Engineering Aspects of Extrusion

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2.1
Mass Flow and Temperature Distribution in a Single-Screw Extruder

There are two main categories of factors that play a key role during the extrusion-cooking process: those related to the type of processed material and those derived from the operational and technical characteristics of the extruder.

The most important factors in the first category are: moisture, viscosity and chemical composition of the raw material; in the second category they are: the compression ratio and configuration of the screw, its rotational speed, process temperature and the pressure range applied in the barrel. The range of adjustable factors includes flow intensity, process temperature and the size of the die openings. All these factors are in close correlation with each other and have a decisive impact on the quality of the extruded product. Their mutual relations are the subject of several theoretical deliberations. This is true especially for vegetable material where, for example, the presence of Newtonian flows is limited due to the large variety of rheological characteristics of the raw material.

2.1.1
The Theory of Mass Flow and Temperature Distribution

To be able to comprehend the mechanism of mixing, flow rate, pressure distribution, and so on, it is necessary to determine the type of flow in the extruder barrel. The basic problem in describing flow patterns inside the extrusion-cooker is that the flows in the compression and metering section are non-Newtonian and non-isothermal.

The most important simplifications are the assumptions of steady state, negligible inertia and gravity forces and fully developed incompressible fluid flow. With these assumptions the flow of material in the barrel of the extruder is reduced to a movement in a slot having width w and height h and one wall moving with a speed of πND against a gradient pressure $\partial p/\partial z$ (Figure 2.1). The flow of the material is analyzed in relation to the rectangular coordinate system x, y, z, which rotates at the angular velocity around the system axis linked with the axis of the extruder barrel.

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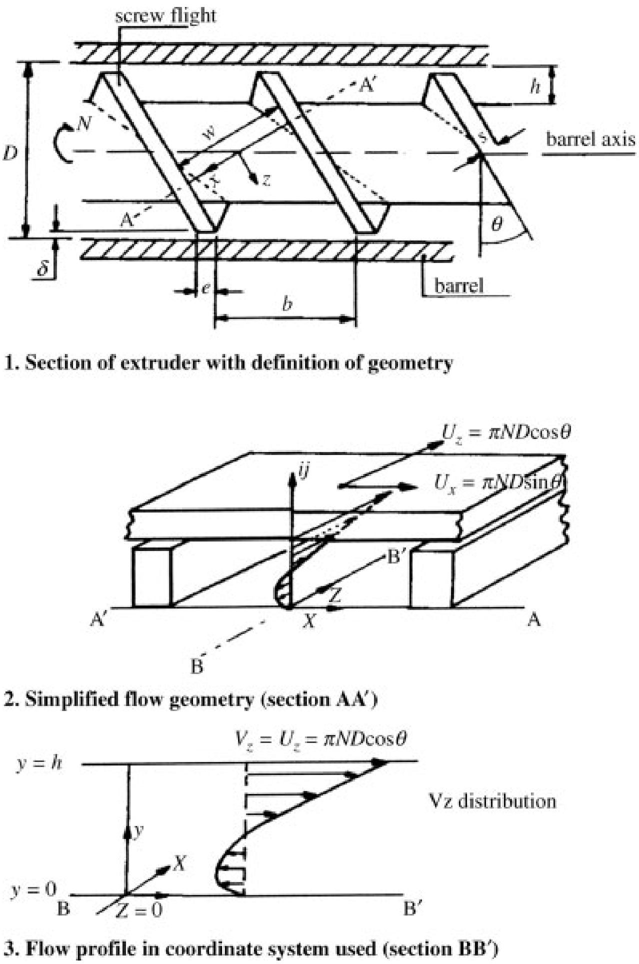

Figure 2.1 A graphic interpretation of the theory of material movement in a single-screw extruder [1].

The velocity of the barrel wall can now be divided into two components: in the cross channel direction $U_x = \pi ND\sin\theta$ and in the channel direction $U_z = \pi ND\cos\theta$ (where N is the screw rotational speed, D the internal diameter of the extruder barrel, and θ the angle of screw flight). The flow in the z-direction is the result of two forces: the drag caused by the z component of the relative velocity of the barrel (U_z) and the pressure gradient in the direction of z, since the value of pressure p gradually grows in this direction [32, 33].

The simplest case arises when the material behaves as a Newtonian fluid with temperature-independent viscosity and when the velocity components in the y-direction near the flights are neglected. This assumption is a reasonable approximation for shallow channels ($h/w < 0.1$).

Theoretical calculations of the distribution of flow speeds V_x and V_z, and the calculation of temperature distribution during extrusion are given in more extensive studies discussing the fundamental dependences, formulas and ways of calculating the above-mentioned values [1–5]. The formula for the calculation of speeds V_x and V_z is presented in Section 2.3, in Figure 2.3.

The material flow rate Q_v can be determined from the component of velocity V_z by integration over the entire section of the inter screw flight space and by multiplying the result by the number of screw flights, k:

$$Q_v = k\int_0^w\int_0^h V_z \mathrm{d}x\mathrm{d}y \tag{2.1}$$

Inserting the velocity component defined according to Figure 2.3 gives:

$$Q_v = \frac{1}{2}hkwU_z\left(1-\frac{1}{3}P\right) = \frac{1}{2}hkwU_z - \frac{1}{12}\frac{h^3kw}{\mu}\left(\frac{\partial p}{\partial z}\right) \tag{2.2}$$

where: μ is the Newtonian coefficient of material viscosity and:

$$P = \frac{h^2}{2\mu U_z}\left(\frac{\partial p}{\partial z}\right) \tag{2.3}$$

Griffith [2] identified Q_v for the material whose viscosity depends on temperature. The conditions in which the temperature is fully dependent on the coordinates x and z are relatively rare. These solutions can be considered valid if they are used for general calculations, that is, much simplified.

As shown above in the formula for Q_v, no material flow has been taken into account between the flights of the screw and the walls of the barrel. If Equation 2.1 is supplemented by this third element of flow, then we get:

$$Q_v = \frac{1}{2}hkwU_z - \frac{1}{12}\frac{h^3kw(\partial p)}{\mu(\partial z)} - \frac{kwU_x\delta}{2tg\theta} \tag{2.4}$$

where δ is the distance between the barrel wall and the screw flight.

