Hybrid Lateral Transshipments in a Multi-Location Inventory System

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February 19, 2010

Abstract

In managing networks of stock holding locations, two approaches to the pooling of inventory have been proposed. Reactive transshipments respond to stockouts at a location by moving inventory from elsewhere within the network, while proactive redistribution of stock seeks to minimise the chance of future shocks. This paper is the first to propose a hybrid approach in which transshipments are viewed as an opportunity for stock redistribution. We adopt a quasi-myopic approach to the development of a strongly performing hybrid transshipment policy. Numerical studies which utilise dynamic programming and simulation testify to the benefits of using transshipments proactively. In comparison to a purely reactive approach to transshipment, service levels are improved while a reduction in safety stock levels is achieved. The aggregate costs incurred in managing the system are significantly reduced, especially so for large networks facing high levels of demand.

Keywords: Inventory Control, Lateral Transshipments, Dynamic Programming

1 Introduction

Lateral transshipments (LTs) are stock movements between locations in the same echelon of an inventory system. They provide a valuable tool to supply chain managers who are looking to reduce the penalties associated with a lack of stock at one or more inventory points. By

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strategically reallocating excess stock it can be possible to improve the systemwide service levels and/or lower the cost of operating the system. These goals have traditionally been sought within spare part networks, where there is a high penalty attached to a stockout. However the benefits of LTs have also been realised in sectors ranging from retail to energy generation. The challenge that LTs bring is in managing when and where it is beneficial to instigate a stock movement. An LT may reduce the short term stockout risk at the receiving location but it inevitably increases the longer term risk at the sending location. A transshipment policy must therefore balance these contrasting risks and decide when the cost of transshipment is outweighed by the benefit it is expected to deliever.

The suitability of a given LT policy will often depend on the attributes of the inventory system in which it is employed. However a key distinction within the literature on LTs is that between reactive and proactive policies. Reactive LTs are performed when a stockout or potential stockout occurs, by shipping either the whole demand or the number of items short from a different location. Proactive transshipments are performed periodically to rebalance the whole system's stock levels. This paper's principle motivation is in considering a policy which falls between these two distinct sets so as to maximise the benefit each transshipment can deliver.

1.1 Literature

Reactive LTs have been studied in the literature under both a periodic and continuous order review setting. For periodic review models, Krishnan and Rao (1965) develop optimal transshipments in a single period for a system with two locations. This is expanded to a multi-location, multi-period setting by Robinson (1990), although here the optimal solution can only be determined for either two locations or identical locations. This highlights the complexity of determining optimal transshipment policies. These papers perform LTs once all demand is known but before it has to be satisfied. In contrast, Archibald (2007) and Archibald et al. (2009, 2010) develop approximately optimal policies which can respond to continuous demand within each period. The former proposes heuristics to deal with the transshipment decision process, while the latter papers look to improve upon this and relax some of the restrictions using dynamic programming policy improvement techniques. The results obtained from these policies show them to be reasonably close to optimal when used in small networks. This method of validation is one which this paper looks to emulate. The above models focus on single echelon centralized models. However, additional research in the periodic setting considers two echelon models [eg. Dong and Rudi (2004)] and decentralized models [eg. Rudi et al. (2001)].

Much of the literature on reactive LTs in a continuous order review setting is motivated by applications in the spare parts industry. Here, practical settings include electronic component manufacturing and electricity generation companies. Building on the METRIC repairs model of Sherbrooke (1968), Lee (1987) proposes a model which uses complete pooling within preset groups of identical locations. This shows the benefit of LTs within the area and the model is expanded by Axsäter (1990) to allow non-identical locations. Several papers have been written which further expand these ideas by relaxing or tightening some constraints such as making repair capacity finite [Jung et al. (2003)] or using lost sales rather than backordering [Dada (1992)]. Alternatively, Sherbrooke (1986) considers a model where backorders have to be minimized rather than costs. All of these papers assume an order-up-to replenishment policy for each location. Kukreja and Schmidt (2005) consider the more general (s, S) policy, but have to resort to a simulation based approach to determine the optimal order policy.

Away from spare parts, Archibald et al. (1997) shows that in a periodic review model without fixed order costs, an order-up-to policy is optimal. However, positive order costs or minimum order quantities often suggest that an (R, Q) policy is more appropriate in practice. Several papers take this approach. Evers (2001) and Minner et al. (2003) develop heuristics that can be used to determine when and how much to transship for systems with lost sales. Axsäter (2003) does the same, but for a model with backorders. He proposes a decision rule which is constructed to make optimal decisions under an assumption that no further transshipments will be made. This assumption enables the exact myopic benefit of transshipping to be calculated and optimised.

Research on proactive LTs explores their use to rebalance an entire system's stock on hand. This rebalancing is done at a set point during a review period and before all demand has been realized. Allen (1958) and Agrawal et al. (2004) consider this problem independently of replenishment decisions. Allen (1958) looks to perform the transshipments at the start of the demand period, whilst Agrawal et al. (2004) devise a method to calculate the best time to redistribute stock during the period.

Other authors study proactive transshipment and replenishment decisions together. Gross (1963) provides optimality results for a two-location system, where both ordering and redistribution decisions take place at the beginning of the review period. This idea is further developed by Das (1975), who allows the redistribution point to occur at an arbitrary time during the review period. Gross and Das both assume negligible transshipment times. Jönsson and Silver (1987) and Bertrand and Bookbinder (1998) allow positive transshipment times. The main difference between these two studies is that Jönsson and Silver (1987) consider how best to meet service levels whilst Bertrand and Bookbinder (1998) examine the goal of cost reduction.

Both reactive and proactive LTs have been shown to provide cost benefits. In this study, we analyze the first 'hybrid' transshipment policy which tries to secure the benefits of both. Our policy can quickly react to shortages by allowing transshipments at any time when they occur, as for previously proposed reactive LT policies. However, the policy also seeks to proactively redistribute stock between the sending and receiving locations whenever such an

LT is triggered. This will allow maximum benefit to be extracted from each transshipment instance and will be especially beneficial in systems where there is a significant fixed cost involved in carrying out a transshipment.

The specific setting that we consider is as in Axsäter (2003), with backordering and an arbitrary number of stocking locations which all apply (R, Q) ordering policies. Axsäter (2003) derives an algorithm that determines near-optimal reactive transshipment decisions. These are shown in a simulation study on small networks (with two and three locations) to provide a significant cost benefit compared both to not transshipping at all and to applying a simpler transshipment policy. In this paper, we generalize this algorithm with the goal of determining an approximately optimal hybrid transshipment policy that allows stock redistribution. The results of a comparative numerical study show that, for small networks, the hybrid policy significantly outperforms the original Axsäter reactive proposal, achieving an average 1.6% cost saving over 600 experiments. Such a recurrent saving is of major practical importance, considering that inventory costs typically account for a substantial proportion of a business's total turnover. To analyze the closeness to optimality of our hybrid policies, we also develop a dynamic programming (DP) approach to finding an ϵ-optimal hybrid transshipment policy. Numerical results show that the optimality gap is closed by over 65% on average compared to a policy of not transshipping and by 42% compared to the original reactive policy. This is strong evidence that our development of a hybrid approach makes an important step towards closing the optimality gap.

In a further numerical study, we compare the reactive and adapted hybrid algorithms for larger networks with 5-20 locations. The exact DP algorithm is too numerically intensive to be applied in these experiments. The results of a comparison of the policies show that the improvement of the adapted hybrid over the reactive policy is even larger than for small networks, with a cost reduction of over 6.6% on average. It also provides an average saving of 14.5% over not transshipping at all. A sensitivity study provides further insights into when the cost reduction is most significant. The study also highlights the additional benefits that an improved transshipment policy can deliver. The average service level within the large network study is improved by 1.2 percentage points by the hybrid policy over the reactive policy and the amount of safety stock required is reduced by over half when compared to a policy of no transshipments.

The remainder of the paper is organized as follows. In the next section, we describe the model and the adapted algorithm for computing our approximately optimal hybrid transshipment policy. The cost benefit of the adaption is tested numerically in Section 3 for networks with two locations. Section 4 describes the exact DP algorithm, and the optimality gap for small networks is investigated in Section 5. Larger networks are explored in Section 6. We end with conclusions and directions for further research in Section 7.

2 A Hybrid Transshipment Policy

An approximately optimal policy for determining reactive transshipment decisions in an inventory system, in which all locations follow a continuous review (R, Q) ordering policy is derived by Axsäter (2003). The algorithm determining the policy is constructed by making use of an assumption that the considered transshipment will be the last one ever made. Whenever a location experiences a stockout, the algorithm calculates the most cost efficient amount and location to transship from under this assumption. However, the derived policy only looks to react to a stockout, not to be proactive in future stockout prevention. The proposed adjusted hybrid policy is allowed to transship more stock than is needed to meet the immediate shortage. This permits the two locations which are parties to a transshipment to redistribute their stock and balance future risk. Table 1 provides a list of notation needed to define the inventory system.

N	Number of demand locations: Location $i \in \{1, ..., N\}$.
R_i	Reorder point at i.
Q_i	Batch size of orders placed at i.
$t_{s,i}$	Time until s^{th} unit of stock becomes available at i.
IL_i	Inventory level at i.
IP_i	Inventory position at i.
L_i	Lead time of orders placed at i.
A_i	Order cost at i (per order).
h_i	Holding cost at i (per item per unit time).
b_i	Backorder cost at i (per item per unit time).
$tran_{f:(i,\tilde{i})}$	Fixed transshipment cost per movement from i to $\tilde{i}$.
$tran_{u:(i,\tilde{i})}$	Transshipment cost per unit transshipped from i to $\tilde{i}$.
λ_i	Arrival rate at i.
μ_i	Average size of each demand at i.
$f_{i,j}$	Probability that a demand at i will be of size j.
$f_{i,j}^n$	Probability that j units are demanded by n customers at i.
$P_{i,j}^n = P_{i,j-1}^n - f_{i,j-1}^n + f_{i,j-1}^{n-1}$	Probability that n^{th} customer demands the j^{th} unit at i.

Table 1: List of Notation

The system has N stocking locations. Location $i \in \{1, ..., N\}$ places an order of size Q_i at the central supplier whenever its inventory position drops to or below reorder level R_i. This order takes a fixed lead time L_i to arrive. If a location does not have sufficient stock on hand to satisfy a demand, then items may be transshipped to that location from a different location with negligible lead time. Any demand that cannot be met immediately (after transshipping) is backordered. Costs are incurred for ordering (A_i), holding stock on hand (h_i per item and time unit), backordering (b_i per item and per time unit), and transshipping ($tran_{f:(i,\tilde{i})}$ per transshipment and $tran_{u:(i,\tilde{i})}$ per item).

Under the assumption that no transshipments take place, Axsäter (2003) derives an expression for the *bias* associated with each system state. This measures the transient effect on costs of starting the system in that state. Performing a transshipment will instantly move the system to a new state so the benefit of a given transshipment can be identified by comparing the bias of the current state with the aggregate of the bias if the transshipment is enacted and the cost to enact it. By maximizing the difference between these quantities over all possible locations and transshipment quantities the best myopic decision can be identified. The limitation of this formulation is that it does not allow the size of the transshipment to be larger than the shortage. This restriction is mathematically convenient in that it ensures that the inventory position of any location never exceeds $R_i + Q_i$ which is necessary for some calculations. However, when there is a significant fixed cost per transshipment it seems intuitive that allowing larger transshipments may well deliver greater cost benefits. We now describe how the above approach can be developed to yield near-optimal decisions in our hybrid approach.

In steady state under an (R_i, Q_i) replenishment policy without transshipments it is known that location i's inventory position is uniformly distributed over the range $[R_i+1, ..., R_i+Q_i]$. With current (time zero) inventory position k and stochastic lead time demand $D_i(L_i)$ then the mean inventory cost rate at L_i is given by

$$\begin{aligned} C_i(k) &= h_i\mathbb{E}(k - D_i(L_i))^+ + b_i\mathbb{E}(k - D_i(L_i))^- \\ &= (h_i + b_i)\mathbb{E}(k - D_i(L_i))^+ + b_i\mathbb{E}(k - D_i(L_i)) \\ &= (h_i + b_i)e^{-\lambda_i L_i}\sum_{j=0}^{k-1}(k-j)\sum_{n=0}^{j}\frac{(\lambda_i L_i)^n}{n!}f_{i,j}^n + b_i(\lambda_i\mu_i L_i - k). \end{aligned} \tag{2.1}$$

We infer that the steady state cost rate for location i is given by

$$C_i = \frac{1}{Q_i}\sum_{k=R_i+1}^{R_i+Q_i} C_i(k). \tag{2.2}$$

The current state X_i of location i incorporates information on the current inventory position (IP_i) together with the times at which each inventory item becomes available. We write $V_i(X_i, t)$ for the total expected cost incurred under an (R_i, Q_i) ordering policy with no transshipments during the time interval $[0, t]$ when the system is in state X_i at time 0. The *bias* associated with X_i is defined as

$$\gamma(X_i) \triangleq \lim_{t\to\infty}\{V_i(X_i, t) - tC_i\}. \tag{2.3}$$

This can be decomposed as

$$\gamma(X_i) = \alpha_i(X_i) + \beta_i(IP_i), \tag{2.4}$$

where

$$\alpha_i(X_i) \triangleq V_i(X_i, L_i) - L_iC_i, \tag{2.5}$$

and

$$\beta_i(IP_i) \triangleq \lim_{t\to\infty}\left\{\mathbb{E}V_i(X_i^{L_i}, t) - tC_i\right\}. \tag{2.6}$$

Note that in (2.6), $X_i^{L_i}$ is the random state of location i at time L_i. The calculation of $\alpha_i(X_i)$ is given in the Appendix and is unchanged by the fact that here we may have $IP_i > R_i + Q_i$. We now describe the calculation of β_i.

We first observe that, since $X_i^{L_i}$ is stochastically independent of all information in X_i save only IP_i, then β_i depends upon X_i only through IP_i. This is reflected in our notation and we note the relation

$$IL_i^{L_i} = IP_i - D_i(L_i). \tag{2.7}$$

As location i evolves under an (R_i, Q_i) ordering policy with no transshipments, the associated inventory position process regenerates upon every entry into state $R_i + 1$ (or, indeed, any other state - $R_i + 1$ is chosen for convenience). In (2.6) we now deem $X_i^{L_i}$ to be the location i state at time zero and write T for the first subsequent time at which the corresponding inventory position, given by (2.7), is in the regeneration state $R_i + 1$. By standard theory we can then replace (2.6) by

$$\beta_i(IP_i) = \mathbb{E}V_i(X_i^{L_i}, T) - \mathbb{E}(T)C_i. \tag{2.8}$$

Plainly,

$$\beta_i(R_i + 1) = 0. \tag{2.9}$$

Further, by conditioning upon the size of the first demand to occur after time 0 we obtain, for $k > R_i + 1$,

$$\beta_i(k) = \frac{\{C_i(k) - C_i\}}{\lambda_i} + \sum_{d=1}^{\infty} f_{i,d}\beta_i(\langle k-d \rangle) \tag{2.10}$$

where in (2.10),

$$\langle k-d \rangle = \begin{cases} k-d & if\ k-d > R_i \\ R_i + j & if\ k-d \leq R_i \end{cases} \tag{2.11}$$

$$\text{where } k-d \equiv R_i + j \pmod{Q_i} \text{ for some } j \in [1, ..., Q_i].$$

In practice, we recover β_i as the limit of a recursive scheme as follows:

$$\beta_i^0(k) = 0, \qquad k \in [R_i + 1, ..., R_i + Q_i, ..., S] \tag{2.12}$$

$$\beta_i^n(R_i + 1) = 0, \qquad n \geq 0 \tag{2.13}$$

$$\beta_i^{n+1}(k) = \frac{\{C_i(k) - C_i\}}{\lambda_i} + \sum_{d=1}^{\infty} f_{i,d}\beta_i^n(\langle k-d \rangle), \qquad k \in [R_i + 2, ..., R_i + Q_i, ..., S], \tag{2.14}$$

where S is a large inventory position state such that no higher state is reached. The scheme must converge geometrically fast, with β_i the limit.

We can now use the complete bias functions for each location to identify the transshipment that delivers the most cost benefit. Suppose that some demand d occurs at location i when in state X_i and that this causes a stockout. The long term cost benefit of transshipping y units from location j (in current state X_j) to location i in comparison with performing no transshipment and allowing all demand to be absorbed at i is identified as the quantity

$$\Delta(j,y) = \lim_{t\to\infty}\{V_i(X_i'(d),t) + V_j(X_j,t) - V_i(X_i'(d-y),t) - V_j(X_j'(y),t)\}$$
$$- y * tran_{u:(j,i)} - tran_{f:(j,i)} - y(A_j/Q_j - A_i/Q_i). \qquad (2.15)$$

From (2.3) we have

$$\Delta(j,y) = \gamma_i(X_i'(d)) + \gamma_j(X_j) - \gamma_i(X_i'(d-y)) - \gamma_j(X_j'(y))$$
$$- y * tran_{u:(j,i)} - tran_{f:(j,i)} - y(A_j/Q_j - A_i/Q_i). \qquad (2.16)$$

Please note that in (2.15) and (2.16) we have used the notational shorthand $X_i'(d)$ for the resulting state of location i once d units of inventory have been withdrawn. A further minor point is that, prior to (2.15), we had taken no account of the order costs in our calculations. A transshipment of y units from j to i has the effect of adjusting the long-term cost burden from orders by $y(A_j/Q_j - A_i/Q_i)$. We write

$$\Delta^* = \max_{j\neq i} \max_{1\leq y\leq IL_j} \Delta(j,y). \qquad (2.17)$$

Our hybrid policy is as follows: If $\Delta^* \leq 0$, do not transship. If $\Delta^* > 0$ transship y^* units from j^* to i, where y^* and j^* are the maximizers in (2.17).

This policy is quasi-myopic in that it is cost minimizing if no further transshipment is permitted after the current stockout is dealt with. Note that the fact that under this scheme the inventory position at location i can exceed $R_i + Q_i$ not only poses mathematical challenges. It may also have the practical implication of requiring additional warehouse space. Should such space be limited then the range(s) of y in the maximization in (2.17) may need to be constrained further.

