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Peli, G; Nootboom, B

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# Market Partitioning and the Geometry of the Resource Space<sup>1</sup>

Gábor Péli and Bart Nooteboom  
*University of Groningen.*

This article gives a new explanation for generalist and specialist organizations' coexistence in crowded markets. It addresses organizational ecology's resource-partitioning theory, which explains market histories with scale economies and crowding, and it shows that some main predictions of this theory can be restated in terms of structural properties of the  $N$ -dimensional Euclidean space. As resource-space dimensionality increases, the changing niche configurations open opportunities for specialists. The proposed approach draws upon the sphere-packing problem in geometry. The model also explains new observations, and its findings apply to a range of crowding and network models in sociology.

## INTRODUCTION

Multidimensional spaces are well understood tools of social scientists to represent objects with several attributes. They have several names in different crowding models of sociology and economics. The  $N$ -dimensional space is called *sociodemographic space* in affiliation models of individuals' group formation (McPherson 1983; Popielarz and McPherson 1995). *Market topology maps* visualize intermarket transactions, where distance measures similarities in input-output dependencies (Burt and Carlton 1989; Burt 1992). The Euclidean framework is called *competence space* if organizational competencies are in focus (Nooteboom 1994; Péli and Noote-

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Gábor Péli, University of Groningen, Faculty of Management and Organization, P.O.  
Box 800, 9700 AV Groningen, The Netherlands.

boom 1997). It is labeled as *product characteristics space* when spatial axes stand for descriptors of commodities (Lancaster 1966), and its name is *resource space* in organizational ecology, when customers with different tastes constitute the key resource for organizations (Hannan and Freeman 1989; Carroll 1985).

If customer tastes are operationalized in terms of product characteristic preferences, then the last two representations coincide. We adopt this interpretation: customers' purchasing power is the key resource for the organizations in question, and customers intend to buy the products that best match their preferences. Our goal is to give a geometry-based explanation for the long-term coexistence of generalist and specialist organizations in different markets. We demonstrate that most of the main claims of an empirically justified sociological theory, the resource-partitioning model of organizational ecology (Carroll 1985, 1997; Carroll and Hannan 1995), can be explained and extended if some structural features of multidimensional niche configurations are taken into account.

The Euclidean space stands for the market, it is the framework in which interactions take place. We model our entities as spatial objects with certain geometric properties that stand for sociological attributes. Spatial configurations represent relations between organizations, and geometry specifies constraints on their possible market positioning. Organizations are characterized by the customer tastes they address, that is, by the resource space locations they exploit, called niches or catchment areas. Competition is modeled in terms of niche overlap. Under certain conditions, markets are partitioned between a number of organizations, just as the Earth's surface is partitioned between countries. Our work intends to contribute to the understanding of these processes.

The simplest and most widely studied way of market partitioning is to assign the same size and shape of catchment area to each organization. A classic one-dimensional example is Hotelling's (1929) linear city model of product differentiation, also readdressed as *circular city* by Salop (1979). Nooteboom (1993) investigated the possibilities of the multidimensional generalization of the problem: how can the Euclidean  $N$ -space be partitioned between congruent and regular polytopes, the  $N$ -dimensional generalizations of polyhedra? The goal was to specify multidimensional "honeycombs" with spherelike cells that completely fill up the space, or in socioeconomic terms, organizations with equal catchment areas in a product characteristics space. However, our survey of the mathematical literature revealed that the underlying tessellation problem has no regular and spherelike solution beyond two dimensions (Coxeter 1948). Under spherelikeliness, we mean that the catchment areas have similar extensions in each direction. For example, soccer balls are not really spheres,

but spherelike (not regular) polyhedra, their surface being composed of pentagons and hexagons.

We address the space-partitioning problem differently. We assume that catchment areas are  $N$ -dimensional spheres (hyperspheres). A hypersphere is a simple geometric object, its volume and surface depend only on its radius.<sup>2</sup> However, spaces cannot be completely covered by spheres without overlap: if  $N > 1$ , there is always some residual left between the objects. Putting it differently, some demand is always left unsatisfied by organizations with spherical, nonoverlapping niches. The ubiquitous presence of leftout space around spheres may seem a mathematical inconvenience. However, it has an explanatory function if one allows for the existence of specialist organizations. Specialists are characterized by their narrow niches (Brittain and Freeman 1980; Freeman and Hannan 1983; Péli 1997). They can populate residual regions or holes between organizations of broader niches (generalists).

We concentrate on these holes. We show that a field in geometry, the *sphere-packing problem*, provides new insights for the sociology of organizations. Applying this field to resource-partitioning theory, one can explain a lot of the dynamics of generalist-specialist markets. While the original theory explains resource-partitioning processes on the basis of scale economies and market center occupation, now the results will be obtained from the properties of  $N$ -dimensional arrangements.

The original and proposed resource-partitioning explanations are complementary: each explains aspects for which the other, alone, could not give an account. Resource inhomogeneity is assumed in the scale economy-based model: demand has uni- or polymodal distribution in space. On the contrary, the geometric approach assumes that demand is homogeneously distributed.<sup>3</sup> The two models can be seen as two layers of explanation for the same phenomena. The geometric explanation (flat demand distribution) serves as a background. The second layer adds complexity to the first in the form of peaks in the demand distribution.

