# CO2 and O2 equilibria in human blood and interstitial fluid 

Zock, Jacob Philippus

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from
it. Please check the document version below.
Document Version
Publisher's PDF, also known as Version of record

Publication date:
1985

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Zock, J. P. (1985). CO2 and O2 equilibria in human blood and interstitial fluid. s.n.

## Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: https://www.rug.nl/library/open-access/self-archiving-pure/taverneamendment.

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

## $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ equilibria

 in
## human blood and interstitial fluid



Jacob P. Zock
$\mathrm{CO}_{2}$ AND $\mathrm{O}_{2}$ EQUILIBRIA IN
HUMAN BLOOD AND INTERSTITIAL FLUID

## Stellingen

1. Bij de beoordeling van het zuur-base evenwicht tijdens hypothermie moet men uitgaan van bloedgasmetingen verricht bij de normale lichaamstemperatuur.
2. In aanwezigheid van $\mathrm{CO}_{2}$ doen de $N$-terminale valines van de $\beta$-ketens van het humane hemoglobine mee bij het tot stand komen van het $\mathrm{H}^{+}$bohr effect.
3. Bij de berekening van snelle veranderingen van de pH in bloed is het onjuist in de Henderson-Hasselbalch vergelijking de "apparent over-all first dissociation constant" te gebruiken als niet zeker is dat koolzuuranhydrase in voldoende mate aanwezig is.
4. De sterke correlatie tussen de hoeveelheid hemoglobine in de rode bloedcellen en het oppervlak van deze cellen toont de grote functionaliteit van deze struktuur.
5. Een wezenlijke verrijking van de wetenschap door de computer is de mogelijkheid die deze biedt tot synthese van delen analytische kennis.
6. De optimale dagdosis voor vitamine $C$ is voor veel mensen waarschijnlijk een veelvoud van de aanbevolen dagdosis.
7. Op grond van het uitgebreidere huidcontact van urine bij vrouwen is te verwachten dat overgevoeligheid voor sensibiliserende stoffen - aanwezig in voedings- en geneesmiddelen - die grotendeels onveranderd door de nieren worden uitgescheiden vaker bij vrouwen dan bij mannen zal voorkomen.
8. De emotionele en irrationele wijze waarop met het begrip discriminatie veelal wordt omgegaan, maakt het gebruik van dit begrip tot middel van politieke manipulatie en staat oplossingen van interculturele problemen eerder in de weg dan dat zij ertoe bijdraagt.
9. Funktionarissen die hun beroepsmatige bestaansrecht ontlenen aan de eventuele noodzaak tot veranderen van een organisatie, dienen van deze organisatie geen deel uit te maken.
10. Het feit dat Nederlandse staatsburgers niet het recht hebben om door de Kroon uitgevaardigde wetten en besluiten door de onafhankelijke rechter aan de grondwet en internationale verdragen te laten toetsen maakt dat Nederland slechts in beperkte mate als een rechtsstaat kan worden beschouwd.
11. Dikdoeners in de sferen van wetenschap en industrie zijn onder andere te herkennen aan het veelvuldig gebruik van de term "technologie" als er "techniek" bedoeld wordt.
12. Promoveren na je veertigste is als verloven na je zeventigste.

Stellingen behorende bij het proefschrift " $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ equilibria in human blood and interstitial fluid" van J.P. Zock.
Groningen, 1985.

# $\mathrm{CO}_{2}$ AND $\mathrm{O}_{2}$ EQUILIBRIA IN HUMAN BLOOD AND INTERSTITIAL FLUID 

## PROEFSCHRIFT

ter verkrijging van het doctoraat in de Geneeskunde
aan de Rijksuniversiteit te Groningen
op gezag van de Rector Magnificus Dr. E. Bleumink in het openbaar te verdedigen op woensdag 19 juni 1985
des namiddags te 4.00 uur
door
JACOB PHILIPPUS ZOCK
geboren te Dordrecht

1985
DRUKKERIJ VAN DENDEREN B.V.
GRONINGEN

Promotor: Prof. Dr. W.G. Zijlstra
Referent: Dr. P. Rispens

The work presented in this study was done within the framework of the program on blood gas transport of the Foundation for Medical Research (FUNGO), which is a subsidiary of the Netherlands Organization for the Advancement of Pure Research (ZWO).

The publication of this thesis was financially supported by the Netherlands Asthma Foundation.

To Jelly Mieke, Stijntie and Machteld
To my parents

CHAPTER I Introduction ..... 9
CHAPTER II Carbon dioxide loading and the acid-base ..... 21 equilibrium states of human blood
CHAPTER III Human whole blood oxygen affinity: $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ ..... 49 and haldane factors calculated for erythrocyte content and blood
CHAPTER IV The influence of temperature on $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ ..... 79 equilibria in human blood
CHAPTER $V$ Carbon dioxide loading and the acid-base ..... 105 equilibrium states of the human isolated internal environment
CHAFTER VI Concluding remarks ..... 127
SAMENVATTING ..... 133

There are few things about scientific method that I hold more firmly than that accuracy beyond a reasonable factor of safety is a mistake in so far as one is concerned with means to on end.
L.J. Henderson
(as quoted by J. Parascandola)

CHAPTER I

INTRODUCTION

A complex living organism like a human being can, in utmost reduction, be considered to consist of a very large number of the smallest units of life, the cells. These are surrounded by a fluid filling the interstitium - the space between them - and supported by the substance in it, which is produced and maintained by the cells. To keep alive, cells need energy and a suitable environment. Energy is derived from controlled combustion of nutrients. Both the nutrients and the oxygen needed for combustion are withdrawn from the interstitial fluid into which, in return, the cells dispose the degraded products of their metabolism. A suitable environment, therefore, has to contain sufficient nutrients and oxygen. Besides water, the main end product of aerobic energy liberation is carbon dioxide. In the interstitial fluid carbon dioxide is present in the form of the highly soluble $\mathrm{CO}_{2}$ molecule and in the form of the $\mathrm{HCO}_{3}^{-}$ ion into which $\mathrm{CO}_{2}$ is converted. In addition to nutrients and oxygen, the interstitial fluid contains electrolytes, of which the concentrations have to be kept within certain ranges. $\mathrm{HCO}_{3}^{-}$is a buffer anion and thus contributes to the acid-base equilibrium. The composition of the interstitial fluid is continuously threatened by the uptake of nutrients by the cells and by the waste the cells return into it. These exchanges tend to change the composition of the interstitial fluid away from the optimum composition. For its constancy, the interstitial fluid is dependent on supply from and disposal into the circulating blood. Specialized organs like the lungs, the kidneys and the liver in their turn see to it that the composition of the onflowing arterial blood is adequate.

The transport of oxygen and carbon dioxide is the primary task of the blood. The transport of $\mathrm{CO}_{2}$ is the result of close collaboration between the red cells and the blood plasma. The transport of oxygen, however, is almost completely done by the red cells. To this end, the red cells are packed with hemoglobin, a specialized protein. Oxygen, taken up in the lungs, is carried to the tissues where it is released. The release is furthered by the circumstances existing in actively metabolizing tissues to which the hemoglobin molecule is especially sensitive: a lower pH and a higher $\mathrm{pCO}_{2}$ than in the arterial blood, both caused by the $\mathrm{CO}_{2}$ produced by the cells. An additional factor is the rise in temperature. In the lungs, a reverse process is at work because the oxygen release by the blood in the presence of $\mathrm{CO}_{2}$ has its complement in the $\mathrm{CO}_{2}$ release in the presence of a higher than venous $\rho \mathrm{O}_{2}$ in the lungs.

Although these processes have their origin in the special properties of the hemoglobin molecule, the comprehension of this interaction is complicated by the fact that the hemoglobin is not directly confronted with pH changes occurring in blood plasma. It is locked up inside the red cells in which the pH changes are modified by the equilibria across the red cell membrane. The
change in these equilibria, furthermore, includes changes in the distribution of water and electrolytes, measureable as a change in the hematocrit. The interstitial fluid, for its part, is separated from the blood by the capillary wall. The changes around the tissue cells are thus transmitted to the blood in a modified form. This complicates the relation between changes in the neighbourhood of the cells and those in blood.

The interactions between the carbon dioxide and oxygen in blood and the interstitial fluid as outlined above, are the subject of this study. In our department a simple model of blood had been in use earlier. Starting with this model, several extensions and refinements have been introduced. These concern the introduction into the model of the osmotic and donnan equilibria across the red cell membrane and across the capillary wall, ATP and 2,3-DPG as buffer substances in the red cells, and a more detailed description of the functional properties of hemoglobin.

The transport of oxygen and carbon dioxide by the blood and the mutual interaction between them, and their connection with the distribution of water and electrolytes in blood have been the subjects of extensive research during the last two centuries. In the course of this period many fundamental concepts changed and new ones were introduced. In the last century a real revolution in thinking took place in the sense that vitalism was replaced by the notion that biophysical and biochemical processes form the basis of life. These new concepts were only gradually accepted.

In the course of his study on blood gas transport and acid-base equilibrium in the first part of our century, Lawrence J. Henderson came to realize that the complex interplay of the processes concerned forms a coherent phy-sico-chemical system which can be described in terms of mathematical equations. This was a major advance, because experiments are necessarily restricted and can only concern a part of all the interactions since it is virtually impossible to measure all variables simultaneously. A mathematical description of the physico-chemical system would provide insight into its structure. In his famous book Blood, published in 1928, Henderson attempted such a description. The book is a compilation of his work on the subject of blood gas transport and blood acid-base balance (a term invented by Henderson). Besides presenting his own achievements, Henderson, in his book, summarized a great part of the insights gained in the research carried out in the preceding era.

Henderson begins chapter II thus: "The blood of vertebrates is a physi-co-chemical system of great complexity". Since then, this statement has proved to be far more true than Henderson may have imagined. Beside the gas transporting system, there are other systems in blood which illustrate its complexity, e.g. the hemostatic coagulation system. All the substances that are
exchanged between cells are to some extent present in blood and many of them take part in a variety of interactions. The presence of these substances in blood plasma underlies the majority of the determinations in clinical chemistry and allows the monitoring of many of the functions and processes in the body.

Henderson's statement has also remained true for the subject of his book, the gas transport by the blood and its connections with acid-base balance. Obviously, books treating only this subject cannot be given such a comprehensive title any longer and one may even wonder whether at the time of publication the title Blood was not already too wide for the subject treated. Nowadays, blood is a subject more suited to fill a whole library and not just one volume; and within that library a fair amount of room would be needed to accommodate the books on gas transport and acid-base balance. The number of studies concerning hemoglobin alone has grown immensely and today it is no longer possible for a single person to cover the whole field of gas transport and acid-base balance from its roots in the molecular properties of hemoglobin to the other, macroscopic, end of the spectrum concerning the gas transport to and from the tissues and the regulations thereof, together with all the other processes influencing acid-base balance.

The discovery of the role of oxygen in combustion by Lavoisier about two centuries ago can be considered the start of modern investigations into the subject of gas transport in the body. Lavoisier showed that the principle of combustion was not only valid in the anorganic world but in living beings as well. He proved the chemical nature of respiration in experiments on guinea pigs conducted in collaboration with Laplace. They measured the intake of oxygen and the output of $\mathrm{CO}_{2}$ and heat, and showed that the continuous production of "animal heat" is a slow form of combustion.

For a long time it remained an open question at which site in the body heat was actually produced. In 1837, Gustav Magnus showed that this site could not be in the lungs because he found differences in $\mathrm{O}_{2}$ and $\mathrm{CO}_{2}$ content between arterial and venous blood. Around 1850 the common opinion was still that "animal heat" is produced in the red blood cells by combustion of nutrients. The heat was thought to be necessary to sustain the life of the tissues. A sufficient amount of red cells was therefore of prime importance for good health. The relation between the amount of red cells and nutrition had already been noted and good nutrition was considered to be of the utmost importance in illness. Anaemia itself was thought to be a cause of disease and not an eventual consequence of it. This may explain the emphasis put on anaemia even a long time after it had been found that the red cells were not the place where the heat was generated. The problem as to where the combustion occurs was tackled by a number of renowned scientists: Felix Hoppe-

Seyler, Georg Liebrig, Heinrich Magnus and Eduard Pflüger. It was Pflüger who finally resolved the matter around 1870.

During the whole of the 19th century the developments in physiology were so closely connected with those in chemistry and physics that the demarcation between the disciplines was often not clear. Hemoglobin was already extensively being studied when Stokes, in 1864, discovered that the absorption spectrum of a hemoglobin solution changed when oxygen was withdrawn from it. At the time, it was already clear that hemoglobin played an important role in the oxygen transport by the blood. Stokes recognized that the blood's turning dark on passing from the arteries to the veins indicates that oxygen is used up. About ten years later Pflüger showed that this oxygen was given up by the blood and used up in the tissue cells.

In 1884 Hüfner determined the amount of oxygen bound by hemoglobin to be 1.34 mL per gram hemoglobin and in 1885 Zinoffski calculated on the basis of the measured iron content of horse hemoglobin that the molecular weight was 16670. From these values it was readily inferred that at high oxygen pressures one mole of hemoglobin binds one mole of oxygen. It was due to Christian Bohr that this insight was suppressed and had to be rediscovered by Barcroft and Peters some 25 years later. Bohr was convinced that several different forms of hemoglobin were present in one individual, each with a different composition and different properties. Bohr used this idea of multiple hemoglobins to explain the form of the oxygen-hemoglobin equilibrium curve.

Already in 1878 Paul Bert had published his finding that at $40{ }^{\circ} \mathrm{C}$ dog blood saturated with oxygen gave up about half of its oxygen when the partial pressure of oxygen fell to about 25 mmHg . Hüfner, on the other hand, came to a value of about 5 mmHg . His value was strongly biased by his assumption about the reaction equation. The matter was settled by Bohr and coworkers in 1904, who measured the complete relation between oxygen pressure and oxygen saturation and proved conclusively the influence of $\mathrm{CO}_{2}$ on oxygen binding. Although Bohr is the first author, it is quite certain that the work was done by Krogh with the use of the tonometer designed by himself. It is a picture of this apparatus taken from Krogh's paper that is reproduced on the cover.

The measurements by Krogh confirmed the results of Paul Bert but raised the problem as to how the sigmoid curve should be explained. In 1910, Hill suggested that hemoglobin forms aggregates of variable numbers of molecules. Variability of the molecular weight and properties, of which Bohr was convinced, had already been rejected because of the osmotic pressure measured. In 1925 Adair reported new findings on the osmotic pressure exerted by hemoglobin in favour of a fixed molecular weight of four times the originally supposed value. This observation was supported by sedimentation measurements by Svedberg. On the basis of this new insight, Adair proposed that the
binding of oxygen takes place in four steps, each with its own equilibrium constant. In 1955, Roughton and coworkers accurately determined the values of these four binding constants. These values indicated that a strong cooperativeness should exist in the binding of oxygen in the sense that once one oxygen molecule is bound, the affinity for oxygen of the other binding sites strongly increases. This phenomenon was explained by Monod, Changeux and Wyman in 1965 with the hypothesis that the quaternary structure of the hemoglobin molecule can be in one of two possible states. The states are indicated as the $R$ and $T$ state. The $R$ state has a high oxygen affinity, the $T$ state a low one. Perutz and coworkers, in 1971 and following years, showed with the aid of x-ray diffraction that the two states indeed existed.

The distinction between the two states of the hemoglobin molecule was in itself not new, what was new was only the interpretation given to it. Already in 1920 Henderson had made use of the assumption that the pK of some ionizable groups on the hemoglobin molecule would change under the influence of oxygen binding. The two states distinguished were oxyhemoglobin and deoxyhemoglobin (then called reduced hemoglobin). Less exactly formulated, but essentially the same, is the assumption by Christiansen, Douglas and Haldane (1914) that oxyhemoglobin is a stronger acid than hemoglobin. They proposed such a mechanism as a possible explanation of their findings that deoxygenated blood could accommodate more $\mathrm{CO}_{2}$ than oxygenated blood at the same $\mathrm{pCO}_{2}$. This assumption was not new either. Already in 1882, Nathan Zuntz had formulated this hypothesis to explain the disputed effect of $\mathrm{O}_{2}$ on the amount of $\mathrm{CO}_{2}$ expelled from blood.

With their results Christiansen et al. conclusively proved the influence of oxygenation on the $\mathrm{CO}_{2}$ binding in the blood. In doing so they resolved a long-standing vexed question. In 1863 Holmgren, working with Ludwig in Vienna, observed that blood shaken with a gas mixture containing oxygen gives up much more $\mathrm{CO}_{2}$ than when no oxygen is present. Afterwards Ludwig failed to confirm this finding. It was not without reason that Zuntz considered it a realistic possibility. In 1870 Zuntz had become an assistant to Pflüger in Bonn, where Wolffberg in 1871 confirmed the results of Holmgren. Zuntz published the above-mentioned explanation of this phenomenon in 1882. The effect of oxygen on $\mathrm{CO}_{2}$ binding in blood was studied in vivo by Werigo in 1892. He separately ventilated the lungs of a dog and found that in the lung where hydrogen was used instead of oxygen, the $\mathrm{pCO}_{2}$ was systematically lower. He discussed the physiological importance of this effect and postulated that in the tissues the reciprocal effect, i.e. $\mathrm{CO}_{2}$ expelling $\mathrm{O}_{2}$ from the blood, occurs. It was the latter effect which was eventually to be confirmed beyond doubt in vitro by Bohr et al. The experimental results of Werigo were eventually explained by the findings of Christiansen et $a l$. The reciprocity that Werigo
assumed was thus shown to exist.
In 1910 Barcroft and Orbeli found that lactic acid had the same effect on the oxygen equilibrium curve as $\mathrm{CO}_{2}$. Therefore, the effect of $\mathrm{CO}_{2}$ was explained by its properties as an acid. It was thus quite natural that the explanation of the expelling of $\mathrm{CO}_{2}$ on oxygenation was sought in a change of the properties of the weak acid that hemoglobin is. Henderson had developed the theory of buffer action which enabled him to express this explanation in an exact way. It was the discovery by Christiansen et al. which made Henderson consider the blood as a physico-chemical system.

The reciprocity between oxygen binding and $\mathrm{H}^{+}$binding by hemoglobin was demonstrated more fundamentally by Wyman, who used chemical thermodynamics to formulate the relation in mathematical form in 1939. He extended his analysis further to include the influence of all substances that are bound by hemoglobin on the oxygen affinity of hemoglobin. This theory was published in 1964 and is used in the present study.

Wyman's theory could thus also be used to explain other interactions than that with protons. About 1930 it had been shown that $\mathrm{CO}_{2}$ itself binds to hemoglobin. This binding appeared to depend on oxygenation too. It follows from Wyman's relations that $\mathrm{CO}_{2}$ exerts its influence on the oxygen affinity of hemoglobin not only by means of proton release but also in a direct way. This has been confirmed experimentally. Wyman's theory also applies to another ligand of hemoglobin, 2,3-diphosphoglycerate. Its binding to hemoglobin was discovered only fairly recently, i.e. in 1967.
$\mathrm{CO}_{2}$ in the blood is distributed over the red cells and plasma. To be useful for the transport of $\mathrm{CO}_{2}$ the distribution has to occur very rapidly. Several discoveries and new concepts in physical chemistry were needed before this process could be completely understood. Until 1887, when Arrhenius published his theory on the dissociation of electrolytes, transfer of bicarbonate and chloride was thought to be possible only in the combination as a salt as proposed by Zuntz, or an acid as proposed by Gürber. The latter observed that the red cell membrane was impermeable to sodium and potassium, but that exchange of chloride and bicarbonate did take place. In 1897 Koeppe explained this by assuming that $\mathrm{CO}_{2}{ }^{=}$is exchanged against $\mathrm{Cl}^{-}$. The observation that $\mathrm{CO}_{2}$, in combination with deoxygenation, makes red cells swell and take up chloride is known as the Hamburger shift, after Hamburger, who published his findings in 1891. The same observation had already been made in 1878 by Herman Nasse, but this remained unnoticed. Neither he nor Hamburger could properly explain the phenomenon. The fact that this shift has nevertheless been called "Hamburger shift" is probably due to the fact that Hamburger became well known by the book he wrote in 1902 on osmotic pressure and the importance of ionic dissociation. Hamburger himself refers to Koeppe and

Gürber for the explanation of his observations and admits that in 1891 he, too, supposed that the exchange took place in the form of salts.

For the study of the distribution of $\mathrm{CO}_{2}$ between cells and plasma the concept of what is now called the donnan equilibrium was important. Donnan published it in 1911, shortly after Sørensen's concept of pH (1907) and Henderson's theory of buffer action (1908). This made it possible to study the "heterogeneous equilibria", as Henderson called them, between red cell content and plasma. A major improvement was the conversion of Henderson's buffer equation into a logarithmic form by Hasselbalch in 1917. This equation, which underlies the whole description of the acid-base balance, is therefore called the Henderson-Hasselbalch equation. Around 1920, this branch of physiology became more and more quantitative and many scientists devoted long years of work to measure the relations between the quantities relevant to the subject.

The subject of the present study is thus not new in physiology, and it may even be considered to be rather old-fashioned. The goal is the same as that aimed at by Henderson, but with the application of modern means and supplemented with insights gained since then. As already mentioned, Henderson tried to cope with the numerous relations governing the physico-chemical equilibria involved in the gas transport by the blood. He did this by using various nomograms representing these relations. The set of nomograms designed by Henderson can be considered to be a model of the physico-chemical properties of blood as far as gas transport and acid-base balance are concerned. Looked at in this way, the model incorporates the state of knowledge then prevailing about these subjects.

In Henderson's time the word "model" had not yet been coined to cover the notion of a description of a complex system. A model can be regarded as a representation of a certain part of reality with respect to some specific properties of that part of the reality. This representation can take different forms. An animal upon which a disease has been inflicted and which is used for studying the course of the disease as it might occur in a human being, is a model of the infected human being. A lump of tissue or a line of cultivated human or animal cells may sometimes also be used to simulate such a course of events and is then used as a model. In medical education, models of parts of the human body are used to simulate the relevant anatomical relations in order to train the skills needed for certain investigations or interventions. In other disciplines planes, ships, bridges are built to scale in order to study various properties of the real objects such as the influence of their shapes, their behaviour when loaded, etc. River deltas are built to scale, as are chemical plants. All of them are models used to represent and study certain parts of reality.

The models mentioned are real objects themselves, differing from their originals mainly in scale. In quite another way, mathematical formulae can be used to represent the parts of reality we are interested in. The behaviour of a gas, for instance, can to a certain degree be described by the gas law: pV = $n R T$. Just as is the case with equivalent models representing objects to scale, this representation is only accurate within certain limits. Up to these limits only the restricted number of quantities occurring in the equation are needed to represent the part of reality under study. When the values of these quantities are known, the state of this part of reality fixed.

In order to obtain a better description of reality it might be necessary to use more, or more extended, equations. This may entail an increase in the number of variables, but not necessarily so. The gas law, for instance, can be improved in the way given by Van der Waals. The Van der Waals equation differs from the gas law in the number of constants used, not in the number of variables. Though it may seem somewhat exaggerated to call an equation like the gas equation a "model" where it is possible to calculate with simple means every possible combination of values that the quantities may take, it may, nevertheless, be considered an example of a mathematical model. This notion of a mathematical model is more evident in the case where the equations that determine the state of a system increase either in number or in complexity. Obtaining an analytic solution may then become prohibitively laborious, thus necessitating other ways of finding solutions.

Before the surge of the digital computer, several ways to accomplish this were in use. Often another real physical system was built that obeyed equivalent equations. The advantage of the substitute system over the original one was that in it the quantities of interest could be measured more easily or that the scale was more suitable as regards size or costs. One of the most versatile and generally applicable systems of this kind of equivalent system is the analogue computer, in which variables are represented by the electrical quantities potential and current. Although still in use for special purposes, the analogue computer has lost most of its ground to the digital computer, which can be completely manipulated by the input of strings of characters without the need of rewiring and connecting new apparatus on each different application.

Although in many cases not of practical importance, it must be realized that quantities which in reality can take an uncountable infinite number of values are represented in the digital computer by quantities that can take only a countable infinite number of values owing to the way numbers are represented. Consequently, in the end rounding off errors can build up to disturbing levels and lead to unreliable results.


 imitation of reaity expmested in mambers. Tlize is called a compmarer sin 1 rion.
 number of advantuges. When the prafam is ieady in trie remputher zeperiments can be performed many times for dffernat yalses of the paras te with
 special skills are recumad tor fun an experiment. Untike in a reat e.
 quantities sain tee otserved simultannoustiy.

 a mathematical model are not restriciod in ils practical use alone. Ae fai a subject itself is comesmer, a :nathemotical model forces me to define at parts neatly. Latutas in the kroowledge uf the systent which is to p pep sented in the mosit! tronme directly appatet! t. lnteractinns betveen te
 appear. Models are Lhareine metl-suizo in support researci by p

 ledge is compised.

The present stuely is pat of the reseetch going at in the Depatrater: Medical Physiohogy of tan Universizy of Cromingen. Preceomg studizes
 blood as well as in respratory gos mon be memsured. Futhermore, $f$. thesis also deals with the intemperation of those quatities in abon what measured in the stinicsl daboratory, In his stady a number oi combutc.
 calculations were wrimen by thit? gresent athor, who, after is delay of * years, has cortimed mis thre of researeh, which has resulted in the study.

## LITERATURE

## History

 tale. Paris: Pacson.

Edsal JT (1972) Blood and Hemoglobin: The evolution of knowledge of functional adaptation in a biochemical system. J.Hist.Biol. 5: 205-257.
Flanagan D (ed.) (1963) Lives in science. New York, N.Y., Simon and Schuster.
Hamburger HJ (1902) Osmotischer Druck und lonenlehre in den medizinischen Wissenschaften. Wiesbaden, J.F. Bergmann.
Henderson LJ (1928) Blood: A study in General Physiology. New Haven, Conn., Yale University Press.
Krogh A (1904) Apparate und Methoden zur Bestimmung der Aufnahme von Gasen im Blute bei verschiedenen Spannungen der Gase, nebst einer Normalcurve für die Sauerstoffaufnahme des Pferdeblutes bei Spannungen von 0 bis 150 mm . Skand. Arch. für Physiologie 16: 390-401.
Parascandola J (1971) L.J. Henderson and the theory of buffer action. Med.Hist. 6: 297-309.

Rothschuh KE (1973) History of Physiology. Huntington, N.Y., RE Krieger Publ.Cy.
Werigo B (1892) Zur Frage über die Wirkung des Sauerstoffs auf die Kohlensäureausscheidung in den Lungen. Pflügers Arch. 51: 321-361.

## Models

Coleman TG (1981) Human, a comprehensive model for body function. Jackson, Miss.
Dickinson CJ (1977) A computer model of human respiration. Lancaster, U.K., MTP Press.
Min FBM (1982) Computersimulatie en wiskundige modellen in het medisch onderwijs. Thesis, Rijksuniversiteit Limburg, Maastricht, The Netherlands.

## CHAPTER II

CARBON DIOXIDE LOADING AND THE ACID-BASE EQUILIBRIUM STATES OF HUMAN BLOOD

Carbon dioxide loading and the acid-base equilibrium states of human blood*. I

by J.P. Zock, P. Rispens and W.G. Zijlstra

Department of Physiology, University of Groningen, the Netherlands

Communicated at the meeting of March 29, 1980


#### Abstract

To calculate acid-base equilibrium states of human blood in connection with blood gas transport, relations and quantities concerning related physico-chemical properties were collected from the literature. These included the Henderson-Hasselbalch equation, and the amount of charge of plasma protein and of hemoglobin varying with pH , the charge of hemoglobin also being dependent on oxygen saturation and carbon dioxide tension. Also included were the Gibbs-Donnan distribution of hydrogen ion acti-ity and of chloride and bicarbonate ions, and osmotic pressure equilibrium conditions. Noreover, the temperature dependence of all relations was taken into consideration. The expression used for the osmotic pressure of hemoglobin was based on osmotic behaviour of albumin as given by Landis and Pappenheimer rather than on the values of Adair. The equilibrium model formed by these equations and quantities describes two homogeneous solutions, one being plasma and the other the erythrocyte content, the two being separated by a membrane which was assumed to be impermeable to sodium, potassium, proteins and organic phosphates. This model was implemented as a computer program. Various characteristic phenomena concerning blood gas transport and acid-base equilibrium could be calculated with this model, e.g. the plasma bicarbonate- pH relation, the changes of hematocrit with pH , and the carbon dioxide saturation curve of blood - all of them dependent on oxygen saturation and temperature. The results agreed well with experimentally determined relations and values as given in the literature. The results suggest that the hitherto accepted osmotic pressure relation of hemoglobin may not be the true one.


[^0]Gas transport by the blood is closely connected with acid-base balance, because carbon dioxide constitutes the bulk of the acid carried by the blood and because of the interrelationship of oxygen transport and blood pH . Inside the red cells $\mathrm{CO}_{2}$ is rapidly converted into $\mathrm{H}_{2} \mathrm{CO}_{3}$ by carbonic anhydrase. Within the range of pH encountered in the blood virtually all $\mathrm{H}_{2} \mathrm{CO}_{3}$ dissociates into $\mathrm{HCO}_{3}{ }^{-}$and $\mathrm{H}^{+}$. Inside the erythrocytes $\mathrm{H}^{+}$is buffered by hemoglobin and to a lesser extent by organic phosphates; a small part of it is carried across the cell membrane (41) and is buffered by plasma proteins. The excess $\mathrm{HCO}_{3}^{-}$inside the red cells moves across the membrane into the plasma. $\mathrm{CO}_{2}$ is also directly bound to amino groups of hemoglobin transforming them into carbamino groups. Because the formation of carbamate turns a basic group into an acidic group, hemoglobin becomes a stronger acid with the binding of $\mathrm{CO}_{2}$. The equilibrium constants of the reactions involved are dependent on the oxygenation of hemoglobin. Besides, hemoglobin becomes a stronger acid on oxygenation. Therefore, at a certain pH the net charge of hemoglobin is dependent on its oxygen saturation and the amount of bound $\mathrm{CO}_{2}$. This is reflected as a change in the titration curve $(66,68)$. The binding of $\mathrm{CO}_{2}$ by hemoglobin is dependent in a yet not quite elucidated way on the concentration of organic phosphate compounds in the erythrocytes ( 10,44 ). Among these phosphates 2,3-diphosphoglycerate (2,3-DPG, in subscripts DPG) appears to be most important. Moreover, all equilibria are influenced by temperature. Temperature is an important variable in man because homeothermia does not imply that the body is isothermal $(55,57)$. All factors taken together, the net negative charge of hemoglobin depends on temperature, oxygen saturation, $\mathrm{pH}, \mathrm{CO}_{2}$ tension and 2,3-DPG concentration in the erythrocytes.

Hemoglobin cannot pass the cell membrane and therefore its charge is fixed inside the red cells, and as a consequence there is a Donnan distribution of the ions that can freely pass the membrane. The required electroneutrality within the two compartments, plasma and erythrocytes, gives rise to a net displacement of the diffusible anions - mainly $\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}^{-}$- across the cell membrane when changes of pH , oxygen saturation and $\mathrm{CO}_{2}$ tension occur. The kations - mainly $\mathrm{Na}^{+}$in plasma and $\mathrm{K}^{+}$in the erythrocytes - are actively confined to their compartments. The net displacement of diffusible anions tends to change the osmotic pressure inside and outside the cells. A net displacement of water accompanying the anion shift keeps the total osmotic pressure inside and outside the cells equal ( $18,42,69$ ).

The above-mentioned processes influence each other in many ways, which makes it difficult to get an overall picture of the events occurring during loading and unloading of blood with carbon dioxide and oxygen, even when these events are simplified to a succession of equilibrium states. Until recently such calculations were too elaborate to make them practically possible. Calculations of this type have been made earlier $(21,36,78,80)$, but a detailed quantitative treatment of the subject became possible only with the use of computers. Studies of this kind have been made by Lloyd and Michel (48), Roos and

Thomas (64), Dell and Winters (17), Rispens (59), Thomas (75), SiggaardAndersen (72) and Brodda (13) and recently by Rodeau and Malan (63). The model presented in this paper is a continuation and extension of the work of Rispens.

MODEL
In the model the equilibrium conditions of the above-mentioned processes are described in a quantitative way by means of equations with proper values of the parameters. This makes the implementation in a computer program possible.

In the model, blood is a suspension of erythrocytes in plasma. Plasma is thought to be a homogeneous solution of $\mathrm{NaCl}, \mathrm{NaHCO}_{3}$ and proteins in water. The red cells consist of a selectively permeable membrane containing the red cell content. Red cell content is assumed to consist of a homogeneous solution of $\mathrm{KCl}, \mathrm{KHCO}_{3}$, adenosine-triphosphate (ATP), 2,3-DPG and hemoglobin in water $(32,82) . \mathrm{Na}^{+}$represents all kations of plasma, $\mathrm{K}^{+}$stands for all kations actually present in the erythrocytes. $\mathrm{Na}^{+}$and $\mathrm{K}^{+}$are assumed to be completely confined to their compartments. The red cell membrane is taken to be impermeable to plasma proteins, hemoglobin, ATP and 2,3-DPG, and freely permeable to $\mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}, \mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}{ }^{-}$(33). This means that at equilibrium the erythrocytes are in osmotic equilibrium with plasma $(38,56)$. In the model the red cell membrane has no other properties (Fig. 1).

The oxygen saturation of hemoglobin is given a certain value. The amount of dissolved oxygen is neglected. In this way the complications concerning the


Fig. 1. Schematic representation of the equilibria considered in the model. Closed arrows indicate the direction in which the equilibria shift when $\mathrm{CO}_{2}$ is added; open arrows indicate the shift when $\mathrm{CO}_{2}$ is withdrawn.
relati: a berween oxygen pressure and oxygen saturation of hemoglobin and the infle nere of pH, $\mathrm{pCO}_{2}$ and 2,3-DPG thereupon are circumvented. In the model it $i$ smmed that the temperature is the same throughout the blood.
choneutrality of both solutions - plasma and erythrocyte content is 4 . 'rat
(1, $\quad \therefore, \mathrm{Na}^{+}=n_{p, \mathrm{Cl}^{-}}+n_{p, \mathrm{HCO}_{3}}+z_{\mathrm{Pr}} \cdot w_{p, \mathrm{Pr}}$
(2) $\quad n_{c, \mathrm{~K}^{+}}=n_{c, \mathrm{Cl}^{-}}+n_{c, \mathrm{HCO}_{3}}+z_{\mathrm{ATP}} \cdot n_{c, \mathrm{ATP}}+z_{\mathrm{DPG}} \cdot n_{c, \mathrm{DPG}}+$ $+z \mathrm{Hb} \cdot n_{c, \mathrm{Hb}}+z \mathrm{HbO}_{2} \cdot n_{c}, \mathrm{HbO}_{2}$
where $n$ denotes an amount of substance in mole, $w$ a mass in gram. The index $p$ means plasma, $c$ stands for red cell; $z$ is the amount of charge per mole, except for plasma protein where it is the amount of negative charge per gram.
$\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}{ }^{-}$are assumed to be distributed between plasma water and red cell water according to the Gibbs-Donnan equilibrium conditions:

$$
\begin{equation*}
r_{\mathrm{c}, \mathrm{p}}=\frac{c_{\mathrm{cw}^{w}, \mathrm{Cl}^{-}}}{c_{p w, \mathrm{Cl}^{-}}}=\frac{c_{\mathrm{cw}, \mathrm{HCO}_{3}^{-}}}{c_{p w, \mathrm{HCO}_{3}^{-}}^{-}} \tag{3}
\end{equation*}
$$

$r_{c, p}$ is the Donnan ratio and $c_{c w}, c_{p w}$ denote the substance concentration in cell water and plasma water, respectively. The partition of proton activity $a_{\mathrm{H}+}$ is also assumed to be according to the Gibbs-Donnan condition, so that

$$
\begin{equation*}
\log \left(r_{c, p}\right)=\mathrm{pH}_{c}-\mathrm{pH}_{p} \tag{4}
\end{equation*}
$$

The osmotic equilibrium condition is

$$
\left\{\begin{array}{l}
\phi_{i} \cdot R T \cdot\left(\frac{n_{p, \mathrm{Na}^{+}}+n_{p, \mathrm{Cl}}-+n_{p, \mathrm{HCO}_{3}-}}{w_{p, \mathrm{H}_{2} \mathrm{O}}}\right)+\pi_{p, \mathrm{Pr}}=  \tag{5}\\
=\phi_{i} \cdot R T \cdot\left(\begin{array}{c}
\left.n_{c, \mathrm{~K}^{+}}+n_{c, \mathrm{Cl}^{-}-\mathrm{n}_{\mathrm{c}, \mathrm{HCO}_{3}}+n_{c, \mathrm{ATP}}+n_{c, \mathrm{DPG}}}^{w_{c, \mathrm{H}_{2} \mathrm{O}}}\right)+\pi_{c, \mathrm{Hb}_{4}}
\end{array}\right.
\end{array}\right.
$$

where $\phi_{i}$ is the osmotic activity coefficient of the ions, $\pi_{p, \mathrm{Pr}}$ the plasma protein osmotic pressure, $\pi_{c, \mathrm{Hb}_{4}}$ the hemoglobin osmotic pressure and $w_{p, \mathrm{H}_{2} \mathrm{O}}, w_{c, \mathrm{H}_{2} \mathrm{O}}$ the mass of plasma water and of cell water*.