3 Two Location Simulation Study: Reactive vs. Hybrid policy

To analyze the performance of the hybrid transshipment policy, an initial simulation study is conducted. This study is restricted to networks with two identical locations. In a second study, of which the details and results will be presented in later sections, larger and more varied networks will be considered. The two reasons for the more restricted initial exploration are as follows. First, there are many model parameters and varying all of them for larger networks is time consuming. This initial exploration allows us to observe how the parameters impact on costs so that the larger network can focus on key issues. Secondly, the restriction will allow us to determine the optimal transshipment policy using DP in Section 5 for the same set of experiments, and therefore to study the relative reduction of the optimality gap from the original reactive policy to the new hybrid transshipment policy.

Since the two locations are assumed identical, we will drop the location identifying subscript for cost parameters in the remainder of this section. We will do the same for the policy parameters, and assume that both locations use the same replenishment and transshipment policy. Keeping both locations identical ensures that the transshipment policies are the focus of the prime numerical study.

Although the transshipment policy might have some effect on the optimal ordering quantity, in practice there are often fixed or minimal order sizes. For this reason and in line with previous studies, we fix the order cost (\$100) and use the EOQ formula ($\sqrt{(2A\lambda\mu)/(h)}$) to determine the order size. For the sizes of successive demands within the study we use a geometric distribution such that $f_j = p(1-p)^{j-1}$, $j \geq 1$. The holding cost rate (\$1) is used as the unit cost and all other model parameters are varied in a full factorial study. The full range of parameters examined is shown in Table 2. There are 600 parameter combinations and hence 600 experiments in total.

Arrival rate (λ)	0.8, 2.4, 4.0
Geometric distribution parameter for demand size (p)	0.6, 0.8
Lead time (L)	2, 3
Backorder cost (b)	10, 20, 30, 40, 50(\$)
Transshipment cost [per item] ($tran_u$)	1, 2(\$)
Transshipment cost [per transshipment] ($tran_f$)	10, 20, 30, 40, 50(\$)

Table 2: Parameter Values

For both the original reactive and adapted hybrid transshipment policies, the transshipment decisions are determined by the approximate algorithms that were discussed in Section 2. The DP model of Section 5 requires a restricted inventory state space such that the inventory position is at most $R + Q$. Applying the hybrid policy under a similar assumption does not statistically impact the results over the given parameter set, but for consistency in our comparisons it is these results which are considered. To obtain further insights we also consider the no transshipment policy as a benchmark. For all three (no, reactive and hybrid) transshipment policies, the optimal value of R is found post-hoc by conducting simulation studies on the full range of possible values of R.

3.1 Results

Over the 600 experiments an average saving of 1.56% is observed for the hybrid policy when compared to the reactive policy. This is broken down by each parameter in Table 3. For example the 200 experiments which have an arrival rate of 2.4 customers per unit time display an average saving of 1.75%.

λ		p		L		b		$tran_f$		$tran_u$	
0.8	0.83%	0.8	1.42%	2	1.21%	10	0.77%	10	1.42%	1	1.70%
2.4	1.75%	0.6	1.70%	3	1.91%	20	1.45%	20	1.71%	2	1.42%
4.0	2.10%					30	1.73%	30	1.68%		
						40	1.91%	40	1.59%		
						50	1.94%	50	1.39%		

Table 3: Hybrid Rule vs Reactive Rule

The results in Table 3 confirm intuition in several ways. Once a transshipment is instigated, a policy which looks to transship more will see a greater benefit if the marginal cost of adding an extra unit is low. This is supported by the $tran_u$ results. Further, if a policy is performing each transshipment more efficiently then it is natural that the saving will increase when there are more transshipment opportunities. By increasing parameters λ or L, or decreasing parameter p we raise the lead time variability and thus the chance of a stockout. Unsurpisingly, increasing the stockout risk through varying these parameters in the manner described leads to an observed gain in savings.

Considering both the fixed transshipment costs and backorder costs we see that the savings are greater as the penalty for not immediately meeting a demand increases. However, as the relative cost per transshipment increases compared to the cost of backordering, the number of transshipments which are beneficial is impacted and this is reflected in the results.

Table 4 provides the results for a sample of the transshipment cost parameters (27 experiments). All statistical comparisons have used paired t-tests at a 95% confidence level, with common random numbers used for all policies.

λ	b	$tran_f$	Q	R	No Tran	R	Reactive (Err)	Saving	R	Hybrid(Err)	Saving	Improve
0.8	10	10	15	1	29.97	1	28.85 (0.11)	3.72 %	1	28.72 (0.11)	4.15 %	0.45 %
0.8	10	30	15	1	29.97	1	29.79 (0.11)	0.60 %	1	29.72 (0.09)	0.81 %	0.21 %
0.8	10	50	15	1	29.97	1	29.95 (0.09)	0.07 %	1	29.87 (0.09)	0.33 %	0.26 %
0.8	30	10	15	3	33.39	2	30.80 (0.11)	7.75 %	2	30.61 (0.11)	8.33 %	0.63 %
0.8	30	30	15	3	33.39	3	32.54 (0.11)	2.53 %	3	32.09 (0.11)	3.90 %	1.40 %
0.8	30	50	15	3	33.39	3	32.53 (0.11)	2.57 %	3	32.25 (0.11)	3.41 %	0.86 %
0.8	50	10	15	4	34.81	3	31.89 (0.12)	8.38 %	3	31.68 (0.12)	8.98 %	0.65 %
0.8	50	30	15	4	34.81	3	32.91 (0.13)	5.46 %	3	32.46 (0.13)	6.75 %	1.36 %
0.8	50	50	15	4	34.81	4	33.81 (0.11)	2.87 %	3	33.05 (0.11)	5.04 %	2.23 %
2.4	10	10	25	7	51.75	7	50.92 (0.10)	1.62 %	5	49.76 (0.10)	3.86 %	2.28 %
2.4	10	30	25	7	51.75	7	51.90 (0.10)	-0.28 %	6	51.30 (0.10)	0.87 %	1.14 %
2.4	10	50	25	7	51.75	7	51.70 (0.00)	0.10 %	7	51.52 (0.00)	0.45 %	0.35 %
2.4	30	10	25	10	57.15	9	54.26 (0.12)	5.06 %	8	53.11 (0.12)	7.06 %	2.11 %
2.4	30	30	25	10	57.15	10	56.16 (0.12)	1.73 %	9	54.78 (0.12)	4.15 %	2.46 %
2.4	30	50	25	10	57.15	10	57.04 (0.15)	0.20 %	9	55.77 (0.15)	2.42 %	2.22 %
2.4	50	10	25	12	59.51	10	55.30 (0.11)	7.08 %	8	54.30 (0.09)	8.76 %	1.81 %
2.4	50	30	25	12	59.51	10	57.77 (0.19)	2.92 %	10	56.13 (0.18)	5.69 %	2.85 %
2.4	50	50	25	12	59.51	11	58.53 (0.14)	1.65 %	10	56.91 (0.13)	4.36 %	2.76 %
4.0	10	10	32	13	66.83	12	65.75 (0.09)	1.61 %	10	64.02 (0.09)	4.20 %	2.63 %
4.0	10	30	32	13	66.83	13	66.73 (0.10)	0.14 %	12	66.06 (0.10)	1.15 %	1.01 %
4.0	10	50	32	13	66.83	13	66.82 (0.00)	0.01 %	12	66.45 (0.00)	0.57 %	0.55 %
4.0	30	10	32	17	73.56	15	70.43 (0.14)	4.26 %	14	68.38 (0.13)	7.04 %	2.90 %
4.0	30	30	32	17	73.56	16	72.80 (0.14)	1.03 %	15	70.52 (0.14)	4.13 %	3.13 %
4.0	30	50	32	17	73.56	16	73.05 (0.18)	0.70 %	16	71.64 (0.18)	2.61 %	1.92 %
4.0	50	10	32	18	76.39	16	71.84 (0.14)	5.95 %	15	70.11 (0.14)	8.22 %	2.41 %
4.0	50	30	32	18	76.39	17	74.39 (0.18)	2.62 %	16	71.85 (0.19)	5.94 %	3.41 %
4.0	50	50	32	18	76.39	18	75.64 (0.19)	0.98 %	17	73.50 (0.19)	3.78 %	2.83 %

Table 4: Two Location Results: $p = 0.8$, $L = 3$, $tran_u = 1$

Overall, the average saving of the new hybrid policy compared to one which uses no transshipments is 4.23%, reinforcing the view that transshipping is worthwhile. The standard errors of the costs for the reactive and the new hybrid policies are also given in Table 4 (in brackets) and show that the improvement of the new over the reactive policy is almost always statistically significant. There are a few exceptions in low demand situations where there is no significant difference between the use of either policy, but importantly there were no statistically significant cases where the hybrid policy was outperformed by the reactive policy.

A comparison of the order levels R provides insight into how the cost reduction is achieved. The average reorder level is 9.8 units without transshipments, 8.9 units for the reactive transshipment policy and 8.2 units for the new transshipment policy. Better transshipment decisions reduce the negative cost effects of stockouts, thereby allowing the system to function with lower safety stocks. Further exploration also showed that in general the hybrid transshipment policy is more cost efficient, with transshipments happening less frequently but with larger quantities (4.7 units for the hybrid policy compared to 2.1 units for the reactive policy on average).

The results in this section have clearly shown that the new hybrid transshipment policy significantly outperforms the original reactive transshipment policy. However, it remains of real interest to discover how close (in cost terms) it is to an optimal transshipment policy. To explore this, we will develop a dynamic programming (DP) formulation for finding the optimal transshipment policy in the next section, and then compare its cost to that of the new hybrid transshipment policy in Section 5, using the same set of experiments that were investigated in this section.

4 Dynamic Programming Formulation

In this section, we provide a DP formulation that can be used to find the optimal transshipment policy using value iteration. In our system, the resulting policy determines how much to transship given the locations' inventory levels and remaining lead times of outstanding orders in order to minimize the overall cost rate incurred. We remark that our DP formulation utilises a discrete time approximation of the actual continuous review system. However, for any sufficiently small time quantum the approximation is very good. This will be verified in the numerical investigation of Section 5. For presentational ease, we will refer to the transshipment policy that minimizes the cost under the DP formulation as the optimal transshipment policy.

Our formulation is for a two location system setup. While it is possible to generalize the approach to larger systems, value iteration becomes computationally intractable very quickly due to the rapid growth in the number of states. In order to limit the state space, only one order is allowed to be outstanding at each location at any period in time. This assumption is supported by real world practice and by the simulation results obtained in Section 3.

These showed that if a location had orders outstanding then the conditional probability of multiple orders outstanding was less than 1%. They also showed that limiting a location to a maximum stock of $R_i + Q_i$ resulted in no statistically significant cost difference. Therefore it is feasible to limit the inventory state space to a range of $R_i \pm Q_i$. Recall from Section 3 that we will apply the DP algorithm for the same set of experiments as described there. To develop the DP model we introduce additional notation and formulae in Table 5.

For sufficiently small time quantum δ, we may assume that in a single time slot the system experiences an instance of demand at either one location or neither location. The probability of a demand of size j at location i as $\delta\lambda_i \tilde{f_{i,j}}$ during each period while the probability of no demand in the system is $1 - \delta(\lambda_1 + \lambda_2)$. In the limit $\delta \to \infty$ these probabilities converge to the exact Poisson probabilities.

δ	Time quantum between each state transition.
$\tilde{f_{i,j}}$	The truncated probability of demand j at location i such that $R_i - Q_i < IL_i$ is always true.
Y_i	Number of quanta until the outstanding order arrives at location i.
Z	Location of most recent demand where $Z = 0$ indicates no demand occurred in the preceding period.
I_H	Indicator function where H is a logical statement: if true $I_H = 1$ else $I_H = 0$

Table 5: Additional Notation

State Definition

A five dimensional system state incorporates the inventory level and the time until the outstanding order arrives (if there is one) at both locations. The fifth dimension indicates the location where any current demand has occurred. We write state s as follows

$$s =< IL_1, IL_2, Y_1, Y_2, Z > \tag{4.1}$$

Recall that we do not allow the inventory position to go above $R_i + Q_i$. Further, the decision to allow a maximum of one outstanding order bounds the inventory level below. Hence we have that $R_i - Q_i < IL_i \leq R_i + Q_i$. Moreover, we clearly have $0 \leq Y_j < \frac{L_j}{\delta}$.

Action Space Definition

The action a $(-a)$ is the amount to transship from location 1(2) to location 2(1). For a given state s, the set of actions $Act(s)$ is bounded by zero, and by the minimum of the amount available to transship and the amount that can be stored at the receiving location (i.e. that does not take the inventory position above its maximum). We summarise these constraints by

If $IL_2 > 0$, $IL_1 < 0$ and $Z = 1$:	$a \in \{-min(IL_2, R_1 + I_{(Y_1=0)} * Q_1 - IL_1), ..., 0\}$
If $IL_1 > 0$, $IL_2 < 0$ and $Z = 2$:	$a \in \{0, ..., min(IL_1, R_2 + I_{(Y_2=0)} * Q_2 - IL_2)\}$
Else:	$a \in \{0\}$.

Cost Function

The cost ($\zeta_s(a)$) associated with being in state s and choosing action a is obtained by aggregating the cost of holding (or backordering) the current level of inventory after the transshipping action has taken place with the cost of the action itself. There is also the additional cost of placing any replenishment order which is included in the period directly after the order has been instigated. Hence we have

$$\begin{aligned}\zeta_s(a) &= h_1(IL_1 - a)^+ + h_2(IL_2 + a)^+ + b_1(IL_1 - a)^- + b_2(IL_2 - a)^- \\ &\quad + A_1 I_{(Y_1 = \frac{L_1}{\delta} - 1)} + A_2 I_{(Y_2 = \frac{L_2}{\delta} - 1)} + tran_u|a| + tran_f I_{|a|>0} \qquad (4.2)\end{aligned}$$

State Transitions

If the current state is $s =< IL_1, IL_2, Y_1, Y_2, Z >$, action a is undertaken and demand d occurs at location 1 then the new state will be $s' =< IL'_1, IL'_2, Y'_1, Y'_2 >$ where

$$IL'_1 = IL_1 - a - d + Q_1 I_{(Y_1=1)}$$

$$IL'_2 = IL_2 + a + Q_2 I_{(Y_2=1)}$$

$$Y'_1 = (Y_1 - 1) I_{(1 \leq Y_1 \leq \frac{L_1}{\delta} - 1)} + (\frac{L_1}{\delta} - 1) I_{(IL_1 + a \leq R_1 \& Y_1 = 0))}$$

$$Y'_2 = (Y_2 - 1) I_{(1 \leq Y_2 \leq \frac{L_2}{\delta} - 1)} + (\frac{L_2}{\delta} - 1) I_{(IL_2 + a \leq R_2 \& Y_2 = 0))}$$

Similar transitions can be identified for the cases when a demand occurs at location 2 and when no demand occurs anywhere in the system.

Value Iteration

The above are deployed in a value iteration in which the value function ($V_n(s)$) is the minimal cost incurred over an n-period horizon from initial state s. If we write $\psi(s, s')$ for the probability of moving from state s to s' then the optimality principle gives

$$V_n(s) = \min_{a \in Act(s)} \left(\zeta_s(a) + \sum_{s' \in S} \psi(s, s') V_{n-1}(s') \right)$$

We develop the $(V_n)_{n \geq 1}$ using backwards induction and utilise the stopping criterion recommended by Tijms (1986). The minimizing actions in the final iteration yield the ϵ-optimal policy.

5 Optimality gap

For each of the 600 experiments, the dynamic programming model was used to determine the optimal transshipment policy. This was then used within the above simulation model along with the two approximate policies. This ensured that like for like comparisons could be made using the same set of randomly generated events. It was necessary to gather results for a range of values of the reorder point R so that the optimal value could be used for each parameter set.

5.1 Results

λ				p				L			
0.8	2.67%	1.86%	**30%**	0.8	3.07%	1.69%	**45%**	2	2.72%	1.54%	**43%**
2.4	3.64%	1.93%	**47%**	0.6	4.22%	2.57%	**39%**	3	4.57%	2.72%	**41%**
4.0	4.64%	2.60%	**47%**								
b				$tran_f$				$tran_u$			
10	2.11%	1.35%	**36%**	10	4.77%	3.40%	**29%**	1	3.95%	2.29%	**42%**
20	3.51%	2.10%	**40%**	20	4.15%	2.49%	**40%**	2	3.35%	1.97%	**42%**
30	4.01%	2.33%	**42%**	30	3.56%	1.92%	**46%**				
40	4.26%	2.40%	**46%**	40	3.10%	1.53%	**50%**				
50	4.35%	2.46%	**43%**	50	2.66%	1.29%	**51%**				

Sub-column Headings: [Parameter Value][Reactive vs. Opt][Hybrid vs. Opt][Improvement]

Table 6: Optimality Gap Analysis

The benefit of using the hybrid policy can now be seen in the large step it takes towards the performance of the ϵ-optimal policy. Table 6 shows that over a range of parameter values there is a consistent level of improvement in cost performance achieved by taking a proactive approach to stock rebalancing. Compared to a policy of no transshipments the hybrid policy closes the suboptimality gap by nearly 75% in cases of large backorder costs with an average of 66% observed over the entire data set. Table 7 looks at the optimality results for the sample of problems previously presented in Table 4.

