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Published in:
Proceedings of the 2012 American Control Conference

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2012

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Dirksz, D. A., Scherpen, J. M. A., van der Schaft, A. J., & Steinbuch, M. (2012). Notch filters for port-Hamiltonian systems. In *Proceedings of the 2012 American Control Conference* (pp. 238-243). University of Groningen, Research Institute of Technology and Management.

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2012 American Control Conference
Fairmont Queen Elizabeth, Montréal, Canada
June 27-June 29, 2012

Notch filters for port-Hamiltonian systems

D.A. Dirksz, J.M.A. Scherpen, A.J. van der Schaft and M. Steinbuch

Abstract— **In this paper a standard notch filter is modeled in the port-Hamiltonian framework. By having such a port-Hamiltonian description it is proven that the notch filter is a passive system. The notch filter can then be interconnected with another (nonlinear) port-Hamiltonian system, while preserving the overall passivity property. By doing so we can combine a frequency-based control method, the notch filter, with the nonlinear control methodology of passivity-based control.**

I. Introduction

Network modeling of lumped-parameter physical systems naturally leads to a geometrically defined class of systems, i.e., port-Hamiltonian (PH) systems [14], [20]. The PH modeling framework describes a large class of (nonlinear) systems including passive mechanical systems, electrical systems, electromechanical systems, mechanical systems with nonholonomic constraints, thermal systems and distributed parameter systems with boundary control [15]. The popularity of PH systems can be largely accredited to its application for analysis and control design of multi-input multi-output (MIMO) physical systems, as shown in [1], [4], [5], [15], [16], [17], [20] and many others. Control can be realized by energy-shaping and damping injection. The Hamiltonian (often the total energy of the system) is shaped into a new function with desired equilibrium while damping injection influences the convergence to the desired equilibrium point.

Nonlinear modeling and control methods, including the PH framework, have the great disadvantage of not including any frequency information in the modeling and control design. In practice, the frequency-based control methods are preferred because of the importance of the system behavior in the frequency domain. In the frequency domain transfer functions, Nyquist/Bode plots, PID controllers, lag-lead compensators and filters are some examples of powerful tools for analysis and control [3], [12]. However, such methods are only theoretically justified for linear systems. Many physical systems are actually nonlinear, for which linearization is then first required. However, the results only hold locally, (very) close to the linearization point. On the other hand, nonlinear control often offers methods to steer a nonlinear system to a desired state for any initial condition. When global convergence cannot be proven, techniques exist to define a region of attraction [10], [13]. With the modern technological advances systems are becoming more complex, with higher performance demands. Linearization alone for control design is not always enough. However, as mentioned before, behavior and performance in the frequency domain is often ignored. The literature on including some frequency methods into nonlinear control is rather limited. For some nonlinear systems the *describing function analysis* [21] method can be used to approximately analyze and predict nonlinear behavior. In [9] a dissipativity framework is presented for power system stabilizer design. The framework is based on including frequency dependent weights into the dissipation inequality. Frequency can then be included into the study of internal stability and disturbance attenuation of power systems. [19] provides a frequency domain perspective on feedforward friction compensation of nonlinear single-input single-output (SISO) systems.

This paper describes a standard notch filter in the port-Hamiltonian form. In control applications a notch filter is often used for control of systems with lightly damped flexible modes (resonance frequencies) [3], [12]. A notch filter is a filter that passes signals of all frequencies except those in a stop band centered around a center frequency. Furthermore, by a specific choice for the notch parameters it is possible to realize an *inverse* notch filter. An inverse notch filter simply does the opposite of a notch filter, it only passes signals which are centered around a center frequency. Inverse notch filters are usually used for disturbance attenuation since high feedback gain is applied at the disturbance frequency [22].

