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# Power-based adaptive and integral control of standard mechanical systems

D.A. Dirksz and J.M.A. Scherpen

**Abstract**—Recently a power-based modeling framework was introduced for mechanical systems, based on the Brayton-Moser framework. In this paper it is shown how this power-based framework is used for control of standard mechanical systems. For systems which are affected by parameter uncertainty or other unknown disturbances adaptive control and integral control are also described in this framework. The power-based control approach is also compared with the energy-shaping control of port-Hamiltonian systems. The most interesting difference is the possibility of having adaptive and integrator dynamics depending on position errors, while preserving the physical structure.

## I. INTRODUCTION

After its introduction the port-Hamiltonian (PH) framework [11] has received a considerable amount of interest because of its insightful physical structure. It is well known that a large class of (nonlinear) physical systems can be described in the PH framework. The popularity of PH systems can be largely accredited to its application for analysis and control design of physical systems [4], [5], [13], [14], [16]. Although many results have been presented in the area of control, the performance is obviously affected by unknown disturbances or model uncertainties. In the case of stabilization uncertainties and/or modeling errors can cause the well known problem of steady-state errors. Such errors are traditionally eliminated by adding an integrator to the system. Adaptive control is another alternative, when errors are caused by parameter uncertainty and parameter estimation is desired. However, and as will be shown in a later section, for mechanical systems it is not possible to define adaptive laws or integrator dynamics based on position errors *and* still preserve the PH structure. In [6] integral control was presented for PH systems, however the integrator states were not directly used for control. A direct use of integrator states would destroy the PH structure, as mentioned before. Another possibility for integral control in the PH framework without losing the structure is presented in [2], but depends on a coordinate transformation.

To be able to define either adaptive laws or integrator dynamics based on position errors and still keep some sort of physical structure we leave the notion of PH systems and look at systems described in the power-based framework. In the power-based framework a system is described by Brayton-Moser (BM) equations [1]. They were originally developed for a large class of nonlinear electrical RLC networks. The use of the BM equations to realize stabilization

of such networks via *power-shaping* was introduced in [15]. Power-based modeling and control of other systems followed and were presented in [3], [7], [8], [9].

The main contribution of this paper is to extend results from [7] for power-based control to include integral and adaptive control. We focus on standard mechanical systems and show that, with power-based control, adaptive and integral control can be realized based on position errors. In this setup the original power-balance for the closed-loop system is preserved. The power-based description of standard mechanical systems [8] is recent and its application and advantages for control have not been explored yet.

Section II first describes a standard mechanical system in the PH framework. Then it is shown how adaptive stabilization and integral control, based on position errors, destroys the PH structure. Section III describes a standard mechanical system by BM equations and presents stabilization via power-shaping. Section IV then shows how to realize adaptive stabilization and integral control of the standard mechanical system. An example of the proposed control method is then shown in section V. Concluding remarks are then given in section VI.

**Notation.** All vectors are column vectors, including the gradient of a scalar function.

## II. PORT-HAMILTONIAN MECHANICAL SYSTEMS

Consider the PH system described by

$$\begin{aligned}\dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g(x)^\top \frac{\partial H}{\partial x}(x)\end{aligned}\quad (1)$$

with  $J(x) \in \mathbb{R}^{n \times n}$  the skew-symmetric interconnection matrix,  $R(x) \in \mathbb{R}^{n \times n}$  the symmetric, positive-semidefinite, damping matrix,  $x \in \mathbb{R}^n$ , the Hamiltonian  $H(x)$ , and  $u, y \in \mathbb{R}^m$  with  $m \leq n$ . A standard mechanical system described by (1) takes the form

$$\begin{aligned}\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -I & -D \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u \\ y &= G^\top \frac{\partial H}{\partial p}\end{aligned}\quad (2)$$

with  $q = (q_1, \dots, q_k)^\top$  the vector of generalized configuration coordinates,  $p = (p_1, \dots, p_k)^\top$  the vector of generalized momenta,  $I$  the identity matrix,  $D \in \mathbb{R}^{k \times k}$  the damping matrix,  $G$  the input matrix and  $y$  the output vector. The Hamiltonian of the system is equal to the sum of kinetic and potential energy:

$$H(q, p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q) \quad (3)$$

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where  $M(q) = M^\top(q) > 0$  is the system mass matrix and  $V(q)$  the potential energy. For fully actuated systems the input matrix can be taken equal to the identity matrix without loss of generality,  $G = I$ . Usually an input  $u = u_{es} + u_{di}$  is defined, which shapes the Hamiltonian of the system into a desired form,  $u_{es}$ , and injects damping,  $u_{di}$ . Assume we are dealing with systems where we are only shaping the potential energy. An easy way of shaping the potential energy of the plant is by substituting it with another function of the generalized positions which has suitable properties. The input signal then takes the form

$$u = \frac{\partial V}{\partial q}(q) - \frac{\partial V_d}{\partial q}(q) - K_d \dot{q} \quad (4)$$

where  $V_d(q)$  is the new, desired, potential energy and  $u_{di} = -K_d \dot{q}$  with  $K_d$  a positive definite matrix. Typically we choose a quadratic potential [12]

$$V_d(q) = \frac{1}{2} \bar{q}^\top K_p \bar{q} \quad (5)$$

with  $\bar{q} = q - q_d$ ,  $q_d$  being the desired position, and  $K_p$  a positive definite matrix. Notice how (4) requires the knowledge of the vector of the potential energy forces,  $\frac{\partial V}{\partial q}$ . In the case of parameter uncertainty, this vector is not accurately known. An adaptive version of (4) can then be applied, to compensate for errors caused by the uncertainties. Assume that the potential energy forces can be linearly parametrized in the form:

$$\frac{\partial V}{\partial q}(q) = \Delta(q)z \quad (6)$$

where  $\Delta(q)$  is a matrix of known functions and  $z = (z_1, \dots, z_m)^\top$  the vector of unknown parameters. An adaptive version of (4) is obtained by

$$u = -K_p \bar{q} - K_d \dot{q} + \Delta(q) \hat{z} \quad (7)$$

where  $\hat{z}$  is the estimate of  $z$ . Now, in the PH framework, if we want to define an update law for  $\hat{z}$  which depends on the position error  $\bar{q}$  we get

$$\dot{\hat{z}} = -K_z \Delta(q)^\top G^\top \frac{\partial H_d}{\partial q}(q, p) \quad (8)$$

with  $K_z$  the positive definite diagonal matrix of adaptive gains and  $H_d(q, p)$  the new Hamiltonian, i.e.,

$$H_d(q, p) = \frac{1}{2} p^\top M^{-1}(q) p + V_d(q) \quad (9)$$

The input (7) with update law (8) result in the closed-loop system

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\bar{z}} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -I & -\bar{D} & G \Delta K_z \\ -K_z \Delta^\top G^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial q} \\ \frac{\partial \bar{H}}{\partial p} \\ \frac{\partial \bar{H}}{\partial \bar{z}} \end{bmatrix} \quad (10)$$

where  $\bar{D} = D + K_d$ ,  $\bar{z} = \hat{z} - z$  and the closed-loop Hamiltonian

$$\bar{H}(q, p, \bar{z}) = H_d(q, p) + \frac{1}{2} \bar{z}^\top K_z^{-1} \bar{z} \quad (11)$$

It is easy to see that the interconnection matrix of (10) is not skew-symmetric and the closed-loop system is not anymore PH. We can realize a skew-symmetric interconnection matrix for the closed-loop system by replacing  $\frac{\partial H_d}{\partial q}$  in (8) by  $\frac{\partial H_d}{\partial p}$ . However, that means that the update law for  $\hat{z}$  is driven by the system velocity. Convergence of the velocity to zero does not mean that  $\bar{q} \rightarrow 0$  as  $t \rightarrow \infty$ , so we may still end up with a steady-state error.