λ	b	$tran_f$	Q	Opt	No Tran	Gap	Reactive	Gap	Hybrid	Gap
0.8	10	10	15	28.33	29.97	5.45 %	28.85	1.80 %	28.72	1.36 %
0.8	10	30	15	29.38	29.97	1.97 %	29.79	1.38 %	29.72	1.17 %
0.8	10	50	15	29.80	29.97	0.55 %	29.95	0.49 %	29.87	0.22 %
0.8	30	10	15	29.86	33.39	10.58 %	30.80	3.06 %	30.61	2.45 %
0.8	30	30	15	31.42	33.39	5.88 %	32.54	3.44 %	32.09	2.06 %
0.8	30	50	15	31.95	33.39	4.31 %	32.53	1.79 %	32.25	0.93 %
0.8	50	10	15	30.88	34.81	11.30 %	31.89	3.18 %	31.68	2.55 %
0.8	50	30	15	31.74	34.81	8.82 %	32.91	3.55 %	32.46	2.22 %
0.8	50	50	15	32.60	34.81	6.35 %	33.81	3.58 %	33.05	1.38 %
2.4	10	10	25	48.34	51.75	6.59 %	50.92	5.05 %	49.76	2.84 %
2.4	10	30	25	50.65	51.75	2.14 %	51.90	2.41 %	51.30	1.28 %
2.4	10	50	25	51.21	51.75	1.06 %	51.70	0.96 %	51.52	0.61 %
2.4	30	10	25	51.39	57.15	10.07 %	54.26	5.28 %	53.11	3.24 %
2.4	30	30	25	53.62	57.15	6.17 %	56.16	4.52 %	54.78	2.11 %
2.4	30	50	25	54.90	57.15	3.93 %	57.04	3.74 %	55.77	1.55 %
2.4	50	10	25	52.17	59.51	12.34 %	55.30	5.66 %	54.30	3.92 %
2.4	50	30	25	54.83	59.51	7.86 %	57.77	5.09 %	56.13	2.30 %
2.4	50	50	25	56.01	59.51	5.88 %	58.53	4.30 %	56.91	1.58 %
4.0	10	10	32	60.86	66.83	8.94 %	65.75	7.44 %	64.02	4.94 %
4.0	10	30	32	64.81	66.83	3.02 %	66.73	2.89 %	66.06	1.89 %
4.0	10	50	32	65.98	66.83	1.26 %	66.82	1.25 %	66.45	0.70 %
4.0	30	10	32	63.80	73.56	13.27 %	70.43	9.41 %	68.38	6.70 %
4.0	30	30	32	68.49	73.56	6.90 %	72.80	5.93 %	70.52	2.88 %
4.0	30	50	32	70.35	73.56	4.36 %	73.05	3.69 %	71.64	1.80 %
4.0	50	10	32	64.95	76.39	14.98 %	71.84	9.60 %	70.11	7.36 %
4.0	50	30	32	69.73	76.39	8.72 %	74.39	6.26 %	71.85	2.95 %
4.0	50	50	32	71.93	76.39	5.84 %	75.64	4.91 %	73.50	2.14 %

Table 7: Two Location Results: Optimality Comparison $p = 0.8$, $L = 3$, $tran_u = 1$

One consistent difference between the the ϵ-optimal policy and the heuristic transshipment policies is its lower average reorder point. Another interesting feature of the results is that the ϵ-optimal policy appears to transship more frequently. As the heuristic policies are quasi-myopic it could be that the lack of foresight results in a more conservative approach to the triggering of a transshipment.

Figures 1 and 2 clearly demonstrate the superiority in performance of the hybrid policy over the reactive policy, more so for larger values of the fixed transshipment cost.

5.2 Accuracy of Discrete Time Assumption & Computation Time

The above results arising from the DP implementation all use discrete time quantum $\delta = \frac{1}{8}$. We confirm the acceptability of this choice by resolving with $\delta = \frac{1}{16}$ and observing that the resulting changes in cost rates are minimal and nowhere statistically significant.

While using the optimal policy may reduce inventory costs its development is computationally expensive when compared to the heuristic policies. These can be obtained very rapidly in real time. All of the above experiments were conducted on the Lancaster High Performance Cluster (HPC). The time taken for each experiment was recorded.

Arrival Rate	$\delta = \frac{1}{8}$	$\delta = \frac{1}{16}$	Multiple
0.8	3.2	24.2	7.6
2.4	33.6	222.7	6.6
4.0	154.9	759.7	4.9
Overall	63.9	335.5	5.3

Table 8: Computational Time (mins)

Table 8 gives a breakdown of the time needed to develop an optimal policy by arrival rate. It displays how halving the size of δ from $\delta = \frac{1}{8}$ increases the computational time by a factor of more than 5 on average. Moreover, the computation time increases rapidly with the arrival rate. These figures indicate that while value iteration is useful in testing the performance of the heuristic policies, it is computationally intractable other that in small cases.

6 Large Network Study

Having shown that the hybrid policy improves the original reactive policy and makes a significant step towards closing the optimality gap while reducing some of the variation in performance, the next step is to consider its performance in larger networks. In a simulation study, designed in a similar way to the small network study, inventory systems with 5, 10 and 20 location are considered. Rather than a full factorial study the focus is now put on how the size of the network and the arrival rate (and hence the lead time demand variability) influences policy performance. Eight different arrival rates are considered in networks with

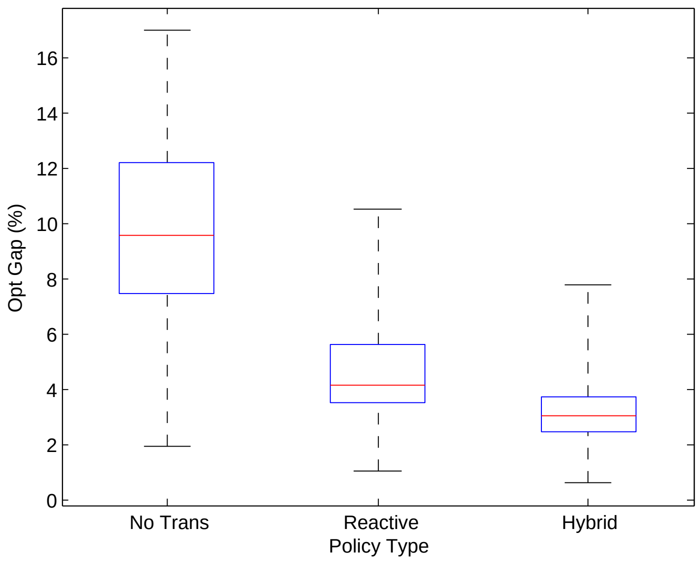

Figure 1: Sensitivity Analysis for $tran_f = 10$

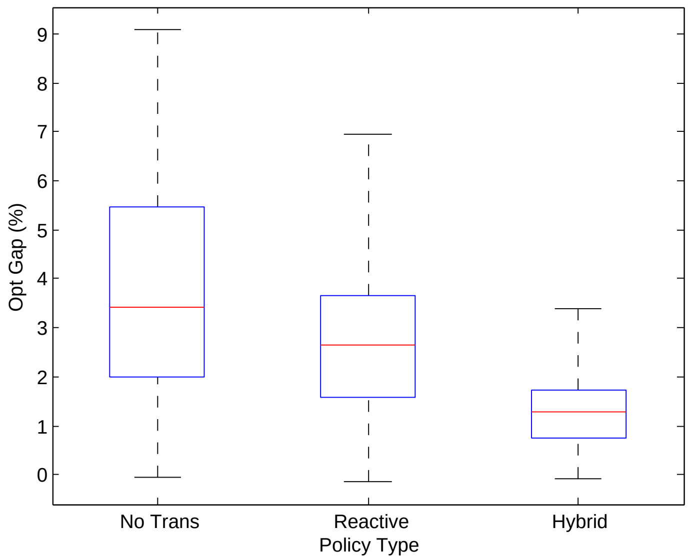

Figure 2: Sensitivity Analysis for $tran_f = 50$

identical locations. Additionally, networks which had two different levels of arrivals are considered. These networks have 40% of the locations at a higher demand rate than the other 60%. In the latter case, the overall system arrival rate is set equal to a corresponding identical location configuration so that comparisons could be fairly drawn.

Number of locations	5, 10, 20
Arrival rate [Identical Locations](λ)	1.4, 2.2, 3.0, 3.8, 4.6
Low arrival rate [Different Locations](λ_l)	1.0, 1.8, 2.6, 3.4, 4.2
High arrival rate [Different Locations](λ_h)	2.0, 2.8, 3.6, 4.4, 5.2
Distribution of order size (p)	0.8
Lead time (L)	3
Backorder cost (b)	30($)
Transshipment cost [per item] ($tran_u$)	1($)
Transshipment cost [per transshipment] ($tran_f$)	10($)

Table 9: Parameter Values

A full set of results is given in Tables 11 and 12. The average percentage savings, broken down by assigned values of the arrival rate, are given in Table 10. The overall average results show a saving over the reactive policy of 6.58% for identical locations and 6.71% for the networks with a two tier arrival rate structure. As for smaller networks, the greatest savings occur when the arrival rate is large.

The difference in results between the identical location setup and the two tier setup is small. Ideally a system with many different arrival rates could be considered but this is challenging to implement due to the necessity of determining suitable values of R for each location via a post-hoc optimisation.

Average λ	Identical	Two Tier
1.4	4.50	4.25
2.2	6.29	6.09
3.0	7.24	7.08
3.8	7.71	7.91
4.6	7.14	8.20

Table 10: Percentage cost savings for the hybrid policy over the reactive policy: large networks

No.	λ	Q	R	No Tran	R	Reactive(Err)	Saving	R	Hybrid(Err)	Saving	Improve
5	1.4	19	6	109.66	3	98.44 (10.23)	10.43 %	2	94.96 (0.13)	13.40 %	3.53 %
5	2.2	24	9	136.96	6	125.07 (8.68)	8.80 %	4	118.85 (0.19)	13.23 %	4.98 %
5	3.0	28	13	159.52	10	147.36 (7.62)	7.59 %	6	139.10 (0.21)	12.80 %	5.60 %
5	3.8	31	16	179.33	13	166.90 (6.93)	6.99 %	9	157.09 (0.17)	12.40 %	5.88 %
5	4.6	34	19	197.12	16	185.14 (6.08)	6.30 %	11	173.56 (0.19)	11.95 %	6.25 %
10	1.4	19	6	219.31	3	195.06 (11.06)	11.16 %	2	186.05 (0.22)	15.16 %	4.62 %
10	2.2	24	9	273.92	6	247.55 (9.63)	9.66 %	3	231.32 (0.23)	15.55 %	6.55 %
10	3.0	28	13	319.03	9	292.00 (8.47)	8.40 %	5	270.00 (0.22)	15.37 %	7.53 %
10	3.8	31	16	358.65	13	331.92 (7.45)	7.57 %	7	305.30 (0.21)	14.87 %	8.02 %
10	4.6	34	19	394.24	16	367.08 (6.89)	7.04 %	10	337.66 (0.19)	14.35 %	8.01 %
20	1.4	19	6	438.62	3	388.18 (11.50)	11.77 %	2	367.37 (0.24)	16.24 %	5.36 %
20	2.2	24	9	547.85	6	493.36 (9.95)	9.86 %	3	457.09 (0.25)	16.57 %	7.35 %
20	3.0	28	13	638.06	9	581.81 (8.82)	8.71 %	5	531.79 (0.22)	16.66 %	8.60 %
20	3.8	31	16	717.30	12	661.75 (7.74)	7.88 %	6	600.72 (0.30)	16.25 %	9.22 %
20	4.6	34	19	788.47	16	732.62 (7.08)	7.24 %	13	680.25 (0.26)	13.72 %	7.15 %

Table 11: Large network results: identical locations

The hybrid policy offers a consistent level of cost improvement. Average costs are reduced

No.	λ	Q	R	No Tran	R	Reactive(Err)	Saving	R	Hybrid(Err)	Saving	Improve
5	1.0 - 2.0	16 - 23	4 - 9	108.23	2 - 6	96.94 (0.11)	10.43 %	1 - 4	93.88 (0.13)	13.26 %	3.16 %
5	1.8 - 2.8	22 - 27	8 - 12	136.03	4 - 9	124.07 (0.18)	8.80 %	3 - 6	118.14 (0.16)	13.15 %	4.78 %
5	2.6 - 3.6	26 - 30	11 - 15	159.03	8 - 12	146.96 (0.20)	7.59 %	5 - 8	138.89 (0.18)	12.66 %	5.49 %
5	3.4 - 4.4	30 - 34	14 - 18	178.75	11 - 15	166.25 (0.17)	6.99 %	6 - 10	156.17 (0.21)	12.63 %	6.06 %
5	4.2 - 5.2	33 - 37	17 - 21	196.73	14 - 19	184.34 (0.18)	6.30 %	9 - 12	172.61 (0.20)	12.26 %	6.37 %
10	1.0 - 2.0	16 - 23	4 - 9	216.45	1 - 5	192.29 (0.23)	11.16 %	1 - 3	183.59 (0.21)	15.18 %	4.52 %
10	1.8 - 2.8	22 - 27	8 - 12	272.07	4 - 8	245.77 (0.24)	9.66 %	2 - 5	230.04 (0.22)	15.45 %	6.40 %
10	2.6 - 3.6	26 - 30	11 - 15	318.05	7 - 12	291.34 (0.24)	8.40 %	4 - 7	269.64 (0.20)	15.22 %	7.45 %
10	3.4 - 4.4	30 - 34	14 - 18	357.50	10 - 15	330.44 (0.26)	7.57 %	5 - 8	302.95 (0.21)	15.26 %	8.32 %
10	4.2 - 5.2	33 - 37	17 - 21	393.46	14 - 18	365.77 (0.16)	7.04 %	7 - 10	334.57 (0.22)	14.97 %	8.53 %
20	1.0 - 2.0	16 - 23	4 - 9	432.91	2 - 6	381.96 (0.26)	11.77 %	1 - 3	362.62 (0.26)	16.24 %	5.07 %
20	1.8 - 2.8	22 - 27	8 - 12	544.13	5 - 8	490.49 (0.22)	9.86 %	2 - 4	455.66 (0.22)	16.26 %	7.10 %
20	2.6 - 3.6	26 - 30	11 - 15	636.10	8 - 12	580.70 (0.24)	8.71 %	4 - 7	532.40 (0.21)	16.30 %	8.32 %
20	3.4 - 4.4	30 - 34	14 - 18	715.01	11 - 15	658.69 (0.28)	7.88 %	5 - 8	597.22 (0.28)	16.47 %	9.33 %
20	4.2 - 5.2	33 - 37	17 - 21	786.93	14 - 18	729.95 (0.37)	7.24 %	7 - 10	659.05 (0.33)	16.25 %	9.71 %

Table 12: Large network results: two tier locations

by between 11% and 17% in comparison with no transshipment. In the case of the reactive policy, the cost saving can be as little as 6.3%. The greater stability of the hybrid policy is illustrated by Figure **??**.

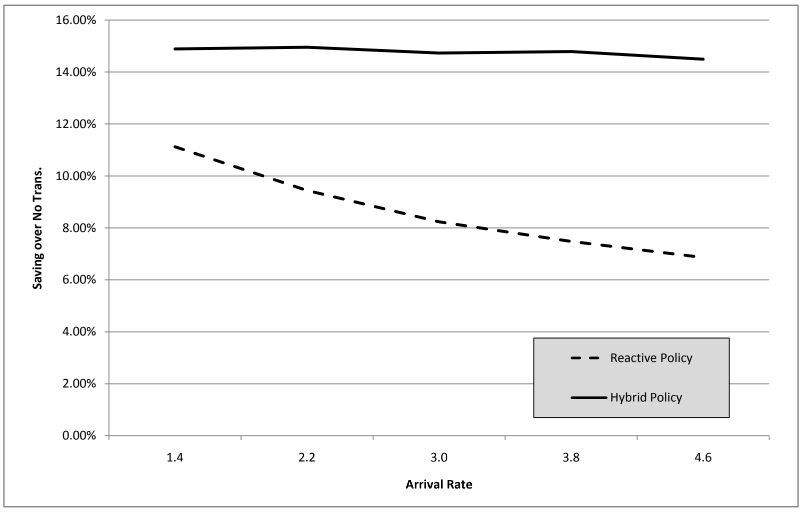

Figure 3: Savings varying by arrival rate

While costs are an important part of inventory systems it is not the only performance measure of interest. Service levels within a system are also a key consideration and the fill rates (the percentage of demand filled immediately from stock on hand or via transshipment) were also recorded for the large network study. For the reactive policy a service level of 96.8% was achieved but the hybrid policy increased this to 98.0%.

The large network study also reinforces other findings from the smaller network results. For identical locations, safety stock is reduced from 12 units on average with no transshipments to 10.5 units under the reactive policy and to 8 units under the new hybrid policy. Similar results are obtained for the two tier networks. The average size of transshipment again increases from 1.84 units to 5.66 units. These results illustrate the greater efficiency possible from anticipating stockouts rather than merely responding to them

7 Conclusions

We have shown that the benefits of reactive transshipments can be enhanced by the development of hybrid policies which incorporate a proactive element. System costs can be reduced and the efficiency of the transshipment process improved. This has been observed through an extensive study of both small and large (R, Q) replenishment policy inventory networks, with the benefits growing with the number of stock holding locations. Moreover the improvements that this hybrid transshipment policy can bring have been shown to significantly reduce the optimality gap.

The comparison to optimality has been achieved thorough a dynamic programming model that enables the calculation of an ϵ-optimal transshipment policy and the resulting costs. Whilst this formulation is restricted to small systems it is an important step in understanding the transshipment process and in evaluating the performance of the more easily developed hybrid policy.

One possible way to further enhance the hybrid policy is to relax the myopic assumption that underpins it. Another avenue would be to develop the redistribution element by considering transshipments at times other that at those when stockouts occur. One clear limitation within batch ordering systems with transshipments is the challenge to find appropriate replenishment policy parameter values, even more so in systems with non-identical locations. Our results have shown that the hybrid transshipment policy can significantly alter the optimal reorder point when compared to no-transshipments. Indeed part of the savings achieved is a consequence of being able to lower the amount of safety stock required throughout the system. Future work could develop analytical approaches to the determination of reorder points. This would enable the full benefits of the improvements in the transshipment policy to be realized in more complex inventory systems with a larger number of stocking locations and non-identical demand rates.