The relation between plasma pH , plasma $p \mathrm{CO}_{2}$ and plasma $\mathrm{HCO}_{3}{ }^{-}$ concentration is given by the Henderson-Hasselbalch equation:

$$
\begin{equation*}
\mathrm{pH}_{p}=\mathrm{p} K_{1}^{\prime}+\log ^{-} \frac{c_{p, \mathrm{HCO}_{3}^{-}}}{S^{\prime} \cdot p \mathrm{CO}_{2}} \tag{6}
\end{equation*}
$$

where $c_{p, \mathrm{HCO}_{3}{ }^{-}}$is the plasma bicarbonate concentration in $\mathrm{mmol}^{\cdot l^{-1}}$ and $S^{\prime}$ the solubility of carbon dioxide in plasma in $\mathrm{mmol}^{\cdot 1^{-1}}{ }^{\mathrm{kPa}}{ }^{-1}$. The values of $\mathrm{p} K_{1}^{\prime}$ and $S^{\prime}$ are dependent on temperature. According to Rispens ei al. (60) this dependence is fairly approximated by

$$
\begin{equation*}
\mathrm{p} K_{\mathrm{i}}^{\prime}=-4.7416+1840.11 / T+0.015906 T-\log \left(1+\frac{0.020682}{10^{(7-\mathrm{pH} p(7))}}\right) \tag{7}
\end{equation*}
$$

[^1]where $T$ is, temperature in K. A similar equation has been giver by Reeves (5\%).
Gn!y in fhe lower temperature range the two equations give slighty different velters of peif. The solubility of carbon dioxide in retation 10 femperature is given ios
(i) $\quad s=7.50\left(-3.246+536.73 / T+0.004985{ }^{\prime}\right)$

Risper, (59) arives at this expression using values gian by Austin et al. (6).
Oiven suiable expressions for the titration carres of plasma proteins, hrmogebin, ATP and 2,3-DPG in relation to the varables on which hey dejend and given the osmotic activity coefficients and the relation between motein concentration and water content of a solution, the above equations begerher with proper starting values are sulficient for calculating the interiependence of oxygen saturation ( $\mathrm{S}_{\mathrm{O}_{2}}$ ), carbon dicide imisim ( $\mathrm{j}_{\mathrm{CO}}^{2} \mathrm{O}_{2}$ ), plasmaty $\mathrm{f}\left(\mathrm{p} \mathrm{H}_{p}\right)$, blond $\mathrm{CO}_{2}$ content $\left(c_{\left.b, \mathrm{CO}_{2}\right)}\right)$ and temperature ( $7_{6}$ ).
 given liy the tiration curve of the protein. This cuive is usinatly assumed to be a straight line in the pH range $6.8-8.0$. The slope of this line is $0.10 \%$ mole of harge fer gram per pH unit (47). The iso-electric point of plasma protein bsed in the model is 5.1 at $37^{\circ} \mathrm{C}$ (77). The titration curve at $37^{\circ} \mathrm{C}$ is thus: represented b
(@) $\quad$ 阬 $=0.107\left(\mathrm{pH}_{p}-5.10\right)$
The slope of this curve is commonly assumed to be indencinden of temperature. Vituse a change in iso-electric pH of -0.014 per ${ }^{\circ} \mathrm{C}$, ancording io Rispens $\{59$ ).

The lliaion curve of hemoglobin depends on oxyen sattration, carbois doxide temsion, 2,3-DPG concentration and temperature (lo, sid). The inmurer of oxygenation and carbon dioxide binding on the litation are of bemogionin are treated as independent effects, supermpesed upon the timation curve of dersygenated hemoglobin in the absence of CO.. The titis, is fairly appoximated by a straight line. Various values have been upofer for the isoUE,ftic print of deoxygenated hemoglobin and the slope of his liue (\%7, 28, 35, 24, 65, 7\%). In the model we use an iso-electric pH of ( 83 and a slope of 2.1 mole of cliarge per mole of deoxygenated hemoglobin pe will pH: Thus in the absence of $\mathrm{CO}_{2}$ the titration curve of deoxygenated hem , glotin is cerseribed by
(1) ZHit $=2.7\left(\mathrm{pH}_{\mathrm{C}}-6.81\right)$

That lemperature dependence of this slope is $-0.009 \mathrm{p}_{2} \mathrm{u}^{-1} \mathrm{Cl}$ (5). The iso-


The tination curve of oxygenated hemoglobin differs slighty from that of drovygunted hemoglobin. This difference can be atributed we chatsges in e nitibrime constants of alkaline and acidic Bohr groups on the inemogiobin molecuic with oxygenation $(5,64,66,84)$. Therefore, eash ighe of group is

[^2]

Fig. 2. Net negative charge per mole $\mathrm{Hb}(z)$ in the absence of $\mathrm{CO}_{2}$ plotted against pH ; curve 1: deoxygenated hemoglobin, curve 2 : oxygenated hemoglobin.
characterized by two equilibrium constants, one for deoxygenated hemoglobin and one for oxygenated hemoglobin. The difference in the charge on one mole deoxygenated hemoglobin and one mole oxygenated hemoglobin is calculated with the use of these four constants. The charge on one mole oxygenated hemoglobin in the absence of $\mathrm{CO}_{2}$ is then obtained by the addition of this difference to the amount of charge on one mole deoxygenated hemoglobin (Fig. 2). The values for the constants are for the acidic group $\mathrm{p} K_{\mathrm{Hb}}=5.4$, $\mathrm{p} K_{\mathrm{HbO}_{2}}=5.9$, and for the alkaline group $\mathrm{p} K_{\mathrm{Hb}}=7.5, \mathrm{p} K_{\mathrm{HbO}_{2}}=6.45(5,44)$. The temperature dependence of the $\mathrm{p} K^{\prime}$ of the alkaline Bohr groups is $-0.0167 \mathrm{~K}^{-1}$ for oxygenated hemoglobin and $-0.0265 \mathrm{~K}^{-1}$ for deoxygenated hemoglobin (44). The temperature dependence of the acidic Bohr groups is $0.004 \mathrm{~K}^{-1}(5)$.

The binding of $\mathrm{CO}_{2}$ by hemoglobin influences the titration curve of hemoglobin, because an alkaline amino group is turned into an acidic carbamino group. From the equations describing these reaction, it follows that (66)

$$
\begin{equation*}
\mathrm{Z}=\frac{K_{c}^{\prime \cdot} K_{z}^{\prime} \cdot c_{c, \mathrm{CO}_{2}}}{K_{c}^{\prime \cdot} \cdot K_{z}^{\prime} \cdot c_{c, \mathrm{CO}_{2}}+K_{z}^{\prime} \cdot a_{\mathrm{H}}++\left(a_{\mathrm{H}}+\right)^{2}} \tag{11}
\end{equation*}
$$

where $Z$ is the $\mathrm{CO}_{2}$ saturation of hemoglobin (Fig. 3, Fig. 4), $c_{c, \mathrm{CO}_{2}}$ is the concentration of dissolved $\mathrm{CO}_{2}\left(\mathrm{~mol}^{\cdot 1^{-1}}\right)$ and $a_{\mathrm{H}^{+}}$the proton activity (mol $\cdot 1^{-1}$ ). The equilibrium constants $K_{c}$ and $K_{z}$ are different for oxygenated and deoxygenated hemoglobin. We resolved the problem of the partially oxygenated hemoglobin molecules by considering them as a mixture of fully saturated and completely deoxygenated hemoglobin molecules and treating both components as being mutually independent. The reactions are presently the subject of much research and at the moment there are no generally accepted values for the equilibrium constants. Perella et al. (53) do not give values for oxygenated hemoglobin. The values stated by Bauer and Schröder (11) result in


Fig. 3. A. Amount of carbamate per mole $\mathrm{Hb}(Z)$ plotted against pH ; curve 1 : deoxygenated hemoglobin, curve 2: oxygenated hemoglobin. B. Difference between the amounts of carbamate of deoxygenated and oxygenated hemoglobin per mole $(\Delta Z)$ plotted against pH .


Fig. 4. Difference in net negative charge between one mole of oxygenated and one mole of deoxygenated hemoglobin $(\Delta z)$ plottted against pH ; curve 1: $p \mathrm{CO}_{2}=0$, curve 2: $p \mathrm{CO}_{2}=5.33$, curve 3: $p \mathrm{CO}_{2}=10.7 \mathrm{kPa}(0,40,80 \mathrm{mmHg})$.
very low catbanino-liemoglobin amounts in oxygenated bloci. We uned tire

 not neminised. Therefore, the temperature coefficient of $\mathrm{p} K_{c, \text { th }}$ and fix: 1 the . was supposed 10 be equal to the one of bovine hemoglobin, i.e. 0.019$\}_{1}^{-1}$ ( 67 ). Because the correspotiding values of $\mathrm{p} K_{z}$ are not mentioned, the lemperathe depentience was provisionally taken to be equal to that of thie $p K$ of the alkaline Boht eroup i.e. $-0.022 \mathrm{~K}^{-1}$ (5).

The manter is rurther complicated by the influence exerted ly ormsuie phosphate compounds in the erythrocytes upon the formation of cratheminohemoglobin. At the moment there is no generally accepted opinion mearling the reaction equations and equilibrium constants describing the influsice of 2,3-DPC on the titation curve of hemoglobin. In a tentative fommation be Kilmartin and Rosi-Eernardi (44) no values of the reaction constants ate presented. Hence, the influence of $2,3-D P G$ on carbamino formation was implicity included in the model.

The concemutivon of dissolved $\mathrm{CO}_{2}$ in the erythrocytes was caleulaled anirie a solubility of 165 mombl $]^{-1} \cdot \mathrm{~Pa}^{-1}\left(0.0260 \mathrm{mmol} \cdot 1^{-1} \cdot \mathrm{mmHg}^{-1}\right)$ at $37^{\circ} \mathrm{C}$ The temperature variation of this value was supposed to be in proportion mith the temperature varation of the $\mathrm{CO}_{2}$ solubility in plasma.

The charges on ATP and 2,3-DPG are represented by $z_{\text {ATP }}$ and zomita equation 2. The values for the corresponding $\mathrm{p} K^{\prime} \mathrm{s}$ and their temperature dependence were taken from Reeves (58). In the physiological range of pry both ATP and 2,3-DPG have three groups with fixed negative charge. ATP bas one dissociable acidic group with $\mathrm{p} K^{\prime}=7.04$ and $\mathrm{dp} K^{\prime} / \mathrm{d} T=-0.0124$, and 2,3. DPG has two dissociable acidic groups both with $\mathrm{p} K^{\prime}=6.73$ and dy $K^{\prime}$ d7 $=$ $=0.0026$.

The red cell is in osmotic equilibrium with plasma $(38,56)$. Osmctic pressure is the sum of the colloid osmotic pressure and the osmotic pressure cansed by the cristalloids in the solution. The latter is

$$
\begin{equation*}
\pi_{c}=R^{\prime} \cdot \gamma^{\circ} \cdot \Sigma\left(\phi_{i} \cdot m_{i}\right) \tag{12}
\end{equation*}
$$

where $\pi_{6}$ is the osmotic pressure caused by the cristalloids, $R$ the gas constant, $T$ temperature, $\phi_{i}$ the molal osmotic coefficient and $m_{i}$ the molalicy of substance i. For a 0.120 mol/ NaCl solution in water, $\phi=0.92$ and for a 0.98 mol 1 k ? solution in water, $\phi=0.913$ at $310 \mathrm{~K}(16)$. As the influence of HCO , these values is uncertain, $\phi_{i}=0.92$ is provisionally used in equation 1 ? for all ions in solution.

The osmotic pressure caused by proteins in solution is not lineaty related to protein concentration $(19,46,49,70)$. The osmotic coefficient is larger than I and depencis on comentration. This can partly be explained by the fate that the protein molecules are much bigger than water molecules and thas jurevent a fraction of the voilme from being occupied by the solvent $(39,71)$.

Landis and Pappenheimer (46) give empirical expressions for the osinotic
pressure of albumin solutions and plasma over the range of 0 to $25 \varepsilon^{*}$ (11 - . based on data from several sources. These expressions are

$$
\begin{equation*}
\pi_{\mathrm{Alb}}=2.8 c_{\mathrm{Alb}}+0.18 c^{2} \mathrm{Alb}+0.012 c^{3} \mathrm{Alb} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
\pi_{\text {plastina }}=2.1 c_{p, \mathrm{Pr}_{r}}+0.16 c_{p, \mathrm{Pr}}^{2}+0.009 c_{p, \mathrm{Pr}}^{3} \tag{14}
\end{equation*}
$$

where $\pi$ is pressure in mmHg and $c$ is protein concentration in $g^{\prime} d l^{-1}$. The dependence of the osmotic coefficient on concentration is inplicil. Changing over to concentration in $\mathrm{g} \cdot 1^{-1}$ and pressure in kPa , and writing the osmonis coefficient between brackets, gives for albumin solutions:

$$
\begin{equation*}
\pi_{\mathrm{Alb}}=0.037 c_{\mathrm{Alb}}\left(1+6.410^{-3} c_{\mathrm{Alb}}+42.910^{-6} c^{2} \mathrm{Alb}\right) \tag{15}
\end{equation*}
$$

At infinite dilution the osmotic coefficient approaches 1 , and $\pi_{\text {, 频 }} \rightarrow$ $R \cdot T \cdot c_{\text {Alb }} / M_{\text {Alb }}$, where $M_{\text {Alb }}$ is the relative molecular mass of albumin, and $T=310 \mathrm{~K}$. The molecular mass of hemoglobin being 64 458, the retation between the osmotic pressure caused by hemoglobin at infinite clilution and the concentration is

$$
\begin{equation*}
\pi_{\mathrm{Hb}_{4}}=R \cdot T \cdot c_{\mathrm{Hb}_{4}} / M_{\mathrm{Hb}_{4}}=0.040 c_{\mathrm{Hb}_{4}}(\text { at } 310 \mathrm{~K}) \tag{16}
\end{equation*}
$$

At finite dilution the osmotic coefficient of hemoglobin has to be taken into account. Because of the small difference between the molecular masses of albumin ( 69100 ) and hemoglobin ( 64458 ) and because both molecules are spherical, we used the expression for the osmotic coefficient of althumin. The osmotic pressure of a hemoglobin solution is thus calculated according to

$$
\begin{equation*}
\pi_{\mathrm{Hb}_{4}}=0.040 c_{\mathrm{Hb}_{4}}+2.5710^{-4} c_{\mathrm{Hb}}^{4} 2+1.7110^{-6} c^{3} \mathrm{Hb}_{4} \tag{17}
\end{equation*}
$$

The colloid osmotic pressure of plasma is calculated with an adapled version of equation 14 (Fig. 5). The total osmotic pressure of a solution of electrolyies and


Fig. 5. Relations between colloid osmotic pressure ( $\pi$ ) and concentration (c) as used in the model; curve 1: plasma protein, curve 2: hemoglobin, curve 3: albumin (46).
proteins in water is obtained as the sum of both. Osmotic pressure caused by crystalloids depends on molality, colloid osmotic pressure is according to the above-mentioned expressions calculated with concentration given as mass per volume of solution. It is therefore necessary to know the amount of water contained in a protein solution. The water fraction in normal erythrocytes has been determined by several investigators. Some older values are 0.73 mentioned by Van Slyke et al. (79) and 0.72 which can be derived from values given by Dill et al. (21). Savits et al. (70) find a water fraction of 0.717 I water per liter solution. One of the seven determinations by Hald et al. (33) has obviously been done on blood of an anaemic person. Omitting this value and taking the mean of the remaining ones also gives 0.717 . The water content found by Kessler et al. (43) as well as by Eisenman et al. (22) is $713 \mathrm{~g} \cdot \mathrm{l}^{-1}$. The density of water at $37^{\circ} \mathrm{C}$ being 0.993 , the water fraction thus determined is 0.718 .

The water content of plasma containing $70 \mathrm{~g} \cdot 1^{-1}$ protein is $934 \mathrm{~g} \cdot \mathrm{l}^{-1}(22)$, corresponding to a water fraction of 0.9406 . At $37^{\circ} \mathrm{C}$ the amount of water in one liter of a $160 \mathrm{mmol}^{-1} 1^{-1} \mathrm{NaCl}$ solution differs insignificantly from the amount of water in one liter of pure water. In a physiological protein solution the non-water volume is thus completely occupied by the protein. The protein volume fraction in the erythrocyte is, consequently, 0.283 and in plasma containing $70 \mathrm{~g} \cdot \mathrm{I}^{-1}$ protein 0.0594 . In the erythrocytes this includes nonhemoglobin protein and in plasma the plasma lipids. Their volumes are supposed to be included in the hemoglobin volume and plasma protein volume, respectively. The mean hemoglobin concentration in the erythrocytes of healthy men is about $333 \mathrm{~g} \cdot \mathrm{l}^{-1}(4,31,54,81)$. The volume occupied by 1 g of hemoglobin is thus $283 / 333=0.850 \mathrm{ml}$. The volume occupied by 1 g of plasma protein is $59.4 / 70=0.849 \mathrm{ml}$.
These results mean that in the model the volume occupied by 1 g of protein in plasma as well as by 1 g of hemoglobin in the erythrocyte can be taken to be equal and independent of protein concentration. The relation between water fraction ( $F_{\mathrm{W}}$ ) and mass concentration of protein is thus given by

$$
\begin{equation*}
F_{W}=1-0.8510^{-3} c_{\mathrm{Pr}} \tag{18}
\end{equation*}
$$

where $c_{\mathrm{Pr}}$ is the mass concentration protein in $\mathrm{g} \cdot \mathrm{l}^{-1}$ of solution. The water content ( $c_{w}$ ) of such a protein solution is

$$
\begin{equation*}
c_{w}=0.993 F_{w} \tag{19}
\end{equation*}
$$

kg water per liter of solution (at 310 K ).
The above-mentioned equations link the variables so that only a restricted number of them can be freely chosen or varied independently. To start the computations, a boundary condition must be given; to assess the consequences of variations, a reference or standard state is needed. We took standard oxygenated blood, i.e. blood with $\mathrm{SO}_{2}=1, p \mathrm{CO}_{2}=5.33 \mathrm{kPa}(40 \mathrm{mmHg})$, $\mathrm{pH}_{p}=7.410$ (71) and $T=310 \mathrm{~K}$ as the boundary condition and as the reference state. Once these variables were fixed, an independent value could be given to only six other variables. In the model at the standard state we took the
following values: $\mathrm{pH}_{c}=7.202$ (81), $c_{p, \mathrm{Na}}=0.153 \mathrm{~mol}^{\cdot 1^{-1}}$ ( Na represents all kations) (Table 2), $c_{p, \mathrm{Pr}}=70 \mathrm{~g} \cdot \mathrm{I}^{-1}, c_{b, \mathrm{Hb}}=150 \mathrm{~g} \cdot \mathrm{l}^{-1}, c_{c, \mathrm{Hb}}=333 \mathrm{~g} \cdot \mathrm{l}^{-1}$, $c_{C w, \text { ATP }}=1.5 \mathrm{mmol}^{\bullet} 1^{-1}(9)$. Insertion of these values into the equations fixes the standard state. Changes following variation of one or more variables were computed and compared with the standard state.

## CALCULATIONS

With the aid of the presented equations and the chosen values the amounts of blood chloride, cell potassium and cell $2,3-$ DPG were calculated for the standard state. Once the standard state was known, one or more quantities were varied and the resulting change of the equilibria calculated. E.g. changing the $p \mathrm{CO}_{2}$ resulted in a change in plasma pH and in plasma bicarbonate concentration. A plot of these two quantities is the blood bufferline in the plasma bicarbonate-pH (Van Slyke-Davenport) diagram. The results of the calculations are presented in the next section. The set of equations could not always be solved directly. In that case the solution was found by consecutive approximations. The maximal permitted error in the solutions corresponded with 0.0002 pHu . The calculations were interrupted when the conditions set by the equations were satisfied to the chosen precision.

## RESULTS

The calculated composition of human blood at the standard state is given in Table 1. In Table 2 the concentrations of the anions in plasma and of the kations and anions in the erythrocytes as calculated with our model as well as the results calculated by others $(13,63)$ are compared with experimental values reported in the literature. In Table 3 the osmolality of plasma and erythrocytes as calculated with our model as well as the results as calculated by others (13) are compared with experimental values reported in the literature. With the standard state as the boundary condition, a variation of one or more variables induces changes in the equilibrium state. If the amount of $\mathrm{CO}_{2}$ is changed, while keeping the amounts of all other substances constant, the resulting changes are those of the titration of blood with $\mathrm{CO}_{2}$. Thus a bufferline of blood is obtained (Fig. 6A). Rispens (59) found the slope of this bufferline to be -30.7 slyke ( $\mathrm{mmol} \cdot \mathrm{pHu}^{-1}$ ) at $c_{b, \mathrm{Hb}}=150 \mathrm{~g} \cdot \mathrm{l}^{-1}$. Our model gives a value of -28.3 slyke. However, this value is in good agreement with the slope of -28.2 slyke which can be derived from figure 12 in reference 72.

In calculating the composition of venous blood it was assumed that the respiratory quotient is 0.83 and venous $p \mathrm{CO}_{2}=6.13 \mathrm{kPa}(46 \mathrm{mmHg})$. The results of this calculation are given in Table 4, together with the values of the considered quantities in the standard state and with the difference between the two states. Bagott (8) reconsidered the values by Dill et al. (21) with other parameters for the carbamate equation and obtained results which are in good agreement with our values.

Table 1. Adopted standard state of human blood

|  | Plasma | Erythrocytes | Blood |  |
| :---: | :---: | :---: | :---: | :---: |
| Quantity |  | Input values |  | Unit |
| $\mathrm{Na}+$ | 153 |  |  | mmol $\mathrm{l}^{-1}$ |
| Protein | 70 |  |  | $g \cdot 1^{-1}$ |
| Hemoglobin |  | 333 | 150 | $\mathrm{g} \cdot 1^{-1}$ |
| ATP |  | 1.5 |  | mmol ${ }^{-1}$ |
| $\mathrm{SO}_{2}$ |  | 1.0 |  |  |
| $p \mathrm{CO}_{2}$ | 5.33 | 5.33 | 5.33 | $k \mathrm{~Pa}$ |
| pH | 7.410 | 7.202 |  |  |
| Temperature | 310 | 310 | 310 | K |
|  |  | d values per li |  |  |
| K + |  | 54 |  | mmol |
| $\mathrm{HCO}_{3}^{-}$ | 13.7 | 5.3 | 19.0 | mmol |
| $\mathrm{Cl}^{-}$ | 61.0 | 23.5 | 84.5 | mmol |
| 2,3-DPG |  | 1.76 |  | mmol |
| Carbamate |  | 1.13 |  | mmol |
| Dissolved $\mathrm{CO}_{2}$ | 0.67 | 0.47 | 1.14 | mmol |
| Total $\mathrm{CO}_{2}$ | 14.37 | 6.88 | 21.25 | mmol |
| Volume | 0.550 | 0.450 | 1.000 | 1 |
| $\mathrm{H}_{2} \mathrm{O}$ | 0.517 | 0.323 | 0.840 | 1 |
| $\pi_{\text {coll }}$ | 3.4 | 106.9 |  | ${ }^{\text {PPa }}$ |
| $\pi$ | 737 | 737 |  | $k \mathrm{~Pa}$ |

Variation of pH is accompanied by variation of anion concentrations, which in turn leads to a change in osmotic pressure and in osmolality of plasma (Fig. 6B). Because of the Donnan distribution the changes are not equally divided between plasma and red cells. This leads to a changing hematocrit with plasma pH (Fig. 6C). The slope of the calculated curve is $-0.032 \mathrm{pHu}^{-1}$. The value found by Reeves (58) is $-0.031 \pm 0.002$.
Titration of blood with $\mathrm{CO}_{2}$ also gives a change in total $\mathrm{CO}_{2}$ content. The total amount of $\mathrm{CO}_{2}$ in blood has been calculated for oxygenated and deoxygenated blood (Fig. 7). The difference between the two curves is the Haldane effect. At $p \mathrm{CO}_{2}=5.33 \mathrm{kPa}(40 \mathrm{mmHg})$ this difference was calculated to be $0.31 \mathrm{mmol} \cdot 1^{-1}$ in fair agreement with experimental values (72).
At constant $p \mathrm{CO}_{2}$ and constant intracellular pH the amount of $\mathrm{CO}_{2}$ bound as carbamate depends on oxygen saturation. With $p \mathrm{CO}_{2}=5.33 \mathrm{kPa}(40 \mathrm{mmHg})$ and $\mathrm{pH}_{c}=7.2$ Bauer and Schröder (11) have found an increase of 0.081 mol $\mathrm{CO}_{2}$ per mol Hb when oxygen saturation changes from one to zero, while plasma pH drops by 0.047 pHu . Under these circumstances we calculated an increase of $0.083 \mathrm{~mol} \mathrm{CO}_{2}$ per mol Hb , while plasma pH had to be lowered by 0.037 pHu to keep the intracellular pH constant.

The Donnan distribution of ions and hydrogen activity between plasma and erythrocytes has been much investigated. In our calculations the Donnan ratios

Table 2. Amounts of electrolytes in plasma and erythrocytes ( $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{H}_{2} \mathrm{O}$ )

| Experimental values | Plasma |  | Erythrocytes |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\Sigma$ kat | $\mathrm{Cl}^{-}$ | $\Sigma$ kat | $\mathrm{Cl}^{-}$ |
| Clauvel et al. (16) | $164 \pm 3^{2}$ |  | $173{ }^{3}$ |  |
| Gleichmann et al. (26) ' | $162 \pm$ |  | 170 |  |
| Hald et al. (33) ${ }^{\text {b }}$ | $156{ }^{2}$ |  | 164 |  |
| Valberg et al. (76) ${ }^{1}$ | $164 \pm 4$ |  | 158 |  |
| Diem and Lentner (20) | 164 | 108 |  |  |
| Albrition (4) | 158 | 109 | 167 | 73 |
| Bernstein (12) | $166^{2}$ | 113 | $153{ }^{3}$ | 79 |
| Dill et al. (21) ${ }^{\text {I }}$ | 164 | 112 | 155 | 75 |
| Erickson et al. (23) | $160{ }^{2}$ | 111 | 1893 | 74 |
| Funder and Wieth (25) |  | 110 |  | 73 |
| Gram (30) |  | 107-109 |  | 73-79 |
| Hastings et al. (34) |  | 104-111 |  | 73-84 |
| Von Bubnoff and Riecker (15) |  |  |  | 73-91 |
| Models | $\Sigma$ kat | CI-4 | $\Sigma \mathrm{kat}$ | $\mathrm{Cl}^{-}$ |
| Brodda (13) I | 161 | $116^{5}$ | 177 | 90 |
| Rodeau and Malan (63) ${ }^{\text {I }}$ | 161 | 114 | 188 | 74 |
| Our model ${ }^{1}$ | 164 | 118 | 168 | 73 |

${ }^{1}$ Arterial and arterialized venous blood.
${ }^{2} 5 \mathrm{mmol}^{\cdot} \mathrm{kg}^{-1}$ added because $\mathrm{Ca}^{2+}$ and $\mathrm{Mg}^{2+}$ were not included.
${ }^{3} 7 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$ added because $\mathrm{Mg}^{2+}$ was not included.
${ }^{4}$ Besides cloride the important anions in plasma are organic acid $6 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$, phosphate $2 \mathrm{mmol}^{\cdot} \mathrm{kg}^{-1}$ and sulfate $1 \mathrm{mmol}^{\cdot} \mathrm{kg}^{-1}$. Rodeau and Malan (63) consider phosphate separately in their model, so their amount of chloride includes $7 \mathrm{mmol}{ }^{\prime} \mathrm{kg}^{-1}$ of other anions. In the model of Brodda and in our own model all these anions are included in the amount of chloride, which thus has to be diminished by $9 \mathrm{mmol}^{\cdot} \mathrm{kg}^{-1}$ for it to be comparable with the experimental value.
5 Calculated values in italics.

Table 3. Osmolality of human blood plasma and erythrocytes

|  | Plasma | Erythrocytes |
| :--- | :---: | :--- |
| Experimental values | $\mathrm{mosm}^{\circ} \mathrm{kg}^{-1}$ | $\mathrm{mosm} \cdot \mathrm{kg}^{-1}$ |
| Diem and Lentner (20) | 287 |  |
| Funder and Wieth (26) | 289 |  |
| Hendry (37, 38) | $289 \pm 4$ | $290 \pm 4$ |
| Models |  |  |
| Brodda (13) | 3111.2 | $311^{2}$ |
| Rodeau and Malan (63) | 290 | 290 |
| Our model | $286^{2}$ | $286^{2}$ |

${ }^{1}$ Calculated values in italics.
2 Not included is the contribution of non-electrolytes such as glucose and ureum. When their total amount is taken to be $10 \mathrm{mmol} \cdot 1^{-1}$ of blood, the calculated values have to be augmented by 12 mosm ${ }^{\bullet} \mathrm{kg}^{-1}$. Brodda's value then becomes 323 mosm ${ }^{\circ} \mathrm{kg}^{-1}$ and our value 298 mosm ${ }^{\circ} \mathrm{kg}^{-1}$.

Table 4. C:Iculated veno-arterial differences in one liter of human blood assuming a respiratory quotient of 0.83 and venous blood with $p \mathrm{CO}_{2}=6.13 \mathrm{kPa}$ $(46 \mathrm{mmHg})$

| Quantity | Arterial blood |  |  | Venous Blood |  |  | Veno-arterial Difference |  |  | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Plasma | Erythrocytes | Blood | Plasma | Erythrocytes | Blood | Plasma | Erythrocytes | Blood |  |
| $\mathrm{S}_{\mathrm{O}_{2}}$ |  | 1 |  |  | 0.76 |  |  | -0.24 |  |  |
| $p \mathrm{CO}_{2}$ | 5.33 | 5.33 |  | 6.13 | 6.13 |  | 0.80 | 0.80 |  | kPa |
| pH | 7.410 | 7.202 | $r_{c . p}=0.619$ | 7.378 | 7.184 | $r_{c, \rho}=0.638$ | -0.032 | -0.018 |  | pHu |
| Total $\mathrm{O}_{2}$ |  | 9.31 | 9.31 |  | 7.07 | 7.07 |  | -2.23 | -2.23 | mmol |
| Total $\mathrm{CO}_{2}$ | 14.36 | 6.89 | 21.25 | 15.31 | 7.81 | 23.11 | 0.95 | 0.92 | 1.86 | mmol |
| Dissolved $\mathrm{CO}_{2}$ | 0.67 | 0.47 | 1.14 | 0.77 | 0.54 | 1.31 | 0.10 | 0.07 | 0.17 | mmol |
| Carbamate |  | 1.13 | 1.13 |  | 1.42 | 1.42 |  | 0.29 | 0.29 | mmol |
| $\mathrm{HCO}_{3}^{-}$ | 13.69 | 5.29 | 18.98 | 14.53 | 5.85 | 20.38 | 0.84 | 0.56 | 1.40 | mmol |
| $\mathrm{Cl}-$ | 60.90 | 23.58 | 84.48 | 60.20 | 24.28 | 84.48 | -0.70 | 0.70 |  | mmol |
| Charge of protein | 9.52 |  |  | 9.39 |  |  | -0.13 |  |  | mmol |
| Charge of hemoglobin |  | 15.47 |  |  | 14.24 |  |  | -1.23 |  | mmol |
| Charge of DPG |  | 7.93 |  |  | 7.90 |  |  | -0.03 |  | mmol |
| Charge of ATP |  | 1.74 |  |  | 1.73 |  |  | -0.01 |  | mmol |
| Volume | 0.550 | 0.450 | 1.000 | 0.548 | 0.452 | 1.000 | -0.002 | 0.002 |  | 1 |
| Water | 0.517 | 0.323 | 0.840 | 0.515 | 0.325 | 0.840 | -0.002 | 0.002 |  | 1 |
| Osmolality | 286.1 |  |  | 287.5 |  |  | 1.4 |  |  | mosm $\cdot \mathrm{kg}$-1 |
| Colloid osmotic pressure | 3.44 | 106.9 |  | 3.46 | 105.6 |  | 0.02 | $-1.3$ |  | kPa |
| Total osmotic pressure | 736.8 | 736.8 |  | 740.6 | 740.6 |  | 3.8 | 3.8 |  | kPa |



Fig. 6. Calculated relations between some variables and plasma pH when blood is titrated with $\mathrm{CO}_{2}$; curves 1 : oxygen saturation $S_{\mathrm{O}_{2}}=0$; curves 2 : oxygen saturation $S_{\mathrm{O}_{2}}=1$. A. Plasma bicarbonate $\left(c_{p, \mathrm{HCO}_{3}}\right)$. B. Plasma osmolality $\left(c_{p, o s m}\right), 10 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$ added for glucose and ureum. C. Relative cell volume (hematocrit $H$ ).


Fig. 7. Total a mount of $\mathrm{CO}_{2}$ per liter blood ( $c_{\left.l,(1)_{2}\right)}$ ) plotted against $\left.p \mathrm{CO}\right)_{2}, c_{\%}, 1 \mathrm{H}=150 \mathrm{~g} \cdot \mathrm{~F}^{-1}$; curve 1: $S_{\mathrm{O}_{2}}=0$, curve 2: $S_{\mathrm{O}_{2}}=1$.
of $\mathrm{Cl}^{-}, \mathrm{HCO}_{3}{ }^{-}$and $\mathrm{H}^{+}$were supposed to equal (25). The calculated Donnan ratio is given in Fig. 8, together with relations taken from the literature.

The Donnan ratio and the hematocrit of oxygenated blood were calculated over the temperature range of $25^{\circ}$ to $40^{\circ} \mathrm{C}$ under two supposed regimens (Fig. 9). It is clear that the temperature induced changes are less disturbing when the total $\mathrm{CO}_{2}$ content of the blood is kept constant. This agrees with the experimental results of Reeves (58), who has found that in blood with constant total $\mathrm{CO}_{2}$ content hematocrit and Donnan ratio hardly change with temperature. However, with constant $\mathrm{CO}_{2}$ content of the blood the calculated rate of change of pH with temperature is only $-0.010 \mathrm{pHu} \cdot \mathrm{K}^{-1}$ in contrast with experimental values of $-0.015 \mathrm{pHu} \cdot \mathrm{K}^{-1}(7)$ and $-0.0144 \mathrm{pHu} \cdot \mathrm{K}^{-1}$ (1). Furthermore, when $T=300 \mathrm{~K}$ the calculated $p \mathrm{CO}_{2}$ is $3.69 \mathrm{kPa}(27.7 \mathrm{mmHg})$, while the reported value is $3.44 \mathrm{kPa}(25.8 \mathrm{mmHg})(52)$. The calculated results depend on the value used for the rate of change of the iso-electric pH of deoxygenated hemoglobin with temperature for which $-0.0122 \mathrm{~K}^{-1}$ was taken (5). When we changed this value to $-0.018 \mathrm{~K}^{-1}$ the calculated rate of change of blood pH with temperature was $-0.0144 \mathrm{pHu} \cdot \mathrm{K}^{-1}$ and $p \mathrm{CO}_{2} 3.34 \mathrm{kPa}(25.1 \mathrm{mmHg})$ at 300.15 K.

Van Slyke and Sendroy (78) have constructed a nomogram with which the ratio $c_{p, \mathrm{CO}_{2}} / c_{b, \mathrm{CO}_{2}}$ as depending on pH and $\mathrm{S}_{\mathrm{O}_{2}}$ can be found. McHardy ( 50 ) has derived an empirical relation from values found with this nomogram. Rispens (59) has modified this relation to include temperature. Fig. 10 shows McHardy's relation together with the curves representing the ratio as calculated with the model. Within the range where Van Slyke and Sendroy checked the nomogram experimentally ( $7.2<\mathrm{pH}_{p}<7.7$ ), the agreement of the calculated values with those of McHardy lies within $1 \%$.

## DISCUSSION

We have tried to derive the acid-base properties of human blood from the relevant physico-chemical properties of its constituents. Until about fifteen


Fig. 8. Donnan ratios ( $r_{c . p}$ ) as calculated for the titration of blood with $\mathrm{CO}_{2}$ compared with relations from the literature; curves $I$ : relations as calculated with the help of our model, curves 2 : Bromberget al. (14), curves 3: Fitzsimons and Sendroy (24), curves 4: Funder and Wieth (25), curve 5: Hilpert et al. (40), curves 6: Takano et al. (74), curve 7: Reeves (58). A. Donnan ratio of hydrogen ion activity in oxygenated blood plotted against plasma pH . B. Donnan ratio of chloride anion concentration in oxygenated blood plotted against plasma pH. C. Donnan ratio of hydrogen ion activity (curves $1,3,6$ ) and chloride anion concentration (curve $3^{\prime}$ ) in deoxygenated blood.


Fig. 9. Donnan ratio $\left(r_{c, j, p}\right)$ and hematocrit $(H)$ as calculated over the temperature range $25-40^{\circ} \mathrm{C}$ under two regimens, curves 1: $p \mathrm{CO}_{2}$ is kept constant at $5.33 \mathrm{kPa}(40 \mathrm{mmHg})$, curves 2: total $\mathrm{CO}_{2}$ content kept constant at $21.2 \mathrm{mmol}^{-\rho^{-1}}$. A. Donnan ratio plotted against temperature. B. Hematocrit plotted against temperature.


Fig. 10. Ratio of plasma total $\mathrm{CO}_{2}$ concentration to blood $\mathrm{CO}_{2}$ concentration plotted against plasma pH . Curves 1 and 3 : oxygenated blood, curves 2 and 4 : deoxygenated blood. Thin lines according to McHardy (50), thick lines as calculated with the help of our model. The range of pH over which the determinations were carried out, and upon which the McHardy formula is based, lies between a and b .
years ago acid-base models were presented as nomograms in which the relations between the variables are given in a graphical way ( $36,73,78,80$ ). Thereafter, more complex models have been published which necessitated the use of a computer. Examples are the models of Lloyd and Michel (48), Roos and Thomas (64), Dell and Winters (17), Rispens (59), Thomas (75), SiggaardAndersen (72), and more recently Brodda (13) and Rodeau and Malan (63). In two of them $(17,64)$ blood is only a part of the models which describe acid-base behaviour of blood in contact with interstitial fluid. The models of blood as given by Dell and Winters (17), Roos and Thomas (64), and Thomas (75) are not only based on the properties of the basic constituents of blood but also on experimentally determined relations between some subsystems. Some variables are assumed to be constant in these models.

In their model Rodeau and Malan (63) have used quite a different approach. They relate the amount of imidazole groups of proteins to the buffer power of


Fig. 11. A. Fraction of cell volume occupied by cell water plotted against plasma pH when blood is titrated with $\mathrm{CO}_{2}$. Drawn line as calculated with the help of our model; asterisks: values given by Murphy (51). B. Relation between relative hematocrit of blood and plasma pH when blood is titrated with CO2. Drawn line as calculated with the help of our model; asteriks: values given by Murphy' (51), open circles: Dill el al. (21), crosses: Reeves (58).
proteins and do not include the influence of oxygenation and carbamate formation on the titration curve of hemoglobin. Oxygen saturation in their model has a fixed value of one. Carbamate as a factor influencing total $\mathrm{CO}_{2}$ content of blood is also neglected. In contrast to the other models they include bivalent ions as such. In all other models only the charge contribution of these ions is considered. Rodeau and Malan suppose that plasma has an osmotic coefficient of one and they include 2,3-DPG, ATP and other phosphorylated compounds as buffers.
Our model is in the same line of reasoning as the models of Lloyd and Michel (48) and Brodda (13). In these models the acid-base properties of blood are as far as possible derived from the properties of the constituents. The main difference between our model and the other ones is that we use a more detailed description of the titration curve of hemoglobin in which the effects of oxygenation and carbamate formation have been included. This became possible through results achieved and published in recent years. Another important difference with the other models is the relation we use to describe the osmotic pressure caused by the hemoglobin in erythrocytes. Yet another, but less important difference was the use of an osmotic coefficient for the plasma protein. In the model we have also included 2,3-DPG and ATP as buffers inside the erythrocyte. In this way we got rid of the "non-diffusible, non-buffer anions", which were needed to balance electric charges and osmotic forces in the models of Lloyd and Michel (48), and Rispens (59).
The use of a different osmotic pressure relation for hemoglobin turned out to be essential to get results that are consistent with experimental findings. Results that are directly dependent on this relation are the amounts of kations and chloride in the erythrocyte as calculated for the standard state (Table 2) and the variations of cell water and hematocrit with plasma pH as given in Fig. 11.


