Cepheid models based on self-consistent stellar evolution and pulsation calculations

Baraffe, I; Alibert, Y; Mera, D; Charbrier, G; Beaulieu, Jean-Philippe

THE ASTROPHYSICAL JOURNAL, 499:L205–L209, 1998 June 1

CEPHEID MODELS BASED ON SELF-CONSISTENT STELLAR EVOLUTION AND PULSATION CALCULATIONS: THE RIGHT ANSWER?

ISABELLE BARAFFE, YANN ALIBERT, DOMINIQUE MÉRA, AND GILLES CHABRIER
Centre de Recherche Astronomique de Lyon (Unité Mixte de Recherche 5574 of CNRS), Ecole Normale Supérieure, 69364 Lyon Cedex 07, France; ibaraffe@ens-lyon.fr, yalibert@ens-lyon.fr, dmera@ens-lyon.fr, chabrier@ens-lyon.fr
AND
JEAN-PHILIPPE BEAULIEU
Kapteyn Astronomical Institute, Postbus 800, 9700 AV Groningen, The Netherlands; beaulieu@astro.rug.nl
Received 1998 February 13; accepted 1998 April 1; published 1998 May 8

ABSTRACT

We have computed stellar evolutionary models for stars in a mass range characteristic of Cepheid variables ($3 < m/M_{\odot} < 12$) for different metallicities representative of the Galaxy and the Magellanic Clouds populations. The stellar evolution calculations are coupled to a linear nonadiabatic stability analysis in order to get self-consistent mass-period-luminosity relations. The period-luminosity (*PL*) relation as a function of metallicity is analyzed and compared with the recent EROS observations in the Magellanic Clouds. The models reproduce the observed width of the instability strips for the SMC and LMC. We determine a statistical *PL* relationship, taking into account the evolutionary timescales and a mass distribution given by a Salpeter mass function. Excellent agreement is found with the SMC *PL* relationship determined by Sasselov et al. The models reproduce the change of slope in the *PL* relationship near $P \sim 2.5$ days discovered recently by the EROS collaboration (Bauer; Bauer et al.) and thus explain this feature in terms of stellar evolution. Some discrepancy, however, remains for the LMC Cepheids. The models are also in good agreement with beat Cepheids observed by the MACHO and EROS collaborations. We show that most of the first-overtone/second-overtone beat Cepheids have not yet ignited central helium burning; they are just evolving off the main sequence toward the red giant branch.

Subject headings: Cepheids — stars: evolution

1. INTRODUCTION

Over the past 5 years, tremendous efforts have been devoted to the search for dark matter through microlensing effects. This systematic search has provided two large databases of variable stars in the Magellanic Clouds due, respectively, to the EROS group (Renault et al. 1996, Beaulieu & Sasselov 1996, and references therein) and the MACHO group (Welch et al. 1996 and references therein). The wealth of data collected for Cepheids by both projects provides a unique tool for analyzing period-luminosity relations and their sensitivity to metal abundances. A significant number of beat Cepheids, oscillating in both the fundamental (F) and first-overtone (1H) modes or the first- and second-overtone (2H) modes have also been observed in the Magellanic Clouds.

The analysis of the EROS Cepheids (Sasselov et al. 1997, hereafter S97) in the small (SMC) and Large (LMC) Magellanic Clouds has recently revived the old debate regarding the metallicity dependence of the period-luminosity (*PL*) relationship (see Tanvir 1996 for a review). A careful comparison of the observed *PL* relations in both clouds enabled S97 to disentangle from differential distance and reddening a significant metallicity effect, which is not predicted by theoretical analysis (Chiosi, Wood, & Capitano 1993).

Analysis of beat Cepheids based on the OPAL opacity data set (Iglesias & Rogers 1991, 1996) now yield good agreement for Galactic bump and beat Cepheids (Moskalik, Buchler, & Marom 1992; Christensen-Dalsgaard 1993; Christensen-Dalsgaard, & Petersen 1995), but discrepancies still remain regarding the Magellanic Clouds (Christensen-Dalsgaard & Petersen 1995; Buchler et al. 1996; Morgan & Welch 1997, hereafter MW97).

