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## Final State Interactions and $\varepsilon'/\varepsilon$

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I discuss the rôle of strong final state interactions (FSI) in  $K \rightarrow 2\pi$  decays. In this case strong FSI effects can be resummed [1,2] by solving the Omnès problem for  $K \rightarrow 2\pi$  amplitudes. Implications for the CP conserving  $\Delta I = 1/2$  ratio and the direct CP violation parameter  $\varepsilon'/\varepsilon$  are also discussed.

### 1. Introduction

Strong final state interaction (FSI) effects are very important in the phenomenology of K meson decays. In the non-leptonic two-body  $K \rightarrow \pi\pi$  decay the dominant FSI contribution is given by the elastic (soft) rescattering of the two pions in the final state.

At centre-of-mass energies around the kaon mass, the strong S-wave  $\pi\text{-}\pi$  scattering generates a large phase shift difference  $(\delta_0^0 - \delta_0^2)(m_K^2) = 45^\circ \pm 6^\circ$  between the  $I = 0$  and  $I = 2$  partial waves [3]. This effect is taken into account by factoring out those phases in the usual decomposition of  $K \rightarrow \pi\pi$  amplitudes with definite isospin  $I = 0$  and  $I = 2$ :

$$A_I \equiv A[K \rightarrow (\pi\pi)_I] \equiv A_I e^{i\delta_I^0} . \quad (1)$$

The presence of such a large phase shift difference also signals a large dispersive FSI effect in the moduli of the isospin amplitudes, since their imaginary and real parts are related by analyticity and unitarity. Intuitively, the behaviour of the  $I = 0$  and  $I = 2$  S-wave phase shifts as a function of the total energy of the two pions as reported in figure (1) suggests a large enhancement of the  $I = 0$  amplitude and a tiny suppression of the  $I = 2$  amplitude. The numerical estimate of this dispersive FSI effect is a difficult task, since it is dominated by long-distance (soft) contributions and reduces to a non-perturbative problem.

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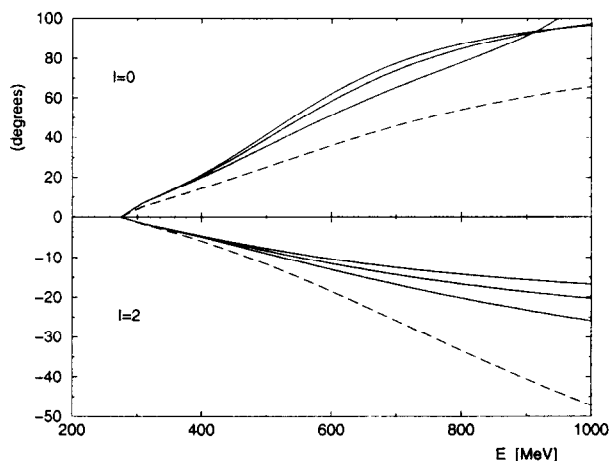


Figure 1. Phase shifts  $\delta_0^{0;2}(s)$  with  $I = 0$  and 2, according to a fit [4] of experimental data and used in the numerical analyses [1,2]. Solid lines enclose the range covered by the experimental data, while dashed lines show the unitarized lowest-order ChPT prediction.

The size of the FSI effect can be roughly estimated at one loop in Chiral Perturbation Theory (ChPT), where the rescattering of the two pions in the final state produces an enhancement of about 40% in the  $A_0$  amplitude [5–7]. However, the fact that the one loop calculation still underestimates the observed  $\delta_0^0$  phase shift indicates that a further enhancement should be produced

by higher orders. It is then necessary to resum FSI effects.