The same problem appears when we want to add an integrator to system (2). Take

$$u = \xi \quad (12)$$

with  $\xi$  the integrator state and

$$\dot{\xi} = -K_i G^\top \frac{\partial H}{\partial q}(q, p) \quad (13)$$

where  $K_i$  is a positive definite diagonal matrix. The closed-loop system can be described by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -I & -D & G K_i \\ -K_i G^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}}{\partial q} \\ \frac{\partial \hat{H}}{\partial p} \\ \frac{\partial \hat{H}}{\partial \xi} \end{bmatrix} \quad (14)$$

with the Hamiltonian

$$\hat{H}(q, p, \xi) = H(q, p) + \frac{1}{2} \xi^\top K_i \xi \quad (15)$$

Just like with the adaptive control we again do not have a skew-symmetric interconnection matrix. We lose both passivity of the closed-loop system and the advantages of having power-ports for control. Similar to with adaptive control we can make it skew-symmetric by letting the  $\dot{\xi}$  dynamics depend on  $\frac{\partial \hat{H}}{\partial p}$ . Like before, this will not compensate for steady-state errors since the dynamics are then driven by the velocity.

As an alternative to deal with this problem we define adaptive and integral control schemes based on the power-based modeling framework. The following section first introduces the concept of power-based modeling and power-shaping.

### III. BRAYTON-MOSER EQUATIONS AND POWER-SHAPING

This section introduces the concept of power-based modeling of standard mechanical systems, as presented in [8]. We then show the notion of power-shaping, introduced in [15] for RLC networks, applied for stabilization of mechanical systems.

#### A. Power-based description of standard mechanical systems

The BM (Brayton-Moser) equations, as introduced for nonlinear electrical RLC networks, take the special gradient form

$$Q(x) \dot{x} = \frac{\partial P}{\partial x}(x) + B(x)u \quad (16)$$

with  $x \in \mathbb{R}^n$  the vector of system states,  $Q(x)$  a symmetric matrix,  $B(x) \in \mathbb{R}^{n \times m}$  the input matrix with  $m \leq n$  and  $P(x)$  the mixed-potential function (which has the units of power). A practical advantage of the BM equations for electrical systems is that they describe the dynamics of a

system in terms of "easily" measurable quantities [7]. That is, inductor currents and capacitor voltages, instead of fluxes and charges as normally used in PH electrical systems.

In [8] it is shown how the PH system described by (2) can be described by BM equations. The standard mechanical system (2) can be described in the form of (16) by  $x = (q, p)^\top$ , the matrix

$$Q(q, p) = \begin{bmatrix} \frac{\partial^2 V}{\partial q^2} + \frac{1}{2} \frac{\partial^2 (p^\top M^{-1}(q)p)}{\partial q^2} & -\frac{\partial p^\top M^{-1}(q)}{\partial q} \\ \frac{\partial M^{-1}(q)p}{\partial q} & -M^{-1}(q) \end{bmatrix} \quad (17)$$

input matrix

$$B = \begin{bmatrix} 0 \\ -M^{-1}(q)G \end{bmatrix} \quad (18)$$

and mixed-potential function

$$P(q, p) = \frac{\partial V^\top}{\partial q} M^{-1}(q)p + \frac{1}{2} \left( \frac{\partial p^\top M^{-1}(q)p}{\partial q} \right)^\top M^{-1}(q)p + \frac{1}{2} p^\top M^{-1}(q) D M^{-1}(q)p \quad (19)$$

which has the units of power.

As described in [1], [9] stability of a BM system is proven by finding an alternative pair  $\tilde{Q}(x)$  and  $\tilde{P}(x)$ , which equivalently characterize the system (16) and where  $\tilde{P}(x)$  can be used as a candidate Lyapunov function. The generation of candidate Lyapunov functions is based on the following theorem

**Lemma 1 ([1]):** For any arbitrary constant  $\lambda$  and any constant symmetric matrix  $K$ , the pair

$$\tilde{Q}(x) = \lambda Q(x) + \frac{\partial^2 P}{\partial x^2}(x) K Q(x) \quad (20)$$

$$\tilde{P}(x) = \lambda P(x) + \frac{1}{2} \left( \frac{\partial P}{\partial x}(x) \right)^\top K \frac{\partial P}{\partial x}(x) \quad (21)$$

equivalently characterizes the dynamics (16).  $\triangleleft$

### B. Stabilization via power-shaping

For stabilization control we want to define an input  $u$  which shapes the mixed-potential function  $P(x)$  into a function of desired form,  $P_d(x)$ . In [7] it is shown how a general system of the form (16) can be asymptotically stabilized by power-shaping. This power-shaping approach can be related to the energy-shaping approach in which the Hamiltonian  $H(x)$  of a system is shaped into a desired Hamiltonian  $H_d(x)$ . We now describe a similar approach for standard mechanical systems, however, the difference with [7] is that for a mechanical system the matrix (17) also changes. Assume that:

**A. 1:** There exists a scalar function  $P_a(x)$  such that:

- $B^\perp(x) \frac{\partial P_a}{\partial x} = 0$ , where  $B^\perp(x)$  is a full-rank left annihilator of  $B(x)$ , i.e.,  $B^\perp(x)B(x) = 0$ .
- $\frac{\partial P_d}{\partial x}(x_d) = 0$ , with  $P_d(x) = P(x) + P_a(x)$  and  $x_d$  a minimum of  $P_d(x)$ .  $\triangleleft$

Define the input signal

$$u = (B^\top(x)B(x))^{-1} B^\top(x) \frac{\partial P_a}{\partial x} \quad (22)$$

which for a mechanical system in the form of (16) results in

$$Q_d(x)\dot{x} = \frac{\partial P_d}{\partial x} \quad (23)$$

with  $Q_d(x)$  a symmetric matrix. For mechanical systems the matrix  $Q(x)$  and the mixed-potential function  $P(x)$  are defined in terms of the kinetic and potential energy. For that reason shaping the power of the system means shaping the energy of the system, changing the matrix  $Q(x)$  too.

Assume that we have a fully actuated mechanical system with no friction, i.e.,  $G = I$  and  $D = 0$ . Potential energy shaping and damping injection comes down to having the desired mixed-potential function

$$P_d(q, p) = \frac{\partial V_d^\top}{\partial q} M^{-1}(q)p + \frac{1}{2} \left( \frac{\partial p^\top M^{-1}(q)p}{\partial q} \right)^\top M^{-1}(q)p + \frac{1}{2} p^\top M^{-1}(q) K_d M^{-1}(q)p \quad (24)$$

with  $V_d(q)$  as in (5). Satisfying assumption A.1 and since  $P_a = P_d - P$  we have

$$-M^{-1}(q)u = \frac{\partial P_a}{\partial p} \quad (25)$$

$$u = \frac{\partial V}{\partial q} - \frac{\partial V_d}{\partial q} - K_d M^{-1}(q)p \quad (26)$$

where it is known that  $M^{-1}(q)p = \dot{q}$ . We then have the same potential energy-shaping and damping injection input shown in (4). Assume for simplicity that we also have a constant mass matrix  $M(q) = M$ . We now have the BM mechanical system (23) with potential function as in (24) and

$$Q_d(q, p) = \begin{bmatrix} \frac{\partial^2 V_d}{\partial q^2} & 0 \\ 0 & -M^{-1} \end{bmatrix} \quad (27)$$

From energy-shaping control we know that potential energy shaping and damping injection results in an asymptotically stable system. We now apply lemma 1 to show this result for the BM mechanical system, which means finding a matrix  $\tilde{Q}_d$

$$\tilde{Q}_d(x) = \lambda Q_d(x) + \frac{\partial^2 P_d}{\partial x^2}(x) K Q_d(x) \quad (28)$$

such that  $\tilde{Q}_d(x) + \tilde{Q}_d^\top(x) < 0$  and a function

$$\tilde{P}_d(x) = \lambda P_d(x) + \frac{1}{2} \left( \frac{\partial P_d}{\partial x}(x) \right)^\top K \frac{\partial P_d}{\partial x}(x) \quad (29)$$