None of the aforementioned analyses are really consistent since the stability analysis is based on envelope models using input physics that is different from the physics entering the stellar evolution calculations. Moreover, the width of the instability strip and, in particular, the position of the blue edge are deduced from the stability analysis, arbitrarily varying $T_{\rm eff}$ for a given mass-luminosity (*ML*) relationship, independently of any stellar evolution constraint on $T_{\rm eff}$. The aim of this Letter is to present fully consistent stellar and pulsation calculations based on the most recent opacity data set. In § 2, we present the stellar and pulsation calculations. Section 3 is devoted to the comparison with the observed *PL* relationships for fundamental mode pulsators in the SMC and LMC. The beat Cepheids are analyzed in § 4, and conclusions are given in § 5.

2. STELLAR EVOLUTION AND PULSATION CALCULATIONS

Our stellar evolution calculations include the updated OPAL opacities (Iglesias & Rogers 1996) for $T > 6000$ K. For lower temperatures, we use the Alexander & Fergusson (1994) opacities. Convection is based on the mixing-length theory with a mixing-length parameter $l_{\rm mix} = 1.5H_p$. The onset of convective instability is based on the Schwarzschild criterion, without either overshooting or semiconvection prescriptions.

The linear nonadiabatic stability analysis is based on a radial pulsation code originally developed by Lee (1985; Lee & Saio 1989). The convective flux $F_{\rm conv}$ is assumed to be frozen-in; i.e., the perturbations $\delta F_{\rm conv}$ are neglected in the linearized energy equation.

The stellar evolution and pulsation codes are coupled. This yields a fully consistent mass-luminosity-period-evolutionary time relationship. We have computed stellar models in the mass range 3–12 $M_{\odot}$ with various chemical composition $(Z, Y) = (0.02, 0.28)$, $(0.01, 0.25)$, and $(0.004, 0.25)$, representative of, respectively, the Galactic, LMC, and SMC environments. The

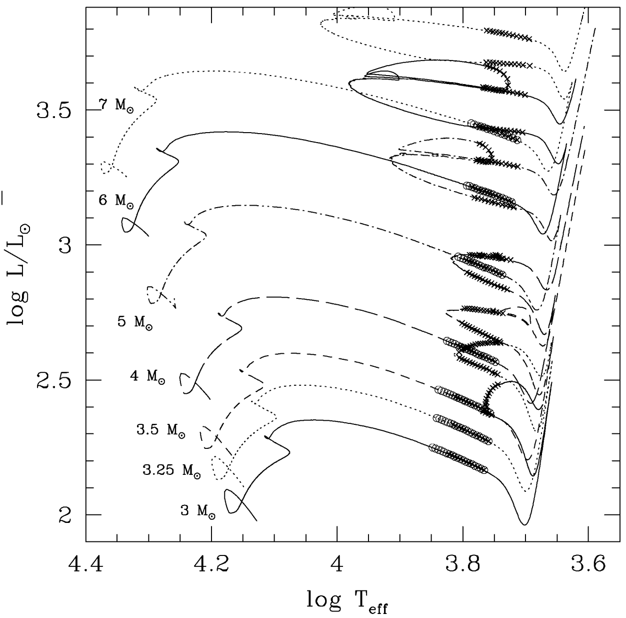

FIG. 1.—Theoretical HRD for stars with $Z = 0.004$, $Y = 0.25$, and different masses. The open circles and the crosses indicate the location of fundamental unstable modes. The distinction between both symbols is explained in the text (§ 2).

detailed evolutionary tracks will be published elsewhere, and in the present Letter, we focus on the main results. Figure 1 shows the evolutionary tracks of $Z = 0.004$ stars in a theoretical Hertzsprung-Russell diagram (HRD), for $m = 3$–7 $M_\odot$. Unstable models are indicated by either open circles or crosses. The distinction is purely artificial: the crosses refer to the usual Cepheid "blue loop" instability that occurs during the central helium-burning phase, while the open circles correspond to the first rapid crossing of the HRD when the stars evolve off the main sequence toward the red giant branch on a thermal timescale. This phase, of typically $\sim 10^4$ yr, is referred to in the following as the "first crossing instability." Since this phase is about 1/100 shorter than the blue loop one, the large database now available may allow the detection of some Cepheids in the first crossing instability.

