Unsteady profile theory in incompressible flow

van de Vel, Hugo

SUMMARY

A general theory is given for the calculation of the pressure distribution and the forces acting on an oscillating thick aerofoil in incompressible flow. The profile is assumed to be symmetric and to perform harmonic oscillations about a mean position, which corresponds to the position at zero angle of incidence. In particular a vertical translation and a rotation about the mid-chord point are considered. In order to solve the boundary value problem a time-dependent conformal transformation is applied, which at any moment maps the profile onto a circle. Then the boundary condition can be applied exactly at the circle. The theory is linearized in the amplitude of oscillation, but the steady flow is taken into account exactly.

The flow in the plane of the circle is separated into three parts:

(i) a flow with zero normal velocity at the circle but with the right behaviour at infinity; there the velocity corresponds in magnitude and direction to the undisturbed speed in the case of translation, but in the case of rotation this direction is rotated through an angle which is opposite to the angle between the positive real axis and the instantaneous position of the profile chord in the physical plane;

(ii) a flow of which the normal velocity at the circle is determined by the motion of the profile; the formula for this velocity is derived for an arbitrary motion, and in particular for a vertical translation and a rotation;

(iii) a flow with circulation, the value of which is opposite to the total strength of the wake vortices; this strength is fixed by application of the Kutta condition which does not allow an infinite pressure at the trailing edge of the aerofoil; the wake vortices are carried off with the flow with the local, steady, velocity; the value of this velocity is, as distinct from the case of an infinitely thin aerofoil, not constant and tends asymptotically towards the value of the undisturbed speed; this influences the determination of the local vortex strength.

The reduced frequency is a parameter in the formula for the pressure distribution; the limit value of the pressure for zero frequency is, in the case of rotation equal to the pressure distribution due to a steady flow with non-zero incidence.

The theory is valid for a profile with arbitrary trailing edge angle. In order to examine the influence of the

trailing edge angle on the behaviour of the pressure, series expansions valid near the trailing edge are derived for the various terms appearing in the pressure formula. It is found that for a zero value of this angle the velocity at the trailing edge remains finite, and the pressure is equal to zero. If this angle differs from zero, the Kutta condition can only cause the velocity term with the highest singularity to vanish. There remains another, weaker, term with infinite velocity at the trailing edge. This means that the rear stagnation point will oscillate around this edge. Moreover, there remains a finite pressure difference above and beneath the wake at the trailing edge. This is due to the local failure of the linearized theory (only in the case of a non-zero trailing edge angle). However, this will not affect the total force and moment.

The general theory is applied to a specific family of profiles; this family contains two parameters; one determining the magnitude of the trailing edge angle, and one determining the position of maximum thickness. The calculations for profile characteristics, velocities, pressure distributions, lift and moment have been carried out for one value of both parameters and for various values of the reduced frequency. A description is given of the methods involved in the numerical calculations; the results are given in graphs and/or tables.