This article is organized as follows. The following section summarizes the original resource-partitioning theory. What are the main predictions and explanatory elements? The next section comes up with the geometric model: similar outcomes but a different explanatory structure. The last section assesses the methodological benefits of the proposed explanation,

<sup>2</sup> We use the words *hypersphere* and *sphere* synonymously, though literally the second denotes 3-D objects.

<sup>3</sup> Actually, Carroll (1985) used first circular niches and homogeneous resource distributions in two dimensions in his first resource-partitioning work.

and it gives empirical illustrations for the outcomes. We also propose some further sphere-packing applications in other models of sociology.

## RESOURCE PARTITIONING WITH SCALE ECONOMIES

### Model and Ramifications

Carroll (1985) analyzed newspaper publishing in several local American markets, explaining the temporal dynamics of markets composed of generalist and specialist organizations. The market is an  $N$ -dimensional Euclidean space with axes that denote taste descriptors; thus, each point in space stands for a certain customer taste. Generalist organizations make appeals to a broad range of customer tastes, while specialists address specific ones. Accordingly, a generalist's niche is a broad region in the resource space, while specialists occupy small spots. The taste distribution is uneven over the population, and the market has a center (or a few centers) composed of mainstream tastes. Resources are abundant in the center, so the organizations there can grow to be large.

This setting gives rise to the following population history. Early in the market, the surviving firms are mainly generalists. To increase sales, generalists tend to differentiate themselves by differentiating their product offers, positioning their niches apart from each other. Product differentiation is a way to reduce competition (Eaton and Lipsey 1989), since niche overlap yields intense price competition. The occupant organizations of the resourceful central regions grow bigger than the others, and the induced increase in size yields scale economy advantages: the big firms get even bigger, forcing medium-size generalists out from the market (Rosse 1980). The number of generalist organizations falls, while their average size grows. Market concentration increases.

A crucial element in the model is that the life chances of the emerging small specialists are attached to the concentration level of generalists: high concentration opens little resource pockets for specialists. This happens as follows. As the relatively smaller generalists disappear, resources become unutilized. The surviving big generalists take the best chunks of the residual space, positioning themselves into the market centers. As the fight between generalists dies out, product differentiation loses its importance. The winner organizations now adjust their offers to the mainstream needs at the center. The surviving generalists increase their niche width, taking over the best parts of the extinct competitors' market segments. But as they move toward the market centers, they leave some customers unsatisfied at the edges. Small specialist organizations establish footholds in these market pockets. Taste distributions also often get flatter as markets develop, thus further increasing resource abundance at the edges. In the end,

there is no competition between the survivor generalists and the newcomer specialists.<sup>4</sup>

### Empirical Evidence and Questions

A rapidly growing research program in a broad variety of industries gives empirical support to the outlined theoretical picture. Predicted effects of resource partitioning were detected in the brewing industry (Carroll and Swaminathan 1992, 1993; Swaminathan and Carroll 1995), in banking cooperatives (Freeman and Lomi 1994; Lomi 1995), in wine production (Swaminathan 1995), in medical diagnostic imaging (Mitchell 1995), and in microprocessor production (Wade 1996). Earlier, Barnett and Carroll (1987) observed a symbiotic relation between telephone companies occupying different niches in the same location: these organizations exerted a positive influence on each other's fate. Recently, Dobrev (1997) analyzed the restructuring process of the Bulgarian newspaper industry during the era of political transition, applying the resource-partitioning framework to an environment substantially different from American markets. The research of Torres (1995) and Seidel (1997) found, respectively, resource-partitioning processes in the British automobile and American airline industries.

Studies on size-localized competition also display similar effects to those claimed by resource-partitioning theory (Hannan and Freeman 1977; Hannan and Ranger-Moore 1990; Hannan, Ranger-Moore, and Banaszak-Holl 1990). Organizations of very different sizes usually also differ in structure, and competition tends to be stronger among structurally similar organizations. Since generalists and specialists are usually quite different in size and in structure, the losers of size-localized competition are mostly the medium-size generalists.<sup>5</sup>

Some market effects are hard to explain only with scale economies. For example, scale economy effects magnify even minor size differences between generalists, finally leaving a single organization in place. In reality, more than one big generalist can be sustained in several types of markets. If we assume polymodal taste distributions, then a handful of generalists may survive in the resulting landscape, each occupying one market center.

<sup>4</sup> Boone and Witteloostuijn (1995), in analyzing the connection between organizational ecology and industrial organization, find similarities between the resource-partitioning model and Sutton's (1991) dual structure theory of industry concentration.

<sup>5</sup> Taking into account other aspects, medium-size generalists may have their chances. Investigating the Californian savings and loan industry, Haveman (1993) found that medium-size organizations are the most willing to diversify their activities into new markets.

The low-demand ditches around the centers keep the incumbent organizations away from appropriating the neighbor's domain. Though such outcomes do occur, still each local center can be occupied by only one player. More general solutions would require further aspects that affect survival, for example, institutional settings (Meyer and Scott 1983; DiMaggio and Powell 1983) or status and network positions (Podolny, Stuart, and Hannan 1996).