As shown in Figure 1, the extension of the 3 $M_\odot$ blue loop (*crosses*, *solid line*) is significantly reduced, compared with higher mass stars. In this case, the blue edge is determined by the *turnover of the evolutionary track*. This reduction of the blue loop extension is expected from stellar evolution since, during core helium-burning, blue loops start at a minimum mass $m_{\rm min}$ and vanish for $m \gtrsim 0$ $M_\odot$, depending on the initial composition and the convective mixing treatment (cf. El Eid 1994). Note that $m_{\rm min}$ decreases with metallicity. We find $m_{\rm min} \sim 3$ $M_\odot$ for $Z = 0.004$, $m_{\rm min} \sim 3.75$ $M_\odot$ for $Z = 0.01$, and $m_{\rm min} \sim 4.5$ $M_\odot$ for $Z = 0.02$.

3. PERIOD-LUMINOSITY RELATIONSHIPS

In Figures 2*a* and 2*b*, we compare our calculations with observed fundamental mode pulsators in the LMC and SMC in a P-M_V diagram. The EROS (B_E, R_E) magnitudes were transformed into the Johnson V magnitude according to Beaulieu et al. (1995). We adopt the extinction $E(B-V) = 0.10$ for the LMC and $E(B-V) = 0.125$ for the SMC, with $R_V = 3.3$ and distance moduli $(m-M)_0 = 18.5$ for the LMC and $(m-M)_0 = 19.13$ for the SMC (cf. Beaulieu et al. 1997a). A comparison is also made with the Laney & Stobie (1994, hereafter LS94) data, which extend to longer periods than the EROS data. The transformation of the theoretical quantities (L, $T_{\rm eff}$) into M_V are based on Allard & Hauschildt's (1998) most recent atmosphere models. These models, originally developed for M dwarfs, do not extend below $\log g < 3.5$. We therefore use the bolometric corrections at constant $\log g = 3.5$ but take metal-