8 Appendix

Calculating $\alpha_i(X_i)$

If $t_{s,i}$ is the time when the s^{th} unit becomes available at location i then at each point in time a location's state can be described by a variable X_i, where

$$X_i = (IP_i, t_{1,i}, t_{2,i}, ...). \tag{8.1}$$

For location i the pdf and cdf of the distribution of the time when the n^{th} demand instant occurs can be respectively given as:

$$g_i^n(t) = \frac{\lambda_i^n t^{n-1} e^{-\lambda_i t}}{(n-1)!}, \tag{8.2}$$

$$G_i^n(t) = 1 - \sum_{j=1}^{n-1} \frac{(\lambda_i t)^j}{k!} e^{-\lambda_i t}. \tag{8.3}$$

Using these distributions it is possible to obtain the pdf and cdf of the time when the j^{th} unit is demanded at location i:

$$r_i^j(t) = \sum_{n=1}^{j} P_{i,j}^n g_i^n(t), \tag{8.4}$$

$$R_i^j(t) = \sum_{n=1}^{j} P_{i,j}^n G_i^n(t), \tag{8.5}$$

We define quantity $U_i^j(t)$ as

$$U_i^j(t) = \int_0^\infty r_i^j(u) u \, du = \sum_{n=1}^{j} P_{i,j}^n G_{i+1}^n(t) \frac{n}{\lambda_i} \tag{8.6}$$

It is now possible to calculate $\alpha_i(X_i)$. We let $x_i(t_{s,i})$ be the expected holding and backorder costs associated with the s^{th} item of stock demanded during the lead time L_i. If $s \leq 0$ then the item has already been demanded, with $s = 0$ the most recently demanded item. This gives four specific cases:

For $s \leq 0, t_{s,i} \leq L_i$,

$$x_i(t_{s,i}) = b_i t_{s,i}. \tag{8.7}$$

For $s \leq 0, t_{s,i} > L_i$,

$$x_i(t_{s,i}) = b_i L_i. \tag{8.8}$$

For $s > 0, t_{s,i} \leq L_i$,

$$\begin{aligned} x_i(t_{s,i}) &= h_i\Big(\int_{t_s}^{L_i} r_i^s(u)(u - t_{s,i})du + (1 - R_i^s(L_i))(L_i - t_{s,i})\Big) \\ &\quad + b_i \int_0^{t_{s,i}} r_i^s(u)(t_{s,i} - u)du \\ &= h_i[U_i^s(L_i) - U_i^s(t_{s,i}) - t_s(1 - R_i^s(t_{s,i})) + L_i(1 - R_i^s(L_i))] \\ &\quad + b_i[R_i^s(t_{s,i}) - U_i^s(t_{s,i})]. \end{aligned} \tag{8.9}$$

For $s > 0, t_{s,i} > L_i$,

$$x_i(t_{s,i}) = b_i[R_i^s(L_i)L_i - U_i^s(L_i)]. \tag{8.10}$$

We calculate $\alpha_i(X_i)$ using

$$\alpha_i(X_i) = \sum_{s=1-(IL)^-}^{IP} x_i(t_s) + \sum_{s=IP+1}^{\infty} x_i(s, L_i) - L_iC_i, \tag{8.11}$$

where L_iC_i is the steady state cost incurred during the lead time period.

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Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
0.8	0.8	2	10	1	10	15	0	29.00	0	28.42	0	28.12	0	28.32	2.36 %	0.35 %	0.68 %
0.8	0.8	2	10	2	10	15	0	29.00	0	28.77	0	28.44	0	28.62	1.32 %	0.54 %	0.62 %
0.8	0.8	2	10	1	20	15	0	29.00	0	29.02	0	28.65	0	28.74	0.89 %	0.96 %	0.33 %
0.8	0.8	2	10	2	20	15	0	29.00	0	28.90	0	28.76	0	28.83	0.59 %	0.23 %	0.24 %
0.8	0.8	2	10	1	30	15	0	29.00	0	28.96	0	28.78	0	28.88	0.40 %	0.25 %	0.36 %
0.8	0.8	2	10	2	30	15	0	29.00	0	29.04	0	28.97	0	28.96	0.15 %	0.30 %	-0.03 %
0.8	0.8	2	10	1	40	15	0	29.00	0	29.01	0	28.91	0	28.94	0.20 %	0.22 %	0.09 %
0.8	0.8	2	10	2	40	15	0	29.00	0	29.09	0	28.99	0	29.09	-0.31 %	-0.01 %	0.33 %
0.8	0.8	2	10	1	50	15	0	29.00	0	28.87	0	28.90	0	28.88	0.41 %	-0.02 %	-0.06 %
0.8	0.8	2	10	2	50	15	0	29.00	0	28.96	0	29.00	0	29.00	0.01 %	-0.12 %	-0.02 %
0.8	0.8	2	20	1	10	15	1	30.97	1	29.84	0	29.09	1	29.69	4.13 %	0.52 %	2.03 %
0.8	0.8	2	20	2	10	15	1	30.97	1	30.04	0	29.53	1	29.93	3.34 %	0.36 %	1.35 %
0.8	0.8	2	20	1	20	15	1	30.97	1	30.41	1	29.83	1	30.17	2.58 %	0.77 %	1.13 %
0.8	0.8	2	20	2	20	15	1	30.97	1	30.37	1	29.86	1	30.19	2.50 %	0.58 %	1.09 %
0.8	0.8	2	20	1	30	15	1	30.97	1	30.56	1	30.18	1	30.49	1.55 %	0.24 %	1.02 %
0.8	0.8	2	20	2	30	15	1	30.97	1	30.70	1	30.37	1	30.63	1.08 %	0.22 %	0.86 %
0.8	0.8	2	20	1	40	15	1	30.97	1	30.72	1	30.32	1	30.48	1.56 %	0.76 %	0.54 %
0.8	0.8	2	20	2	40	15	1	30.97	1	30.88	1	30.58	1	30.75	0.71 %	0.42 %	0.55 %
0.8	0.8	2	20	1	50	15	1	30.97	1	30.63	1	30.46	1	30.61	1.15 %	0.05 %	0.49 %
0.8	0.8	2	20	2	50	15	1	30.97	1	30.93	1	30.72	1	30.86	0.35 %	0.22 %	0.46 %
0.8	0.8	2	30	1	10	15	2	31.96	1	30.31	1	29.60	1	30.10	5.81 %	0.69 %	1.67 %
0.8	0.8	2	30	2	10	15	2	31.96	1	30.45	1	29.76	1	30.36	5.02 %	0.29 %	1.98 %
0.8	0.8	2	30	1	20	15	2	31.96	1	31.16	1	30.30	1	30.85	3.48 %	1.00 %	1.77 %
0.8	0.8	2	30	2	20	15	2	31.96	1	31.07	1	30.33	1	30.86	3.45 %	0.68 %	1.72 %
0.8	0.8	2	30	1	30	15	2	31.96	2	31.53	1	30.72	1	31.30	2.07 %	0.73 %	1.85 %
0.8	0.8	2	30	2	30	15	2	31.96	2	31.54	1	30.98	2	31.44	1.62 %	0.32 %	1.48 %
0.8	0.8	2	30	1	40	15	2	31.96	2	31.67	1	31.19	2	31.43	1.67 %	0.74 %	0.75 %
0.8	0.8	2	30	2	40	15	2	31.96	2	31.86	1	31.42	2	31.61	1.09 %	0.76 %	0.60 %
0.8	0.8	2	30	1	50	15	2	31.96	2	31.87	2	31.48	2	31.68	0.88 %	0.59 %	0.64 %
0.8	0.8	2	30	2	50	15	2	31.96	2	31.61	2	31.37	2	31.60	1.12 %	0.01 %	0.76 %
0.8	0.8	2	40	1	10	15	2	32.81	1	30.66	1	29.72	1	30.53	6.93 %	0.42 %	2.68 %
0.8	0.8	2	40	2	10	15	2	32.81	1	30.81	1	29.88	1	30.61	6.70 %	0.65 %	2.39 %
0.8	0.8	2	40	1	20	15	2	32.81	2	31.36	1	30.41	1	31.00	5.52 %	1.17 %	1.89 %
0.8	0.8	2	40	2	20	15	2	32.81	2	31.55	1	30.72	1	31.36	4.42 %	0.61 %	2.04 %
0.8	0.8	2	40	1	30	15	2	32.81	2	31.80	1	31.18	2	31.57	3.78 %	0.73 %	1.23 %
0.8	0.8	2	40	2	30	15	2	32.81	2	31.76	1	31.27	2	31.62	3.61 %	0.42 %	1.12 %
0.8	0.8	2	40	1	40	15	2	32.81	2	32.11	2	31.58	2	31.89	2.80 %	0.68 %	0.97 %
0.8	0.8	2	40	2	40	15	2	32.81	2	31.95	2	31.53	2	31.74	3.27 %	0.66 %	0.66 %
0.8	0.8	2	40	1	50	15	2	32.81	2	32.22	2	31.74	2	31.98	2.52 %	0.74 %	0.75 %
0.8	0.8	2	40	2	50	15	2	32.81	2	32.34	2	31.89	2	32.09	2.20 %	0.77 %	0.63 %
0.8	0.8	2	50	1	10	15	3	33.35	1	30.95	1	29.84	1	30.90	7.35 %	0.18 %	3.42 %
0.8	0.8	2	50	2	10	15	3	33.35	2	31.06	1	30.15	1	31.08	6.80 %	-0.07 %	3.01 %
0.8	0.8	2	50	1	20	15	3	33.35	2	31.72	1	30.83	2	31.55	5.38 %	0.52 %	2.31 %
0.8	0.8	2	50	2	20	15	3	33.35	2	31.67	1	30.94	2	31.54	5.41 %	0.40 %	1.91 %
0.8	0.8	2	50	1	30	15	3	33.35	2	32.12	1	31.40	2	31.86	4.47 %	0.83 %	1.42 %
0.8	0.8	2	50	2	30	15	3	33.35	2	32.18	1	31.58	2	31.87	4.43 %	0.96 %	0.93 %
0.8	0.8	2	50	1	40	15	3	33.35	2	32.44	2	31.76	2	32.07	3.83 %	1.15 %	0.98 %
0.8	0.8	2	50	2	40	15	3	33.35	2	32.34	2	31.61	2	32.04	3.91 %	0.94 %	1.36 %
0.8	0.8	2	50	1	50	15	3	33.35	2	32.86	2	32.21	2	32.50	2.55 %	1.11 %	0.88 %
0.8	0.8	2	50	2	50	15	3	33.35	2	32.79	2	32.24	2	32.52	2.48 %	0.83 %	0.87 %

Table 13: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
0.8	0.8	3	10	1	10	15	1	29.97	1	28.85	1	28.33	1	28.74	4.10 %	0.39 %	1.41 %
0.8	0.8	3	10	2	10	15	1	29.97	1	29.22	1	28.62	1	29.09	2.92 %	0.43 %	1.62 %
0.8	0.8	3	10	1	20	15	1	29.97	1	29.73	1	29.13	1	29.46	1.70 %	0.93 %	1.13 %
0.8	0.8	3	10	2	20	15	1	29.97	1	29.69	1	29.18	1	29.50	1.56 %	0.64 %	1.10 %
0.8	0.8	3	10	1	30	15	1	29.97	1	29.79	1	29.38	1	29.67	0.98 %	0.38 %	1.01 %
0.8	0.8	3	10	2	30	15	1	29.97	1	29.80	1	29.55	1	29.66	1.02 %	0.47 %	0.37 %
0.8	0.8	3	10	1	40	15	1	29.97	1	29.96	1	29.70	1	29.84	0.43 %	0.40 %	0.48 %
0.8	0.8	3	10	2	40	15	1	29.97	1	30.05	1	29.78	2	30.03	-0.20 %	0.09 %	0.82 %
0.8	0.8	3	10	1	50	15	1	29.97	1	29.95	1	29.80	1	29.91	0.20 %	0.13 %	0.35 %
0.8	0.8	3	10	2	50	15	1	29.97	2	30.03	2	29.90	1	30.05	-0.27 %	-0.05 %	0.50 %
0.8	0.8	3	20	1	10	15	3	32.08	2	30.53	1	29.38	2	30.31	5.51 %	0.71 %	3.08 %
0.8	0.8	3	20	2	10	15	3	32.08	2	30.64	2	29.72	2	30.36	5.36 %	0.89 %	2.12 %
0.8	0.8	3	20	1	20	15	3	32.08	2	31.03	2	30.04	2	30.61	4.59 %	1.35 %	1.86 %
0.8	0.8	3	20	2	20	15	3	32.08	2	31.31	2	30.35	2	30.92	3.62 %	1.26 %	1.85 %
0.8	0.8	3	20	1	30	15	3	32.08	2	31.53	2	30.59	2	31.12	3.00 %	1.30 %	1.70 %
0.8	0.8	3	20	2	30	15	3	32.08	3	31.76	2	30.79	2	31.32	2.37 %	1.38 %	1.68 %
0.8	0.8	3	20	1	40	15	3	32.08	3	31.87	2	31.15	2	31.57	1.59 %	0.95 %	1.32 %
0.8	0.8	3	20	2	40	15	3	32.08	3	31.93	2	31.14	2	31.53	1.73 %	1.26 %	1.23 %
0.8	0.8	3	20	1	50	15	3	32.08	3	31.95	2	31.40	2	31.74	1.07 %	0.67 %	1.07 %
0.8	0.8	3	20	2	50	15	3	32.08	3	31.87	2	31.34	2	31.69	1.23 %	0.57 %	1.09 %
0.8	0.8	3	30	1	10	15	3	33.39	2	30.80	2	29.86	2	30.61	8.33 %	0.63 %	2.45 %
0.8	0.8	3	30	2	10	15	3	33.39	2	31.43	2	30.26	2	31.21	6.52 %	0.70 %	3.04 %
0.8	0.8	3	30	1	20	15	3	33.39	3	31.70	2	30.70	2	31.47	5.74 %	0.73 %	2.47 %
0.8	0.8	3	30	2	20	15	3	33.39	3	31.99	2	30.87	2	31.72	5.00 %	0.87 %	2.68 %
0.8	0.8	3	30	1	30	15	3	33.39	3	32.54	2	31.42	3	32.09	3.90 %	1.40 %	2.06 %
0.8	0.8	3	30	2	30	15	3	33.39	3	32.44	2	31.54	3	32.07	3.95 %	1.15 %	1.66 %
0.8	0.8	3	30	1	40	15	3	33.39	3	32.76	3	31.86	3	32.27	3.35 %	1.49 %	1.27 %
0.8	0.8	3	30	2	40	15	3	33.39	3	32.75	2	31.91	3	32.27	3.36 %	1.48 %	1.10 %
0.8	0.8	3	30	1	50	15	3	33.39	3	32.53	3	31.95	3	32.25	3.41 %	0.86 %	0.93 %
0.8	0.8	3	30	2	50	15	3	33.39	3	32.74	3	31.99	3	32.33	3.16 %	1.24 %	1.08 %
0.8	0.8	3	40	1	10	15	4	34.13	3	31.56	2	30.43	2	31.33	8.18 %	0.70 %	2.89 %
0.8	0.8	3	40	2	10	15	4	34.13	3	31.83	2	30.61	2	31.44	7.87 %	1.22 %	2.63 %
0.8	0.8	3	40	1	20	15	4	34.13	3	32.11	2	31.09	3	31.85	6.67 %	0.81 %	2.37 %
0.8	0.8	3	40	2	20	15	4	34.13	3	32.07	2	31.17	3	31.81	6.78 %	0.81 %	2.02 %
0.8	0.8	3	40	1	30	15	4	34.13	3	32.82	3	31.67	3	32.31	5.31 %	1.54 %	2.00 %
0.8	0.8	3	40	2	30	15	4	34.13	3	32.95	3	31.89	3	32.51	4.73 %	1.32 %	1.93 %
0.8	0.8	3	40	1	40	15	4	34.13	3	32.92	3	32.05	3	32.40	5.07 %	1.59 %	1.07 %
0.8	0.8	3	40	2	40	15	4	34.13	3	33.48	3	32.22	3	32.74	4.05 %	2.19 %	1.59 %
0.8	0.8	3	40	1	50	15	4	34.13	3	33.51	3	32.36	3	32.86	3.70 %	1.93 %	1.52 %
0.8	0.8	3	40	2	50	15	4	34.13	3	33.35	3	32.41	3	32.75	4.02 %	1.79 %	1.05 %
0.8	0.8	3	50	1	10	15	4	34.81	3	31.89	3	30.88	3	31.68	8.98 %	0.65 %	2.55 %
0.8	0.8	3	50	2	10	15	4	34.81	3	31.91	3	31.04	3	31.78	8.70 %	0.40 %	2.33 %
0.8	0.8	3	50	1	20	15	4	34.81	3	32.74	3	31.71	3	32.38	6.98 %	1.11 %	2.05 %
0.8	0.8	3	50	2	20	15	4	34.81	3	32.70	3	31.62	3	32.28	7.25 %	1.29 %	2.05 %
0.8	0.8	3	50	1	30	15	4	34.81	3	32.91	3	31.74	3	32.46	6.75 %	1.36 %	2.22 %
0.8	0.8	3	50	2	30	15	4	34.81	3	32.92	3	31.92	3	32.70	6.04 %	0.66 %	2.40 %
0.8	0.8	3	50	1	40	15	4	34.81	3	33.55	3	32.38	3	32.74	5.94 %	2.43 %	1.10 %
0.8	0.8	3	50	2	40	15	4	34.81	4	33.68	3	32.64	3	33.25	4.48 %	1.28 %	1.84 %
0.8	0.8	3	50	1	50	15	4	34.81	4	33.81	3	32.60	3	33.05	5.04 %	2.23 %	1.38 %
0.8	0.8	3	50	2	50	15	4	34.81	4	33.75	3	32.77	3	33.35	4.18 %	1.19 %	1.76 %