Two other effects to explain are specialists' presence in early markets and also in nonmarginal market segments. How can specialists make a foothold before the winning big generalists position to market centers and leave behind resources at the margins? If there is a market center, then how can some specialists persist in this region tightly controlled by big generalists? We will explain these phenomena as well as other resource-partitioning processes on the basis of niche positioning in densely packed markets.

### THE GEOMETRIC RESOURCE-PARTITIONING MODEL

We recapitulate the market history phases to explain why mainly generalists compete in the market, why their number decreases with time, how the surviving generalists broaden their niche, what resource pockets open up for specialists, and how generalists and specialists coexist in the market.

Competing generalists seek to cover as much resource space as possible under some restrictions. The first restriction is that generalist niches are symmetric in all directions. The second restriction is that the market is sufficiently resourceful to recover entry costs of setting up production and distribution. The third restriction is that generalist niches do not overlap as a free-entry equilibrium is reached. A fourth assumption adds dynamics to the model: customer tastes get elaborated with time. We address the conditions under which these restrictions are met and show how they lead to resource partitioning. Our model applies mathematical results from the sphere-packing field of geometry that we summarize next. Then, we address the conditions of niche sphericity and explain resource-partitioning processes in terms of the proposed model.

### The Sphere-Packing Problem

This field in geometry is concerned with ways of filling up  $N$ -dimensional Euclidean space with hyperspheres of equal size (Conway and Sloane 1988).<sup>6</sup> The main issue is to find dense packings, configurations where the

<sup>6</sup> Currently, see also on the Internet at <http://www.astro.virginia.edu/~eww6n/math/Hypersphere.html>

proportion of space occupied by spheres is high. The efficiency of a sphere packing is measured by *packing density* ( $\Delta$ ), the ratio of the volume occupied by the spheres to total space volume ( $0 \leq \Delta \leq 1$ ). Unfortunately, the solution of the sphere-packing problem is not known beyond three dimensions. In one dimension, the hyperspheres are sections of equal length along a line. If the neighboring sections meet, then packing density is unity (fig. 1a). In two dimensions, hyperspheres are circles. The densest packing has  $\Delta = 0.9069$  (fig. 1b). In three dimensions, the cannonball packing is the densest (fig. 1c). Fortunately, upper bounds on packing density can be calculated for each  $N$ , and the known densest packings approximate these bounds quite well (table 1). Maximal packing density converges fast to zero with  $N$ . This finding will play a central role in our argument.

### Niche Symmetry

Stinchcombe (1991) calls *mechanisms* those pieces of scientific reasoning that connect lower- and higher-level entities in theories. He argues that objects at the lower level can be conceptualized as very simple if this characterization sufficiently explains the higher-level outcomes (as molecules are modeled as little balls in classical gas theory). Resource-partitioning theory has two levels: organizational events appear as cumulative outcomes at the population level. We represent our lower-level objects (organizations) with spheres, thus obtaining a simple and powerful explanatory mechanism. However, the assumption that the organizations under investigation have spherical niches needs justification.

Organizational ecology makes a distinction between *fundamental* and *realized* niche (Hannan and Freeman 1989). The fundamental niche represents those resource configurations under which organizations persist in lack of competition. The realized niche is the subset of the fundamental niche in which organizations are perceived in case of competition.

The symmetry of the fundamental niche is a consequence of isotropy, the invariance of spatial directions. Isotropy follows from the homogeneity of the resource distribution. If spatial directions do not count, then other things being equal, organizations develop the same niche breadth in any direction. In reality, taste descriptors do differ in importance. This fact can be incorporated into the model by assigning a set of weights to the dimensions and performing affine transformations along each axis with these weights. Instead of having hyperspheres, then we arrive to the  $N$ -dimensional analogues of ellipses (in 3-D: rugby balls). Affine transformations do not affect volume ratios or the topology of the spatial arrangements, therefore the forthcoming geometric arguments also apply when