Lattice determinations of  $K \rightarrow \pi\pi$  amplitudes could in principle take into account *automatically* strong FSI effects. However, the direct measurement of  $K \rightarrow \pi\pi$  amplitudes on the lattice is still afflicted by a series of problems. More recently, a possible solution to those problems which overcomes the Maiani–Testa theorem [8] has been proposed [9]. In the meanwhile, most of the attempts up to date are based on the so called *indirect* method, i.e. a two–steps procedure where first the simpler  $K \rightarrow \pi$  matrix element is measured on the lattice and second, the physical  $K \rightarrow 2\pi$  matrix elements are obtained by using a lowest–order ChPT relation between  $K \rightarrow 0$ ,  $K \rightarrow \pi$  and  $K \rightarrow 2\pi$  [10]. Neither the first nor the second step include FSI effects. Recently, the inclusion of one–loop ChPT contributions has been investigated in this context [11].

Approaches based on effective low–energy models [12] or the  $1/N_c$  expansion [13] do include some one–loop corrections and find larger values for the  $A_0$  amplitude. However, the drawback in these cases might be the possible model dependence of the matching procedure with short–distance.

Here, I discuss an approach to FSI effects in  $K \rightarrow \pi\pi$  decays that has been recently proposed [1,2]. It is based on the Omnès solution [14] for  $K \rightarrow \pi\pi$  amplitudes, which permits the resummation of strong FSI effects to all orders in ChPT. Intuitively, what the Omnès solution does is to correct a local weak  $K \rightarrow \pi\pi$  transition with an infinite chain of pion–loop bubbles, incorporating the strong  $\pi\pi \rightarrow \pi\pi$  rescattering to all orders in ChPT.

A few properties of strong FSI effects can be useful in order to understand how they enter the prediction of  $K \rightarrow \pi\pi$  decays:

- the production of the two pions and their subsequent rescattering are two independent processes. The rescattering process only depends on the quantum numbers (total isospin  $I$  and total angular momentum  $J$ ) of the two pions in the final state and on their total energy.
- At centre–of–mass energies of the order of

the  $K$  meson mass the elastic (soft) rescattering of the two pions is the dominant strong FSI effect. Hence, strong FSI effects in  $K \rightarrow \pi\pi$  amplitudes can be treated in a fully non–perturbative way. This implies that in the usual description of weak  $\Delta S = 1$  decays with Operator Product Expansion (OPE), strong FSI effects can be treated without introducing any dependence on the factorization scale which separates short–distance and long–distance contributions.

The last point means that the strong FSI problem in  $K \rightarrow \pi\pi$  decays can be solved independently of the matching problem between short–distance Wilson coefficients and long–distance weak matrix elements. The situation can be different for higher energy processes like rare  $B$  decays such as  $B \rightarrow \pi\pi$ , where, at the  $B$  meson mass, hard rescattering and soft inelastic rescattering contributions are expected to be the dominant strong FSI effects.

In Section 2 the general Omnès problem is formulated, while in Section 3 I review the Omnès solution for the  $K \rightarrow \pi\pi$  amplitudes [2], mainly in the CP conserving sector. A few results for the CP violating amplitudes, relevant for the prediction of  $\epsilon'/\epsilon$  are also discussed. A new Standard Model prediction of  $\epsilon'/\epsilon$  with the inclusion of FSI effects is discussed in [15].

## 2. The Omnès problem

Let us consider a generic amplitude (or form factor)  $A_J^I(s)$ , with two pions in the final state which have total angular momentum and isospin given by  $J$  and  $I$ , respectively, and invariant mass  $s \equiv q^2 \equiv (p_1 + p_2)^2$ . The amplitude is analytic everywhere except for a cut on the real positive  $s$  axis  $L = [4m_\pi^2, \infty)$ .

Below the first inelastic threshold, only the  $2\pi$  intermediate state contributes to the absorptive part of the amplitude and Watson’s theorem [16] implies that the phase of the amplitude is equal to the phase of the  $\pi\pi$  partial–wave scattering amplitude, so that

$$\text{Im } A_J^I = (\text{Im } A_J^I)_{2\pi} = e^{-i\delta_J^I} \sin \delta_J^I A_J^I =$$

$$e^{i\delta_J^I} \sin \delta_J^I A_J^{I*} = \sin \delta_J^I |A_J^I| = \tan \delta_J^I \operatorname{Re} A_J^I. \quad (2)$$