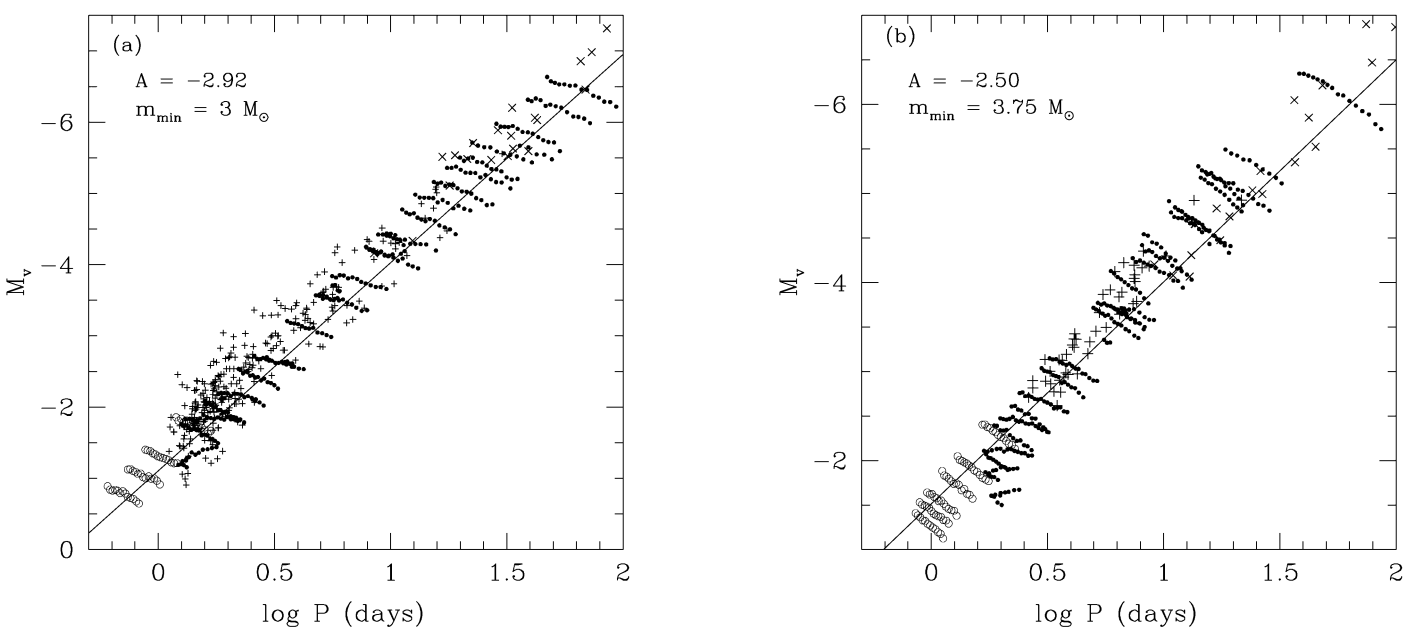

FIG. 2.—(*a*) Period-magnitude relationship for the SMC. Observations are from EROS (*plus signs*) and LS94 (*crosses*). The models are for $Z = 0.004$ and $Y = 0.25$. The filled circles correspond to the core He-burning unstable models. The open circles correspond to unstable models in the first crossing instability phase (see text). The straight line is the average statistical P-M_V relation derived from the present models (see text). The slope and minimum mass are indicated in the upper left-hand corner. (*b*) Same as (*a*), but for LMC data and models with $Z = 0.01$ and $Y = 0.25$.

