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**Comment on 'Estimation and prediction of the HIV-AIDS-epidemic under conditions of HAART using mixtures of incubation time distributions' (letter)**

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*Published in:*  
Statistics in Medicine

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2008

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Heisterkamp, S. H., de Vries, R., Sprenger, H. G., Hubben, G. A. A., & Postma, M. J. (2008). Comment on 'Estimation and prediction of the HIV-AIDS-epidemic under conditions of HAART using mixtures of incubation time distributions' (letter). *Statistics in Medicine*, 27(11), 2035-2036.

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## LETTER TO THE EDITOR

## Comment on ‘Estimation and prediction of the HIV-AIDS-epidemic under conditions of HAART using mixtures of incubation time distributions’

by S. H. Heisterkamp, R. de Vries, H. G. Sprenger, G. A. A. Hubben, M. J. Postma, *Statistics in Medicine*, DOI: 10.1002/sim.2974

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The recent paper by Heisterkamp *et al.* [1] derived the incubation time distributions for HIV treated with the HAART regimen from a cohort study. This entailed the study of the convolution  $Z = \sum_{i=1}^k X_i$ , where  $X_i$ ,  $i = 1, 2, \dots, k$  are independent Gumbel random variables specified by the cumulative distribution functions (cdfs):

$$F_{X_i}(x) = \exp \left\{ -\exp \left( -\frac{x - \alpha_i}{\beta} \right) \right\}$$

for  $-\infty < x < \infty$ ,  $-\infty < \alpha_i < \infty$  and  $\beta > 0$ . Heisterkamp *et al.* stated that the distribution of  $Z$  looks ‘intractable for convolutions  $k > 2$ ’, and proposed an approximation based on the fact ‘that for  $k$  not too large the form of the distribution will remain Gumbel’. This fact does not appear to be true even for the case  $k = 2$ , see Figure 1. Heisterkamp *et al.* also derived an elementary expression for the cdf of  $Z$  for the very restrictive case  $k = 2$  and  $\alpha_1 = \alpha_2$ , a result that is not at all new: it follows from the well-known fact that the difference between two independent and identically distributed Gumbel random variables is a logistic random variable, see Johnson *et al.* [2].

The main point of this letter is to point out that explicit expressions for the probability density function (pdf) of  $Z = \sum_{i=1}^k X_i$  have been derived in the most general form when  $X_i$  are specified by the cdfs

$$F_{X_i}(x) = \exp \left\{ -\exp \left( -\frac{x - \alpha_i}{\beta_i} \right) \right\}$$

for  $-\infty < x < \infty$ ,  $-\infty < \alpha_i < \infty$  and  $\beta_i > 0$ . For the case  $k = 2$ , Nadarajah [3, Theorem 1] derived an expression for the pdf of  $Z$  in terms of hypergeometric functions. For  $k > 2$ , Nadarajah [4, Theorem 1 and Corollary 1] derived an expression for the pdf in terms of the Meijer  $G$  function. The hypergeometric and Meijer  $G$  functions are well known and well established (see Sections 9.23

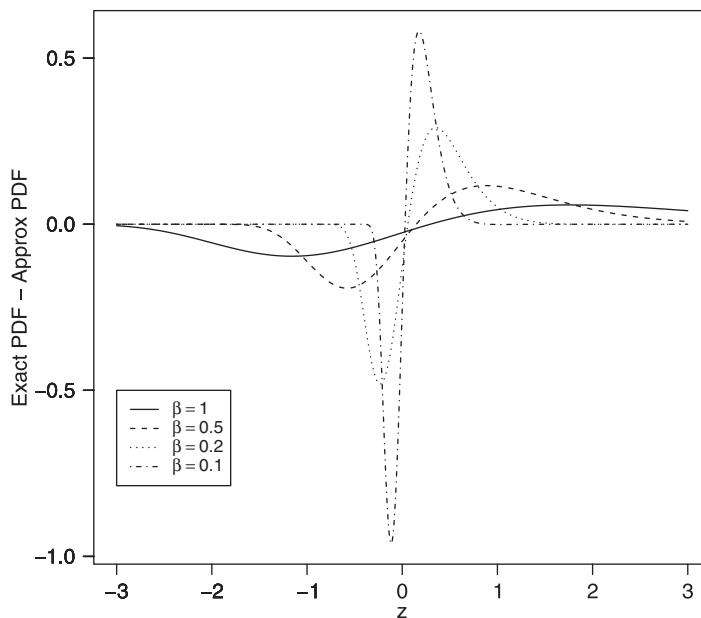


Figure 1. Difference between the exact and approximate pdfs of  $X_1 + X_2$  for  $\alpha_1 = 0$ ,  $\alpha_2 = 0$  and  $\beta = 0.1, 0.2, 0.5, 1$ . The approximation proposed by Heisterkamp *et al.* is used.

and 9.3 of Gradshteyn and Ryzhik [5]) and in-built numerical routines for them are widely available (see Maple and Mathematica).

#### REFERENCES

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