (in min)	# Meetings scheduled in week
School 1	17	52	8	50	32–34
School 2	10	52	9	50	32–37
School 3	14	52	5	90	19–21
School 4	10	55	9	50	34–36
School 5	28	78	9	45	32–41

Table 5
Average number of classes impacted by a disturbance in the experiments.

Experiments	# Teachers absent	Period of absence	School 1	School 2	School 3	School 4	School 5
1–100	2	Monday	3.7	4.2	4.0	5.3	5.6
101–200	3	Monday	5.4	5.7	5.7	7.2	7.8
201–300	4	Monday	6.9	6.9	7.1	8.1	10.4
301–400	5	Monday	7.9	7.6	8.3	8.5	11.9
401–500	6	Monday	9.1	8.6	9.3	9.1	13.7
501–600	10	Monday	12.5	9.4	12.1	9.9	18.9
601–700	2	Full week	5.8	6.5	7.4	6.1	9.8
701–800	3	Full week	8.1	7.8	9.5	8.1	13.7
801–900	4	Full week	9.9	8.8	10.8	8.9	16.8
901–1000	5	Full week	11.1	9.4	12.2	9.4	19.4
1001–1100	6	Full week	12.7	9.7	12.8	9.7	21.3
1101–1200	10	Full week	14.8	10.0	13.8	10.0	25.1

Table 6
Small disturbances (experiments 1–600): comparison between optimal solution, heuristic and rule-of-thumb.

	Overall	School 1	School 2	School 3	School 4	School 5
% Optimal – rule-of-thumb	13.1	20.7	14.7	22.2	5.8	2.3
% Optimal – heuristic	50.2	58.2	50.3	66.8	25.5	50.1
% Rule-of-thumb outperforms heuristic	0.5	1.2	0.3	0.2	1.0	0.0
% Heuristic outperforms rule-of-thumb	82.5	73.2	80.0	75.8	86.5	97.2

Table 7
Small disturbances: Gap between rule-of-thumb and heuristic, and optimal solution.

Experiments	Gap between rule-of-thumb and optimal solution (%)	Gap between heuristic and optimal solution (%)	Gap between rule-of-thumb and heuristic (%)
Batch 1 (1–100)	36.2	4.9	30.3
Batch 2 (101–200)	49.9	9.0	38.7
Batch 3 (201–300)	57.5	9.5	44.7
Batch 4 (301–400)	65.1	11.2	48.6
Batch 5 (401–500)	79.0	14.2	57.1
Batch 6 (501–600)	125.0	17.6	92.1
Average	68.8	11.1	51.9

Table 8a
Small disturbances: number of eliminated empty periods, % eliminated number of empty periods, number of shifts needed and number of shifts over a day in an optimal solution.

Experiments	Optimal			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
Batch 1 (1–100)	4.0	97.3	4.7	0.6
Batch 2 (101–200)	5.7	98.3	6.6	0.7
Batch 3 (201–300)	7.5	98.5	8.5	0.9
Batch 4 (301–400)	8.5	97.6	9.4	0.9
Batch 5 (401–500)	10.1	98.6	11.0	0.9
Batch 6 (501–600)	14.3	99.2	13.6	1.0
Average	8.4	98.3	9.0	0.8

Table 8b
Small disturbances: % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the heuristic.

Experiments	Heuristic			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
Batch 1 (1–100)	3.9	95.1	4.3	0.8
Batch 2 (101–200)	5.5	95.7	6.0	1.1
Batch 3 (201–300)	7.3	95.7	7.6	1.4
Batch 4 (301–400)	8.2	94.4	8.6	1.5
Batch 5 (401–500)	9.9	95.6	10.0	1.7
Batch 6 (501–600)	14.0	96.6	12.7	2.1
Average	8.2	95.5	8.2	1.4

Table 8c
Small disturbances: % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the rule-of-thumb.