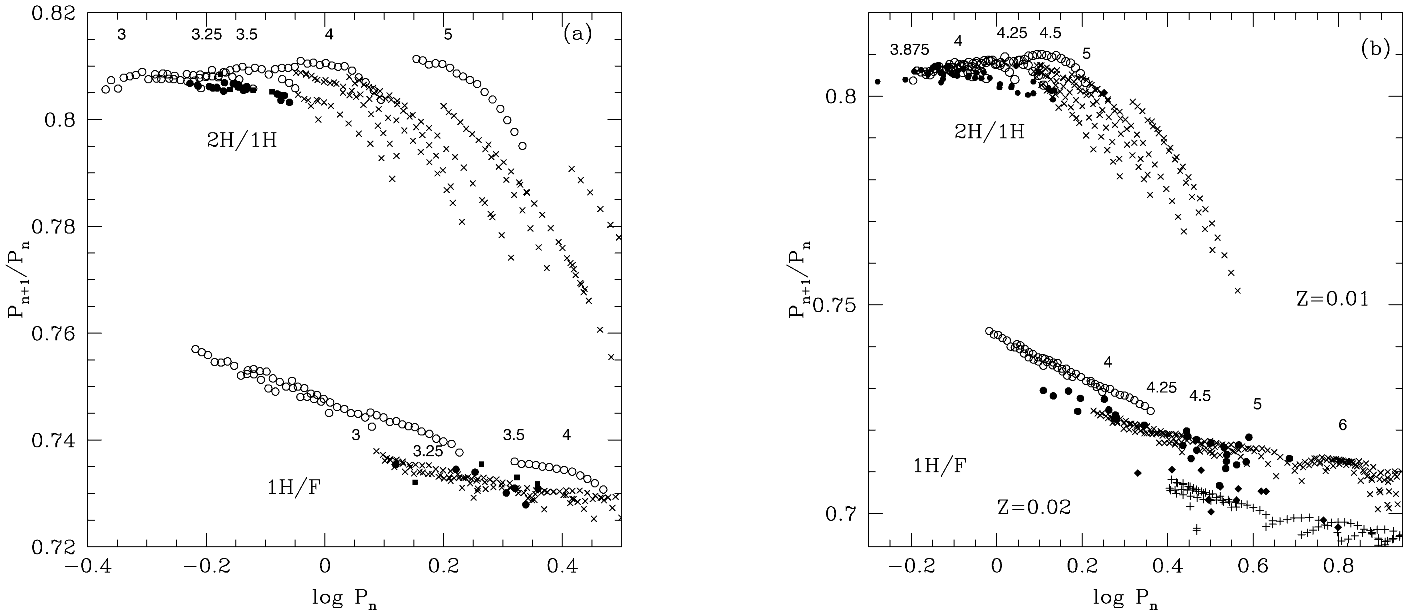

FIG. 3.—(*a*) Petersen diagram for SMC models with $Z = 0.004$. The observations are from MACHO (*filled circles*) and EROS (*filled squares*). The crosses correspond to unstable models during the He-burning blue loop. The open circles correspond to unstable models in the first crossing instability (see text). The corresponding masses are indicated for both F/1H (lower sequence) and 1H/2H (upper sequence) mode pulsators. (*b*) Same as (*a*), but for LMC models with $Z = 0.01$ and $Y = 0.25$. The F/1H Galactic pulsators of Szabados (1988) are added (*filled diamonds*) and compared with $Z = 0.02$ models (the lowest sequence, *plus signs*). The corresponding masses are indicated for $Z = 0.01$. The $Z = 0.02$ models are only shown during the He blue loop and correspond to 4.5–6 $M_\odot$ from left to right.

licity effects into account. We verified, however, that gravity effects in the *V* band are small.

We determine a theoretical statistical P-M_V relationship under the form $M_V = A \log P + B$ with a weighted least-squares fit by minimizing the quantity:

$$Q = \sum_i \alpha_i \, (M_{V_i} - A \log P_i - B)^2 ,$$

where the summation extends over all the stellar masses. The coefficients $\alpha_i(m_i,t)$ depend on the mass distribution and on the evolutionary times. We adopt a Salpeter mass function (MF) $dN/dm \propto m^{-2.35}$, and the time dependence derives directly from the coupled evolution and pulsation calculations. Although the present-day MF in the Magellanic Clouds may differ from an initial Salpeter MF, we verified that variations of the slope of the MF between -2 and -4 barely affect the slope of the *PL* relationship. Such a steep MF favors the lowest mass stars because of (i) the number of stars itself and (ii) the longer time spent in the instability strip as the mass decreases. The slope of the *PL* relationship is thus hardly affected by stars with $m \geq 7\ M_\odot$.

In Figure 2*a*, a comparison is made between the SMC observations and the $Z = 0.004$ models with masses $m = 3$–12 $M_\odot$. The first crossing unstable models are indicated by open circles, but only for $m \leq 4\ M_\odot$. Although included in the calculations, the first crossing instability phase for $m \geq 5\ M_\odot$ is statistically insignificant, since the time spent during this phase is more than 300 times smaller than the time spent by a 4 $M_\odot$ in the instability strip (first crossing and blue loop). We thus predict the existence of Cepheids with periods $P \lesssim 1$ day, as the signature of the first instability strip, but the observation of these objects requires larger statistics. The models agree reasonably well with the observed width of the instability strip, although they do not reach the observed blue edge of the EROS data. The faintest objects ($M_V \geq -1.5$, $\log P \leq 0.4$) seem to indicate a change of slope in the *PL* relationship, becoming steeper for $\log P \lesssim 0.4$, as discovered in the EROS-2 Cepheid sample and analyzed carefully by Bauer (1997) and Bauer et al. (1998). We note that such a trend is observed in the theoretical relation near ~3 $M_\odot$. This change of slope is thus real and stems from stellar evolution, illustrating the reduction of the He blue loop as mass decreases (cf. § 2). Note also that the period of the minimum mass undergoing a blue loop (~3 $M_\odot$) is consistent with the faintest observed SMC Cepheids. Finally, the average slope of our P-M_V relationship is $A = -2.92$, in excellent agreement with the slope derived by S97.

Figure 2*b* shows the results for the LMC with $Z = 0.01$ and masses from 3.75 to 12 $M_\odot$. As in Figure 2*a*, the models reproduce correctly the observed width of the instability strip, and the overall agreement with observations is good. We note that the minimum theoretical "unstable" mass undergoing a blue loop $m_{\rm min} \sim 3.75\ M_\odot$ for $Z = 0.01$ does not correspond to the faintest objects observed, which correspond to ~4.25 $M_\odot$. We predict that fundamental pulsators with $\log P \lesssim 0.3$ should be in the first crossing instability. The slope of the *PL* relationship with a minimum mass of 4.25 $M_\odot$ is $A = -2.50$, shallower than the one observed in the LMC by S97. In order to test the influence of chemical composition, we have recomputed the whole grid of models with ($Z = 0.008$, $Y = 0.25$) and ($Z = 0.01$, $Y = 0.28$), without any substantial modification of the theoretical slope. For solar metallicity models, we also find a slope $A = -2.55$ shallower than the one observed in the Milky Way. A detailed analysis of the possible sources for such discrepancies is in progress.

