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## Strongly and slightly flavored gauge theories

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We review some recent progress in our understanding of the phase diagram of non abelian gauge theories, by varying their flavor content – fermion representations and the number of flavors. In particular, we explore the way conformal symmetry can be restored before the loss of asymptotic freedom, and through a subtle interplay of perturbation theory, chiral dynamics and confining forces. It is with the combination of numerical lattice studies and theoretical insights into gauge theories with and without supersymmetry that we may successfully attempt to clarify the missing pieces of this puzzle.

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\*Speaker.

## 1. The phase diagram of gauge theories and conformal symmetry

This review explores non abelian gauge theories beyond what is the best known, yet challenging example among others, quantum chromodynamics (QCD).  $SU(N)$  Yang Mills theory, with  $N$  gauge degrees of freedom – the colors – and no matter content, is asymptotically free and confines. Fermionic matter, added in various representations of  $SU(N)$ , effectively screens the strength of gauge interactions and eventually modifies the fundamental properties of the theory. First, confinement will disappear and chiral symmetry restores, then asymptotic freedom will be lost. This change of properties can be attributed to the emergence of fixed points of the renormalization group flow of the theory – in the infrared or the ultraviolet – accompanied by the occurrence of phase transitions in parameter space. Uncovering such change of properties thus consists in building a detailed map of the phase diagram of these theories in terms of their relevant parameters: these are the temperature  $T$ , the number of fermionic flavors  $N_f$  and the gauge coupling constant  $g$ .

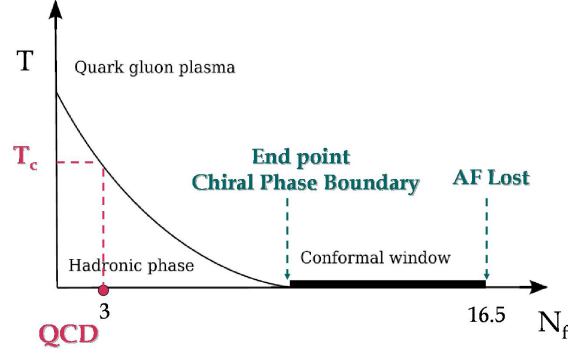
Conformal symmetry plays as protagonist in the zero-temperature plane. This symmetry is easily lost in four dimensional field theories, but can eventually be restored by varying their matter content. The presence of a conformal fixed point with anomalous dimensions can be a relevant ingredient for models of electroweak interactions at energies beyond the Standard Model. The running of gauge couplings is slowed down in a nearly conformal region of the parameter space, and it is this feature that has inspired new versions of technicolor models, such as tumbling [1] and walking technicolor [2]. On the other hand, understanding the way conformal symmetry is recovered, the relevance of the AdS/CFT correspondence for this mechanism, and the differences and similarities of theories with and without supersymmetry in this context, has a theoretical appeal in its own. As may be expected and based on our knowledge of QCD, chiral dynamics plays a major role in the game, and eventually renders a perturbative study insufficient. Then, a lattice formulation of gauge theories can in principle provide a non perturbative answer to the problem.

In the search for conformality, a distinction can be made between  $SU(N)$  gauge theories with fermions in the fundamental representation and those with fermions in higher dimensional representations, such as the adjoint or two-index symmetric. Properties of the first ones need many flavors to significantly change and recover conformality; I thus call these theories strongly flavored. Properties of the latter change when just a few flavors are added, and I call them slightly flavored theories.

The rest of this review is organized as follows. Section 2 is devoted to strongly flavored theories, meaning the case of a QCD-like theory with an enlarged flavor content. I will review some basic knowledge of the perturbative  $\beta$  function, formulate a few paradigms that might serve as guidance for lattice studies, and summarize recent lattice results and analytical conjectures. Section 3 treats the case of  $SU(N)$  gauge theories with fermions in higher dimensional representations. Possible scenarios for the zero-temperature phase diagram are discussed and recent lattice studies reviewed. I conclude in Section 4 with discussing future prospects, and connections to the physics at the Large Hadron Collider (LHC).

## 2. Strongly flavored theories: $SU(3)$ with fundamental fermions

We start with QCD with massless fermions and increase its flavor content. The first instructive



**Figure 1:** A qualitative view of the phase diagram of a QCD-like gauge theory with varying flavor content, in the  $T, N_f$  plane.

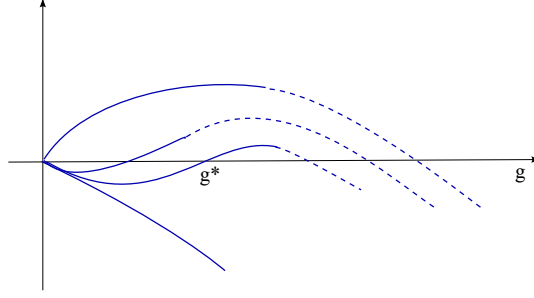
step is to project the three-dimensional phase diagram onto the plane temperature  $T$  and number of flavors  $N_f$ . Fig. 1 provides a qualitative summary of what we know and what we reasonably expect to be true. We know that QCD with three light flavors, at zero temperature, is in the hadronic phase: chiral symmetry is broken and the theory confined. At a critical temperature  $T_c \sim 120$  MeV, predicted by lattice QCD calculations, a phase transition occurs to quark-gluon plasma (QGP), where chiral symmetry is restored and the theory deconfined. Properties of QGP can be inferred from conformal symmetry to a good approximation. On the right hand side of Fig. 1, at zero temperature and larger  $N_f$ , we know that asymptotic freedom is lost at  $N_f = 16\frac{1}{2}$  [3]. In the intermediate region of this phase diagram, we envisage the existence of a continuous line of finite temperature phase transitions between a chirally broken (low- $T$ ) and a chirally symmetric (high- $T$ ) phase. The end point of this chiral phase boundary signals the opening of the *conformal window* at zero temperature<sup>1</sup>. The structure itself of the phase diagram suggests that the appearance of a conformal window is likely to result from the interplay between perturbation theory and genuinely non-perturbative chiral dynamics. We are thus challenged with a series of interesting questions where perturbative and non perturbative dynamics are interwound, and lattice field theory can provide useful answers. In particular, where is the end point of the chiral phase boundary located, what is the extent of the conformal region at large  $N_f$  is connected to the high temperature, small  $N_f$  quark-gluon plasma? The nature of chiral and deconfining phase transitions at various points of the phase diagram is also an historically challenging problem.