Experiments	Rule-of-thumb			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
Batch 1 (1–100)	3.5	86.9	4.1	1.1
Batch 2 (101–200)	4.9	83.8	5.6	1.6
Batch 3 (201–300)	6.3	81.4	7.2	2.0
Batch 4 (301–400)	7.0	79.6	8.0	2.2
Batch 5 (401–500)	8.3	79.4	9.2	2.8
Batch 6 (501–600)	10.8	71.6	11.6	3.5
Average	6.8	80.5	7.6	2.2

few seconds. A comparison between the optimal results, the results of the heuristic and the results of the rule-of-thumb will be shown. Secondly, we present the outcomes if large disturbances (set 2) occur. For those instances, 12 h of computation time was not sufficient to provide optimal solutions for some instances. Consequently, we compare the outcomes of the heuristic with the outcomes obtained with the rule-of-thumb. Those solution methods provided a solution to each instance within a few seconds. In Section 6.4 we show insights for implementation of the methods in practice.

6.1. Small disturbances

As explained in Section 5, in set 1 we distinguish six batches of experiments with respectively 2, 3, 4, 5, 6, or 10 teachers being absent. The meetings of those teachers on Monday are removed from the schedules, and the availability of those teachers is set to zero for all time periods on Monday. The heuristic approach is able to find an optimal schedule for 50.2% of the instances. For the rule-of-thumb, an optimal schedule is found for only 13.1% of the instances. For 82.5% of the instances the heuristic outperforms the rule-of-thumb, whereas for 0.5% of the instances this is the other way round. These results and a subdivision of the results for the different schools can be found in Table 6. A more detailed comparison of the different schools is given in Section 6.3. As indicated our goal is to present three methods to eliminate disturbances from schedules and show their mutual relations. Therefore, we present in Table 7 for each set of experiments the relative differences between the various methods. On average the gap between the rule of thumb and the optimal solution is 68.8%, between the rule of thumb and the heuristic 51.9% and the heuristic and the optimal solution is 11.1%.

To find an explanation for those relative differences, we now take a closer look into the solutions obtained. We will elaborate on the empty periods eliminated and the shifts made in order to reduce the number of empty periods (refer to Tables 8a–8c). We note that the number of empty periods eliminated by the rule-of-thumb is 6.8 on average overall (i.e., over all schools and all experiments), which is 80.5% of the number of empty periods present before starting the procedure. This is an average gap of 1.6 empty periods with the optimal solution. The difference in the number of empty periods solved between the optimal and heuristic procedure overall is only 0.2. The average number of empty periods eliminated grows over the batches, since the number of absent teachers also increases over the batches. For the rule-of-thumb, the overall performance measured in the proportion of empty periods solved decreases from 86.9% to 71.6% for set 6. Contrary to the rule-of-thumb, we note that both the optimal procedure and the heuristic procedure show a stable performance over the various batches of experiments with respect to the percentage of eliminated empty periods. The absolute number of total shifts needed increases over the batches for all three solution methods. If we look at the relation between the total number of shifts and empty periods solved, we note that on average, the rule-of-thumb, the heuristic, and the optimal procedure need, respectively, 1.1, 1.0, and 1.1 shifts to solve 1 empty period in the schedule. On average 28.9% of the shifts made by the rule-of-thumb concerns a shift to another day. For the optimal and heuristic procedure this is on average 8.9% and 17.1%, respectively. As a result, the structure of the optimal procedure and the heuristic enable better use of the shifts on a given day avoiding more expensive shifts to another day.

6.2. Large disturbances

Large disturbances are defined by the absence of a specified number of teachers for a whole week. If a teacher is absent, all the meetings corresponding to this teacher are removed from the schedules of all classes involved. We compare the schedules found by the rule-of-thumb and the heuristic in Table 9. On average, for 93.4% of the instances the heuristic outperforms the rule-of-thumb. Only for 3.7% of the instances this is the other way round. From the results it can be noted that for Schools 2–5, the heuristic outperforms the rule-of-thumb in almost all instances. However, in the experiments of School 1, for 18.0% of the instances the schedule obtained by the rule-of-thumb is better than the one generated by the heuristic. In Section 6.3 we will present a more detailed analysis on this and the other schools.