The processed material flows through the die. Assuming that the die opening is in the shape of a cylinder with diameter d and length l, then Q_v can be written as:

$$Q_V = \frac{\pi d^4 \Delta p}{128\mu l} \tag{2.5}$$

Assuming that the pressure in the die opening is identical to that at the end of the barrel, then replacing $\partial p/\partial z$ by $\Delta p/(L/\sin\theta)$ in Equation 2.4, where L is the length of the screw, it will be possible to exclude Δp in Equation 2.5. Then the flow rate of extruded material can be shown as a non-dimensional equation [1, 3, 6]:

$$\frac{Q_v \sin\theta}{kwhNL} = \frac{\pi}{2}\frac{\left(1-\frac{\delta}{h}\right)\left(\frac{D}{L}\right)\sin\theta\cos\theta}{1+\frac{32}{3\pi}\left(\frac{h}{d}\right)^3\frac{l}{d}\frac{wk}{L}\sin\theta} \equiv \Lambda \tag{2.6}$$

As mentioned at the outset, during the extrusion-cooking of vegetable materials, the processed material often behaves like a non-Newtonian liquid. The main reason for this is the volatility of the rheological features of the processed material which, in turn, depend on the conditions of material compression and the scope of physical and chemical transformations within the process. In the available literature, one can indeed identify some information about these features but it is incomplete and relates only to some selected vegetable materials. Hence, only after having collected the appropriate data, will it be possible to propose a flow model for the material in question.

2.1.2
Residence Time Distribution of the Material in the Extruder

When describing the extrusion process, it is of great importance to define the residence time for material particles in the extruder. On the basis of this time distribution, it is possible to establish the degree of mixing of the material, anticipate the course of plasticization as well as the extent and degree of uniformness in the deformation of the stream of liquid material during extrusion. Residence time is largely the result of the distribution of the velocities inside the device and the length of the screw. Although it is possible to calculate the residence time distribution for particular zones in the extruder from the flow velocities, practice shows that it is empirical evidence that provides the best results.

The theory of the distribution of mixing time has been developed by process engineering [7]. Many authors have explored the characteristics of flow and mixing dynamics in extrusion processes, in particular the extrusion of plastics [3, 4, 6, 8–10].

The average residence time of the material in a fully filled extruder can be calculated from the ratio between the volume and the volumetric throughput:

$$\bar{t} = \frac{kwhL}{Q_v \sin\theta} \tag{2.7}$$

Because the throughput is proportional to the rotation rate (at constant end pressure) the average residence time for particles in the extruder is proportional to the inverse of the rotational speed of the screw N.

Particular attention should be given to the proper study of the function of time distribution $E(t)$ and $F(t)$.

$E(t)$ is called the internal age distribution and can be defined as the fraction of the material that flows through the die opening after a residence time between t and $t + \mathrm{d}t$. The function $F(t)$ is defined by integration of $E(t)$, being the cumulative function of the internal age distribution:.

$$F(t) = \int_0^t E(t)\mathrm{d}t \tag{2.8}$$

The function $(1 - F(t))$ is called the exit age distribution and is particularly important in food extrusion, because it represents the fraction of material that remains longer than a given time t in the extruder. The larger this fraction, the greater chance that the product will show signs of burning.

The simplest case, for which the average residence time of material elements in the extruder can be determined, is the flow of Newtonian fluids. Pinto and Tadmor [6] examining the flow of material in a single-screw extruder and assuming Newtonian flow and a constant material viscosity μ defined the average residence time as the first moment of the exit age distribution:

$$\bar{t} = \int_{o}^{\infty} tE(t)\mathrm{d}t \tag{2.9}$$

For a single-screw food extruder $\bar{t} = {}^{4}/_{3}t_0$, where t_0 is the minimum residence time for the particles inside the extruder or the break through time. This residence time occurs for fluid elements that remain in the channel at the level of $y/h = 2/3$.

The literature offers relatively limited data on the extrusion time of vegetable materials. Zuilichem *et al.* [9] using the technique of radioactivity studied residence time distribution for maize in a single-screw extrusion-cooker. As a result of multiple trials they found that in the case of maize the residence time for its particles inside that type of extruder ranged from 30 to 45 s.

2.2 Energy Balance

It is a widespread opinion that the extrusion technique is highly energy consuming. However, this is not justified in the light of research, especially in relation to most conventional methods of heat treatment of vegetable material. Let us have a closer look at the basic issues of energy consumption of extrusion-cooking processes.

The rotation of the extruder screw provides input of mechanical energy. Through viscous dissipation, most of this energy is converted to heat in the processed material but some goes to increase the pressure in the material and its kinetic energy. The total energy balance dE consists of the following components:

$$\mathrm{d}E = \mathrm{d}E_H + \mathrm{d}E_p + \mathrm{d}E_k + \mathrm{d}E_\delta \tag{2.10}$$

Where: dE_H is the viscous energy dissipation in the channel, dE_p the energy consumed to increase the pressure of the processed material, dE_k the energy consumed to increase kinetic energy, dE_δ the increase in heat energy in the area between the screw and the cylinder. Each of the above items will be discussed separately with the assumption that the velocities of mass flow in the extruder are small, and consequently dE_k is negligible.

2.2.1 Components of Energy Balance

In a channel element between screw flights, with dimensions $wh\mathrm{d}z$, the energy condition may be expressed by the following equation [5, 11]:

$$\mathrm{d}E_H + \mathrm{d}E_p = k\mu(\pi ND)^2 \frac{w}{h}(\cos^2\theta + 4\sin^2\theta)\mathrm{d}z + k\pi ND \frac{wh}{2}\frac{\partial p}{\partial z}\cos\theta dz \quad (2.11)$$

The dissipation of heat energy in this channel element is proportional to μ and N^2. The energy requirements for the pressure increase are directly proportional to N, which is related to Q and the pressure increase along the channel. In most cases, the energy to increase the pressure of the processed material is very small compared to the viscous dissipation term due to the relatively low Δp and high μ involved.

$\mathrm{d}E_\delta$ in an area of the flight clearance due to drag flow can be expressed by the following equation:

$$\mathrm{d}E_\delta = k\left[\int_0^e \tau V \mathrm{d}x\right]\mathrm{d}z \quad (2.12)$$

where e is the screw flight tip width and the shear stress τ can safely be approximated by:

$$\tau = \mu_\delta \frac{\pi ND}{\delta} \quad (2.13)$$

The viscosity in the clearance is μ_δ to denote the fact that it may be significantly different from the viscosity in the channel because of temperature and shear effects.

Using $V = \pi DN$ and integrating gives:

$$\mathrm{d}E_\delta = k\mu_\delta \frac{(\pi DN)^2 e}{\delta}\mathrm{d}z \quad (2.14)$$

This equation shows the high significance of the clearance between the screw and the barrel surface. The smaller the gap the higher the power needed to the drive the extruder.

