

University of Groningen

On the 1

Krauskopf, Bernd

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2009

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Krauskopf, B. (2009). *On the 1: 4 resonance problem*. s.n.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

SUMMARY

The central question is: what happens near a bifurcation where a closed orbit of a vector field loses its stability in a 1:4 resonance? This leads to the study of a classical codimension-two bifurcation, a Hopf bifurcation of a planar diffeomorphism with 1:4 resonance. The diffeomorphism can be approximated up to flat terms by a \mathbb{Z}_4 -equivariant planar vector field composed with the rotation by $\pi/2$. It is a conjecture by Arnol'd that the family

$$\dot{z} = \varepsilon z + Az|z|^2 + B\bar{z}^3,$$

where $\varepsilon, A, B \in \mathbb{C}$, is a versal model, which means that it contains all versal unfoldings in the parameter ε for given constants A and B .

This thesis deals with this conjecture. By scaling, we reduce the above family to the system

$$\dot{z} = e^{i\alpha}z + e^{i\varphi}z|z|^2 + b\bar{z}^3,$$

where $b \in \mathbb{R}^+$, $\varphi \in [\pi, 3\pi/2]$ and $\alpha \in (-\pi, \pi]$. Our point of view is to treat the unfolding parameter α and the constants b and φ as parameters on equal terms. We describe the bifurcation set in (b, φ, α) -space, using a combination of analytical, numerical and geometrical methods. The bifurcation set, together with the 15 types of phase portraits, describes all known phenomena in a condensed way and gives more insight into the problem.

In particular we study the bifurcations at ∞ along the line $b = 1$, $\varphi = 3\pi/2$ and $\alpha \in (-\pi, \pi]$. A special role is played by the point $b = 1$, $\varphi = 3\pi/2$ and $\alpha = 0$, which we call an organizing center, because all types of phase portraits can be found near it. We give an unfolding in a neighborhood of the corresponding codimension-three singularity at ∞ .

The study of the bifurcations at ∞ together with numerical results on global phenomena strongly suggests that there are no unfoldings apart from the known ones. This is evidence for the conjecture of Arnol'd.

For completeness, the appendix contains computer generated figures of all known unfoldings, together with an explanation how they can be translated to the dynamics near a closed orbit of a vector field in 1:4 resonance.