The main contribution of this paper is to include frequency filtering into nonlinear control design by describing a standard notch filter in the PH framework. In a similar way we also present a PH description of the inverse notch filter. By having the notch filter in PH form we can interconnect it, in a passive way, with another PH system without losing global passivity and without losing the PH structure. We can then combine the nonlinear control methodology of passivity based-control (PBC) [13], [20] with a notch filter, which is tuned to reduce (or increase in the case of an inverse notch) the feedback gains at a specific frequency. Compared to [9] we deal with a larger class of systems and we describe the dynamics of the notch filter which is tuned to have a specific frequency behavior. In [8] the internal model principle [2] is generalized to nonlinear systems. Compared to [8], we deal with the specific class of PH systems, for which the structure implies global results. Furthermore, we only need to know the resonance and/or disturbance frequencies, i.e, a model of an exosystem is not required.

Section II briefly summarizes the PH modeling framework

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978-1-4577-1096-4/12/$26.00 ©2012 AACC

and PBC. In section III we present the PH models for both a notch filter and the inverse notch filter. An inverse notch filter is simply a normal notch filter with a different choice of parameters. However, in the PH framework this is not so straightforward. For this reason a different PH model is presented for the inverse notch filter. Furthermore, the disturbance attenuation properties of the inverse notch filter for nonlinear systems are also analyzed. In section IV PBC is applied for control of a nonlinear mechanical system, while the inverse notch filter is used for disturbance attenuation. Final remarks are then given in section V.

II. Port-Hamiltonian Modeling and Control

In the PH framework [14] a general (time-invariant) system is described by

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g(x)^\top \frac{\partial H}{\partial x}(x) \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$, $J(x) \in \mathbb{R}^{n\times n}$ the skew-symmetric interconnection matrix, $R(x) \in \mathbb{R}^{n\times n}$ the symmetric, positive-semidefinite, damping matrix, the Hamiltonian $H(x)$, input u and output y, with $u, y \in \mathbb{R}^m$, $m \leq n$. Systems described in the form of (1) satisfy the energy-balance

$$\begin{aligned} \frac{dH}{dt} &= u^\top y - \frac{\partial H}{\partial x}^\top (x) R(x) \frac{\partial H}{\partial x}(x) \\ &\leq u^\top y \end{aligned} \tag{2}$$

and are called *passive* systems [20], [23]. Passive systems are a class of dynamical systems in which the rate at which energy flows into the system is not less than the increase in storage. The passivity property holds for a large class of physical systems. Otherwise, PBC can be applied to make a system passive with respect to a storage function which has the desired equilibrium point. PBC is a control methodology for nonlinear MIMO systems that achieves stabilization of a system by passivation of the closed-loop dynamics. Briefly summarized, by PBC a smooth state-feedback law

$$u = \alpha(x) + v \tag{3}$$

is found which transforms (1) into

$$\begin{aligned} \dot{x} &= [J_d(x) - R_d(x)] \frac{\partial H_d}{\partial x}(x) + g(x)v \\ \bar{y} &= g(x)^\top \frac{\partial H_d}{\partial x}(x) \end{aligned} \tag{4}$$

with $H_d(x)$ the new Hamiltonian (storage function) having strict minimum at the desired equilibrium point x_d. Notice in (4) that besides shaping the Hamiltonian it is also possible to change (however not always necessary) the interconnection and damping matrices, from $J(x)$ into $J_d(x)$ and $R(x)$ into $R_d(x)$ respectively. The function $H_d(x)$ is characterized by the partial differential equation (PDE)

$$g^\perp(x) \left([J_d(x) - R_d(x)] \frac{\partial H_d}{\partial x}(x) - [J(x) - R(x)] \frac{\partial H}{\partial x}(x) \right) = 0 \tag{5}$$

where $g^\perp(x)$ is a full rank left annihilator of $g(x)$, i.e., $g^\perp(x)g(x) = 0$. When $R_d = 0$ and system (4) is zero-state detectable the new input $v = -C(x)\bar{y}$, with $C(x)$ a positive definite matrix, realizes asymptotic stability of (4). For more cases of PBC we refer to e.g. [4], [7], [13], [16], [17], [20].