Table 14: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
0.8	0.6	2	10	1	10	17	1	34.84	1	33.98	0	33.12	0	33.83	2.88 %	0.42 %	2.12 %
0.8	0.6	2	10	2	10	17	1	34.84	0	33.92	0	33.02	0	33.85	2.84 %	0.20 %	2.45 %
0.8	0.6	2	10	1	20	17	1	34.84	1	34.40	0	33.87	1	34.13	2.02 %	0.79 %	0.76 %
0.8	0.6	2	10	2	20	17	1	34.84	1	34.47	0	33.94	1	34.44	1.12 %	0.08 %	1.46 %
0.8	0.6	2	10	1	30	17	1	34.84	1	34.39	1	33.97	1	34.23	1.73 %	0.45 %	0.78 %
0.8	0.6	2	10	2	30	17	1	34.84	1	34.58	1	34.32	1	34.53	0.87 %	0.14 %	0.60 %
0.8	0.6	2	10	1	40	17	1	34.84	1	34.67	1	34.45	1	34.50	0.95 %	0.47 %	0.17 %
0.8	0.6	2	10	2	40	17	1	34.84	1	34.70	1	34.44	1	34.60	0.68 %	0.30 %	0.45 %
0.8	0.6	2	10	1	50	17	1	34.84	1	34.70	1	34.61	1	34.66	0.52 %	0.13 %	0.14 %
0.8	0.6	2	10	2	50	17	1	34.84	1	34.95	1	34.80	1	34.94	-0.28 %	0.05 %	0.38 %
0.8	0.6	2	20	1	10	17	3	37.74	1	35.52	1	34.18	1	35.41	6.17 %	0.30 %	3.46 %
0.8	0.6	2	20	2	10	17	3	37.74	2	35.66	1	34.45	1	35.58	5.73 %	0.22 %	3.16 %
0.8	0.6	2	20	1	20	17	3	37.74	2	36.36	1	35.16	2	36.07	4.42 %	0.80 %	2.52 %
0.8	0.6	2	20	2	20	17	3	37.74	2	36.27	1	35.17	2	36.15	4.21 %	0.34 %	2.71 %
0.8	0.6	2	20	1	30	17	3	37.74	2	36.84	2	35.95	2	36.61	2.99 %	0.61 %	1.80 %
0.8	0.6	2	20	2	30	17	3	37.74	2	37.03	1	36.24	2	36.75	2.61 %	0.76 %	1.41 %
0.8	0.6	2	20	1	40	17	3	37.74	2	37.12	2	36.29	2	36.82	2.42 %	0.80 %	1.45 %
0.8	0.6	2	20	2	40	17	3	37.74	2	37.02	2	36.36	2	36.78	2.53 %	0.65 %	1.15 %
0.8	0.6	2	20	1	50	17	3	37.74	2	37.13	2	36.49	2	37.04	1.84 %	0.23 %	1.49 %
0.8	0.6	2	20	2	50	17	3	37.74	2	37.20	2	36.53	2	36.97	2.04 %	0.63 %	1.19 %
0.8	0.6	2	30	1	10	17	3	39.35	2	36.13	1	34.81	2	36.02	8.47 %	0.32 %	3.34 %
0.8	0.6	2	30	2	10	17	3	39.35	2	36.44	1	35.11	2	36.44	7.40 %	0.02 %	3.63 %
0.8	0.6	2	30	1	20	17	3	39.35	2	37.13	2	35.82	2	36.93	6.16 %	0.54 %	2.99 %
0.8	0.6	2	30	2	20	17	3	39.35	2	37.13	2	36.02	2	36.87	6.29 %	0.68 %	2.31 %
0.8	0.6	2	30	1	30	17	3	39.35	3	37.54	2	36.22	3	37.19	5.49 %	0.92 %	2.60 %
0.8	0.6	2	30	2	30	17	3	39.35	3	37.96	2	36.65	2	37.69	4.22 %	0.72 %	2.76 %
0.8	0.6	2	30	1	40	17	3	39.35	3	37.85	2	36.72	2	37.44	4.85 %	1.09 %	1.92 %
0.8	0.6	2	30	2	40	17	3	39.35	3	38.43	2	37.45	3	37.96	3.53 %	1.23 %	1.34 %
0.8	0.6	2	30	1	50	17	3	39.35	3	38.64	2	37.64	3	38.27	2.74 %	0.95 %	1.65 %
0.8	0.6	2	30	2	50	17	3	39.35	3	38.61	2	37.63	3	38.29	2.70 %	0.85 %	1.70 %
0.8	0.6	2	40	1	10	17	4	40.39	2	36.69	2	35.39	2	36.66	9.25 %	0.09 %	3.47 %
0.8	0.6	2	40	2	10	17	4	40.39	3	36.82	2	35.58	2	36.85	8.77 %	-0.10 %	3.46 %
0.8	0.6	2	40	1	20	17	4	40.39	2	37.72	2	36.47	2	37.50	7.17 %	0.59 %	2.74 %
0.8	0.6	2	40	2	20	17	4	40.39	2	37.81	2	36.39	2	37.59	6.93 %	0.57 %	3.21 %
0.8	0.6	2	40	1	30	17	4	40.39	3	38.29	2	37.08	3	37.93	6.09 %	0.93 %	2.24 %
0.8	0.6	2	40	2	30	17	4	40.39	3	38.08	2	37.18	3	37.94	6.07 %	0.34 %	2.02 %
0.8	0.6	2	40	1	40	17	4	40.39	3	39.01	3	37.87	3	38.39	4.96 %	1.58 %	1.35 %
0.8	0.6	2	40	2	40	17	4	40.39	3	38.78	3	37.66	3	38.36	5.04 %	1.10 %	1.82 %
0.8	0.6	2	40	1	50	17	4	40.39	3	39.15	3	37.98	3	38.71	4.18 %	1.13 %	1.89 %
0.8	0.6	2	40	2	50	17	4	40.39	4	39.56	3	38.34	3	38.98	3.50 %	1.47 %	1.65 %
0.8	0.6	2	50	1	10	17	5	41.38	3	37.46	2	35.99	2	37.38	9.67 %	0.23 %	3.72 %
0.8	0.6	2	50	2	10	17	5	41.38	3	37.06	2	35.80	2	36.90	10.81 %	0.42 %	3.00 %
0.8	0.6	2	50	1	20	17	5	41.38	3	38.02	2	36.63	3	37.75	8.78 %	0.73 %	2.97 %
0.8	0.6	2	50	2	20	17	5	41.38	3	37.88	2	36.87	3	37.79	8.66 %	0.23 %	2.44 %
0.8	0.6	2	50	1	30	17	5	41.38	4	39.12	2	37.59	3	38.50	6.95 %	1.59 %	2.38 %
0.8	0.6	2	50	2	30	17	5	41.38	3	38.33	3	37.27	3	38.15	7.81 %	0.48 %	2.31 %
0.8	0.6	2	50	1	40	17	5	41.38	3	39.48	3	38.12	3	38.80	6.23 %	1.71 %	1.76 %
0.8	0.6	2	50	2	40	17	5	41.38	3	39.56	3	38.34	3	39.03	5.67 %	1.33 %	1.78 %
0.8	0.6	2	50	1	50	17	5	41.38	3	39.61	3	38.40	3	38.93	5.91 %	1.71 %	1.36 %
0.8	0.6	2	50	2	50	17	5	41.38	4	39.66	3	38.41	3	39.31	4.99 %	0.88 %	2.31 %

Table 15: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
0.8	0.6	3	10	1	10	17	3	36.32	2	34.41	1	33.06	2	34.21	5.81 %	0.58 %	3.37 %
0.8	0.6	3	10	2	10	17	3	36.32	2	34.55	1	33.33	2	34.37	5.38 %	0.52 %	3.04 %
0.8	0.6	3	10	1	20	17	3	36.32	2	35.59	1	34.40	2	35.24	2.99 %	0.98 %	2.37 %
0.8	0.6	3	10	2	20	17	3	36.32	2	35.63	1	34.49	2	35.40	2.55 %	0.66 %	2.55 %
0.8	0.6	3	10	1	30	17	3	36.32	2	35.86	2	34.83	2	35.62	1.95 %	0.67 %	2.22 %
0.8	0.6	3	10	2	30	17	3	36.32	2	35.84	2	34.84	2	35.63	1.91 %	0.57 %	2.21 %
0.8	0.6	3	10	1	40	17	3	36.32	3	36.16	2	35.22	2	35.88	1.21 %	0.76 %	1.86 %
0.8	0.6	3	10	2	40	17	3	36.32	3	35.98	2	35.37	2	35.81	1.42 %	0.48 %	1.20 %
0.8	0.6	3	10	1	50	17	3	36.32	3	36.10	2	35.48	2	35.88	1.21 %	0.61 %	1.12 %
0.8	0.6	3	10	2	50	17	3	36.32	3	36.25	2	35.67	3	36.06	0.72 %	0.51 %	1.08 %
0.8	0.6	3	20	1	10	17	5	39.57	3	36.27	2	34.67	3	35.99	9.07 %	0.78 %	3.66 %
0.8	0.6	3	20	2	10	17	5	39.57	3	36.57	2	34.98	3	36.34	8.17 %	0.63 %	3.76 %
0.8	0.6	3	20	1	20	17	5	39.57	3	37.60	3	35.74	3	37.07	6.32 %	1.39 %	3.59 %
0.8	0.6	3	20	2	20	17	5	39.57	4	37.73	3	36.14	3	37.33	5.66 %	1.04 %	3.19 %
0.8	0.6	3	20	1	30	17	5	39.57	4	38.16	3	36.82	4	37.81	4.45 %	0.91 %	2.63 %
0.8	0.6	3	20	2	30	17	5	39.57	4	37.97	3	36.59	4	37.53	5.17 %	1.16 %	2.51 %
0.8	0.6	3	20	1	40	17	5	39.57	4	38.49	3	37.17	4	38.07	3.81 %	1.11 %	2.35 %
0.8	0.6	3	20	2	40	17	5	39.57	4	38.25	3	37.07	4	37.90	4.23 %	0.92 %	2.19 %
0.8	0.6	3	20	1	50	17	5	39.57	4	38.66	3	37.57	4	38.30	3.21 %	0.92 %	1.92 %
0.8	0.6	3	20	2	50	17	5	39.57	4	39.00	3	37.72	4	38.43	2.88 %	1.46 %	1.86 %
0.8	0.6	3	30	1	10	17	6	41.40	4	37.53	3	35.99	4	37.36	9.75 %	0.44 %	3.67 %
0.8	0.6	3	30	2	10	17	6	41.40	4	37.55	3	36.05	4	37.29	9.91 %	0.68 %	3.34 %
0.8	0.6	3	30	1	20	17	6	41.40	4	38.28	3	36.51	4	37.73	8.85 %	1.42 %	3.25 %
0.8	0.6	3	30	2	20	17	6	41.40	4	38.31	3	36.71	4	37.89	8.46 %	1.10 %	3.14 %
0.8	0.6	3	30	1	30	17	6	41.40	5	38.81	4	37.30	4	38.27	7.55 %	1.38 %	2.55 %
0.8	0.6	3	30	2	30	17	6	41.40	4	39.17	4	37.74	4	38.69	6.55 %	1.23 %	2.44 %
0.8	0.6	3	30	1	40	17	6	41.40	5	39.53	4	38.00	4	38.97	5.86 %	1.40 %	2.49 %
0.8	0.6	3	30	2	40	17	6	41.40	5	39.77	4	38.22	4	38.92	5.98 %	2.13 %	1.80 %
0.8	0.6	3	30	1	50	17	6	41.40	5	40.12	4	38.54	4	39.27	5.14 %	2.12 %	1.87 %
0.8	0.6	3	30	2	50	17	6	41.40	5	39.82	4	38.37	4	39.27	5.15 %	1.39 %	2.28 %
0.8	0.6	3	40	1	10	17	6	42.60	4	37.81	3	36.08	4	37.53	11.90 %	0.75 %	3.86 %
0.8	0.6	3	40	2	10	17	6	42.60	4	38.19	4	36.87	4	38.16	10.41 %	0.07 %	3.38 %
0.8	0.6	3	40	1	20	17	6	42.60	4	39.10	4	37.32	4	38.63	9.30 %	1.19 %	3.39 %
0.8	0.6	3	40	2	20	17	6	42.60	4	38.93	4	37.45	4	38.53	9.55 %	1.03 %	2.81 %
0.8	0.6	3	40	1	30	17	6	42.60	5	39.68	4	38.11	5	39.16	8.07 %	1.30 %	2.67 %
0.8	0.6	3	40	2	30	17	6	42.60	5	39.71	4	38.17	4	39.15	8.09 %	1.40 %	2.52 %
0.8	0.6	3	40	1	40	17	6	42.60	5	40.03	4	38.25	5	39.31	7.72 %	1.81 %	2.68 %
0.8	0.6	3	40	2	40	17	6	42.60	5	40.34	4	38.74	5	39.83	6.48 %	1.24 %	2.75 %
0.8	0.6	3	40	1	50	17	6	42.60	5	40.53	4	38.90	5	39.70	6.81 %	2.06 %	2.01 %
0.8	0.6	3	40	2	50	17	6	42.60	5	40.54	4	39.14	5	39.93	6.25 %	1.49 %	2.00 %
0.8	0.6	3	50	1	10	17	7	43.58	5	38.35	4	36.68	4	38.10	12.57 %	0.66 %	3.72 %
0.8	0.6	3	50	2	10	17	7	43.58	5	38.99	4	37.44	5	38.84	10.88 %	0.38 %	3.60 %
0.8	0.6	3	50	1	20	17	7	43.58	5	39.67	5	38.23	5	39.49	9.39 %	0.47 %	3.18 %
0.8	0.6	3	50	2	20	17	7	43.58	5	39.35	4	38.04	5	39.10	10.27 %	0.63 %	2.73 %
0.8	0.6	3	50	1	30	17	7	43.58	5	39.89	4	38.29	5	39.39	9.61 %	1.25 %	2.79 %
0.8	0.6	3	50	2	30	17	7	43.58	5	40.35	5	38.79	4	39.80	8.67 %	1.36 %	2.55 %
0.8	0.6	3	50	1	40	17	7	43.58	5	40.75	5	39.20	5	40.18	7.81 %	1.41 %	2.42 %
0.8	0.6	3	50	2	40	17	7	43.58	5	40.49	4	38.87	5	39.82	8.62 %	1.65 %	2.38 %
0.8	0.6	3	50	1	50	17	7	43.58	5	41.51	4	39.94	5	40.55	6.95 %	2.31 %	1.50 %
0.8	0.6	3	50	2	50	17	7	43.58	5	41.05	4	39.62	5	40.25	7.64 %	1.95 %	1.57 %

Table 16: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
2.4	0.8	2	10	1	10	25	4	50.33	3	49.92	3	48.82	3	49.33	1.99 %	1.19 %	1.04 %
2.4	0.8	2	10	2	10	25	4	50.33	3	49.96	3	49.16	3	49.57	1.51 %	0.78 %	0.83 %
2.4	0.8	2	10	1	20	25	4	50.33	4	50.26	3	49.59	3	49.95	0.75 %	0.61 %	0.72 %
2.4	0.8	2	10	2	20	25	4	50.33	4	50.27	3	49.74	3	50.01	0.64 %	0.52 %	0.54 %
2.4	0.8	2	10	1	30	25	4	50.33	4	50.20	3	49.92	4	50.14	0.38 %	0.12 %	0.43 %
2.4	0.8	2	10	2	30	25	4	50.33	4	50.37	4	50.23	4	50.35	-0.03 %	0.03 %	0.22 %
2.4	0.8	2	10	1	40	25	4	50.33	3	50.21	3	50.08	3	50.17	0.32 %	0.08 %	0.17 %
2.4	0.8	2	10	2	40	25	4	50.33	4	50.35	4	50.28	4	50.36	-0.05 %	-0.02 %	0.15 %
2.4	0.8	2	10	1	50	25	4	50.33	4	50.44	3	50.36	4	50.38	-0.09 %	0.13 %	0.05 %
2.4	0.8	2	10	2	50	25	4	50.33	3	50.34	3	50.24	3	50.37	-0.07 %	-0.05 %	0.26 %
2.4	0.8	2	20	1	10	25	5	53.41	5	52.13	4	50.13	4	51.25	4.04 %	1.68 %	2.20 %
2.4	0.8	2	20	2	10	25	5	53.41	5	52.27	3	50.74	4	51.61	3.38 %	1.27 %	1.67 %
2.4	0.8	2	20	1	20	25	5	53.41	5	52.73	4	51.22	4	51.99	2.66 %	1.41 %	1.48 %
2.4	0.8	2	20	2	20	25	5	53.41	5	52.96	4	51.62	5	52.29	2.09 %	1.26 %	1.28 %
2.4	0.8	2	20	1	30	25	5	53.41	5	53.39	5	52.18	5	52.70	1.33 %	1.28 %	0.99 %
2.4	0.8	2	20	2	30	25	5	53.41	5	53.12	5	52.31	5	52.76	1.22 %	0.68 %	0.85 %
2.4	0.8	2	20	1	40	25	5	53.41	5	53.31	5	52.42	5	52.90	0.96 %	0.77 %	0.89 %
2.4	0.8	2	20	2	40	25	5	53.41	5	53.18	5	52.42	5	52.84	1.06 %	0.64 %	0.80 %
2.4	0.8	2	20	1	50	25	5	53.41	5	53.44	5	52.76	5	53.03	0.71 %	0.77 %	0.50 %
2.4	0.8	2	20	2	50	25	5	53.41	5	53.34	5	52.88	5	53.00	0.77 %	0.63 %	0.22 %
2.4	0.8	2	30	1	10	25	6	55.06	5	52.91	4	50.82	4	52.07	5.43 %	1.58 %	2.40 %
2.4	0.8	2	30	2	10	25	6	55.06	5	53.28	4	51.38	5	52.58	4.50 %	1.31 %	2.30 %
2.4	0.8	2	30	1	20	25	6	55.06	6	54.01	5	52.22	5	53.10	3.56 %	1.68 %	1.66 %
2.4	0.8	2	30	2	20	25	6	55.06	6	54.02	5	52.36	5	53.17	3.44 %	1.57 %	1.52 %
2.4	0.8	2	30	1	30	25	6	55.06	6	54.48	5	52.82	5	53.57	2.70 %	1.66 %	1.41 %
2.4	0.8	2	30	2	30	25	6	55.06	6	54.52	5	53.15	6	53.85	2.20 %	1.24 %	1.30 %
2.4	0.8	2	30	1	40	25	6	55.06	6	54.78	5	53.41	6	54.06	1.82 %	1.32 %	1.20 %
2.4	0.8	2	30	2	40	25	6	55.06	6	54.88	6	53.71	6	54.18	1.59 %	1.27 %	0.88 %
2.4	0.8	2	30	1	50	25	6	55.06	6	54.81	6	53.96	6	54.35	1.29 %	0.85 %	0.71 %
2.4	0.8	2	30	2	50	25	6	55.06	6	54.94	6	53.98	6	54.40	1.20 %	0.99 %	0.78 %
2.4	0.8	2	40	1	10	25	7	56.11	5	53.46	4	51.46	5	52.69	6.09 %	1.43 %	2.34 %
2.4	0.8	2	40	2	10	25	7	56.11	6	53.89	5	51.98	5	53.04	5.48 %	1.58 %	2.00 %
2.4	0.8	2	40	1	20	25	7	56.11	6	54.64	5	52.68	5	53.69	4.32 %	1.74 %	1.87 %
2.4	0.8	2	40	2	20	25	7	56.11	6	54.76	5	53.08	6	53.93	3.88 %	1.51 %	1.58 %
2.4	0.8	2	40	1	30	25	7	56.11	7	55.50	5	53.63	6	54.42	3.02 %	1.95 %	1.45 %
2.4	0.8	2	40	2	30	25	7	56.11	7	55.39	6	53.68	6	54.32	3.20 %	1.94 %	1.17 %
2.4	0.8	2	40	1	40	25	7	56.11	7	55.70	6	54.11	6	54.83	2.28 %	1.55 %	1.31 %
2.4	0.8	2	40	2	40	25	7	56.11	7	55.87	6	54.46	6	55.07	1.85 %	1.42 %	1.11 %
2.4	0.8	2	40	1	50	25	7	56.11	7	55.85	6	54.53	6	54.94	2.09 %	1.63 %	0.75 %
2.4	0.8	2	40	2	50	25	7	56.11	7	55.93	6	54.88	7	55.39	1.28 %	0.96 %	0.93 %
2.4	0.8	2	50	1	10	25	8	57.05	6	53.97	5	52.04	5	53.13	6.87 %	1.55 %	2.06 %
2.4	0.8	2	50	2	10	25	8	57.05	6	53.89	5	52.23	5	53.25	6.66 %	1.19 %	1.91 %
2.4	0.8	2	50	1	20	25	8	57.05	6	55.06	5	52.93	6	53.99	5.37 %	1.95 %	1.97 %
2.4	0.8	2	50	2	20	25	8	57.05	6	55.51	5	53.55	6	54.36	4.72 %	2.07 %	1.49 %
2.4	0.8	2	50	1	30	25	8	57.05	7	55.84	6	53.88	6	54.58	4.33 %	2.26 %	1.29 %
2.4	0.8	2	50	2	30	25	8	57.05	7	56.00	6	54.19	6	54.95	3.69 %	1.89 %	1.37 %
2.4	0.8	2	50	1	40	25	8	57.05	7	56.27	6	54.53	7	55.32	3.03 %	1.69 %	1.43 %
2.4	0.8	2	50	2	40	25	8	57.05	7	56.39	6	54.82	6	55.33	3.03 %	1.88 %	0.91 %
2.4	0.8	2	50	1	50	25	8	57.05	7	56.65	6	55.10	6	55.72	2.34 %	1.64 %	1.10 %
2.4	0.8	2	50	2	50	25	8	57.05	7	56.62	6	55.18	7	55.60	2.55 %	1.80 %	0.76 %

