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VECTOR-ATTRIBUTE FILTERS

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Abstract A variant of morphological attribute filters is developed, in which the attribute on which filtering is based, is no longer a scalar, as is usual, but a vector. This leads to new granulometries and associated pattern spectra. When the vector-attribute used is a shape descriptor, the resulting granulometries filter an image based on a shape or shape family instead of one or more scalar values.

Keywords: Mathematical morphology, connected filters, multi-scale analysis, granulometries, pattern spectra, vector-attributes, shape filtering

Introduction

Attribute filters [2, 12], which preserve or remove components in an image based on the corresponding attribute value, are a comparatively new addition to the image processing toolbox of mathematical morphology. Besides binary and gray-scale 2-D images [2, 12], these filters have also been extended to handle vector images, like color images [5, 7] and tensor-valued data [3], and 3-D images. So far the attributes used in all of these cases have been scalars. Although the set of scalar attributes used in multi-variate filters and granulometries [14] can also be considered as a single vector-attribute, these multi-variate operators can always be written as a series of uni-variate scalar operators, which is not the case for vector-attribute filters.

In this paper vector-attribute filters and granulometries will be introduced, whose attributes consists of vectors instead of scalar values, followed by a discussion on their use as filters and in granulometries where the parameter is a single shape image or a family of shape images instead of a threshold value.

1. Theory

The theory of granulometries and attribute filters is presented only very briefly here. For more detail the reader is referred to [2, 9, 12, 16]. In the following discussion binary images X and Y are defined as subsets of the image domain $\mathbf{M} \subset \mathbb{R}^n$ (usually $n = 2$), and gray-scale images are mappings from \mathbf{M} to \mathbb{R} .

Let us define a scaling X_λ of set X by a scalar factor $\lambda \in \mathbb{R}$ as

$$X_\lambda = \{x \in \mathbb{R}^n \mid \lambda^{-1}x \in X\}. \quad (1)$$

An operator ϕ is said to be *scale-invariant* if

$$\phi(X_\lambda) = (\phi(X))_\lambda \quad (2)$$

for all $\lambda > 0$. A scale-invariant operator is therefore sensitive to shape rather than to size. If an operator is scale, rotation and translation invariant, we call it a *shape operator*. A *shape filter* is simply an idempotent shape operator. In the digital case, pure scale invariance will be harder to achieve due to discretization artefacts, but a good approximation may be achieved.

Attribute openings and thinnings

Attribute filters, as introduced by Breen and Jones [2], use a criterion to remove or preserve connected components (or flat zones for the gray-scale case) based on their attributes. The concept of trivial thinnings Φ_T is used, which accepts or rejects connected sets based on a non-increasing criterion T . A criterion T is increasing if the fact that C satisfies T implies that D satisfies T for all $D \supset C$. The binary connected opening $\Gamma_x(X)$ of set X at point $x \in \mathbf{M}$ yields the connected component of X containing x if $x \in X$, and \emptyset otherwise. Thus Γ_x extracts the connected component to which x belongs, discarding all others. The trivial thinning Φ_T of a connected set C with criterion T is just the set C if C satisfies T , and is empty otherwise. Furthermore, $\Phi_T(\emptyset) = \emptyset$.

DEFINITION 1 *The binary attribute thinning Φ^T of set X with criterion T is given by*

$$\Phi^T(X) = \bigcup_{x \in X} \Phi_T(\Gamma_x(X)) \quad (3)$$

It can be shown that this is a thinning because it is idempotent and anti-extensive [2]. The attribute thinning is equivalent to performing a trivial thinning on all connected components in the image, i.e., removing all connected components which do not meet the criterion. It is trivial to show that if criterion

T is scale-invariant:

$$T(C) = T(C_\lambda) \quad \forall \lambda > 0 \wedge C \subseteq \mathbf{M}, \quad (4)$$

so are Φ_T and Φ^T . Assume $T(C)$ can be written as $\tau(C) \geq r$, $r \in \Lambda$, with τ some scale-invariant attribute of the connected set C . Let the attribute thinnings formed by these T be denoted as Φ_r^τ . It can readily be shown that

$$\Phi_r^\tau(\Phi_s^\tau(X)) = \Phi_{\max(r,s)}^\tau(X). \quad (5)$$

Therefore, $\{\Phi_r^\tau\}$ is a shape granulometry, since attribute thinnings are anti-extensive, and scale invariance is provided by the scale invariance of $\tau(C)$. An attribute thinning with an increasing criterion is an attribute opening.