Fig. 12. Osmotic coefficient of hemoglobin ( $\phi_{\mathrm{Hb}_{4}}$ ) plotted against concentration ( $c_{\mathrm{H}_{4}}$ ); curve 1: as used in our model i.e. equal to that of albumin according to Landis and Pappenheimer (46), curve 2: according to Dick and Lowenstein (19), curve 3: calculated with values given by McConaghey and Maizels (49), standard deviation indicated by vertical bars, curve 4: as found by Savitz et al. (70).

There is a good agreement between the calculated relations and the experimental findings from the literature. We are aware of the fact that in the way we calculate the osmotic pressure of hemoglobin, generally accepted experimental values are put aside. However, as indicated in the derivation, the relation used is not completely devoid of a physical basis. The multiple interactions in which hemoglobin is involved give rise to many pitfalls in the measurement of the osmotic pressure, some of which were not known in the time of Adair $(2,3)$. Albumin does not have this drawback to the same extent and is a spherical protein of about the same molecular mass as hemoglobin. So it is rather unlikely that their osmotic coefficients will be very different. The various findings of the osmotic coefficient of hemoglobin and the one assumed by us are given in Fig. 12. Although the results obtained with the presented model should not be seen as proof that the supposed osmotic coefficient of hemoglobin is correct, at least the requirement of Occam's razor is met because a minimal number of assumed essentials makes the model give results which cover the acid-base properties of blood to a remarkable degree.

## CONCLUSION

The acid-base and related interactions of blood are fairly well described by this model. The consequences of titration with $\mathrm{CO}_{2}$ have been presented here. However, the results of titration with HCl and NaOH can also be calculated with the help of this model, as also the changes induced by variation of $S_{\mathrm{O}_{2}}$, hemoglobin concentration, plasma protein concentration and other parameters. The model is a means to provide insight into the way the variables are related and to check whether the properties of particular constituents fit in with the acid-base properties of blood as they are known from experiments. We find a difference between the osmotic properties of hemoglobin in solution as given in the literature and the properties needed to complete the model in such a way that it gives results that are consistent with experimental findings.

## REFERENCES

1. Adamson, K., S.S. Daniel, G. Gandy and L.S. James - Influence of temperature on blood pH of the human adult and newborn. J. Appl. Physiol. 19, 897-900 (1964).
2. Adair, G.S. - A theory of partial osmotic pressures and membrane equilibria with special reference to the application of Dalton's law to haemoglobin solutions in the presence of salts. Proc. Roy. Soc. (London) A120, 573-603 (1928).
3. Adair, G.S. - The thermodynamic analysis of the observed osmotic pressure of protein salts in solutions of finite concentration. Proc. Roy. Soc. (London) A126, 16-24 (1929).
4. Albritton, E.C. - Standard values in blood. Philadelphia: Sanders 1952.
5. Antonini, E., J. Wyman, M. Brunori, C. Fronticelli, E. Bucci and A. Rossi-Fanelli Studies on the relations between the molecular and functional properties of hemoglobin. V. The influence of temperature on the Bohr effect in human and horse hemoglobin. J. Biol. Chem. 240, 1096-1 103 (1965).
6. Austin, W.H., E. Lacombe, P.W. Rand and M. Chatterjee - Solubility of carbon dioxide in serum from $15-38^{\circ} \mathrm{C}$. J. Appl. Physiol. 18, 301-304 (1963).
7. Austin, W.H., E.H. Lacombe and P.W. Rand - pH-temperature conversion factors and $\mathrm{pCO}_{2}$ factors for hypothermia. J. Appl. Physiol. 19, 893-896 (1964).
8. Bagott, J. - The contribution of carbamate to physiological carbon dioxide transport. TIBS - September 1978, N 207.
9. Bartlett, G.R. - Human red cell glycolytic intermediates. J. Biol. Chem. 234, 449-458 (1959).
10. Bauer, C. - On the respiratory function of haemoglobin. Rev. Physiol. Biochem. Pharmacol. 70, 1-31 (1974).
11. Bauer, C. and E. Schröder - Carbamino compounds of haemoglobin in human adull and foetal blood. J. Physiol. 227, 457-471 (1972).
12. Bernstein, R.E. - Potassium and sodium balance in mammalian red cells. Science 120, 459-460 (1954).
13. Brodda, K. - Zur Theorie des Säure-Basen-Haushalts von menschlichem Blut. Funktionsanalyse biologischer Systeme 2. Wiesbaden: Franz Steiner Verlag Gmbh 1975.
14. Bromberg, P.A., J. Theodore, E.D. Robin and W.N. Jensen - Anion and hydrogen ion distribution in human blood. J. Lab. Clin. Med. 66, 464-475 (1965).
15. Bubnoff, M.v. and G. Riecker - Zur potentiometrischen Chloridbestimmung in eiweisshalligen Lösungen. Biochem. Z. 331, 577-579 (1959).
16. Clauvel De Mendonca, M., K. Schwartz and E. Terrier - Sodium, potassium and osmolality of human and canine erythrocytes - interest of trapped plasma and water content for their full significance. Comp. Biochem. Physiol. 34, 147-161 (1970).
17. Dell, R.B. and W.W. Winters - A model for the in vivo $\mathrm{CO}_{2}$ equilibration curve. Am. J. Physiol. 219, 37-44 (1970).
18. Dick, D.A.T. - Cell water. London: Butterworths 1966.
19. Dick, D.A.T. and L.M. Lowenstein - Osmotic equilibria in human erythrocytes studied by immersion refractometry. Proc. Roy. Soc, B148, 241-256 (1956).
20. Diem, K. and C. Lentner, eds: Scientific Tables. Basel: J.R. Geigy A.G., 1968.
21. Dill, D.B., H.T. Edwards and W.V. Consolazio - Blood as a physicochemical system, XI. Manat rest. J. Biol. Chem. 118, 635-648 (1937).
22. Eisenmann, A.J., L.B. Mackenzie and J.P. Peters - Protein and water of serum and cells of human blood, with a note on the measurement of red blood cell volume. J. Biol. Chem. 116, 33-45 (1936).
23. Erickson, B.N., H.H. Williams, F.C. Hummel, P. Lee and I.G. Macy - The lipid and miniral distribution of the serum and erythrocytes in the hemolytic and hypochromic anemias of childhood. J. Biol. Chem. 118, 569-598 (1937).
24. Fitzsimons, E.J. and J. Sendroy, Jr. - Distribution of electrolytes in human blood. J. Biol. Chem. 236, 1595-1601 (1961).
25. Funder, J. and J.O. Wieth - Chloride and hydrogen ion distribution between human red cells and plasma. Acta Physiol. Scand. 68, 234-245 (1966).
26. Funder, J. and J.O. Wieth - Potassium, sodium and water in normal human red blood cells. Scan. J. Clin. Lab. Invest. 18, 167-180 (1966).
27. Gary-Bobo, C.M. and A.K. Solomon - Properties of hemoglobin solution in red cells. J. Gen. Physiol. 52, 825-853 (1968).
28. German, B. and J. Wyman - The titration curves of oxygenated and reduced hemoglobin. J. Biol. Chem. 117, 533-550 (1937).
29. Gleichmann, U., H.v. Stuckrad and M. Zindler - Intracellularer Säurebasen- und Elektrolythaushalt. Experimentelle und klinische Untersuchungen an Erythrocyten. Z. รes. exp. Med. 139, 255-266 (1965).
30. Gram, H.C. - Chlorides of serum, blood, and corpuscles in various pathological conditions. J. Biol. Chem. 61, 337-343 (1924).
31. Greendyke, R.M., W.A. Meriwether, E.T. Thomas, J.D. Flintjer and M.W. Bayliss - A suggested revision of normal values for hemoglobin, hematocrit, an erythrocyte count in healthy adulı men. Am. J. Clin. Path. 37, 429-436 (1962).
32. Gros, G., H.S. Rollema, W. Jelkman, H. Gros, C. Bauer and W. Moll - Net charge and oxygen affinity of human hemoglobin are independent of hemoglobin concentration. J. Gen. Physiol. 72, 765-773 (1978).
33. Hald, P.M., M. Tulin, T.S. Danowski, P.H. Lavietes and J.P. Peters - The distribution
of sodium and potassium in oxygenated human blood and their effects upon the movements of water between cells and plasma. Am. J. Physiol. 149, 340-349 (1947).
34. Hastings, A.B., J. Sendroy Jr., J.F. Mclntosh and D.D. van Slyke - Studies of gas and electrolyte cquilibria in blood. Xlll. The distribution of chloride and bicarbonate in the blood of normal and pathological human subjects. J. Biol. Chem. 79, 193-209 (1928).
35. Hastings, A.B., J. Sendroy Jr., C.D. Murray, M. Heidelberger and C.R. Harrington Studies of gas and electrolyte equilibria in blood. V1. The acid properties of reduced and oxygenated hemoglobin. J. Biol. Chem. 60, 89-153 (1924).
36. Henderson, L.J.H. - Blood, a study in general physiology. New Haven: Yale University Press 1928.
37. Hendry, E.B. - Osmolality of human serum and of chemical solutions of biologic importance. Clin. Chem. 7, 156-164 (1961).
38. Hendry, E.B. - The osmofic pressure and chemical composition of human body fluids. Clin. Chem. 8, 246-265 (1962).
39. Hildebrand, J.H. - The entropy of solution of molecules of different size. J. Chem. Phys. 15, 225-228 (1947).
40. Hilpert, P., R.G. Fleischmann, D. Kempe and H. Bartels - The Bohr effect related to blood and erythrocyte pH. Am. J. Physiol. 205, 337-340 (1963).
41. Jacobs, MI.H. and D.R. Stewart - The role of carbonic anhydrase in certain ionic exchanges involving the erythrocyte. J. Gen. Physiol. 25, 539-552 (1942).
42. Jacobs, M.H. and D.R. Stewart - Osmotic properties of the erythrocyte. XII. Ionic and osmotic equilibria with a complex external solution. J. Cell. Comp. Physiol. 30, 79-103 (1947).
43. Kessler, E., M.R. Levy and R.L. Allen - Red cell electrolytes in patients with edema. J. Lab. Clin. Med. 57, 32-41 (1961).
44. Kilmartin, J.V., L. Rossi-Bernardi - Interaction of hemoglobin with hydrogen ions, carbon dioxide, and organic phosphates. Physiol. Rev. 53, 836-890 (1973).
45. Klocke, R.A. - Mechanism and kinetics of the Haldane effect in human erythrocytes. J. Appl. Physiol. 35, 673-681 (1973).
46. Landis, E.M. and Pappenheimer, J.R. - Exchange of substance through capillary walls. In: Handbook of Physiology, Circulation II (Hamilıon, W.F., Dow, P., eds.), pp. 972-976. Washington D.C.: Am. Physiol. Soc. 1963.
47. Leeuwen, A.M. van - Net cation equivalency of the plasma proteins. Amsterdam: Scheltema \& Holkema 1964.
48. Lloyd, B.B. and C.C. Michel - A theoretical treatment of the carbon dioxide dissociation curse of true plasma in vitro. Resp. Physiol. 1, 107-137 (1966).
49. McConaghey, P.D. and M. Maizels - The osmotic coefficients of haemoglobin in red cells under varying conditions. J. Physiol. 155, 28-45 (1961).
50. Mc.Hardy, G.J.R. - The relationship between the difference in pressure and content of carbonic dioxide in arterial and venous blood. Clin. Sci. 32, 299-309 (1967).
51. Murphy, J.R. - The influence of pH on physical properties of the erythrocyte. In: Metabolism and membrane permeability of erythrocytes and thrombocytes. (E. Deutsch, E. Gerlach, K. Moser, eds.), pp. 452-456. Stuttgart: Georg Thieme Verlag 1968.
52. Nunn, J.F., N.A. Bergman, A. Bunatyan and A.J. Coleman - Temperature coefficients for $\mathrm{PCO}_{2}$ and $\mathrm{P}_{\mathrm{O}_{2}}$ of blood in vitro. J. Appl. Physiol. 20, 23-26 (1965).
53. Perrella, M., G. Guglielmo and A. Mosca - Determination of the equilibrium constants for oxygen-linked $\mathrm{CO}_{2}$ binding to human hemoglobin. FEBS Letters 78, 287-290 (1977).
54. Plum, C.M. - Discussion. In: Standardization, documentation and normal values in haematology. (Boroviczeny, C.G. de, ed.) p. I66. Basel: S. Karger 1965.
55. Rahn, H., R.B. Reeves and B.J. Howell - Hydrogen ion regulation, temperature, and evolution. Am. Rer: Resp. Disease 112, 165-172 (1975).
56. Rand, R.P. and A.C. Burton - Mechanical properties of the red cellmembrane. I. Membrane stiffness and intracellular pressure. Biophys. J. 4, 115-135 (1964).
57. Reeves, R.B. - Temperature-induced changes in blood acid-base status: pH and $\mathrm{P}_{\mathrm{CO}_{2}}$ in a
binary buffer. J. Appl. Physiol. 40, 752-761 (1976).
58. Reeves, R.B. - Temperature-induced changes in blood acid-base status: Donnan $\mathrm{r}_{\mathrm{Cl}}$ and red cell volume. J. Appl. Physiol. 40, 762-767 (1976).
59. Rispens, P. - Significance of plasma bicarbonate for the evaluation of $\mathrm{H}^{+}$homeostasis. Thesis, Groningen 1970. Assen: Van Gorcum and Comp. 1970.
60. Rispens, P., J.R. Brunsting, J.P. Zock and W.G. Zijlstra - A modified Singer-Hastings nomogram. J. Appl. Physiol. 34, 377-382 (1973).
61. Rispens, P., C.W. Dellebarre, D. Eleveld, W. Helder and W.G. Zijlsira - The apparent first dissociation constant of carbonic acid in plasma between 16 and $42.5^{\circ} \mathrm{C}$. Clin. Chim. Acta 22, 627-637 (1968).
62. Robinson, R.A. and R.H. Stokes - Electrolyte solutions. London: Butterwoths 1968.
63. Rodeau, J.L. and A. Malan - A two-compartment model of blood acid-base state at constant or variable temperature. Resp. Physiol. 36, 5-30 (1979).
64. Roos, A. and L.J. Thomas Jr. - The in-vitro and in-vivo dioxide dissociation curves of true plasma. Anesthesiol. 28, 1048-1063 (1967).
65. Rossi, L., J.R. Chipperfield and F.J.W. Roughton - The effect of temperature on the titration curve of human oxygenated and reduced haemoglobin. Biochem. J. 87, 33P (1963).
66. Rossi-Bernardi, L. and F.J.W. Roughton - The effect of temperature on the oxygen-linked ionization of hemoglobin. J. biol. Chem. 242, 784-792 (1967).
67. Rossi-Bernardi, L. and F.J.W. Roughton - The specific influence of carbon dioxide and carbamate compounds on the buffer power and Bohr effects in human hemoglobin solutions. J. Physiol. 189, 1-29 (1967).
68. Roughton, F.J.W. - Some recent work on the interactions of oxygen, carbon dioxide and haemoglobin. Biochem. J. 117, 801-812 (1970).
69. Sachs, J.R., P.A. Knauf and P.B. Dunham - Transport through red cell membranes. In: The red blood cell, vol. II. (Surgenor, D. MacN., ed.) pp. 613-695. New York: Acad. Press, 1975.
70. Savitz, D., V.W. Sidel and A.K. Solomon - Osmotic properties of human red cells. J. Gen. Physiol. 48, 79-94 (1964).
71. Schulz, G.V. - Statistische Ableitung der Grenzgesetze für verdïnnte Lösungen bei Verschiedenheit der Molvolumina. Z. Naturf. 2A, 27-38 (1927).
72. Siggaard-Andersen, O. - The acid-base status of the blood. Copenhagen: Munksgaard 1974.
73. Singer, R.B. and A.B. Hastings - An improved clinical method for the estimation of disturbances of the acid-base balance of human blood. Medicine 27, 223-242 (1948).
74. Takano, N., E. Hayashi and K. Matsue: Effect of oxygen saturation on $\mathrm{H}^{+}$and $\mathrm{Cl}^{-}$ distribution across the red cell membrane in human and ruminant blood. Pflïgers Arch. 336, 285-288 (1976).
75. Thomas, L.J., Jr. - Algorithms for selected blood acid-base and blood gas calculations. J. Appl. Physiol. 33, 154-1 58 (1972).
76. Valberg, L.S., J.M. Holt, E. Paulson and J. Szivek - Spectrochemical analysis of sodium, potassium, calcium, magnesium, copper, and zinc in normal human erythrocytes. J. Clin. Invest. 44, 379-389 (1965).
77. Van Slyke, D.D., A.B. Hastings, A. Hiller and J. Sendroy - Studies of gas and electrolyte equilibria in blood. XIV. The amounts of alkali bound by serum albumin and globulin. J. Biol. Chem. 79, 769-780 (1928).
78. Van Slyke, D.D. and J. Sendroy Jr. - Studies of gas and electrolyte equilibria in blood. XV. Line charts for graphic calculations by the Henderson-Hasselbalch equation, and for calculating plasma carbon dioxide content from whole blood content. J. Biol. Chem. 79, 781-798 (1928).
79. Van Slyke, D.D., J. Sendroy, A.B. Hastings and J.M. Neill: Studies of gas and electrolyte equilibria in blood. X. The solubility of carbon dioxide at $38^{\circ}$ in water, salt solution, serum, and blood cells. J. Biol. Chem. 78, 765-799 (1928).
80. Van Slyke, D.D., H. Wu and F.C. Mclean - Studies of gas and electrolyte equilibria in
blood. V. Factors controlling the electrolyte and water distribution in the blood. J. Biol. Chem. 56, 765-849 (1923).
81. Waddell, W.J. and R.G. Bates - Intracellular pH. Physiol. Rev. 49, 285-329 (1969).
82. Warth, J. and J.F. Desforges - Intraerythrocyte pH and physiochemical homogeneity. Proc. Soc. Exp. Biol. Med. 159, 136-138 (1978).
83. Wintrobe, M.M. - Clinical hematology, Sth ed., p. 104. Philadelphia: Lea \& Febiger 1961.
84. Wyman, J. - The heat of oxygenation of hemoglobin. J. Biol. Chem. 127, 581-599 (1939).
```
hUMAN Whole blood oxycen affinity: \(\mathrm{H}^{+}\)AND \(\mathrm{CO}_{2}\) BOHR AND HALDANE FACTORS CALCULATED FOR ERYTHROCYTE CONTENT AND BLOOD
```

Human whole-blood oxygen affinity: $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr and haldane factors calculated for erythrocyte content and blood
by J.P. Zock

Department of Physiology, Bloemsingel 10, University of Groningen, Groningen, the Netherlands

Communicated by Prof. W.G. Zijlstra at the meeting of October 29, 1984


#### Abstract

Oxygen affinity of hemoglobin is influenced by $\mathrm{pH}, p \mathrm{CO}_{2}$, and temperature. A mathematical description of the mutually dependent influences of $\mathrm{pH}, \mathrm{pCO}_{2}$, and temperature was formulated by Wyman (1964). In this paper an expression is presented based on Wyman's theory and on some assumptions about proton and carbon dioxide binding to hemoglobin. This expression relates the oxygen affinity of hemoglobin to the intra-erythrocytic value of pH , and to $\mathrm{pCO} \mathrm{CO}_{2}$ and temperature. In addition to this the model of blood we developed earlier (Zock et al., 1980) was used to relate pH in the red cells to that in plasma. As $p \mathrm{CO}_{2}$ and temperature were assumed to be the same both inside and outside the erythrocytes, this model enabled us to relate the oxygen affinity of hemoglobin inside the erythrocytes to plasma $\mathrm{pH}, \mathrm{pCO}_{2}$, and temperature. From this the bohr and haldane factors of blood were calculated. A comparison of the values calculated with those mentioned in the literature concerning whole blood shows good agreement.


## INTRODUCTION

In a previous paper (33) we presented a set of equations modelling some physico-chemical properties of blood. In that model the oxygen saturation of hemoglobin was one of the variables to which a certain value had to be assigned. However, the normal situation in the body is that the oxygen saturation of hemoglobin is determined by several quantities, of which oxygen tension $\left(p \mathrm{O}_{2}\right)$ is the main one. Quantities with a significant influence on the oxygen affinity of hemoglobin are pH , carbon dioxide tension $\left(p \mathrm{CO}_{2}\right)$, 2,3 -DPG concentration in the erythrocytes and temperature. To bring the model closer to reality, we have added equations relating the oxygen saturation $\left(s \mathrm{O}_{2}\right)$ of human hemoglobin to $p \mathrm{O}_{2}, \mathrm{pH}, p \mathrm{CO}_{2}$, and temperature. This
extended model was used to calculate the oxygen affinity of hemoglobin as the $p \mathrm{O}_{2}$ at $s \mathrm{O}_{2}=0.5\left(p_{50}\right)$ in relation to plasma $\mathrm{pH}, p \mathrm{CO}_{2}$, and temperature. The results thus obtained can be compared with those obtained by measuring oxygen affinity in whole blood. The model is therefore open to experimental verification.

Research concerning hemoglobin roughly falls apart into two approaches, the biochemical and the physiological. In the biochemical approach hemoglobin is regarded as a special protein and is studied under conditions which are for the greater part outside the normal, physiological range. This is, of course, a necessity because for many of the properties the physiological range of conditions is too narrow to allow reliable conclusions to be made. Physiological research, on the other hand, concerns the transport capabilities of hemoglobin in blood and experiments are done with blood under physiological conditions. Under these circumstances the hemoglobin solution is locked up inside the erythrocytes, so that information concerning the direct environment of the hemoglobin molecules is more difficult to obtain than in the biochemical studies. This applies mainly to intracellular pH , as the 2,3 -DPG concentration changes more slowly and can be determined afterwards. Temperature, $p \mathrm{O}_{2}$, and $p \mathrm{CO}_{2}$ can be considered to be the same as outside the erythrocytes. Because the intracellular pH is not uniquely related to plasma conditions, referring to extracellular values, as is of ten done, is prone to errors.

The biochemical approach has resulted in a more or less detailed knowledge of the binding of oxygen, protons, carbon dioxide and 2,3 -DPG to the hemoglobin molecule. Combination with chemical thermodynamics has led to mathematical formulations of the mutual interactions in the binding of these ligands (31). With this theory and using characteristic values for the ligand binding, it is possible to calculate changes in the oxygen affinity of hemoglobin caused by changes in intracellular $\mathrm{pH}, \mathrm{pCO}_{2}$ and temperature. The appropriate detailed quantitative knowledge of the mutual interactions of 2,3 -DPG with the other ligands is at present still lacking. This makes it impossible to calculate the effects of changes in 2,3 -DPG concentration.

On the basis of the above, the assigned value of $s \mathrm{O}_{2}$ in the model was replaced by expressions relating $s \mathrm{O}_{2}$ to intracellular $p \mathrm{O}_{2}, \mathrm{pH}, p \mathrm{CO}_{2}$ and temperature. A normal 2,3-DPG concentration inside the erythrocytes was assumed throughout.

The model, which has been described earlier (33), provides the relation between the compositions of the intra-erythrocytic compartment and the blood plasma compartment. Changes in the oxygen affinity of hemoglobin resulting from changes inside the erythrocytes can thus be related to concomitant changes in the plasma composition. The calculated values are compared with results of measurements in whole blood as given in the literature.

## THE OXYGEN AFFINITY OF HEMOGLOBIN

The reversible binding of oxygen to hemoglobin is influenced by several quantities defining the state of the solution. Quantities exerting major in-
fluences are $\mathrm{pH}, p \mathrm{CO}_{2}, 2,3$-DPG concentration in the solution, and temperature. These will be called affinity modifiers. Other less powerful influences are ionic strength and the concentrations of certain ions. We shall assume these to have normal physiological values throughout. When the modifiers are kept at constant values, a unique relation exists between $p \mathrm{O}_{2}$ and $s \mathrm{O}_{2}$ of hemoglobin. From its graphical representation, this relation is commonly referred to as the oxygen dissociation curve (ODC) or the oxygen equilibrium curve (OEC), the term preferred here. The effect of the modifiers on the oxygen affinity of hemoglobin is usually measured through the amount of change induced with respect to the OEC of hemoglobin under standard conditions.
The influence of the modifiers mainly consists of a shift of the OEC along the $\log \left(p \mathrm{O}_{2}\right)$ axis with only minor or no changes in its shape. The shape seems to be particularly indifferent to changes in temperature (35). Invariance of the shape means that the OEC is completely defined by only one parameter, for which $p_{50}$ is the generally accepted quantity. Under the assumption of invariance of shape, the description of the influence of the affinity modifiers on the OEC is reduced to the description of their influence on $p_{50}$ only. However, the modifiers with the exception of temperature also slightly change the shape of the OEC. As a consequence of this, $p_{50}$ is theoretically not a good measure of the oxygen affinity of hemoglobin. Instead, Wyman (31) has defined a more general measure of affinity for ligands bound to large molecules like hemoglobin which he calls the median ligand activity. For the reversible binding of oxygen to hemoglobin the median ligand activity $\left(p_{m}\right)$ is defined by

$$
\begin{equation*}
\int_{0}^{p_{m}} s \mathrm{O}_{2} d\left(\log \left(p \mathrm{O}_{2}\right)\right)=\int_{p_{m}}^{\infty}\left(1-s \mathrm{O}_{2}\right) d\left(\log \left(p \mathrm{O}_{2}\right)\right) \tag{1}
\end{equation*}
$$

in which the $s \mathrm{O}_{2}$ is a function of $p \mathrm{O}_{2}$ ranging from 0 to 1 and $\log \left(p \mathrm{O}_{2}\right)$ is the logarithm of $p \mathrm{O}_{2}$ to the base 10 .

From this, it is clear that $p_{50}=p_{\mathrm{m}}$ if the relation between $s \mathrm{O}_{2}$ and $\log \left(p \mathrm{O}_{2}\right)$ would be symmetrical around $p_{50}$ in the sense that

$$
\begin{equation*}
s \mathrm{O}_{2}\left(p \mathrm{O}_{2}=\frac{1}{\mathrm{a}} \cdot p_{50}\right)=1-s \mathrm{O}_{2}\left(p \mathrm{O}_{2}=\mathrm{a} \cdot p_{50}\right) \tag{2}
\end{equation*}
$$

In that case the hill plot, i.e. $\log \left(s \mathrm{O}_{2} /\left(1-s \mathrm{O}_{2}\right)\right)$ versus $\log \left(\mathrm{O}_{2}\right)$, would also be symmetrical around $p_{50}$. In practice, the difference between $p_{\mathrm{m}}$ and $p_{50}$ is small and as long as no changes in shape occur the change in $\log \left(p_{\mathrm{m}}\right)$ is equal to that in $\log \left(p_{50}\right)$. Therefore, in this paper derivations that are valid for $p_{\mathrm{m}}$ will be considered applicable to $p_{50}$ as well. will be considered applicable to $p_{50}$ as well.

The linkage equations derived by Wyman (31) relate the change in affinity of large molecules such as hemoglobin for a certain ligand under the influence of a change in the activity of a second ligand to the amount of the latter ligand that is bound to the molecule. The interactions between oxygen and the other ligands arise through the influence which the combination of hemoglobin with
oxygen exerts on the equilibrium constants of the reactions of the other ligands with hemoglobin. Temperature directly influences the equilibrium constants.

According to this, the relation between $p_{\mathrm{m}}$ as a measure of the oxygen affinity of hemoglobin and pH is

$$
\begin{gather*}
\partial \log \left(p_{\mathrm{m}}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)\right)  \tag{3}\\
\partial \mathrm{pH}
\end{gather*}=-\Delta z\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)
$$

in which $\Delta z$ is the amount of protons released per hemoglobin monomer when hemoglobin passes from the deoxygenated state $\mathrm{Hb}_{4}$ to the completely oxygenated state $\mathrm{Hb}_{4} \mathrm{O}_{8}$, while the modifiers remain constant. It was assumed that this amount is equal to the increase in the net negative charge of the hemoglobin monomer:

$$
\begin{equation*}
\Delta z\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)=z_{d}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)-z_{0}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right) \tag{4}
\end{equation*}
$$

where the indices " $d$ " and " $o$ " refer to deoxyhemoglobin and oxyhemoglobin, respectively.

In the same way, the relation between the amount of $\mathrm{CO}_{2}$ bound to hemoglobin (as carbaminohemoglobin) and $p_{\mathrm{m}}$ is

$$
\begin{gather*}
\partial \log \left(p_{\mathrm{m}}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)\right)  \tag{5}\\
\partial \log \left(p \mathrm{CO}_{2}\right)
\end{gather*}=\Delta \mathrm{Z}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)
$$

in which $\Delta Z$ is the amount of $\mathrm{CO}_{2}$ released per hemoglobin monomer when hemoglobin passes from the deoxygenated to the completely oxygenated state and the modifiers are kept constant.

At $p \mathrm{CO}_{2}=0$ the protons released on oxygenation originate from the acidic and alkaline bohr groups only. Thus

$$
\begin{equation*}
\Delta z=\Delta z_{A}+\Delta z_{B} \tag{6}
\end{equation*}
$$

in which $\Delta z_{A}$ refers to the acidic and $\Delta z_{B}$ to the alkaline groups and from which the arguments $\mathrm{pH}, p \mathrm{CO}_{2}$, and $T$ have been omitted. In line with the original proposal of Wyman (30), it was assumed that a hemoglobin monomer has one acidic bohr group, so that $\Delta z_{A}$ is equal to the difference between the saturation of the acidic group with protons in the deoxygenated state $\left(s \mathrm{H}_{a d}^{+}\right)$ and the saturation in the oxygenated state $\left(\mathrm{H}_{a o}^{+}\right)$. Saturation ranges from zero to one. Thus:

$$
\begin{equation*}
\Delta z_{A}=s \mathrm{H}_{a d}^{+}-s \mathrm{H}_{a o}^{+} \tag{7}
\end{equation*}
$$

This can conveniently be written as

$$
\begin{equation*}
\Delta z_{A}=\left[s \mathrm{H}_{a x}^{+}\right]_{x=0}^{x=d} \tag{8}
\end{equation*}
$$

in which the state variable $x$ can take one of the two values " $d$ " and " $o$ ". The degree of ionization of an acidic group equals one minus the saturation with protons. Consequently,

$$
\begin{equation*}
\left(1-s \mathrm{H}_{a . x}^{+}\right)=\frac{1}{1+10^{\mathrm{p} K_{a x}-\mathrm{pH}}} \tag{9}
\end{equation*}
$$

Because the $\mathrm{p} K$ values of the acidic group ( $\mathrm{p} K_{a r}$ ) depend on the state of oxygenation the same holds for the saturation with protons. In deoxyhemoglobin $\mathrm{p} K_{a x}=\mathrm{p} K_{a d}$, in oxyhemoglobin $\mathrm{p} K_{a x}=\mathrm{p} K_{a 0}$. The equations 7 and 8 can be written as

$$
\begin{equation*}
\Delta z_{A A}=-\left(1-s \mathrm{H}_{a d}^{+}\right)+\left(1-s \mathrm{H}_{a, 0}^{+}\right) \equiv\left[-\left(1-s \mathrm{H}_{a, \mathrm{~A}}^{+}\right)\right]_{A=0}^{+=d} \tag{10}
\end{equation*}
$$

The difference in charge caused by the alkaline bohr groups $\Delta z_{B}$, is the result of changes in saturation with protons of several amino groups on a hemoglobin molecule when hemoglobin passes from the deoxygenated to the oxygenated state (16). This change in proton saturation is brought about by the changes in $\mathrm{p} K$ of these groups that accompany changes in the state of oxygenation. Some of the $\mathrm{CO}_{2}$ is bound to hemoglobin by combination with some of the alkaline bohr groups. In the model we have approached this by assuming that two alkaline bohr groups reside on each hemoglobin monomer: one that does not combine with $\mathrm{CO}_{2}$ (indicated as $b$ ), and one that does (indicated as $z$ ). At $p \mathrm{CO}_{2}=0$ all of the latter groups contribute to the alkaline bohr effect. Accordingly,

$$
\begin{equation*}
\Delta z_{B}=\Delta s \mathrm{H}_{b}^{+}+\Delta s \mathrm{H}_{z}^{+} \tag{11}
\end{equation*}
$$

which can be written as

$$
\begin{equation*}
\Delta s \mathrm{H}_{b}^{+}=s \mathrm{H}_{b d}^{+}-s \mathrm{H}_{b o}^{+} \equiv\left[s \mathrm{H}_{b .}^{+}\right]_{y=0}^{x=d} \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
\angle \mathrm{S} \mathrm{H}_{z}^{+}=\left[s \mathrm{H}_{z+1}^{+}\right]_{x=0}^{x=d} \tag{13}
\end{equation*}
$$

In alkaline groups ionization is caused by proton binding, so that the degree of ionization is equal to the saturation with protons. Thus

$$
\begin{equation*}
s \mathrm{H}_{b x}^{+}=\frac{1}{1+10^{\mathrm{pH}-\mathrm{p} K_{b x}}} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
s \mathrm{H}_{z x}^{+}=\frac{1}{1+10^{\mathrm{pH}-\mathrm{p} K_{z x}}} \tag{15}
\end{equation*}
$$

The values of both $\mathrm{p} K$ 's as well as their changes with temperature depend on the state of oxygenation.

At $p \mathrm{CO}_{2}=0$ the change in the amount of bound protons as caused by oxygenation is described by

$$
\begin{equation*}
\Delta z=\left[-\left(1-s \mathrm{H}_{a x}^{+}\right)+s \mathrm{H}_{b . x}^{+}+s \mathrm{H}_{z x}^{+}\right]_{x=o}^{x=d} \tag{16}
\end{equation*}
$$

According to equation 3 , changes in $p_{\mathrm{m}}$ caused by changes in pH are calculated by integration of $\Delta z$ with respect to pH . Integration from $\mathrm{pH}_{1}$ to $\mathrm{pH}_{2}$ using equations 9,14 and 15 leads to

$$
\left\{\begin{align*}
\Delta \log \left(p_{\mathrm{m}}\right)= & {\left[\left[\operatorname { l o g } \left\{\left(1+10^{\mathrm{pH}-\mathrm{p} K_{u \mathrm{r}}}\right) \cdot\left(1+10^{\mathrm{p} K_{h \mathrm{r}}-\mathrm{pH}}\right)\right.\right.\right.}  \tag{17}\\
& \left.\left.\left.\cdot\left(1+10^{\mathrm{p} K_{\mathrm{zt}}-\mathrm{pH}}\right)\right\}\right]_{x=o}^{x=d}\right]_{\mathrm{pH}_{1}}^{\mathrm{pH}_{2}}
\end{align*}\right.
$$

In this expression the order of the substitution of $x$ and pH can be interchanged. This corresponds with the notion that when a combination of a reversible oxygenation and a reversible change in pH occurs, the change in $z$ is the same irrespective of the order of the oxygenation and the change in pH . Substitution of $x$ and pH gives four terms, each with one of the possible combinations of $x$ and pH . The combinations $\left(d, \mathrm{pH}_{2}\right)$ and ( $o, \mathrm{pH}_{1}$ ) take a positive sign, the other two a negative one.
When $\mathrm{CO}_{2}$ is present in the solution, the effect of carbamino formation must be taken into account. This effect is three-fold. Firstly, $\mathrm{CO}_{2}$ is a ligand and changes $p_{\mathrm{m}}$ according to equation 5 . Secondly, with the binding of $\mathrm{CO}_{2}$, alkaline bohr groups disappear. Thirdly, the carbamino groups formed with the binding of $\mathrm{CO}_{2}$ are strongly acidic; they contribute to the amount of protons released from hemoglobin when it passes from the deoxygenated to the completely oxygenated state, because carbamino formation strongly depends on oxygenation.