Table 17: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
2.4	0.8	3	10	1	10	25	7	51.75	7	50.92	5	48.34	5	49.76	3.86 %	2.28 %	2.84 %
2.4	0.8	3	10	2	10	25	7	51.75	6	51.20	5	49.12	6	50.37	2.67 %	1.62 %	2.49 %
2.4	0.8	3	10	1	20	25	7	51.75	7	51.63	6	49.97	6	50.85	1.76 %	1.52 %	1.73 %
2.4	0.8	3	10	2	20	25	7	51.75	7	51.52	6	50.27	6	50.93	1.59 %	1.15 %	1.29 %
2.4	0.8	3	10	1	30	25	7	51.75	7	51.90	6	50.65	6	51.30	0.87 %	1.14 %	1.28 %
2.4	0.8	3	10	2	30	25	7	51.75	7	51.61	6	50.76	7	51.25	0.98 %	0.70 %	0.95 %
2.4	0.8	3	10	1	40	25	7	51.75	7	51.79	6	51.08	7	51.51	0.48 %	0.55 %	0.82 %
2.4	0.8	3	10	2	40	25	7	51.75	7	51.63	6	51.02	7	51.38	0.73 %	0.48 %	0.69 %
2.4	0.8	3	10	1	50	25	7	51.75	7	51.70	7	51.21	7	51.52	0.45 %	0.35 %	0.61 %
2.4	0.8	3	10	2	50	25	7	51.75	7	51.76	6	51.41	7	51.66	0.18 %	0.18 %	0.48 %
2.4	0.8	3	20	1	10	25	9	55.23	8	53.37	5	50.24	7	52.02	5.82 %	2.53 %	3.43 %
2.4	0.8	3	20	2	10	25	9	55.23	8	53.37	7	50.77	7	52.33	5.25 %	1.94 %	2.98 %
2.4	0.8	3	20	1	20	25	9	55.23	9	54.49	7	51.65	7	53.12	3.83 %	2.52 %	2.77 %
2.4	0.8	3	20	2	20	25	9	55.23	9	54.67	7	52.27	8	53.40	3.32 %	2.33 %	2.11 %
2.4	0.8	3	20	1	30	25	9	55.23	9	54.77	7	52.54	8	53.44	3.24 %	2.42 %	1.69 %
2.4	0.8	3	20	2	30	25	9	55.23	9	54.75	8	52.84	8	53.62	2.92 %	2.07 %	1.44 %
2.4	0.8	3	20	1	40	25	9	55.23	9	54.76	8	53.14	8	54.07	2.10 %	1.26 %	1.74 %
2.4	0.8	3	20	2	40	25	9	55.23	9	54.94	8	53.53	8	54.25	1.78 %	1.26 %	1.33 %
2.4	0.8	3	20	1	50	25	9	55.23	9	55.13	8	53.63	9	54.49	1.34 %	1.15 %	1.58 %
2.4	0.8	3	20	2	50	25	9	55.23	9	54.99	8	53.94	9	54.43	1.46 %	1.03 %	0.89 %
2.4	0.8	3	30	1	10	25	10	57.15	9	54.26	7	51.39	8	53.11	7.06 %	2.11 %	3.24 %
2.4	0.8	3	30	2	10	25	10	57.15	8	54.30	7	51.72	8	53.42	6.53 %	1.63 %	3.19 %
2.4	0.8	3	30	1	20	25	10	57.15	9	55.65	8	52.75	8	54.16	5.24 %	2.68 %	2.59 %
2.4	0.8	3	30	2	20	25	10	57.15	9	55.76	8	53.02	9	54.35	4.90 %	2.54 %	2.44 %
2.4	0.8	3	30	1	30	25	10	57.15	10	56.16	8	53.62	9	54.78	4.15 %	2.46 %	2.11 %
2.4	0.8	3	30	2	30	25	10	57.15	10	56.17	8	53.83	9	54.97	3.82 %	2.14 %	2.07 %
2.4	0.8	3	30	1	40	25	10	57.15	10	56.49	9	54.28	9	55.14	3.51 %	2.39 %	1.56 %
2.4	0.8	3	30	2	40	25	10	57.15	10	56.68	9	54.77	9	55.49	2.90 %	2.10 %	1.30 %
2.4	0.8	3	30	1	50	25	10	57.15	10	57.04	9	54.90	9	55.77	2.42 %	2.22 %	1.55 %
2.4	0.8	3	30	2	50	25	10	57.15	10	56.66	9	55.04	9	55.72	2.50 %	1.65 %	1.23 %
2.4	0.8	3	40	1	10	25	11	58.43	9	54.83	7	51.74	8	53.74	8.04 %	1.99 %	3.72 %
2.4	0.8	3	40	2	10	25	11	58.43	9	55.25	7	52.62	9	54.16	7.31 %	1.98 %	2.84 %
2.4	0.8	3	40	1	20	25	11	58.43	10	56.08	8	53.21	9	54.51	6.71 %	2.80 %	2.39 %
2.4	0.8	3	40	2	20	25	11	58.43	10	56.50	8	53.84	9	54.98	5.91 %	2.69 %	2.07 %
2.4	0.8	3	40	1	30	25	11	58.43	10	56.90	8	54.16	9	55.18	5.57 %	3.01 %	1.86 %
2.4	0.8	3	40	2	30	25	11	58.43	10	57.28	9	54.79	9	55.79	4.52 %	2.59 %	1.80 %
2.4	0.8	3	40	1	40	25	11	58.43	11	57.72	9	54.98	10	56.18	3.86 %	2.67 %	2.13 %
2.4	0.8	3	40	2	40	25	11	58.43	11	57.56	9	55.18	10	56.34	3.58 %	2.12 %	2.06 %
2.4	0.8	3	40	1	50	25	11	58.43	11	58.00	9	55.52	10	56.43	3.43 %	2.70 %	1.61 %
2.4	0.8	3	40	2	50	25	11	58.43	11	57.93	9	55.92	10	56.71	2.94 %	2.09 %	1.41 %
2.4	0.8	3	50	1	10	25	12	59.51	10	55.30	7	52.17	8	54.30	8.76 %	1.81 %	3.92 %
2.4	0.8	3	50	2	10	25	12	59.51	10	55.67	8	53.18	9	54.84	7.86 %	1.50 %	3.01 %
2.4	0.8	3	50	1	20	25	12	59.51	10	56.67	9	53.77	9	55.35	6.99 %	2.33 %	2.85 %
2.4	0.8	3	50	2	20	25	12	59.51	10	56.67	9	54.13	9	55.53	6.69 %	2.01 %	2.53 %
2.4	0.8	3	50	1	30	25	12	59.51	10	57.77	9	54.83	10	56.13	5.69 %	2.85 %	2.30 %
2.4	0.8	3	50	2	30	25	12	59.51	11	57.77	9	55.09	10	56.28	5.44 %	2.58 %	2.11 %
2.4	0.8	3	50	1	40	25	12	59.51	11	58.12	10	55.72	10	56.69	4.75 %	2.48 %	1.70 %
2.4	0.8	3	50	2	40	25	12	59.51	11	58.17	10	55.91	10	56.63	4.85 %	2.65 %	1.28 %
2.4	0.8	3	50	1	50	25	12	59.51	11	58.53	9	56.01	10	56.91	4.36 %	2.76 %	1.58 %
2.4	0.8	3	50	2	50	25	12	59.51	11	58.30	10	56.21	10	56.97	4.28 %	2.29 %	1.34 %

Table 18: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
2.4	0.6	2	10	1	10	29	6	60.15	5	58.81	4	56.72	4	58.07	3.46 %	1.27 %	2.33 %
2.4	0.6	2	10	2	10	29	6	60.15	5	59.12	4	57.50	5	58.69	2.43 %	0.74 %	2.02 %
2.4	0.6	2	10	1	20	29	6	60.15	6	59.60	5	58.07	5	58.92	2.04 %	1.13 %	1.45 %
2.4	0.6	2	10	2	20	29	6	60.15	5	59.92	5	58.69	5	59.32	1.38 %	1.01 %	1.06 %
2.4	0.6	2	10	1	30	29	6	60.15	5	59.91	5	58.91	5	59.45	1.15 %	0.76 %	0.91 %
2.4	0.6	2	10	2	30	29	6	60.15	6	60.02	5	59.32	6	59.73	0.69 %	0.48 %	0.68 %
2.4	0.6	2	10	1	40	29	6	60.15	6	60.22	5	59.51	6	59.91	0.41 %	0.52 %	0.66 %
2.4	0.6	2	10	2	40	29	6	60.15	6	60.04	5	59.58	5	59.96	0.32 %	0.13 %	0.63 %
2.4	0.6	2	10	1	50	29	6	60.15	6	60.18	5	59.79	6	60.04	0.19 %	0.24 %	0.42 %
2.4	0.6	2	10	2	50	29	6	60.15	6	60.14	5	59.83	5	60.09	0.10 %	0.09 %	0.43 %
2.4	0.6	2	20	1	10	29	8	64.47	7	61.98	5	59.27	6	60.87	5.58 %	1.79 %	2.64 %
2.4	0.6	2	20	2	10	29	8	64.47	7	61.85	5	59.62	6	61.18	5.10 %	1.07 %	2.55 %
2.4	0.6	2	20	1	20	29	8	64.47	7	63.01	6	60.48	7	61.97	3.88 %	1.66 %	2.39 %
2.4	0.6	2	20	2	20	29	8	64.47	8	63.24	6	61.04	7	62.21	3.51 %	1.63 %	1.87 %
2.4	0.6	2	20	1	30	29	8	64.47	8	63.78	6	61.73	7	62.69	2.75 %	1.70 %	1.54 %
2.4	0.6	2	20	2	30	29	8	64.47	8	63.96	7	62.16	7	62.97	2.32 %	1.54 %	1.29 %
2.4	0.6	2	20	1	40	29	8	64.47	8	63.90	7	62.14	8	63.00	2.27 %	1.41 %	1.37 %
2.4	0.6	2	20	2	40	29	8	64.47	8	64.20	6	62.66	8	63.55	1.42 %	1.01 %	1.40 %
2.4	0.6	2	20	1	50	29	8	64.47	8	64.10	7	62.49	7	63.39	1.68 %	1.11 %	1.41 %
2.4	0.6	2	20	2	50	29	8	64.47	8	64.13	7	63.06	8	63.63	1.31 %	0.79 %	0.89 %
2.4	0.6	2	30	1	10	29	10	66.89	8	63.14	6	60.35	7	62.14	7.10 %	1.58 %	2.87 %
2.4	0.6	2	30	2	10	29	10	66.89	8	63.20	7	60.77	7	62.65	6.34 %	0.86 %	3.00 %
2.4	0.6	2	30	1	20	29	10	66.89	8	64.34	7	61.69	7	62.99	5.83 %	2.09 %	2.06 %
2.4	0.6	2	30	2	20	29	10	66.89	8	64.78	7	62.23	8	63.46	5.13 %	2.04 %	1.94 %
2.4	0.6	2	30	1	30	29	10	66.89	9	65.41	8	62.96	8	64.05	4.25 %	2.09 %	1.69 %
2.4	0.6	2	30	2	30	29	10	66.89	9	65.56	8	63.35	8	64.45	3.65 %	1.70 %	1.70 %
2.4	0.6	2	30	1	40	29	10	66.89	9	65.80	7	63.54	8	64.64	3.37 %	1.77 %	1.70 %
2.4	0.6	2	30	2	40	29	10	66.89	9	65.78	8	63.73	9	64.77	3.16 %	1.53 %	1.61 %
2.4	0.6	2	30	1	50	29	10	66.89	9	66.30	8	64.19	8	65.00	2.82 %	1.96 %	1.25 %
2.4	0.6	2	30	2	50	29	10	66.89	9	66.27	8	64.52	9	65.35	2.30 %	1.39 %	1.27 %
2.4	0.6	2	40	1	10	29	11	68.56	8	63.88	7	61.21	8	63.13	7.92 %	1.16 %	3.05 %
2.4	0.6	2	40	2	10	29	11	68.56	8	63.70	7	61.48	8	63.10	7.97 %	0.95 %	2.57 %
2.4	0.6	2	40	1	20	29	11	68.56	9	65.35	7	62.63	8	63.94	6.74 %	2.15 %	2.06 %
2.4	0.6	2	40	2	20	29	11	68.56	9	65.60	7	63.00	8	64.38	6.11 %	1.86 %	2.13 %
2.4	0.6	2	40	1	30	29	11	68.56	9	66.49	8	63.69	9	64.98	5.22 %	2.26 %	2.00 %
2.4	0.6	2	40	2	30	29	11	68.56	10	66.56	8	64.24	9	65.28	4.79 %	1.92 %	1.60 %
2.4	0.6	2	40	1	40	29	11	68.56	10	67.16	9	64.45	9	65.66	4.23 %	2.23 %	1.84 %
2.4	0.6	2	40	2	40	29	11	68.56	9	67.20	8	64.98	9	65.90	3.88 %	1.93 %	1.40 %
2.4	0.6	2	40	1	50	29	11	68.56	10	67.48	9	65.06	9	66.04	3.68 %	2.13 %	1.50 %
2.4	0.6	2	40	2	50	29	11	68.56	10	67.34	9	65.14	9	65.94	3.83 %	2.08 %	1.21 %
2.4	0.6	2	50	1	10	29	11	69.78	9	64.53	8	61.69	8	63.74	8.66 %	1.23 %	3.20 %
2.4	0.6	2	50	2	10	29	11	69.78	9	64.88	8	62.52	9	64.09	8.16 %	1.22 %	2.44 %
2.4	0.6	2	50	1	20	29	11	69.78	9	65.75	7	63.05	8	64.66	7.33 %	1.66 %	2.49 %
2.4	0.6	2	50	2	20	29	11	69.78	9	66.12	8	63.53	9	65.03	6.81 %	1.66 %	2.30 %
2.4	0.6	2	50	1	30	29	11	69.78	10	67.16	8	63.97	9	65.40	6.27 %	2.61 %	2.19 %
2.4	0.6	2	50	2	30	29	11	69.78	10	67.25	9	64.72	9	65.88	5.59 %	2.04 %	1.76 %
2.4	0.6	2	50	1	40	29	11	69.78	10	67.87	9	64.96	9	66.04	5.35 %	2.69 %	1.65 %
2.4	0.6	2	50	2	40	29	11	69.78	10	68.04	9	65.57	10	66.88	4.16 %	1.70 %	1.95 %
2.4	0.6	2	50	1	50	29	11	69.78	10	68.10	9	65.42	9	66.48	4.73 %	2.39 %	1.59 %
2.4	0.6	2	50	2	50	29	11	69.78	11	68.44	9	65.79	9	66.72	4.38 %	2.51 %	1.39 %