4. BEAT CEPHEIDS

Figures 3*a* and 3*b* display the Petersen diagrams for SMC (Fig. 3*a*), LMC, and Galactic (Fig. 3*b*) beat Cepheids. The data are taken from EROS (Beaulieu et al. 1997b) and MACHO (Alcock et al. 1995, 1997) collaborations. A comparison is also made with Galactic F/1H pulsators (Szabados 1988). An agreement with the F/1H mode pulsators is excellent for the three metallicities, except for the shortest periods of the LMC objects. The first crossing unstable models are also displayed but predict slightly too high 1H/F period ratios and extremely short fundamental periods. A variation of Y from 0.25 to 0.28 does not affect significantly the period ratios, and the $Z = 0.008$ models give slightly higher period ratios than observed in the LMC, by 1%–2%. The present models, however, yield a better agreement with observations than the one obtained by MW97 using the Chiosi (1990) *ML* relationship with overshooting.

The most striking new result shown in Figure 3 concerns the 1H/2H pulsators, which are reproduced by models in the first crossing instability. Indeed, low luminosities are required to reproduce the observed period ratios at very short periods, as noted by MW97. The luminosities during the He-burning blue loop are already too large and yield too high P_1, as indicated by the crosses in Figure 3 and as already noted by MW97. In order to reproduce the observed shortest periods, they need masses $m < 2\ M_\odot$. Figure 1 shows, however, that the evolution during the first crossing takes place at a significantly fainter luminosity and reaches the shortest periods observed with masses $m \geq 3\ M_\odot$. We find that at least four of the faintest 1H/2H beat Cepheids of the Beaulieu et al. (1997b) sample can be explained by models in the first crossing instability. This is statistically reasonable, considering that the total number of single-mode Cepheids (F or 1H) found by EROS in the SMC is 400. Since the lowest masses contribute dominantly to this total number, the probability of having four beat Cepheids in this sample in the first crossing instability, i.e, $4/400 \sim 1\%$, is consistent with the ratio of the time spent in the first crossing instability phase to the total time spent in the instability strip (F and 1H instability strips included) (cf. § 2).

5. CONCLUSIONS

We have computed self-consistent calculations between the stellar evolution and the linear stability analysis for SMC, LMC, and Galactic chemical compositions. This first consistent analysis yields a good general agreement with the observed P-M_V diagrams in the SMC and LMC and reproduces reasonably well the width of the instability strip in both cases.

The theoretical slope of the *PL* relationship is in excellent agreement with the empirical one derived by S97 for the SMC. The puzzling trend of a change of slope observed for log $P \lesssim 0.4$ and discovered by Bauer (1997) is reproduced by the models and thus can be explained by *purely evolutionary* effects due to the reduced extension of the blue loop with decreasing mass.

For the LMC, the theoretical slope is ~15% less steep than the observed one. The minimum mass that is found to undergo a blue loop ($M = 3.75\ M_\odot$, $P_{\rm blue\,loop} \sim 2$ days) corresponds to periods shorter by 20% than the ones of the faintest observed LMC Cepheids ($P_{\rm obs} \sim 2.5$ days). Variations of the metallicity and initial helium abundance do not solve this discrepancy. This shows the limitation of the present calculations and points out a remaining puzzle for the LMC Cepheids. An investigation of remaining uncertainties in the current stellar evolution models is in progress but requires numerous lengthy calculations. Indeed, we note that overshooting can change the minimum mass undergoing a blue loop and thus the minimum period. However, the good agreement found for the SMC seems to indicate that the present standard evolutionary models are already basically correct.

Regarding beat Cepheids, we show that 1H/2H pulsators with the shortest periods can *only* be reproduced by stars in the *first crossing instability phase*, with luminosity significantly lower than the one in the usual blue loop phase. This new result is supported by statistical arguments (cf. §4).