Carbamino-hemoglobin is formed by the reversible combination of $\mathrm{CO}_{2}$ with certain aminogroups on the hemoglobin molecule according to

$$
\begin{align*}
& \mathrm{HbNH}_{3}^{+} \leftrightarrows \mathrm{HbNH}_{2}+\mathrm{H}^{+} \quad: \mathrm{p} K_{z x}  \tag{18}\\
& \mathrm{HbNH}_{2}+\mathrm{CO}_{2} \leftrightarrows \mathrm{HbNHCOO}^{-}+\mathrm{H}^{+} \quad: \mathrm{p} K_{c x} \tag{19}
\end{align*}
$$

It is a generally accepted assumption that the carbamino groups on hemoglobin are completely dissociated. Therefore, the amount of protons released on carbamino formation equals, according to equation 19, the amount of $\mathrm{CO}_{2}$ bound. In the presence of $\mathrm{CO}_{2}, \Delta z$ thus contains additional terms to account for the latter two of the three effects above mentioned. It follows from these equilibria that the ratio $Z_{x}$ between the amount of carbamino-hemoglobin and the total amount of hemoglobin monomer is given by

$$
\begin{equation*}
Z_{x}=\frac{S \cdot p \mathrm{CO}_{2}}{S \cdot p \mathrm{CO}_{2}+10^{\mathrm{p} K_{c x}-\mathrm{pH}} \cdot\left(1+10^{\mathrm{p} K_{z x}-\mathrm{pH}}\right)} \tag{20}
\end{equation*}
$$

in which $S$ is the solubility of $\mathrm{CO}_{2}$ in the hemoglobin solution. The ratio $Z_{x}$ is different for deoxyhemoglobin and oxyhemoglobin. This is because $\mathrm{p} K_{c x}$ and $\mathrm{p} K_{z \gamma}$ have different values for deoxyhemoglobin and oxyhemoglobin. The amount of $\mathrm{CO}_{2}$ released per hemoglobin monomer on oxygenation at constant $\mathrm{pH}, p \mathrm{CO}_{2}$, and $T$ is given by

$$
\begin{equation*}
\Delta Z=Z_{d}-Z_{o} \equiv\left[Z_{x}\right]_{x=0}^{K=d} \tag{21}
\end{equation*}
$$

As already mentioned, the combination of $\mathrm{CO}_{2}$ with hemoglobin causes the disappearance of one alkaline bohr group. When a fraction $Z_{x}$ of the available groups is occupied by $\mathrm{CO}_{2},\left(1-Z_{x}\right)$ groups remain. It is assumed that the saturation with protons of the $\mathrm{CO}_{2}$ binding amino groups that do not carry
$\mathrm{CO}_{2}$ is unchanged. The amount of protons per hemoglobin monomer is thus $\left(1-Z_{x}\right) \cdot s \mathrm{H}_{z r}^{+}$. The expression for $z_{B}$ then becomes:

$$
\begin{equation*}
\Delta z_{B}=\left[s \mathrm{H}_{b x}^{+}+\left(1-\mathrm{Z}_{x}\right) \cdot s \mathrm{H}_{z x}^{+}-Z_{x}\right]_{x=0}^{x=d} \tag{22}
\end{equation*}
$$

and therefore

$$
\begin{equation*}
\Delta z=\left[-\left(1-s \mathrm{H}_{a x}^{+}\right)+s \mathrm{H}_{b x}^{+}+\left(1-Z_{x}\right) \cdot s \mathrm{H}_{z x}^{+}-Z_{x}\right]_{x=0}^{x=d} \tag{23}
\end{equation*}
$$

In accordance with equation 5 , the effect of $\mathrm{CO}_{2}$ proper on $p_{\mathrm{m}}$ can be obtained by integrating equation 21 with respect to $\log \left(p \mathrm{CO}_{2}\right)$. Integration from $p \mathrm{CO}_{2,1}$ to $p \mathrm{CO}_{2,2}$ gives
(24) $\Delta \log \left(p_{\mathrm{m}}\right)=\left[\left[\log \left\{1+\frac{S \cdot p \mathrm{CO}_{2}}{10^{\mathrm{P} K_{\mathrm{cr}}-\mathrm{pH}} \cdot\left(1+10^{\mathrm{p} K_{z, x}-\mathrm{pH}}\right)}\right\}\right]_{x=0}^{x=d}\right]_{p \mathrm{CO}_{2,1}}^{\rho \mathrm{CO}_{2,2}}$

Combination of this equation with equation 17 gives the expression for the case where changes in pH and $p \mathrm{CO}_{2}$ occur simultaneously:

$$
\left\{\begin{align*}
\Delta \log \left(p_{\mathrm{m}}\right)= & {\left[\left[\operatorname { l o g } \left\{\left(1+10^{\mathrm{pH}-\mathrm{p} K_{a x}}\right) \cdot\left(1+10^{\mathrm{p} K_{b x}-\mathrm{pH}}\right) \cdot\left(1+10^{\mathrm{p} K_{z x}-\mathrm{pH}}\right)\right.\right.\right.} \\
& \left.\left.\left.\cdot\left(1+\frac{S \cdot p \mathrm{CO}_{2}}{10^{\mathrm{p} K_{c x}-\mathrm{pH}} \cdot\left(1+10^{\mathrm{p} K_{z x}-\mathrm{pH}}\right)}\right)\right\}\right]_{x=o}^{x=d}\right]_{\mathrm{pH}_{1}, p \mathrm{CO}_{2,1}}^{\mathrm{pH}_{2}, p \mathrm{CO}_{2,2}} \tag{25}
\end{align*}\right.
$$

Consequently,

$$
\left\{\begin{align*}
\log \left(p_{\mathrm{m}}\right)= & \text { constant }+\left[\operatorname { l o g } \left\{\left(1+10^{\mathrm{pH}-\mathrm{p} K_{a x}}\right) \cdot\left(1+10^{\mathrm{p} K_{b x}-\mathrm{pH}}\right)\right.\right.  \tag{26}\\
& \left.\left.\cdot\left(1+10^{\mathrm{p} K_{z x}-\mathrm{pH}}\right) \cdot\left(1+\frac{S \cdot p \mathrm{CO}_{2}}{10^{\mathrm{p} K_{c x}-\mathrm{pH}} \cdot\left(1+10^{\mathrm{p} K_{z x}-\mathrm{pH}}\right)}\right)\right\}\right]_{x=o}^{x=d}
\end{align*}\right.
$$

The value of the constant is the logarithm of $p_{\mathrm{m}}$ in some reference state with the reference values $\mathrm{pH}_{r}, p \mathrm{CO}_{2, r}$, and $T_{r}$, minus the value of the function between the square brackets in this reference state. The part of the right hand side of equation 26 with the square brackets will be henceforth referred to as $P$.

Changes in $p_{\mathrm{m}}$ occurring with changes in temperature $(T)$ are related to the apparent heat of combination of oxygen with hemoglobin according to

$$
\begin{gather*}
\partial \log \left(p_{\mathrm{m}}\right)-\Delta Q  \tag{27}\\
\partial(1 / T)
\end{gather*}-R \cdot \ln (10)
$$

in which $\Delta Q$ is the total heat of the reaction and $R$ is the universal gas constant $8.314 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K}$. The total heat is the result of the binding of oxygen and the concomitant changes in ionization and in the amounts of other substances bound to hemoglobin:

$$
\begin{equation*}
\Delta Q=\Delta Q_{i}+\Delta Q_{\mathrm{O}_{2}} \tag{28}
\end{equation*}
$$

in which $\Delta Q_{\mathrm{O}_{2}}$ is the heat of the binding of oxygen and $\Delta Q_{i}$ is the heat of the accompanying ionizations including carbon dioxide binding. Besides that of oxygen $\Delta Q_{\mathrm{O}_{2}}$ was assumed to include the heat of combination with hemoglobin of the substances not accounted for in $\Delta Q_{i}$ like e.g. 2,3-DPG.

In the equation 26, temperature is implicitly present in the temperature dependence of the various $\mathrm{p} K$ 's and in the solubility of $\mathrm{CO}_{2}$. With these dependences inserted, the function $P$ is a function of temperature as well, and can be used to calculate that part of $\Delta Q$ that is caused by changes in ionization and carbon dioxide binding with temperature:

$$
\begin{equation*}
\Delta Q_{i}=R \cdot \ln (10) \cdot \frac{\partial P}{\partial(1 / T)} \tag{29}
\end{equation*}
$$

According to this

$$
\left\{\begin{align*}
\Delta Q_{i} & =R \cdot \ln (10) \cdot\left[-\left(1-s \mathrm{H}_{a x}^{+}\right) \cdot \frac{d \mathrm{p} K_{a x}}{d(1 / T)}+s \mathrm{H}_{b x}^{+} \cdot \frac{d \mathrm{p} K_{b x}}{d(1 / T)}+\right.  \tag{30}\\
& \left.+\left(1-Z_{x}\right) \cdot s \mathrm{H}_{z x}^{+} \cdot \frac{d \mathrm{p} K_{z x}}{d(1 / T)}+Z_{x} \cdot\left(\frac{1}{S \cdot \ln (10)} \cdot \frac{d S}{d(1 / T)}-\frac{d \mathrm{p} K_{c x}}{d(1 / T)}\right)\right]_{x=0}^{x=d}
\end{align*}\right.
$$

From the equations 27 and 28 it follows that
(31) $\quad \partial \log \left(p_{\mathrm{m}}\right) \quad=\frac{1}{\partial(1 / T)} \overline{R \cdot \ln (10)} \cdot\left(\Delta Q_{\mathrm{o}_{2}}+\Delta Q_{i}\right)$

Over a restricted range of temperature, $\Delta Q_{\mathrm{O}_{2}}$ can be taken to be constant. Because the combination of oxygen with hemoglobin is exothermic, $\Delta Q_{\mathrm{O}_{2}}$ is negative. A part of the energy of the reaction is used in the change of ionization; this energy $\Delta Q_{i}$ is positive because energy is used up to increase ionization. The total heat of ionization is the weighted sum of the heats of ionization of the various groups. The term $\Delta Q_{\mathrm{O}_{2}}$ represents that part of the heat of oxygenation which is not accounted for in $\Delta Q_{i}$. This is the reaction with oxygen itself and, if present, that with $2,3-\mathrm{DPG}$. Possibly, the binding of other ions also contributes to $\Delta Q_{\mathrm{O}_{2}}$ to a lesser degree. From the known dependence of $p_{\mathrm{m}}$ on temperature it follows that all these heats add up to $-43 \mathrm{~kJ} / \mathrm{mol}$ at $\mathrm{pH}=7.4, p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$, and $T=310 \mathrm{~K}$ (35). With the adopted temperature dependencies of $\mathrm{p} K$ and $S$, the heat of ionization $\Delta Q_{i}$ calculated according to equation 30 amounts to about $51 \mathrm{~kJ} / \mathrm{mol}$. In order to obtain a temperature dependence of $p_{\mathrm{m}}$ in accordance with experimental values $\Delta Q_{\mathrm{O}_{2}}$ was taken to be $-94 \mathrm{~kJ} / \mathrm{mol}$. A comparison with known thermodynamic values is beyond the scope of this paper.

The relation giving $p_{\mathrm{m}}$ including its temperature dependence follows from the integration of equation 31 with respect to $(1 / T)$. Considering that $\Delta Q_{i}$ is $R \cdot \ln (10)$ times the derivative of $P$, this gives

$$
\begin{equation*}
\log \left(p_{\mathrm{m}}\right)=\log \left(p_{\mathrm{mr}}\right)+\left[\frac{\Delta Q_{\mathrm{O}_{2}}}{R \cdot T \cdot \ln (10)}+P\right]_{\left(\mathrm{PH},, \rho C \mathrm{O}_{2, r}, T_{r}\right)}^{\left(\mathrm{pH}, \rho C \mathrm{O}_{2}, T\right)} \tag{32}
\end{equation*}
$$

in which $p_{\mathrm{m} r}$ is the median oxygen affinity at reference conditions $\left(\mathrm{pH}_{r}\right.$, $\left.p \mathrm{CO}_{2, r}, T_{r}\right)$. This function represents the oxygen affinity of hemoglobin dependent upon $\mathrm{pH}, \mathrm{pCO}_{2}$, and $T$ under the restrictions imposed by the assumptions adopted.

To be able to compare values from the literature with those calculated with the relations derived here, $\log \left(p_{\mathrm{m}}\right)$ can be expanded in a taylor series about the reference point:

$$
\left\{\begin{align*}
\log \left(p_{\mathrm{m}}\left(\mathrm{pH}, p \mathrm{CO}_{2}, T\right)\right)= & \log \left(p_{\mathrm{m}}\left(\mathrm{pH}_{r}, p \mathrm{CO}_{2, r}, T_{r}\right)\right)+  \tag{33}\\
& +A \cdot \Delta \mathrm{pH}+B \cdot \Delta \log \left(p \mathrm{CO}_{2}\right)+ \\
& +C \cdot \Delta(1 / T)+D \cdot \Delta \mathrm{pH} \cdot \Delta \log \left(p \mathrm{CO}_{2}\right)+ \\
& +E \cdot \Delta \log \left(p \mathrm{CO}_{2}\right) \cdot \Delta(1 / T)+ \\
& +F \cdot \Delta(1 / T) \cdot \Delta \mathrm{pH}+\ldots .
\end{align*}\right.
$$

in which $\Delta \mathrm{pH}=\mathrm{pH}-\mathrm{pH}_{r}, \Delta \log \left(p \mathrm{CO}_{2}\right)=\log \left(p \mathrm{CO}_{2} / p \mathrm{CO}_{2, r}\right)$, and $\Delta(\mathrm{I} / T)=$ $=1 / T-1 / T_{r}$. The coefficients of $\Delta \mathrm{pH}, \Delta \log \left(p \mathrm{CO}_{2}\right)$, and $\Delta(1 / T)$ are equal to the first partial derivatives of equation 32:

$$
\left\{\begin{array}{l}
A=\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \mathrm{pH}}=-\Delta z  \tag{34}\\
B=\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \log \left(p \mathrm{CO}_{2}\right)}=\Delta Z \\
C=\frac{\partial \log \left(p_{\mathrm{m}}\right)-\Delta Q}{\partial(1 / T)}-R \cdot \ln (\overline{10})
\end{array}\right.
$$

The derivative with respect to $\mathrm{pH}, A$, gives the influence of pH , i.e. $\mathrm{H}^{+}$ activity, on $p_{\mathrm{m}}$ and is therefore called the proton or $\mathrm{H}^{+}$bohr factor. The derivative with respect to $\log \left(p \mathrm{CO}_{2}\right), B$, representing the $\mathrm{CO}_{2}$ influence on $p_{\mathrm{m}}$, is accordingly called the $\mathrm{CO}_{2}$ bohr factor. The second derivatives give the mutual dependencies of the first derivatives. These mutual influences are sometimes called interdependent ligand effects.

Table 1. Values of pK 's and their temperature dependence as used in the calculations.

|  | deoxyhemoglobin |  | oxyhemoglobin |  |
| :--- | :--- | :---: | :--- | :---: |
| index | $\mathrm{p} K$ | $\mathrm{p} K / d(1 / T)$ | $\mathrm{p} K$ | $d \mathrm{p} K / d(1 / T)$ |
| $a$ | 5.13 | -846 | 5.60 | 306 |
| $b$ | 7.56 | 2340 | 6.65 | 1224 |
| $z$ | 7.30 | 1500 | 7.00 | 1500 |
| $c$ | 4.60 | -1828 | 5.00 | -1828 |

In our model they are:

$$
\left\{\begin{align*}
D= & \frac{\partial}{\partial \log \left(p \mathrm{CO}_{2}\right)}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \mathrm{pH}}\right)=\frac{\partial}{\partial \mathrm{pH}}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \log \left(p \mathrm{CO}_{2}\right)}\right)= \\
= & \ln (10) \cdot\left[Z_{x} \cdot\left(1-Z_{x}\right) \cdot\left(1+s \mathrm{H}_{z x}^{+}\right)\right]_{x=o}^{x=d} \\
E= & \frac{\partial}{\partial(1 / T)}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \log \left(p \mathrm{CO}_{2}\right)}\right)=\frac{\partial}{\partial \log \left(p \mathrm{CO}_{2}\right)}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial(1 / T)}\right)= \\
= & \ln (10) \cdot\left[Z_{x} \cdot\left(1-Z_{x}\right) \cdot\left(\frac{1}{S \cdot \ln (10)} \cdot \frac{d S}{d(1 / T)}-\frac{d \mathrm{p} K_{c x}}{d(1 / T)}-s \mathrm{H}_{z x}^{+} \cdot \frac{d \mathrm{p} K_{z x}}{d(1 / T)}\right)\right]_{x=0}^{x=d} \\
F= & \frac{\partial}{\partial(1 / T)}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial \mathrm{pH}}\right)=\frac{\partial}{\partial \mathrm{pH}}\left(\frac{\partial \log \left(p_{\mathrm{m}}\right)}{\partial(1 / T)}\right)=  \tag{35}\\
= & -\ln (10) \cdot\left[\left(1-s \mathrm{H}_{a x}^{+}\right) \cdot s \mathrm{H}_{a x}^{+} \cdot \frac{d \mathrm{p} K_{a x}}{d(1 / T)}+\left(1-s \mathrm{H}_{b x}^{+}\right) \cdot s \mathrm{H}_{b x}^{+} \cdot \frac{d \mathrm{p} K_{b x}}{d(1 / T)}+\right. \\
& +\left(1-s \mathrm{H}_{z x}^{+}\right) \cdot s \mathrm{H}_{z x}^{+} \cdot \frac{d \mathrm{p} K_{z x}}{d(1 / T)}+ \\
& \left.-Z_{x} \cdot\left(1-Z_{x}\right) \cdot\left(\frac{1}{S \cdot \ln (10)} \cdot \frac{d(S)}{d(1 / T)}-\frac{d \mathrm{p} K_{c x}}{d(1 / T)}-s \mathrm{H}_{z x}^{+} \cdot \frac{d \mathrm{p} K_{z x}}{d(1 / T)}\right)\right]_{x=o}^{x=d}
\end{align*}\right.
$$

Table 1 presents the values of $\mathrm{p} K$ 's and their temperature dependence as used in the calculations. The values of $\mathrm{p} K_{a}$ and $\mathrm{p} K_{b}$ are as given by Rossi-Bernardi and Roughton (24). The values of $\mathrm{p} K_{c}$ and $\mathrm{p} K_{z}$ are reasonable estimates based on the values given by Klocke (17), Perrella (20), Bauer (5), and those given by Rossi-Bernardi and Roughton (23) based on values of Ferguson (8). The values chosen fulfill the requirement that they comply with experimental values and lead to generally accepted values concerning the bohr effects and carbamino hemoglobin in the deoxy and oxy form as well. The solubility $S$ and other quantities not mentioned here have the same values as in the previous paper (33).

## the oxygen equilibrium curve

The standard OEC of blood is related to the conditions $T=310 \mathrm{~K}$, $\mathrm{pH}_{p}=7.4, p \mathrm{CO}_{2}=5.33 \mathrm{kPa}$, and a normal 2,3-DPG concentration in the erythrocytes of $0.92 \mathrm{~mol} 2,3-\mathrm{DPG}$ per $\mathrm{mol} \mathrm{Hb}_{4}$. Numerous expressions to describe the OEC have been presented. In 1910 Hill published his well-known equation (11). The Adair equation (2) is based on the four successive steps in the oxygen binding reaction. Recent values of the four association constants are given by Roughton and Severinghaus (24). A number of expressions have been put forward that are not based on a reaction mechanism but are intended to approximate measured curves as closely as possible ( $10,15,18,26,28$ ). The equation presented by Lobdell (18) is both simple and readily invertible. This equation, which is used in our model, is

$$
\begin{equation*}
s \mathrm{O}_{2}=\frac{\mathrm{a} \cdot p_{\mathrm{r}}^{\mathrm{n}}+\mathrm{b} \cdot p_{\mathrm{r}}^{2 \mathrm{n}}}{1+\mathrm{c} \cdot p_{\mathrm{r}}^{\mathrm{n}}+\mathrm{b} \cdot p_{\mathrm{r}}^{2 \mathrm{n}}} \tag{36}
\end{equation*}
$$

The inverse equation is

$$
p_{\mathrm{r}}=\left\{\begin{array}{c}
\mathrm{a}-\mathrm{c} \cdot s \mathrm{O}_{2}-\sqrt{\left(\mathrm{a}-\mathrm{c} \cdot s \mathrm{O}_{2}\right)^{2}-4 \cdot \mathrm{~b} \cdot\left(s \mathrm{O}_{2}-1\right) \cdot s \mathrm{O}_{2}}  \tag{37}\\
2 \cdot \mathrm{~b} \cdot\left(s \mathrm{O}_{2}-1\right)
\end{array}\right\}^{1 / n}
$$

in which $s \mathrm{O}_{2}$ is the oxygen saturation, and $p_{\mathrm{r}}$ is the reduced oxygen tension, i.e. $p \mathrm{O}_{2} / p_{50}$, and $\mathrm{a}, \mathrm{b}, \mathrm{c}$, and n are constants. Their values are given in the caption of table 2 , which gives a comparison between measured values of $p \mathrm{O}_{2}$ at fixed values of $s \mathrm{O}_{2}$ (34) and the values of $p \mathrm{O}_{2}$ resulting from equation 37, using these values for the constants. Calculation of $p_{\mathrm{m}}$ using equation 36 with the values of the constants as given shows $p_{\mathrm{m}}$ to be about 1.06 times $p_{50}$.

Table 2. Comparison of $p \mathrm{O}_{2}$ calculated according to Lobdell (18) with values measured (34). Standard conditions $\mathrm{pH}=7.4, p \mathrm{CO}_{2}=5.33 \mathrm{kPa}$, and $T=310 \mathrm{~K}$. The values of the parameters used were: $\mathrm{n}=1.58, \mathrm{a}=0.355, \mathrm{~b}=0.645$, and $\mathrm{c}=0.355$.

| $s \mathrm{O}_{2}$ | $p \mathrm{O}_{2}$ measured <br> $(\mathrm{kPa})$ | $p \mathrm{O}_{2}$ calculated <br> $(\mathrm{kPa})$ |
| :--- | :---: | :--- |
| 0.1 | 1.39 | 1.37 |
| 0.2 | 2.01 | 2.00 |
| 0.3 | 2.52 | 2.52 |
| 0.4 | 3.01 | 3.01 |
| 0.5 | 3.53 | 3.53 |
| 0.6 | 4.12 | 4.12 |
| 0.7 | 4.84 | 4.84 |
| 0.8 | 5.86 | 5.87 |
| 0.9 | 7.76 | 7.77 |
| 0.95 | 10.14 | 9.98 |

## MODEL

In the model blood was assumed to consist of two compartments, each containing a homogeneous solution. The compartment representing plasma was assumed to contain a solution of $\mathrm{NaCl}, \mathrm{NaHCO}_{3}$, and protein in water. The other compartment, representing the red blood cells, was assumed to contain a solution of $\mathrm{KCl}, \mathrm{KHCO}_{3}$, ATP, 2,3-DPG, and hemoglobin in water. The two compartments were taken to be separated by a membrane which was assumed to be impermeable to $\mathrm{Na}^{+}, \mathrm{K}^{+}$, ATP, 2,3-DPG, hemoglobin, and plasma protein. It was assumed to be permeable to $\mathrm{O}_{2}, \mathrm{CO}_{2}, \mathrm{HCO}_{3}^{-}, \mathrm{Cl}^{-}$, $\mathrm{H}^{+}$, and water. The two compartments were assumed to be in osmotic equilibrium. These aspects of the model have been described in detail earlier (33). Changes with respect to the earlier version of the model are the slightly different values of some $\mathrm{p} K^{\prime}$ 's and their temperature dependencies (Table 1).
The present version of the model has been extended with the OEC and the dependence of $p_{\mathrm{m}}$ on $\mathrm{pH}, p \mathrm{CO}_{2}$, and temperature in the hemoglobin solution representing red cell content. A further change is the notion that $\mathrm{p} K_{z}$ belongs
to an alkaline bohr group that not only binds $\mathrm{CO}_{2}$ but also contributes to $\mathrm{H}^{+}$ binding.

The reference value of $p_{\mathrm{m}}$ was taken to be 3.53 kPa at $\mathrm{pH}_{c}=7.192$, $p \mathrm{CO}_{2}=5.33 \mathrm{kPa}$, and $T=310 \mathrm{~K}$. The 2,3-DPG concentration used resulted from the various conditions that had to be satisfied at the reference state. In the model it amounts to $0.92 \mathrm{~mol} / \mathrm{mol}_{\mathrm{Hb}}^{4}$.

## CALCULATIONS

The equations derived above allow the calculation of the change in the $p_{\mathrm{m}}$ of human hemoglobin in solution with changes in $\mathrm{pH}, \mathrm{pCO}_{2}$, and temperature in this solution. The results presented were obtained by substituting the values of $\mathrm{pH}, p \mathrm{CO}_{2}$ and $T$ in the relevant equations.

The changes in $p_{\mathrm{m}}$ under the influence of changes in plasma values cannot be calculated by direct substitution in the equations. Instead, the plasma values were used to calculate the intracellular pH at $s \mathrm{O}_{2}=0.5$ since temperature and $p \mathrm{CO}_{2}$ were the same both inside and outside the red cell, these values and the intracellular pH were substituted in equation 32 , thus giving $p_{\mathrm{m}}$ under these conditions. From these sets of values the whole blood bohr factors were calculated.

To calculate the whole blood $\mathrm{H}^{+}$haldane factor at a chosen $p \mathrm{CO}_{2}$, plasma pH was controlled by the simulated addition of HCl or NaOH . At the same values of plasma $\mathrm{pH}, p \mathrm{CO}_{2}$, and $T$, but with $s \mathrm{O}_{2}=0$ and $s \mathrm{O}_{2}=1$ the amounts of NaOH or HCl needed to keep plasma pH constant were calculated as well. The difference between the amount of HCl and NaOH at $s \mathrm{O}_{2}=0$ and that at $s \mathrm{O}_{2}=1$, divided by the amount of hemoglobin monomer, gives the whole blood $\mathrm{H}^{+}$haldane effect at these plasma conditions, i.e. the amount of fixed acid that has to be withdrawn or added to keep plasma pH constant during the oxygenation of blood.

Using the same set of values of $\mathrm{pH}, \mathrm{CO}_{2}$, and $T$, with $s \mathrm{O}_{2}=0$ and $s \mathrm{O}_{2}=1$, the total amounts of $\mathrm{CO}_{2}$ present in blood were also calculated. The difference between the amount of $\mathrm{CO}_{2}$ at $s \mathrm{O}_{2}=0$ and that at $s \mathrm{O}_{2}=1$, divided by the amount of hemoglobin gives the whole blood $\mathrm{CO}_{2}$ haldane factor.

By taking into consideration only those protons and that $\mathrm{CO}_{2}$ that bind to hemoglobin, the contributions of hemoglobin to the whole blood $\mathrm{H}^{+}$haldane and the whole blood $\mathrm{CO}_{2}$ haldane factors were calculated, i.e. $\Delta z$ and $\Delta Z$. These values differ from those of a hemoglobin solution because intracellular pH is not constant under these circumstances.

The calculations were made for values of plasma pH ranging from 6.7 to 8.1 in steps of 0.05 pHu at $p \mathrm{CO}_{2} 0.16$ to 29 kPa in steps of $\log \left(p \mathrm{CO}_{2}\right)=0.25$ for the temperatures 290 to 320 K in steps of 5 K .

The program for these calculations was run on a HP9845A desk top computer. The program was written in BASIC. It contained a number of iterative procedures. By controlled variation of some variables, the values were made to converge to an end state in which all the conditions set by the equations were satisfied to within $0.001 \mathrm{mmol} / \mathrm{L}$.

In blood the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr factors were calculated from the values of $\log \left(p_{\mathrm{m}}\right)$ at the concomitant values of plasma $\mathrm{pH}, \mathrm{CO}_{2}$, and temperature. This was done by fitting a taylor series over the 26 neighbouring points using a least square method. The taylor series contained the three first derivatives with respect to plasma $\mathrm{pH}, \log \left(p \mathrm{CO}_{2}\right)$ and $(1 / T)$, and three partial second derivatives as coefficients. Only three second derivatives were used because it was assumed that the result of differentiation is independent of the order. The first derivative with respect to pH represents the whole blood $\mathrm{H}^{+}$bohr factor, that with respect to $\log \left(p \mathrm{CO}_{2}\right)$ the whole blood $\mathrm{CO}_{2}$ bohr factor. The derivative with respect to $(1 / T)$ represents the temperature dependence of $\log \left(p_{\mathrm{m}}\right)$ in whole blood. The second derivatives give the mutual influences of the variables on these factors analogous to those derived for hemoglobin solutions in equation 35 .

## RESULTS AND DISCUSSION

The discovery of what is now called the bohr effect by Bohr et al. (6) concerned the influence of $\mathrm{CO}_{2}$ on the position of the OEC of blood. This effect can be observed when blood is titrated with $\mathrm{CO}_{2}$ without controlling pH . In fact, the shift is the result of a combination of two effects, the origins of which lie in the nature of the interactions of hemoglobin with protons and $\mathrm{CO}_{2}$. Titration of blood with $\mathrm{CO}_{2}$ liberates protons and $\mathrm{CO}_{2}$ itself interacts with hemoglobin by forming carbamino-hemoglobin.

The original haldane effect (7) is the complement of the original bohr effect, $i . e$. the influence of the oxygen saturation on the equilibrium concentration of all $\mathrm{CO}_{2}$ present in blood at a certain $p \mathrm{CO}_{2}$. This, too, is a combination of two effects. The increased $\mathrm{CO}_{2}$ content of deoxygenated blood is caused by the decrease in acidity of hemoglobin in additon to the increased $\mathrm{CO}_{2}$ binding by hemoglobin. The property that these two affinities of hemoglobin change on oxygenation is a thermodynamical necessity complementary to the two bohr effects as was first shown by Adair (1) and further elaborated by Wyman (31). Thus, for hemoglobin the $\mathrm{H}^{+}$bohr factor related to the median oxygen tension is equal to the $\mathrm{H}^{+}$haldane factor. The same equality applies to the $\mathrm{CO}_{2}$ bohr and haldane factors. This complementariness is expressed in equations 3 and 5 and underlies the equations derived from them.

At a time that little was known about the structure of hemoglobin, Wyman (30) showed that the titration data of hemoglobin could be explained by the assumption that each hemoglobin monomer has two oxygen-linked titrable groups. These groups were supposed to change their $\mathrm{p} K$ 's on oxygenation, the $\mathrm{p} K$ of one group lying in the acidic range of pH (acid bohr group), the other $\mathrm{p} K$ in the alkaline range (alkaline bohr group). It is clear now, that these oxygen-linked groups are not evenly distributed over the four hemoglobin monomers of the $\mathrm{Hb}_{4}$ molecule and that the bohr effect may involve complex interactions between the four chains as well. It has been shown (16) that most of the alkaline bohr effect can be ascribed to the $\alpha$-amino part of the N -terminal valines of the $\alpha$-chains and the imidazole part of the histidines 146 of the
$\beta$-chains. In the presence of 2,3 -DPG the proton bohr effect increases because the binding of 2,3-DPG includes binding of $\mathrm{H}^{+}(22)$. The amount of bound 2,3 -DPG diminishes on oxygenation. The 2,3 -DPG is bound to the $\alpha$-amino parts of the N -terminal valines of the two $\beta$-chains. Because these valines can also bind $\mathrm{CO}_{2}, 2,3$-DPG and $\mathrm{CO}_{2}$ are competitive here. These valines do not contribute to the proton bohr effect (16). The acid bohr effect may be the result of interactions between many groups which have not been identified yet. It seems likely that the alkaline and acid parts of the bohr effect are independent (16).

The $\alpha$-amino parts of the N -terminal valines of both the $\alpha$-and the $\beta$-chains participate in the oxygen-linked binding of $\mathrm{CO}_{2}$. This oxygen-linked $\mathrm{CO}_{2}$ binding is the cause of the shift in $p_{50}$ occurring when $p \mathrm{CO}_{2}$ changes at a fixed pH , i.e. the $\mathrm{CO}_{2}$ bohr effect.

For the sake of simplicity, the differences between the proton and $\mathrm{CO}_{2}$ binding properties of the $\alpha$ - and $\beta$-chains of the hemoglobin molecule were neglected, as were the interactions between them. The monomers were assumed to behave identically, in line with the original assumption of Wyman (30). The affinities to protons and $\mathrm{CO}_{2}$ as used in the model can be considered to represent mean values of the affinities of the proton and $\mathrm{CO}_{2}$ binding groups to the $\alpha$ - and $\beta$-chains in the presence of a normal 2,3-DPG concentration. To account for the fact that in the N -terminal valines of the $\alpha$-chains $\mathrm{CO}_{2}$ binding partly interferes with proton binding, each monomer was assumed to have two alkaline bohr groups instead of one. To one of these groups oxygen-linked $\mathrm{CO}_{2}$ and proton binding was ascribed, the other group exhibited only oxygenlinked proton binding.

As already mentioned, the classical bohr effect concerns whole blood and is due to the change in $p \mathrm{CO}_{2}$ and the concomitant change in pH . To separate these two influences one of the two variables, $p \mathrm{CO}_{2}$ or pH , was varied and the other was kept constant. Thus, the $\mathrm{H}^{+}$bohr factor of blood with respect to $p_{50}$ is measured as the change in $\log \left(p_{50}\right)$ divided by the change in blood (plasma) pH at constant $p \mathrm{CO}_{2}$. The $\mathrm{CO}_{2}$ bohr factor of blood is accordingly determined at constant blood (plasma) pH . The relation between the classical bohr factor and the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr factors is

$$
\begin{equation*}
\frac{d \log \left(p_{50}\right)}{\bar{d} \log \left(p \mathrm{CO}_{2}\right)}=\left.\frac{\partial \log \left(p_{50}\right)}{\partial \log \left(p \mathrm{CO}_{2}\right)}\right|_{\mathrm{pH}_{p}}+\left.\frac{\partial \log \left(p_{50}\right)}{\partial \mathrm{pH}_{p}}\right|_{p \mathrm{CO}_{2}} \cdot \frac{d \mathrm{pH}_{p}}{d \log \left(p \mathrm{CO}_{2}\right)} \tag{38}
\end{equation*}
$$

Thus in contrast with the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr factors of blood, the classical bohr factor also depends on the $\mathrm{CO}_{2}$ buffer capacity of the blood. Direct calculation of the classical bohr factor by simulating titration of blood with $\mathrm{CO}_{2}$ gave -0.33 at $\mathrm{pH}_{p}=7.4, p \mathrm{CO}_{2}=5.33 \mathrm{kPa}$, and $T=310 \mathrm{~K}$. The classical bohr factor is often expressed with respect to the uncontrolled change in blood pH concomitant with the change in $p \mathrm{CO}_{2}$. The reason for this is that the measurement of pH is easier, more accurate, and more reliable than that of $p \mathrm{CO}_{2}$. The value calculated for this quantity was -0.51 at $\mathrm{pH}_{p}=7.4$, $p \mathrm{CO}_{2}=5.33 \mathrm{kPa}$, and $T=310 \mathrm{~K}$. This agrees with comparable experimental


Fig. 1. Change in $\log \left(p_{\mathrm{m}}\right)$ of a hemoglobin solution versus pH in the solution at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$. Panel A: $32{ }^{\circ} \mathrm{C}(305 \mathrm{~K})$; panel B: $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$; panel C: $42{ }^{\circ} \mathrm{C}(315 \mathrm{~K})$.
values given in the literature: $-0.54 \pm 0.06$ (12), -0.53 (9), and -0.51 (21). Hilpert et al. (12) also determined the ratio between $\Delta \log \left(p_{50}\right)$ and the change in red cell pH . They have found this to be $-0.66 \pm 0.08$. The value of this quantity as calculated with our model was -0.65 .

Some properties of hemoglobin in a solution representing erythrocyte content as calculated with the equations derived are given in figs. 1 to 5 . Figs. 1 to 3 show the change in $\log \left(p_{\mathrm{m}}\right)$ dependent on the $\mathrm{pH}, \mathrm{pCO}_{2}$, and temperature of the hemoglobin solution. Figs. 4 and 5 give the $\mathrm{H}^{+}$and the $\mathrm{CO}_{2}$ bohr factors, respectively. Because these curves concern a hemoglobin solution they also represent the change in $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bound per mol hemoglobin monomer, i.e. the haldane factors for hemoglobin. The absolute value of the $\mathrm{H}^{+}$bohr factor at low $p \mathrm{CO}_{2}$ is 0.57 . As already mentioned, this value was reached by


Fig. 2. Change in $\log \left(p_{\mathrm{m}}\right)$ of a hemoglobin solution versus $p \mathrm{CO}_{2}$ at six values of pH in the solution. Panel A: $32{ }^{\circ} \mathrm{C}(305 \mathrm{~K})$; panel B: $37^{\circ} \mathrm{C}(310 \mathrm{~K})$; panel C: $42^{\circ} \mathrm{C}(315 \mathrm{~K}) . p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at $\Delta \log \left(p \mathrm{CO}_{2}\right)=0$.


Fig. 3. Change in $\log \left(p_{\mathrm{m}}\right)$ of a hemoglobin solution versus temperature. Panel $\mathrm{A}: \mathrm{pH}=7.2$ at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel $\mathrm{B}: p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at three values of pH .
a suitable choice of $\mathrm{p} K_{z o}$ and $\mathrm{p} K_{z d}$. The $\mathrm{H}^{+}$haldane factor thus calculated is in agreement with that experimentally determined in erythrolysate by SiggaardAndersen (27) and is slightly lower than found by Horvath et al. (14) for the $\mathrm{H}^{+}$bohr factor for a concentrated hemoglobin solution ( 0.64 ). On the whole, fig. 4A is in good agreement with the experimental results concerning the influence of $\mathrm{CO}_{2}$ on the proton haldane factor as given by Siggaard-Andersen (27).

Figs. 6, 7, 8, and 9 show the relations between $p_{\mathrm{m}}$ and plasma values for blood in a similar way as figs. $1,2,3$, and 4 show these for the hemoglobin solution. The difference with the latter is that here plasma pH is referred to, instead of the pH of the solution which contains the hemoglobin. The curves for blood are related to those of the hemoglobin solution through a transfor-



Fig. 4. Proton haldane factor $(\Delta z)$ of hemoglobin in solution at $37^{\circ} \mathrm{C}(310 \mathrm{~K})$. Panel A as a function of pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel B as a function of $\log \left(p \mathrm{CO}_{2}\right)$ at six values of the pH . The proton bohr factor is equal to the proton haldane factor (equation 3 ). $p \mathrm{CO}_{2}=5.3$ kPa at $\Delta \log \left(p \mathrm{CO}_{2}\right)=0$.


Fig. 5. $\mathrm{CO}_{2}$ haldane factor $(\triangle Z)$ of hemoglobin in solution at $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$. Panel A as a function of pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel B as a function of $\log \left(p \mathrm{CO}_{2}\right)$ at six values of the pH . The $\mathrm{CO}_{2}$ bohr factor is equal to the $\mathrm{CO}_{2}$ haldane factor (equation 4). $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at $\Delta \log$ $\left(p \mathrm{CO}_{2}\right)=0$.
mation of the pH . The plasma pH differs from the pH of the intracellular hemoglobin solution by an amount equal to the 10 logarithm of the donnan ratio between the plasma and the intracellular solution. The donnan ratio depends on pH and $p \mathrm{CO}_{2}$ and was calculated with the set of equations presented earlier (33).

In blood the bohr and haldane factors are not equal. This is shown in figs. 9 and 10. The curves give the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr and haldane factors as calculated for human blood. Fig. 9A shows that the difference between the $\mathrm{H}^{+}$bohr and the $\mathrm{H}^{+}$haldane factors in blood increases with increasing $p \mathrm{CO}_{2}$. In the presence of $\mathrm{CO}_{2}$ the $\mathrm{H}^{+}$haldane effect is smaller than the $\mathrm{H}^{+}$bohr effect. In contrast with this, the $\mathrm{CO}_{2}$ haldane effect in blood is larger than the $\mathrm{CO}_{2}$ bohr effect (fig. 10).


Fig. 6. Change in $\log \left(p_{m}\right)$ of blood versus plasma pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$. Panel A : $32{ }^{\circ} \mathrm{C}(305 \mathrm{~K})$; panel B: $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$; panel C : at $42{ }^{\circ} \mathrm{C}(315 \mathrm{~K})$.