2.2.2 Total Power Input to a Screw

Starting with the total Equation 2.10 for power input, the following equation emerges:

$$\mathrm{d}E = k\left[\mu(\pi ND)^2 \frac{w}{h}(\cos^2\theta + 4\sin^2\theta) + \pi NDw\frac{h}{2}\frac{\partial p}{\partial z}\cos\theta + \mu_\delta(\pi ND)^2\frac{e}{\delta}\right]\mathrm{d}z \quad (2.15)$$

Assuming that:

$$\mathrm{d}L = \mathrm{d}z\sin\theta \quad (2.16)$$

after integration we obtain the total power input as:

$$E = k\frac{(\pi ND)^2 L}{\sin\theta}\left[\mu\frac{w}{h}(\cos^2\theta + 4\sin^2\theta) + \mu_\delta\frac{e}{\delta}\right] + k\frac{\pi NDwh}{2}\Delta p\cos\theta \quad (2.17)$$

This equation is sometimes rewritten as:

$$E = G_2 N^2 \left[\mu \frac{w}{h} (\cos^2 \theta + 4\sin^2 \theta) + \mu_\delta \frac{e}{\delta} \right] + G_1 N \Delta p \tag{2.18}$$

where:

$$G_2 = \frac{k(\pi D)^2 L}{\sin \theta} \tag{2.19}$$

and

$$G_1 = \frac{\pi}{2} D^2 h \left(1 - \frac{ek}{\pi D \sin \theta} \right) \sin \theta \cos \theta \tag{2.20}$$

In extrusion theory we often make use of the specific mechanical energy (SME), that is, the energy consumption per unit of mass of processed material. In many food extrusion processes where the pressure is low the flow rate is almost equal to the drag flow or $G_1 N$. In this case it is expected that the SME will increase linearly with the speed of the screw N for Newtonian fluids at a constant temperature T. For pseudoplastic materials, the viscosity is proportional to the shear rate $\dot{\gamma}$ which is proportional to N. Therefore the viscosity is proportional to N^{n-1} and the SME can be expected to increase as N^n.

Strictly speaking, Equations 2.17 and 2.18 describe the power input for an extruder operating with Newtonian fluids only. These equations can be used to approximate the energy balance for non-Newtonian fluids if the apparent viscosity η is substituted for the Newtonian viscosity μ. In this case, η needs to be evaluated for the shear rate $\dot{\gamma}_H$ and temperature in the channel. Although the shear rate varies with the location in the channel, for low-pressure processes the average shear rate can be approximated by:

$$\dot{\gamma}_H = \frac{\pi D N}{h} \tag{2.21}$$

Similarly, the viscosity in the clearance between the screw tip and the barrel wall needs to be evaluated at $\dot{\gamma} = \pi N D / \sigma$ and the temperature in the clearance.

Bruin *et al.* [1] divided both parts of Equation 2.15 by $\mu k N^2 D^3$ and obtained a power number similar to that used in the analysis and design of mixing systems in the viscous flow region. It should be noted that the power number is a constant for a given extruder and die design. In the literature there are insufficient examples of practical verification of the energy consumption models during extrusion. However, these models give a useful approximation for solving problems with baro-thermal treatment of biopolymers.

The application of Equation 2.18 requires a thorough knowledge of the rheology of food materials as a function of process temperature. Most of them are non-Newtonian fluids, therefore it is important to know the shear ratio $\dot{\gamma}$.

Harmann and Harper [12] calculated the torque requirements for extrusion-cooking a rehydrated pregelatinized corn dough and compared the results with actual measurements. Their experiment was conducted in isothermal conditions with $\dot{\gamma}$ calculated according to Equation 2.21. The values of torque obtained by dividing Equation 2.18 by N were 17 to 73% higher than the values actually measured. They assigned the inability to predict the current values in this equation to insufficient data on the material temperature at the surface of the barrel and the filling degree of the screw. The equation might have over predicted the theoretical power demand, because the temperature at the surface of the barrel was probably higher than the temperature of the entire material, with a small degree of filling of the screw.

According to the investigations of Moscicki *et al.* [8, 11] the demand for energy during extrusion of vegetable material decreases with increase in the process temperature and moisture of the processed material. Generally speaking, it can be assumed that good results are obtained at an energy consumption (SME) with an average of 0.1 to 0.2 $\mathrm{kWh\,kg^{-1}}$.

The impact of an increase in speed of mass flow on energy consumption during extrusion-cooking has been investigated by Tsao [13], who used 14 different screws using a high moisture food dough. His results showed specific power increasing approximately as N^n within a range of a threefold increase in screw speed, as expected. Actual power requirements were less than those calculated and could be attributed to the actual dough temperature being higher than that measured at the barrel surface.

A number of SME measurements for cereal–soybean mixtures for a simple autogenous extruder at a moisture content of 12 to 20% have been made by Harper [14]. These data show ranges in SME of 0.08 to 0.16 $\mathrm{kWh\,kg^{-1}}$. He also confirmed that energy consumption did not increase in proportion to N as anticipated. There is an urgent need for a more accurate understanding of the effects of moisture and shear rate on viscosity and the correct measurement of material temperature.

Bruin *et al.* [1] examined energy consumption during the extrusion of cereals at different moisture contents in a single-screw extrusion-cooker fitted with different dies and screws at a single screw speed. Their data showed not only a reduction in SME with increased moisture, but also an increase at higher moisture. They attributed these results to increased gelatinization at higher moistures.

Above we have analyzed the energy of the extruder's screw drive related to the transformation of mechanical energy into heat in the channel and flight clearance. During extrusion-cooking, this mechanical energy is principally dissipated in the form of heat, which causes temperature, chemical and perhaps phase changes in the processed material. The amount of energy dissipated may be insufficient in relation to the thermal requirements, so very often heat is added with a heating system for the barrel and sometimes for the screw. Direct addition of steam to the processed material is also used in order to raise its temperature and hydration.

The energy balance per unit of time can be expressed as follows:

$$\frac{E_t}{\Delta t} = \frac{E}{\Delta t} + q \qquad (2.22)$$

and

$$\frac{E_t}{\Delta t} = Q_v \left[\int_{T_1}^{T_2} c_p \mathrm{d}T + \int_{p_1}^{p_2} \frac{\mathrm{d}p}{\varrho} + \Delta H^\circ + \Delta H_{s1} \right] \tag{2.23}$$

where E_t is the total net energy added to the extruder, E the mechanical energy dissipated (from Equation 2.6), Δt the process time, q the heat flux to dough $(+)$ or loss $(-)$, c_p the specific heat, ΔH° the heat of reaction/unit mass dough, ΔH_{s1} the latent heat of fusion/unit mass of dough, ϱ the density of the material, l – material feeder hole, 2 just behind the die.