III. Notch Filter

In this section we present the PH models for the notch and inverse notch filters.

A. Port-Hamiltonian modeling of the notch filter

A notch filter is in general described by the transfer function

$$T(s) = \frac{s^2 + 2\beta_1\omega_0 s + \omega_0^2}{s^2 + 2\beta_2\omega_0 s + \omega_0^2} \tag{6}$$

where $\beta_1 > 0, \beta_2 > 0$ are the constant notch parameters which determine the depth and width of the notch and ω_0 is the center frequency. Figure 1 shows the Bode plot for $\omega_0 = 10$ rad/s and two different combinations for β_1 and β_2.

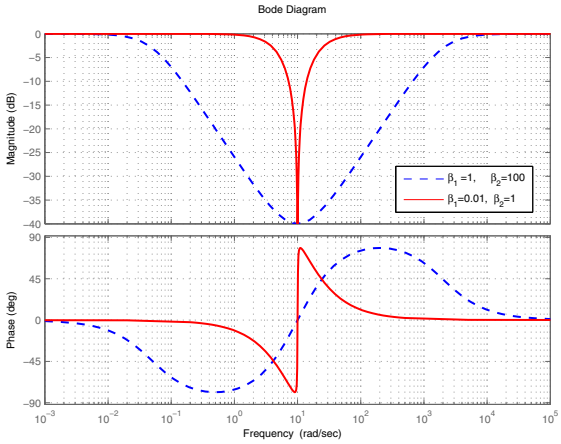

Fig. 1. Bode plots for $\omega_0 = 10$ rad/s.

The notch filter (6) can be described in state-space form by

$$\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} -2\beta_2\omega_0 & -\omega_0^2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_z \\ y_z &= \begin{bmatrix} 2\omega_0(\beta_1 - \beta_2) & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + u_z \end{aligned} \tag{7}$$

where $z = (z_1, z_2)^\top$ are the system states, u_z the notch filter input and y_z the notch filter output. Notice that the notch filter is a system with feedthrough.

In [1], [6], [11] a more general description of PH systems is given which includes a direct feedthrough channel. A PH (time-invariant) system with feedthrough is described in the PH framework by

$$\begin{aligned} \dot{z} &= [J_z(z) - R_z(z)] \frac{\partial H_z}{\partial z}(z) + [g_z(z) - P_z(z)] u_z \\ y_z &= [g_z(z) + P_z(z)]^\top \frac{\partial H_z}{\partial z}(z) + [M_z(z) + S_z(z)] u_z \end{aligned} \tag{8}$$

with state vector $z \in \mathbb{R}^k$ and similar to (1) the skew-symmetric interconnection matrix $J_z(z)$, damping matrix $R_z(z)$, Hamiltonian $H_z(z)$, input vector u_z and output vector y_z. Additionally, a PH system with feedthrough has a skew-symmetric matrix M_z, positive semidefinite matrix $S_z(z)$

and matrix $P_z(z)$. As shown in [1], [11], a PH system with feedthrough (8) has the power-balance equation given by

$$\dot{H}_z = u_z^\top y_z - \begin{bmatrix} \frac{\partial H_z}{\partial z} \\ u_z \end{bmatrix}^\top \begin{bmatrix} R_z(z) & P_z(z) \\ P_z^\top(z) & S_z(z) \end{bmatrix} \begin{bmatrix} \frac{\partial H_z}{\partial z} \\ u_z \end{bmatrix} \tag{9}$$