Fig. 7. Change in $\log \left(p_{m}\right)$ of blood versus $\log \left(p \mathrm{CO}_{2}\right)$ at six values of plasma pH . Panel A : $32{ }^{\circ} \mathrm{C}(305 \mathrm{~K})$; panel $\mathrm{B}: 37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$; panel $\mathrm{C}: 42^{\circ} \mathrm{C}(315 \mathrm{~K}) . p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at $\Delta \log$ $\left(p \mathrm{CO}_{2}\right)=0$.

The difference between the bohr and haldane factors in blood results from the fact that the buffering of $\mathrm{H}^{+}$not only takes place in the hemoglobin solution inside the erythrocytes but also in plasma. Hemoglobin, however, which is intracellular, only "sees" intracellular pH . This pH is related to plasma pH through the osmotic and donnan equilibrium conditions. The consequence of this is that although plasma pH is kept fixed while changing $s \mathrm{O}_{2}$, intracellular pH is not. Changes in intracellular pH give rise to changes in the amounts of both protons and $\mathrm{CO}_{2}$ bound to hemoglobin. For blood therefore, it is not possible to separate the bohr effect of protons from that of $\mathrm{CO}_{2}$ on the guidance of plasma values. So, when measured in whole blood, the $\mathrm{H}^{+}$bohr factor includes a $\mathrm{CO}_{2}$ bohr effect and the $\mathrm{CO}_{2}$ bohr factor includes a proton effect. Furthermore, as the buffer properties of hemoglobin depend


Fig. 8. Change in $\log \left(p_{\mathrm{m}}\right)$ of blood versus temperature. Panel A: plasma pH 7.4 at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel B : $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ and three values of pH .


Fig. 9. Proton bohr factor (broken lines) and proton haldane factor (drawn lines) in blood at $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$. Panel A as a function of plasma pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel B as a function of $\log \left(p \mathrm{CO}_{2}\right)$ at three values of plasma pH . The haldane factor is smaller than the bohr factor, the difference getting larger at increasing $p \mathrm{CO}_{2} . p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at $\Delta \log \left(p \mathrm{CO}_{2}\right)=0$.
on oxygen saturation, the bohr factors in whole blood also depend on oxygen saturation, because at different degrees of saturation equal changes in plasma pH do not give rise to changes of equal magnitude in red cell pH . This phenomenon is reflected in the fact that in whole blood the unconstrained change in plasma pH following upon a change in $p \mathrm{CO}_{2}$ is oxygen saturation



Fig. 10. $\mathrm{CO}_{2}$ bohr factor (broken lines) and $\mathrm{CO}_{2}$ haldane factor (continuous lines) in blood at $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$. Panel A as a function of plasma pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$; panel B as a function of $\log \left(p \mathrm{CO}_{2}\right)$ at three values of plasma pH . The $\mathrm{CO}_{2}$ haldane factor is larger than the $\mathrm{CO}_{2}$ bohr factor. $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ at $\Delta \log \left(p \mathrm{CO}_{2}\right)=0$.


Fig. 11. Proton bohr factor of blood (broken line), proton haldane factor of blood (continuous line), and bohr and haldane factors of a hemoglobin solution representing red cell content at $s \mathrm{O}_{2}=0.5$ (continuous thick line), and the change in $z$ of hemoglobin between deoxygenated and oxygenated blood (broken thick line) versus plasma pH at $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ and $T=310 \mathrm{~K}$.
dependent. From fig. 9A it can be read that at $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ and $\mathrm{pH}_{p}=7.4$ the $\mathrm{H}^{+}$haldane factor calculated for blood is -0.36 . This value is in good agreement with the value of -0.35 measured by Arczynska and Held (3). The bohr factor calculated under these conditions was -0.40 .

A comparison of bohr factors in whole blood from the literature with those calculated here is given in table 3. As far as possible the mutual influences of pH and $\mathrm{pCO}_{2}$ on the bohr factors were calculated. These are also given in table 3. There is a reasonable agreement between the experimental and the calculated values. Figs. 11 to 16 give the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr and haldane factors as calculated for whole blood. In figs. 12 and 14 the whole blood $\mathrm{H}^{+}$ and $\mathrm{CO}_{2}$ haldane factors are given together with the contributions made by hemoglobin.


Fig. 12. Proton haldane factor of blood (continuous lines) and the change in $z$ of hemoglobin between oxygenated and deoxygenated blood (broken thick lines) versus plasma pH at three values of $p \mathrm{CO}_{2}(\mathrm{kPa})$.

Table 3. $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr effects and their mutual dependence as measured in and calculated for human blood at $37{ }^{\circ} \mathrm{C}$ with $p \mathrm{CO}_{2}$ or pH as indicated.

| Source | $\begin{gathered} \Delta \log \left(p_{50}\right) \\ \Delta \mathrm{pH}_{\rho} \end{gathered}$ | $\frac{\Delta}{\Delta \log \left(p \mathrm{CO}_{2}\right)}\binom{\Delta \log \left(p_{\mathrm{SO}}\right)}{\Delta \mathrm{pH}_{p}}$ | $\frac{\Delta \log \left(p_{50}\right)}{\bar{\Delta} \log \left(p \mathrm{CO}_{2}\right)}$ | $\frac{\Delta}{\Delta \mathrm{pH}_{p}}\left(\frac{\Delta \log \left(p_{\mathrm{s} 0}\right)}{\Delta \log \left(p \mathrm{CO}_{2}\right)}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Arturson et al. (4) | -0.39 ( 5.3 kPa ) | 0.149 | 0.06 (7.4) | 0.10 |
| Garby et al. (9) | -0.38 ( 4.5 kPa ) | 0.19 | $0.10 \quad$ (7.4) | 0.20 |
|  |  |  | 0.06 (7.2) |  |
| Hlastala and Woodson (13) | -0.399 ( 5.6 kPa ) |  | 0.0514 (7.4) |  |
| Horvath et al. (14) | -0.395 ( 5.6 kPa ) |  |  |  |
| Winslow et al. (29) | -0.36 ( 5.3 kPa ) | 0.123 | 0.038 (7.36) | 0.14 |
|  | -0.33 ( 9.3 kPa ) |  | 0.045 (7.51) |  |
| Zijlstra et al. (32) | -0.396 ( 5.3 kPa ) | 0.038 | 0.068 (7.4) | 0.038 |
| Calculated (this paper) | -0.40 ( 5.3 kPa ) | 0.18 | 0.068 (7.4) | 0.18 |



Fig. 13. Explanation of the fact that the amount of protons released per hemoglobin monomer ( $\Delta z$ ) is smaller when blood is oxygenated at constant pH and $p \mathrm{CO}_{2}$ than when the same is done with a hemoglobin solution. Panel A: change in $z$ of hemoglobin in solution (continuous thick line) and in blood as a function of plasma pH (broken thick line). Panel B: $z$ as a function of intracellular $\mathrm{pH}\left(\mathrm{pH}_{c}\right)$ for oxygenated and deoxygenated hemoglobin. The thin arrow indicates the change in $z$ at constant pH in the solution, the thick arrow at constant plasma pH . In the latter case the pH inside the red cells changes in such a way that $\Delta z$ is smaller than when no such a change in pH occurs.

In fig. 11 besides the $\mathrm{H}^{+}$bohr and haldane curves, two other curves are given. The uppermost curve gives the bohr factor in a hemoglobin solution at the intracellular pH and $p \mathrm{CO}_{2}$ that result from the plasma pH at that $p \mathrm{CO}_{2}$ with $s \mathrm{O}_{2}=0.5$. Because it concerns a solution, the bohr factor is equal to the haldane factor. Below the broken thin line of the blood $\mathrm{H}^{+}$bohr factor and above the continuous thin line of the $\mathrm{H}^{+}$haldane factor of blood, the broken thick line represents the change in $z$ of hemoglobin between $s \mathrm{O}_{2}=0$ and $s \mathrm{O}_{2}=1$. In fig. 12 the change in $z$ is compared with the $\mathrm{H}^{+}$haldane effect of


Fig. 14. $\mathrm{CO}_{2}$ bohr factor of blood (broken thin line), $\mathrm{CO}_{2}$ haldane factor of blood (continuous thin line), $\mathrm{CO}_{2}$ bohr and haldane factors of a hemoglobin solution representing red cell content at $\mathrm{sO}_{2}=0.5$ (continuous thick line), and the change in $Z$ of hemoglobin between deoxygenated and oxygenated blood (broken thick line) versus plasma pH at $p \mathrm{CO}_{2}=5.3 \mathrm{kPa}$ and $T=310 \mathrm{~K}$.


Fig. 15. $\mathrm{CO}_{2}$ haldane factor of blood (drawn lines) and the change in $Z$ of hemoglobin between oxygenated and deoxygenated blood (broken thick lines) versus plasma pH at three values of $p \mathrm{CO}_{2}$ (kPa).
blood for three different values of $p \mathrm{CO}_{2}$. The haldane factor is smaller than the change in the amount of protons bound to hemoglobin. This is due to the action of other buffers in blood, notably plasma protein, ATP and 2,3-DPG. Fig. 13A presents the curve of the bohr factor of hemoglobin in solution and that of $\Delta z$ between $s \mathrm{O}_{2}=0$ and $s \mathrm{O}_{2}=1$. In a solution the bohr factor equals $\Delta z$, but in blood it does not because then intracellular pH is not constant. This is illustrated in fig. 11B. In figs. 14, 15, and 16 similar pictures are given for the effects of $\mathrm{CO}_{2}$. These, however, are larger in blood than in a hemoglobin


Fig. 16. Explanation of the fact that the amount of $\mathrm{CO}_{2}$ released per hemoglobin monomer ( $\Delta \mathrm{Z}$ ) is larger when blood is oxygenated at constant pH and $p \mathrm{CO}_{2}$ than when the same is done with a hemoglobin solution. Panel A : change in $Z$ as a function of plasma pH in a hemoglobin solution (continuous thick line) and in blood (broken thick line). Panel B: $Z$ as a function of intracellular $\mathrm{pH}\left(\mathrm{pH}_{c}\right)$ for oxygenated and deoxygenated blood. The thin arrow indicates the change in Z at constant pH in the solution, the thick arrow at constant plasma pH . In the latter case the pH inside the red cells decreases which leads to an increase in $\Delta Z$.
solution. The mechanism which makes the $\mathrm{H}^{+}$effects in blood smaller than in a solution, works in the case of $\mathrm{CO}_{2}$ just the other way round. The buffering by plasma proteins, ATP, and 2,3-DPG makes that more $\mathrm{CO}_{2}$ can be accomodated in blood than in hemoglobin solution. The shift in $\mathrm{pH}_{c}$ also acts in favour of $\mathrm{CO}_{2}$ transport, as is illustrated in fig. 16B.

The titration curve of hemoglobin depends on $s \mathrm{O}_{2}$. When $s \mathrm{O}_{2}$ changes, the donnan ratio between intra-erythrocytic solution and plasma changes as well. Thus, although plasma $\mathrm{pH}, p \mathrm{CO}_{2}$, and temperature are kept constant, intraerythrocytic pH changes with $s \mathrm{O}_{2}$. This also causes $p_{50}$ to change. Under these conditions, points of the OEC measured in whole blood do not lie on one and the same true hemoglobin-OEC because, as far as pH is concerned, the intraerythrocytic environment for hemoglobin is not constant. An OEC measured in blood is therefore not a "true" OEC, for which all intracellular quantities except $p \mathrm{O}_{2}$ remain constant.

With the assumption that the OEC calculated with equation 36 is correct for blood, the OEC for constant intra-erythrocytic values was calculated. $\mathrm{sO}_{2}=0.5$ was taken as the reference value. The difference between the OEC for blood and the "true" OEC of hemoglobin is small. There are two extremes in the difference, one between $s \mathrm{O}_{2}=0$ and $s \mathrm{O}_{2}=0.5$, in which case it was about -0.02 kPa at about $s \mathrm{O}_{2}=0.18$, and a positive, infinitely large one at $s \mathrm{O}_{2}=1$ and $p \mathrm{O}_{2}$ infinite. The lower value was calculated with $\mathrm{pH}_{p}=7.4, p \mathrm{CO}_{2}=5.3$ kPa , and $T=310 \mathrm{~K}$. On the whole, the OEC measured in blood is less curved than that of the intra-erythrocytic hemoglobin solution, but for all practical purposes the difference is negligibly small.

The buffer capacity of blood depends on the oxygen saturation of the hemoglobin. Because of this the position of the $\mathrm{CO}_{2}$ bufferline of blood is $s \mathrm{O}_{2}$ dependent. In the $\mathrm{HCO}_{3}^{-} / \mathrm{pH}_{p}$ diagram $\mathrm{CO}_{2}$ bufferlines of blood with different $s \mathrm{O}_{2}$ are almost parallel to each other. The bufferline with $s \mathrm{O}_{2}=0$ is the uppermost bufferline, that with $s \mathrm{O}_{2}=1$ the lowest one. However, at constant $p \mathrm{O}_{2}$, the $s \mathrm{O}_{2}$ depends on pH and $p \mathrm{CO}_{2}$ as well. Blood exposed to a certain $p \mathrm{O}_{2}$


Fig. 17. Slope of $\mathrm{CO}_{2}$ bufferline of blood at constant $p \mathrm{O}_{2}$ versus the $p \mathrm{O}_{2}$ at which the bufferline is calculated.
in the range $0-10 \mathrm{kPa}$ that is titrated with $\mathrm{CO}_{2}$ to measure the $\mathrm{CO}_{2}$ bufferline does not have a constant $s \mathrm{O}_{2}$. It has a lower $s \mathrm{O}_{2}$ in the lower pH range i.e. at a high $p \mathrm{CO}_{2}$, and a higher $s \mathrm{O}_{2}$ in the higher pH range, i.e. at a low $p \mathrm{CO}_{2}$. Such a $\mathrm{CO}_{2}$ bufferline thus crosses the $\mathrm{CO}_{2}$ bufferlines with constant $s \mathrm{O}_{2}$. The effect is that at certain $p \mathrm{O}_{2}$ in the range $0-10 \mathrm{kPa}$, the slope of the bufferline is dependent on $p \mathrm{O}_{2}$. The slope as calculated for these conditions is given in fig. 17.

The model presented does not represent the most recent state of affairs concerning research on oxygen binding by hemoglobin. In the MWC (Monot, Wyman, Changeux) model of hemoglobin (19), the molecule is thought to switch over between a restricted number of states. The most important switch is that between two quaternary structures, the R state, with a high oxygen affinity, and the T state, with a low affinity. At increasing $p \mathrm{O}_{2}$, the ratio between the number of $R$ state molecules and the number of $T$ state molecules increases, because the stability of the R state increases on oxygen binding. Thus, the average affinity to oxygen increases.

The model of hemoglobin used here, however, perhaps does not differ so much from the two state MWC model as it may seem. The hemoglobin molecule was assumed to be in one of the two states: oxygenated or deoxygenated, each state characterized by its affinities to $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$. Instead of the oxygenated and the deoxygenated states, one could substitute the R and T state. In terms of this model, the oxygen affinity modifiers $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ can be considered to influence the average oxygen affinity expressed as $p_{\mathrm{m}}$ by changing the ratio between the amounts of molecules in the R and the T state. They could induce this change by influencing either the affinities of the monomers for oxygen or the free energies in the two states or both. In this way, the description of the properties of hemoglobin presented can be interpreted as conforming to a two state model. For the description of the macroscopic behaviour of hemoglobin under equilibrium conditions, these differences in interpretation are not very relevant.

In the model presented, a number of assumptions concerning the properties of hemoglobin were discussed. Apart from the simplifications applied, the main disadvantage of the model presented is that $2,3-\mathrm{DPG}$ only acts as a buffer substance that is osmotically active but not as a modifier of oxygen affinity. This disadvantage was met in so far that the values of $\mathrm{p} K$ 's used lead to a description of the properties of hemoglobin as these are in the presence of normal amounts of $2,3-\mathrm{DPG}$. Moreover, the assumption that the kations other than $\mathrm{H}^{+}$cannot leave their compartment may not always lead to completely realistic results. Nevertheless, the model provides useful insight into the interactions between $\mathrm{pH}, \mathrm{pCO}_{2}$ and the resulting oxygen affinity of blood.

## ACKNOWLEDGEMENTS

The help of Miss H. Anthonio and Mr. V. Sonak in preparing the manuscript, and of Mr. J. Pleiter and Mr. B.L. Tebbes in preparing the figures, is gratefully acknowledged.

## REFERENCES

1. Adair, G.S. - Thermodynamical proof of the reciprocal relationship of oxygen and carbon dioxide in the blood. J. Physiol. (London) 58, iv-v (1923).
2. Adair, G.S. - The hemoglobin system. VI. The oxygen dissociation curve of hemoglobin. J. Biol. Chem. 63, 529-545 (1925).
3. Arczynska, W. and D.R. Held - The amount of $\mathrm{H}^{+}$released on iso-pH oxygenation of human whole blood. Resp. Physiol. 12, 7-16 (1971).
4. Arturson, G., L. Garby, M. Robert and B. Zaar - The oxygen dissociation curve of normal human blood with special reference to the influence of physiological effector ligands. Scand. J. Clin. Lab. Invest. 34, 9-13 (1974).
5. Bauer, C. and E. Schröder - Carbamino compounds of haemoglobin in human adult and foetal blood. J. Physiol. (London) 227, 457-471 (1972).
6. Bohr, C., K.A. Hasselbalch and A. Krogh - Ueber einen in biologischer Beziehung wichtigen Einflusz, den die Kohlensäurespannung des Blutes auf dessen Sauerstoffbindung übt. Skand. Arch. Physiol. 16, 402-4 12 (1904).
7. Christiansen, J., C.C. Douglas and J.S. Haldane - The adsorption and dissociation of carbon dioxide by human blood. J. Physiol. (London) 48, 244-277 (1914).
8. Ferguson, J.K.W. - Carbamino compounds of $\mathrm{CO}_{2}$ with human haemoglobin and their role in the transport of $\mathrm{CO}_{2}$. J. Physiol. (London) 88, 49-55 (1936).
9. Garby, L., M. Robert and B. Zaar - Proton- and carbamino-linked oxygen affinity of normal human blood. Acta Physiol. Scand. 84, 481-492 (1972).
10. Heineken, F.G. and D.D. Lobdell - Calculations of pH during $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ exchange with blood. Respir. Physiol. 28, 277-287 (1976).
11. Hill, A.V. - The possible effects of the aggregation of the molecules of hemoglobin on its dissociation curves. J. Physiol. (London) 40, IV (1910).
12. Hilpert, P., R.G. Fleischmann, D. Kempe and H. Bartels - The bohr effect related to blood and erythrocyte pH. Am. J. Physiol. 205, 337-340 (1963).
13. Hlastala, M.P. and R.D. Woodson - Bohr effect data for blood gas calculations. J. Appl. Physiol. Respirat. Environ. Exercise Physiol. 55, 1002-1007 (1983).
14. Horvath, S.M., A. Malenfant, F. Rossi and L. Rossi-Bernardi - The oxygen affinity of concentrated human hemoglobin solutions and human blood. Am. J. Hematol. 2, 343-354 (1977).
15. Kelman, G.R. - Digital computer subroutine for the conversion of oxygen tension into saturation. J. Appl. Physiol. 21, 1375-1376 (1966).
16. Kilmartin, J.V. and L. Rossi-Bernardi - Interaction of hemoglobin with hydrogen ions, carbon dioxide, and organic phosphates. Physiol. Rev. 53, 836-890 (1973).
17. Klocke, R.A. - Mechanism and kinetics of the Haldane effect in human erythrocytes. J. Appl. Physiol. 35, 673-681 (1973).
18. Lobdell, D.B. - An invertible simple equation for computation of blood $\mathrm{O}_{2}$ dissociation relations. J. Appl. Physiol.: Respirat.-Environ. Exercise Physiol. 50, 971-973 (1981).
19. Monod, J., J. Wyman and J.P. Changeux - On the nature of allosteric transitions: a plausible model. J. Molec. Biol. 12, 88-118 (1965).
20. Perrella, M., G. Guglielmo and A. Mosca - Determination of the equilibrium constants for oxygen-linked $\mathrm{CO}_{2}$ binding to human hemoglobin. FEBS Letters 78, 287-290 (1977).
21. Reeves, R.B. - The effect of temperature on the oxygen equilibrium curve of human blood. Respir. Physiol. 42, 317-328 (1980).
22. Riggs, A. - Mechanism of the enhancement of the Bohr effect in mammalian hemoglobin by diphosphoglycerate. Proc. Natn. Asad. Sci. USA 68, 2062-2065 (1971).
23. Rossi-Bernardi, L. and F.J.W. Roughton - The specific influence of carbon dioxide and carbamate compounds on the buffer power and bohr effects in human haemoglobin solutions. J. Physiol. 189, 1-29 (1967).
24. Rossi-Bernardi, L. and F.J.W. Roughton - The effect of temperature on the oxygen-linked ionizations of hemoglobin. J. Biol. Chem. 242, 784-792 (1967).
25. Roughton, F.J.W. and J.W. Severinghaus - Accurate determination of $\mathrm{O}_{2}$ dissociation curve of human blood above $98.7 \%$ saturation with data on $\mathrm{O}_{2}$ solubility in unmodified human blood from 0 to $37{ }^{\circ} \mathrm{C}$. J. Appl. Physiol. 35, 861-896 (1973).
26. Severinghaus, J.W. - Simple accurate equation for human blood $\mathrm{O}_{2}$ dissociation computations. J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. 46, 599-602 (1979).
27. Siggaard-Andersen, O. - Oxygen-linked hydrogen ion binding of human hemoglobin. Effects of carbon dioxide and 2,3-diphosphoglycerate. I. Studies on erythrolysate. Scand. J. Clin. Lab. Invest. 27, 351-360 (1971).
28. Thomas, B.J. - Algorithms for selected blood acid-base and blood gas calculations. J. Appl. Physiol. 33, 154-158 (1972).
29. Winslow, R.M., M. Samaja, N.J. Winslow, L. Rossi-Bernardi and R.I. Schrager - Simulation of continuous blood $\mathrm{O}_{2}$ equilibrium curve over physiological pH, DPG, and $\mathrm{P}_{\mathrm{CO}}$ range. J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. 54, 524-529 (1983).
30. Wyman, J. - The heat of oxygenation of hemoglobin. J. Biol. Chem. 127, 591-599 (1939).
31. Wyman, J. - Linked functions and reciprocal effects in hemoglobin: a second look. Adv. Protein Chem. 19, 223-266 (1964).
32. Zijlstra, W.G., B. Oeseburg, G. Kwant and A. Zwart - Determination of interdependent ligand effects on human red cell oxygen affinity. Scand. J. Clin. Lab. Invest. 42, 339-345 (1982).
33. Zock, J.P., P. Rispens and W.G. Zijlstra - Carbon dioxide loading and the acid-base equilibrium states of human blood. Proc. Kon. Ned. Akad. Wet. C 83, 307-332 (1980).
34. Zwart, A., G. Kwant, B. Oeseburg and W.G. Zijlstra - Oxygen dissociation curves for whole blood, recorded with an instrument that continuously measures $\rho_{\mathrm{O}_{2}}$ and $S_{\mathrm{O}_{2}}$ independently at constant $t, p_{C O}$, and pH. Clin. Chem. 28, 1287-1292 (1982).
35. Zwart, A., G. Kwant, B. Oeseburg and W.G. Zijlstra - Human whole-blood oxygen affinity: effect of temperature. J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. 57, 429-434 (1984).

## INFLUENCE OF TEMPERATURE ON $\mathrm{CO}_{2}$ AND $\mathrm{O}_{2}$ EQUILIBRIA IN HUMAN BLOOD

## ABSTRACT


#### Abstract

In order to study how various physico-chemical quantities in blood interact, an equilibrium model of blood has been formulated. The set of equations with appropriate values for a number of quantities, together comprising the model, have been implemented in a computer program. This program makes it possible to calculate, among other things, the acid base equilibria of blood in vilro in closed system or in isocapnic conditions at temperatures other than $37{ }^{\circ} \mathrm{C}$. Results of these calculations are presented in the form of diagrams.

The results show that when the temperature decreases and carbon dioxide tension is kept constant, pH of plasma remains almost constant at the starting value of 7.41 at $37{ }^{\circ} \mathrm{C}$. Under conditions where the total $\mathrm{CO}_{2}$ content of the blood remains constant, the pH of plasma rises with decreasing temperature. This rise is parallel with the increase in the pH of pure water ( pN ) with decreasing temperature, the pH of plasma being about 0.6 unit above pN . The slope of the plasma pH vs temperature curve was calculated to be $-0.0169 / \mathrm{K}$ when $\mathrm{cHb}=150 \mathrm{~g} / \mathrm{L}$, which is slightly higher than the experimental values.

When the total $\mathrm{CO}_{2}$ content is kept constant, the decrease in $\rho \mathrm{CO}_{2}$ with decreasing temperature is fairly approximated by a linear relation between $\log \left(\rho \mathrm{CO}_{2}\right)$ and $T$, with a slope of $0.023 / K$. This slope is in fair agreement with experimental values. It follows from this relationship that after anaerobic cooling to $22{ }^{\circ} \mathrm{C}$, the $\rho \mathrm{CO}_{2}$ of the blood that is normocapnic at $37^{\circ} \mathrm{C}(5.3 \mathrm{kPa})$ is about 2 kPa .

The results also show that under the condition of a closed system the donnan ratio between plasma and erythrocytes hardly changes with changes in temperature, while when $\rho \mathrm{CO}_{2}$ in the blood is kept constant during cooling, the donnan ratio increases with decreasing temperature. The same applies to the hematocrit. These results are in agreement with experimental findings from the literature. Furthermore, it was calculated that when total $\mathrm{CO}_{2}$ content is kept constant, the fall in $\rho \mathrm{CO}_{2}$ with decreasing temperature is almost independent of the hematocrit. This finding supports the theory by Severinghaus (1959) and by Rahn, Reeves and Howell (1975) that in a patient under induced hypothermia the total $\mathrm{CO}_{2}$ content of arterial blood should be kept constant by way of a suitable alveolar ventilation.


## INTRODUCTION

The influence of temperature on the gas transport by and the acid-base balance of the blood regained interest with the theories put forward by Rahn, Reeves, and Howell (32). They put forward a rather convincing hypothesis about which blood gas values are "normal" for homeotherms at temperatures other than the normal body temperature. Their conclusion was based on observations made in ectothermic vertebrates. These animals regulate their pH by means of the arterial $\mathrm{COO}_{2}$ in such a way that, at any temperature in the
physiological range, the difference between plasma pH and the neutral point of water, pN , is maintained at a constant value of about 0.6. Some years ago, we formulated an equilibrium model of human blood in order to study interactions between various physico-chemical quantities in blood (52). The set of equations with the appropriate values for a number of constants have been implemented in a computer program. This computer program offers the possibility to calculate the acid-base equilibria of human blood in vitro under a variety of conditions including temperatures other than $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$. Two types of equilibrium conditions will be compared here: blood cooled down or warmed up at a constant $\mathrm{CO}_{2}$ and blood cooled down or warmed up with a constant total $\mathrm{CO}_{2}$ content $\left(t \mathrm{CO}_{2}\right)$. In both conditions a number of quantities were calculated for temperatures between 17 and $42{ }^{\circ} \mathrm{C}$ (290-315 K) with intervals of $5{ }^{\circ} \mathrm{C}$. The two conditions may be seen as representing two policies advocated for the ventilation of patients under hypothermia. The results show that in blood in closed system conditions, the donnan ratio and the hematocrit are nearly independent of temperature, which is in contrast with the results calculated for constant $\mathrm{pCO}_{2}$. Moreover, under these conditions, the difference between plasma pH and pN is also almost constant at 0.6 in the range of temperature from 17 to $42{ }^{\circ} \mathrm{C}$. This supports the extension of the original hypothesis by Rahn (31) put forward by Rahn, Reeves and Howell (32) which states that the "normal" blood gas values at temperatures other than $37{ }^{\circ} \mathrm{C}$ are those at which the arterial $\mathrm{CO}_{2}$ content is equal to that at $37{ }^{\circ} \mathrm{C}$.

The study of the deterioration of physiological functions in homeotherms at low body temperatures goes back at least as far as 1862. In that year Walther (50) published his "Beitrage zur Lehre von der thierischen Wärme". He found that rabbits at a body temperature of $20^{\circ} \mathrm{C}$ did not shiver and would have died if left in a cool place, but that they could be revived by means of artificial ventilation, with their body temperature slowly rising, even if the air temperature was only $10^{\circ} \mathrm{C}$. With these experiments Walther contributed to the discussion on the matter of the "animal heat". He claimed that this was the first proof that the generation of animal heat depends on the entrance of air into the lungs.

The systematic study of the influence of body temperature on the physiological functions of homeotherms started only eighty years later with the study by Dill and Forbes (18) on the respiratory and metabolic effects of hypothermia. The impetus to this study was the need for insight into such matters as adaptation to and protection against cold and the nature of the breakdown resulting from prolonged exposure to cold. This need was born from experiences of troops in Finland. Norway and Greece and of armed forces in the arctic and at high altitude. During World War 11 it became clear from the ob-
servation of cases of drowning in cold sea-water that the most important threat to life for those who kept afloat was the cooling down, leading to certain death (23). Answers to questions raised about hypothermia called for knowledge of metabolic, respiratory and cardiovascular response to cold. In 1940 Smith and Fay (46), in an attempt to treat cancer, for the first time induced in man body temperatures as low as $24^{\circ} \mathrm{C}$. As an outgrowth of this, an experimental hypothermic treatment of schizophrenic patients was instituted. In retrospect, it is clear that the application of the - at the time - new technique was pure magic. It makes one wonder how years hence today's modern medicine will be looked upon. The observations made by Dill and Forbes (18) concern these schizophrenic patients. The patients were cooled down externally as far as a lowering of skin temperature by $20{ }^{\circ} \mathrm{C}$. They were lightly anaesthetized; after 1 to 4 hours cold itself provided the only anaesthesia. The patients were breathing spontaneously and besides a glucose solution given by stomach tube, no measures were taken to control the state of the patient.

Dill and Forbes found that the ventilation had increased out of proportion to the increased oxygen consumption. Arterial blood samples were interpreted as showing the patients to be in a state of metabolic acidosis caused by the shivering and the struggling of the patients and partially compensated for by hyperventilation. This interpretation was more or less a guess, because to judge the acid-base status of blood at temperatures other than $37{ }^{\circ} \mathrm{C}$ one will first have to decide what state should be considered "normal" and thus be used as the reference. Until now, no consensus of opinion has been reached about this.

The question of which blood gas values and which acid-base status should be taken as optimal at lower body temperatures became even more important when controlled hypothermia was introduced into regular medical practice. The clinical application of hypothermia was the sequel to experimental cardiotomy during inflow occlusion on dogs under deep hypothermia by Bigelow et al. in 1950 (9). Similar experiments were done by Boerema et al. (11) in 1951. In 1953 Swan et al. (48) and Lewis and Taufic (25) reported successful open heart surgery with circulatory occlusion under hypothermia. Since then, cardiac surgery under deep hypothermia has developed into a technique with its own specific indications. When the optimal blood gas values and acid-base status in hypothermia are known, a part of the control effort can be aimed at maintaining or approaching this optimal condition in the patient as closely as possible.

In the absence of a clear-cut idea about which condition is the normal, or strickly speaking the optimal, blood gas and acid-base condition, many policies were advocated for the regulation of ventilation and plasma pH under hypothermia. These were often at variance with each other, indicating the lack of a
generally accepted theory. Considering the application, one can understand that the behaviour of the heart shortly before and after the arrest during hypothermia was taken as an important indicator for judging the quality of the policy used. Two properties were considered especially important: irritability and cardiac performance. The first as indicating whether the heart will be able to regain a stable and normal rhythm, the second as a necessity to sustain life. The findings concerning these properties did anything but lead to a conclusive answer as to which $\mathrm{pCO}_{2}$ and pH should be aimed at in deep hypothermia.

For the clinical application of hypothermia, the question of which oxygen tension $\left(\rho \mathrm{O}_{2}\right)$ should be considered normal at a lower body temperature is probably less important. As long as the oxygen saturation $\left(\mathrm{s}_{2}\right)$ of the arterial blood is almost complete, the oxygen supply of the tissues only depends on their perfusion. This again stresses the importance of acid-base conditions that allow optimal functioning of the heart, which, in turn, is necessary to protect so vulnerable an organ as the brain from a shortage of oxygen. As for the brain, a correct choice of the $\mathrm{CCO}_{2}$ is important in another way too. The vascular resistance of the brain, and therefore its perfusion and thus the oxygen supply, strongly depends on the $\rho \mathrm{CO}_{2}$. Regarding the control of the patient, the maintenance of complete oxygen saturation practically means that alveolar $\mathrm{pO}_{2}$ has to be kept sufficiently high.

It is important to know whether under hypothermia the $\rho \mathrm{O}_{2}$ at capillary oxygen saturations between 0.7 and 1 is sufficiently high to guarantee an adequate supply of oxygen to even the most vulnerable tissue cells. Some studies (10) indicate that this is indeed the case, so that as far as the oxygen supply is concerned, full saturation of arterial blood should be aimed at in patients with controlled ventilation. If necessary, it is possible, although difficult, to raise the $\mathrm{OO}_{2}$ in the tissues above that concomitant with normal values of the oxygen saturation of hemoglobin, i.e. $\mathrm{sO}_{2}$ between 0.7 and 0.95 . In that case $s \mathrm{O}_{2}=1$ and the oxygen supply has to come from dissolved $\mathrm{O}_{2}$ so that the arterial $\rho \mathrm{O}_{2}$ must be rather high. A high $\rho \mathrm{O}_{2}$ in the tissues is hard to control because the differential storing capacity of blood is low at full saturation. During prolonged application of a high $\rho \mathrm{O}_{2}$ the possibility of oxygen toxicity should be considered.

In patients breathing spontaneously, alveolar ventilation will diminish under the anaesthetic action of hypothermia, and therefore alveolar $\rho \mathrm{O}_{2}$ tends to diminish as well. In accidental hypothermia this may lead to an insufficient supply of oxygen. To be able to judge at which $\mathrm{\rho O}_{2}$ one should intervene with assisted ventilation, the critical value of the arterial $\rho \mathrm{O}_{2}$ should be known. Assuming that a normal arterial oxygen saturation complies with a $p \mathrm{O}_{2}$ that is sufficient to provide the necessary oxygen to the tissues, this $\mathrm{pO}_{2}$ can be
taken from the oxygen equilibrium curve (OEC) at the prevailing temperature, provided the corresponding optimal pH and $\mathrm{pCO}_{2}$ are known. Thus, we are again confronted with the question of what pH and $\mathrm{pCO}_{2}$ values are to be considered optimal at deviating body temperatures.

As early as 1959, Severinghaus (45), in considering that either pH or $\rho \mathrm{CO}_{2}$ might arbitrarily be held constant, reasoned that the rapid change in both pH and $\mathrm{pCO}_{2}$ in anaerobically cooled blood detracts from the rationality of fixing one of them in controlling the acid-base status of arterial blood in artificially ventilated patients under hypothermia. He proposed that ventilation in hypothermic patients should be of such a rate that carbon dioxide elimination equals its rate of metabolic production. This rate will diminish as cooling progresses. Severinghaus concluded from this that the total $\mathrm{CO}_{2}$ content of the blood should remain constant and that, by consequence, the pH of the arterial blood in vivo should rise at the same rate as in blood anaerobically cooled in vitro, i.e. by the rosenthal factor of $0.0147 / \mathrm{K}$ (44). Furthermore, arterial $\rho \mathrm{CO}_{2}$ should fall at the same rate as the $\mathrm{pCO}_{2}$ of normal arterial blood cooled anaerobically in vitro, reaching about $3 \mathrm{kPa}\left(22.5 \mathrm{mmHg}\right.$ ) at $25{ }^{\circ} \mathrm{C}$ (297 K). This $p \mathrm{CO}_{2}$ should be used to define the ventilation. The arterial $p \mathrm{CO}_{2}$ should therefore be equal to the $p \mathrm{CO}_{2}$ of blood of normal composition at $37{ }^{\circ} \mathrm{C}$ which has been anaerobically cooled to the prevailing temperature. Metabolic acidosis would present itself as a fall in $t \mathrm{CO}_{2}$ and a less than expected rise in pH . Severinghaus stressed that this ventilation is not necessarily the best ventilation to ensure optimal protection of the circulation in hypothermia. He considered this, therefore, to be a separate problem which should be solved experimentally.

Severinghaus found that the desired decrease in arterial $\mathrm{pCO}_{2}$ could be reached by the use of approximately the same tidal volume and rate of ventilation as needed to maintain a $\mathrm{pCO}_{2}$ of $5.3 \mathrm{kPa}(40 \mathrm{mmHg})$ at normal body temperature. Surprisingly, this paper by Severinghaus has not been quoted by Rahn or his coworkers in their papers concerning this matter.

The consequences of three control policies for the acid-base status of arterial blood under hypothermia were calculated by Albers in 1962 (3). The policies concerned were: constant ventilation, constant $\mathrm{CO}_{2}$ content, and constant $\mathrm{pCO}_{2}$. In contrast with the findings of Severinghaus, Albers' calculations predict a fall in $t \mathrm{CO}_{2}$ at constant ventilation. To keep $t \mathrm{CO}_{2}$ constant, ventilation should be about halved at $20{ }^{\circ} \mathrm{C}$; to keep pCO 2 constant the ventilation should even have to be lowered to about one fourth.

To be able to calculate the ventilation, Albers assumed that the metabolic production of $\mathrm{CO}_{2}$ depends exponentially on temperature. A fall in temperature of $10{ }^{\circ} \mathrm{C}$ would diminish $\mathrm{CO}_{2}$ production by a factor of 2.5 . At constant $t \mathrm{CO}_{2}$ the arterial pH was calculated to increase linearly with the diminishing tem-
perature to about 7.65 at $20^{\circ} \mathrm{C}$. At constant ventilation the pH rose even higher with falling temperature. A constant $p \mathrm{CO}_{2}$ gave an almost constant pH , nearly independent of temperature. This result is in agreement with the experimental findings of Brewin et al. (13). Besides these mutual relations between $t \mathrm{CO}_{2}, \mathrm{pCO}_{2}$, and pH , Albers calculated the consequences of the three policies on the ratio of $\left(\mathrm{OH}^{-}\right)$to $\left(\mathrm{H}^{+}\right)$. This ratio was the least influenced by changes in temperature when $t \mathrm{CO}_{2}$ was held constant. For the importance of this quantity Albers referred to Winterstein (51), who reasoned that at a deviating body temperature the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio should be taken as the measure of the acid-base status instead of pH . The ratio $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$is equivalent to $\mathrm{pH}-\mathrm{pN}$ as an indicator of the deviation from neutrality, as can be easily shown ${ }^{1}$ ). The importance of the properties of water in relation to changes in temperature was clear to Winterstein, who referred to Henderson (21) for this matter. He even quoted Henderson to show that the latter had been well aware that the change in pH , or rather its complement, the "alkalinity", with temperature should be considered in connection with the concomitant change in the dissociation constant of water $K_{w}$. This reasoning was used by Winterstein as an argument to reject pH as a measure of the acid-base status of the blood when changes in temperature take place in favour of the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio.