A certain amount of heat is also generated as a result of chemical reactions such as starch gelatinization, protein denaturation, browning, and so on. Sahagun and Harper, [15] using blends of 70% maize and 30% soybean, found that approximately 15% of total energy added to the autogenous extruder was unaccounted for by sensible heat changes, pressure increase and heat loss. These differences are attributed to the heat of reaction associated with chemical changes occurring in food materials and heat losses to the surroundings (radiation and convection).

The latent heat of fusion is also included in the energy balance to account for a relatively small amount of energy associated with the melting of solid lipids materials that may be part of a food formulation. To prepare a comprehensive model of energy balance in the extrusion process still requires much work and many experiments.

2.3
Mass and Heat Transfer in a Twin-Screw Extruder

There are two main types of food extruders: single- and twin-screw. The second type is divided into the so-called co-rotating and counter-rotating extruders. Counter-rotating twin-screw extruders work as positive-displacement pumps with closed C-shaped chambers between the screws [4] – this minimizes mixing but also backflow caused by the increase in pressure. In co-rotating extruders, material is transported steadily from one screw to another. The flow mechanism can be described by a combination of drag flow and a positive displacement caused by the pushing action of the screw-set in the intermeshing region. Co-rotating extruders usually work at a higher screw speed ranging from 300 to 600 rpm.

Although currently designed screws have a modular structure of different geometry, they are generally divided according to three different screw sections:

- the feed section, which ensures that sufficient solid material is transported into the screw;
- the compression section, in which the material is heated and processed into a dough-like mass,
- the metering section, in which the screw configuration feeds the die constantly with material.

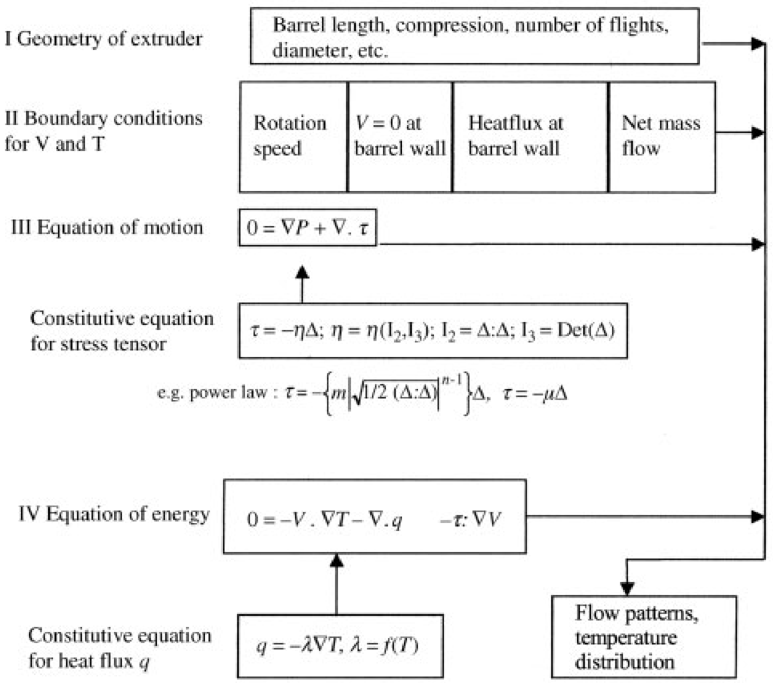

Figure 2.2 Schematic approach for calculation of flow patterns in an extruder [1].

The basic problem in the description of the model of flow inside the extruder is that the flow in the compression section and in the metering section is non-Newtonian and not isothermal. Therefore, the equation of movement and the equation of energy, should be presented so as to obtain a realistic model of flow. The main procedure is shown in the diagram in Figure 2.2. The set of equations can be solved only with a few approximations.

The most important are the assumptions of steady states, negligible inertia and gravity forces and of fully developed incompressible fluid flow. Other assumptions are set out in Section 2.1. With these assumptions and simplifications the solution of the equation of motion and the energy equation is possible. The most simple case arises when the material behaves as a Newtonian fluid with temperature-independent viscosity and when the velocity component in the y-direction near the flights is neglected. This second assumption is a reasonable approximation for shallow channels ($h/w < 0.1$).

Figure 2.3 illustrates how velocity profiles could be derived and, successively, the heat flux at the barrel surface and the screw root can be calculated. It is interesting to note that the equation of energy is fairly difficult to solve even for this simple rheological behavior because of the convection term. Only when adiabatic extrusion is assumed and the temperature gradient in the y-direction is assumed to be negligible, can a relatively simple result be obtained.