Figure 1a



Figure 1b

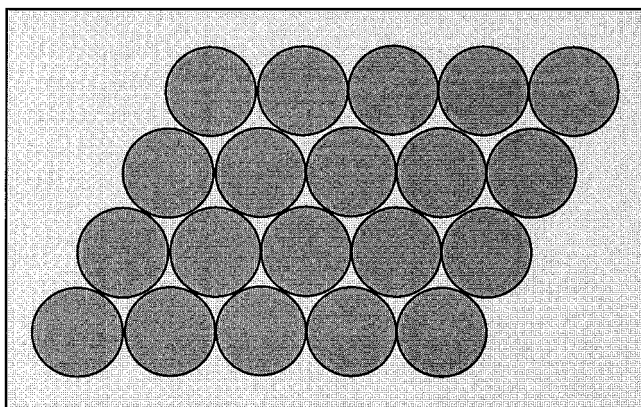


Figure 1c

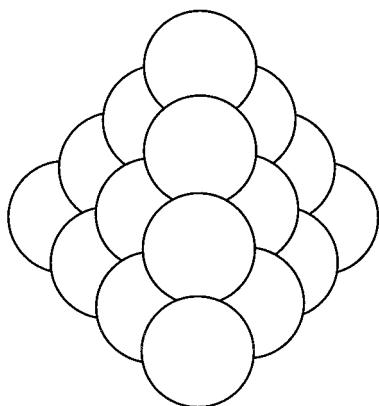


FIG. 1.—Dense sphere packings in 1–3 dimensions

TABLE 1

THE KNOWN DENSEST PACKINGS WITH KISSING  
NUMBERS AND THE KNOWN THINNEST COVERINGS

Dimensions ( <i>N</i> )	Packing Density ( $\Delta$ )	Kissing Number ( $\tau$ )	Thinnest Covering ( $\Theta$ )
1 .....	1	2	1
2 .....	.90690	6	1.2092
3 .....	.74048	12	1.4635
4 .....	.61685	24	1.7655
5 .....	.46526	40	2.1243
6 .....	.37295	72	2.5511
7 .....	.29530	126	3.0596
8 .....	.25367	240	3.6658
9 .....	.14577	272	4.3889
10 .....	.09962	372	5.2517
11 .....	.06624	519.78	6.2813
12 .....	.04945	756	7.5101
13 .....	.03201	1,060.67	8.9768
14 .....	.02162	1,422	10.727
15 .....	.01686	2,340	12.817
16 .....	.01471	4,320	15.311
17 .....	.008811	5,346	18.288
18 .....	.005958	7,398	21.841
19 .....	.004121	10,668	26.082
20 .....	.003226	17,400	31.143

SOURCE.—Conway and Sloane (1988, pp. 15, 38).

NOTE.— $\tau$  is not always an integer. If hyperspheres have different number of neighbors in a packing, then  $\tau$  is calculated as an average.

taste aspects differ in importance. For the sake of convenience, we assume that all taste variables are standardized, and then we can proceed with spheres instead of ellipses.

The shape of the realized niche is affected by the neighboring organizations that compete for the same resources. Realized niches also take a symmetric shape in case of a free-entry equilibrium. First, we assume that price discrimination is prohibited: one cannot offer the same product for different prices for different customers at the same time. So, the lowest price anywhere in the niche applies throughout the niche. Second, we assume that to the price for the product consumers add a cost related to the distance between their position in resource space (which represents the characteristics of their “ideal product”) and the position of the product as represented by the center of the niche. This cost reflects the compromise

consumers make in taking a product that does not exactly match their preferences. Given the impossibility of price discrimination, the price throughout the niche is now determined by the maximum product price plus distance cost that is still acceptable to the most distant customer. The bigger the distance to the most distant consumer, the lower the product price has to be to compensate for the higher distance cost. The trade-off in niche extension between a larger niche volume and a lower price (to “pull in” more distant customers) yields an optimal niche size, having the given number of spatial dimensions. Further niche extension reduces profit. Asymmetric niche extension is even more unprofitable: the price reduction needed to extend “reach” is insufficiently compensated by additional sales only in selected directions. For this reason, generalists tend to maintain symmetric niches.

### Dimensional Expansion: New Opportunities for Specialists

Now, we apply the sphere-packing field to organizational markets to explain resource partitioning. We assume that total demand is constant in time. When demand expands rapidly, then organizations easily find free resource, making the partitioning problem irrelevant. If demand decreases in time, then all forthcoming arguments hold a fortiori. As approximating their optimal niche breadth, generalists arrange their niches in a way to minimize competition. So generalist niches realize a sphere packings. Later, we will specify forces that penalize deviations from such an arrangement. Moreover, we consider tight market packings, when generalists’ niche configuration is close to the optimal. If huge market segments were left unexploited by generalists due to loose configuration, then specialists’ presence would be obvious.

We consider the increasing number of spatial dimensions as an explanatory variable. Since axes stand for taste descriptors, an increase in  $N$  reflects customer taste elaboration.<sup>7</sup> As customer demand gradually becomes more diversified, the resource space extends into new dimensions. Organizational bids also fold out into the extended space. For example, circular niches take spherical shape in moving from two to three dimensions. Maximal packing density persistently falls with  $N$  (fig. 2). This means that the percentage of total resource accessible for generalists becomes less and less. This is in line with the observation that the number of generalists decreases in the market with time.

<sup>7</sup> This article does not address other forms of demand elaboration like scale extension (offering extra size products) or scale refinement (intermediate sizes, qualities).