Take  $\lambda = -1$  and

$$K = \begin{bmatrix} 0 & 0 \\ 0 & M^2 \end{bmatrix} \quad (30)$$

We then have that

$$\tilde{Q}_d = \begin{bmatrix} -\frac{\partial^2 V_d}{\partial q^2} & -\frac{\partial^2 V_d}{\partial q^2} \\ 0 & M^{-1}(I - K_d) \end{bmatrix} \quad (31)$$

and  $\tilde{Q}_d + \tilde{Q}_d^\top < 0$  for a sufficiently large  $K_d$  and by choosing  $V_d$  such that  $\frac{\partial^2 V_d}{\partial q^2}$  is positive definite. It can be easily verified that

$$\frac{d\tilde{P}_d}{dt} = \frac{1}{2} \dot{x}^\top \left( \tilde{Q}_d(x) + \tilde{Q}_d^\top(x) \right) \dot{x} \quad (32)$$

$$\leq 0 \quad (33)$$

which then means that  $\dot{q}, \dot{p} \rightarrow 0$  as time  $t \rightarrow \infty$ . The function  $\tilde{P}_d(x)$  is bounded from below and LaSalle's invariance principle [10] can be used to prove asymptotic stability in  $\bar{q} = 0$ . If the function  $\tilde{P}_d$  is radially unbounded, asymptotic stability is global.

A similar approach can be taken to show asymptotic stability of systems with a non-constant matrix  $M(q)$ .

#### IV. INTEGRAL AND ADAPTIVE CONTROL OF BRAYTON-MOSER MECHANICAL SYSTEMS

##### A. Integral control

In the previous section we showed the idea of potential energy shaping and damping injection via power-shaping in the BM framework. However, potential energy shaping is usually realized by canceling the potential energy terms of the original system and replacing them by a desired function. A mismatch between real potential energy and the potential energy used for control can cause steady-state errors. It is well known that an integrator can compensate for steady-state errors caused by unknown disturbances or model uncertainties. The results of [7] can be extended to realize an integral control scheme for stabilization of standard mechanical systems, where  $\tau$  denotes the integrator state.

**Theorem 1:** Consider a standard mechanical system with a constant input disturbance  $d$ , described in BM form by

$$Q(x)\dot{x} = \frac{\partial P}{\partial x} + B(x)(u - d) \quad (34)$$

Assume that assumption A.1 holds and that

A. 2: There exists a constant  $\lambda$  and a matrix  $K$  in (28) such that  $\tilde{Q}_d(x) + \tilde{Q}_d^\top(x) < 0$ .

A. 3: The largest invariant set contained in the set

$$\{x \in \mathbb{R}^n \mid \dot{x}^\top \tilde{Q}_d(x) \dot{x} = 0\}$$

equals  $\{x_d\}$ .

Then, the power-shaping plus integral control input

$$u = (B^\top(x)B(x))^{-1} B^\top(x) \left( \frac{\partial P_a}{\partial x}(x) + \tau \right) \quad (35)$$

and integrator dynamics

$$\dot{\tau} = -K_i B^\top(x) Q_d^{-1}(x) \frac{\partial P_d}{\partial x}(x) \quad (36)$$

with  $K_i$  a constant, positive definite, diagonal matrix, asymptotically stabilizes the system (34) in the point  $x_d$  with domain of attraction given by the set

$$\{(x, \bar{\tau}) \in \mathbb{R}^{n+k} \mid \tilde{P}_i(x, \bar{\tau}) \leq c_1\} \quad (37)$$

with  $\bar{\tau} = \tau - d$ ,

$$\tilde{P}_i(x, \bar{\tau}) = \tilde{P}_d(x) + \bar{\tau}^\top K_i^{-1} \bar{\tau} \quad (38)$$

$\tilde{P}_d(x)$  as in (29) and constant  $c_1 > 0$ .