Cauchy's theorem implies instead that  $A_J^I(s)$  can be written as a dispersive integral along the physical cut:

$$A_J^I(s) = \frac{1}{\pi} \int_L dz \frac{\operatorname{Im} A_J^I(s)}{z - s - i\epsilon} + \text{subtractions}. \quad (3)$$

Inserting eq. (2) in the dispersion relation (3), one obtains an integral equation for  $A_J^I(s)$  of the Omnès type, which has the well-known Omnès solution [14] (for  $n$  subtractions with subtraction point  $s_0$  outside the physical cut):

$$A_J^I(s) = Q_{J,n}^I(s, s_0) \exp \{I_{J,n}^I(s, s_0)\}, \quad (4)$$

where

$$I_{J,n}^I(s, s_0) \equiv \frac{(s - s_0)^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dz}{(z - s_0)^n} \frac{\delta_J^I(z)}{z - s - i\epsilon} \quad (5)$$

and

$$\log \{Q_{J,n}^I(s, s_0)\} \equiv \sum_{k=0}^{n-1} \frac{(s - s_0)^k}{k!} \frac{d^k}{ds^k} \ln \{A_J^I(s)\} \Big|_{s=s_0}, \quad (6)$$

for ( $n \geq 1$ ) and with  $Q_{J,0}^I(s, s_0) \equiv 1$ . The dispersive integral  $I_{J,n}^I(s, s_0)$  is uniquely determined up to a polynomial ambiguity (that does not produce any imaginary part of the amplitude), which depends on the number of subtractions and the subtraction point. The simple iterative relation for the real part of  $I_{J,n}^I(s, s_0)$

$$\operatorname{Re} I_{J,n}^I(s, s_0) = \operatorname{Re} I_{J,n-1}^I(s, s_0) - (s - s_0)^{n-1} \lim_{s \rightarrow s_0} \frac{\operatorname{Re} I_{J,n-1}^I(s, s_0)}{(s - s_0)^{n-1}}, \quad (7)$$

shows that only a polynomial part of  $I_{J,n}^I(s, s_0)$  does depend on the subtraction point  $s_0$  and the number of subtractions  $n$ , while the non-polynomial part of  $I_{J,n}^I(s, s_0)$ , the one containing the infrared chiral logarithms, is universal (i.e.  $s_0$  and  $n$  independent). Thus, the Omnès solution predicts the chiral logarithmic corrections in a universal way and provides their exponentiation to all orders in the chiral expansion. The

polynomial ambiguity of  $I_{J,n}^I(s, s_0)$  and the subtraction function  $Q_{J,n}^I(s, s_0)$  can be fixed, at a given order in the chiral expansion, by matching the Omnès formula (4) with the ChPT prediction of  $A_J^I(s)$ . It remains a polynomial ambiguity at higher orders. Notice that in the presence of a zero of the amplitude the Omnès solution can be found for the factorized amplitude  $\overline{A_J^I(s)}$ , such that  $A_J^I(s) = (s - \zeta)^p \overline{A_J^I(s)}$ , where  $\zeta$  is a zero of order  $p$ .

### 3. $K \rightarrow \pi\pi$ matrix elements

The usual OPE description of  $K \rightarrow \pi\pi$  decays is realized by a three-flavour short-distance effective Lagrangian with  $\Delta S = 1$  [17,18],

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\nu) Q_i(\nu), \quad (8)$$

where  $G_F$  is the Fermi coupling and  $V_{ij}$  are the appropriate CKM matrix elements. The sum is over the product of local four-fermion operators  $Q_i$  and the short-distance Wilson coefficients  $C_i(\nu)$ . The renormalization (or factorization) scale  $\nu$  separates the short- and long-distance contributions contained in  $C_i(\nu)$  and  $Q_i$  respectively. The long-distance realization of matrix elements among light pseudoscalar mesons such as  $K \rightarrow \pi\pi$  can be realized with ChPT, as an expansion in powers of momenta of the external particles and light quark masses. At lowest order in the chiral expansion, the most general effective bosonic Lagrangian, with the same  $SU(3)_L \otimes SU(3)_R$  transformation properties as the short-distance Lagrangian (8), contains three terms [2]:

$$\begin{aligned} \mathcal{L}_2^{\Delta S=1} = & -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 f^4 \langle \lambda L_\mu L^\mu \rangle \right. \\ & + g_{27} f^4 \left( L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) \\ & \left. + e^2 f^6 g_{EM} \langle \lambda U^\dagger Q U \rangle \right\} + \text{h.c.} \quad (9) \end{aligned}$$

The flavour-matrix operator  $L_\mu = -iU^\dagger D_\mu U$  represents the octet of  $V - A$  currents at lowest order in derivatives, where  $U = \exp(i\sqrt{2}\phi/f)$  is

the exponential representation of the light pseudoscalar meson field with  $\phi$  the flavour octet matrix.  $\mathcal{Q} = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$  is the quark charge matrix,  $\lambda \equiv (\lambda^6 - i\lambda^7)/2$  projects onto the  $\bar{s} \rightarrow \bar{d}$  transition [ $\lambda_{ij} = \delta_{i3}\delta_{j2}$ ] and  $\langle A \rangle$  denotes the flavour trace of  $A$ .

At generic values of the squared centre-of-mass energy  $s = (p_{\pi 1} + p_{\pi 2})^2$ , the  $I = 0, 2$  amplitudes generated by the lowest-order lagrangian in eq. (9) are given by

$$A_0(s) = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sqrt{2} f \left\{ \left( g_8 + \frac{1}{9} g_{27} \right) (s - M_\pi^2) - \frac{2}{3} f^2 e^2 g_{EM} \right\},$$

$$A_2(s) = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{2}{9} f \left\{ 5 g_{27} (s - M_\pi^2) - 3 f^2 e^2 g_{EM} \right\} \quad (10)$$

where the usual isospin decomposition

$$A[K^0 \rightarrow \pi^+ \pi^-] \equiv A_0 + \frac{1}{\sqrt{2}} A_2,$$

$$A[K^0 \rightarrow \pi^0 \pi^0] \equiv A_0 - \sqrt{2} A_2, \quad (11)$$

$$A[K^+ \rightarrow \pi^+ \pi^0] \equiv \frac{3}{2} A_2,$$

has been used. In the absence of  $e^2 g_{EM}$  corrections, the amplitudes in eq. (10) have a zero at  $s = M_\pi^2$ , because the on-shell amplitudes should vanish in the SU(3) limit [19]. This is not the case for the amplitudes mediated by the electroweak penguin operator  $Q_8$ , since its lowest-order ChPT realization is given by the term proportional to  $e^2 g_{EM}$ . The lowest order chiral contribution to those amplitudes is a constant of order  $e^2 p^0$  (which anyway counts as  $O(p^2)$  in the usual chiral power counting).

The derivation of the Omnès solution for  $K \rightarrow \pi\pi$  decays has been discussed in detail in ref. [2]. Here, I focus on some relevant aspects of the problem. Our aim is to resum the strong FSI effects due to soft rescattering of the two pions in the final state. The study of the scalar pion form factor and its comparison with  $K \rightarrow \pi\pi$  amplitudes in ref. [2] has clarified various facts:

- soft FSI in the  $I = 0$  channel generate large infrared logarithms dependent on the pion

mass which need to be resummed to all orders in ChPT.

- Those infrared logarithms are universal, i.e. only depend on the quantum numbers of the  $\pi\pi$  system in the final state.
- The Omnès solution provides an evolution of the given amplitude from low energy values, where the ChPT momentum expansion can be trusted, to higher energy values, through the exponentiation of the infrared effects due to FSI.