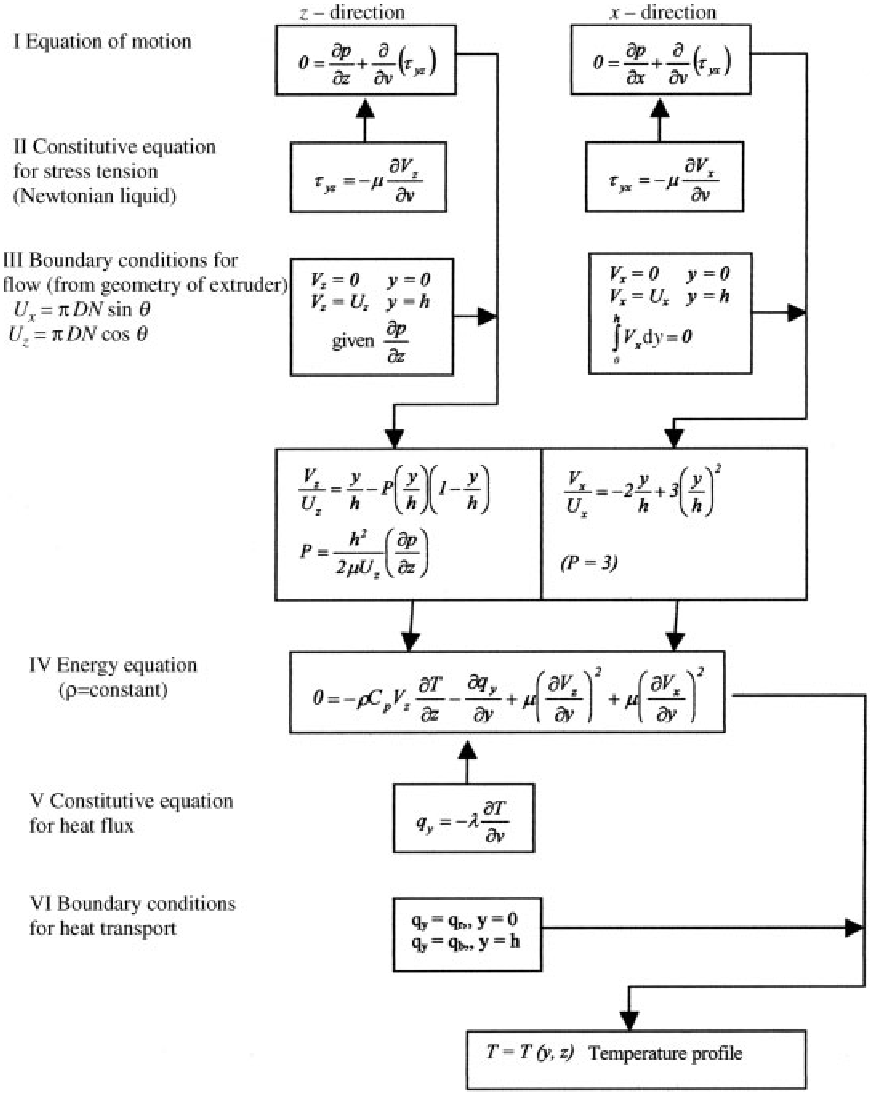

Figure 2.3 Calculation scheme for flow patterns in a single screw extruder for an incompressible Newtonian fluid with constant heat conductivity and viscosity [1].

2.3.1 Heat Transfer

The main problem in modeling heat transfer in food extruders is the inclusion of all variables in a complete analysis. This seems to be quite impossible because of the computational effort involved although in many cases some of the variables can be ignored as unimportant for the practical set of conditions under consideration. This means that it is possible to make some assumptions and to derive a solution which is useful under certain operating conditions. Care has to be taken, however, that the model is not used for an unjustified extrapolation to other circumstances.

As commonly known, most food materials behave like non-Newtonian fluids. Griffith [16], Pearson [17], Yankov [18] and Yacu [19] developed models which give a better understanding of the extrusion of such materials. Griffith [16] used a numerical analysis to solve differential equations for fully developed velocity and temperature profiles for single screw extrusion of power-law fluids. Pearson [17] solved the equations of motion and energy for transverse channel flow and a superimposed temperature profile for power-law fluids. Yankov [18] proposed equations for movement of non-Newtonian fluids, assuming that the temperature did not change along the channel. Unfortunately, these models are doubtful because of the assumptions used in the description of the phenomenon, especially when it comes to extrusion of food. The most thorough description of the non-Newtonian behavior of mass was proposed by Yacu [19], which has been taken into account in developing the model proposed by Zuilichem [5, 20, 21].

The first examination of two-phase flow in the melting zone in a single-screw extruder was carried out by Maddok [22]. He concluded that solid particles in contact with the hot surface of the barrel partially melt and smear a film of molten polymer over the barrel surface. This film, and probably some particles, are dragged partly along the barrel surface, and when they meet the advancing flight, they are mixed with previously molten material. The molten material collects at an area in the pushing flight (meltpool), whereas the forward portion of the channel is filled with solid particles. The width of a solid bed, *X*, gradually decreases toward the outlet of the screw. The melting process ends when the solid bed disappears.

Based on the observations of Maddock [22], Tadmor and Klein [23] proposed a model which provided the equations that are required to calculate the length of the melting zone. This length depends on the physical properties of the polymer and the operating conditions. The only melting mechanism takes place between the hot barrel and the solid bed. The molten mass is transported by the movement of the barrel relative to the solid bed, until it reaches a screw flight. The leading edge of the advancing flight scrapes the melt of the barrel surface and forces it into a meltpool. This meltpool needs space and moves into the solid bed; the solid bed deforms and its width decreases. It is assumed that the melt film thickness profile does not change in the melting zone and that the melt film thickness equals the flight clearance. Shapiro [24] and Vermeulen *et al.* [25] concluded that the melt film thickness cannot be constant and will increase. This decreases the heat flux to the solid bed and, consequently, increases the melting length.

Over the years Tadmors model has been improved and adjusted by himself and many other authors, among others, by Dekker [7] and Lindt [26].

For the heat transfer coefficient two different mechanisms have to be distinguished, heat transfer at the barrel and heat transfer at the screw. At the barrel the flight of the rotating screw wipes a certain area on the inner barrel surface, a fresh layer of polymer becomes attached to the same region and remains there for, approximately, one revolution. The amount of heat penetrating into this layer during that time by pure conductivity heat transfer is then removed with the polymer layer and is thought to be homogeneously distributed throughout the bulk of the polymer in the screw channel [27]. This process of heat penetration during extrusion can often be considered as a non-stationary penetration in a semi-infinite medium. The heat balance is given by the following equation:

$$\varrho c_p \frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{2.24}$$

or in one dimension:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = a\frac{\mathrm{d}^2 T}{\mathrm{d}y^2} \tag{2.25}$$

Where λ is the thermal conductivity, and a the thermal diffusivity. The solution of this differential equation gives the temperature distribution in the thermal boundary layer and can be written as:

$$T - T_0 = 1 - \mathrm{erf}\left[\frac{x}{\sqrt{\pi a t}}\right]$$

From this expression the time average heat transfer coefficient at the barrel wall can be calculated to be [4]:

$$\langle \alpha \rangle = \sqrt{\frac{4}{\pi}\lambda \varrho c_p m N}$$

Where ϱ and c_p are the density and specific heat of the dough and m and N are the number of flights starts and the rotation rate of the screws.

In contrast to the heat transfer with surface renewal that occurs at the barrel wall the heat transfer at the screw surface has to be described as heat transfer to a semi-infinite flowing medium, for which the energy balance can be approximated by:

$$V_x \frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial y^2} \tag{2.26}$$

with boundary conditions:

$$\begin{array}{rcl} x < 0 & & T = T_o \\ x > 0 & x \to \infty & T = T_o \\ x > 0 & y = 0 & T = T_w \end{array}$$

with the assumption that:

$$V_x = \dot{\gamma} y \tag{2.27}$$

Where: $y = 0$ indicates the location on the barrel wall at uniform temperature T_w.