**Proof.** The control input (35) with integrator dynamics (36) results in the closed-loop system<sup>1</sup>

$$\begin{bmatrix} Q_d & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} I & BK_i \\ -K_i B^\top Q_d^{-1} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial P_i}{\partial x} \\ \frac{\partial P_i}{\partial \tau} \end{bmatrix} \quad (39)$$

with

$$P_i(x) = P_d(x) + \frac{1}{2} \bar{\tau}^\top K_i^{-1} \bar{\tau} \quad (40)$$

This closed-loop system can be rewritten in the form

$$\begin{bmatrix} \dot{x} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} Q_d^{-1} & Q_d^{-1} BK_i \\ -K_i B^\top Q_d^{-1} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial P_i}{\partial x} \\ \frac{\partial P_i}{\partial \tau} \end{bmatrix} \quad (41)$$

Take (38) as Lyapunov candidate function. It can be verified, with assumption A.2, that

$$\frac{d\tilde{P}_i}{dt} = \frac{d\tilde{P}_d}{dt} \quad (42)$$

$$\leq 0 \quad (43)$$

Asymptotic stability of  $x$  then follows by invoking LaSalle's invariance principle together with assumption A.3, which also implies that  $\bar{\tau}$  converges to zero.  $\square$

**Remark 1:** From (42) it can be seen that the power-balance obtained by stabilization via power-shaping, inequality (33) in the previous section, is preserved when adding integral control as described above.  $\triangleleft$

**Remark 2:** It can be noticed that for a mechanical system the original matrix  $Q(x)$  may not be invertible, i.e., systems with no potential energy. However, the power-shaping approach as shown above can assign a (virtual) potential energy function such that the matrix  $Q_d(x)$  of the closed-loop system becomes invertible.  $\triangleleft$

For illustration purposes, take a fully actuated mechanical system with a constant matrix  $M$  and no friction. Application of theorem 1 gives

$$u = \frac{\partial V}{\partial q}(q) - K_p \bar{q} - K_d \dot{q} + \tau \quad (44)$$

and the integrator dynamics (36)

$$\dot{\tau} = -K_i M^{-1} (K_p \bar{q} + K_d M^{-1} p) \quad (45)$$

In the previous section it was already shown that a matrix  $\tilde{Q}_d$  exists which proves that  $\dot{q}, \dot{p} \rightarrow 0$  as time  $t \rightarrow \infty$ . We then know that  $q$  becomes constant and since  $\dot{q} = M^{-1} p$  we know that  $p \rightarrow 0$  too. The dynamics then reduce to

$$\dot{p} = K_p \bar{q} + \tau \quad (46)$$

$$= 0 \quad (47)$$

as  $t \rightarrow \infty$ , showing that  $\tau$  has to be constant too. Assumption A.3 can be verified by noting that, if  $q$  is constant in another

<sup>1</sup>Since  $d$  is constant,  $\dot{\tau} = \dot{\tau}$ .

point other than the desired equilibrium point,  $\bar{q} \neq 0$  and from (45) we know that  $\dot{\tau} \neq 0$ . A non-constant  $\tau$  contradicts the fact that  $\dot{p} \equiv 0$ . This shows that the largest invariant set where  $\frac{d\tilde{P}_d}{dt} = 0$  is the set containing  $\bar{q} = 0, p = 0$ , and we have a zero steady-state error.

A similar approach can be taken for integral control of mechanical systems with a non-constant matrix  $M(q)$  and/or systems with friction  $D \geq 0$ . It can be seen, however, that the integrator dynamics (36) become more complicated since kinetic and dissipation energy terms have to be included too. The matrix  $Q(x)$  also becomes more complex. However, the method can still be applied for such systems, which are more complex.

### B. Adaptive control

In the previous subsection we extended results of power-shaping by adding an integrator to the system. In the special case that we only have parameter uncertainty also adaptive control can be applied to compensate for errors caused by using uncertain parameter values in the control input. Adaptive control, compared to integral control, becomes interesting when parameter estimation is desired. Theorem 1 will now be modified to define an adaptive control theorem.

**Theorem 2:** Consider a standard mechanical system of the form (16) and assume that assumptions A.1-A.3 hold. Assume furthermore that

A. 4: We can write

$$\frac{\partial P_a}{\partial x}(x) = \alpha(x) + \Phi(x)z \quad (48)$$

where  $\alpha(x)$  is a known vector function,  $\Phi(x)$  a matrix of known functions and  $z$  the vector of unknown system parameters (as before).