Finally, the present Letter shows that consistent calculations between stellar evolution and pulsation analysis may give the right answer to some unexplained features of Cepheids.

We are grateful to the EROS group and in particular to F. Bauer for discussing the EROS-2 Cepheid sample prior to publication. We are also grateful to U. Lee for providing the original pulsation code and to W. Glatzel for helpful discussions. The calculations have been performed on the Cray T3E at the Centre d'Etudes Nucléaires de Grenoble.

REFERENCES

Alcock, C., et al. 1995, AJ, 109, 1653
———. 1997, AJ, submitted (astro-ph/9709025)
Alexander, D. R., & Fergusson, J. W. 1994, ApJ, 437, 879
Allard, F., & Hauschildt, P. H. 1998, ApJ, submitted
Bauer, F. 1997, Ph.D. thesis, Univ. Paris VII
Bauer, F., et al. 1998, A&A, in preparation
Beaulieu, J. P., et al. 1995, A&A, 301, 137
———. 1997a, A&A, 318, L47
———. 1997b, A&A, 321, L5
Beaulieu, J. P., & Sasselov, D. 1996, in 12th IAP Astrophys. Meeting, Variable Stars and the Astrophysical Returns of Microlensing Surveys, ed. R. Ferlet, J.-P. Maillard, & B. Raban (Gif-sur-Yvette: Editions Frontières), 193
Buchler, J. R., Kollath, Z., Beaulieu, J. P., & Goupil, M. J. 1996, ApJ, 462, L83
Chiosi, C. 1990, in ASP Conf. Ser. 11, Confrontation between Stellar Pulsation and Evolution, ed. C. Cacciari & G. Clementini (San Francisco: ASP), 158
Chiosi, C., Wood, P. R., & Capitano, N. 1993, ApJS, 86, 541
Christensen-Dalsgaard, J. 1993, in ASP. Conf. Ser. 40, Inside the Stars: IAU Colloq. 137, ed. W. W. Weiss & A. Baglin (San Francisco: ASP), 483
Christensen-Dalsgaard, J., & Petersen, J. O. 1995, A&A, 299, L17
El Eid, M. F. 1994, MNRAS, 275, 983
Iglesias, C. A., & Rogers, F. J. 1991, ApJ, 371, L73
———. 1996, ApJ, 464, 943
Laney, C. D., & Stobie, R. S. 1994, MNRAS, 266, 441 (LS94)
Lee, U. 1985, PASJ, 37, L261
Lee, U., & Saio, H. 1989, MNRAS, 237, L875
Morgan, S. M., & Welch, D. L. 1997, AJ, 114, 1183 (MW97)
Moskalik, P., Buchler, R., & Marom, A. 1992, ApJ, 385, 685
Renault, C., et al. 1996, in 12th IAP Astrophys. Meeting, Variable Stars and the Astrophysical Returns of Microlensing Surveys, ed. R. Ferlet, J.-P. Maillard, & B. Raban (Gif-sur-Yvette: Editions Frontières), 5
Sasselov, D., et al. 1997, A&A, 324, 471 (S97)
Szabados, L. 1988, in Multimode Stellar Pulsations, ed. G. Kovacs, L. Szabados, & B. Szeidl (Kultura: Kunkoly Obs.), 1
Tanvir, N. R. 1996, in Proc. STScI May Symp., The Extragalactic Distance Scale, ed. M. Livio, M. Donahue, & N. Panagia (Cambridge: Cambridge Univ. Press), 91
Welch, D. L., et al. 1996, in 12th IAP Astrophys. Meeting, Variable Stars and the Astrophysical Returns of Microlensing Surveys, ed. R. Ferlet, J.-P. Maillard, & B. Raban (Gif-sur-Yvette: Editions Frontières), 205

Note added in proof.—After these calculations were completed, the MACHO collaboration published its complete sample in the LMC (D. Alves et al. [astro-ph/9804003 (1998)]) and reported very short period Cepheids with $\log P < 0.4$. We predict that these Cepheids are in the first crossing instability (see Fig. 2*b*) with a period variation over 4 yr of $\Delta P < 10^{-5}$ days, not observable in the measured 4 yr light curves, as suggested by Alves et al.