Tables 10a and 10b show the number of empty periods eliminated and the number of shifts needed to do so, for both the rule-of-thumb and the heuristic. If we compare the results in Tables 10a and 10b to the results in Tables 8a–8c we note that the number of empty periods in the schedules of the disturbed classes is larger resulting from the fact that teachers are not present for 5 days instead of one single day. Consequently, more shifts are needed to eliminate the empty periods in the schedule. The heuristic eliminates on average 97.2% of the empty periods compared to 77.6% with the rule-of-thumb. This gap is even bigger than we noticed in the situation with the small disturbances. The performance of the heuristic is quite stable over the various experiments. Contrary to that, the performance measured for the rule-of-thumb varies between 79.7% and 75.6%, and decreases if the number of absent teachers increases. On average 5.4 more empty periods can be solved with the heuristic by applying only on average 0.9 shifts more. The rule-of-thumb needs on average 1.1 shifts to solve an empty period. The heuristic on average needs 0.9 shifts for each empty period. For the large disturbances, the amount of shifts over a day is quite comparable for both procedures and equals on average around 26% of the total number of shifts needed.

6.3. General applicability of solution method: comparing different schools

In this section we discuss the relations and differences in outcomes for the different schools. As mentioned in Section 1, most papers available show the applicability of the method designed in the context of a single school. In this paper, we test the method over a variety of schools by taking the data of five different high schools in The Netherlands (refer to Table 4). We follow the same order with first discussing the insights obtained from the experiments with the small disturbances and secondly with the large disturbances.

Tables 6, 11 and 12a–12c show detailed results for schools for the experiments with small disturbances. In Table 6 the objective values of the schedules generated by the rule-of-thumb and the heuristic are compared for different schools. In Table 11 we show similar to Table 7 the relative differences between the various methods. Similar patterns can be noted as in Table 7 for each of the schools. This demonstrates that a somewhat more complex method enables better solutions. In Table 6 it can be noted that the heuristic outperforms the rule-of-thumb most of the times. There are no large differences between the different schools. More specifically, the results in Table 6 demonstrate that the methods can be used for different schools with their own characteristics. Similar conclusions as already discussed in Section 6.1 can be presented for each of the individual schools. For school 5 the heuristic in almost all cases outperforms the rule-of-thumb. School 3 only allows scheduling of 5 meetings on a daily base. Consequently, less options are available for alterations in the schedule enabling heuristics to find optimal solutions earlier. For school 4 the results show that only in 5.8% of the cases the rule-of-thumb is optimal. The gap between optimal and the rule of thumb is the highest for this school, namely 105.8%. For this school the relative

Table 9
Large disturbances (experiments 601–1200): comparison between heuristic and rule-of-thumb.

	Overall	School 1	School 2	School 3	School 4	School 5
% Rule-of-thumb outperforms heuristic	3.7	18.0	0.5	0.0	0.0	0.0
% Heuristic outperforms rule-of-thumb	93.4	76.8	98.0	93.7	98.7	100.0

Table 10a
Large disturbances: % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the heuristic.

Experiments	Heuristic			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
Batch 7 (601–700)	12.1	96.7	12.4	3.3
Batch 8 (701–800)	17.5	97.1	17.8	4.7
Batch 9 (801–900)	23.5	97.1	23.2	6.2
Batch 10 (901–1000)	28.8	97.0	27.3	7.3
Batch 11 (1001–1100)	33.8	97.4	31.3	8.4
Batch 12 (1101–1200)	50.7	97.9	42.7	11.4
Average	27.7	97.2	25.8	6.9

Table 10b
Large disturbances: % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the rule-of-thumb.