Table 19: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
2.4	0.6	3	10	1	10	29	10	62.53	9	60.61	8	56.68	8	59.26	5.23 %	2.22 %	4.35 %
2.4	0.6	3	10	2	10	29	10	62.53	9	60.60	8	57.68	8	59.63	4.64 %	1.61 %	3.26 %
2.4	0.6	3	10	1	20	29	10	62.53	10	61.71	8	58.51	9	60.23	3.67 %	2.40 %	2.86 %
2.4	0.6	3	10	2	20	29	10	62.53	10	61.71	9	59.18	9	60.66	2.99 %	1.70 %	2.44 %
2.4	0.6	3	10	1	30	29	10	62.53	10	62.04	9	59.67	10	61.13	2.23 %	1.47 %	2.39 %
2.4	0.6	3	10	2	30	29	10	62.53	10	61.91	10	60.13	10	61.18	2.16 %	1.19 %	1.71 %
2.4	0.6	3	10	1	40	29	10	62.53	10	62.03	9	60.24	10	61.37	1.84 %	1.05 %	1.84 %
2.4	0.6	3	10	2	40	29	10	62.53	10	62.04	10	60.78	10	61.65	1.40 %	0.63 %	1.41 %
2.4	0.6	3	10	1	50	29	10	62.53	10	62.39	9	60.73	10	61.83	1.12 %	0.91 %	1.77 %
2.4	0.6	3	10	2	50	29	10	62.53	10	62.51	9	61.40	10	61.99	0.85 %	0.82 %	0.96 %
2.4	0.6	3	20	1	10	29	13	67.45	11	63.36	9	59.22	10	62.03	8.04 %	2.10 %	4.53 %
2.4	0.6	3	20	2	10	29	13	67.45	12	64.03	10	60.27	11	62.79	6.91 %	1.93 %	4.02 %
2.4	0.6	3	20	1	20	29	13	67.45	12	64.99	11	61.07	11	63.14	6.39 %	2.84 %	3.29 %
2.4	0.6	3	20	2	20	29	13	67.45	12	65.26	11	61.77	11	63.77	5.46 %	2.28 %	3.14 %
2.4	0.6	3	20	1	30	29	13	67.45	12	65.91	10	62.34	11	64.19	4.83 %	2.61 %	2.88 %
2.4	0.6	3	20	2	30	29	13	67.45	12	65.76	10	62.61	12	64.32	4.64 %	2.19 %	2.66 %
2.4	0.6	3	20	1	40	29	13	67.45	13	66.63	11	63.25	11	64.96	3.69 %	2.50 %	2.64 %
2.4	0.6	3	20	2	40	29	13	67.45	13	66.71	12	63.88	12	65.19	3.35 %	2.28 %	2.01 %
2.4	0.6	3	20	1	50	29	13	67.45	13	66.62	12	64.04	12	65.43	3.00 %	1.79 %	2.12 %
2.4	0.6	3	20	2	50	29	13	67.45	13	67.03	12	64.53	12	65.81	2.44 %	1.83 %	1.94 %
2.4	0.6	3	30	1	10	29	15	70.18	12	64.98	11	61.10	12	63.61	9.36 %	2.11 %	3.96 %
2.4	0.6	3	30	2	10	29	15	70.18	13	65.60	11	61.81	12	64.21	8.51 %	2.12 %	3.73 %
2.4	0.6	3	30	1	20	29	15	70.18	13	66.39	11	62.13	12	64.56	8.01 %	2.76 %	3.76 %
2.4	0.6	3	30	2	20	29	15	70.18	13	66.88	11	63.54	13	65.32	6.92 %	2.32 %	2.72 %
2.4	0.6	3	30	1	30	29	15	70.18	14	67.29	12	63.42	12	65.52	6.64 %	2.63 %	3.20 %
2.4	0.6	3	30	2	30	29	15	70.18	14	67.94	12	64.16	13	66.21	5.65 %	2.54 %	3.10 %
2.4	0.6	3	30	1	40	29	15	70.18	14	68.60	12	64.79	12	66.50	5.24 %	3.06 %	2.58 %
2.4	0.6	3	30	2	40	29	15	70.18	14	68.52	12	65.13	13	66.67	4.99 %	2.70 %	2.32 %
2.4	0.6	3	30	1	50	29	15	70.18	14	68.85	12	65.31	13	67.07	4.44 %	2.59 %	2.62 %
2.4	0.6	3	30	2	50	29	15	70.18	14	68.89	12	66.09	14	67.15	4.32 %	2.53 %	1.58 %
2.4	0.6	3	40	1	10	29	16	72.06	13	65.98	11	61.94	13	64.65	10.29 %	2.01 %	4.19 %
2.4	0.6	3	40	2	10	29	16	72.06	13	66.47	12	62.70	13	65.39	9.26 %	1.62 %	4.11 %
2.4	0.6	3	40	1	20	29	16	72.06	14	67.65	12	63.71	13	65.88	8.57 %	2.61 %	3.30 %
2.4	0.6	3	40	2	20	29	16	72.06	13	67.81	12	64.33	13	66.36	7.91 %	2.13 %	3.07 %
2.4	0.6	3	40	1	30	29	16	72.06	14	69.31	12	65.06	13	67.03	6.99 %	3.29 %	2.93 %
2.4	0.6	3	40	2	30	29	16	72.06	15	68.98	12	64.99	13	66.63	7.54 %	3.41 %	2.45 %
2.4	0.6	3	40	1	40	29	16	72.06	14	69.62	13	65.47	13	67.14	6.84 %	3.56 %	2.48 %
2.4	0.6	3	40	2	40	29	16	72.06	14	69.77	13	66.24	14	67.70	6.06 %	2.98 %	2.15 %
2.4	0.6	3	40	1	50	29	16	72.06	15	70.36	13	66.63	14	68.28	5.25 %	2.95 %	2.41 %
2.4	0.6	3	40	2	50	29	16	72.06	15	70.04	13	66.80	14	68.22	5.33 %	2.59 %	2.08 %
2.4	0.6	3	50	1	10	29	17	73.52	13	66.54	11	62.23	13	65.33	11.14 %	1.82 %	4.75 %
2.4	0.6	3	50	2	10	29	17	73.52	14	66.90	12	63.68	13	65.94	10.31 %	1.43 %	3.43 %
2.4	0.6	3	50	1	20	29	17	73.52	14	68.41	12	63.97	13	66.82	9.12 %	2.34 %	4.26 %
2.4	0.6	3	50	2	20	29	17	73.52	15	68.86	13	65.14	14	67.21	8.58 %	2.39 %	3.08 %
2.4	0.6	3	50	1	30	29	17	73.52	15	69.25	12	65.11	13	67.09	8.75 %	3.13 %	2.94 %
2.4	0.6	3	50	2	30	29	17	73.52	14	69.62	12	65.89	14	67.57	8.09 %	2.94 %	2.50 %
2.4	0.6	3	50	1	40	29	17	73.52	15	70.47	13	66.74	14	68.28	7.13 %	3.10 %	2.26 %
2.4	0.6	3	50	2	40	29	17	73.52	16	70.84	13	66.52	14	68.47	6.87 %	3.35 %	2.84 %
2.4	0.6	3	50	1	50	29	17	73.52	16	71.36	14	67.49	15	69.08	6.04 %	3.20 %	2.30 %
2.4	0.6	3	50	2	50	29	17	73.52	15	70.94	14	67.42	15	68.76	6.48 %	3.07 %	1.95 %

Table 20: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
4	0.8	2	10	1	10	32	7	64.94	7	64.46	5	62.36	5	63.34	2.46 %	1.73 %	1.55 %
4	0.8	2	10	2	10	32	7	64.94	7	64.58	6	63.06	6	63.83	1.71 %	1.15 %	1.20 %
4	0.8	2	10	1	20	32	7	64.94	7	64.75	6	63.60	7	64.17	1.19 %	0.90 %	0.88 %
4	0.8	2	10	2	20	32	7	64.94	7	64.92	6	64.18	7	64.56	0.59 %	0.56 %	0.60 %
4	0.8	2	10	1	30	32	7	64.94	7	64.87	7	64.34	7	64.61	0.50 %	0.39 %	0.42 %
4	0.8	2	10	2	30	32	7	64.94	7	65.00	7	64.75	7	64.86	0.12 %	0.21 %	0.17 %
4	0.8	2	10	1	40	32	7	64.94	7	64.98	7	64.68	7	64.82	0.19 %	0.25 %	0.21 %
4	0.8	2	10	2	40	32	7	64.94	7	65.02	7	64.87	7	64.89	0.08 %	0.20 %	0.02 %
4	0.8	2	10	1	50	32	7	64.94	7	64.96	7	64.76	7	64.88	0.10 %	0.12 %	0.19 %
4	0.8	2	10	2	50	32	7	64.94	7	64.95	7	64.92	7	64.87	0.11 %	0.13 %	-0.09 %
4	0.8	2	20	1	10	32	10	68.81	9	67.51	7	64.52	7	66.04	4.03 %	2.17 %	2.31 %
4	0.8	2	20	2	10	32	10	68.81	9	67.63	7	65.23	8	66.42	3.48 %	1.80 %	1.79 %
4	0.8	2	20	1	20	32	10	68.81	9	68.51	7	65.87	9	67.13	2.44 %	2.02 %	1.88 %
4	0.8	2	20	2	20	32	10	68.81	10	68.46	8	66.35	9	67.44	2.00 %	1.50 %	1.61 %
4	0.8	2	20	1	30	32	10	68.81	10	68.57	8	66.83	9	67.62	1.73 %	1.38 %	1.17 %
4	0.8	2	20	2	30	32	10	68.81	10	68.63	9	67.28	9	67.98	1.21 %	0.95 %	1.03 %
4	0.8	2	20	1	40	32	10	68.81	10	68.58	8	67.38	9	67.95	1.26 %	0.92 %	0.83 %
4	0.8	2	20	2	40	32	10	68.81	10	68.69	9	67.79	9	68.18	0.92 %	0.74 %	0.58 %
4	0.8	2	20	1	50	32	10	68.81	10	68.60	9	67.65	9	68.14	0.97 %	0.66 %	0.72 %
4	0.8	2	20	2	50	32	10	68.81	10	68.64	9	67.94	9	68.22	0.87 %	0.61 %	0.40 %
4	0.8	2	30	1	10	32	11	70.82	9	68.64	7	65.26	8	67.00	5.40 %	2.38 %	2.59 %
4	0.8	2	30	2	10	32	11	70.82	10	68.84	8	65.99	9	67.63	4.51 %	1.76 %	2.43 %
4	0.8	2	30	1	20	32	11	70.82	10	69.96	9	67.03	9	68.19	3.72 %	2.53 %	1.71 %
4	0.8	2	30	2	20	32	11	70.82	10	69.83	8	67.50	9	68.67	3.04 %	1.67 %	1.69 %
4	0.8	2	30	1	30	32	11	70.82	11	70.46	9	68.21	10	69.11	2.42 %	1.92 %	1.31 %
4	0.8	2	30	2	30	32	11	70.82	10	70.43	9	68.44	9	69.29	2.16 %	1.61 %	1.24 %
4	0.8	2	30	1	40	32	11	70.82	11	70.32	9	68.59	10	69.34	2.10 %	1.40 %	1.08 %
4	0.8	2	30	2	40	32	11	70.82	10	70.49	9	69.01	10	69.69	1.60 %	1.13 %	0.98 %
4	0.8	2	30	1	50	32	11	70.82	11	70.84	10	69.41	10	69.96	1.21 %	1.23 %	0.80 %
4	0.8	2	30	2	50	32	11	70.82	11	70.54	10	69.54	10	70.10	1.02 %	0.63 %	0.80 %
4	0.8	2	40	1	10	32	12	72.27	10	69.40	7	66.13	9	67.95	5.98 %	2.08 %	2.68 %
4	0.8	2	40	2	10	32	12	72.27	10	69.68	9	66.72	9	68.45	5.29 %	1.77 %	2.53 %
4	0.8	2	40	1	20	32	12	72.27	11	70.70	8	67.60	10	69.02	4.49 %	2.37 %	2.06 %
4	0.8	2	40	2	20	32	12	72.27	11	70.89	9	68.23	10	69.45	3.90 %	2.03 %	1.77 %
4	0.8	2	40	1	30	32	12	72.27	11	71.48	9	68.96	10	69.94	3.22 %	2.15 %	1.41 %
4	0.8	2	40	2	30	32	12	72.27	11	71.57	10	69.40	10	70.22	2.84 %	1.89 %	1.16 %
4	0.8	2	40	1	40	32	12	72.27	11	71.62	10	69.37	10	70.25	2.80 %	1.91 %	1.26 %
4	0.8	2	40	2	40	32	12	72.27	11	71.67	10	69.83	10	70.58	2.34 %	1.52 %	1.06 %
4	0.8	2	40	1	50	32	12	72.27	11	71.96	10	70.13	11	70.72	2.14 %	1.72 %	0.84 %
4	0.8	2	40	2	50	32	12	72.27	12	71.84	10	70.54	10	70.99	1.78 %	1.19 %	0.64 %
4	0.8	2	50	1	10	32	12	73.25	10	69.81	8	66.46	9	68.67	6.26 %	1.64 %	3.22 %
4	0.8	2	50	2	10	32	12	73.25	10	70.16	9	67.28	9	69.04	5.75 %	1.60 %	2.54 %
4	0.8	2	50	1	20	32	12	73.25	11	71.47	9	68.19	10	69.67	4.89 %	2.52 %	2.12 %
4	0.8	2	50	2	20	32	12	73.25	11	71.58	9	68.85	10	70.11	4.29 %	2.05 %	1.80 %
4	0.8	2	50	1	30	32	12	73.25	11	72.39	9	69.45	11	70.51	3.74 %	2.60 %	1.50 %
4	0.8	2	50	2	30	32	12	73.25	11	72.24	10	70.01	11	70.76	3.41 %	2.05 %	1.05 %
4	0.8	2	50	1	40	32	12	73.25	12	72.65	11	70.27	11	71.03	3.03 %	2.22 %	1.08 %
4	0.8	2	50	2	40	32	12	73.25	12	72.52	10	70.51	11	71.34	2.61 %	1.63 %	1.16 %
4	0.8	2	50	1	50	32	12	73.25	12	72.78	10	70.84	11	71.57	2.30 %	1.66 %	1.02 %
4	0.8	2	50	2	50	32	12	73.25	12	72.98	11	71.15	11	71.96	1.77 %	1.40 %	1.12 %