The condition for passivity is then given by

$$\begin{bmatrix} R_z(z) & P_z(z) \\ P_z^\top(z) & S_z(z) \end{bmatrix} \geq 0 \tag{10}$$

for all z and admissible inputs u_z. Notice that $R_z(z), S_z(z)$ and $P_z(z)$ determine a kind of dissipation structure of (8). The notch filter (7) with $z = (z_1, z_2)^\top$ can be described in the PH form (8) by choosing the Hamiltonian to be

$$H_z(z) = \frac{\omega_0}{4(\beta_2 - \beta_1)} z_1^2 + (\beta_2 - \beta_1)\omega_0 z_2^2 \tag{11}$$

and

$$J_z = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},\ R_z = \begin{bmatrix} 4\beta_2(\beta_2 - \beta_1) & 0 \\ 0 & 0 \end{bmatrix},$$

$$g_z = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ P_z = \begin{bmatrix} 2(\beta_1 - \beta_2) \\ 0 \end{bmatrix},\ M_z = 0,\ S_z = 1 \tag{12}$$

For a standard notch filter $\beta_2 > \beta_1$. All the principal minors of the matrix in (10) can then be verified to be non-negative. Non-negativity of all the principal minors implies that (10) always holds and proves passivity of the notch filter.

***Remark** 1:* The choices for the PH parameters of the notch filter are such that the interconnection matrix J_z agrees with the interconnection matrix for standard mechanical systems. The interconnection structure for standard mechanical systems [20] is described by

$$J = \begin{bmatrix} 0 & -I_j \\ I_j & 0 \end{bmatrix} \tag{13}$$

where j are the degrees of freedom. The same interconnection matrix as for standard mechanical systems provides a physical interpretation for the PH notch filter. ◁

B. Port-Hamiltonian modeling of the inverse notch filter

In section I we also mentioned another kind of notch filter, namely the inverse notch filter. The inverse notch filter is actually the standard notch filter (6) but with $\beta_1 > \beta_2$. The result is a filter that only passes and amplifies signals containing the center frequency ω_0. However, when $\beta_1 > \beta_2$ the PH formulation of the notch filter described above by (11) and (12) is not anymore adequate. Since $\beta_1 > \beta_2$ the Hamiltonian (11) becomes negative definite and the matrix R negative semidefinite. In other words, while for (6) and (7) the choice of the notch parameters has no implications for the system description, the PH formulation of a notch filter is not anymore PH when $\beta_1 > \beta_2$. For the inverse notch filter we then need a different PH formulation, i.e, take

$$J_z = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},\ R_z = \begin{bmatrix} 4\beta_2(\beta_1 - \beta_2) & 0 \\ 0 & 0 \end{bmatrix},$$

$$g_z = \begin{bmatrix} 2(\beta_1 - \beta_2) \\ 0 \end{bmatrix},\ P_z = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ M_z = 0,\ S_z = 1 \tag{14}$$

and Hamiltonian

$$H_z(z) = \frac{\omega_0}{4(\beta_1 - \beta_2)} z_1^2 + (\beta_1 - \beta_2)\omega_0 z_2^2 \tag{15}$$

The most important difference with the standard notch filter is that for the inverse notch filter P_z becomes the zero vector. It is then easier to see that (10) is satisfied, since the matrix in (10) becomes diagonal and $\beta_1 > \beta_2$.

The most important and interesting reason for having a PH description is the passivity property. It is known that the interconnection of two passive systems is again passive [20], [23]. Consider two passive systems, system $\sum_a$ with input u_a and output y_a, and system $\sum_b$ with input u_b with input u_b and output y_b. Figure 2 shows the standard passive interconnection between system $\sum_a$ and system $\sum_b$, where r_a and r_b are new input signals. From the theory of

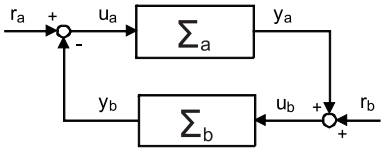

Fig. 2. Passive interconnection of two passive systems.

dissipative systems the closed-loop system shown in figure 2 with inputs (r_a, r_b) and outputs (y_a, y_b) is also passive.

By having a passive description of a notch filter we can now interconnect it with a nonlinear passive system and/or combine it with PBC, without losing passivity. We are then able to also include frequency requirements to nonlinear control and still preserve the global properties and advantages of the nonlinear control method.