Experiments	Rule-of-thumb			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
Batch 7 (601–700)	10.0	78.2	11.6	2.8
Batch 8 (701–800)	14.5	79.7	16.8	4.1
Batch 9 (801–900)	19.1	78.0	22.1	5.4
Batch 10 (901–1000)	23.2	77.1	26.3	6.7
Batch 11 (1001–1100)	27.0	76.8	30.3	7.8
Batch 12 (1101–1200)	39.7	75.6	42.6	11.8
Average	22.3	77.6	24.9	6.4

Table 11
Small disturbances: Gap between rule-of-thumb and heuristic, and optimal solution over all schools.

Experiments	Gap between rule-of-thumb and optimal solution (%)	Gap between heuristic and optimal solution (%)	Gap between rule-of-thumb and heuristic (%)
School 1	65.9	10.8	51.2
School 2	78.6	12.4	58.9
School 3	49.6	7.1	39.7
School 4	105.8	19.8	73.0
School 5	43.9	5.2	36.8

Table 12a
Small disturbances: per school number of eliminated empty periods, % eliminated number of empty periods, number of shifts needed and number of shifts over a day in an optimal solution.

Experiments	Optimal			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
School 1	5.5	95.2	5.7	0.6
School 2	7.0	99.8	7.3	0.3
School 3	9.4	99.1	9.5	0.5
School 4	15.4	98.8	17.6	1.5
School 5	8.4	98.3	9.0	0.8

Table 12b
Small disturbances: per school % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the heuristic.

Experiments	Heuristic			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
School 1	5.2	91.2	5.0	0.9
School 2	6.9	98.2	6.7	0.9
School 3	9.0	94.1	8.4	1.3
School 4	15.2	97.8	16.7	2.2
School 5	8.2	95.5	8.2	1.4

Table 12c
Small disturbances: per school % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the rule-of-thumb.

Experiments	Rule-of-thumb			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
School 1	4.4	78.6	4.6	1.5
School 2	5.8	85.8	6.0	1.9
School 3	7.1	77.2	8.0	2.4
School 4	13.7	88.0	16.3	3.6
School 5	6.8	80.5	7.6	2.2

Table 13a
Large disturbances: per school % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the heuristic.

Experiments	Heuristic			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
School 1	20.7	95.7	22.4	6.6
School 2	26.8	99.2	23.3	5.5
School 3	12.5	94.5	11.8	5.3
School 4	30.7	98.0	25.3	8.3
School 5	48.0	98.6	46.1	8.6

Table 13b
Large disturbances: per school % eliminated number of empty periods, number of shifts needed and number of shifts over a day for the rule-of-thumb.

Experiments	Rule-of-thumb			
	# Empty periods eliminated	% Eliminated	# Shifts	# Shifts over a day
School 1	16.5	77.7	17.9	4.5
School 2	22.5	83.8	25.0	5.7
School 3	9.0	69.3	9.7	3.3
School 4	22.9	73.2	24.0	9.5
School 5	40.5	83.8	48.1	9.1

difference between the heuristic and optimal is also higher than for the other schools. However, the results in Table 12 show that the heuristic eliminates 97.8% of the empty periods. A specific characteristic of this school is that the number of affected classes (see Table 5) in relation to the number of classes (Table 4) is much higher than the other schools. For this school rescheduling is more complex, demonstrating the need for a more advanced method which results in 25.5% of the cases in an optimal answer.

In Tables 12a–12c we examine the schedules resulting from the optimal method, the heuristic and the rule-of-thumb compared to the initial schedules of each school in more detail.