## 2.1 The perturbative running of the gauge coupling

The first step to understand the emergence, or loss, of conformality is to investigate the perturbative renormalization group (RG) flow of the theory in parameter space, in this case the running of the gauge coupling  $g$  in the weak coupling regime. It suffices to consider the Callan-Symanzik beta function to two loops, calculated in [5]:

$$\beta(g) = -b_0 \frac{g^3}{16\pi^2} - b_1 \frac{g^5}{(16\pi^2)^2} + O(g^7)$$

<sup>1</sup>For an analytical study of the scaling of chiral observables near the end point of the chiral phase boundary see [4].



**Figure 2:** The behavior of the two-loop beta function  $\beta(g)$  as a function of the coupling  $g$  for  $SU(3)$ , and increasing (bottom to top) the number of massless flavors  $N_f$  in the fundamental representation. For  $N_f < 8.05$  the beta function stays negative. For  $8.05 < N_f < 16.5$  the  $\beta$  function develops a zero (IRFP) at a non zero coupling  $g^*$ . For  $N_f > 16.5$ , the  $\beta$  function becomes positive, implying that  $g = 0$  is not anymore an UV stable fixed point and asymptotic freedom is lost. The beta function might develop an additional zero (UVFP) at strong coupling (dashed lines), a property to be investigated in future studies.

$$\begin{aligned} b_0 &= \frac{11}{3}C_2(G) - \frac{4}{3}T(R)N_f \\ b_1 &= \frac{34}{3}C_2(G)^2 - \frac{20}{3}C_2(G)T(R)N_f - 4C_2(R)T(R)N_f, \end{aligned} \quad (2.1)$$

for  $N_f$  massless Dirac fermions in the representation  $R$  of the compact Lie gauge group  $G$ .  $C_2(G)$  and  $C_2(R)$  are the quadratic Casimir operators of the adjoint and fermion representations, respectively, and  $T(R)$  is the trace of  $R$ . The coefficients  $b_0$  and  $b_1$  are universal, meaning they are renormalization scheme independent and their flavor dependence makes evident how the matter content affects, depending on the representation, the sign and zeros of the  $\beta$  function, modifying the RG flow of the theory. For  $G = SU(3)$  and  $N_f$  Dirac fermions in the fundamental representation the first two coefficients read  $b_0 = 11 - 2N_f/3$  and  $b_1 = 102 - 38N_f/3$  – see Table 1. For  $N_f^{AF} = 11C_2(G)/4T(R) = 16.5$ ,  $b_0$  becomes negative and asymptotic freedom is lost, see Fig. 2. It should be noted that, if  $b_0 = 0$ , the first non zero contribution to the beta function is

$$\beta(g) = \frac{g^5}{(16\pi^2)^2} (7C_2(G)^2 + 11C_2(G)C_2(R)), \quad (2.2)$$

which is positive for any group  $G$ . Thus, at least perturbatively, we should not expect the occurrence of a zero at finite coupling, signaling an ultraviolet fixed point (UVFP) in the non asymptotically free theory. Its non perturbative existence in the continuum is however not excluded, it was already

$R$	$C_2(R)$	$T(R)$	$R$	$C_2(R)$	$T(R)$
Fund	$\frac{N^2-1}{2N}$	$1/2$	2S	$\frac{(N-1)(N+2)}{N}$	$\frac{N+2}{2}$
Adj(G)	$N$	$N$	2A	$\frac{(N+1)(N-2)}{N}$	$\frac{N-2}{2}$

**Table 1:** The quadratic Casimir operator  $C_2(R)$  and the trace  $T(R)$  for various irreducible representations  $R$  of  $SU(N)$ : fundamental, adjoint, two-index symmetric (2S) and two-index antisymmetric (2A).

discussed in early works [6], recently reconsidered [7] and treated in Section 2.4. The second coefficient  $b_1$  changes its sign at  $N_f^* < N_f^{AF}$ , given by

$$N_f^* = \frac{34}{3} \frac{C_2(G)^2}{T(R) \left( \frac{20}{3} C_2(G) + 4C_2(R) \right)} \quad (2.3)$$

and  $N_f^* \simeq 8.05$  for  $SU(3)$  with fundamental fermions. It implies the emergence of a simple zero of the two-loop beta function at finite coupling, i.e. a stable infrared fixed point (IRFP) where the theory is conformally invariant with anomalous dimensions and the coupling approaches its fixed-point value with a power law.