DEFINITION 2 *A binary shape granulometry is a set of operators $\{\beta_r\}$ with r from some totally ordered set Λ , with the following three properties*

$$\beta_r(X) \subset X \quad (6)$$

$$\beta_r(X_\lambda) = (\beta_r(X))_\lambda \quad (7)$$

$$\beta_r(\beta_s(X)) = \beta_{\max(r,s)}(X), \quad (8)$$

for all $r, s \in \Lambda$ and $\lambda > 0$.

Thus, a shape granulometry consists of operators which are anti-extensive, and idempotent, but not necessarily increasing. Therefore, the operators must be thinnings, rather than openings. To exclude any sensitivity to size, we add property (7), which is just scale invariance for all β_r .

Size and shape pattern spectra

Size pattern spectra were introduced by Maragos [8]. Essentially they are a histogram containing the number of pixels, or the amount of image detail over a range of size classes. If r is the scale parameter of a size granulometry, the size class of $x \in X$ is the smallest value of r for which $x \notin \alpha_r(X)$. Shape pattern spectra can be defined in a similar way [15]. The pattern spectra $s_\alpha(X)$ and $s_\beta(X)$ obtained by applying size and shape granulometries $\{\alpha_r\}$ and $\{\beta_r\}$ to a binary image X are defined as

$$(s_\alpha(X))(u) = - \left. \frac{dA(\alpha_r(X))}{dr} \right|_{r=u} \quad (9)$$

and

$$(s_\beta(X))(u) = - \left. \frac{dA(\beta_r(X))}{dr} \right|_{r=u} \quad (10)$$

in which $A(X)$ denotes the Lebesgue measure in \mathbb{R}^n , which is just the area if $n = 2$.

In the discrete case, a pattern spectrum can be computed by repeatedly filtering an image by each β_r , in ascending order of r . After each filter step, the sum of gray levels S_r of the resulting image $\beta_r(f)$ is computed. The pattern spectrum value at r is computed by subtracting S_r from S_{r^-} , with r^- the scale immediately preceding r . In practice, faster methods for computing pattern spectra can be used [2, 10, 11]. These faster methods do not compute pattern spectra by filtering an image by each β_r . However, for methods using structuring elements this is usually unavoidable [1].

2. Vector-attribute granulometries

Attribute filters as described by Breen and Jones [2] filter an image based on a criterion. Much work has been done since: uni- and multi-variate granulometries [1, 14] and their use on different types of images, such as binary, gray-scale, and vector images. Although the original definition of the attribute filters was not limited to scalar attributes, the attributes used so far have always been based on scalar values.

A multi-variate attribute thinning $\Phi^{\{T_i\}}(X)$ with scalar attributes $\{\tau_i\}$ and their corresponding criteria $\{T_i\}$, with $1 \leq i \leq N$, can be defined such that connected components are preserved if they satisfy at least one of the criteria $T_i = \tau_i(C) \geq r_i$ and are removed otherwise:

$$\Phi^{\{T_i\}}(X) = \bigcup_{i=1}^N \Phi^{T_i}(X). \quad (11)$$

The set of scalar attributes $\{\tau_i\}$ can also be considered as a single vector-attribute $\vec{\tau} = \{\tau_1, \tau_2, \dots, \tau_N\}$, in which case a vector-attribute thinning is needed with a criterion:

$$T_{\vec{\tau}} = \exists i : \tau_i(C) \geq r_i \quad \text{for } 1 \leq i \leq N. \quad (12)$$

Although a thinning using this definition of $T_{\vec{\tau}}$ and $\vec{\tau}$ can be considered as a multi-variate thinning with scalar attributes, and thus be decomposed into a series of uni-variate thinnings (see definition 11), this is not the case with the vector-attributes and their corresponding filters for binary and gray-scale 2-D images that will be discussed below.