Although Austin and Cullen (5) appear to have been the first to propose that evaluation of the acid-base balance at different temperatures requires a consideration of the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio, they were not the first who used this ratio. In 1919 Moore (28) introduced the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio in a paper in which he discussed the extreme sensitivity of living systems to small changes in pH . At the time it was supposed that proteins reacted with both $\mathrm{H}^{+}$and $\mathrm{OH}^{-}$ions. From this, Moore deduced that not the concentration of one of these ions was important but their ratio. He used this ratio as a general measure of the deviation from neutrality, nọt explicitly for cases in which changes in tem-
${ }^{1}$ ) Dissociation of water, equilibrium condition: $\left(\mathrm{OH}^{-}\right) \cdot\left(\mathrm{H}^{+}\right)=K_{w}$.
Taking the ${ }^{10} \log$ of this and changing signs gives:

$$
\mathrm{pOH}+\mathrm{pH}=\mathrm{pK}_{\mathrm{w}}=2 \mathrm{pN} \text { and } \mathrm{pH}-\mathrm{pN}=\mathrm{pOH}-\mathrm{pN} .
$$

Thus: $\log \left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)=\mathrm{pH}-\mathrm{pOH}=\mathrm{pH}-\mathrm{pN}+\mathrm{pN}-\mathrm{pOH}=2(\mathrm{pH}-\mathrm{pN})$.
Before the introduction of pH , two measures were used to indicate deviations from neutrality: the acidity, concerning the hydrogen ion concentration, and the alkalinity, concerning the hydroxyl ion concentration. The $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio was therefore called the relative alkalinity by Rahn (31). Being equivalent to $\mathrm{pH}-\mathrm{pN}$, the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio is a superfluous quantity.
perature occurred. It is, nevertheless, possible that Austin and Cullen knew this paper.

Austin et al. (6) showed that in the alligator the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio was preserved after acclimatization to $9{ }^{\circ} \mathrm{C}$ and $35^{\circ} \mathrm{C}$. They found that the change in pH concomitant with changes in temperature was of such an extent that the net charge of plasma protein remained nearly constant. Later results obtained by Reeves and Wilson (34) indicated that the preservation of the protein net charge also applies to the intracellular protein of the frog skeletal muscle.

In addition to the results of Austin et al. (6), Robin (42), in 1962, found that in the blood of the turtle in vitro, of which the total $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ contents were kept approximately constant, the changes in pH and $\mathrm{pCO}_{2}$ with temperature were roughly the same as those in vivo. From this, Howell et ol. (22) concluded that pH in ectotherms is regulated in such a way that the $\left(\mathrm{OH}^{-}\right) /\left(\mathrm{H}^{+}\right)$ratio is preserved, i.e. that a constant difference between plasma pH and pN is maintained. Such a relationship between the pH of blood and pN could be caused by a buffer system of which the pK is about 7 and the $d(p K) / d(1 / T)$ is about 1500, as was pointed out by Alberry and Lloyd (2). Reeves and Wilson (34), in their paper on the frog muscle, suggested that this buffer action would be exerted by the imidazole group of the histidine residues of proteins. In further work Reeves $(35,37)$ accumulated much material to strengthen the evidence that the imidazole group is indeed responsible for this property of proteins in blood and in cells. Reeves called this property the "imidazole alphastat" because of the preservation of the degree of ionization a of the imidazole group. In the ectotherms the regulation of arterial pH is executed by control of the ventilation leading to the proper $p \mathrm{CO}_{2}$ (35).

The facts presented above apply to ectotherms in their adaptation to variations in the temperature of the environment. Yet another question is in how far the conclusions concerning normal pH and $\mathrm{pCO}_{2}$ in ectotherms can be extrapolated to homeotherms under hypothermia. In addition to what is known about ectotherms, there is a rather convincing argument which is directly applicable to homeotherms. This argument is so straightforward that, in retrospect, one wonders why this argument was not used earlier. It short-circuits the whole detour over the ectotherms.

In the discussions about the influence of temperature on the blood gases and the acid-base status of the blood in homeotherms it was generally overlooked that homeotherms control their core temperature but that they are not isothermal, the extremities mostly being colder than the core of the body. It was only when it became clear that neither pH nor $\mathrm{pCO}_{2}$ should be kept constant but rather $\mathrm{pH}-\mathrm{pN}$, and that this coincides with a constant composition of the blood on cooling, that a homeotherm's not being isothermal was recognized as being important for reaching conclusions about blood gases in hypothermia.

In 1975 Rahn reviewed the results and insights gained by himself and his coworkers in his J. Burns Amberson Lecture (32). In that lecture he remarked that on its way to the periphery the blood coming from the core of the body may cool down by more than $20^{\circ} \mathrm{C}$. This cooling occurs while the blood is flowing through arteries in which virtually no exchange of substance takes place, so that the total $\mathrm{O}_{2}$ and $\mathrm{CO}_{2}$ contents remain the same. This cooling is, in fact, equivalent to the process Rosenthal (44) used when he determined the in vitro dependence of blood pH on temperature in a closed system. From this notion it follows that, at least for peripheral tissues, the composition of the arterial blood at temperatures other than $37{ }^{\circ} \mathrm{C}$ should be the same as that of normal blood arterialized at $37{ }^{\circ} \mathrm{C}$, irrespective of the changes in temperature and the concomitant changes in $\mathrm{pH}, \mathrm{pCO}_{2}$, and $\rho \mathrm{O}_{2}$. It is, of course, not certain whether a condition of the arterial blood well-suited for peripheral tissues is good for the internal organs as well. But with respect to this, the ectotherm argument is of just as little or even less value as the direct argument given above.

Contrary to the opinion expressed above about the probably optimal blood gas values for patients under hypothermia, other investigators argue that arterial $p \mathrm{CO}_{2}$ should be kept constant when the temperature varies. This opinion is based on observations of blood gas values and acid-base control in hibernating animals during winter still active. In these animals $\rho \mathrm{CO}_{2}$ can reach high levels and appears not to be very strictly controlled. A high $\mathrm{pCO}_{2}$, however, leads to accumulation of $\mathrm{CO}_{2}$ in the tissues, a situation not normally encountered in non-hibernating animals. At lower body temperatures even a constant arterial $\rho \mathrm{CO}_{2}$ leads to an accumulation of $\mathrm{CO}_{2}$.

In so far as the above questions concern the behaviour of blood under hypothermia, they are within the range of possibilities of the model developed earlier $(52,53)$. This model includes all the relevant temperature dependencies necessary to calculate blood gas and acid-base changes in human blood at temperatures other than $37{ }^{\circ} \mathrm{C}(310 \mathrm{~K})$. The model was used to calculate the effects of two hypothetical policies of ventilation on blood gas and acid-base equilibria in arterial blood when its temperature is changed. One policy is that the total $\mathrm{CO}_{2}$ content, $t \mathrm{CO}_{2}$, and the total oxygen content, $t \mathrm{O}_{2}$, are kept constant, the other that $\rho \mathrm{CO}_{2}$ and $t \mathrm{O}_{2}$ are kept constant. The quantities considered especially are plasma and red cell $\mathrm{pH}, \mathrm{pCO}_{2}$, hematocrit, donnan ratio, $\mathrm{pO}_{2}$, and $\mathrm{sO}_{2}$. The influence of hemoglobin content on the course of these quantities with temperature is also considered, as well as the influence of the initial $t \mathrm{CO}_{2}$ and $t \mathrm{O}_{2}$ on this course.

THE MODEL

In the model the blood is assumed to consist of two compartments, each containing a homogeneous solution. The compartment representing plasma is assumed to contain a solution of $\mathrm{NaCl}, \mathrm{NaHCO}_{3}$, and protein in water. The other compartment, representing the red blood cells, is assumed to contain a solution of $\mathrm{KCl}, \mathrm{KHCO}_{3}, \mathrm{ATP}, 2,3-\mathrm{DPG}$, and hemoglobin in water. The two compartments are thought to be separated by a membrane, which is assumed to be impermeable to the kations, ATP, 2,3-DPG, hemoglobin, and plasma proteins. It is assumed to be permeable to $\mathrm{O}_{2}, \mathrm{CO}_{2}, \mathrm{HCO}_{3}^{-}, \mathrm{Cl}^{-}, \mathrm{H}^{+}$, and water. The two compartments are in osmotic equilibrium. The model has been described in detail previously $(52,53)$.

Temperature influences practically all the equilibria in the blood. The influence on the solubility of $\mathrm{CO}_{2}$ and on the various pK 's have also been discussed in the earlier paper (52); that on the oxygen equilibrium curve of hemoglobin in a recent one (53). To calculate some of the results presented in this paper, the temperature dependences of two additional quantities were used: the solubility of oxygen in blood and the pH of pure water ( pN ).

To calculate changes in blood occurring when the oxygen content of the blood is constant and temperature is low, the amount of oxygen dissolved must be taken into account. The partitioning of oxygen between that combined with hemoglobin and that dissolved depends on the oxygen affinity of hemoglobin prevailing and on the solubility of oxygen in blood. The solubility of oxygen in blood is usually related to the solubility of oxygen in water of the same temperature. To calculate the latter a third degree polynomial was fitted through solubility values for water over the range $10-45{ }^{\circ} \mathrm{C}(283-318 \mathrm{~K}$ ) as given in (4). After transformation of these values to the quantities mmol , liter, and kPa the following expression results:

$$
\begin{aligned}
\alpha_{\mathrm{O} 2}= & 10.75 \cdot 10^{-3}-1.43 \cdot 10^{-4} \cdot(T-308)+3.30 \cdot 10^{-6} \cdot(T-308)^{2} \\
& -20.2 \cdot 10^{-8} \cdot(T-308)^{3}
\end{aligned}
$$

Different findings have been published concerning the temperature dependence of the ratio between the solubilities of oxygen in water and in blood. The Biological Handbook (4) gives a relative increase in oxygen solubility in blood in proportion to that in water with decreasing temperature. Roughton and Severinghaus (43), instead, found a decrease in this ratio. Christoforides and Hedley-Whyte (17) and Hedley-Whyte and Laver (20) found a constant ratio between the two solubilities. They found that the solubility of oxygen in blood slightly depends on hemoglobin concentration. In this paper a constant
ratio of 0.935 between the solubility of oxygen in blood and that in water is used. This ratio is slightly higher than that given for plasma (16).

To calculate the values of pN at various temperatures, a second degree polynomial was fitted over the values of pN between $10^{\circ} \mathrm{C}$ and $40^{\circ} \mathrm{C}$ :

$$
\mathrm{pN}=6.998-0.0167 \cdot(T-298)+8.5 \cdot 10^{-5} \cdot(T-298)^{2}
$$

## CALCULATIONS

The temperature dependences of the relevant quantities were studied by calculating their equilibrium values with the model at five temperatures besides $37{ }^{\circ} \mathrm{C}$ : $42,32,27,22$, and $17{ }^{\circ} \mathrm{C}$. Each calculation started with an initial condition defined at $37{ }^{\circ} \mathrm{C}$. The initial conditions differed with respect to hemoglobin content, $\mathrm{CO}_{2}$ content, and $\mathrm{O}_{2}$ content. Calculations were done for blood with a hemoglobin content of $150,50,10,0.5$, and $0 \mathrm{~g} / \mathrm{L}$. The $\mathrm{CO}_{2}$ content was varied by changing the $\rho \mathrm{CO}_{2}$ at $37{ }^{\circ} \mathrm{C}$. Three values were considered: $2.66,5.33$, and 10.66 kPa . To vary the intitial $\mathrm{O}_{2}$ content four initial values of $\mathrm{sO}_{2}$ were used: $0.98,0.74,0.50$, and 0.26 . In all, sixty different initial conditions were considered. For each case the course of the relevant quantities with temperature was calculated twice: once with $p \mathrm{CO}_{2}$ and $t \mathrm{O}_{2}$ constant and once with $t \mathrm{CO}_{2}$ and $t \mathrm{O}_{2}$ constant. In the latter series the equilibrium values of $\rho \mathrm{O}_{2}, \mathrm{sO}_{2}$ and $\rho \mathrm{CO}_{2}$ at each temperature were calculated too.

The calculations were carried out on an HP 9845 A computer. The program was written in the extended version of BASIC implemented in this machine. It contained a number of iterative procedures. By controlled variation of some variables, the values were made to converge to an end state in which all the conditions set by the equations were satisfied to within $0.001 \mathrm{mmol} / \mathrm{L}$.

## RESULTS AND DISCUSSION

The results of the calculations are to a great extent given in the figures. Figure 1 shows the course of plasma pH with temperature when the total $\mathrm{CO}_{2}$ content of blood with a hemoglobin concentration of $150 \mathrm{~g} / \mathrm{L}$ and $\mathrm{pCO}_{2}=5.33$ kPa and $\mathrm{sO}_{2}=0.98$ at $37{ }^{\circ} \mathrm{C}$ is kept constant. The slope of the plasma pH curve is $-0.0169 / \mathrm{K}$, that of the pN curve at $32^{\circ} \mathrm{C}-0.0156 / \mathrm{K}$, which shows that they are almost parallel to each other. In the literature several experimentally determined values of $\Delta \mathrm{pH} / \Delta T$ have been reported: -0.017 (47), -0.0147 (44), -0.0140 (14), and ranging from -0.0137 to -0.0167 (7). The


Fig. 1. A. Influence of temperature on plasma pH in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions: $t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$ (thick line), and $\mathrm{p} \mathrm{CO}_{2}$ constant 5.33 kPa (broken line). The course of pN with temperature is given in addition (thin line). B. Plasma pH minus pN versus temperature. Same conditions as in $A$.
result that, when $\mathrm{pCO}_{2}$ is kept constant, plasma pH hardly changes with temperature is also in agreement with experimental findings (13, 41).

In the case of constant $t \mathrm{CO}_{2}$ the course of intra-erythrocytic pH is also almost parallel to both plasma pH and pN (figure 2). This agrees with the fact that at constant $t \mathrm{CO}_{2}$ the donnan ratio hardly changes with temperature (figure 3A). At constant $\rho \mathrm{CO}_{2}$ plasma pH remains nearly constant, but red cell pH slightly rises with decreasing temperature. This is caused by the rise in the donnan ratio. These results agree with those found by Reeves (36). The changes in the donnan ratio indicate the changes in the distribution of $\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}{ }^{-}$between plasma and red cells. The exchanges of $\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}^{-}$lead to a new equilibrium in the distribution of water between plasma and red cells. This results in a change in the hematocrit with a change in temperature (figure 3 B ). This change does not occur when $t \mathrm{CO}_{2}$ is constant.

The slopes of the curves calculated for plasma pH and red cell pH depend on the assumed initial conditions of the blood. They are affected by the hemoglobin concentration, the initial oxygen saturation and the initial $\mathrm{pCO}_{2}$. The hemoglobin concentration has the greatest influence on the slope (figures 4 and 5). This is caused by the buffer capacity of hemoglobin, which is larger and more temperature dependent than that of plasma protein. This effect was experimentally studied by Rispens (39). His data points are widely scattered, possibly indicating that quantities other than those considered contribute to the temperature dependence of pH . The line fitted through his data and pivoting around the value of -0.0118 on the ordinate at zero hemoglobin concentra-


Fig. 2. A. Influence of temperature on red cell pH in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions: $t \mathrm{CO}_{2}$ is constant at its value at $37{ }^{\circ} \mathrm{C}$ (thick line), and $\mathrm{pCO} \mathrm{C}_{2}$ is constant 5.33 kPa (broken line). Thin line: pN . B. Red cell pH minus pN versus temperature. Same conditions as in A .


Fig. 3. Influence of temperature on the donnan ratio between red cells and plasma (panel A) and on the hematocrit (panel B) of blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions: $p \mathrm{CO}_{2}$ constant 5.33 kPa (thin lines) and $t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$ (thick lines). In both conditions $t \mathrm{O}_{2}$ is kept constant at its initial value at $37^{\circ} \mathrm{C}$ and $s \mathrm{O}_{2}=0.98$.


Fig. 4. Influence of the hemoglobin content of blood on the temperature dependence of plasma pH in two conditions. Panel $\mathrm{A}: ~ p \mathrm{CO}_{2}$ constant 5.33 kPa ; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$. Hemoglobin contents: $150 \mathrm{~g} / \mathrm{L}$ (thick lines), $50 \mathrm{~g} / \mathrm{L}$ (broken lines), 0.5 $\mathrm{g} / \mathrm{L}$ (dotted lines). Thin lines: pN .
tion, i.e. plasma, gives the following relation between $\Delta \mathrm{pH} / \Delta T$ and cHb :

$$
\frac{\Delta \mathrm{pH}}{\Delta T}=-\left(11.8+2.84 \cdot 10^{-2} \cdot \mathrm{cHb}\right) \cdot 10^{-3}
$$

Substitution of $c \mathrm{Hb}=150 \mathrm{~g} / \mathrm{L}$ gives $-0.0161 / \mathrm{K}$, while the slope of the calculated curve is $-0.0169 / \mathrm{K}$. For $\mathrm{cHb}=10 \mathrm{~g} / \mathrm{L}$ the slope of the calculated curve is equal to that resulting from equation $3:-0.0121 / \mathrm{K}$. Adamson et al. (1) and Rispens (39) found that $\Delta \mathrm{pH} / \Delta T$ is also dependent on the initial pH . Rispens found that it made no difference whether this pH was attained by titration with $\mathrm{CO}_{2}$ or with non-volatile acid. Adamson et al. found that $\Delta \mathrm{pH} / \Delta T$ increased by 0.005 per unit change in the initial pH ; the results of Rispens lead to a value of 0.0058 . This latter value was also found for the change in the slope of the calculated curves shown in figure 6B. In the calculations the change in the initial pH was attained by changing the intitial $\mathrm{pCO}_{2}$ and oxygen saturation. Figures 6 and 7 give the curves for blood with a hemoglobin concentration of $150 \mathrm{~g} / \mathrm{L}$, figures 8 and 9 those for "blood" with $0.5 \mathrm{~g} / \mathrm{L}$ hemoglobin. The latter concentration is so small that the oxygen saturation no longer has a discernable effect on the plasma pH curves. Of course, there is an effect on the curves calculated for the red cell compartment (figure 9B).

The changes in $p \mathrm{CO}_{2}$ and $\log \left(p \mathrm{CO}_{2}\right)$ in blood with constant $t \mathrm{CO}_{2}$ are shown in figure 10. For blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin, the slope of the curves of $\log \left(p \mathrm{CO}_{2}\right)$ versus temperature between 27 and $37{ }^{\circ} \mathrm{C}$ is $0.023 / \mathrm{K}$. This value is in fair agreement with the experimental findings: $0.019 / \mathrm{K}$ by Nunn et al.


Fig. 5. Influence of the hemoglobin content of blood on the temperature dependence of red cell pH in two conditions. Panel $\mathrm{A}: \rho \mathrm{CO}_{2}$ constant 5.33 kPa ; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$. Hemoglobin contents: $150 \mathrm{~g} / \mathrm{L}$ (thick lines), $50 \mathrm{~g} / \mathrm{L}$ (broken lines), $0.5 \mathrm{~g} / \mathrm{L}$ (dotted lines). Thin lines: pN .


Fig. 6. Influence of the oxygen saturation and the total carbon dioxide content at $37{ }^{\circ} \mathrm{C}$ on the temperature dependence of plasma pH in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions. Panel A: $\rho \mathrm{CO}_{2}$ constant; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37^{\circ} \mathrm{C}$. Continuous lines: initial oxygen saturation 0.98 ; broken lines: initial oxygen saturation 0.26 . The values of $\rho \mathrm{CO}_{2}$ at $37^{\circ} \mathrm{C}$ are given in the figure. In both conditions $2 \mathrm{O}_{2}$ is kept constant at its initial value at $37^{\circ} \mathrm{C}$.


Fig. 7. Influence of the oxygen saturation and the total carbon dioxide content at $37{ }^{\circ} \mathrm{C}$ on the temperature dependence of red cell pH in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions. Panel $\mathrm{A}: \rho \mathrm{CO}_{2}$ constant; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$. Continuous lines: initial $s \mathrm{O}_{2}$ 0.98; broken lines: initial $\mathrm{sO}_{2} 0.26$. The values of $\rho \mathrm{CO}_{2}$ at $37{ }^{\circ} \mathrm{C}$ are given in the figure. In both conditions $t \mathrm{O}_{2}$ is kept constant at its initial value at 37 ${ }^{\circ} \mathrm{C}$.
(29), 0.021 by Castaing and Pocidalo (14), and 0.024 as derived from the nomogram by Rispens et $a l$. (40). Bradley et $a /$. (12) calculated 0.0185 lK for $\Delta \log \left(\rho \mathrm{CO}_{2}\right) / \Delta T$. The $\rho \mathrm{CO}_{2}$ calculated for blood at $20{ }^{\circ} \mathrm{C}$ is about 2 kPa . It follows from the course of $\mathrm{pCO}_{2}$ with temperature (figure 10 A ) at several values of the hemoglobin concentration that in peripheral arterial blood the $\rho \mathrm{CO}_{2}$ resulting after cooling only slightly depends on the hemoglobin concentration of the blood. Consequently, as long as the central $\rho \mathrm{CO}_{2}$ is normal, it is the temperature in the peripheral tissues which determines the arterial $\rho \mathrm{CO}_{2}$ in the peripheral tissues.

Figure 11 shows the effects of keeping either $\rho \mathrm{CO}_{2}$ or $t \mathrm{CO}_{2}$ constant on the amounts of the several forms in which $\mathrm{CO}_{2}$ is present in the blood when the temperature varies. When $\mathrm{CCO}_{2}$ is kept constant, the total amount of $\mathrm{CO}_{2}$ increases to a considerable extent. The greater part of the increase is in the bicarbonate fraction, both in plasma and in the erythrocytes. When $t \mathrm{CO}_{2}$ is kept constant (figure 11B), only slight changes in the partitioning of $\mathrm{CO}_{2}$ appear. This is coherent with the constancy of the donnan ratio.

Figures 12 and 14 show the changes in the oxygen equilibrium with temperature when either $\rho \mathrm{CO}_{2}$ or $t \mathrm{CO}_{2}$ is kept constant. Figure 12 shows that except for very small hemoglobin concentrations, the changes in oxygen saturation do not differ much between the two policies. The changes in $p \mathrm{O}_{2}$, however, are different in the two cases (figure 13). When $p \mathrm{CO}_{2}$ is kept constant, the decrease in $\mathrm{\rho O}_{2}$ is almost independent of the hemoglobin concentration (panel $A$ ), but with $t \mathrm{CO}_{2}$ constant the decrease is clearly smaller at


Fig. 8. Influence of the oxygen saturation and the total carbon dioxide content at $37{ }^{\circ} \mathrm{C}$ on the temperature dependence of plasma pH in blood with $0.5 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions. Panel A: $p \mathrm{CO}_{2}$ constant; panel $\mathrm{B}: ~ t \mathrm{CO}_{2}$ constant at its value at $37^{\circ} \mathrm{C}$. The values of $p \mathrm{CO}_{2}$ at $37{ }^{\circ} \mathrm{C}$ are given in the figure. In both conditions $t \mathrm{O}_{2}$ is kept constant at its initial value at $37{ }^{\circ} \mathrm{C}$. The curves with the initial $\mathrm{sO}_{2}=0.98$ coincide with those of $\mathrm{sO}_{2}=0.26$ because of the excess of plasma.


Fig. 9. Influence of the oxygen saturation and the total carbon dioxide content at $37{ }^{\circ} \mathrm{C}$ on the temperature dependence of red cell pH in blood with $0.5 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions. Panel $\mathrm{A}: ~ p \mathrm{CO}_{2}$ constant; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$. Continuous lines: initial ${s O_{2}}^{0}$. 98; broken lines: initial $s \mathrm{O}_{2} \quad 0.26$. The values of $\rho \mathrm{CO}_{2}$ at $37{ }^{\circ} \mathrm{C}$ are given in the figure. In both conditions $t \mathrm{O}_{2}$ is kept constant at its initial value at 37 ${ }^{\circ} \mathrm{C}$.


Fig. 10. Influence of hemoglobin content on the temperature dependence of $\mathrm{pCO}_{2}$ (panel A ) and $\log \left(\rho \mathrm{CO}_{2}\right)$ (panel B ) in blood of which $t \mathrm{CO}_{2}$ and $t \mathrm{O}_{2}$ are kept constant at their values at $37^{\circ} \mathrm{C}$ with $\rho \mathrm{CO}_{2}=5.33 \mathrm{kPa}$ and $s \mathrm{O}_{2}=0.98$. Hemoglobin content $150 \mathrm{~g} / \mathrm{L}$ (thick lines), $50 \mathrm{~g} / \mathrm{L}$ (broken lines), $10 \mathrm{~g} / \mathrm{L}$ (dotted lines), and $0 \mathrm{~g} / \mathrm{L}$ (thin line).
smaller hemoglobin concentrations. For the case with $t \mathrm{CO}_{2}$ constant, Nunn et al. (29) found that the initial oxygen saturation had a marked influence. The slope of the $\log \left(p \mathrm{O}_{2}\right)$ versus temperature curve was $0.0187 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.98$, $0.0293 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.91$, and $0.032 / \mathrm{K}$ at $\mathrm{sO}_{2}=0.83$. The calculated curves were also clearly influenced by the initial $\mathrm{SU}_{2}$, but this influence was greatly dependent on the hemoglobin concentration. In blood with $\mathrm{cHb}=150 \mathrm{~g} / \mathrm{L}$ the calculated slope is $0.033 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.98$, and $0.034 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.74$ and 0.50. In blood with $c \mathrm{Hb}=50 \mathrm{~g} / \mathrm{L}$ this slope is $0.020 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.98,0.031$ $/ \mathrm{K}$ at $s \mathrm{O}_{2}=0.74$, and $0.032 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.50$. In blood with $\mathrm{cHb}=10 \mathrm{~g} / \mathrm{L}$ the slope is $0.0096 / \mathrm{K}$ at $s \mathrm{O}_{2}=0.98,0.028$ at $s \mathrm{O}_{2}=0.74$, and 0.029 at $s \mathrm{O}_{2}=$ 0.50 . At $c \mathrm{Hb}=50 \mathrm{~g} / \mathrm{L}$ our calculated values agree reasonably well with those found by Nunn et al. (29); they, however, report a hemoglobin concentration of $150-180 \mathrm{~g} / \mathrm{L}$.

Figure 14 shows the changes in the logarithm of the median oxygen tension, $\log \left(p_{m}\right)$, (53), with temperature. As far as changes are concerned, $p_{m}$ can be considered equivalent to $p_{50}$, the oxygen tension at $s \mathrm{O}_{2}=0.5$. Panel B shows the changes when $t \mathrm{CO}_{2}$ is constant. The value calculated from the curve with $\mathrm{cHb}=150 \mathrm{~g} / \mathrm{L}$ is $0.034 / \mathrm{K}$. From the experimental data of Reeves (38) follows that in blood with constant $t \mathrm{CO}_{2}$ the slope of $\log \left(p_{50}\right)$ versus temperature is $0.032 / \mathrm{K}$. The experiments reported in (54) and (55) yield a value of $0.033 / \mathrm{K}$. Castaing and Sinet ( 15 ) have measured this quantity in blood with $7.5 \mathrm{~g} / \mathrm{L}$ hemoglobin. They found a value of $0.027 / \mathrm{K}$. The calculated curve of $\log \left(\mathrm{p}_{\mathrm{m}}\right)$ versus temperature for blood with $10 \mathrm{~g} / \mathrm{L}$ hemoglobin has a slope of $0.030 / \mathrm{K}$. In both cases, the calculated values are slightly


Fig. 11. Influence of temperature on the total carbon dioxide content and its partitioning in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin in two conditions. Panel $\mathrm{A}: \rho \mathrm{CO}_{2}$ is constant at 5.33 kPa ; panel $\mathrm{B}: 1 \mathrm{CO}_{2}$ constant its value at $37{ }^{\circ} \mathrm{C}$ with $\rho \mathrm{CO}_{2}=5.33 \mathrm{kPa}$. In both conditions $t \mathrm{O}_{2}$ is kept constant at its value at $37{ }^{\circ} \mathrm{C}$ with $\mathrm{sO}_{2}=0.98$.


Fig. 12. Influence of hemoglobin content on the temperature dependence of the oxygen saturation in two conditions. In both conditions the total oxygen content $t \mathrm{O}_{2}$ is kept at its value at $37{ }^{\circ} \mathrm{C}$. Panel $\mathrm{A}: \rho \mathrm{CO}_{2}$ constant 5.33 kPa . Each curve represents three values of the hemoglobin content: the three lines coincide. Panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37^{\circ} \mathrm{C}$ with $\rho \mathrm{CO}_{2} 5.33 \mathrm{kPa}$. Hemoglobin content: $150 \mathrm{~g} / \mathrm{L}$ (continuous lines), $10 \mathrm{~g} / \mathrm{L}$ (broken lines), and $0.5 \mathrm{~g} / \mathrm{L}$ (dotted lines).


Fig. 13. Influence of hemoglobin content on the temperature dependence of the oxygen tension $\mathrm{pO}_{2}$ in blood in two conditions. In both conditions the total oxygen content $1 \mathrm{O}_{2}$ is kept at its value at $37{ }^{\circ} \mathrm{C}$. Panel $\mathrm{A}: \rho \mathrm{CO}_{2}$ constant 5.33 kPa ; panel $\mathrm{B}: t \mathrm{CO} 2$ constant at its value at $37{ }^{\circ} \mathrm{C}$ with $\rho \mathrm{CO}_{2} 5.33 \mathrm{kPa}$. Hemoglobin content: $150 \mathrm{~g} / \mathrm{L}$ (thick lines). $50 \mathrm{~g} / \mathrm{L}$ (broken lines), $10 \mathrm{~g} / \mathrm{L}$ (dotted lines), and $0 \mathrm{~g} / \mathrm{L}$ (thin lines). In panel $A$ the dotted tine cannot be distinguished from the broken line.
higher than the experimental ones.
Overall, the values calculated for blood gas and acid-base quantities agree fairly well with values found experimentally. This indicates that the model used offers a good representation of the equilibria that determine blood gas and acid-base parameters of blood. This also includes the oxygen equilibria, the calculations of which are based on the relations between the $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ binding properties of hemoglobin and the concomitant changes in $\log \left(p_{m}\right)$ (53). In the model the carbon dioxide and oxygen equilibria are treated in a coherent and consistent way. As far as the temperature dependence of oxygen equilibria other than those pertaining to the OEC is concerned, experimental data are scarce and this subject deserves further attention.

The question of how to interpret blood gas and acid-base values at other temperatures has not been settled yet $(19,33)$. This paper cannot provide conclusive arguments for the use of a specific procedure, but it shows that complex interactions should be taken into account when calculating from $37{ }^{\circ} \mathrm{C}$ to patient temperature. This complexity is not duly considered in most cases and rules of thumb are used. But even conversion factors (7), nomograms (24, 27, 40) or algorithms (49) are not reliable as long as they do not consider all possible relations in blood in combination with all the relevant concentrations, which, however, is virtually impossible. Moreover, the fact that every acidbase status of the blood at any temperature in the physiological range has its unique representation at $37^{\circ} \mathrm{C}$ allows the assessment of the acid-base status of


Fig. 14. Influence of hemoglobin content on the temperature dependence of the change in $\log \left(\rho_{m}\right)$ in blood in two conditions. In both conditions the total oxygen content is kept at its value at $37{ }^{\circ} \mathrm{C}$. Panel $\mathrm{A}: ~ p \mathrm{CO}_{2}$ constant 5.33 kPa ; panel $\mathrm{B}: t \mathrm{CO}_{2}$ constant at its value at $37{ }^{\circ} \mathrm{C}$ with $\mathrm{pCO}_{2} 5.33 \mathrm{kPa}$. Hemoglobin content $150 \mathrm{~g} / \mathrm{L}$ (continuous lines) and $0.5 \mathrm{~g} / \mathrm{L}$ (broken lines).
blood from measurements at $37{ }^{\circ} \mathrm{C}$ unambiguously and makes conversion to bódy temperature superfluous. To prevent confusion, blood gas and acid-base values should be measured at and reported for $37{ }^{\circ} \mathrm{C}$.

The policy to be adopted concerning the arterial $\rho \mathrm{CO}_{2}$ in patients under hypothermia cannot be determined on the basis of the study of the blood gases and acid-base equilibria in blood in vitro. This study shows the differences in blood gas values and acid-base equilibria resulting from the two policies of ventilation used. The fact that no major changes occur when $t \mathrm{CO}_{2}$ is kept constant is a plea in favour of this policy if this would turn out to be the case not only in blood but in other tissues as well. The body of evidence showing that this is in fact the case appears to be growing. Confirmation is found in experiments on animals $(8,26,30)$ as well as in clinical situations where surgical intervention takes place in patients under hypothermia (10).

## ACKNOWLEDGEMENT

The author is indebted to dr. P. Rispens for many fruitful discussions on the subject. The help of Miss H . Anthonio and Mr. V. Sonak in preparing the manuscript, and of Mr. J. Pleiter and Mr. B.L. Tebbes in preparing the figures, is gratefully acknowledged.