The temperature profile in the thermal boundary layer is approximated by a parabolic function:

$$\frac{T - T_o}{T_w - T_o} = \left(1 - \frac{y}{\delta}\right)^2 \tag{2.28}$$

where δ is the (unknown) boundary layer thickness. After combining this with Equation 2.26 and integrating, the boundary layer thickness is given by:

$$\delta = \left(\frac{36ax}{\dot{\gamma}}\right)^{\frac{1}{3}} \tag{2.29}$$

And for the heat flux we can write:

$$\Phi''_{\mathrm{w}} = 2\lambda\left(\frac{36ax}{\dot{\gamma}}\right)^{-\frac{1}{3}}(T_{\mathrm{w}} - T_{\mathrm{o}}) \tag{2.30}$$

Therefore the average heat transfer coefficient at the screw on location x can be approximated by:

$$\alpha_x = 2\lambda\left(\frac{36ax}{\dot{\gamma}}\right)^{-\frac{1}{3}} \tag{2.31}$$

Inspection of the two different heat transfer coefficients shows that, in general food extrusion processes, the heat transfer at the wall is significantly more important than that at the screw. However, an important exception has to be made. In the derivation of the expression for the heat transfer coefficient at the barrel wall it is wiped clean completely. In practice, where there exists a gap between the screw flight and the barrel wall a small layer of material will remain at the wall, forming an insulating layer that can decrease the heat transfer considerably. Solving Equation 2.26 for this case leads to a correction for the heat transfer coefficient at the wall:

$$\langle\alpha\rangle = \sqrt{\frac{4}{\pi}\lambda\varrho c_p mN \times \exp\left(\frac{\delta}{2}\sqrt{\frac{mN}{a}}\right)}$$

where δ stands for the clearance between screw flight and barrel wall [28].

2.3.2 Model by Yacu

Yacu [19] divided the twin-screw extrusion-cooker into three main sections: solid conveying zone, melt pumping zone and melt shearing zone. The temperature and pressure profiles are predicted for each section separately. The following assumptions were made:

- The rheology of the molten material is described by a non-Newtonian, non-isothermal viscosity model taking into account the effect of moisture and fat content. The extruder is operating under steady state conditions.
- Steady state behavior and uniform conditions exist.
- The melt flow is highly viscous and in the laminar flow regime.
- Gravity effects are negligible and ignored.
- The screw is assumed to be adiabatic.
- The degree of cooking is assumed to be uniform over the cross-section.

2.3.2.1 Solid Conveying Section

Twin-screw food extruders operate mostly under starved feed conditions and the throughput is determined by the feeding unit, and the extruder's screw speed and torque. Therefore the screws in the feeding section are partially filled and no pressure is developed. The dispersion of mechanical energy is negligible because of these conditions. Heat is transferred by conduction from the barrel to the food material.

To simplify the analysis and taking into account that mixing within the channel is reasonably good in a co-rotating extruder, the heat transfer is assumed to be controlled only by convection.

Constructing a heat balance across an element normal to the axial direction and solving for T with the boundary condition at $x = 0$ and $T = T_f$, results in the equations:

$$Q_m c_{ps} T + F U_s A (T_b - T) dx = Q_m c_{ps} (T + dT) \tag{2.32}$$

which gives:

$$T = T_b - (T_b - T_f) e^{\frac{-F U_s A x}{Q_m c_{ps}}} \tag{2.33}$$

where Q_m is the feed rate, c_{ps} the heat capacity of the solid matter, F the degree of fill, A the surface area, T_b the temperature of the barrel, T_f the temperature of the feed material, and U_s the pseudo heat transfer coefficient.

The heat transfer coefficient U_s to the powder in the feed zone, for practical reasons estimated as the heat transfer coefficient based on the penetration theory, would give unrealistic values. Phase discontinuity and the existence of additional resistance to heat transfer between the solid particles are the main reasons for this.

2.3.2.2 Melt Pumping Section

In this section the change of the material from solid food-powder to a fluid melt is assumed to take place abruptly. Martelli [29] assumed that the energy was dissipated within the channel and due to the leakage flows in the various gaps. He defined four locations where energy was converted:

- in the channel:

$$Z_1 = \frac{\pi^4 D_e^3 D tg\theta}{2h} \mu N^2 \tag{2.34}$$

where h is the channel depth and D_e the equivalent twin-screw diameter.

- between the flight tip of the screw and the inner surface of the barrel:

$$Z_2 = \frac{\pi^2 D^2 e C_e}{\delta} k \mu N^2 \tag{2.35}$$

where C_e is the equivalent twin-screw circumference

- between the flight tip of one screw and the bottom of the channel of the other screw:

$$Z_3 = \frac{8\pi^2 I^3 e}{\varepsilon} k \mu N^2 \tag{2.36}$$

Where ε is the clearance between the flight tip channel bottom of two opposite screws, and I is the distance between the screw shafts.

- between the flights of opposite screws parallel to each other:

$$Z_4 = \frac{\pi^2 I^2 h\sqrt{D^2 - I^2}}{2\sigma} k\mu N^2 \tag{2.37}$$

where σ is the clearance between flights of opposite screws parallel to each other.

The total energy converted per channel per screw turn was therefore expressed as:

$$Z_\mathrm{p} = Z_1 + Z_2 + Z_3 + Z_4 = C_{1\mathrm{p}}\mu_\mathrm{p} N^2 \tag{2.38}$$

Where: μ_p is the viscosity of the product; $C_{1\mathrm{p}}$ can be defined as the pumping section screw geometry-factor and is described by:

$$C_{1\mathrm{p}} = \frac{\pi^4 D_\mathrm{e}^3 D tg\theta}{2h} + k\left(\frac{\pi^2 De^2 C_e}{\delta} + \frac{8\pi^2 I^3 e}{\varepsilon}\frac{\pi^2 I^2 h\sqrt{D^2 - I^2}}{2\sigma}\right) \tag{2.39}$$

Because the length of the screw channel per screw turn equals $\pi D tg\theta$, the average amount of heat generated within an element of thickness dx can therefore be evaluated as:

$$\mathrm{d}Z_\mathrm{p} = C_{1\mathrm{p}}\mu_\mathrm{p} N \frac{\mathrm{d}x}{\pi D tg\theta} \tag{2.40}$$

The overall shear rate on the product, while passing through the pumping zone, including the amount taken up by leakage flows, can be estimated as:

$$\dot{\gamma} = N\frac{\sqrt{C_{1\mathrm{p}}}}{V_\mathrm{p}} \tag{2.41}$$

The viscosity μ_p is dependent on shear rate ($\dot{\gamma}$), temperature (T), moisture content (MC) and fat content (FC):

$$\mu_\mathrm{p} = \mu\dot{\gamma}^{-n_1}\mathrm{e}^{-a_1(MC - MC_\mathrm{o})}\mathrm{e}^{-a_2 FC}\mathrm{e}^{-b_1\Delta T} \tag{2.42}$$

where a_1 is the moisture coefficient of viscosity, a_2 the fat coefficient of viscosity, b_1 the temperature coefficient of viscosity, and n_1 the power-law index.