A binary vector-attribute thinning $\Phi_{\vec{r}, \epsilon}^{\vec{\tau}}(X)$, with d -dimensional vectors from a space $\Upsilon \subseteq \mathbb{R}^d$, removes the connected components of a binary image X whose vector-attributes differ more than a given quantity from a reference vector $\vec{r} \in \Upsilon$. For this purpose we need to introduce some dissimilarity measure $d : \Upsilon \times \Upsilon \rightarrow \mathbb{R}$, which quantifies the difference between the attribute vector $\vec{\tau}(C)$ and \vec{r} . A connected component C is preserved if its vector-attribute

$\vec{\tau}(C) \in \Upsilon$ satisfies criterion $T_{\vec{r},\epsilon}^{\vec{\tau}}(C) = d(\vec{\tau}(C), \vec{r}) \geq \epsilon$ and is removed otherwise, with ϵ some threshold. Thus it satisfies $T_{\vec{r},\epsilon}^{\vec{\tau}}$ if the dissimilarity $d(\vec{\tau}(C), \vec{r})$ between vectors $\vec{\tau}(C)$ and \vec{r} is at least ϵ . The simplest choice for d is the Euclidean distance: $d(\vec{u}, \vec{v}) = \|\vec{v} - \vec{u}\|$, and any other distance measure (such as Mahalanobis) could be used. However, d need not be a distance, because the triangle inequality $d(a, c) \leq d(a, b) + d(b, c)$ is not required.

More formally, the vector-attribute thinning can be defined as:

DEFINITION 3 *The vector-attribute thinning $\Phi_{\vec{r},\epsilon}^{\vec{\tau}}$ of X with respect to a reference vector \vec{r} and using vector-attribute $\vec{\tau}$ and scalar value ϵ is given by*

$$\Phi_{\vec{r},\epsilon}^{\vec{\tau}}(X) = \{x \in X \mid T_{\vec{r},\epsilon}^{\vec{\tau}}(\Gamma_x(X))\}. \quad (13)$$

This equation can be derived from definition 1 of the binary attribute thinning [2] by substituting T with $T_{\vec{r},\epsilon}^{\vec{\tau}}$ in the definition of the trivial thinning.

Although a multi-variate thinning $\Phi^{\{T_i\}}$ can be defined as a vector-attribute thinning $\Phi_{\vec{r},\epsilon}^{\vec{\tau}}$ with $T_{\vec{r},\epsilon}^{\vec{\tau}} = T_{\vec{r},\epsilon}^{\vec{\tau}}$, equation 13 cannot be decomposed in a similar way, unless $d(\vec{\tau}(C), \vec{r})$ is the L_∞ norm.

It should be noted here that vector-attribute openings are vector-attribute thinnings with an increasing criterion $T_{\vec{r},\epsilon}^{\vec{\tau}}$. Although it is easy to define an increasing criterion based on scalar attributes, this is much harder for vector-attributes, i.e. a criterion using a vector-attribute consisting of only increasing scalar attributes is not necessarily increasing. Furthermore, since all of these scalar attributes are increasing, they will generally be strongly correlated. For this reason we restrict our attention to thinnings.

The reference vector \vec{r} in the definition of vector-attribute thinnings can be computed using a given shape S : $\vec{r} = \vec{\tau}(S)$. This way a binary vector-attribute thinning with respect to a given shape S can be constructed:

DEFINITION 4 *The binary attribute thinning with respect to a shape $S \in \mathcal{C}$ can be defined as:*

$$\Phi_{S,\epsilon}^{\vec{\tau}} = \Phi_{\vec{\tau}(S),\epsilon}^{\vec{\tau}} \quad (14)$$

In Fig. 1 the effect of ϵ in the criterion $T_{\vec{r},\epsilon}^{\vec{\tau}}(C) = d(\vec{\tau}(C), \vec{r}) \geq \epsilon$ is demonstrated, with $d(\vec{\tau}(C), \vec{r}) = \|\vec{r} - \vec{\tau}(C)\|$. The reference vector \vec{r} was computed from an image of the letter A. Three values were (manually) chosen for ϵ : the maximum (rounded) value that removes exactly one letter, one value that removes nearly all letters, and one value in between.

The extension of binary attribute filters and granulometries to gray-scale has been studied extensively [2, 10–12]. Extending our vector-attribute thinnings and granulometries can be done in a similar fashion. Gray-scale thinning with respect to a shape is demonstrated in Fig. 2.