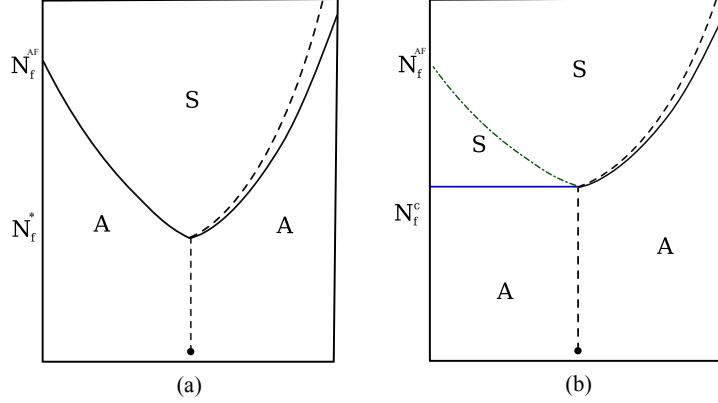
A perturbative expansion in the small parameter  $N_f^{AF} - N_f$  was suggested in [6]. Such an expansion implicitly assumes that we are allowed to consider non integer  $N_f$ : we can indeed continue  $N_f$  to non integer values since it appears analytically in the path integral of the theory. For  $N_f \lesssim N_f^{AF} = 16.5$  perturbation theory thus implies the existence of an entire family of asymptotically free theories with a finite zero of the beta function and conformal behavior. Approaching  $N_f = 8$  the task of establishing the emergence of conformality becomes increasingly non perturbative.

## 2.2 The theory beyond perturbation theory

Physics considerations, together with early results from explorative lattice studies, inspired the scenario proposed by Banks and Zaks [6] and reported in Fig. 3(a), for an  $SU(N)$  gauge theory with fundamental fermions. Projecting the phase diagram onto the zero-temperature plane  $(g, N_f)$  – where  $g$  is the bare lattice coupling and the phase diagram can eventually be mapped into the continuum –, the following features were envisaged. The line to the left in Fig. 3(a) is the location of the infrared fixed points associated with the zero of the perturbative beta function, where larger flavors correspond to weaker couplings. The line starts at  $N_f^*$  and ends at  $N_f^{AF}$ . Chiral symmetry should be exact on the right of this line, and the theory deconfined. The authors of [6] thus inferred that a deconfinement first order phase transition should accompany the emergence of the IRFP, together with a chiral transition from a broken to a symmetric phase. If driven by instabilities, the latter could also presumably be a first order transition. This picture, however, would lead to the counterintuitive conclusion that, for a given number of flavors, a phase transition from a chirally broken to a symmetric phase is taking place while moving towards stronger couplings.

On the right side of Fig. 3(a), a second order confining transition corresponding to the emergence of an UVFP – an additional zero of the continuum beta function – was also envisaged. This line extends to non asymptotically free theories with  $N_f > N_f^{AF}$ , and carries flavor dependence, since larger  $N_f$  produce more effective screening of confining forces, pushing the transition to stronger couplings. A chiral phase transition should also occur: fermions in the fundamental representation do feel confining forces, thus the chiral condensate cannot be zero as long as the transition to the massless gluon phase (deconfined phase) has not occurred. Thus the chiral line can only be to the left of the confinement line. Chiral symmetry breaking ( $\chi$ SB) and confinement are entangled for fundamental fermions, and lattice studies to date favor the conclusion that the two transitions actually coincide.

It is also true that the lattice regularized theory at zero temperature will undergo a transition to a chirally broken phase at sufficiently strong coupling, usually referred to as a bulk transition.



**Figure 3:** (a) The Banks-Zaks scenario for conformal symmetry restoration [6] by varying the gauge coupling  $g$  and flavors  $N_f$ . S and A refer to chirally symmetric and asymmetric, respectively. Dashed lines are chiral transitions and solid lines are confinement transitions. They coincide on the left branch, location of the IRFP. (b) The modified scenario by [8]. The horizontal (blue) line is the conformal phase transition (CPT) that opens up the conformal window. The dot-dashed (green) line is the location of the IRFPs.

It is a task for the lattice to distinguish between a scenario with a strong coupling UVFP in the continuum theory, and thus accompanied by second order phase transitions, and a scenario with a lattice bulk transition to a phase with no continuum limit – see also Section 2.4. The two lines in Fig. 3(a) plausibly coalesce at one point  $(N_f^*, g^*)$ , below which the theory always confines and is chirally broken. A first order bulk transition, a feature dependent on the lattice action, between two chirally broken phases can occur at lower number of flavors, and should be absent in the pure gauge theory.

It is the study of the dynamics of chiral symmetry that brought to the discovery of the *conformal window* in [9, 8]. In [8] a line is added to the phase diagram of Fig. 3(a), providing the new scenario in Fig. 3(b), and avoiding the occurrence of a chiral restoration transition at the IRFP. The key ingredient is the observation that there exists a critical coupling, solution of the Schwinger-Dyson gap equation, above which chiral symmetry must be broken. This translates into a critical number of flavors  $N_f^c$  – the horizontal line of Fig. 3(b) – at which a transition from a chirally symmetric many-flavor phase to a chirally broken phase occurs. Such a zero-temperature phase transition [9, 8] is not of second order, nor first order, and named *conformal phase transition* (CPT): it is continuous in the chiral order parameter with an abrupt change in the spectrum of the theory. A CPT might also enter in the dynamics of strongly coupled condensed matter systems, such as a non-Fermi liquid, which might be relevant for high-temperature superconductivity.