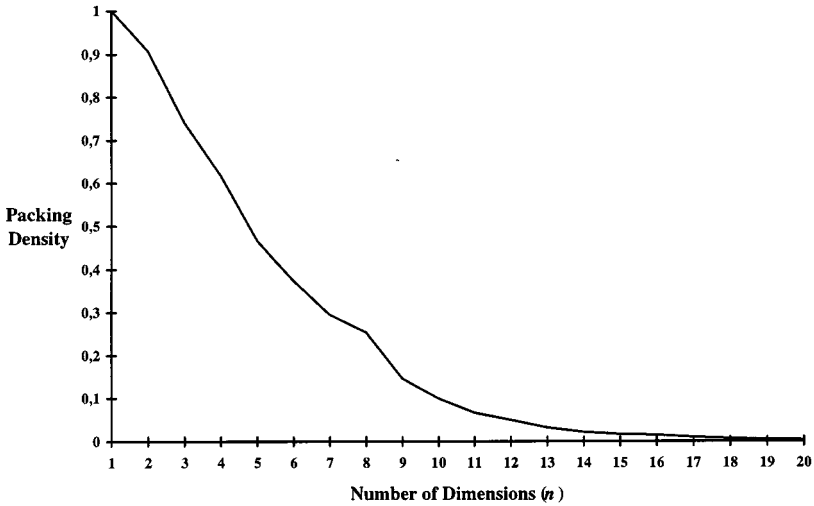


FIG. 2.—The density of the known densest packings

The most surprising conclusion in resource-partitioning theory, specialists' emergence in tightly packed markets, can be also explained with dimensional change. As  $N$  increases, pockets open between the generalist niches in which specialists can take footholds. The portion of residual space between spheres increases steeply at each dimension change (fig. 2). In one dimension, spheres are linear sections that can fill up the space without a residue: there are usually no specialists in markets where products are differentiated only in one dimension (or in none, like in some classical shortage economies, Kornai 1980). Moving to two dimensions, generalists' resource utilization decreases to about 90% (table 1). Since tastes develop in time, this prediction is in line with specialists occasional presence in early markets. When moving from two to three dimensions, generalists' maximal resource share falls to 75%. The next few dimension shifts yield roughly a 20% loss per step. The decline even gets steeper from  $N = 8$ . In 10 dimensions, generalist niches would cover less than 10% of the resource space.

Note that the generalist organizations leave unabsorbed resources in every market segment. So, specialists can make footholds at any taste regions, not only at the margins. However, because we assumed flat resource distribution, our approach does not explain why specialists' occurrence is more frequent at nonmainstream tastes. Here, the original and the geometric resource-partitioning explanations complement each other. Imagine the homogeneous resource distribution as an elastic membrane in two di-

mensions. Add inhomogeneities to the flat surface; then humps will stand for market centers. According to Carroll's (1985) model, competition is stronger at the center, and generalists are better competitors than specialists due to size effects. So, opportunities do open for specialists in every market segment (geometric explanation). However, not all these opportunities are necessarily taken by specialists, because their survival chances are much better at the margins than in the center (original explanation).

### Broader and Nonoverlapping Generalist Niches

Resource-partitioning theory claims that the survivor generalists' increase their niche breadth. Niche extension is unprofitable beyond a limit having a given number of spatial dimensions. However, generalists have to increase their niche width if the number of spatial dimensions increases. In higher  $N$ s, customer demand is distributed along a higher number of taste "cells" (just like a hectare, a square with 100 meter edges contains  $100^2$  square meters). Resource density gradually thins out having constant total demand and more spatial dimensions. A spherical niche in the  $N + 1$  dimensional market typically occupies a much lower share of total resource than a niche of the same radius in  $N$  dimensions. Losing volume percentage means losing sales. To preserve market share, generalist organizations have to increase their niche width after each dimensional shift. In the original resource-partitioning model, generalists occupy new territories as a reward of winning the competition. In the new model, generalists have to increase their niches not to lose sales.

Resource-partitioning theory also claims that competition between generalists lessens with time. Thick niche overlap could exclude specialists, since any space can be completely covered by overlapping hyperspheres. But niche overlap causes head-on price competition, which further reduces price. In pure price ("Bertrand") competition, profits are eroded to zero, which, as a result of the impossibility of price discrimination, applies throughout the niche. In "Cournot" competition, on the basis of sales volume, profits are still positive but less than when niche overlap is evaded. Here profit declines with an increasing number of competitors in the overlap.<sup>8</sup>

Overlapping generalists face a steeply increasing number of competitors as space dimensionality increases, and the overlap also becomes thicker with  $N$ . If generalists cover the whole resource space by overlapping niches, then another mathematical notion, the *thickness of the covering* ( $\Theta$ ) applies. While the sphere-packing problem is about densely filling up

<sup>8</sup> See more on Bertrand and Cournot competition in Shapiro (1989).

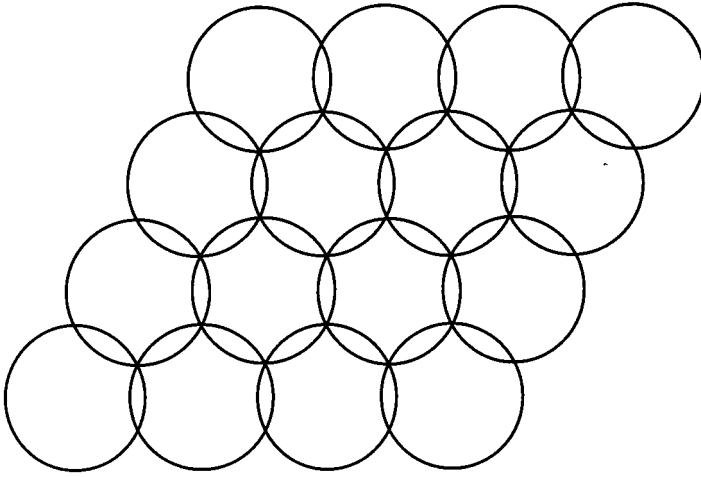


FIG. 3.—The thinnest space covering in two dimensions

the space with touching spheres, the space-covering problem searches the thinnest covering with overlapping hyperspheres of equal size. Covering thickness ( $\Theta$ ) measures overlap, telling the average number of spheres that contain a given point in space. If each point is occupied by exactly one sphere, then  $\Theta = 1$ . But beyond one dimension,  $\Theta > 1$ .