## REFERENCES

1. Adamson, K., S.S. Daniel, G. Gandy and L.S. James - Influence of temperature on blood pH of the human adult and newborn. J.Appl. Physiol. 19, 896-900 (1964).
2. Alberry, W.J. and B.B. Lloyd - Variation of chemical potential with temperature. In: Development of the lung. A.V.S. de Renck and R. Porter eds. Boston: Little, Brown (1967), pp. 30-33.
3. Albers, C. - Die ventilatorische Kontrolle des Säure-Basen-Gleichgewichts in Hypothermie. Anaesthesist 11, 43-51 (1962).
4. Altman, P.L. and S. Dittmers (eds.) - Biological Handbooks: Respiration and Circulation. Fed. Am. Soc.Exp.Biol. Bethesda, Maryland (1971).
5. Austin, J.H. and G.E. Cullen - Hydrogen ion concentration of the blood in health and disease. Medicine 4, 275-343 (1925).
6. Austin, J.H., F.W. Sunderman and J.G. Camack - Studies in serum electrolytes. II. The electrolyte composition and the pH of serum of a poikilothermous animal at different temperatures. J.Biol.Chem. 72: 677-685 (1927).
7. Austin, W.H., E.H. Lacombe and P.W. Rand - pH-temperature conversion factors and $\mathrm{pCO}_{2}$ factors for hypothermia. J. Appl.Physiol. 19, 893-896 (1964).
8. Becker, H., J. Vinten-Johansen and J.V. Maloney Jr. et al. - Effect of pH adjustment in deep hypothermia and circulatory arrest. Chir. Forum Exp. Klin. Forsch. 80, 291-294 (1980).
9. Bigelow, W.G., J.C. Calloghan and J.A. Hopps - General hypothermia for experimental intracardiac surgery. Ann.Surg. 132, 531-541 (1950).
10. Blayo, M.C., Y. Lecompte and J.J. Pocidalo - Control of acid-base status during hypothermia in man. Resp. Physiol. 42, 287-298 (1980).
11. Boerema, I., A. Wildschut, W.J.H. Schmidt and L. Broekhuysen - Experimental researches into hypothermia as an aid in surgery of the heart. Arch. Chir. Neerl. 3, 25-34 (1951).
12. Bradley, A.F., M. Stupfel and J.W. Severinghaus - Effect of temperature on $\mathrm{pCO}_{2}$ and $\mathrm{pO}_{2}$ of blood in vitro. J.Appl.Physiol. 9, 201-204 (1956).
13. Brewin, E.G., R.P. Gould, F.S. Nashat and E. Neil - An investigation of problems of acid-base equilibrium in hypothermia. Guys Hospital Report 104, 177-214 (1955).
14. Castaing, M. and J.J. Pocidalo - Temperature and acid-base status of human blood at constant and variable total $\mathrm{CO}_{2}$ content. Respir. Physiol. 38, 243-256 (1979).
15. Castaing, $M$. and $M$. Sinet - Temperature and oxygenation of human blood at constant total $\mathrm{CO}_{2}$ content. Pflügers Arch. 386, 135-140 (1980)
16. Christoforides, C., L.H. Laasberg and J. Hedley-Whyte - Effect of temperature on solubility of $\mathrm{O}_{2}$ in plasma. J.Appl. Physiol. 26, 56-60 (1969).
17. Christoforides, C. and J. Hedley-Whyte - Effect of temperature and hemogiobin concentration on solubility of $\mathrm{O}_{2}$ in blood. J.Appl.Physiol. 27, 592-596 (1969).
18. Dill, D.B. and W.H. Forbes - Respiratory and metabolic effects of hypothermia. Am. J. Physiol.132, 685-697 (1941).
19. Hansen, J.E. and D. Y. Sue - Should blood gas measurements be corrected for the patient's temperature ? N.Eng.J.Med. 303, 341 (1980).
20. Hedley-Whyte, J. and M.B. Laver $-\mathrm{O}_{2}$ solubility in blood and temperature correction factors for $\mathrm{P}_{\mathrm{O} 2}$. J.Appl. Physiol. 19, 901-906 (1964).
21. Henderson, L.J. - Das Cleichgewicht zwischen Basen und Säuren im tierischen Organismus. Ergebnisse der Physiologie. Jahrgang VIII, 254-325 (1909).
22. Howell, B.J., F.W. Baumgardner, K. Bondi and H. Rahn - Acid-base balance in coldblooded vertebrates as a function of body temperature. Am.J. Physiol. 218, 600-606 (1970).
23. Keatinge, W.R. - Survival in cold water. Oxford, Blackwell Sc. Publ. (1969).
24. Kelman, G.R. and J.F. Nunn - Nomograms for correction of blood $\mathrm{pO}_{2}, \mathrm{pCO}_{2}$ and base excess for time and temperature. J.Appl.Physiol. 21, 1484 (1966).
25. Lewis, F.J. and M. Taufic - Closure of atrial septal defects with aid of hypothermia: experimental accomplishments and report of one successful case. Surgery 33, 52-61 (1953).
26. McConnell, D.H., F. White, R. Nelson, et al. - Importance of alkalosis in maintenance of ideal blood pH during hypothermia. Surg. Forum 26, 263-265 (1975).
27. Malan, A. - Blood acid-base state at a variable temperature. A graphical representation. Respir. Physiol. 31: 259-275 (1977).
28. Moore, B. - The cause of the exquisite sensitivity of living cells to changes in hydro-gen- and hydroxyl-ion concentration. J. Physiol. (London) 53: LVII-LVIII (1919).
29. Nunn, J.F., N.A. Bergman, A. Bunatyan and A.J. Coleman - Temperature coefficients for $\mathrm{P}_{\mathrm{CO} 2}$ and $\mathrm{P}_{\mathrm{O} 2}$ of blood in vitro. J. Appl. Physiol. 20, 23-26 (1965).
30. Ohmura, A.. K.C. Wong and D.R. Westenskow - Effects of hypocarbia and normocarbia on cardiovascular dynamics and regional circulation in the hypothermic dog. Anesthesiology 50, 293-298 (1979).
31. Rahn, H. - Gas transport from the external environment to the cell. In: Development of the lung. A.V.S de Renck and R. Porter (eds.). Boston: Little, Brown (1967), pp. 3-23.
32. Rahn, H., R.B. Reeves and B.J. Howell - Hydrogen ion regulation, temperature and evolution. Am. Rev.Resp. Dis. 112, 165-172 (1975).
33. Ream, A.K., B.A. Reitz and G. Silverberg - Temperature correction of $\mathrm{P}_{\mathrm{CO}}$ and pH in estimating эcid-base status: an example of the emperor's new clothes ? Anesthesiology 56, 41-44 (1982).
34. Reeves. R.B. and T.L. Wilson - Intracellular pH in bullfrog striated and cardiac muscle as a function of body temperature. Fed. Proc. 28, 782 (1969).
35. Reeves, R.B. - An imidazole alphastat hypothesis for vertebrate acid-base regulation: tissue carbon dioxide content and body temperature in bullfrogs. Respir. Physiol. 14. 219-236 (1972).
36. Reeves, R.B. - Temperature-induced changes in blood acid-base status. Donnan ${ }^{r} \mathrm{Cl}$ and red cell volume. J.Appl.Physiol. 40, 762-767 (1976).
37. Reeves, R.B. - The interaction of body temperature and acid-base balance in ectothermic vertebrates. Ann. Rev. Physiol. 39, 559-586 (1977).
38. Reeves, R.B. - The effect of temperature on the oxygen equilibrium curve of human blood. Respir. Physiol. 42, 317-328 (1980).
39. Rispens, P. - Significance of plasma bicarbonate for the evaluation of $\mathrm{H}^{+}$homeostasis. Thesis, Groningen 1970.
40. Rispens, P., J.R. Brunsting, J.P. Zock and W.G. Zijlstra - A modified SingerHastings nomogram. J.Appl.Physiol. 34, 377-382 (1973).
41. Rispens, P., J.P. Zock and W.G. Zijlstra - Quantitative relationships between total $\mathrm{CO}_{2}$ concentration in blood and plasma, plasma bicarbonate concentration, plasma pH and carbon dioxide tension between $16-42{ }^{\circ} \mathrm{C}$. In: Blood pH, Gases and Electrolytes. NBS Special publication 450, 39-45 (1977).
42. Robin, E.G. - Relationship between temperature and plasma pH and carbon dioxide tension in the turtle. Nature 195, 249-251 (1962).
43. Roughton, F.J.W. and J.W. Severinghaus - Accurate determination of $\mathrm{O}_{2}$ dissociation curve of human blood above $98.7 \%$ saturation with data on $\mathrm{O}_{2}$ solubility in unmodified human blood from $0^{\circ}$ to $37{ }^{\circ} \mathrm{C}$. J. Appl.Physiol. 35, 861-896 (1973).
44. Rosenthal, T.B. - The effect of temperature on the pH of blood and plasma in vitro. J. Biol. Chem. 173, 25-30 (1948).
45. Severinghaus, J.W. - Respiration and hypothermia. Ann.NY. Acad. Sci 80, 384-394 (1959).
46. Smith, L.W. and T. Fay - Observations on human beings with cancer maintained at reduced temperatures of $75^{\circ}-90^{\circ}$ Fahrenheit. Am.J.Clin. Pathol. 10, 1-11, (1940).
47. Stadie, W.C., J.H. Austin, H.W. Robinson - The effect of temperature on the acidbase protein equilibrium and its influence on the $\mathrm{CO}_{2}$ absorption curve of whole blood, true and separated serum. J.Biol. Chem. 66, 901-920 (1925).
48. Swan, H., I. Zeavin and J.H. Holmes - Cessation of circulation in general hypothermia. I. Physiologic changes and their control. Ann. Surg. 138, 360-376 (1953).
49. Thomas, L.J. Jr. - Algorithms for selected blood acid-base and blood gas calculations. J.Appl.Physiol.33, 154-158 (1972).
50. Walther, A. - Beitrage zur Lehre von der thierischen Wärme. Virchow's Arch. 25, 414-417 (1862).
51. Winterstein, H. - Der Einflusz der Körpertemperatur auf das Säure-Basen-Gleichgewicht im Blut. Naunyn Schmiedeberg's Arch.exptl. Pathol.u. Pharmakol 223, 1-18 (1954).
52. Zock, J.P., P. Rispens and W.G. Zijlstra - Carbon dioxide loading and the acid-base equilibrium states of human blood. Proc. Kon. Ned. Akad. Wet. C83, 307-332 (1980).
53. Zock, J.P. - Human whole blood oxygen affinity: $\mathrm{H}^{+}$and $\mathrm{CO}_{2}$ bohr and haldane factors calculated for erythrocyte content and blood. Proc. Kon. Ned.Akad. Wet. (In the press).
54. Zwart, A., G. Kwant, B. Oeseburg and W.G. Zijlstra - Human whole blood oxygen affinity: efect of temperature. J.Appl.Physiol. 57, 429-434 (1984).
55. Zijlstra, W.G., B. Oeseburg, G. Kwant and A. Zwart - Determination of interdependent ligand effects on human red cell oxygen affinity. Scand.J. Clin.Lab.Invest. 42 , 339-345, 1982.

## CHAPTER V

CARBON DIOXIDE LOADING AND THE ACID-BASE EQUILIBRIUM STATES OF THE HUMAN ISOLATED INTERNAL ENVIRONMENT

# Carbon dioxide loading and the acid-base equilibrium states of the human isolated internal environment* 

by J.P. Zock, P. Rispens and W.G. Zijlstra<br>Department of Physiology, University of Groningen, the Netherlands


#### Abstract

The changes in plasma pH and plasma bicarbonate concentration accompanying changes in $p \mathrm{CO}_{2}$ are different for blood in yivo and blood in vitro. This difference is caused by buffering and dilution in the interstitial fluid, by the effects exerted by the tissue cells and the gut, and by the regulatory actions of the kidneys. Because of the intricate intermingling of the interstitial fluid with the tissuecells it is not possible to separate experimentally the contribution of the interstitialfluid to and the effects of the tissue cells on the $\mathrm{CO}_{2}$ bufferline in vivo. In order to get an impression of the influence of the interstitial fluid on the in vivo buffering, a model was developed with which equilibrium states of the blood of the body in venous composition with all the interstitial fluid can be calculated. The model consists of a number of equations describing, first, the relevant physicochemical properties of the homogeneous solutions contained in the three supposed compartments (red cells, plasma and interstitial fluid) and, second, the equilibrium conditions on the two boundaries between the compartments (red cell-plasma and plasma-interstitial fluid). The plasma compartment is the central one, in equilibrium on one side with the erythrocyte content, and on the other side with the interstitial fluid. The equilibrium conditions and properties of the blood have been described previously (28). The interstitial fluid was assumed to reach equilibrium with plasma through exchange of all its supposed constituents except protein. This condition, together with the need to assign an arbitrary value to the volumes of the compartments, necessitated the introduction of a hydrostatic pressure difference between the blood and the interstitial fluid compartment. The model was implemented as a computer program. The results as calculated with this model can, however, only indirectly be compared with measured values since the interstitial fluid is not accessible to an experimental approach.


[^3]
## INTRODUCTION

Cells in the body are surrounded by the interstitial fluid. This fluid forms the environment from which the cells are supplied with nutrients and into which they dispose of the end products of their metabolism. Both the uptake of nutrients and the disposal of end products tend to change the composition of the interstitial fluid. For optimal functioning, cells require a well adjusted composition of the surrounding interstitial fluid. This composition is maintained by supply from and disposal into the blood. Constancy of the composition of the blood is attained by regulatory actions of the body. Interstitial fluid is a filtrate of blood plasma and by diffusion most of its constituents exchange very rapidly with plasma (19). In this way the composition of the interstitial fluid is related to the composition of blood plasma and, as far as oxygen and carbon dioxide are concerned, also to the composition of whole blood. The extracellular fluid (i.e. interstitial fluid and blood plasma) and the erythrocytes together constitute the internal environment supplying the tissue cells. Although erythrocytes are living cells too, they are considered as being part of the internal environment because their functions in the maintenance of the constancy of the internal environment are operative throughout the body in close interaction with plasma, at a very short distance from the interştitial fluid.

One of the important variables representing the state of the interstitial fluid is pH . This is partly determined by the balance between the production and the removal of $\mathrm{CO}_{2}$, which, besides water, is the main end product of oxidative cell metabolism. In the body $\mathrm{CO}_{2}$ is present as dissolved $\mathrm{CO}_{2}$, as $\mathrm{HCO}_{3}{ }^{-}$ions, and, bound to proteins - mainly hemoglobin - , as carbamate. It is, furthermore, a constituent of bone in which it is complexly bound. As such it only participates in very slow exchanges of $\mathrm{CO}_{2}$ and is usually not considered in this context (11). The amount of dissolved $\mathrm{CO}_{2}$ depends on the solubility and the partial pressure of $\mathrm{CO}_{2}$. In the tissues the latter is the result of the partial pressure of $\mathrm{CO}_{2}$ in arterial blood, the perfusion and the rate of production of $\mathrm{CO}_{2}$. With the formation of $\mathrm{HCO}_{3}{ }^{-}$ions and also with carbamate formation, protons are released which are buffered both inside and outside the tissue cells. Inside the tissue cells the main buffers are phosphates and proteins, while outside the tissue cells these buffers are hemoglobin in the red cells and the proteins of plasma and interstitial fluid. Plasma is in very close contact with interstitial fluid and the acid-base status of venous plasma shortly after the exchange in the tissue reflects the acid-base status of the perfused tissue.
Besides the influence of $\mathrm{CO}_{2}$ on the acid-base status of the interstitial fluid, that of non-volatile ionic substances is also important. Abnormal concentrations of these can result from metabolism, as e.g. lactic acid in hypoxia, as well as from disturbances in the balance between uptake and excretion of electrolytes, as e.g. in kidney failure or after prof use vomiting or diarrhoea.
Acid-base disturbances which follow a period of imbalance between production and removal of $\mathrm{CO}_{2}$ are caused by inadequate respiration and are thus called respiratory disturbances, while all others are called non-respiratory
disturbances. Disturbances of overall acid-base status can be best detected in the plasma of arterial blood, because the composition of this blood is subjected to the regulatory action of the body as effected in the lungs. Any deviation in the acid-base status remaining after passage of the blood through the lungs indicates a disturbance of homeostasis.

In isolated plasma of constant temperature which is equilibrated with a gas mixture containing $\mathrm{CO}_{2}$, the acid-base status is characterized by two of the three variables: $p \mathrm{CO}_{2}, \mathrm{pH}$ and plasma bicarbonate concentration ( $c_{p, \mathrm{HCO}_{3}}{ }^{-}$). The third variable is related to the other two by the Henderson-Hasselbalch relation (Eq. 9). This plasma is called "separated plasma". Its acid-base status can be unambiguously represented as a point in a diagram with two of the variables plotted along the axes. Varying the $p \mathrm{CO}_{2}$ is equivalent to titration of the plasma with carbonic acid. The line in the $c_{p,} \mathrm{HCO}_{3}^{-} / \mathrm{pH}_{p}$ diagram which is found when varying the $p \mathrm{CO}_{2}$, is called the bufferline of separated plasma. The buffer value is the slope of this line and depends upon the buffering by nonbicarbonate buffers.

When whole blood of constant temperature is equilibrated with a gas mixture containing $\mathrm{O}_{2}$ and $\mathrm{CO}_{2}$, the buffering by the hemoglobin inside the erythrocytes and the redistribution of $\mathrm{HCO}_{3}{ }^{-}$and $\mathrm{Cl}^{-}$across the red cell membrane influences the acid-base status of the plasma. This results in a steeper bufferline than that of separated plasma. Such plasma is called "true plasma in vitro". The oxygen saturation of hemoglobin has some influence on the position of the bufferline, that of deoxygenated normal blood lying about $3 \mathrm{mmol} \cdot \mathrm{l}^{-1}$ above that of fully oxygenated blood.
Plasma circulating in the body as constituent of the blood is called "true plasma in vivo". This plasma is in direct contact with erythrocytes as well as with the tissues. Tissue cells can readily exchange substances with plasma by way of the interstitial fluid. Because of this, the acid-base status of mixed venous plasma reflects the average acid-base status of the various tissues of the body. By changing the balance between removed and produced $\mathrm{CO}_{2}$, the whole body can be titrated with carbonic acid. The result of this titration is represented in the bufferline as found for mixed venous plasma. Apart from the buffering by the internal environment, the slope of this bufferline depends on shifts of electrolytes between the internal environment and the tissue cells, on the buffering which takes place inside the tissue cells, on the production and utilization of organic ions and on renal regulations. The quantitative assessment of these contributions to the buffering in the body requires that they be distinguished from the buffering by the internal environment alone. Experimentally, this separation cannot be realized. To cope with this difficulty the existing model of blood in vitro (28) was extended by the addition of the interstitial fluid, thus forming a model of the "isolated internal environment" i.e. the blood in exchange with the interstitial fluid.
The model consists of a set of equations describing the behaviour of the isolated internal environment under the assumption of a number of simplifying idealizations. It opens up the possibility of calculating to what extent buffering
takes place by the internal environment and thus to estimate the contributions of other influences on the bufferline of true plasma in vivo.

MODEL
If the body could be in a completely steady state there would be a balance between production and removal of $\mathrm{CO}_{2}$, local tissue concentrations would not vary with time and the composition of arterial and venous blood would be constant. In the model a completely steady state was assumed. It was furthermore assumed that diffusion and the reactions and processes related to $\mathrm{CO}_{2}$ transport are so fast that in this steady state the deviations from equilibrium could be neglected. This steady state of an open system is then identical with an equilibrium state of a closed system.

In the model the internal environment consists of all the blood of the body as modelled by Zock et al. (28) in a venous state in equilibrium with all the interstitial fluid of the body across a selectively permeable capillary wall (fig. 1). Blood is a suspension of erythrocytes in plasma. Plasma is assumed to be a homogeneous solution of $\mathrm{NaCl}, \mathrm{NaHCO} 3$ and proteins in water. The red cells consist of a selectively permeable membrane containing the red cell content and separating it from the plasma. Red cell content is assumed to consist of a homogeneous solution of $\mathrm{KCl}, \mathrm{KHCO}_{3}$, ATP, 2,3-DPG and hemoglobin in water. Interstitial fluid is assumed to be a homogeneous solution of NaCl , $\mathrm{NaHCO}_{3}$ and plasma protein. $\mathrm{Na}^{+}$represents all kations of plasma and inter-


Fig. I. Schematic representation of the equilibria considered in the model of the isolated internal environment. Closed arrows indicate the direction in which the equilibria shift when $\mathrm{CO}_{2}$ is added; open arrows indicate the shift when $\mathrm{CO}_{2}$ is withdrawn.
stitial fluid, $\mathrm{K}^{+}$stands for all kations present in the erythrocytes. $\mathrm{K}^{+}$is assumed to be confined to the erythrocyte compartment, $\mathrm{Na}^{+}$to stay completely out of it.

The red cell membrane is taken to be impermeable to $\mathrm{Na}^{+}, \mathrm{K}^{+}$, plasma proteins, hemoglobin, ATP and 2,3-DPG, and freely permeable to $\mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}$, $\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}{ }^{-}$. This means that at equilibrium the erythrocytes are in osmotic equilibrium with plasma. In the model the red cell membrane has no other properties. The property ascribed to the capillary wall is that it is impermeable to proteins, letting all other substances exchange freely between plasma and interstitial fluid.

The oxygen saturation of hemoglobin is given a certain value. The amount of dissolved oxygen is neglected. In the model it is assumed that the temperature is the same throughout the isolated internal environment.

The electroneutrality of the three solutions - plasma, erythrocyte content and interstitial fluid - requires that

$$
\begin{array}{ll}
\text { (1) } & n_{p, \mathrm{Na}^{+}}=n_{p, \mathrm{Cl}^{-}}+n_{p, \mathrm{HCO}_{3}}-+z_{\mathrm{Pr}} \cdot w_{p, \mathrm{Pr}} \\
\text { (2) } & n_{c, \mathrm{~K}^{+}}=n_{c, \mathrm{Cl}^{-}}+n_{c, \mathrm{HCO}_{3}-}+z_{\mathrm{ATP}} \cdot n_{c, \mathrm{ATP}}+z_{\mathrm{DPG}} \cdot n_{c, \mathrm{DPG}}+ \\
& +z_{\mathrm{Hb}} \cdot n_{c, \mathrm{Hb}}+z_{\mathrm{HbO}_{2}} \cdot n_{c, \mathrm{HbO}}^{2}
\end{array}
$$

where $n$ denotes an amount of substance in mole, $w$ a mass in gram. The index $p$ means plasma, $c$ is red cell, $i$ stands for interstitial fluid; $z$ is the amount of charge per mole, except for plasma and interstitial protein where it is the amount of negative charge per gram protein.
$\mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}{ }^{-}$are assumed to be distributed between plasma water and red cell water according to the Gibbs-Donnan equilibrium conditions:
(4) $\quad r_{c, p}=\frac{c_{c w, \mathrm{Cl}^{-}}}{c_{p w, \mathrm{Cl}^{-}}}=\frac{c_{c w, \mathrm{HCO}_{3}-}}{c_{p w, \mathrm{HCO}_{3}{ }^{-}}}$
$r_{c, p}$ is the Donnan ratio and $c_{c w}$ and $c_{p w}$ denote the substance concentration in cell water and plasma water, respectively. The ratio of proton activity $a_{\mathrm{H}^{+}}$in plasma water and in red cell water is also assumed to be according to the GibbsDonnan condition, so that
(5) $\quad \log \left(r_{c, p}\right)=\mathrm{pH}_{c}-\mathrm{pH}_{p}$.

The supposed distribution of $\mathrm{Na}^{+}, \mathrm{Cl}^{-}$and $\mathrm{HCO}_{3}^{-}$between plasma water and the water of the intersiitial fluid according to the Gibbs-Donnan condition gives:

$$
\begin{equation*}
r_{i, p}=\frac{c_{i w, \mathrm{Cl}^{-}}}{c_{p w, \mathrm{Cl}^{-}}}=\frac{c_{i w, \mathrm{HCO}_{3}^{-}}}{c_{p w, \mathrm{HCO}_{3}^{-}}}=\frac{c_{p w, \mathrm{Na}^{+}}}{c_{i w, \mathrm{Na}^{+}}} \tag{6}
\end{equation*}
$$

where $c_{i w}$ is the substance concentration in interstitial water. Applied to $a_{\mathrm{H}^{+}}$this gives
(7) $\quad \log \left(r_{i, p}\right)=\mathrm{pH}_{i}-\mathrm{pH}_{p}$.

The conditions of osmotic equilibrium between erythrocytes and plasma and the balance between osmotic and hydrostatic pressures between blood and interstitium leads to

$$
\begin{align*}
& \int \phi_{e} \cdot R T \cdot\left(\frac{n_{i, \mathrm{Na}^{+}}+n_{i, \mathrm{Cl}^{-}}+n_{i, \mathrm{HCO}_{3}^{-}}}{w_{i, \mathrm{H}_{2} \mathrm{O}}}\right)+\pi_{i, \mathrm{Pr}}+\Delta p_{h}= \\
& \left\{=\phi_{e} \cdot R T \cdot\left(\frac{n_{p, \mathrm{Na}^{+}}+n_{p, \mathrm{Cl}^{-}}+n_{p, \mathrm{HCO}_{3}^{-}}}{w_{p, \mathrm{H}_{2} \mathrm{O}}}\right)+\pi_{p, \mathrm{Pr}}=\right.  \tag{8}\\
& =\phi_{e} \cdot R T \cdot\left(\frac{n_{c, \mathrm{~K}^{+}}+n_{c, \mathrm{Cl}^{-}}+n_{c, \mathrm{HCO}_{3}-}+n_{c, \mathrm{ATP}}+n_{c, \mathrm{DPG}}}{w_{c, \mathrm{H}_{2} \mathrm{O}}}\right)+\pi_{c, \mathrm{Hb}_{4}}
\end{align*}
$$

where $\phi_{e}$ is the osmotic activity coefficient of the electrolytes, $\pi_{i, \mathrm{Pr}}$ the osmotic pressure of the protein in the interstitial fluid, $\pi_{p, \mathrm{Pr}}$ the osmotic pressure of plasma protein, $\pi_{c, \mathrm{Hb}_{4}}$ the osmotic pressure of hemoglobin, $w_{i, \mathrm{H}_{2} \mathrm{O}}, w_{p, \mathrm{H}_{2} \mathrm{O}}$, $w_{c, \mathrm{H}_{2} \mathrm{O}}$ the masses of interstitial, plasma and cell water, and $\Delta p_{h}$ the difference in hydrostatic pressure between capillary lumen and the interstitium necessary to get equilibrium under the conditions imposed by the other relations mentioned, while still allowing the freedom to choose the volumes of the three compartments arbitrarily. The relation between plasma $\mathrm{pH}, p \mathrm{CO}_{2}$ and plasma $\mathrm{HCO}_{3}{ }^{-}$concentration is given by the Henderson-Hasselbalch equation:

$$
\begin{equation*}
\mathrm{pH}_{p}=\mathrm{p} K_{1}^{\prime}+\log \frac{c_{p, \mathrm{HCO}_{3}^{-}}^{-}}{S^{\prime} \cdot p \mathrm{CO}_{2}} \tag{9}
\end{equation*}
$$

where $c_{p, \mathrm{HCO}_{3}}{ }^{-}$is the plasma $\mathrm{HCO}_{3}{ }^{-}$concentration in mmol $\cdot 1^{-1}$ and $S^{\prime}$ the solubility of $\mathrm{CO}_{2}$ in plasma in mmol $\cdot 1^{-1} \cdot \mathrm{kPa}^{-1}$.

The temperature dependences of $\mathrm{p} K_{1}^{\prime}$ and $S^{\prime}$ are according to Rispens (22):

$$
\begin{equation*}
\mathrm{p} K_{1}^{\prime}=-4.7416+1840.11 / T+0.015906 T-\log \left(1+\frac{0.020682}{\left.10^{\left(7-\mathrm{pH}_{p}(T)\right)}\right)}\right) \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
S^{\prime}=7.50(-3.246+536.73 / T+0.004985 T) \tag{11}
\end{equation*}
$$

where $T$ is temperature in K . Provided that suitable expressions are available for the titration curves of the proteins, hemoglobin, ATP and 2,3-DPG and that the osmotic activity coefficients and the relation between protein and water content of a solution are given, the above equations with proper starting values are sufficient for calculating the interdependence of the many variables such as oxygen saturation ( $\mathrm{S}_{\mathrm{O}_{2}}$ ), carbon dioxide tension $\left(p \mathrm{CO}_{2}\right)$, plasma $\mathrm{pH}\left(\mathrm{pH}_{p}\right)$, $\mathrm{CO}_{2}$ content of the internal environment, the slope of the bufferline of true plasma in vivo and temperature $\left(T_{b}\right)$.

The titration curve of plasma protein at $37^{\circ} \mathrm{C}$ is given by

$$
\begin{equation*}
z_{p, \mathrm{Pr}}=0.107\left(\mathrm{pH}_{p}-5.10\right) \tag{12}
\end{equation*}
$$

The slope of this curve is commonly assumed to be independent of temperature. We used a change in iso-electric pH of $-0.014 \mathrm{pHu} \cdot \mathrm{K}^{-1}$, according to Rispens (22).

The titration curve of hemoglobin depends on oxygen saturation, carbon dioxide tension, 2,3-DPG concentration and temperature (18). The influences of oxygenation and carbon dioxide binding on the titration curve of hemoglobin are treated as independent effects, superimposed upon the titration curve of deoxygenated hemoglobin in the absence of $\mathrm{CO}_{2}$. At $37^{\circ} \mathrm{C}$ the amount of charge on one mole of Hb monomer is fairly approximated by

$$
\begin{equation*}
z_{\mathrm{Hb}}=2.7\left(\mathrm{pH}_{c}-6.81\right) \tag{13}
\end{equation*}
$$

The temperature dependence of the slope is $-0.009 \mathrm{mmol} \cdot \mathrm{pHu}^{-1} \cdot \mathrm{~K}^{-1}$ and the iso-electric point changes by $-0.0122 \mathrm{pHu} \cdot \mathrm{K}^{-1}$ (1).

The titration curve of oxygenated hemoglobin differs slightly from that of deoxygenated hemoglobin. This is caused by changes in equilibrium constants of alkaline and acidic Bohr groups of the hemoglobin molecule which occur with oxygenation ( $1,18,25$ ), each group thus being characterized by two constants. The difference in charge between one mole deoxygenated and one mole oxygenated hemoglobin is calculated with the use of these constants. They are for the acidic group $\mathrm{p} K_{\mathrm{Hb}}=5.4$ and $\mathrm{p} K_{\mathrm{HbO}_{2}}=5.9$, for the alkaline group $\mathrm{p} K_{\mathrm{Hb}}=7.5$ and $\mathrm{p} K_{\mathrm{HbO}_{2}}=6.45$. The temperature dependence of the alkaline group is $-0.0265 \mathrm{~K}^{-1}$ and $-0.0167 \mathrm{~K}^{-1}$, respectively, and that of the acidic Bohr groups is $0.004 \mathrm{~K}^{-1}(1,18)$.

The binding of $\mathrm{CO}_{2}$ by hemoglobin influences the titration curve of hemoglobin, because an alkaline amino group is changed into an acid carbamino group. From the equations describing these reactions, it follows that (24)

$$
\begin{equation*}
Z=\frac{K_{c}^{\prime} \cdot K_{z}^{\prime} \cdot c_{c, \mathrm{CO}_{2}}}{K_{c}^{\prime} \cdot K_{z}^{\prime} \cdot c_{c, \mathrm{CO}_{2}}+K_{z}^{\prime} \cdot a_{\mathrm{H}}++\left(a_{\mathrm{H}^{+}}\right)^{2}} \tag{14}
\end{equation*}
$$

where $Z$ is the $\mathrm{CO}_{2}$ saturation of hemoglobin, $c_{c, \mathrm{CO}_{2}}$ the concentration of dissolved $\mathrm{CO}_{2}\left(\mathrm{~mol} \cdot \mathrm{l}^{-1}\right)$ and $a_{\mathrm{H}^{+}}$the proton activity ( $\mathrm{mol} \cdot \mathrm{l}^{-1}$ ). The equilibrium constants $K_{c}^{\prime}$ and $K_{z}^{\prime}$ are different for oxygenated and deoxygenated hemoglobin. Partially oxygenated hemoglobin is considered to be a mixture of fully saturated and completely deoxygenated hemoglobin, mutually independent. The values of the constants used are $\mathrm{p} K_{c, \mathrm{Hb}}=4.6, \mathrm{p} K_{c, \mathrm{HbO}_{2}}=5.0$ with temperature coefficient $0.019 \mathrm{~K}^{-1}$ and $\mathrm{p} K_{z, \mathrm{Hb}}=7.0, \mathrm{p} K_{z, \mathrm{HbO}_{2}}=6.5$ with temperature coefficient $-0.022 \mathrm{~K}^{-1}$.

The concentration of dissolved $\mathrm{CO}_{2}$ in the erythrocytes was calculated using a solubility of $0.195 \mathrm{mmol} \cdot \mathrm{l}^{-1} \cdot \mathrm{kPa}^{-1}$ at $37{ }^{\circ} \mathrm{C}$ (28). The temperature variation of this value was assumed to be in proportion with the temperature variation of the $\mathrm{CO}_{2}$ solubility in plasma.

The charges on ATP and 2,3-DPG as represented by $z_{\text {ATP }}$ and $z_{\text {DPG }}$ were calculated with $\mathrm{p} K^{\prime}=7.04$ and $\mathrm{dp} K^{\prime} / \mathrm{d} T=-0.0124$ for ATP and with $\mathrm{p} K^{\prime}=6.73$ and $\mathrm{dp} K^{\prime} / \mathrm{d} T=0.0026$ for $2,3-\mathrm{DPG}(21)$.

The osmotic coefficient for the ions was supposed to be 0.92 . The colloid osmotic pressure of plasma proteins was taken as

$$
\begin{equation*}
\pi_{\text {plasma }}=0.028 c_{p, \mathrm{Pr}}+2.13310^{-4} c_{p, \mathrm{Pr}}^{2}+1.20010^{-6} c_{p, \mathrm{Pr}}^{3} \tag{15}
\end{equation*}
$$

where $\pi$ is the pressure in kPa and $c_{p, \mathrm{P}_{\mathrm{r}}}$ is the plasma protein concentration in $\mathrm{g} \cdot \mathrm{l}^{-1}$. The equivalent expression used for the colloid osmotic pressure of hemoglobin was

$$
\begin{equation*}
\pi_{\mathrm{Hb}_{4}}=0.040 c_{\mathrm{Hb}_{4}}+2.5710^{-4} c^{2} \mathrm{Hb}_{4}+1.7110^{-6} c^{3} \mathrm{Hb}_{4} . \tag{16}
\end{equation*}
$$

The volume occupied by 1 g of hemoglobin and that by 1 g of protein was assumed to be 0.85 ml . This volume was assumed to include the volumes occupied by non-hemoglobin proteins in the erythrocytes and by lipids in plasma and interstitial fluid (28).

## ISOLATED INTERNAL ENVIRONMENT (IIE)

The equations given above interrelate the variables; because of this only a restricted number of variables can be chosen independently. The values which are chosen determine the composition and the state of the isolated internal environment. The interstitial fluid is considered to be in equilibrium with venous blood plasma, the composition of venous blood being calculated from arterial blood. Standard arterial blood is determined by assuming the following

Table I. Calculated composition of venous blood at $R Q=0.83$, compaired with arterial blood.

| Quantity | Unit | Arterial blood |  |  | Venous blood |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Plasma | Erythrocytes | Blood | Plasma | Erythrocytes | Blood |
| $S_{\mathrm{O}_{2}}$ |  |  | 1* |  |  | 0.76 * |  |
| $p \mathrm{CO}_{2}$ | kPa | 5.33* | 5.33* |  | 6.13* | 6.13* |  |
| pH | pHu | 7.410* | 7.202* |  | 7.378 | 7.184 |  |
| $r_{c, p}$ |  |  |  | 0.619 |  |  | 0.638 |
| Total $\mathrm{O}_{2}$ | mmol |  | 9.31 | 9.31 |  | 7.07 | 7.07 |
| Total $\mathrm{CO}_{2}$ | mmol | 14.36 | 6.89 | 21.25 | 15.31 | 7.81 | 23.11 |
| Dissolved $\mathrm{CO}_{2}$ | mmol | 0.67 | 0.47 | 1.14 | 0.77 | 0.54 | 1.31 |
| Carbamate | mmol |  | 1.13 | 1.13 |  | 1.42 | 1.42 |
| $\mathrm{HCO}_{3}{ }^{-}$ | mmol | 13.69 | 5.29 | 18.98 | 14.53 | 5.85 | 20.38 |
| $\mathrm{Cl}^{-}$ | mmol | 60.90 | 23.58 | 84.48 | 60.20 | 24.28 | 84.48 |
| Charge of protein | mmol | 9.52 |  |  | 9.39 |  |  |
| Charge of hemoglobin | mmol |  | 15.47 |  |  | 14.24 |  |
| Charge of DPG | mmol |  | 7.93 |  |  | 7.90 |  |
| Charge of ATP | mmol |  | 1.74 |  |  | 1.73 |  |
| Volume | 1 | 0.550 | 0.450 | 1.000* | 0.548 | 0.452 | 1.000* |
| Water | 1 | 0.517 | 0.323 | 0.840 | 0.515 | 0.325 | 0.840 |
| Osmolality | mosm/kg | 286.1 |  |  | 287.5 |  |  |
| Colloid osmotic pressure | kPa | 3.44 | 106.9 |  | 3.46 | 105.6 |  |
| Total osmotic pressure | kPa | 736.8 | 736.8 |  | 740.6 | 740.6 |  |

* Assumed values.
values: $T=310 \mathrm{~K}, S_{\mathrm{O}_{2}}=1, p \mathrm{CO}_{2}=5.33 \mathrm{kPa}(40 \mathrm{mmHg}), \mathrm{pH}_{p}=7.410$, $\mathrm{pH}_{c}=7.202, \quad c_{p, \mathrm{Na}^{+}}=153 \mathrm{mmol} \cdot \mathrm{l}^{-1}, \quad c_{p, \mathrm{Pr}}=70 \mathrm{~g} \cdot \mathrm{l}^{-1}, \quad c_{b, \mathrm{Hb}}=150 \mathrm{~g} \cdot \mathrm{l}^{-1}$, $c_{c, \mathrm{Hb}}=333 \mathrm{~g} \cdot 1^{-1}$ and $c_{c w, \mathrm{ATP}}=1.5 \mathrm{mmol} \cdot 1^{-1}$. To calculate the composition of venous plasma it was assumed that in the venous blood $S_{\mathrm{O}_{2}}=0.76$ and the respiratory quotient is 0.83 with respect to standard arterial blood, which corresponds in the model with a $p \mathrm{CO}_{2}=6.13 \mathrm{kPa}$. The calculated amounts of the substances in arterial and venous blood are given in table I. Additional values needed to fix the remaining degrees of freedom are the concentration of protein in the interstitial fluid and the volumes of blood and interstitial fluid. The concentration of protein in the interstitial fluid was assumed to be $35 \mathrm{~g} \cdot 1^{-1}$ (4). The volumes were based on the assumptions that the volume of the extracellular water is 200 ml per kg of body mass and that blood volume is 76 ml per kg of body mass. These generally accepted values applying to young men were used to calculate the volumes of blood and interstitial fluid for the "physiological standard man'", which means a young man of 70 kg body mass.


## CALCULATIONS

The above-mentioned equations were implemented in a computer program. Values for the quantities which can be chosen independently were provided in order to calculate the composition of the isolated internal environment. When the above-mentioned values were put into the program the standard composition followed. The insertion of various values of the independent quantities makes it possible to study their effects. The variation of one or more of the variables defining the state of the system, e.g. like $p \mathrm{CO}_{2}, \mathrm{~S}_{\mathrm{O}_{2}}$ and temperature, results in relations between the variables of the system. One of our main interests was in the effect of the titration of the isolated internal environment with $\mathrm{CO}_{2}$. This titration results in a change in plasma pH and plasma bicarbonate concentration. A plot of these two quantities is the $\mathrm{CO}_{2}$ bufferline. Other relations can be calculated in a similar way. The results of these calculations are presented in the next section.

The equations were solved by consecutive approximations. The maximum permitted error in the solutions was 0.0001 pHu . The approximation was stopped when the conditions set by the equations were satisfied to the chosen precision.

## RESULTS

The values of a number of quantities as calculated with the IIE model for standard conditions and standard composition are given in table II. The Donnan distribution of $\mathrm{Na}^{+}, \mathrm{HCO}_{3}^{-}, \mathrm{Cl}^{-}$and $\mathrm{H}^{+}$between plasma and interstitial fluid leads to a difference in pH of 0.013 . The calculated difference in pressure between the blood compartment and the interstitial fluid counteracting the osmotic pressure difference is $3.2 \mathrm{kPa}(24 \mathrm{mmHg})$. This pressure difference represents the mean hydrostatic pressure difference occurring there where the blood exchanges substance with the interstitial fluid, i.e. in the capillaries.

Loading and unloading of the IIE with $\mathrm{CO}_{2}$ is attained by changing the $p \mathrm{CO}_{2}$

Table II. Calculated values of some quantities in the isolated internal environment at standard composition and conditions.

| Quantity | Unit | Ery | Plasma | ISF | IIE |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{S}_{\mathrm{O}_{2}}$ |  | $0.76^{*}$ |  |  |  |
| $p \mathrm{CO}_{2}$ | kPa | $6.13^{*}$ | $6.13^{*}$ | $6.13^{*}$ | $6.13^{*}$ |
| T | K | $310^{*}$ | $310^{*}$ | $310^{*}$ | $310^{*}$ |
| pH |  | 7.184 | 7.379 | 7.392 |  |
| Volume | 1 | 2.406 | 2.914 | 11.605 | 16.925 |
| $\mathrm{H}_{2} \mathrm{O}$ |  | 1.727 | 2.740 | 11.260 | 15.727 |
| Hemoglobin | $\mathrm{g} \cdot \mathrm{l}^{-1}$ | 332 |  |  |  |
| Protein | $\mathrm{g} \cdot \mathrm{l}^{-1}$ |  | 70.3 | 35.0 |  |
| ATP | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ | 1.3 |  |  |  |
| $2,3-\mathrm{DPG}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ | 5.4 |  |  |  |
| $\mathrm{Na}^{+}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ |  | 164.5 | 159.6 |  |
| $\mathrm{~K}^{+}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ | 167.4 |  |  |  |
| $\mathrm{Cl}^{-}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ | 75.2 | 117.7 | 121.3 |  |
| $\mathrm{HCO}_{3}^{-}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ | 18.1 | 28.4 | 29.3 |  |
| $\pi_{\text {coll }}$ | kPa | 105.6 | 3.5 | 1.3 |  |
| $\Delta \mathrm{P}_{\mathrm{h}}$ | kPa | 3.2 | 3.2 | $0^{*}$ |  |
|  |  |  |  |  |  |

* Assumed values.