The CPT at  $N_f^c$  signals the opening of the conformal window, the interval  $N_f^c < N_f < N_f^{AF}$  where theories develop a conformal IRFP with anomalous dimensions, they do not confine and chiral symmetry is exact. The green line in Fig. 3(b) is the location of the IRFPs where, differently from Fig. 3(a), no phase transition occurs. It separates two chirally symmetric and deconfined theories which differ in their short distance behavior: an asymptotically (ultraviolet) free theory to

the weak coupling side, and a coulomb theory (infrared free) to the strong coupling side. Besides separating two symmetric phases, the phase diagram of Fig. 3(b) differs from Fig. 3(a) because the lower end point of the new phase occurs at  $N_f^c$  and not at  $N_f^*$ , the value predicted by the perturbative beta function; chiral dynamics has been incorporated, and for  $N_f < N_f^c$  the infrared stable fixed point is washed out by the generation of a dynamical fermion mass.

The first quantitative prediction for the lower end-point of the conformal window was obtained [10] by solving the Schwinger-Dyson gap equation in the ladder approximation, and demanding that the conformal window should close when the IRFP coupling  $\alpha(N_f)$  – predicted by the perturbative beta function – reaches the critical value at which a mass gap arises; this happens when the anomalous dimension of the fermion mass operator  $\bar{\psi}\psi$  is  $\gamma = 1$ .  $N_f^c$  can thus be determined at a given order in the combined perturbative expansion of the beta function and the anomalous dimension  $\gamma$ .

To leading order and for  $SU(3)$  with fundamental fermions  $N_f^c \simeq 11.9^2$ . As the authors of [10] interestingly observed, the ladder approximation becomes gradually more accurate for higher dimensional representations, and suggest that the estimate for fundamental fermions is good to about 20%. The question remains, if the mechanism advocated above and entirely driven by chiral dynamics is actually providing the complete picture. The uncertainties involved in the analytical predictions, and the subtle interplay of chiral dynamics and confinement invite for a lattice enterprise.

### 2.3 Lattice studies

Two complementary strategies can pave the way for lattice studies, and have in fact been implemented in recent works. One can reconstruct the theory renormalization group flow on the lattice by means of the discretised beta function, followed by the recovery of the continuum limit. This can be done with the Schroedinger functional and step scaling function technique, as in [11]. Similarly, a Monte Carlo RG can be formulated on the lattice [12]. The aim of this type of studies is the mapping of the zero's of the beta function; the emergence of conformality – alias the existence of a stable IRFP – is probed by the flattening of the continuum running coupling, reached from the far ultraviolet and the far infrared.

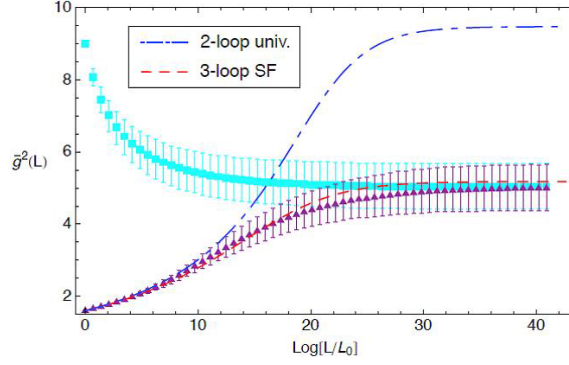
The other strategy, complementary to the previous, is inspired by the physics of phase transitions, and is driven by the general scope of uncovering the complete phase diagram of gauge theories. This approach has been recently adopted in [13], and an early study can be found in [14]. Chiral symmetry and its breaking pattern is also monitored by the distribution of the eigenvalues of the Dirac operator<sup>3</sup>; this strategy has been investigated in [17].

Recent studies have established that  $N_f = 8$  lies within the hadronic phase of QCD [18], resolving a long time contention. Delicate task remains the one of locating the lower-end point of the conformal window. Analytical studies favor a value around  $N_f = 12$  [10, 4], reason for recent

<sup>2</sup>An enhancement of the chiral condensate w.r.t. the ordinary QCD phase is also predicted in the broken phase and in the vicinity of the critical point  $N_f \lesssim N_f^c$  [10].

<sup>3</sup>The assumption of preserved maximal flavor symmetry implies a unique breaking pattern for all irreducible representations[15]. In particular  $SU(N_f) \times SU(N_f) \rightarrow SU(N_f)_V$  for complex,  $SU(2N_f) \rightarrow O(2N_f)$  for real, and  $SU(2N_f) \rightarrow Sp(2N_f)$  for pseudoreal representations. Staggered fermions on coarse lattices show an inverted pattern for real and pseudoreal fermions [16].





**Figure 4:** The running gauge coupling for  $SU(3)$  with twelve flavors of fermions in the fundamental representation from [11].

lattice investigations with twelve flavors [11, 13, 12, 17, 19]. The result reported by [11] and summarized in Fig. 4 beautifully shows a flattening of the running coupling from the far infrared (from above) and the far ultraviolet (from below), thus suggesting that twelve flavors are in the conformal window. The same conclusion is reached by the authors of [13], based on the physics of phase transitions and a strategy that uses as a paradigm the scenario of Fig. 3(b). The strategy allows to explore the theory in different regimes and regions of the phase diagram, and infer the existence of the IRFP from outside its basin of attraction. Three ingredients are used: moving from strong to weak coupling one encounters i) a bulk transition in the chiral order parameter, ii) a high-statistics, infinite volume chiral extrapolation of the condensate strongly favors exact chiral symmetry on the weak coupling side of the transition and iii) the mass spectrum is compatible with a positive beta function, signature of a Coulomb phase. This result agrees with Fig. 3(b), while the exclusion of Fig. 3(a) further needs the confirmation that no  $\chi$ SB transition is encountered at weaker couplings. Other numerical studies [12, 17, 19] have challenged the conclusion of [11, 13]. This is hardly surprising, given that  $N_f = 12$  is likely to be close to  $N_f^c$ , making a numerical study particularly delicate. In this context, I would like to observe that a comparison of the light spectrum and decay constants of the theory with the form predicted by chiral perturbation theory – at large or small volumes – might lead to incomplete or misleading interpretations and needs to be corroborated by additional evidences such as phase transitions and high precision chiral extrapolations.