For each of the schools similar analyses can be presented as described in Section 6.1. Here we discuss the differences we note between the various schools. School 1 relatively has the lowest score on the percentage of empty periods that can be eliminated for the heuristic and optimal method. As shown in Table 4 school 1 distinguishes itself from the other schools with a similar amount of teachers for more classes and 8 instead of 9 meetings per day. Table 5 shows that at school 1 on average the lowest number of classes is affected by the disturbances, resulting together with less hours per day in fewer options for changes in the schedule. The new schedules obtained for school 4 show that a large number of empty periods can be eliminated compared to the initial schedule resulting in a relatively large number of moves.

Tables 9, 13a and 13b show the results for each school in the experiments with large disturbances. From the results in Table 9 we conclude that for schools 2–5 the heuristic outperforms the rule-of-thumb with at least 93.7% of all cases with large disturbances. In case of school 1, the rule-of-thumb outperforms the heuristic in 18% of the cases. To find an explanation for this difference we study the changes performed to the initial schedule in more detail in Tables 13a and 13b. The results in those tables show that for schools 1 and 3 the heuristic makes on average more shifts over a day compared to the rule-of-thumb. However, the total number of empty periods eliminated is on average respectively 4.2 and 3.5 higher for the heuristic than for the rule of thumb. The overall performance will depend on the setting of the weights for shifts over days in the objective as set by a school. Overall we conclude that the heuristic also in the case of large disturbances is able to solve for any school at least 94.5% of the empty periods. For the rule-of-thumb the minimum obtained is 69.3%. For both policies the lowest performance is obtained for school 3. School 3 only has 5 meetings per day resulting in fewer options for shifting scheduled meetings.

The different outcomes show that, the optimal approach and heuristic generate robust high quality results in an efficient way independent of school characteristics as the number of classes, number of teachers and number of meetings per day.

6.4. Insights for implementation

The results discussed in the previous sections demonstrate the applicability of the heuristic and rule-of-thumb designed in varying practical contexts. Below we show the main insights derived that can help schools in their decision making process of method selection and method implementation. With regard to method selection we have derived the following insights as based on the differences in performances of the various methods:

- outcomes illustrate that the heuristic outperforms on average the rule-of-thumb in most experiments. Still we recommend schools to have a test phase to allow thorough testing of both methods in the specific context of that school, before selecting one of the two methods.
- Schools that aim for a simple rule that can be easily implemented can safely select the rule-of-thumb to get satisfactory results. A more complex method however results in better outcomes.
- Schools searching for a more robust approach that generates good results independent of the conditions might prefer the heuristic method which shows a more stable performance over the different instances.
- Schools that typically have a large number of classes affected by a disturbance, resulting in more complex rescheduling, should opt for a more complex and advanced method, such as the heuristic.

Summarizing, the newly designed heuristic methods generate efficiently effective results for adjusting schedules due to disturbances. The methods can be implemented, by using the pseudocodes in Appendix A, and run independently of the scheduling software used. The penalties for empty periods and shifts can be freely selected by school management. To this end, we suggest school management to follow the next steps in setting weights.

1. Obtain expert estimates on penalties from stakeholders in the school.
2. Define several test instances based on disturbances that took place in the past weeks and perform a sensitivity analysis with the different weights obtained from step 1.
3. Have a discussion session to analyze and evaluate effects of the weights and derive one conclusion that shows the weights that reflect the opinion on the quality of schedules as expressed by the mutual relation of penalties on shifts and empty periods.

