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SUMMARY

The central question is: what happens near a bifurcation where a closed orbit of a vector field looses its stability in a 1:4 resonance? This leads to the study of a classical codimension-two bifurcation, a Hopf bifurcation of a planar diffeomorphism with 1:4 resonance. The diffeomorphism can be approximated up to flat terms by a $\mathbb{Z}_4$-equivariant planar vector field composed with the rotation by $\pi/2$. It is a conjecture by Arnol'd that the family

$$\dot{z} = \varepsilon z + Az|z|^2 + B\bar{z}^3,$$

where $\varepsilon, A, B \in \mathbb{C}$, is a versal model, which means that it contains all versal unfoldings in the parameter ε for given constants A and B.

This thesis deals with this conjecture. By scaling, we reduce the above family to the system

$$\dot{z} = e^{i\alpha}z + e^{i\varphi}z\,|z|^2 + b\bar{z}^3,$$

where $b \in \mathbb{R}^+$, $\varphi \in [\pi, 3\pi/2]$ and $\alpha \in (-\pi, \pi]$. Our point of view is to treat the unfolding parameter α and the constants b and φ as parameters on equal terms. We describe the bifurcation set in (b, φ, α)-space, using a combination of analytical, numerical and geometrical methods. The bifurcation set, together with the 15 types of phase portraits, describes all known phenomena in a condensed way and gives more insight into the problem.

In particular we study the bifurcations at ∞ along the line $b = 1$, $\varphi = 3\pi/2$ and $\alpha \in (-\pi, \pi]$. A special role is played by the point $b = 1$, $\varphi = 3\pi/2$ and $\alpha = 0$, which we call an organizing center, because all types of phase portraits can be found near it. We give an unfolding in a neighborhood of the corresponding codimension-three singularity at ∞.

The study of the bifurcations at ∞ together with numerical results on global phenomena strongly suggests that there are no unfoldings apart from the known ones. This is evidence for the conjecture of Arnol'd.

For completeness, the appendix contains computer generated figures of all known unfoldings, together with an explanation how they can be translated to the dynamics near a closed orbit of a vector field in 1:4 resonance.