Alternatively, one can isolate optimal observables that are directly sensitive to the presence of an infrared fixed point and work within its basin of attraction; recalling from Fig. 3(b) that theories on the sides of the infrared fixed point differ in their ultraviolet behavior – the presence or absence of asymptotic freedom – one should expect that observables sensitive to the far ultraviolet will show a change in behavior across the fixed point. This might be the case for the plaquette, or particular combinations of its spatial and temporal components – entangled to the trace anomaly [20]. The inclusion of additional scales, mass and temperature, can be used to measure the anomalous dimensions of the relevant operators in the surroundings of the would be conformal fixed point. A power-law scaling of the chiral condensate  $\langle \bar{\psi}\psi \rangle \sim A m^\delta$ , determined by its anomalous dimensions, should be expected in the vicinity of the infrared fixed point [21]; this behavior is in fact observed in [13].

Finally, a direct numerical study of the realization of a conformal phase transition by varying the number of flavors would be welcome; it is in principle feasible, provided the lattice fermion action can be extrapolated to the correct continuum limit for any value of the flavor number.

## 2.4 Theory and conjectures: recent developments

A closer comparison with supersymmetric gauge theories, the application of the AdS/CFT correspondence, and attempts to better account for the confining dynamics are all at the base of most recent developments in the theoretical description of conformality in non abelian non supersymmetric gauge theories. The phase diagram in Fig. 3(b), with the prediction of a conformal window, resembles what happens in supersymmetric theories; there, electromagnetic duality allows to establish the existence of a conformal window over the interval  $3/2N_c < N_f < 3N_c$  for supersymmetric QCD [22], while the presence of supersymmetry provides an exact beta function also in the presence of matter multiplets [23] – the NSVZ beta function. Crucial ingredients, consequences of supersymmetry, are the cancellation of non zero modes and non renormalization theorems, which guarantee for the anomalous dimensions  $\gamma_{gluino} = \gamma_{gluon} = \beta(g)/\alpha(g)$ . Without supersymmetry, an NSVZ-like beta function is in general not expected, and one has to rely on truncations of the perturbative expansion, on particular simplifying limits such as the large- $N$  limit, or conjectures. One exception is the large- $N$  limit of  $SU(N)$  with one Dirac flavor in the two-index antisymmetric or symmetric representation [24], due to planar equivalence with supersymmetric Yang-Mills. In all other examples, it is instructive to look for analogies and differences with the supersymmetric case.

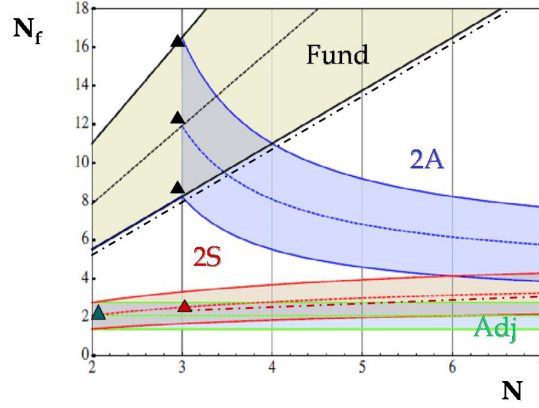
The beta function of Yang Mills theories in the large- $N$  limit recently derived in [25], is a potentially important result. The absence of supersymmetry – and the presence of non zero modes – is responsible for the appearance of an anomalous dimension term in the running of the canonical coupling  $g_c = \sqrt{N}g$  and a renormalization scheme dependence, contrary to supersymmetric Yang-Mills. It would be interesting to extend the result of [25] to include matter fields, for example in the Veneziano limit in which  $N_f \rightarrow \infty$  and  $N_c \rightarrow \infty$  with  $N_f/N_c$  finite<sup>4</sup>. On the other hand, the supersymmetry inspired beta function for  $SU(N)$  gauge theories conjectured in [26] and used to estimate a bound for the lower end of the conformal window, does not account for the presence of an anomalous dimension term arising from the lack of supersymmetry. The presence of such a term would possibly modify the constraint on the lower end of the conformal window predicted in [26] and would force an analysis order by order in the gauge coupling.

The upper part of Fig. 5 summarizes the bounds on the conformal window obtained for fundamental fermions by using i) the ladder solution to the gap equation [10], ii) deformation theory (DT) [27], and iii) a supersymmetry inspired conjecture for the beta function [26] to which the unitary bound on the dimension of  $\bar{\psi}\psi$ ,  $D_{\bar{\psi}\psi} = 3 - \gamma \geq 1$ , is imposed. The latter and the deformation theory analysis of [27] predict a lower bound around  $N_f \sim 8$ , similar to the two-loop beta

<sup>4</sup>A possible generalization that reproduces the two-loop beta function in the Veneziano limit can be

$$\beta(g_c) = \frac{-\beta_0^\infty g_c^3 + \frac{\beta_j}{4} g_c^3 \left( \frac{\partial \log Z}{\partial \log \Lambda} + c_F \frac{g_c^2}{16\pi^2} \right) + c_F \frac{g_c^3}{16\pi^2} (1 + \gamma(g_c^2)/2)}{1 - \beta_j g_c^2}, \quad (2.4)$$