The arrangement with least overlap in two dimensions has  $\Theta = 1.2092$  (fig. 3). Just like densest packings, the thinnest coverings are only known up to two dimensions. But lower bounds on  $\Theta$  are given for each dimension, and the known best values well approximate these bounds if  $N$  is not very high. Covering thickness steeply increases with  $N$  (table 1; fig. 4). Beyond four dimensions,  $\Theta > 2$ ; that is, more than two producers compete for a customer on the average. Since there is no overlap near the sphere centers, this means much more than two offers for a customer in the overlap. Complete market covering with generalists ignites an increasingly strong competition as the resource-space dimensionality increases.

Generalists also have many more potential competitors as the number of taste dimensions increases with  $N$ . The *kissing number* ( $\tau$ ) denotes the number of neighboring spheres that touch a certain sphere in a packing. The kissing number that now measures the number of competitors increases extremely fast with  $N$ . For example, for the known best packings, the kissing number is 24 at  $N = 4$ , but it is 240 at  $N = 8$  (table 1; fig. 5). Massively growing covering thickness and high kissing numbers with increasing resource-space dimensionality: these effects make the occupa-

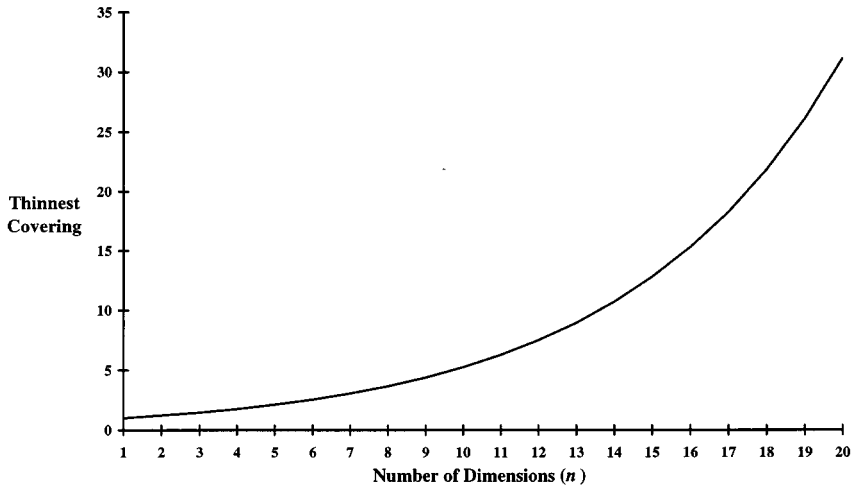


FIG. 4.—The known thinnest coverings

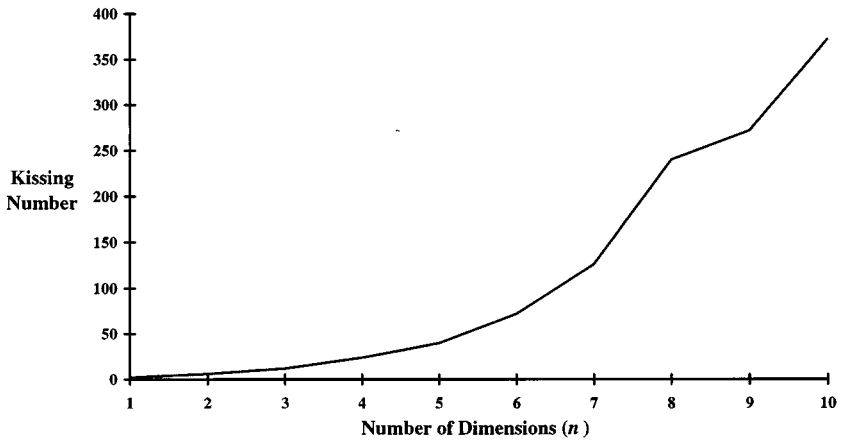


FIG. 5.—Kissing numbers in the known densest packings

tion of residual space increasingly disadvantageous for generalists. This allows for some specialists to utilize the resource pockets and to survive.

#### DISCUSSION

We address three topics in this section. First, we give empirical illustrations to specialist organizations' emergence when new spatial dimensions

open. Second, we compare the proposed model to other space models in sociology and sketch some applications of the sphere-packing approach beyond the ecology of organizations. Finally, we summarize the core conclusions and mention some future research directions.

### Empirical Illustrations

One aspect for the empirical testing of the model is the measurement of niche shape: is it symmetric when market equilibrium is reached? This task would require detailed sales distribution data along the relevant taste dimensions for each firm. The empirical studies on the standard resource-partitioning model do not address niche sphericalness. However, our model can be tested by checking if its main predictions conform with the empirical findings in this field. This section illustrates some core predictions with examples from industries in which resource-partitioning processes have been pointed out. Since the predictions of the original and the geometric explanations mostly coincide, the forthcoming examples support both models. But there is one important aspect in which the predictions of the two models differ: the geometric approach predicts resource pockets also at nonmarginal tastes. The forthcoming examples support this picture: specialists can be observed even at mainstream positions but much less frequently than at marginal tastes.