This rheological model was developed by Yacu [19] for a wheat starch. The various indices are determined by multiple regression analysis to fit the rheological model.

Constructing a heat balance across an element in the melt pumping zone, coupled with the boundary conditions: $T = T_\mathrm{m}$ at $x = X_\mathrm{m}$, gives the relation:

$$Q_\mathrm{m} c_{pm} T + \alpha A(T_\mathrm{b} - T)\mathrm{d}x + \frac{C_{1\mathrm{p}}\mu_\mathrm{p} N^2 \mathrm{e}^{-b_1 T}}{\pi D tg\theta}\mathrm{d}x = Q_\mathrm{m} c_{pm}(T + \mathrm{d}T) \tag{2.43}$$

or

$$\frac{\mathrm{d}T}{\mathrm{d}x} = C_{2\mathrm{p}}\mathrm{e}^{-b_1 T} + C_{3\mathrm{p}}(T_\mathrm{b} - T) \tag{2.44}$$

where:

$$C_{2\mathrm{p}} = \frac{C_{1\mathrm{p}}\mu_{\mathrm{p}}N^2}{\pi D tg\theta Q_{\mathrm{m}} c_{pm}} \tag{2.45}$$

and

$$C_{3\mathrm{p}} = \frac{FA}{Q_{\mathrm{m}} c_{pm}} \tag{2.46}$$

In these formulas c_{pm} is the heat capacity of the molten material, α the heat transfer coefficient, and T_{m} the temperature of the molten material.

One of the assumptions made by Yacu [19] was that the moment the mass enters the melt pumping section the screws become completely filled. In this calculation the mass is subjected to the total shear stress. However, most twin-screw extruders are starved fed which results in a partially low filling degree. It would, therefore, be reasonable to conclude that the shear the product receives is lower than Yacu assumes.

2.3.3 Model by van Zuilichem

The unique feature of the model developed by van Zuilichem is that it calculates the total transferred heat in the extruder for every position along the screw axis. The model is composed of two major parts. The first part calculates the heat transferred from the barrel to the extrudate. This part of the calculation method is based on several of Zuilichem's works [5, 20, 21, 30]. The second part of the model calculates the heat generated by viscous dissipation (Figure 2.4).

The following assumptions were made for the description:

1) Input parameters of the model are:
   a) the temperature profile imposed at the barrel,
   b) torque,
   c) feedflow.
2) Torque is converted into: pressure energy, phase transition energy and temperature increase. Hence:
   a) the torque and the heat convection are linearly related to the degree of fill,
   b) the viscosity changes of the material are described by a non-Newtonian, temperature-time dependent power law model,
   c) the amount of energy required to gelatinize the starchy food product is negligible compared to the energy consumed to increase the temperature of the product,
   d) the meltflow is highly viscous and in the laminar flow regime,
   e) only the net flow in the axial direction is considered.

A computer model, following a stepwise procedure, was developed with the following process and material variables for the input:

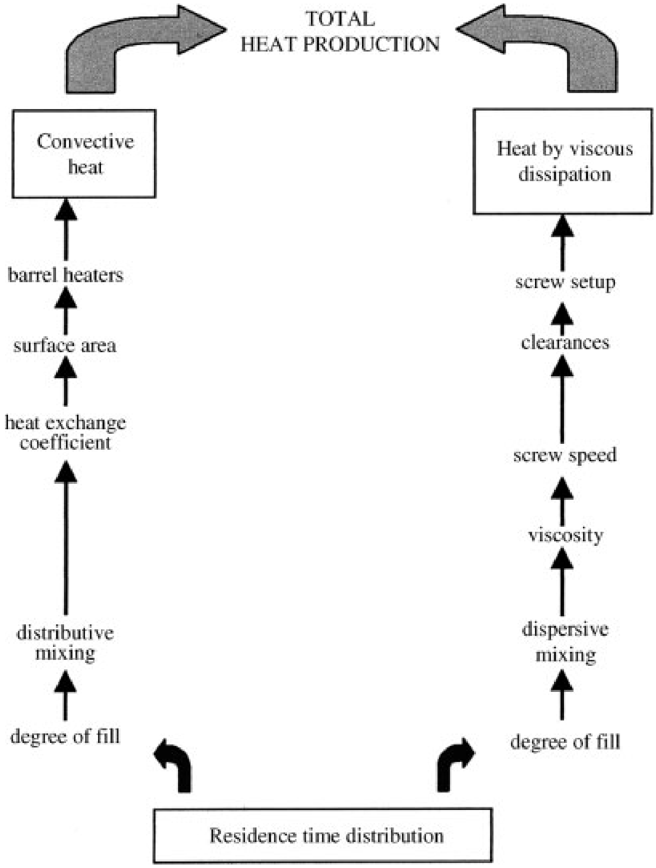

Figure 2.4 The structure of thermal energy inside the extruder [5].

- material variables: C_p, initial viscosity, melting temperature, density and moisture content,
- system variables: throughput, initial material temperature, length of extruder, screw geometry and temperature profile.

The output of this computer program is a plot of the temperature of the food product versus the axial distance in the extrusion-cooker. It is also possible to plot the temperature increase, due to penetration of heat and dissipation, versus the axial distance in the extruder.

The function of many operations in polysaccharide extrusion-cooking is to rupture the starch particles and to gelatinize their contents to a certain extent for different applications in human food and animal feed. During this process considerable changes in the rheology of the material are occurring due to the formation and breakage of gel bonds. These changes can have an impact on processing parameters and stability criteria. Janssen and van Zuilichem [31] suggested an equation especially for applications in food extrusion technology. This method is used in van Zuilichem's model and describes the effect of the mechanism of gel-formation and gel-breakage on the viscosity as a function of shear rates and fat and moisture content. They combined a power law behavior with a temperature dependence and an extra term for the formation of the network depending on the activation energy and the residence time. Now the apparent viscosity, μ_p, can be written as:

$$\mu_p = \mu_o \dot{\gamma}_p^{-n_1} e^{-a_1(MC-MC_o)} e^{-a_2 FC} e^{-b_1 \Delta T} e^{\left(\int_0^\tau e^{-\frac{\Delta E}{RT(t)}} Dt\right)} \tag{2.47}$$

where Dt is the convective derivative accounting for the fact that the coordinate system is attached to a material element as it moves through the extruder. With this model it became possible to explain certain instabilities as they occur during the extrusion-cooking of starch.