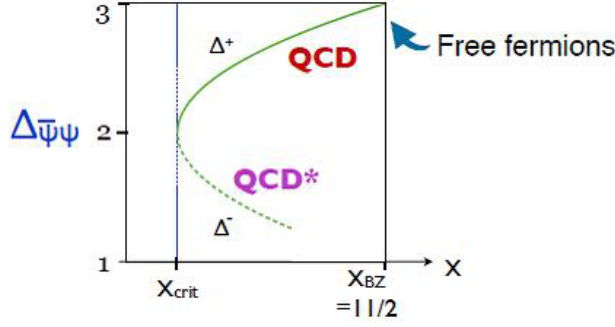
where we assumed the presence of matter fields only in the numerator in this limit,  $c_F = 4T(R)N_f/(3N)$ , while  $\beta_j$  and  $\partial \log Z / \partial \log \Lambda$  are derived in [25].



**Figure 5:** Plot of the predicted conformal windows, adapted from [26]. For each representation, fundamental (Fund), two-index asymmetric (2A), two-index symmetric (2S) and adjoint (Adj), the lowest curve is the lower bound predicted by a supersymmetry inspired conjecture [26], the middle curve (dashed) is the lower bound from the gap equation in the ladder approximation [10] and the dot-dashed line is the prediction from deformation theory in [27]. A more recent analysis [28] moves the dot-dashed line for fundamental fermions to  $\sim 4N$ , close to the ladder estimate. Triangles are points simulated on the lattice.

function. The solution to the gap equation is more restrictive and provides a lower bound  $N_f \sim 12$  that captures the bulk of chiral dynamics. No surprise that the unitarity bound is loosely constraining, since it corresponds to an anomalous dimension  $\gamma = 2$  of the fermion mass operator, while the ladder solution of the gap equation corresponds to  $\gamma = 1$ ; the description provided by the latter might well be the closest approximation to reality. In fact, a more recent and complete analysis with deformation theory [28], moves the estimate of the lower end – most likely an upper bound of it – of the conformal window for fundamental fermions to  $\sim 4N$ , close to the ladder prediction. The use of worldline formalism and large- $N$  [29] leads to a similar estimate for fundamental fermions. The rest of Fig. 5 will be treated in Section 3.

Already in [6] the coalescence of an infrared and ultraviolet fixed point was embedded in scenarios for conformality, although no conformal window or underlying mechanism for their merging was envisaged. Recently, the authors of [7] have proposed a scenario for QCD at large number of flavors inspired by the AdS/CFT correspondence, where the closure of the conformal window is due to the merging of the Banks-Zaks IR fixed point and an UV fixed point, most plausibly appearing at strong coupling. The theories on the infrared and ultraviolet branch of Fig. 6 are the boundary realization of a bulk scalar theory in  $\text{AdS}_5$ . The dimension of the chiral condensate on the two branches is thus constrained by the profile of the bulk scalar field through AdS/CFT, so that the sum is fixed  $\Delta_+ + \Delta_- = d = 4$ . A relevant consequence is that the dimension of the chiral condensate on the IR branch ranges from 3 (the free limit) to 2, and never approaches the unitary limit  $\Delta_+ = 1$ ; this would create difficulties for those models of electroweak symmetry breaking demanding large – greater than one – anomalous dimensions. The merger mechanism, if in place, would distinguish QCD-like theories from their supersymmetric version, where the disappearance of the IRFP happens because  $g_{IRFP} \rightarrow 0$  on one side of the conformal window where  $N_f/N_c = 3 - \varepsilon$ , and  $g_{IRFP} \rightarrow \infty$  on the other side where  $N_f/N_c = 3/2 + \varepsilon$ .



**Figure 6:** Picture for the disappearance of conformality from [7], by varying  $x = N_f/N_c$ . The lines denote the dimension of the chiral condensate at the IR fixed point (upper) and the UV fixed point (lower).

Various elements hint at the possibility that the merger scenario would be intimately connected with the scenario derived from the Schwinger-Dyson gap equation. In fact, known examples of merging (e.g. defect QFT) do produce a Berezinskii-Kosterlitz-Thouless (BKT) scaling of the chiral condensate [7], nothing but the Miransky scaling [8] of a conformal phase transition. In addition [7], the point at which the anomalous dimension of the fermion mass operator becomes  $\gamma = 1$  is also the point at which four-fermi interactions  $(\bar{\psi}\psi)^2$  become (marginally) relevant. This would suggest that QCD\*, the dual theory of QCD on the UV branch, contains this relevant operator. As a consequence, the parameter space of the continuum effective theory at strong coupling is enlarged.

A lattice study can in principle establish or exclude the existence of an ultraviolet stable fixed point at strong coupling. One example is the MBL [30] ultraviolet fixed point in quenched strongly coupled QED, studied on the lattice in [31]. The first step of a lattice search would be to uncover the nature of the bulk transition observed in the vicinity of the lower end point of the conformal window – this might be the case of [13] for  $N_f = 12$ : a first order transition would exclude the occurrence of an UVFP in the continuum theory. One comment is in order about the presence of new relevant operators at strong coupling in the continuum effective lagrangian. There, the RG flow must be studied in a multi-dimensional parameter space. The lattice regularized theory, however, has only one adjustable bare parameter – the gauge coupling. It is reasonable to expect that four-fermion like interactions will anyway be induced on the lattice by the gauge dynamics at sufficiently strong coupling; thus, we should not explicitly account for the effective low energy realization of the continuum theory on the lattice. In other words, the physics of phase transitions extracted from the lattice regularized fundamental theory, should provide a complete description of the continuum strongly coupled theory.