7. Conclusions

In this paper we have presented three different solution approaches (rule-of-thumb, heuristic and optimal approach) to allow users of already published school timetables to rapidly make minor changes to respond to disturbances. Disturbances are described in terms of teachers' absence due to, for example, illness or extra-curricular activities. Meetings of absent teachers need to be removed from the initial schedule and result in empty periods for classes. The main goal of rescheduling activities is to decrease the number of empty periods in the schedule with a limited number of shifts. We derive a new optimal model, a rule-of-thumb and a heuristic approach to avoid using complex mathematical approaches for designing schedules from scratch each time a disturbance occurs. Data collection has been performed at five different high schools in The Netherlands enabling testing of the methods designed over various schools with each their own characteristics. The schools are categorized in terms of number of classes, number of teachers and number of meetings per day. Overall, it can be concluded that the heuristic eliminates for respectively small and large disturbances 95.5% and 97.2% of the empty periods. The rule-of-thumb only eliminates respectively 80.5% and 77.6% of the empty periods. The average number of shifts required to eliminate a single empty period is quite comparable with respectively 1.1 and 1.0 for the rule-of-thumb and the heuristic. For small disturbances the heuristic generates optimal solutions for 50% of the instances and on average solves 0.2 empty periods less than in the optimal situation. Experiments with both small and large disturbances show that the method designed generates robust solutions of high quality for each type of school.

Acknowledgements

The authors thank Wim Kokx, president of the board of Openbare Scholengroep Vlaardingen Schiedam, for providing expert insights, feedback and enabling contacts in the field.

The authors thank the schedulers of the following schools for providing their data: Picasso Lyceum Zoetermeer, Stedelijk Gymnasium Schiedam, Mavo Schravenlant XL, Het College Vos, and Het Lyceum Vos.

Appendix A

Algorithm 1. Pseudocode for the simple rule-of-thumb

S: Initial schedule (after the removal of disturbed meetings), where $S(c, d, p)$ equals the teacher of the meeting of class c at period p of day d, and 0 if there is no meeting;

$E = \{e_1, \ldots, e_\delta\}$: List of empty time periods, ordered by ascending time, where $\forall e \in E, c(e), p(e)$ and $d(e)$ are the corresponding class, time period and day, respectively;

for $i = 1$ to δ **do**
  $found = 0; class = c(e_i); period = p(e_i); day = d(e_i)$;
  $firstteacher = S(class, day, \overline{a}_{class,day})$; $lastteacher = S(class, day, \underline{a}_{class,day})$;
  if found = 0 **then**
    if $\tilde{t}_{lastteacher,day,period} = 1$ **then**
      $S(class, day, period) = lastteacher$; $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
    end
  end
  if found = 0 **then**
    if $\tilde{t}_{firstteacher,day,period} = 1$ **then**
      $S(class, day, period) = firstteacher$; $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
    end
  end
  for $j = \overline{a}_{class,day} + 1$ to $\underline{a}_{class,day} - 1$ **do**
    $teacher = S(class, day, j)$;
    if $found = 0$ & $teacher \neq 0$ **then**
      if $\tilde{t}_{lastteacher,day,j} = 1$ & $\tilde{t}_{teacher,day,period} = 1$ **then**
        $S(class, day, period) = teacher; S(class, day, j) = lastteacher$;
        $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
      end
    end
  end
  for $j = \overline{a}_{class,day} + 1$ to $\underline{a}_{class,day} - 1$ **do**
    $teacher = S(class, day, j)$;
    if $found = 0$ & $teacher \neq 0$ **then**
      if $\tilde{t}_{firstteacher,day,j} = 1$ & $\tilde{t}_{teacher,day,period} = 1$ **then**
        $S(class, day, period) = teacher; S(class, day, j) = firstteacher$;
        $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
      end
    end
  end
  for $j = day + 1$ to $|D|$ **do**
    $day_o = j$; $teacher = S(class, day_o, \underline{a}_{class,day_o})$;
    if $found = 0$ **then**
      if $\tilde{t}_{teacher,day,period} = 1$ **then**
        $S(class, day, period) = teacher$; $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
      end
    end
    $teacher = S(class, day_o, \overline{a}_{class,day_o})$;
    if $found = 0$ **then**
      if $\tilde{t}_{teacher,day,period} = 1$ **then**
        $S(class, day, period) = teacher$; $found = 1$; update $\underline{a}, \overline{a}, \widetilde{T}$;
      end
    end
  end
end
Output: S