C. Disturbance attenuation of the inverse notch filter

Consider two PH systems, a SISO plant $\sum_P$ in the form (1) with input u and output y and an inverse notch filter $\sum_N$ in the form of (8), with input u_z and output y_z. The parameters for the PH inverse notch are given by (14). A standard passive interconnection between plant and filter is realized by

$$u = v - K_d y_z \tag{16}$$

$$u_z = K_d y \tag{17}$$

with positive constant K_d and new input v. From the theory of dissipative systems [20], [23] the closed-loop system realized by (16) and (17) is passive, given by [1]

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} J - R - gK_d^2 g^\top & -gK_d g_z^\top \\ g_z K_d g^\top & J_z - R_z \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ \frac{\partial H_z}{\partial z} \end{bmatrix} + \begin{bmatrix} g \\ 0 \end{bmatrix} v$$

$$y = g^\top \frac{\partial H}{\partial x} \tag{18}$$

For comparison, when the plant (1) is controlled by the simple feedback

$$u = -K_d y + v \tag{19}$$

[1]For simplicity of notation we leave out the arguments of $J(x), R(x), g(x), J_z(z), R_z(z)$ and $g_z(z)$

and no inverse notch filter the closed-loop system becomes

$$\begin{aligned} \dot{x} &= \left[J(x) - R(x) - g(x)K_d g^\top(x)\right] \frac{\partial H}{\partial x}(x) + g(x)v \\ y &= g(x)^\top \frac{\partial H}{\partial x}(x) \end{aligned} \tag{20}$$

Compared to (20), in the closed-loop system (18) the inverse notch filter has increased the feedback gain from K_d to K_d^2.

To show the disturbance attenuation properties of the inverse notch filter we analyze the L_2-gain of the closed-loop system (18) with respect to an input disturbance v. For a general nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)v \\ y &= h(x) \end{aligned} \tag{21}$$

the L_2-gain bound $\gamma > 0$ is found if for γ there exists a smooth nonnegative solution $\mathcal{W}(x)$ to the Hamilton-Jacobi inequality (HJI) [20],

$$\frac{\partial \mathcal{W}}{\partial x}^\top f(x) + \frac{1}{2}\frac{1}{\gamma^2}\frac{\partial \mathcal{W}}{\partial x}^\top b(x)b^\top(x)\frac{\partial \mathcal{W}}{\partial x} + \frac{1}{2}h^\top(x)h(x) \leq 0 \tag{22}$$

Consider the closed-loop system (18) and function $\mathcal{W}(x) = H(x) + H_z(z)$. The HJI (22) for system (18) is then given by

$$\begin{aligned} -\frac{\partial H}{\partial x}^\top R(x)\frac{\partial H}{\partial x} - \frac{\partial H_z}{\partial z}^\top R_z(z)\frac{\partial H_z}{\partial z} - y^\top K_d^2 y \\ + \frac{1}{2}\frac{1}{\gamma^2}y^\top y + \frac{1}{2}y^\top y \leq 0 \end{aligned} \tag{23}$$

Since the first two terms of (23) are negative, the inequality (23) is satisfied when

$$-K_d^2 y^\top y + \frac{1}{2}\frac{1}{\gamma^2}y^\top y + \frac{1}{2}y^\top y \leq 0 \tag{24}$$

and so we have

$$\gamma^2 \geq \frac{1}{2K_d^2 - 1}, \qquad K_d > \sqrt{\frac{1}{2}} \tag{25}$$

In a similar way, the HJI (22) for system (20) with $\mathcal{W}(x) = H(x)$ yields

$$\gamma^2 \geq \frac{1}{2K_d - 1}, \qquad K_d > \frac{1}{2} \tag{26}$$

Notice that the bound on γ is smaller in (25) compared to the bound on γ in (26), where no inverse notch is applied. When γ is smaller the influence of an input disturbance on the system output is smaller. It shows that the inverse notch filter improves the disturbance attenuation. However, the analysis above does not immediately show the frequency properties of the inverse notch filter. Notice from (16) and (17) that the output of the plant enters the inverse notch filter and the (negative) output of the notch filter enters the plant. We know from frequency analysis that the inverse notch filter amplifies signals with a frequency around a specified center frequency ω_0, by the factor $\frac{\beta_1}{\beta_2}$. By choosing ω_0 equal (or close) to the disturbance frequency, the feedback gain is further increased (by the factor $\frac{\beta_1}{\beta_2}$) only when close to the disturbance frequency, which further improves the disturbance attenuation.