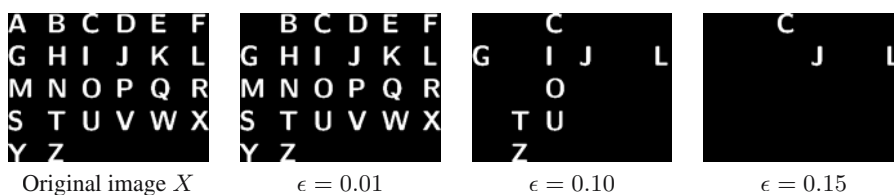


Figure 1. Filtering using a vector-attribute thinning $\Phi_{\vec{r},\epsilon}^{\vec{\tau}}(X)$ with increasing values of ϵ

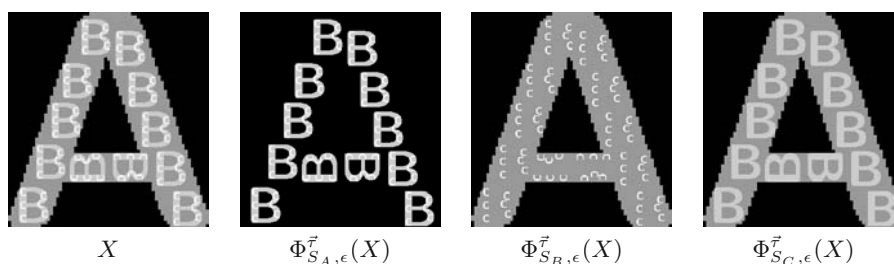


Figure 2. Removal of letters using $\Phi_{S_i,\epsilon}^{\vec{\tau}}(X)$ in gray-scale image (left) of letters A, B, C with S_i being respectively the shapes S_A , S_B , and S_C

DEFINITION 5 A granulometry with respect to reference vector $\vec{r} \in \Upsilon$, using scale, rotation and translation invariant vector-attribute $\vec{\tau} \in \Upsilon$ is given by the family of vector-attribute thinnings $\{\Phi_{\vec{r},\epsilon}^{\vec{\tau}}\}$ with ϵ from \mathbb{R} .

It is obvious from (13) that $\Phi_{\vec{r},\epsilon}^{\vec{\tau}}$ is anti-extensive and idempotent, and more importantly that

$$\Phi_{\vec{r},\eta}^{\vec{\tau}}(\Phi_{\vec{r},\epsilon}^{\vec{\tau}}(X)) = \Phi_{\vec{r},\max(\epsilon,\eta)}^{\vec{\tau}}(X) \quad \epsilon, \eta \in \mathbb{R} \quad (15)$$

Furthermore, if $\vec{\tau}$ is scale, rotation, and translation invariant, $\Phi_{\vec{r},\epsilon}^{\vec{\tau}}$ is a shape filter and $\{\Phi_{\vec{r},\epsilon}^{\vec{\tau}}\}$ is a shape granulometry [15].

An example of a suitable vector-attribute for shape granulometries are moment invariants. Hu's moment invariants [6] are invariant to rotation, scaling and translation, and are therefore suitable as shape attribute. Recently, new sets of moment invariants have been presented, such as the Krawtchouk moment invariants [17], which form a set of discrete and orthogonal moment invariants, and a set of complete and independent moment invariants by Flusser and Suk [4]. A problem that occurs with Krawtchouk moment invariants when the reference shape is not rotationally symmetric, like most letters, is that the angle used in the definitions of these moment invariants is defined by the orientation instead of the direction of the shape, which means that a 180 degrees rotated version of a shape S will generate a different vector-attribute than S does. The sensitivity of the moment invariants of Hu and Krawtchouk to rotation and

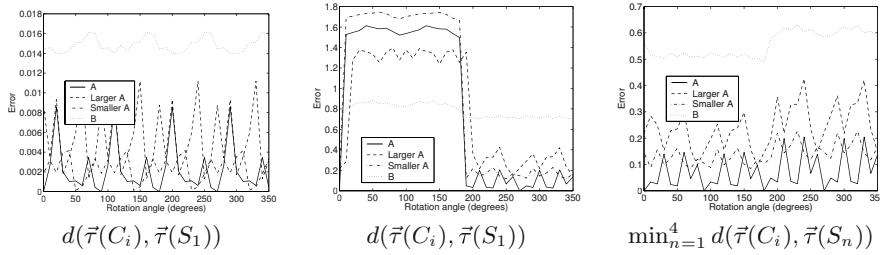


Figure 3. Effect of orientation on the distance between the vector-attributes of a connected component C_i and a given reference image S_j for Hu (left) and Krawtchouk (middle and right) moment invariants, where C_i represents the letter A, double-sized A, half-sized A, and B for $i = 1, 2, 3, 4$ respectively; S_j represents the letter A for $j = 1, 2$ at 0 and 180 degrees rotation respectively