Table 21: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
4	0.8	3	10	1	10	32	13	66.83	12	65.75	8	60.86	10	64.02	4.20 %	2.63 %	4.94 %
4	0.8	3	10	2	10	32	13	66.83	12	66.22	9	62.87	11	64.77	3.08 %	2.19 %	2.93 %
4	0.8	3	10	1	20	32	13	66.83	12	66.50	10	63.36	11	65.22	2.40 %	1.92 %	2.85 %
4	0.8	3	10	2	20	32	13	66.83	13	66.64	11	64.31	11	65.62	1.81 %	1.53 %	1.99 %
4	0.8	3	10	1	30	32	13	66.83	13	66.73	11	64.81	12	66.06	1.15 %	1.01 %	1.89 %
4	0.8	3	10	2	30	32	13	66.83	13	66.44	11	65.19	12	65.95	1.30 %	0.73 %	1.17 %
4	0.8	3	10	1	40	32	13	66.83	13	66.96	12	65.62	12	66.44	0.58 %	0.79 %	1.22 %
4	0.8	3	10	2	40	32	13	66.83	13	66.60	12	65.73	13	66.29	0.81 %	0.47 %	0.84 %
4	0.8	3	10	1	50	32	13	66.83	13	66.82	12	65.98	12	66.45	0.57 %	0.55 %	0.70 %
4	0.8	3	10	2	50	32	13	66.83	13	66.72	12	66.18	13	66.58	0.36 %	0.21 %	0.60 %
4	0.8	3	20	1	10	32	15	71.19	14	68.89	10	62.46	12	66.77	6.21 %	3.08 %	6.46 %
4	0.8	3	20	2	10	32	15	71.19	14	69.28	11	64.76	13	67.51	5.16 %	2.55 %	4.08 %
4	0.8	3	20	1	20	32	15	71.19	15	70.29	12	65.76	14	68.32	4.04 %	2.81 %	3.74 %
4	0.8	3	20	2	20	32	15	71.19	15	70.09	12	66.49	14	68.55	3.71 %	2.20 %	3.00 %
4	0.8	3	20	1	30	32	15	71.19	15	70.64	12	67.28	14	68.83	3.31 %	2.55 %	2.26 %
4	0.8	3	20	2	30	32	15	71.19	15	70.74	13	67.99	14	69.26	2.71 %	2.09 %	1.84 %
4	0.8	3	20	1	40	32	15	71.19	15	70.83	13	68.28	14	69.40	2.52 %	2.03 %	1.61 %
4	0.8	3	20	2	40	32	15	71.19	15	70.88	14	68.83	15	69.78	1.98 %	1.56 %	1.36 %
4	0.8	3	20	1	50	32	15	71.19	15	71.03	14	68.96	14	69.90	1.81 %	1.58 %	1.35 %
4	0.8	3	20	2	50	32	15	71.19	15	70.87	14	69.27	14	70.06	1.59 %	1.14 %	1.13 %
4	0.8	3	30	1	10	32	17	73.56	15	70.43	10	63.80	14	68.38	7.04 %	2.90 %	6.70 %
4	0.8	3	30	2	10	32	17	73.56	15	70.77	11	65.48	13	69.12	6.03 %	2.32 %	5.26 %
4	0.8	3	30	1	20	32	17	73.56	16	71.98	12	66.85	14	69.59	5.40 %	3.32 %	3.94 %
4	0.8	3	30	2	20	32	17	73.56	16	72.03	13	67.87	15	70.19	4.59 %	2.57 %	3.30 %
4	0.8	3	30	1	30	32	17	73.56	16	72.80	13	68.49	15	70.52	4.13 %	3.13 %	2.88 %
4	0.8	3	30	2	30	32	17	73.56	16	72.67	14	69.18	15	70.79	3.77 %	2.59 %	2.28 %
4	0.8	3	30	1	40	32	17	73.56	16	72.76	15	69.63	15	70.85	3.68 %	2.62 %	1.72 %
4	0.8	3	30	2	40	32	17	73.56	17	73.05	15	70.29	16	71.55	2.73 %	2.05 %	1.76 %
4	0.8	3	30	1	50	32	17	73.56	16	73.05	15	70.35	16	71.64	2.61 %	1.92 %	1.80 %
4	0.8	3	30	2	50	32	17	73.56	17	73.32	16	71.06	16	71.99	2.13 %	1.80 %	1.30 %
4	0.8	3	40	1	10	32	18	75.19	16	71.18	11	64.27	15	69.40	7.70 %	2.50 %	7.40 %
4	0.8	3	40	2	10	32	18	75.19	16	71.50	12	65.93	14	69.86	7.09 %	2.30 %	5.62 %
4	0.8	3	40	1	20	32	18	75.19	16	73.05	12	67.46	15	70.61	6.09 %	3.34 %	4.45 %
4	0.8	3	40	2	20	32	18	75.19	17	73.59	13	68.77	15	71.24	5.25 %	3.20 %	3.47 %
4	0.8	3	40	1	30	32	18	75.19	17	73.80	14	69.36	16	71.34	5.11 %	3.34 %	2.77 %
4	0.8	3	40	2	30	32	18	75.19	17	74.13	15	70.11	15	71.79	4.52 %	3.16 %	2.34 %
4	0.8	3	40	1	40	32	18	75.19	17	74.19	15	70.50	16	71.93	4.33 %	3.05 %	1.99 %
4	0.8	3	40	2	40	32	18	75.19	17	74.47	15	71.26	16	72.63	3.41 %	2.48 %	1.88 %
4	0.8	3	40	1	50	32	18	75.19	17	74.54	15	71.12	16	72.54	3.53 %	2.70 %	1.95 %
4	0.8	3	40	2	50	32	18	75.19	17	74.69	16	71.75	16	72.70	3.31 %	2.67 %	1.31 %
4	0.8	3	50	1	10	32	18	76.39	16	71.84	11	64.95	15	70.11	8.22 %	2.41 %	7.36 %
4	0.8	3	50	2	10	32	18	76.39	17	72.24	12	66.84	15	70.76	7.38 %	2.06 %	5.54 %
4	0.8	3	50	1	20	32	18	76.39	17	73.86	13	68.14	16	71.24	6.74 %	3.55 %	4.36 %
4	0.8	3	50	2	20	32	18	76.39	17	73.49	14	68.89	15	71.59	6.29 %	2.59 %	3.77 %
4	0.8	3	50	1	30	32	18	76.39	17	74.39	15	69.73	16	71.85	5.94 %	3.41 %	2.95 %
4	0.8	3	50	2	30	32	18	76.39	17	74.84	15	70.67	16	72.59	4.97 %	3.01 %	2.65 %
4	0.8	3	50	1	40	32	18	76.39	18	75.09	15	71.25	16	72.77	4.74 %	3.09 %	2.09 %
4	0.8	3	50	2	40	32	18	76.39	18	75.48	15	71.82	17	73.28	4.08 %	2.92 %	1.98 %
4	0.8	3	50	1	50	32	18	76.39	18	75.64	16	71.93	17	73.50	3.78 %	2.83 %	2.14 %
4	0.8	3	50	2	50	32	18	76.39	18	75.61	16	72.48	17	73.76	3.44 %	2.45 %	1.74 %

Table 22: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
4	0.6	2	10	1	10	37	11	77.59	10	76.26	9	72.90	9	74.79	3.61 %	1.94 %	2.53 %
4	0.6	2	10	2	10	37	11	77.59	10	76.64	9	74.16	9	75.66	2.49 %	1.28 %	1.98 %
4	0.6	2	10	1	20	37	11	77.59	11	77.20	9	74.50	9	76.06	1.97 %	1.48 %	2.05 %
4	0.6	2	10	2	20	37	11	77.59	10	77.07	9	75.37	10	76.43	1.49 %	0.83 %	1.39 %
4	0.6	2	10	1	30	37	11	77.59	11	77.49	9	75.88	10	76.97	0.80 %	0.68 %	1.41 %
4	0.6	2	10	2	30	37	11	77.59	11	77.30	10	76.19	11	76.95	0.82 %	0.45 %	0.99 %
4	0.6	2	10	1	40	37	11	77.59	11	77.46	10	76.43	10	77.01	0.75 %	0.59 %	0.75 %
4	0.6	2	10	2	40	37	11	77.59	11	77.45	10	76.76	10	77.23	0.46 %	0.28 %	0.61 %
4	0.6	2	10	1	50	37	11	77.59	11	77.56	10	76.97	10	77.41	0.23 %	0.20 %	0.57 %
4	0.6	2	10	2	50	37	11	77.59	11	77.62	10	77.18	10	77.48	0.15 %	0.19 %	0.38 %
4	0.6	2	20	1	10	37	14	82.92	12	79.82	10	75.63	11	77.95	6.00 %	2.34 %	2.97 %
4	0.6	2	20	2	10	37	14	82.92	12	80.42	11	77.11	12	79.03	4.69 %	1.73 %	2.44 %
4	0.6	2	20	1	20	37	14	82.92	13	81.53	12	77.82	12	79.60	4.01 %	2.36 %	2.24 %
4	0.6	2	20	2	20	37	14	82.92	13	81.68	11	78.44	12	80.19	3.30 %	1.83 %	2.17 %
4	0.6	2	20	1	30	37	14	82.92	14	82.21	11	79.13	12	80.50	2.92 %	2.08 %	1.71 %
4	0.6	2	20	2	30	37	14	82.92	13	82.19	12	79.63	12	80.91	2.43 %	1.56 %	1.58 %
4	0.6	2	20	1	40	37	14	82.92	13	82.39	12	79.69	13	80.99	2.33 %	1.70 %	1.60 %
4	0.6	2	20	2	40	37	14	82.92	14	82.75	12	80.41	13	81.40	1.83 %	1.63 %	1.22 %
4	0.6	2	20	1	50	37	14	82.92	13	82.53	12	80.41	13	81.50	1.72 %	1.26 %	1.33 %
4	0.6	2	20	2	50	37	14	82.92	14	82.50	12	80.82	13	81.75	1.41 %	0.90 %	1.14 %
4	0.6	2	30	1	10	37	16	85.91	13	81.73	11	77.13	12	79.73	7.19 %	2.44 %	3.26 %
4	0.6	2	30	2	10	37	16	85.91	13	82.13	12	78.41	12	80.63	6.15 %	1.82 %	2.75 %
4	0.6	2	30	1	20	37	16	85.91	14	83.61	12	79.27	13	81.42	5.22 %	2.61 %	2.64 %
4	0.6	2	30	2	20	37	16	85.91	14	83.11	11	79.89	14	81.64	4.96 %	1.77 %	2.15 %
4	0.6	2	30	1	30	37	16	85.91	15	84.47	13	80.87	14	82.52	3.94 %	2.31 %	1.99 %
4	0.6	2	30	2	30	37	16	85.91	15	84.67	12	81.25	13	82.81	3.60 %	2.19 %	1.89 %
4	0.6	2	30	1	40	37	16	85.91	15	85.25	13	81.71	14	83.18	3.17 %	2.42 %	1.77 %
4	0.6	2	30	2	40	37	16	85.91	15	84.60	13	82.22	14	83.22	3.13 %	1.64 %	1.19 %
4	0.6	2	30	1	50	37	16	85.91	16	85.20	13	82.19	14	83.42	2.89 %	2.08 %	1.47 %
4	0.6	2	30	2	50	37	16	85.91	15	85.15	14	82.94	14	83.88	2.36 %	1.50 %	1.11 %
4	0.6	2	40	1	10	37	17	87.90	14	82.63	11	78.11	13	80.98	7.87 %	2.00 %	3.54 %
4	0.6	2	40	2	10	37	17	87.90	14	82.89	12	79.30	13	81.58	7.19 %	1.58 %	2.80 %
4	0.6	2	40	1	20	37	17	87.90	15	84.84	12	80.08	14	82.50	6.15 %	2.76 %	2.93 %
4	0.6	2	40	2	20	37	17	87.90	15	85.19	13	81.35	14	83.27	5.27 %	2.25 %	2.31 %
4	0.6	2	40	1	30	37	17	87.90	16	85.58	13	81.60	14	83.23	5.31 %	2.74 %	1.96 %
4	0.6	2	40	2	30	37	17	87.90	16	86.09	14	82.45	15	83.97	4.47 %	2.46 %	1.81 %
4	0.6	2	40	1	40	37	17	87.90	16	86.21	14	82.71	14	84.09	4.33 %	2.45 %	1.65 %
4	0.6	2	40	2	40	37	17	87.90	16	86.72	14	83.57	15	84.82	3.50 %	2.19 %	1.47 %
4	0.6	2	40	1	50	37	17	87.90	16	86.48	13	83.43	14	84.67	3.67 %	2.09 %	1.47 %
4	0.6	2	40	2	50	37	17	87.90	16	86.90	14	83.80	15	85.14	3.14 %	2.03 %	1.57 %
4	0.6	2	50	1	10	37	18	89.45	15	83.54	12	79.02	13	81.91	8.43 %	1.95 %	3.53 %
4	0.6	2	50	2	10	37	18	89.45	15	83.90	13	80.23	15	82.62	7.64 %	1.53 %	2.89 %
4	0.6	2	50	1	20	37	18	89.45	16	85.75	13	81.03	15	83.49	6.67 %	2.64 %	2.95 %
4	0.6	2	50	2	20	37	18	89.45	16	85.77	13	81.95	14	83.74	6.39 %	2.37 %	2.13 %
4	0.6	2	50	1	30	37	18	89.45	16	86.34	13	81.99	14	83.96	6.15 %	2.76 %	2.34 %
4	0.6	2	50	2	30	37	18	89.45	16	86.71	14	82.88	15	84.51	5.53 %	2.54 %	1.93 %
4	0.6	2	50	1	40	37	18	89.45	17	87.75	14	83.63	15	84.96	5.03 %	3.18 %	1.56 %
4	0.6	2	50	2	40	37	18	89.45	17	87.54	15	84.27	16	85.56	4.36 %	2.27 %	1.50 %
4	0.6	2	50	1	50	37	18	89.45	16	88.00	15	84.36	15	85.54	4.38 %	2.80 %	1.38 %
4	0.6	2	50	2	50	37	18	89.45	17	87.85	15	84.80	16	86.09	3.75 %	2.00 %	1.51 %

Table 23: Full Factorial: 2 Location Results

Parameters							Costs								Hybrid Performance		
λ	p	L	b	t_u	t_f	Q	R	No Tran	R	Reactive	R	Opt	R	Hybrid	vs No Tran	vs Reactive	to Opt
4	0.6	3	10	1	10	37	18	80.57	17	78.37	11	70.43	14	75.91	5.79 %	3.14 %	7.22 %
4	0.6	3	10	2	10	37	18	80.57	17	78.85	14	73.33	16	77.07	4.34 %	2.26 %	4.86 %
4	0.6	3	10	1	20	37	18	80.57	18	79.38	13	74.02	16	77.39	3.95 %	2.50 %	4.36 %
4	0.6	3	10	2	20	37	18	80.57	18	79.95	14	75.78	17	78.40	2.70 %	1.94 %	3.34 %
4	0.6	3	10	1	30	37	18	80.57	19	79.94	15	75.74	17	78.34	2.77 %	2.00 %	3.32 %
4	0.6	3	10	2	30	37	18	80.57	18	80.03	17	77.05	17	78.82	2.18 %	1.52 %	2.24 %
4	0.6	3	10	1	40	37	18	80.57	18	80.24	16	77.03	17	79.06	1.88 %	1.47 %	2.57 %
4	0.6	3	10	2	40	37	18	80.57	18	79.84	15	77.79	18	79.24	1.65 %	0.75 %	1.83 %
4	0.6	3	10	1	50	37	18	80.57	18	80.04	17	77.67	18	79.33	1.54 %	0.89 %	2.09 %
4	0.6	3	10	2	50	37	18	80.57	18	80.27	17	78.66	17	79.95	0.77 %	0.40 %	1.61 %
4	0.6	3	20	1	10	37	22	86.71	20	82.59	14	73.83	19	80.04	7.69 %	3.10 %	7.76 %
4	0.6	3	20	2	10	37	22	86.71	20	82.85	16	75.70	19	80.81	6.80 %	2.46 %	6.32 %
4	0.6	3	20	1	20	37	22	86.71	20	84.08	16	76.67	19	81.08	6.48 %	3.56 %	5.44 %
4	0.6	3	20	2	20	37	22	86.71	21	84.60	18	78.78	19	82.41	4.95 %	2.59 %	4.41 %
4	0.6	3	20	1	30	37	22	86.71	21	85.11	16	79.00	19	82.52	4.82 %	3.04 %	4.27 %
4	0.6	3	20	2	30	37	22	86.71	22	85.60	17	80.79	20	83.31	3.92 %	2.68 %	3.02 %
4	0.6	3	20	1	40	37	22	86.71	22	85.82	18	80.52	20	83.29	3.94 %	2.95 %	3.32 %
4	0.6	3	20	2	40	37	22	86.71	22	86.12	18	81.55	21	84.10	3.00 %	2.34 %	3.04 %
4	0.6	3	20	1	50	37	22	86.71	22	85.74	18	81.60	20	83.96	3.17 %	2.07 %	2.81 %
4	0.6	3	20	2	50	37	22	86.71	22	86.01	19	82.45	21	84.48	2.56 %	1.77 %	2.40 %
4	0.6	3	30	1	10	37	24	90.09	21	84.81	15	75.33	20	82.08	8.89 %	3.22 %	8.22 %
4	0.6	3	30	2	10	37	24	90.09	22	85.33	16	77.64	21	83.10	7.76 %	2.61 %	6.57 %
4	0.6	3	30	1	20	37	24	90.09	22	86.27	17	78.24	20	83.38	7.45 %	3.35 %	6.17 %
4	0.6	3	30	2	20	37	24	90.09	22	86.86	18	80.39	21	84.12	6.62 %	3.15 %	4.44 %
4	0.6	3	30	1	30	37	24	90.09	23	87.53	19	80.60	20	84.28	6.45 %	3.71 %	4.37 %
4	0.6	3	30	2	30	37	24	90.09	23	87.57	19	81.87	22	85.27	5.36 %	2.63 %	3.98 %
4	0.6	3	30	1	40	37	24	90.09	23	88.12	19	81.70	21	85.12	5.52 %	3.40 %	4.02 %
4	0.6	3	30	2	40	37	24	90.09	22	88.57	19	83.51	22	86.12	4.41 %	2.76 %	3.03 %
4	0.6	3	30	1	50	37	24	90.09	23	88.59	20	83.46	21	86.09	4.44 %	2.82 %	3.06 %
4	0.6	3	30	2	50	37	24	90.09	23	88.96	20	84.28	22	86.57	3.91 %	2.69 %	2.65 %
4	0.6	3	40	1	10	37	25	92.40	22	85.74	16	75.86	20	83.18	9.98 %	2.99 %	8.80 %
4	0.6	3	40	2	10	37	25	92.40	22	86.33	19	78.86	21	84.26	8.81 %	2.40 %	6.41 %
4	0.6	3	40	1	20	37	25	92.40	23	87.87	19	80.13	21	84.98	8.04 %	3.29 %	5.70 %
4	0.6	3	40	2	20	37	25	92.40	23	87.98	20	81.66	22	85.47	7.51 %	2.85 %	4.46 %
4	0.6	3	40	1	30	37	25	92.40	23	89.24	19	81.80	22	85.78	7.17 %	3.88 %	4.64 %
4	0.6	3	40	2	30	37	25	92.40	24	89.32	20	82.98	22	86.29	6.62 %	3.39 %	3.83 %
4	0.6	3	40	1	40	37	25	92.40	23	90.13	20	83.31	22	86.60	6.28 %	3.91 %	3.80 %
4	0.6	3	40	2	40	37	25	92.40	24	90.13	21	84.12	22	87.05	5.79 %	3.41 %	3.36 %
4	0.6	3	40	1	50	37	25	92.40	25	90.77	20	84.46	22	87.42	5.39 %	3.69 %	3.39 %
4	0.6	3	40	2	50	37	25	92.40	25	90.89	22	85.66	23	88.13	4.62 %	3.03 %	2.81 %
4	0.6	3	50	1	10	37	26	94.15	23	87.12	18	77.22	22	84.83	9.90 %	2.63 %	8.97 %
4	0.6	3	50	2	10	37	26	94.15	23	87.28	17	80.01	21	85.31	9.39 %	2.26 %	6.22 %
4	0.6	3	50	1	20	37	26	94.15	24	88.82	19	80.63	23	85.97	8.69 %	3.21 %	6.21 %
4	0.6	3	50	2	20	37	26	94.15	24	89.15	20	82.86	23	86.69	7.93 %	2.76 %	4.41 %
4	0.6	3	50	1	30	37	26	94.15	24	90.18	20	82.94	22	87.14	7.45 %	3.37 %	4.82 %
4	0.6	3	50	2	30	37	26	94.15	24	90.40	21	83.87	22	87.13	7.46 %	3.61 %	3.75 %
4	0.6	3	50	1	40	37	26	94.15	25	91.22	21	84.26	22	87.60	6.97 %	3.97 %	3.81 %
4	0.6	3	50	2	40	37	26	94.15	24	91.04	21	85.05	23	87.90	6.65 %	3.45 %	3.24 %
4	0.6	3	50	1	50	37	26	94.15	25	91.90	22	85.65	23	88.64	5.85 %	3.55 %	3.38 %
4	0.6	3	50	2	50	37	26	94.15	25	92.03	20	86.74	23	89.13	5.33 %	3.15 %	2.68 %

Table 24: Full Factorial: 2 Location Results