Denote the estimation of  $z$  by  $\zeta$ . Then the power-shaping and adaptive control input

$$u = (B^\top(x)B(x))^{-1} B^\top(x) (\alpha(x) + \Phi(x)\zeta) \quad (49)$$

with update law

$$\dot{\zeta} = -K_z \Phi(x) B^\top(x) Q_d^{-1}(x) \frac{\partial P_d}{\partial x}(x) \quad (50)$$

and  $K_z$  a positive definite diagonal matrix asymptotically stabilizes the system (16) in the point  $x_d$  with domain of attraction given by the set

$$\{(x, \tilde{z}) \in \mathbb{R}^{n+m} | \tilde{P}_z(x) \leq c_2\} \quad (51)$$

with  $\tilde{z} = \zeta - z$ ,

$$\tilde{P}_z(x, \tilde{z}) = \tilde{P}_d(x) + \frac{1}{2} \tilde{z}^\top K_z^{-1} \tilde{z} \quad (52)$$

$\tilde{P}_d(x)$  as in (29) and constant  $c_2 > 0$ .

**Proof.** The proof follows the same steps as in the proof of theorem 1. The only difference is that the closed-loop system is now described by

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} Q_d^{-1} & Q_d^{-1} B \Phi K_z \\ -K_z \Phi^\top B^\top Q_d^{-1} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial P_z}{\partial x} \\ \frac{\partial P_z}{\partial \tilde{z}} \end{bmatrix} \quad (53)$$

with  $\tilde{z} = \zeta - z$  and mixed-potential function

$$P_z(x, \tilde{z}) = P_d(x) + \frac{1}{2} \tilde{z}^\top K_z^{-1} \tilde{z} \quad (54)$$

The same  $\tilde{Q}_d(x)$  and  $\tilde{P}_d(x)$  as before can be used, with (52) as Lyapunov candidate function.  $\square$

However, for adaptive control we have the requirement that the matrix  $\Phi(x)$  has to be known. This means that only parameter values can be uncertain. Furthermore, the uncertainties should not impede the computation of (50). Take again the standard mechanical system with constant matrix  $M$  and no friction ( $D = 0$ ), then

$$P_a = \left( \frac{\partial V_d}{\partial q} - \frac{\partial V}{\partial q} \right)^\top M^{-1} p + \frac{1}{2} p^\top M^{-1} K_d M^{-1} p \quad (55)$$

with  $V_d(q)$  as in (5). The shaped potential function becomes

$$P_d(q, p) = \frac{\partial V_d}{\partial q}^\top M^{-1} p + \frac{1}{2} p^\top M^{-1} K_d M^{-1} p \quad (56)$$

and we have that the update law (50) becomes

$$\dot{\zeta} = -K_z \Phi(q)^\top M^{-1} (K_p \bar{q} + K_d M^{-1} p) \quad (57)$$

It is possible to replace  $M^{-1} p$  by  $\dot{q}$ , since this is what is actually measured. However, we will still have that the update law depends on  $M$ . This can complicate the application of the adaptive scheme when there is parameter uncertainty in this matrix. In the case of a constant  $M$  matrix there are cases where this is not an issue. Denote  $M_0$  as the (nominal) matrix used for computation of the update law. Instead of (57) we actually have

$$\dot{\zeta} = -K_z \Phi^\top(q) M_0^{-1} (K_p \bar{q} + K_d \dot{q}) \quad (58)$$

In the special case that  $\Phi(q)$  is a diagonal matrix, of equal size as  $M$ , we can write

$$K_z \Phi^\top(q) M_0^{-1} = \Phi^\top(q) K_z M_0^{-1} \quad (59)$$

which means that a constant, positive definite, matrix  $\tilde{K}_z$  exists such that

$$\Phi^\top(q) K_z M_0^{-1} = \Phi^\top(q) \tilde{K}_z M^{-1} \quad (60)$$

We can then say that the update law (58) satisfies (50), however, with a different matrix for the adaptation gains,  $\tilde{K}_z$ . In other words, applying the matrix  $M_0$  with  $K_z$  is equal to applying  $M$  with  $\tilde{K}_z$  (different adaptation gains than originally intended). The simple example in the next section illustrates this. Notice that such problem does not occur when the uncertainty is only present in the potential energy  $V(q)$ . For analysis,  $K_z$  in (53), (54) and (52) can then be replaced by  $\tilde{K}_z$ .