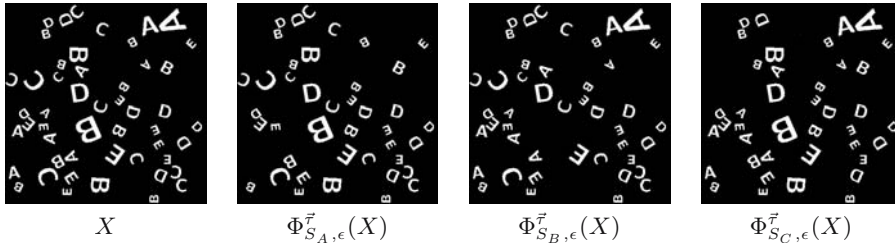


Figure 4. Left to right: original image and letters A, B, and C removed

scaling is demonstrated in Fig. 3, where one would expect the distance d between different orientations and sizes of the same letter A to be smaller than the distance between A and, according to the vector-attribute, the letter closest the A : the B . As can be seen, this is in both cases true for scaling, but it is clear that for Krawtchouk moment invariants rotation-invariance only holds for a certain range of orientations. This problem can be solved by using a filter that removes a connected component C if it matches any of the four orientations of a given shape S . This is demonstrated in Fig. 3(right). Furthermore, the Krawtchouk moments depend on the image size, which means that comparing two vectors requires that the same image size is used for the computation of both vectors and that some form of normalization is necessary. Considering these drawbacks of the Krawtchouk moment invariants we decided to use the well-known moment invariants of Hu for the other experiments described in this paper.

In Fig. 4 an image X consisting of the letters A, B, C, D, and E at different sizes and orientations is filtered with the goal of removing all instances of a certain letter in the image. As can be seen, especially the smallest letters in the image are not always removed when they should have been.

3. Granulometries with respect to a shape family

Let $\Phi_{S,\epsilon}^{\vec{\tau}}$ be defined as above, and let $F = \{S_1, S_2, \dots, S_n\}$ be a shape family with $F \subseteq \mathcal{C}$. The vector-attribute thinning $\Phi_{F,\epsilon}^{\vec{\tau}}$ with respect to shape family F is defined as

DEFINITION 6 *The vector-attribute thinning $\Phi_{F,\epsilon}^{\vec{\tau}}$ of X with respect to a set F , with $F \subseteq \mathcal{C}$ and using vector-attribute thinning with respect to shape $\Phi_{S,\epsilon}^{\vec{\tau}}$ is given by*

$$\Phi_{F,\epsilon}^{\vec{\tau}}(X) = \bigcap_{S \in F} \Phi_{S,\epsilon}^{\vec{\tau}} \quad (16)$$

In other words, connected components are removed if they resemble any member of the shape family F closer than a given amount ϵ and are preserved otherwise. Again we have that $\Phi_{F,\epsilon}^{\vec{\tau}}$ is anti-extensive and idempotent, and scale, rotation, and translation invariance is inherited from $\vec{\tau}$. Furthermore,

$$\Phi_{F,\epsilon}^{\vec{\tau}}(\Phi_{G,\epsilon}^{\vec{\tau}}(X)) = \Phi_{G,\epsilon}^{\vec{\tau}}(\Phi_{F,\epsilon}^{\vec{\tau}}(X)) = \Phi_{F,\epsilon}^{\vec{\tau}}(X) \quad \text{for } G \subseteq F. \quad (17)$$

DEFINITION 7 *Assume we have N shapes S_1, S_2, \dots, S_N and let F_n be a set containing the $n \leq N$ shapes S_1, \dots, S_n . A granulometry $\{\beta_n\}$ with respect to shape family F_N using vector-attribute thinning with respect to shape $\Phi_{S_i,\epsilon}^{\vec{\tau}}(X)$ for $S_i \in F_N$, is given by the family of vector-attribute thinnings with respect to shape family $\{\Phi_{F_n,\epsilon}^{\vec{\tau}}\}$ such that*

$$\beta_n = \Phi_{F_n,\epsilon}^{\vec{\tau}} \quad (18)$$

It is easy to see that if all $\{\Phi_{S_i,\epsilon}^{\vec{\tau}}\}$ are a shape granulometry, then so is $\{\beta_n\}$.