### 3. Slightly flavored theories: Other representations for fermions

The dependence of the two-loop beta function in eq. (2.1) upon the quadratic Casimir operator  $C_2(R)$  and the trace  $T(R)$  of the fermion representation  $R$  shows that higher (than the fundamental) dimensional representations are more effective in i) losing asymptotic freedom ii) acquiring an IRFP. In other words, they push the zero of the coefficients  $b_{0,1}$  to a lower number of flavors: for adjoint fermions in color  $SU(3)$  – see Table 1 – asymptotic freedom is lost for

$N_f^{AF} = 11/4 = 2.75$ , while an IRFP – a second zero of the beta function – would appear for flavors larger than  $N_f^* = 102/96 \simeq 1.06$ . Thus, it appears that slightly flavoring an  $SU(N)$  gauge theory with fermions in higher dimensional representations would be sufficient for the emergence of conformality. The appearance of an infrared attractive fixed point at low  $N_f$  suggests new avenues for strongly-interacting theories beyond the Standard Model: the near appearance of a fixed point, in the proximity of a conformal window, with a beta function hover near the axis and a slowly varying gauge coupling, is an appealing feature for candidates of walking technicolor theories. Phenomenology favors low  $N_f$  and  $N$ .

Turning to chiral dynamics, it is important to observe that the solution of the gap equation predicts, to leading order in the ladder approximation, a critical coupling  $\alpha_c = \pi/(3C_2(R))$  for the emergence of a mass gap; such coupling decreases with increasing Casimirs. This argument shows that fermions in higher dimensional representations of the color group move the breaking of chiral symmetry to weaker couplings; short-distance forces alone become gradually more adequate to drive chiral symmetry breaking. This is also in qualitative agreement with the Casimir scaling of the chiral symmetry breaking coupling  $C_2(R)g_{ren}^2 \simeq const$ , observed in early quenched lattice studies [32] for  $SU(3)$  with sextet and octet fermions.

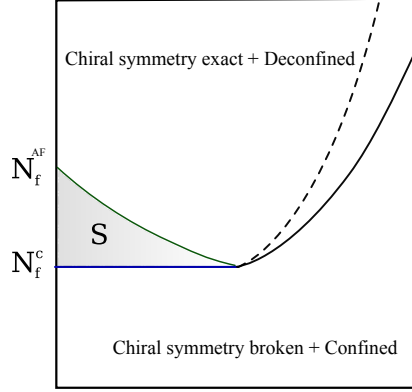
Lattice studies with dynamical fermions in varying representations should provide a better understanding of the interplay between confinement and  $\chi$ SB mechanisms. We must distinguish adjoint fermions, which do not feel confining forces at large distances, from other representations that do feel confining forces: the latter will form bound states, a condition sufficient to induce  $\chi$ SB [33]. In particular, for  $SU(N)$  with fundamental fermions, the 't Hooft anomaly matching conditions show that confinement implies chiral symmetry breaking for  $N_f > 2$  [34]. For adjoint fermions instead, the theory contains two natural scales, the string tension for heavy fundamental quarks and the  $\chi$ SB scale for light adjoint quarks. A separation of scales was in fact observed [35] for  $SU(2)$  with two Dirac adjoint flavors, and interpreted as two thermal transitions with  $T_{deconf} < T_{chiral}$ .

Based on these facts, we can attempt some speculations about the structure of the zero-temperature phase diagram for adjoint fermions and for other representations such as the two-index symmetric (2S) or antisymmetric (2A). For the latter two<sup>5</sup> the formation of bound states will induce  $\chi$ SB, possibly leading to a similar phase diagram to fundamental fermions<sup>6</sup>. Independently of the fermion representation, once  $\chi$ SB has occurred, fermions acquire a dynamical mass and decouple at energies below this mass, leaving the pure gauge theory which confines.

If a conformal window arises through a conformal phase transition at a given  $N_f^c$ , as discussed for fundamental fermions, the phase diagram for adjoint flavors will be different from the scenarios first depicted in [6]. Instead, Fig. 7 shows a possible scenario with a conformal phase transition. We have no theoretical prejudice about the relative order of the chiral and confinement transitions for adjoint fermions. However, at the lower end of the conformal window, call it  $(N_f^c, g^c)$ , the non perturbative beta function should turn negative, and confinement should follow. A plausible picture is that the confinement and chiral lines coincide at least at the critical point  $(N_f^c, g^c)$ . Away from it, the lines can a priori be disjoint, with the constraint that the theory is deconfined and chirally

<sup>5</sup>In some cases representations may coincide, as for adjoint and 2S in  $SU(2)$ , or fundamental and 2A in  $SU(3)$ .

<sup>6</sup>A scale separation might still occur, with the  $\chi$ SB transition happening before the theory confines.