Algorithm 2. Pseudocode for the heuristic

for $t \in T$, $c \in C$, $d \in D$, $p \in P$ **do**
  $x_{tcdp} = \overline{x}_{tcdp}$;
end
for $i = 1$ to $|C|$ **do**
  $\overline{c} = c_i$;
  for $t \in T$, $d \in D$, $p \in P$ **do**
    if $x_{t,\overline{c},d,p} = 1$ **then**
      $\tilde{t}_{tdp} = 1$;
    end
  end
  Solve ILP with $C = \{\overline{c}\}$ and updated availability matrix $\widetilde{T}$;
  Output: $x_{t,\overline{c},d,p} \forall t \in T$, $d \in D$, $p \in P$;
  for $t \in T$, $d \in D$, $p \in P$ **do**
    if $x_{t,\overline{c},d,p} = 1$ **then**
      $\tilde{t}_{tdp} = 0$;
    end
  end
end
Output: X

Appendix B

Table 1
Initial schedule for two classes with each empty slots (in italic) at day 1.

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Class 1								
Day 1	0	4	*0*	15	*0*	28	28	0
Day 2	0	43	20	44	21	9	0	0
Day 3	3	3	36	36	21	39	44	44
Day 4	20	20	39	20	44	44	21	0
Day 5	36	36	20	4	21	15	20	0
Class 2								
Day 1	0	0	4	*0*	4	36	36	0
Day 2	9	44	42	20	36	36	0	0
Day 3	29	29	21	42	44	21	39	9
Day 4	42	42	4	4	21	39	36	36
Day 5	4	15	4	42	15	21	0	0

Table 2
Availability of teachers (T = teacher is occupied/teaching a class not considered in this example; A = teacher is available to teach class 1 or 2; class 1 or class 2: teacher is assigned to the classes under study in the example in the old schedule).

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Availability teacher 3								
Day 1	A	A	T	T	T	A	T	T
Day 2	T	T	T	T	A	A	T	T
Day 3	Class 1	Class 1	T	T	A	T	T	A
Day 4	A	A	A	A	A	T	T	T
Day 5	T	T	T	T	T	T	T	T
Availability teacher 4								
Day 1	T	Class 1	Class 2	A	Class 2	T	T	A
Day 2	A	T	A	T	A	A	T	T
Day 3	A	A	A	T	T	T	T	A
Day 4	T	T	Class 2	Class 2	T	T	T	T
Day 5	Class 2	T	Class 2	Class 1	T	T	A	A
Availability teacher 9								
Day 1	T	T	T	T	T	T	T	T
Day 2	Class 2	T	A	T	T	Class 1	T	T
Day 3	T	T	T	T	T	T	T	Class 2
Day 4	T	T	T	T	T	T	T	T
Day 5	T	T	T	T	T	T	T	T
Availability teacher 15								
Day 1	T	T	T	Class 1	T	T	A	A
Day 2	T	T	A	T	T	A	T	T
Day 3	T	T	T	T	T	T	T	T
Day 4	T	T	T	T	T	T	T	T
Day 5	T	Class 2	T	T	Class 2	Class 1	T	T
Availability teacher 22								
Day 1	T	T	T	T	T	T	T	T
Day 2	T	A	Class 1	Class 2	A	T	T	T
Day 3	A	A	T	T	T	T	T	T
Day 4	Class 1	Class A	A	Class 1	T	T	T	T
Day 5	T	T	Class 1	T	T	T	Class 1	A
Availability teacher 21								
Day 1	T	T	T	T	T	T	T	T
Day 2	T	T	T	A	Class 1	A	T	T
Day 3	T	A	Class 2	A	Class 1	Class 2	T	A
Day 4	T	T	A	T	Class 2	A	Class 1	T
Day 5	T	T	T	T	Class 1	Class 2	T	A
Availability teacher 28								
Day 1	T	T	T	A	A	Class 1	Class 1	A
Day 2	T	T	A	T	T	T	T	T
Day 3	A	T	T	A	T	T	T	A
Day 4	A	A	T	T	T	T	T	T
Day 5	T	T	T	T	T	T	T	A