Analysis of the HJI (22) for system (20) can show that disturbance attenuation is also improved with the simple feedback (19), by choosing K_d high enough. Although a higher feedback gain improves disturbance attenuation, a very high gain for K_d gives a slow transient response, i.e., it results in an overdamped system. Even though our analysis of the HJI (24) only shows better disturbance rejection due to the presence of K_d^2 instead of K_d, the additional dissipation term given by $-\frac{\partial H_z}{\partial z}^\top R_z \frac{\partial H_z}{\partial z}$ includes the frequency information, and implies that the bound (25) is rather conservative. In fact, the inverse notch filter only applies the high gain when necessary, at the specific disturbance frequency, and avoids overdamping at other frequencies. In the next section we apply the inverse notch filter on a mechanical example and show the advantages with respect to transient performance.

***Remark** 2:* Compared to [8] we do not need to solve a PDE which depends on a model of the system generating the reference and/or trajectory which must be tracked and/or rejected (exosystem). Although we do not model an exosystem, we still need to know the specific resonance and/or disturbance frequencies in order to tune the (inverse) notch filter(s). This is in line with the results of [2], that control of a system can be achieved only if the control incorporates some representation of the reference and/or trajectory which must be tracked and/or rejected. ◁

To conclude this section, notice that the notch filter and inverse notch filter are described as SISO systems. However, it is straightforward to describe a PH system consisting of N notches. It is then possible to interconnect a MIMO plant with as many (inverse) notch filters as necessary. The only requirement is that the interconnection is passive, as shown in figure 2. This is illustrated in figure 3. For inverse notch

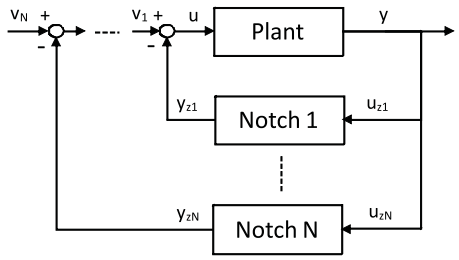

Fig. 3. Passive interconnection of a plant with N notches.

filters the disturbance attenuation can still be analyzed as presented above, however with K_d a positive definite matrix. Many physical systems, like our example in section IV, have several inputs and outputs. Additionally, each input can be influenced by disturbances of different frequencies, which is why it can be useful to describe a PH system consisting of more than one (inverse) notch filter.

IV. Application to standard mechanical systems

In this section we show an example of PBC combined with a PH inverse notch filter for control of a nonlinear mechanical system. The system is a robot manipulator with rigid joints under the influence of sinusoidal input disturbances. The manipulator system belongs to the class of standard mechanical systems, which are described in the PH form (1) by

$$\begin{aligned}\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} &= \begin{bmatrix}0 & I\\ -I & -D\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\end{bmatrix}+\begin{bmatrix}0\\ G\end{bmatrix}(u+\phi)\\ y &= G^\top \tfrac{\partial H}{\partial p}=\dot{q}\end{aligned} \tag{27}$$

with $q=(q_1,...,q_k)^\top$ the vector of generalized configuration coordinates, $p=(p_1,...,p_k)^\top$ the vector of generalized momenta, D the positive semidefinite damping matrix, I the identity matrix, G the input matrix (of rank $m\leq k$), u the input vector, ϕ the input disturbance vector and y the output vector. The Hamiltonian of the system is equal to the sum of kinetic and potential energy:

$$H(q,p)=\frac{1}{2}p^\top M^{-1}(q)p+V(q) \tag{28}$$

where $M(q)=M^\top(q)>0$ is the system mass-inertia matrix and $V(q)$ the potential energy.