ISF = interstitial fluid.
in the IIE. In this way the amount of $\mathrm{CO}_{2}$ in the body can be varied and the $\mathrm{CO}_{2}$ equilibration curve of the IIE obtained (fig. 2). In table III the $\mathrm{CO}_{2}$ contents of the three compartments are given, partitioned into the contributions of the different compounds to the standard composition at the standard state. In table IV concentrations related to $1.0 \mathrm{~kg} \mathrm{H}_{2} \mathrm{O}$ and 1.0 kg body mass are given. From table III it follows that in the blood of the IIE $122.9 \mathrm{mmol} \mathrm{CO}_{2}$ ( 2.741 STPD) is stored. This agrees with the value of 2.70 l reported by Cherniak and Longobardo (11). The $\mathrm{CO}_{2}$ content of the IIE per kg body mass is 6.66 mmol . This value is difficult to compare with experimental values because these always refer to particular tissues or to the whole body. However, to compare the experimental values with the calculated one we can make the following estimation. The calculated concentration of $\mathrm{HCO}_{3}{ }^{-}$in the extracellular fluid (plasma + interstitial fluid) is $28.9 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{H}_{2} \mathrm{O}$, which is about three times the concentration in intracellular water in tissue cells of $9.6 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{H}_{2} \mathrm{O}$ (2). Because the amount of intracellular water is about twice the amount of extracellular water, it contains $2 \times \frac{1}{3}$ times the extracellular amount of $\mathrm{HCO}_{3}{ }^{-}$

Table III. Total carbon dioxide content of the IIE at standard composition and conditions.

| Quantity | Unit | Ery | Plasma | ISF | IIE |
| :--- | :--- | :--- | :--- | :--- | ---: |
| $\mathrm{HCO}_{3}{ }^{-}$ | mmol | 31.1 | 77.3 | 327.2 | 435.9 |
| dissolved $\mathrm{CO}_{2}$ | mmol | 2.9 | 4.1 | 16.3 | 23.3 |
| carbamino $\mathrm{CO}_{2}$ | mmol | 7.5 |  |  | 7.5 |
| total $\mathrm{CO}_{2}$ | mmol | 41.5 | 81.4 | 343.5 | 466.8 |



Fig. 2. Calculated concentrations of the various forms in which $\mathrm{CO}_{2}$ is present in one liter of IIE as functions of $p \mathrm{CO}_{2}$. From bottom to top: bicarbonate in plasria, dissolved $\mathrm{CO}_{2}$ plus carbonic acid in plasma, bicarbonate in erythrocytes, dissolved $\mathrm{CO}_{2}$ plus carbonic acid in erythrocytes, carba-mino-hemoglobin, bicarbonate in interstitial fluid, and dissolved $\mathrm{CO}_{2}$ plus carbonic acid in interstitial fluid. The oxygen saturation of hemoglobin is 0.76 .
which is $\frac{2}{3} \cdot 404.5=270 \mathrm{mmol}$. Together with the total $\mathrm{CO}_{2}$ of the internal environment of 466.5 mmol , this is 736.5 mmol fast exchanging $\mathrm{CO}_{2}$ in the whole body, thus $10.5 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$ body mass. This value is well within the range of experimental results (Irving et al. (17) $10 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$; Wallace and Hastings (27) $11 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$; Fenn (14) $10 \mathrm{mmol} \cdot \mathrm{kg}^{-1}$; Albers et al. (2) 8.6 $\mathrm{mmol} \cdot \mathrm{kg}^{-1}$ ).

To obtain a complete $\mathrm{CO}_{2}$ equilibrium curve of the body, the amounts of $\mathrm{CO}_{2}$ present at different values of $p \mathrm{CO}_{2}$ must be known. Instead of these absolute values, which are difficult to measure, the slope of the equilibrium curve, approximated as a straight line, is measured. Reported values are for humans per kg body mass $0.48 \pm 0.06 \mathrm{ml} / \mathrm{kg}^{-1} \cdot$ torr $^{-1}\left(0.16 \pm 0.02 \mathrm{mmol} \cdot \mathrm{kg}^{-1}\right.$. $\cdot \mathrm{kPa}^{-1}$ ) for equilibrations of short duration ( $1-8 \mathrm{~min}$ ) (11). Values obtained

Table IV. Calculated values of the concentrations of bicarbonate and total $\mathrm{CO}_{2}$ per kg water of the compartment concerned and per kg body mass.

| Quantity | Unit | Ery | Plasma | ISF | IIE |
| :--- | :--- | :---: | :--- | :--- | :---: |
| $\mathrm{HCO}_{3}{ }^{-}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{H}_{2} \mathrm{O}$ | 18.1 | 28.4 | 29.3 | 27.7 |
| total $\mathrm{CO}_{2}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{H}_{2} \mathrm{O}$ | 24.0 | 29.9 | 30.8 | 29.9 |
| $\mathrm{HCO}_{3}{ }^{-}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{BM}$ | 0.44 | 1.10 | 4.67 | 6.22 |
| total $\mathrm{CO}_{2}$ | $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \mathrm{BM}$ | 0.59 | 1.16 | 4.91 | 6.66 |

for longer equilibrations differ even more: $1.8 \pm 0.4 \mathrm{ml} \cdot \mathrm{kg}^{-1} \cdot \operatorname{torr}^{-1}(0.61 \pm 0.14$ $\mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}$ ) after equilibrations of $20-60 \mathrm{~min}$ duration (11). (One greatly deviating value has been omitted by us from the calculation of this mean value.) At the standard state the calculated slope of the $\mathrm{CO}_{2}$ equilibration curve of the IIE is $0.24 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}$, which is about $50 \%$ more than the value mentioned in the literature for short term equilibration. From the slope as calculated for the IIE by us it follows that for a breathholding man of 70 kg with a supposed rate of $\mathrm{CO}_{2}$ production of $10 \mathrm{mmol} / \mathrm{min}$, the $p \mathrm{CO}_{2}$ would rise by 0.6 $\mathrm{kPa} / \mathrm{min}(4.5 \mathrm{mmHg} / \mathrm{min})$ if this $\mathrm{CO}_{2}$ would be buffered in the IIE only. This agrees with the $5 \mathrm{mmHg} / \mathrm{min}$ mentioned by Cherniak and Longobardo (11). Of the amount of $0.24 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}$, about $30 \%\left(0.075 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}\right)$ is buffered by the blood as part of the IIE. It is not justifiable to compare the value of $0.24 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}$ with the reported $0.61 \mathrm{mmol} \cdot \mathrm{kg}^{-1} \cdot \mathrm{kPa}^{-1}$ for long term equilibration.

In our model only fast exchanging parts of the extracellular compartment are incorporated. Compounds like bone and tendon exchange matter only slowly, bone being the largest store of carbon dioxide compounds in the body. Cells contribute to the storing of $\mathrm{CO}_{2}$ in an unknown way, and exchange of $\mathrm{HCO}_{3}^{-}$ in the kidney and the intestines makes that the experimental long term results cannot be compared with the calculated value, except for the conclusion that in the long run the combined renal, interstitial and cellular contribution to $\mathrm{CO}_{2}$ "buffering" is large compared with that of the internal environment.

If in the model the amounts of all substances except $\mathrm{CO}_{2}-$ in all its compounds - are kept fixed, a one-to-one relation exists between plasma pH and


Fig. 3. Relations between $\log \left(p \mathrm{CO}_{2}\right)$ and plasma pH as calculated for blood in vitro and in the isolated internal environment (thick line). In both the oxygen saturation is 0.76.
$\log \left(p \mathrm{CO}_{2}\right)$. This relation is given in fig. 3. The same relation for blood in vitro is also shown. The slope of the line for blood is $-1.64 \cdot \mathrm{pHu}^{-1}$, that of the IIE $-1.23 \mathrm{pHu}^{-1}$. The slope of the curve of blood in vitro as experimentally determined can be compared with the calculated one because the conditions are the same. For in vivo experiments the results can hardly be compared with the calculated curve because of the reasons already mentioned. Experimental values are $-1.60 \pm 0.07 \mathrm{pHu}^{-1}$ for blood in vitro and $-1.45 \pm 0.13 \mathrm{pHu}^{-1}$ for arterial blood in vivo (20). As expected, the measured slope of the in vivo line is larger than that calculated for the IIE.

From the two preceeding relations the relation between the amount of $\mathrm{CO}_{2}$ in the IIE and plasma pH follows. For the standard IIE this relation is given in fig. 4. It shows how the amounts of the various compounds in which $\mathrm{CO}_{2}$ is present are related to plasma pH . From this it can be derived that at $\mathrm{pH}_{p}=7.41$ the change in total $\mathrm{CO}_{2}$ per kg body mass per unit plasma pH if caused by $\mathrm{CO}_{2}$ titration, is -3.43 mmol .

The calculated change in total bicarbonate in the extracellular fluid is $-144.13 \mathrm{mmol} \cdot \mathrm{pHu}^{-1}$, which is $-2.06 \mathrm{mmol} \cdot \mathrm{pHu}^{-1}$ per kg body mass. This quantity was experimentally determined by Albers ef al. (2) who found a value of $-4.0 \mathrm{mmol} \cdot \mathrm{pHu}^{-1} \cdot \mathrm{~kg}^{-1}$.

The influence of the interstitial fluid on the capacity to buffer $\mathrm{CO}_{2}$ is clear from the plasma bicarbonate-plasma pH diagram (fig. 5). This diagram shows


Fig. 4. Calculated concentrations of the various forms in which $\mathrm{CO}_{2}$ is present in one liter of the IIE as functions of plasma pH . From bottom to top: bicarbonate in plasma, dissolved $\mathrm{CO}_{2}$ plus carbonic acid in plasma, bicarbonate in erythrocytes, dissolved $\mathrm{CO}_{2}$ plus carbonic acid in erythrocytes, carbamino-hemoglobin, bicarbonate in interstitial fluid, and dissolved $\mathrm{CO}_{2}$ plus carbonic acid in interstitial fluid. The oxygen saturation of hemoglobin is 0.76 .


Fig. 5. Calculated bufferline of the isolated internal environment (thick line), and of blood in vitro (thin line); $S_{\mathrm{O}_{2}}=0.76$. Several $\mathrm{CO}_{2}$ isobars are given (hatched lines).
the curves calculated for blood in vitro and for blood in exchange with interstitial fluid. The calculated slope of the latter is $-9.7 \mathrm{mmol} \cdot 1^{-1} \cdot \mathrm{pHu}^{-1}$ while that of the former is $-28.3 \mathrm{mmol} \cdot 1^{-1} \cdot \mathrm{pHu}^{-1}$. In table V values are given of the slope of the bufferline in vivo experimentally determined by means of whole body titration with $\mathrm{CO}_{2}$ by various investigators. It must be realized that the in vivo bufferline is not only due to the buffering by the blood and the interstitial fluid but also represents contributions from the tissue cells, the kidneys and the intestines.

Fig. 6 shows the effect of $\mathrm{CO}_{2}$ loading and unloading on the Donnan ratio. The thick line is the Donnan ratio for $\mathrm{Cl}^{-}, \mathrm{HCO}_{3}^{-}$and $\mathrm{H}^{+}$between plasma and red cells when blood exchanges ions and water with the interstitial fluid. The thin line is that calculated for blood in vitro. The influence of the redistribution of ions through the dilution with the interstitial fluid is also demonstrated in the changes in osmolality with pH (fig. 7). The osmotic changes in blood in vitro are substantially larger than those in blood in exchange with the interstitial fluid. This is also reflected in the changes in hematocrit and the volumes with pH as given in figs. 8 and 9.

## DISCUSSION

The results of measurements concerning the change in acid-base status with changes in $p \mathrm{CO}_{2}$ are represented in many forms. Frequently used variables are, first pH or $a_{\mathrm{H}^{+}}$, second $p \mathrm{CO}_{2}$ or $\log \left(p \mathrm{CO}_{2}\right)$, and third $c_{p, \mathrm{HCO}_{3}}$. This makes it

Table V. Experimental values of the slope of the $\mathrm{CO}_{2}$ bufferline in vivo.

| HUMANS | slope <br> $\mathrm{mmol} \cdot \mathrm{l}^{-1} \cdot \mathrm{pHu}^{-1}$ <br> (absolute value) | equilibration <br> time <br> (min) | number of individuals resp. dogs | circumstances |
| :---: | :---: | :---: | :---: | :---: |
| Böning and Heinrich (6) | 8.3 | 15 | 10 | exogenous hypercapnia |
| Böning et al. (7) | 13 | 30 | 33 | exogenous hypercapnia |
|  | 12 | 40 | 18 | voluntary hypocapnia |
| Bracket et al. (8) | 9.7 | 40-90 | 7 | exogenous hypercapnia |
| Bunker (10) | 17.6 | 60 |  |  |
| Ichiyanagi et al. (16) | 21 | 15 | 8 | exogenous hypercapnia and voluntary hypocapnia |
|  | 15 | 15 | 10 | anesthesia, endogenous hypercapnia |
|  | 15 | 60 | 10 | anesthesia, exogenous hypercapnia |
|  | 27 | 20 | 12 | anesthesia, hypocapnia |
| Prys-Roberts et al. (20) | $21 \pm 6$ | 40-60 |  | anesthesia, exogenous hypercapnia, hypocapnia |
| DOGS |  |  |  |  |
| Albers et al. (2) | 23 | 30 | 13 | anesthesia, hyper- and hypocapnia; kidneys ligated |
| Brown and Clancy (9) | 11.8 | 15 | 6 | anesthesia, hypercapnia |
| Brown and Michel* | 9.3 |  |  | venous blood |
|  | 12.2 |  |  | arterial blood |
| Cohen et al. (12) | 19 | 60 | 21 | awake, exogenous |
| Garcia et al. (15) | 12 | 15 | 6 | anesthesia endogenous |
|  | 5 | 120 |  | hypercapnia |
|  | 8 | 10 | 6 | anesthesia, hypocapnia |
|  | 11 | 120 |  |  |
| * as quoted by Garcia et al. (15). |  |  |  |  |



Fig. 6. Relation between plasma pH and the Donnan ratio erythrocyte-plasma as calculated for the isolated internal environment (thick line), and for blood in vitro (thin line); $S_{\mathrm{O}_{2}}=0.76$.


Fig. 7. Relation between the real osmolality of plasma and plasma pH as calculated for the isolated internal environment (thick line) and for blood in vitro (thin line); $S_{\mathrm{O}_{2}}=0.76$. In both cases 10 $\mathrm{mmol} / \mathrm{kg}$ is added to the calculated osmolality to account for the osmotic contribution of glucose and urea.


Fig. 8. Changes in hematocrit versus plasma pH under $\mathrm{CO}_{2}$ titration in the isolated internal environment (thick line) and in blood in vitro (thin line); $S_{\mathrm{O}_{2}}=0.76$.
difficult to compare results, because when a relation between two quantities is represented as linear, this linearity is not preserved after transformation to another set of variables. As far as possible, available results have been transformed to the $c_{p, \mathrm{HCO}_{3}} / \mathrm{pH}_{p}$ diagram to make any comparison possible. An exception to this is the $\log \left(p \mathrm{CO}_{2}\right) / \mathrm{pH}_{p}$ relation shown in fig. 3. As stated earlier, the slope of the $c_{p, \mathrm{HCO}_{3}}-\mathrm{pH}$ p bufferline in vivo in the steady state is the consequence of buffering by the blood, the interstitial fluid, the cells, and eventually of the action of the kidneys and the intestines. Thus the experimental results will probably differ from our calculated values.

From 1932 onwards, when Shaw and Messer (26) discussed the transfer of bicarbonate between the blood and the tissues, the difference in slope between blood in vitro and blood in vivo has been the subject of much debate (10). A simple way to account for the effect of the interstitial fluid on the slope of the bufferline is to assume that the blood is diluted with the interstitial fluid. This was put forward by Armstrong et al. (3) who used the resultant hematocrit in the calculations with the Singer-Hastings nomogram or the $\log \left(p \mathrm{CO}_{2}\right) / \mathrm{pH}_{p}$ diagram. They showed that the outcomes were in agreement with experimental values. This line of reasoning was followed by others in their constructions of models of the isolated internal environment: Roos and Thomas (23), Dell and Winters (13) and Böning (5). In these studies values of buffer capacities are assigned to compartments without a detailed description of the various processes such as water, chloride and bicarbonate exchange. In this way a formula for the buffer capacity of the internal environment is derived in which the


Fig. 9. Changes in plasma volume (lower line) and in blood volume (upper line) versus plasma pH under $\mathrm{CO}_{2}$ titration in the isolated internal environment (thick lines) and in blood in vitro (thin lines); $S_{\mathrm{O}_{2}}=0.76$.
quantities initially used appear. The influence of these quantities upon the buffer value can be assessed. Depending on the values of quantities used, the calculated values of the slope of the $\mathrm{CO}_{2}$ bufferline differ widely in these studies. Roos and Thomas (23) arrive at a value of $-10.1 \mathrm{mmol} \cdot \mathrm{l}^{-1} \cdot \mathrm{pHu}^{-1}$, in agreement with the value of $-9.7 \mathrm{mmol} \cdot \mathrm{l}^{-1} \cdot \mathrm{pHu}^{-1}$ calculated by us. When in the model of Dell and Winters (13) our values of the blood volume and the


Fig. 10. Hydrostatic pressure difference between the blood compartment and the interstitial fluid compartment necessary to maintain constancy of the volumes of these compartments with various protein concentrations in the interstitial fluid; $S_{\mathrm{O}_{2}}=0.76$.
amount of interstitial fluid are substituted, a slope of $-18 \mathrm{mmol} \cdot 1^{-1} \cdot \mathrm{pHu}{ }^{-1}$ results, which is about twice the value calculated by us. The value that follows from the study of Böning (5) is about $-16 \mathrm{mmol} \cdot 1^{-1} \cdot \mathrm{pHu}^{-1}$.

When $p \mathrm{CO}_{2}$ differs from normal, it is difficult to define a steady state, because the rate of exchange of the interstitial compartments with plasma differs widely. This makes it the more difficult to determine whether a steady state has been attained. It may be doubted whether in all of the reported experiments a real steady state was established. Furthermore, Roos and Thomas (23) have shown that changes in cardiac output lead to a variation in the slope of the $\mathrm{CO}_{2}$ bufferline of arterial blood.

The model offers the possibility to study the effects of dilution by the interstitial fluid and the consequences of the addition, withdrawal and redistribution of $\mathrm{CO}_{2}, \mathrm{O}_{2}$, ions, protein and water. As an example a calculation was made in order to determine the hydrostatic pressure difference between capillary lumen and interstitium needed to maintain the partitioning of water between plasma and interstitium at the chosen proportion with various protein concentrations in the interstitial fluid. The outcome is given in fig. 10. It follows from this figure that for an ultrafiltrate from plasma with no protein content, the corresponding hydrostatic pressure difference across the capillary wall would be at least 4.7 $\mathrm{kPa}(35 \mathrm{mmHg})$.

## REFERENCES

1. Antonini, E., J. Wyman, M. Brunori, C. Fronticelli, E. Bucci and A. Rossi-Fanelli - Studies on the relations between the molecular and functional properties of hemoglobin. V. The influence of temperature on the Bohr effect in human and horse hemoglobin. J. Biol. Chem. 240, 1096-1103 (1965).
2. Albers, C., C. Ludwig, W. Usinger and P. Spaich - Indirect determination of mean whole body and intracellular $\mathrm{CO}_{2}$ and buffer capacity. Resp. Physiol. 11, 197-210 (1971).
3. Armstrong, B.W., J.G. Mohler, R.C. Jung and J. Remmers - The in-vivo carbon dioxide titration curve. Lancet 759-761 (1966).
4. Aukland, K. and G. Nicolaysen - Interstitial fluid volume: local regulatory mechanisms. Physiol. Rev. 61, 556-643 (1981).
5. Böning, D. - The in vivo and in vitro $\mathrm{CO}_{2}$-equilibration curve of blood during acute hypercapnia and hypocapnia. II. Theoretical considerations. Pfliigers Arch. 350, 213-222 (1974).
6. Böning, D. and K.W. Heinrich - Veränderungen der $\mathrm{CO}_{2}$-Bindungskurve des Blutes bei akuter respiratorischer Acidose und ihre Ursachen. II. Untersuchungen am Menschen. Pfliigers Arch. 303, 162-172 (1968).
7. Böning, D., U. Schweigart, V. Nutz and J. Stegemann - The in vivo and in vitro $\mathrm{CO}_{2^{-}}$ equilibration curves of blood during acute hypercapnia and hypocapnia. I. Experimental Investigations. P fliugers Arch. 350, 201-212 (1974).
8. Bracket, N.C. Jr., J.J. Cohen and W.B. Schwartz - Carbon dioxide titration curve of normal man. New Engl. J. Med. 272, 6-12 (1965).
9. Brown, E.B. Jr. and R.L. Clancy - In vivo and in vitro $\mathrm{CO}_{2}$ blood buffer curves. J. Appl. Physiol. 20, 885-889 (1965).
10. Bunker, J.P. - The great trans-atlantic acid-base debate. J. Anesthesiology 26, 591-594 (1965).
11. Cherniak, N.S. and G.S. Longobardo - Oxygen and carbon dioxide gas stores of the body. Physiol. Rev. 50, 196-243 (1970).
12. Cohen, J.J., N.C. Bracket, Jr. and W.B. Schwartz - The nature of the carbon dioxide titration curve in the normal dog. J. Clin. Invest. 43, 777-786 (1964).
13. Dell, R.B. and R.W. Winters - A model for the in vivo $\mathrm{CO}_{2}$ equilibration curve. Am. J. Physiol. 219, 37-44 (1970).
14. Fenn, W.O. - Carbon dioxide and intracellular homeostasis. Ann. N.Y. Acad. Sci. 92 , 547-558 (1961).
15. Garcia, A.C., Y.L. Lai, B.A. Allebery and E.B. Brown Jr. - In vivo $\mathrm{CO}_{2}$ buffer curves of arterial and mixed venous blood. Proc.-Soc. Exp. Biol. Med. 137, 237-242 (1971).
16. Ichiyanagi, K., K. Masuko, N. Nishisaka, M. Matsuki, H. Horikawa and R. Watanabe -Acid-base changes of arterial plasma during exogenous and endogenous hypercapnia in man. Resp. Physiol. 7, 310-325 (1969).
17. Irving, L., H.C. Foster and J.K.W. Ferguson - The carbon dioxide dissociation curve of living mammalian muscle. J. Biol. Chem. 95, 95-113 (1932).
18. Kilmartin, J.V. and L. Rossi-Bernardi - Interaction of hemoglobin with hydrogen ions, carbon dioxide, and organic phosphates. Physiol. Rev. 53, 836-890 (1973).
19. Landis, E.M. and J.R. Pappenheimer - Exchange of substance through capillary walls. In: Handbook of Physiology, Circulation II (Hamilton W.F. and P. Dow, eds.) pp. 961-1034. Washington D.C.: Am. Physiol. Soc. 1963.
20. Prys-Roberts, C., G.R. Kelman and J.F. Nunn - Determination of the in vivo carbon dioxide titration curve of anaesthetized man. Brit. J. Anaesth. 38, 500-509 (1966).
21. Reeves, R.B. - Temperature-induced changes in blood acid-base status: Donnan $r_{C l}$ and red cell volume. J. Appl. Physiol. 40, 762-767 (1976).
22. Rispens, P . - Significance of plasma bicarbonate for the evaluation of $\mathrm{H}^{+}$homeostasis. Thesis, Groningen 1970. Assen: Van Gorcum and Comp. 1970.
23. Roos, A. and L.J. Thomas Jr. - The in-vitro and in-vivo carbon dioxide dissociation curves of true plasma. Anesthesiology 28, 1048-1063 (1967).
24. Rossi-Bernardi, L. and F.J.W. Roughton - The specific influence of carbon dioxide and carbamate compounds on the buffer power and Bohr effects in human hemoglobin solutions. J. Physiol. 189, 1-29 (1967).
25. Rossi-Bernardi, L. and F.J.W. Roughton - The effect of temperature on the oxygen-linked ionization of hemoglobin. J. Biol. Chem. 242, 784-792 (1967).
26. Shaw, L.A. and A.C. Messer - The transfer of bicarbonate between the blood and tissues caused by alterations of the carbon dioxide concentration in the lungs. Am. J. Physiol. 100, 122-136 (1932).
27. Wallace, W.M. and A.B. Hastings - The distribution of the bicarbonate in mammalian muscle. J. Biol. Chem. 144, 637-649 (1942).
28. Zock, J.P., P. Rispens and W.G. Zijlstra - Carbon dioxide loading and the acid-base equilibrium states of human blood. Proc. Kon. Ned. Akad. Wet. C 83, 307-332 (1980).

The development of a mathematical model is to some extent comparable with the growth of a scientific theory. On the one hand the comparison of calculated results with experimental values gives an impression of the quality of the model, on the other hand the model provides new results which may lead to new experiments. The better the known results comply with reality, the more reliable new results become.

The results presented in the preceding chapters are to a large extent in agreement with experimental findings. However, there was one assumption necessary that is not in accordance with experimental results. This assumption concerns the osmotic behaviour of hemoglobin in solution. In the second chapter, the osmotic coefficient of hemoglobin was assumed to be equal to that of plasma protein. This value is larger than the highest experimental value, that of McConaghey and Maizels (5) (figure 12, chapter 2).

Recent determinations of the osmotic coefficient of haemoglobin by Wittmann and Gros (8) have confirmed the values of McConaghey and Maizels. However, analysis of the relations among variations in human red cell volume, density, membrane area, hemoglobin content and cation content by Svetina (7) shows much more influence of hemoglobin and less influence of cations on the osmotic behaviour of the red cell than is expected from the generally assumed osmotic behaviour of red cells. Because in the model all the variables are inter-related, it is not possible to change the osmotic properties of hemoglobin without influencifig other characteristic quantities. This discrepancy may be removed by introducing refinements in the model. The hemoglobin solution in the red cell compartment was assumed to fill the whole of the red cell volume, regardless of other voluminous constituents like, for example, the cell membrane. In the model the volume of the latter is reckoned in the volume of hemoglobin by assuming that 1 g of hemoglobin occupies 0.85 mL instead of 0.75 mL . The extra volume of 15 mL in 1 L blood is probably somewhat more than can be attributed to membranes 10 nm thick. Such a membrane would occupy about 8 mL in blood with $150 \mathrm{~g} / \mathrm{L}$ hemoglobin. Taking this volume apart leads to a slight rise of the hemoglobin concentration in the solution filling the red cell compartment. Furthermore, the hematocrit measured in blood includes more than only the red cell volume fraction because of plasma trapping. This means that the actual intracellular hemoglobin concentration comes out higher than assumed in the model. Thus, instead of the higher osmotic coefficient it appears to be more justified to use a higher than supposed hemoglobin concentration in the red cell, i.e. about $340-350 \mathrm{~g} / \mathrm{L}$. This shift in the assumptions indeed leads to more or less the same results as with the original assumption of the higher osmotic coefficient while using a proven relation for the osmotic
pressure of hemoglobin.
The model presented is suited to calculate equilibrium values of quantities but not the course of these values during exchange of substances. Nevertheless, when a process can be split up into a succession of equilibria it is possible to quantitate the effects of certain exchanges by looking at differences between these equilibrium states. The model of blood in an earlier version has been used in this way to estimate the influence of the availability of carbonic anhydrase to blood plasma on the gas exchange in the lungs (9).

Observations of postcapillary changes in blood $\mathrm{pH}(1,4)$ and on pH changes in the aorta (6) strongly indicate that there is no equilibrium in the blood when it leaves the pulmonary capillaries. Two processes are slow enough to cause this: firstly, the uncatalyzed dehydration and hydration of $\mathrm{CO}_{2}$ and, secondly, the $\mathrm{HCO}_{3}^{-} / \mathrm{Cl}^{-}$exchange across the red cell membrane. Until recently, it was supposed that all reactions of the gas exchange in the lungs reached equilibrium at the end of the pulmonary capillary and that the rapid $\mathrm{CO}_{2}$ exchange was due to carbonic anhydrase catalytic activity inside the erythrocytes. The $\mathrm{HCO}_{3}^{-} / \mathrm{Cl}^{-}$exchange was supposed not to be a limiting factor, so that the plasma $\mathrm{HCO}_{3}{ }^{-}$could be converted into $\mathrm{CO}_{2}$ inside the erythrocyte as well. This picture has changed since the observation that the blood pH in arterial blood changes during the passage through the aorta $(1,4,6)$ and the discovery of carbonic anhydrase at the endothelium of the pulmonary capillaries $(2,3)$. The former has been interpreted as indicating that physico-chemical equilibria have not yet settled in arterial blood in the aorta.

In order to study the influence of the availability of carbonic anhydrase to plasma, it was assumed that the uncatalyzed hydration and dehydration of $\mathrm{CO}_{2}$ and the $\mathrm{HCO}_{3}^{-} / \mathrm{Cl}^{-}$exchange proceed so slowly that the progress of these processes during the gas exchange in the lungs is negligible in comparison with their progress afterwards. This makes it possible to add an intermediate equilibrium stage. Venous blood entering the lungs is supposed to be in equilibrium. The gas exchange takes place before the intermediate stage, and the other equilibrations are assumed to occur between the intermediate stage and the final equilibrium stage. The change in pH between these stages represents the maximum change that can occur in arterial blood in the aorta. This value should therefore be compared with experimental results.

In the model the presence of carbonic anhydrase is taken into account by the way plasma was assumed to equilibrate with alveolar gas. With carbonic anhydrase available, both plasma and erythrocyte content reach complete equilibrium with the gas phase, but separately. When no carbonic anhydrase is available to plasma, only the red cell content was assumed to reach complete equilibrium with the alveolar gas, plasma being assumed to keep its venous composition with regard to $\mathrm{HCO}_{3}{ }^{-}$and to equilibrate with regard to dissolved
$\mathrm{CO}_{2}$. In both cases, pH in plasma was calculated. After the intermediate stage the exchange between plasma and red cells was assumed to take place in closed system conditions. For the final stage both pH and $\mathrm{pCO}_{2}$ were calculated. There are, of course, two final states corresponding to the two types of gas exchange assumed. The results obtained with this approach were in line with the experimental results (9). In this way the extreme values of exchange processes in certain conditions can be calculated.

The model described in chapter 5 was used to study the influence of the protein concentration in plasma and the interstitial fluid on the whole body buffer line for $\mathrm{CO}_{2}$. It turned out (10) that these quantities are just as important as hemoglobin concentration, although, generally, more emphasis is put on the latter.

The results discussed in the previous chapters show that many of the physico-chemical properties of blood and interstitial fluid in relation to blood gas and acid-base equilibria can be derived from the properties of the constituents. The results presented are in no way exhaustive, nor can the model in its present state be considered an end point. Besides the problem of the osmotic pressure there are other points that deserve attention in order to improve and extend the model. A desired extension is the introduction of 2,3-DPG binding to hemoglobin. Although rather complex to describe, it should be possible to incorporate this binding into the model as well. Eventually, this model of blood can be used as an educational tool for medical students.

It is our intention to apply the method to calculate exchanges of $\mathrm{O}_{2}$ and $\mathrm{CO}_{2}$ from differences between equilibrium states as presented above for the estimation of carbonic anhydrase influence in the lung capillaries also to the gas exchange in the tissues. To simulate the $\mathrm{O}_{2}$ and $\mathrm{CO}_{2}$ exchanges in the body in steady state, the cardiac output will be introduced into the model. Assuming that equilibria of some kind will be attained both in the lungs and in the tissues, steady state exchanges can be calculated as differences in total $\mathrm{CO}_{2}$ and total $\mathrm{O}_{2}$ contents between arterial and venous blood multiplied by the cardiac output. The equilibria in the tissues can be calculated with the model of the isolated internal environment. The pulmonary gas exchange will be calculated with a simple steady state model of the ventilation.

## REFERENCES

1. Crandall, E.D., A. Bidani and R.E. Forster - Postcapillary changes in blood pH in vivo during carbonic anhydrase inhibition. J.Appl.Physiol. 43: 582-590 (1977).
2. Crandall, E.D. and J.E. O'Brasky - Direct evidence for participation of rat lung carbonic anhydrase in $\mathrm{CO}_{2}$ reactions. J.Clin.Invest. 62: 618-622 (1978).
3. Effros, R.M., L. Shapiro and P. Silverman - Carbonic anhydrase activity of rabbit lungs. J.Appl.Physiol. 49: 589-600 (1980).
4. Hill, E.P., G.G. Power and R.D. Gilbert - Rate of pH changes in blood plasma in vitro and in vivo. J.Appl.Physiol. 42: 928-934 (1977).
5. McConaghey, P.D. and M. Maizels - The osmotic coefficients of haemoglobin in red cells under varying conditions. J.Appl. Physiol. 155: 28-45 (1961).
6. Rispens, P., B. Oeseburg, J.P. Zock and W.G. Zijlstra - Intra-aortic decrease in blood plasma pH . Pflügers Arch. 386: 97-99 (1980)
7. Svetina, S. - Relations among variations in human red cell volume, density, membrane area, hemoglobin content and cation content. J. Theor. Biol. 95: 123-134 (1982).
8. Wittmann, B. and G. Gros - The osmotic properties of hemoglobin under physiological conditions - implications for the osmotic behaviour of red cells. In: Biophysics of water. Franks F. and S. F. Mathias (eds.). John Wiley $\varepsilon$ Sons, Chichester, 1982, pp. 121-124.
9. Zock, J.P., P. Rispens and W.G. Zijlstra - Calculated changes in pH and $p \mathrm{CO}_{2}$ in arterial blood plasma assuming absence of ion and water exchange between plasma and erythrocytes during their equilibration with alveolar gas. Pflügers Arch. 391: 159-161 (1981).
10. Zock, J. P., P. Rispens and W.G. Zijlstra - An equilibrium state model for the acid-base status of blood and interstitial fluid. Proc. 6th IFCC Expert Panel on pH and Blood Gases, Groningen, 1982, pp 166-175.

## Het onderwerp

Een levend wezen als de mens bestaat, schematisch gezien, uit een zeer groot aantal levende cellen die zijn ingebed in een vloeistofhoudende tussenstof die niet "levend" is, maar die door de cellen wordt onderhouden. Hiervoor en voor hun eigen leven moeten de cellen kunnen beschikken over energie. De energie wordt verkregen door beheerste verbranding van voeding. Daartoe heeft de cel zuurstof nodig en ook moet de bij de verbranding vrijkomende kooldioxide worden afgevoerd. De zuurstof komt uit de buitenlucht en het kooldioxide moet daarin worden geloosd. Voor deze uitwisseling zorgen de longen. Voor het verdere transport zorgt het rondstromende bloed. Voor deze functie bevat het bloed speciale cellen, de rode bloedcellen, die daartoe vol zitten met een speciaal eiwit, het hemoglobine. Dit eiwit geeft het bloed zijn karakteristieke, rode kleur. Het transport van zuurstof gebeurt voor meer dan $90 \%$ door binding aan het hemoglobine. Ter plaatse van de verbruikers, de lichaamscellen, wordt de zuurstof losgelaten. Dit loslaten wordt bevorderd door de omstandigheden ter plaatse: een zuurdere omgeving door het koolzuur en een temperatuur en koolzuurgasspanning die hoger is dan in het toestromende bloed. De gevoeligheid van het hemoglobine voor deze faktoren is een zeer functionele bijzonderheid van dit eiwit. Het hemoglobine wordt niet direkt geconfronteerd met de veranderde omstandigheden doordat het in de rode bloedcellen opgesloten zit. Het inwendige van de cel verandert wel met de omgeving mee, maar in een afgezwakte vorm. Doordat er tussen de ruimte om de lichaamscellen en het bloed nog de scheiding door de bloedvaatwand is, werken de veranderingen rondom de cellen ook in gemodificeerde vorm door op het bloed. Het verband tussen veranderingen rondom de cellen, in de interstitiële vloeistof en de veranderingen in bloed is daardoor gecompliceerd.

## Het proefschrift

De bovenbeschreven wisselwerkingen tussen zuurstof en kooldioxide en hun invloed op de verdeling van water en elektrolyten tussen bloedplasma en de rode cellen zijn uitgebreid bestudeerd. Experimenten daarover kunnen niet het gehele samenspel tegelijk betreffen omdat het ondoenlijk is alle variabelen te meten. Om toch inzicht te krijgen in het totale systeem is al snel geprobeerd de samenhang tussen de grootheden in grafieken en vergelijkingen te vangen. Eertijds was het gebruik van nomogrammen de enige methode om deze vergelijkingen met verschillende parameters op te lossen. In 1928 volgde Henderson deze methode in zijn boek "Blood". Toch is het zelfs hiermee zeer bewerkelijk en weinig nauwkeurig om samenhangen vorm te geven. Met de
komst van de rekenmachine kwam een nieuwe mogelijkheid om dit te doen, beschikbaar.

De eerste modellen waren gebaseerd op enige vereenvoudigende veronderstellingen. Z Zo'n model was ook in ons laboratorium in gebruik. Uitgaande daarvan is een uitgebreider en verfijnder model ontwikkeld dat in dit proefschrift wordt beschreven. Verfijningen en uitbreidingen betreffen het invoeren van osmotische en donnan evenwichten, het invoeren van adenosinetrifosfaat (ATP) en 2,3-difosfoglycerat (2,3-DPG) als buffers in de rode cellen, en het uitbreiden van de in rekening gebrachte eigenschappen van hemoglobine. Daarnaast is ook het evenwicht van bloed met de interstitiële vloeistof onderwerp van studie. Een beperking is, dat in het model alleen de bufferwerking van het 2,3-DPG in aanmerking is genomen en de invloed ervan op het hemoglobine slechts impliciet aanwezig is. Voor het overige leiden de modellen in een veelheid van condities tot berekende waarden die goed in overeenstemming zijn met bekende experimentele resultaten. Zij kunnen daarom als een redelijke goede representatie van dit stukje werkelijkheid worden beschouwd.


[^0]:    * This work was supported in part by grants from the Netherlands Organization for the Adrancement of Pure Research (Z.W.O.) received through the Foundation for Medical Research (Fungo)

[^1]:    * Throughout this paper $\mathrm{Hb}_{4}$ means hemoglobin tetramer and Hb hemoglobin monomer.

[^2]:    * Un: und on pH will be abbreviated as pHu .

[^3]:    * This work was supported in part by grants from the Netherlands Organization for the Advancement of Pure Research (Z.W.O.) received through the Foundation for Medical Research (Fungo).