Table 2 (*continued*)

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Availability teacher 29								
Day 1	T	T	T	T	T	A	A	A
Day 2	T	T	A	A	T	T	T	T
Day 3	Class 2	Class 2	A	T	T	T	T	A
Day 4	T	T	T	T	T	T	T	T
Day 5	T	T	T	T	T	A	A	A
Availability teacher 36								
Day 1	A	A	A	T	T	Class 2	Class 2	A
Day 2	T	A	T	T	Class 2	Class 2	T	T
Day 3	A	A	Class 1	Class 1	T	T	T	A
Day 4	T	T	A	A	T	T	Class 2	Class 2
Day 5	Class 1	Class 1	T	T	T	A	A	A
Availability teacher 39								
Day 1	T	T	T	T	T	T	T	T
Day 2	T	T	T	T	A	T	T	T
Day 3	T	T	A	T	T	Class 1	Class 2	T
Day 4	A	T	Class 1	A	T	Class 2	T	A
Day 5	T	T	T	T	T	T	T	T
Availability teacher 42								
Day 1	A	T	T	T	A	A	A	A
Day 2	A	T	Class 2	T	A	A	T	T
Day 3	A	A	A	Class 2	T	T	T	T
Day 4	Class 2	Class 2	T	A	A	A	T	A
Day 5	A	T	A	Class 2	T	T	T	A
Availability teacher 43								
Day 1	T	T	T	T	T	T	T	T
Day 2	A	Class 1	T	T	T	T	T	T
Day 3	T	T	T	T	T	T	T	T
Day 4	T	T	T	T	T	T	T	T
Day 5	T	T	T	T	T	T	T	T
Availability teacher 44								
Day 1	T	T	T	T	T	T	T	T
Day 2	T	Class 2	A	Class 1	A	T	T	T
Day 3	A	A	T	A	Class 2	A	Class 1	Class 1
Day 4	T	T	T	T	Class 1	Class 1	A	A
Day 5	T	T	T	T	T	T	T	T

Table 3a
Solutions for rule of thumb.

	Class 1							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	4	36	15	28	28	0	0
Day 2	0	43	20	44	21	9	0	0
Day 3	3	3	36	36	21	39	44	44
Day 4	20	20	39	20	44	44	21	0
Day 5	0	36	20	4	21	15	20	0
	Class 2							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	0	0	4	4	36	36	0
Day 2	9	44	42	20	36	36	0	0
Day 3	29	29	21	42	44	21	39	9
Day 4	42	42	4	4	21	39	36	36
Day 5	4	15	4	42	15	21	0	0

Table 3b
Solutions for heuristic.

	Class 1							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	0	0	0	28	28	15	4
Day 2	0	43	20	44	21	9	0	0
Day 3	3	3	36	36	21	39	44	44
Day 4	20	20	39	20	44	44	21	0
Day 5	36	36	20	4	21	15	20	0
	Class 2							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	0	0	4	4	36	36	0
Day 2	9	44	42	20	36	36	0	0
Day 3	29	29	21	42	44	21	39	9
Day 4	42	42	4	4	21	39	36	36
Day 5	4	15	4	42	15	21	0	0

Table 3c
Solutions for optimal procedure.

	Class 1							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	0	4	15	28	28	0	0
Day 2	0	43	20	44	21	9	0	0
Day 3	3	3	36	36	21	39	44	44
Day 4	20	20	39	20	44	44	21	0
Day 5	36	36	20	4	21	15	20	0
	Class 2							
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Day 1	0	0	0	4	4	36	36	0
Day 2	9	44	42	20	36	36	0	0
Day 3	29	29	21	42	44	21	39	9
Day 4	42	42	4	4	21	39	36	36
Day 5	4	15	4	42	15	21	0	0

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