In the beer market, production is subject to economy of scale, as is generally the case in process technology industries, so in principle generalist producers have a cost advantage. There is also a scale effect in access to distribution channels and in building up brand names through advertising. In the beer industry, brand image is of great importance and is not easily built up. Evidence for this is given by the fact that Heineken, for many years, produced beer at home and shipped it to the United States, thus incurring the enormous cost of transporting mostly water across the ocean, to maintain its credibility as a Dutch brand. However, an interest in special tastes has emerged, yielding opportunities for specialist producers (Carroll and Swaminathan 1992, 1998; Carroll, Preisendörfer, Swaminathan, and Wiedenmayer 1993). Generalists cannot outcompete specialized tastes by price discrimination, and they have trouble selling specialty beers of their own. Moreover, it is obvious in the U.S. market that this is a temporal coincidence between the rise of specialty brewers on the one hand and an increase in the number of dimensions that customers use in evaluating and purchasing beer on the other hand. Specialty beers reside in an expanded product space that did not exist earlier; new dimensions of flavor, color, and ingredients have become operative in consumer decision making. According to the geometric model discussed here, this expansion makes it easier for specialists to enter and survive.



In retailing, there are economies of scale in unified purchasing and logistics, shop design, and fittings across many shops, which yields an advantage for chain store corporations. This requires a certain standardization in the range of products and qualities offered. Small, specialized independents have a potential advantage in tailoring their product range more finely to local demand. Large shops cannot counter this with price discrimination. And it would cost them too much to obtain similar fine-grained knowledge of all the peculiarities of local demand, and, to cater to it, they would have to compromise too much on the standardization of product range from which they obtain their cost advantage. This explains the existence of small independents next to chain stores. Note, however, that developments in information and communication technology (ICT) offer opportunities for “mass customizing,” whereby large generalists can more easily differentiate their products. This may yield a countervailing effect. Some specialist products of yesterday can be generalist products today, the content of specialization is subject to change in time. Specialists have to come up with new product aspects, thus contributing to the introduction of new dimensions to the market.

Newspaper publishing offers economies of scale in production, news gathering, printing, and distribution. But local publishers can tailor the news to local conditions. Generalist publishers would compromise the economies of large-scale production by tailoring the news to local readers. They would encounter search problems in gathering the news and may encounter problems of acceptance and credibility as a nonlocal producer. Here again, recent developments in information and communication technology bring new elements into the picture. On the one hand, ICT offers flexible combinations of local, national, and global news that are locally offered by generalist producers. On the other hand, generalists might still encounter the problems of local search and credibility. Economies of scale in printing and distribution can be offset by making specialized, idiosyncratic information available on the Internet. The availability of such specialized information might stimulate even greater demand for local news. A recent development is the boost of electronic publishing on the Internet: a new, resourceful spatial dimension is emerging, giving rise to a broad variety of “local papers” in the cyberspace (newsgroups, electronic bulletins, Web sites of specific interest groups) that coexist with the professionally designed and frequently visited sites of reputable “generalist” publishers.

### Sphere-Packing and Crowding Models in Social Science

Bioecologists like Levins (1968), Hutchinson (1978), and Roughgarden (1979) consider niches as the set of environmental conditions, represented

as a part of the resource space, under which populations of a species are sustained. Hannan and Freeman (1977, 1989) adopted this view for organizational populations. Since the objects under investigation (species, organizations) often can be observed in certain parameter ranges (sections) along each resource axis, a convenient way to conceptualize niches is to consider them as  $N$ -dimensional rectangles (Hutchinson 1978; McPherson 1983). We have already emphasized why niche sphericalness is crucial in our model. Now, we argue that an approach that assumes rectangular niches only works if  $N$  is low.

The niche dwellers that stay far from the niche center in several dimensions face multiple disadvantages. For example, if someone is not pleased with the color of a pair of trousers, then that person will be even more displeased if the size does not fit either. Geometrically: if staying close to a niche edge is bad, then staying close to a niche vertex is even worse in rectangular niches. The misfit gets bigger as  $N$  increases. Consider hypercubes of unity edge as special rectangles. In  $N$  dimensions, the Euclidean center-vertex distance is  $0.5\sqrt{N}$ , while the smallest distance from the center to the cube's surface is 0.5. That is, the ratio of the biggest to smallest center-edge distances is  $\sqrt{N}$ , that goes to infinity with  $N$ . This ratio is 1.41 in two dimensions, and it exceeds two beyond  $N = 4$ . Spherical niches are exempt from this problem. Note, however, that niches can be rectangular if the problem under investigation requires some specific non-Euclidean metrics (see more in Freeman 1983).

The application of the sphere-packing problem goes far beyond bioecology and organizational ecology. It can have a bearing on political sociology: how should political parties position their catchment areas in the space of potential voters, minimizing both residual space and overlap? The sphere-packing problem has connections to multidimensional data evaluation. For example, the task of finding appropriate cluster centers in cluster analysis is related to the *quantizer problem* (Conway and Sloane 1988), which also goes back to the search for optimal sphere packings. Here, we sketch the application of sphere packings in two well-known network theories: the McPherson affiliation model (McPherson 1983; Popielarz and McPherson 1995) and the structural hole theory of Burt (1992).