This model was practically verified by van Zuilichem with a co-rotating twin-screw food extruder, type MPF-50 APV Baker. A big advantage of this extruder is the modular screw design allowing one to compose almost every desired screw configuration. As a model food material, standard biscuit flour was used.

The measurements showed the accuracy of the proposed model. There is a slight deviation in calculated and measured temperature values (up to a maximum of 10 °C), giving a measured temperature slightly lower than predicted.

An advantage of the model is that it is capable of calculating the resulting temperature profile of the product over the viscosity-history as the key rheological problem. It must be said that the readings from the thermocouples at high screw speeds and relatively low throughputs are not ideal because a low degree of fill in nearly all parts of the extruder can cause the thermocouples to measure a mixture of melt temperature and steam temperature in the metering section, instead of a homogeneous product temperature.

All in all, there is a deficiency of various rheological data on biopolymers, and there is a need for further thorough examination. This will allow the development of new models which, if verified through experimental studies, will be more reliable and closer to reality.

References

1 Bruin, S., van Zuilichem, D.J., and Stolp, W. (1978) A review of fundamental and engineering aspects of extrusion of biopolymers in a single-screw extruder. *J. Food Process Eng.*, **2**.

2 Griffith, R.M. (1962) Fully developed flow in screw extruder. *Ind. Eng. Chem. Fundam.*, **1** (3), 180.

3 Harper, J.M. (1984) *Extrusion of Foods*, CRT Press Inc., Florida.

4 Janssen, L.P.B.M. (1978) *Twin-Screw Extrusion*, Elsevier, Amsterdam-New York.

5 Van Zuilichem (1992) Extrusion Cooking. Craft or Science? Ph.D. thesis, Wageningen University, Netherlands.

6 Pinto, G. and Tadmor, Z. (1970) Mixing and residence time distribution in melt screw extruder. *Polym. Eng. Sci.*, **10**, 279.

7 Dekker, J. (1976) Verbesserte schneckenkonstruction für das

extrudieren von polypropylen. *Kunststoffe.*, **66**.

8 Mościcki, L., Mitrus, M., and Wojtowicz, A. (2007) *Technika ekstruzji w przetwórstwie rolno-spożywczym (in Polish)*, PWRiL, Warsaw.

9 van Zuilichem, D.J., Swart de, I.G., and Zuiman, G. (1973) Residence time distribution in an extruder. *Lebensm. -Wiss. U. Technol.*, **6**, 184.

10 Wolf, D. and White, D.H. (1976) Experimental study of the residence time distribution in plasticating screw extruders. *AIChEJ*, **22**, 122.

11 Mościcki, L. and Mitrus, M. (2001) Energochøonność procesu ekstruzji (in Polish) Teka Komisji Motoryzacji i Energetyki Rolnictwa, PAN, vol. 1 186–193.

12 Harmann, D.V. and Harper, J.M. (1974) Modelling a forming foods extruder. *J. Food Sci.*, **39**, 1099.

13 Tsao, T.F., Harper, J.M., and Repholz, K.M. (1978) The effects of screw geometry on extruder operational characteristics. *AIChE Symp. Ser.*, **74** (172), 142.

14 Harper, J.M. (1976) Goals and activities of the Colorado State University LEC program. Low- Cost Extrusion Cookers, LEC-1. Colorado State University, Fort Collins.

15 Sahagun, J. and Harper, J.M. (1980) Parameters affecting the performance of a low-cost extrusion cooker. *J. Food Proc. Eng.*, **3** (4), 199.

16 Griffith, R.M. (1962) Fully developed flow in screw extruders, theoretical and experimental. *Ind. Eng. Chem. Fundam.*, **2**.

17 Pearson, J.R.A. (1966) Mechanical principles of polymer melt processing. *Trans. Soc. Rheol.*, **13**.

18 Yankov, V.I. (1978) Hydrodynamics and heat transfer of apparatus for continuous high speed production of polymer solutions. *Heat Transfer Soviet Res.*, **10**.

19 Yacu, W.A. (1983) Modelling of a two-screw co-rotating extruder, in *Thermal Processing and Quality of Foods*, Elsevier Applied Science Publishers, London.

20 van Zuilichem, D.J., Alblas, B., Reinders, P.M., and Stolp, W. (1983) A comparative study of the operational characteristics of single and twin screw extruders, in *Thermal Processing and Quality of Foods*, Elsevier Applied Science Publisher, London.

21 van Zuilichem, D.J., Tempel, W.J., Stolp, W., and van't Riet, K. (1985) Production of high-bolied sugar confectionery by extrusion-cooking of sucrose: liquil glucose mixtures. *J. Food Eng.*, **4**.

22 Maddock, B.H. (1959) A visual analysis of flow and mixing in extruder screws. *SPE J.*, **15**.

23 Tadmor, Z. and Klein, I. (1973) *Engineering Principles of Pacification Extrusion*, Van Nostrand Reinhold, New York.

24 Shapiro, J. (1973) Melting in plasticating extruders. PhD Thesis. Cambridge University.

25 Varmeulen, J., Scargo, P.G., and Beek, W. (1971) The melting of a crystalline polimer in a screw extruder. *Chem. Eng. Sci.*, **46**.

26 Lindt, T.J. (1976) A dynamic melting model for a single screw extruder. *Polym. Eng. Sci.*, **16**.

27 Jephson, C.H. (1953) *Ind. Eng. Chem.*, **45**, 992.

28 Ganzeveld, K.J., Capel, J.E., van der Wal, D.J., and Janssen, L.P.B.M. (1994) *Chem. Eng. Sci.*, **49**, 1639–1649.

29 Martelli, F.B. (1983) *Twin-Screw Extruders*, Van Nostrand-Reinhold, New York.

30 van Zuilichem, D.J., Jager, T., Spaans, E.J., and De Ruigh, P. (1990) The influence of a barrelvalve on the degree of fill in a co-rotating twin screw extruder. *J. Food Eng.*, **10**.

31 van Zuilichem, D.J. and Janssen, L.P.B.M. (1980) *Reology*, vol. **3**, Plenum Press, New York and London.

32 Carley, J.F. and Strub, R.A. (1978) Basic concepts of extrusion. *Ind. Eng. Chem.*, **45**, 970. 1953.

33 Metzner, A.B. and Reed, J.C. (1955) Flow on non-Newtonian fluids – correlation of the laminar, transition and turbulent flow region. *AIChEJ*, **1**, 434.