The use of granulometries with respect to a shape family F for the computation of pattern spectra is demonstrated in Fig. 5, where a pattern spectrum of the input image in Fig. 4(left) is computed using a granulometry with respect to a family $F_n = \{S_1, \dots, S_5\}$, with S_1, \dots, S_5 representing the letters A till E respectively. As a comparison, a histogram was also computed representing the number of occurrences of each letter in the image.

4. Conclusions

A new class of attribute filters was presented, whose attributes are vector instead of scalar values. These vector-attribute filters are a subclass of the attribute filters defined by Breen and Jones. Using Hu's moment invariants, it was shown how thinnings and granulometries could be defined that filter images based on a given shape or a family of shapes.

For discrete images, the rotation- and scale-invariance of the moment invariant attributes is only by approximation. Furthermore, the rotation-invariance

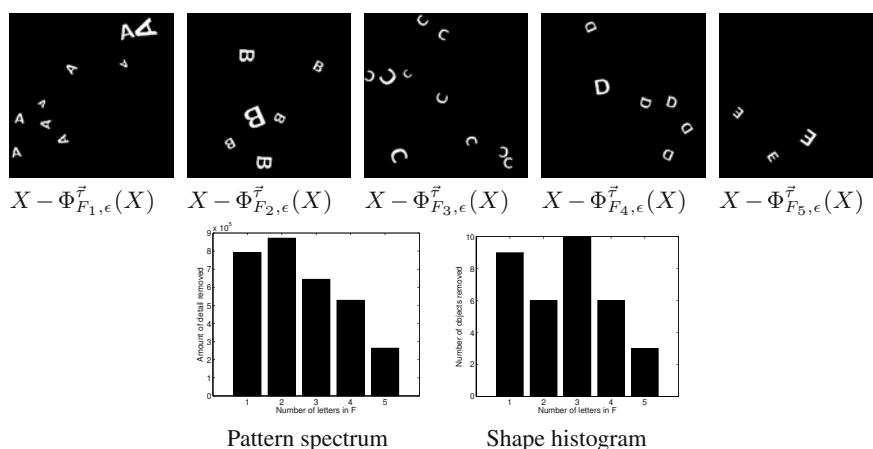


Figure 5. Pattern spectrum and shape histogram computed using $\Phi_{F_n, \epsilon}^{\vec{r}}(X)$ with $n = 1, 2, \dots, 5$, resulting in filtering with family F_n , where F_n is the family of the first n letters in the alphabet. Each F_n includes one more shape to remove (top row)

of the Krawtchouk moment invariants does not hold for all orientations for shapes without rotational symmetry, due to the fact that the angle computed here refers to the orientation instead of the direction of the component. Although this problem can be solved by filtering using a few orientations of one shape, vector-attributes that do not have this problem, like Hu’s moment invariants, are preferred. Future research will also investigate alternatives such as the complex moment invariants of Flusser and Suk [4]. More research is also needed to determine better ways for selecting the parameters like ϵ and the order and the choice of shape classes.

The dissimilarity measure d is also a critical choice. Other dissimilarity measures than the Euclidean distance should be investigated. If an adaptive system like a genetic algorithm would be used for d , an adaptive shape filter would be obtained. If multiple (reference) instances of the target class are available, the Mahalanobis distance is an option. This would lend more weight to directions in the attribute space Υ in which the class is compact, compared to directions in which the class is extended. Because we only use examples of the target class, the filtering problem resembles one-class classification [13]. This can be done with (kernel) density estimates to obtain a likelihood of class membership. The inverse of this probability would also yield a dissimilarity measure. Support-vector domain description could be used in a similar way [13].

An interesting approach would be the use of pattern spectra consisting of three dimensions: shape information from vector-attributes, size information

such as the area, and the orientation of the components. This would be particularly useful in texture classification.

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