The network research of McPherson and his colleagues addresses group formation dynamics. A central explanatory element in their argument is the homophily principle (Blau 1977): people with similar sociodemographic positions are more likely to form voluntary groups. Moreover, group membership duration is positively affected by the similarity of members. To represent individuals' affiliation drives, "social circles" are drawn around potential group members. If a great part of a member's social circle falls into the niche of the focal organization, then membership is stable (Popielarz and McPherson 1995). However, this line of research

applies rectangular niches for the modeled organizations. Applying spherical niches for the studied organizations instead of rectangles, one arrives at a new sphere-packing application, which we briefly indicate.

The affiliation argument says that group members in marginal niche positions typically have a higher number of destabilizing external ties, and therefore they are more likely to leave the organization. To reduce the ratio of uncertain members at the edges, it is in the interest of the organization to minimize its niche boundary surface. Hyperspheres have the minimal surface to volume ratio among  $N$ -dimensional bodies: hence, another argument for spherical niche shape. Niche overlap is also bad for voluntary organizations because it causes head-on competition for members. We showed that the minimal thickness of sphere coverings strongly increases with  $N$ . In higher dimensions, overlap is more disadvantageous, so voluntary groups will be wary of overstretching their niche. But, overlap elimination entails empty pockets in the sociodemographic space. Hence come the following two predictions: (1) High dimensional sociodemographic spaces facilitate compact group formation. (2) High dimensional sociodemographic spaces open opportunities for small human groups with very similar members.

Assuming that the texture of the sociodemographic space becomes more elaborated with time, the second prediction is in line with the perceived upsurge of a broad variety of alternative groups in the last decades. Research on human group formation operates with operationalizable dimensions. Therefore, the affiliation model offers an opportunity to test the empirical relevance of the sphere-packing approach.<sup>9</sup>

Another promising application is Burt's (1992) structural hole theory. Structural holes separate nonredundant ties in a network. Because maintaining ties has costs, agents in the network seek to optimize their connections by minimizing redundancy (thus maximizing the number of structural holes). This brings us to a potential application of sphere packings. In a theoretical paper, Linton Freeman (1983) investigates network embeddings into multidimensional spaces: then nodes are characterized by  $N$  coordinates. A maximal distance,  $\delta$ , is introduced beyond which connection between nodes is not possible. Consider a focal agent (ego) in the network with direct access to a number of others who are all mutually insulated from each other. In Burt's terminology, ego has nonredundant ties under the criterion of cohesion (1992, p. 18). Is there an upper bound for ego's nonredundant ties in the  $N$ -space? Putting it differently, what is the densest *star* graph in  $N$  dimensions? Choosing  $\delta/2$  as sphere radius, the problem reduces to the search for the maximal kissing number in the

<sup>9</sup> We are indebted to a reviewer who drew our attention to this possibility.

given number of dimensions (Freeman 1983): how many touching hyperspheres can be placed around a focal sphere? The kissing number ( $\tau$ ) minus one delivers an upper bound for ego's nonredundant ties. Because each pair of ties is insulated by a hole, the maximal number of structural holes around ego is  $(\tau - 1)(\tau - 2)/2$  in  $N$ -dimensional networks. Note that a similar argument would apply to a variety of network studies (e.g., Podolny and Stuart 1995; Stokman and Zeggelink 1996) were they rephrased in terms of network ties embedded to the  $N$ -space of sociological descriptors.

### Summary and Directions for Future Research

The common underlying theme in this paper is the sociological application of the sphere-packing problem. The key concepts of this geometry domain (packing density, covering thickness, kissing number) can be coupled with some basic features of the addressed sociological models. Maximal sphere-packing density drastically decreases, while the covering thickness and the kissing number steeply increases with  $N$ . So, if space dimensionality increases in the modeled sociological theory, then the new geometric conditions necessitate reconfiguration.

We chose organizational ecology's resource-partitioning theory as a focal application and reconstructed the empirically documented phases of the resource-partitioning process between generalist and specialist organizations. How do resource pockets open for specialists? Why does the number of generalists decrease, and why do generalist niches broaden with the number of spatial dimensions? What makes niche overlap increasingly costly with  $N$ , putting a brake on generalist competition? Beyond resource-partitioning theory, the sphere-packing domain also applies to a variety of crowding models in social science. We found applications to explain voluntary group formation dynamics and to estimate the maximum number of nonredundant ties in  $N$ -dimensional star networks.

We mention three directions for future research. First, one can allow for size differences between generalist niches even if much of the mathematical rigor of the original setting would be lost. The result might resemble soap foam: there are large bubbles of different sizes, and the fluid between them is filled up with little bubbles. Second, not only the densest sphere packings can be interesting for sociology. For example, the relatively simple *square-lattice* packing (partition the space with hypercubes and place a maximal sphere into each) can reflect a copying mechanism in organizational positioning ("copy your neighbors, but differentiate your offer at least in one dimension"). A third research direction can address the reverse of the resource-partitioning story, when  $N$  decreases in time.

Then, the geometric model predicts that collapsing space dimensions will sweep away specialist organizations.

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