However, one difference between the Omnès solution for  $K \rightarrow \pi\pi$  amplitudes and the scalar pion form factor, is that we need to consider an off-shell kaon of mass squared  $s = (p_{\pi 1} + p_{\pi 2})^2$  in the first case, instead of a physical momentum transfer  $s$ . This generates a *local* ambiguity at higher orders in the ChPT expansion (see also [2] for the explicit expressions at the next-to-leading order in ChPT), which however has nothing to do with the Omnès procedure of resumming FSI effects.

The CP conserving  $K \rightarrow \pi\pi$  isospin amplitudes admit a general decomposition in ChPT [2]:

$$\mathcal{A}_I(s) = \bar{a}_I(s) (s - M_\pi^2) + \delta\bar{a}_I(s) (M_K^2 - M_\pi^2), \quad (12)$$

where  $\delta\bar{a}_I(s)$  parameterizes tiny corrections due to the explicit breaking of chiral symmetry via the quark mass matrix and it is zero at lowest order<sup>2</sup>. Since there is a single strong phase, for a given isospin, the unitarity relation (2) is valid for  $\bar{a}_I(s)$  and  $\delta\bar{a}_I(s)$  individually and the Omnès problem can be solved separately for the two pieces. Combining them, one can write the result for the physical on-shell amplitude in the simpler form:

$$\begin{aligned} \mathcal{A}_I &\equiv \mathcal{A}_I(M_K^2) = (M_K^2 - M_\pi^2) a_I(M_K^2) \\ &= (M_K^2 - M_\pi^2) \Omega_I(M_K^2, s_0) a_I(s_0) \quad (13) \\ &= (M_K^2 - M_\pi^2) \Re_I(M_K^2, s_0) a_I(s_0) e^{i\delta_0^I(M_K^2)}, \end{aligned}$$

where  $a_I(s) \equiv \bar{a}_I(s) + \delta\bar{a}_I(s)$ . The Omnès factor  $\Omega_I(M_K^2, s_0)$  can be interpreted as a sort of evolution operator from the subtraction point  $s_0$  to  $M_K^2$ . Its explicit expression for a given number of

<sup>2</sup>To make the decomposition (12) unique, we require the function  $\delta\bar{a}_I(s)$  to depend on  $s$  only logarithmically.

subtractions can be directly derived from eq. (4) and can be split into the dispersive contribution  $\Re_I(M_K^2, s_0)$  and the phase shift exponential. Notice also that the once-subtracted Omnès factor  $\Omega_I^{(1)}(M_K^2, s_0)$  is universal because it only depends on the phase shifts  $\delta_0^I(s)$ , while for two subtractions the Omnès factor depends on  $f'(s_0)/f(s_0)$  for a given amplitude  $f(s)$ . However, given the smallness of the sub-leading  $\delta\tilde{a}_I$  contribution, it remains a good numerical approximation to take a global Omnès factor for  $a_I(s)$  also with two subtractions.

For each of the amplitudes  $a_0^{(8)}$ ,  $a_0^{(27)}$  (the octet and 27-plet  $I = 0$  amplitudes) and  $a_2$  (with  $I = 2$ ), the  $s$  dependence can be written in a simple form:

$$a(s) = a(0) \{1 + g(s) + O(p^4)\}, \quad (14)$$

where the one-loop functions  $g = g_0^{(8)}$ ,  $g_0^{(27)}$  and  $g_2$  have been computed in ref. [2]. The main properties of the  $g(s)$  functions can be summarized as follows:

- the contribution from  $\delta\tilde{a}_I(s)$  is always very small and exclusively due to non-analytic  $\bar{K}K$  and  $\eta\eta$  loop contributions which are numerically suppressed at low values of  $s$ .
- All isoscalar  $g$  functions contain exactly the same infrared  $\ln M_\pi^2$  contribution and the same contribution from the finite one-loop  $\pi\pi$  rescattering function  $\bar{J}_{\pi\pi}(s)$  [2] which generates the absorptive part of the isospin amplitude below the inelastic threshold. This shows the universality of the infrared effects due to FSI.
- The  $s$  dependence of the one-loop correction at low values of  $s$  is dominated by the pure  $SU(2)$  effect of elastic  $\pi\pi \rightarrow \pi\pi$  scattering. These universal infrared effects enhance the  $I = 0$  amplitudes while suppress the  $I = 2$  amplitude.