**Figure 7:** Speculations over the zero-temperature phase diagram for  $SU(N)$  with adjoint fermions, assuming that a conformal phase transition (horizontal blue line) occurs at  $N_f^c$ . At strong coupling (r.h.s.) the ordering of the bulk  $\chi$ SB transition (dashed) and confinement transition (solid) seems favored. At weak coupling (l.h.s.) the green line is the location of IRFPs. The shaded area qualitatively indicates where a deconfinement transition may occur. S stands for chirally symmetric.

symmetric at the IRFP. The ordering of the bulk transitions on the right side of Fig. 7 would be favored by [35] and the occurrence of  $\chi$ SB at relatively weak coupling. At weaker coupling and for  $g < g^c$  the theory is possibly confined for any flavor  $N_f < N_f^c$ . The deconfinement transition might occur in the surroundings of the CPT line, or even coincide with the IRFP line<sup>7</sup>, as originally envisaged by the authors of [6]. Their scenario of Fig. 3(b) in [6] would however be difficult to accommodate in a phase diagram where a conformal phase transition occurs.

The lower part of Fig. 5 reports on the conformal windows predicted for fermions in the adjoint and the two-index symmetric representations. Deformation theory [27, 28] and the gap equation produce similar estimates for the lower end point and differ from the unitary bound prediction. Given that, according to [27], deformation theory takes confinement into account, its agreement with the ladder prediction is suggestive of the fact that chiral dynamics alone, as previously argued, leads to a complete description of the emergence of conformality. For this reason, the initial disagreement of [27] with the ladder prediction for fundamental fermions was rather surprising, and recently resolved in [28]<sup>8</sup>. Additional estimates with the worldline formalism for various gauge groups and matter representations can be found in [29].

Lattice studies are currently being carried out in two specific cases, both candidates for technicolor theories:  $SU(3)$  with two Dirac flavors in the two-index symmetric (2S) representation of the color group [36], and in the quenched approximation [37], and  $SU(2)$  with two Dirac flavors in the adjoint (Adj) representation [38, 39, 40].

The study in [36] favors the presence of an IRFP for  $SU(3)$  with two sextet (2S) fermions, by using the discrete beta function and studying the confinement phase transition in the lattice

<sup>7</sup>In this case the theory would be confining for all  $N_f < N_f^{AF}$ .

<sup>8</sup>Notice that, differently from the gap equation ladder-approach [10], the formalism of [27, 28] can still be applied to chiral gauge theories which exhibit confinement without chiral symmetry breaking.

parameter space with varying volume. The degeneracies of the screening masses and behavior of the pseudoscalar decay constant indicate that the deconfined phase is also a phase in which chiral symmetry is restored. Note that the ladder solution of the gap equation and deformation theory predict a lower end of the conformal window at  $N_f^c \simeq 2.5$  and  $N_f^c \simeq 2.4$ , respectively, while asymptotic freedom is lost at  $N_f^{AF} = 3$ . Hence, a simulation at  $N_f = 2$  is likely to be in the vicinity of the conformal window, rendering a lattice study a delicate task. For  $N_f = 1$  and in the large- $N$  limit the theory is non perturbatively equivalent to supersymmetric Yang-Mills [24], and thus confines.

It has been proposed that four-dimensional QCD with adjoint fermions exhibits volume independence in the large- $N$  limit [41]. This would allow for *reduction*, i.e. the possibility to determine the properties of the theory from just a single-site lattice calculation. Evidence for reduction is found in [42] with one flavor of Dirac adjoint (Wilson) fermions. Another study [43], using overlap as opposed to Wilson fermions, has recently confirmed the evidence for reduction.

Analytical predictions for the lower end of the conformal window suggest, as shown in Fig. 5, that the  $SU(2)$  theory with two Dirac fermions in the adjoint representation is a likely candidate for a near-conformal or walking scenario. Lattice calculations of the spectrum of the theory [38] and the renormalization group flow [40] possibly hint at a conformal or near-conformal behavior, though still requiring an improved control of systematic uncertainties. Recently, the authors of [39] have outlined a strategy based on the combination of gluonic and mesonic observables in order to discriminate between a confining and conformal scenario.

#### 4. Some concluding remarks

The last two years have seen a rapidly growing interest in the yet unexplored aspects of non abelian gauge theories. The list of names, contributing to this enterprise, is getting longer. We interrogate over the emergence, or loss, of conformal symmetry by varying the flavor content of the theory; what is the interplay of chiral dynamics and confining forces? What residual analogy does survive, when going from the supersymmetric to the non supersymmetric theory? Elucidating the way conformal symmetry or its remnants drive particle dynamics will contribute to clarifying the possible connection of field theory to string theory that the AdS/CFT correspondence seems to imply. Lattice simulations are the natural place where to look for a non-perturbative answer and complement analytical predictions and conjectures.

One main motivation, genuinely theoretical, drives these lattice studies: we aim at a better understanding of the phase diagram of gauge theories. This is a reason enough to pursue this goal. Such an understanding is not only important per se, but it is likely to improve our way and tools for describing the critical behavior and the breaking of symmetries in field theory, where relevant examples go from particle physics to condensed matter systems. All the subtleties of field theory, on and off the lattice, come into play in this entertaining game.

A connection to the physics at the Large Hadron Collider (LHC) is also a must, in these times. Needless to say that conformal symmetry might play an important role beyond the electroweak symmetry breaking (EWSB) scale, and as such it is potentially relevant for physics searches at the LHC. Quark-gluon plasma physics at Alice will teach about deviations from conformality, and LHC will say about the mechanism of EWSB: is it induced by a strongly interacting dynamics,

of which walking technicolor is an example, and what gauge groups do realize the phenomenologically allowed symmetry breaking pattern? More in general, and without invoking technicolor, the unification of forces might still be realized in a scenario where gauge groups attain a quasi-conformal dynamics, with or without supersymmetry, for a given interval of energies. The impact on phenomenology, with the LHC activities just starting, added to a theoretically driven curiosity, make it the right time to pursue these studies.

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