Consider a rigid joint robot with 2 links and denote the length of link i by l_i, the angle of link i by θ_i, the distance from the joint to the center of gravity of the link i by r_i, the mass of link i by m_i and the inertia of link i by I_i. The control input is given by $u=(u_1,u_2)^\top$, with u_i the input torque of joint i and the input disturbance vector $\phi=(\phi_1,\phi_2)$. Describe the mass-inertia matrix of the robot by

$$M(q)=\begin{bmatrix}a_1+a_2+2b\cos\theta_2 & a_2+b\cos\theta_2\\ a_2+b\cos\theta_2 & a_2\end{bmatrix} \tag{29}$$

with constants

$$a_1=m_1r_1^2+m_2l_1^2+I_1,\ a_2=m_2r_2^2+I_2,\ b=m_2l_1r_2$$

The damping matrix is assumed to be $D=diag\{d_1,d_2\}$, with positive constants d_1,d_2. For simplicity we assume a constant damping, however non-constant damping can be also included in a PH model, see [7]. The robot in this example is described by coordinates $q=(\theta_1,\theta_2)^\top$, $p=M(q)\dot{q}$, input matrix $G=I$ and potential energy

$$V(q)=m_1gr_1\cos(\theta_1)+m_2g\Big(l_1\cos(\theta_1)+r_2\cos(\theta_1+\theta_2)\Big) \tag{30}$$

with $g=9.81$ m/s^2 the gravity constant. Assume the control input u is described by

$$u=a(q,p)+v \tag{31}$$

with $a(q,p)$ the energy-shaping control input vector for the system (27) when $\phi=0$ and $v=(v_1,v_2)^\top$ the new control input. In the PBC literature [13], [20] the control input

$$a=\frac{\partial V}{\partial q}(q)-K_p(q-q_d) \tag{32}$$

with K_p positive definite, shapes the potential energy from $V(q)$ into a function $V_d(q)$ with desired equilibrium point $q_d=(q_{1d},q_{2d})^\top$, i.e,

$$V_d(q)=\frac{1}{2}(q-q_d)^\top K_p(q-q_d) \tag{33}$$

Global asymptotic stability is then realized by damping injection, i.e.,

$$v = -K_dy \tag{34}$$

with K_d a positive definite matrix.

When there is input disturbance the manipulator can be interconnected in a passive way with the necessary numbers of PH inverse notch filters, in a similar way as shown in (16), (17) and shown in figure 3. In this simulation example we assume the disturbance $\phi_i=c_i\sin(\omega_it)$, with ω_i the disturbance frequency for joint i. For simplicity we take $c_1=c_2=2$ and $\omega_1=\omega_2=3$ rad/s. The manipulator parameters are shown in table IV. The parameters, except for the friction, describe an available experimental manipulator from Quanser. The friction values have not yet been verified on the experimental setup. Figure 4 shows the results for $K_p=diag(10,10)$ and $K_d=diag(4,4)$. Figure 5 compares the results when we add an inverse notch filter with the results when we only use high K_d gains. In both cases the influence of the disturbance is significantly reduced, however, the trajectories converge faster when the inverse notch filter

TABLE I
2R manipulator parameter values

Parameter	Value	Parameter	Value	Parameter	Value
m_1	0.5	l_1	0.2	d_1	0.1
m_2	1	l_2	0.25	d_2	0.1
I_1	0.01	r_1	0.1	q_{1d}	0.5π
I_2	0.01	r_2	0.1	q_{2d}	0.25π