For systems with a non-constant matrix  $M(q)$  the update law (50) also depends on kinetic energy terms. The update law then has more terms depending on  $M(q)$ , impeding its application when there is uncertainty in this matrix.

## V. EXAMPLE

Adaptive stabilization control, as presented in the previous section, is applied on a simple nonlinear system, i.e., a single pendulum. Assume that the pendulum has a massless rod of length  $l$  with a mass  $m$  attached at the end. In the example we have the angle  $q = \theta$  and  $u$  is the input torque. The system can be described by

$$M = ml^2, \quad V(q) = mgl(1 - \cos(q)) \quad (61)$$

with  $g$  the gravitational constant and input matrix  $G = 1$ . This system has a stable equilibrium, the hanging position, and an unstable equilibrium, the upward position. Assume that we want to asymptotically stabilize this system at the angle  $q_d = \frac{\pi}{2}$  rad. Assume also that  $m$  is unknown and a nominal value is used for control,  $m_0 \neq m$ . The power-shaping adaptive control input (49) is given by

$$u = m_0gl \sin(q) - k_p \bar{q} - k_d \dot{q} + gl \sin(q) \zeta \quad (62)$$

where  $\bar{q} = q - q_d$  and the adaptation law (50) takes the form

$$\dot{\zeta} = -\frac{k_z g}{m_0 l} \sin(q) (k_p \bar{q} + k_d \dot{q}) \quad (63)$$

with  $k_p, k_d, k_z$  positive constants. Figure 1 shows simulation results for this example with  $m = 1, l = 1, g = 9.81, m_0 = 1.5$  and control gains  $k_p = 20, k_d = 5, k_z = 0.02$ . The figure

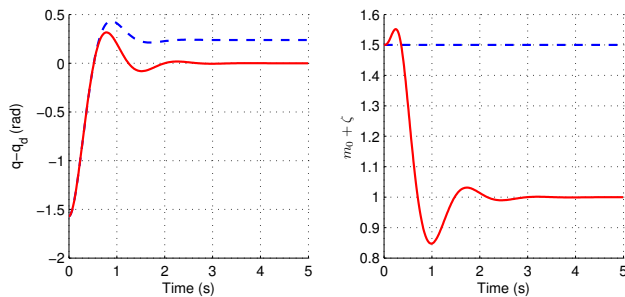


Fig. 1. Trajectories for the pendulum system. Dashed lines: results without adaptive control. Solid lines: results with power-based adaptive control. Initial conditions:  $[q(0) \ p(0) \ \zeta(0)] = [0 \ 0 \ 0]$ .

shows how the adaptive part compensates for the steady-state error in  $\bar{q} = q - q_d$  caused by  $m_0 \neq m$  (dashed line). The figure also shows how  $m_0 + \zeta \rightarrow m$ .

## VI. CONCLUDING REMARKS

Power-based control was applied to be able to define either adaptive control dynamics or integrator dynamics based on position error measurements. Contrary to with PH systems, we preserve the power-balance for the closed-loop system (remark 1). The theories used for generation of Lyapunov functions to prove stability of a Brayton-Moser system can then still be applied for the closed-loop system. The disadvantage, however, is that the update law and integrator dynamics become more complex when the system has a coordinate dependent matrix  $M$  and friction. The reason is that the mixed-potential function for mechanical systems

depends on the kinetic, potential and dissipation energy. The more complex the system is, the more complex the mixed-potential function. The result is more complex adaptive or integrator dynamics. In short, the power-based approach can be applied to more complex systems (e.g. robotic manipulators) but the increased complexity implies more complex adaptive or integrator dynamics. Nevertheless, the dynamics can now depend on position measurements and the original power-balance is preserved. Compared to integral control, adaptive control estimates the values of uncertain parameters. The dynamics are also different, based on how the uncertain parameters influence the system.

The adaptive control scheme was applied on a simple example where parameter uncertainty caused a steady-state error. Simulation results showed how the position error converged to zero and that the estimation of the parameter error converged to the real (unknown) parameter error.

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