The dynamics leading to the  $\pi\pi$  final state, also generates local contributions which are different in each case. For the scalar form factor these contributions are small [2]. For the weak  $K \rightarrow \pi\pi$  amplitudes the knowledge of those contributions

(generated by the ChPT counterterms) is still quite limited and has to be further investigated. In addition, being the kaon off-shell, local off-shell contributions are also allowed, starting at next-to-leading order in the chiral expansion. The usual factorization models [20] predict all the local contributions to the functions  $g(s)$  to be zero at the ChPT renormalization scale  $\mu = M_\rho$ . However, a model-independent analysis still remains affected by the ambiguity due to the presence of *local* contributions, already for the on-shell amplitude. The Omnès factor cannot fix that problem. The role of the Omnès factor remains that of providing an efficient resummation of large infrared effects due to FSI. The advantage of the Omnès exponentiation respect to the usual one-loop ChPT computation is to control the uncertainty coming from higher order ( $>$  two-loops) FSI effects.

Taking a low subtraction point  $s_0 = 0$  where higher-order corrections are expected to be small, we can just multiply the tree-level formulae (10) with the experimentally determined Omnès exponentials [2]. The two dispersive corrections factors thus obtained [2] are  $\Re_0(M_K^2, 0) = 1.55 \pm 0.10$  and  $\Re_2(M_K^2, 0) = 0.92 \pm 0.03$ , where the errors are supposed to take into account a) the uncertainties of the fits to the experimental phase shifts data used in the calculation of the Omnès factor and b) the additional inelastic contributions above the first inelastic threshold.

The corrections induced by FSI in the moduli of the decay amplitudes  $\mathcal{A}_I$  generate an additional enhancement of the  $\Delta I = 1/2$  to  $\Delta I = 3/2$  ratio,  $\Re_0(M_K^2, 0)/\Re_2(M_K^2, 0) = 1.68 \pm 0.12$ . (15)

This factor multiplies the enhancement already found at short distances.

The Omnès procedure can be directly extended to the CP-violating  $K \rightarrow \pi\pi$  amplitudes relevant for the estimate of the direct CP violation parameter  $\varepsilon'/\varepsilon$ . In deriving the structure of the absorptive part of the amplitude in eq. (2) one makes use of Time-Reversal invariance, so that the Omnès procedure as formulated in eq. (4) can be applied only to CP-conserving amplitudes. However, working at the first order in the Fermi coupling, the CP-odd phase is

fully contained in the ratio of CKM matrix elements  $\tau = V_{td} V_{ts}^* / V_{ud} V_{us}^*$  which multiplies the short-distance Wilson coefficients. Decomposing the isospin amplitude as  $\mathcal{A}_I = \mathcal{A}_I^{CP} + \tau \mathcal{A}_I^{C\bar{P}}$ , the Omnès solution can be derived for the two amplitudes  $\mathcal{A}_I^{CP}$  and  $\mathcal{A}_I^{C\bar{P}}$  which respect Time-Reversal invariance. In ref. [1] it has been shown how the inclusion of FSI effects in  $K \rightarrow \pi\pi$  amplitudes can easily enhance previous short-distance based Standard Model predictions of  $\varepsilon'/\varepsilon$  [21,22] by roughly a factor of two. To obtain a complete Standard Model prediction for  $\varepsilon'/\varepsilon$  an exact matching procedure has been proposed [23]. It is inspired by the large- $N_c$  expansion, but only at scales below the charm quark mass  $\mu \leq m_c$  (where the logarithms that enter the Wilson coefficients are small). FSI effects, which are next-to-leading in the  $1/N_c$  expansion but numerically relevant, are taken into account through the multiplicative factors  $\mathcal{R}_I(M_K^2, 0)$  while avoiding any double counting. The Standard Model prediction for  $\varepsilon'/\varepsilon$  has been discussed in [15] at this conference.

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