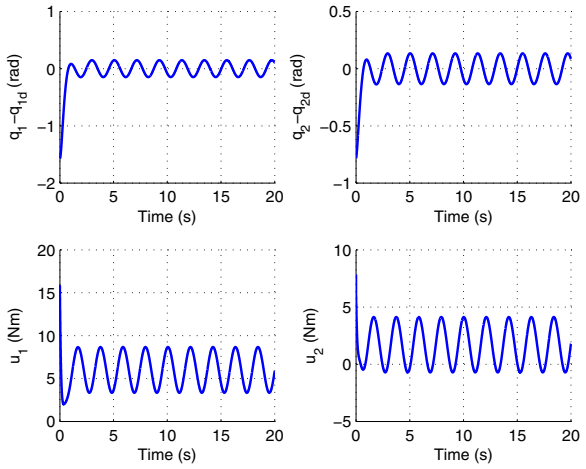

Fig. 4. Trajectories for the 2R manipulator. Initial conditions $[q(0)p(0)]=[0\ 0\ 0\ 0]$.

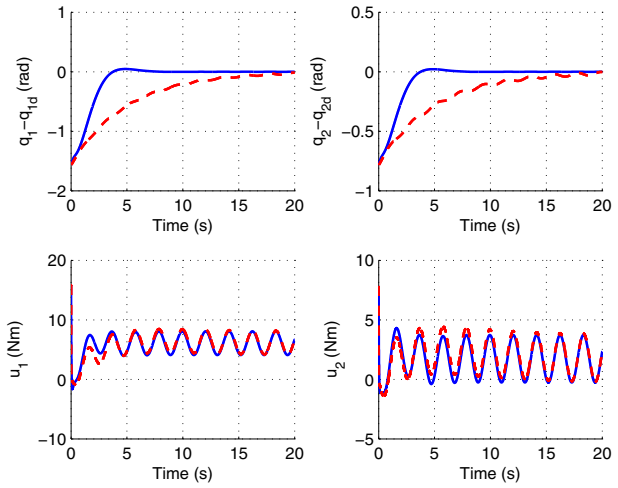

Fig. 5. Trajectories for the 2R manipulator. Initial conditions $[q(0)\ p(0)] = [0\ 0\ 0\ 0]$. Blue lines: trajectories with inverse notch filters with $\omega_0 = 3$ rad/s, $\beta_1 = 1, \beta_2 = 0.01$. Dashed red lines: trajectories without inverse notch and $K_d = diag(50, 50)$.

is applied. As explained in the previous section and shown in this example, only applying high K_d gains increases (overall) damping and slows down the convergence.

We also did simulations for $\omega_0 = 2.5$ rad/s and $\omega_0 = 3.5$ rad/s, while keeping the real disturbance frequency at $\omega_i = 3$ rad/s. The results showed very small differences compared to figure 5, showing the robustness of the inverse notch filter. By an appropriate choice of β_1 and β_2 we can make the inverse notch wide enough to deal with uncertainties in the disturbance frequency.

V. Final remarks

The notch filter is a well-known control tool and popular in many control applications. The contribution of this paper is to describe the standard notch filter in the port-Hamiltonian (PH) framework. By having a PH description of the notch filter it can be interconnected (in a passivity preserving way) to passive nonlinear systems and preserve the passivity property. The passivity property can offer great advantages in the analysis and control of nonlinear systems. It becomes possible to combine a notch filter, which is tuned based on frequency requirements, with passivity-based control. Frequency components are then included into nonlinear control, while preserving global stability for the closed-loop system.

In a similar way we also give a PH description for the inverse notch filter. Contrary to a normal notch filter, the inverse notch filter only passes signals of a specific frequency. We also analyze the disturbance attenuation properties of the inverse notch filter for (nonlinear) PH systems.

A simulation example is given for the application of the inverse notch filter for disturbance attenuation of a robot manipulator. The simulation results show, as expected, disturbance attenuation without overdamping the system. Although only illustrated for an example of a standard mechanical system, the port-Hamiltonian notch/inverse notch filter can be applied in a similar way in other physical domains.

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