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Bergshoeff, E.A.; de Roo, M.; Kerstan, S.F.; Ortin, T.; Riccioni, F.

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## IIA ten-forms and the gauge algebras of maximal supergravity theories

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**Eric A. Bergshoeff, Mees de Roo and Sven F. Kerstan**

*Centre for Theoretical Physics, University of Groningen*

*Nijenborgh 4, 9747 AG Groningen, The Netherlands*

*E-mail: E.A.Bergshoeff@rug.nl, M.de.Roo@rug.nl, S.Kerstan@rug.nl*

**Tomás Ortín**

*Instituto de Física Teórica UAM/CSIC, Facultad de Ciencias C-XVI*

*C.U. Cantoblanco, E-28049-Madrid, Spain*

*E-mail: Tomas.Ortin@cern.ch*

**Fabio Riccioni**

*DAMTP, Centre for Mathematical Sciences, University of Cambridge*

*Wilberforce Road, Cambridge CB3 0WA, U.K., and*

*Dipartimento di Fisica, Università di Roma "Tor Vergata"*

*I.N.F.N. - Sezione di Roma II "Tor Vergata"*

*Via della Ricerca Scientifica, 1 - 00133 Roma - Italy*

*E-mail: F.Riccioni@damtp.cam.ac.uk*

**ABSTRACT:** We show that IIA supergravity can be extended with two independent 10-form potentials. These give rise to a single BPS IIA 9-brane. We investigate the bosonic gauge algebra of both IIA and IIB supergravity in the presence of 10-form potentials and point out an intriguing relation with the symmetry algebra  $E_{11}$ , which has been conjectured to be the underlying symmetry of string theory/M-theory.

**KEYWORDS:** p-branes, M-Theory, D-branes.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. IIA supergravity and ten-form potentials</b>	<b>2</b>
<b>3. Nine-branes of IIA</b>	<b>6</b>
<b>4. The bosonic gauge algebras</b>	<b>7</b>
4.1 The IIB algebra	8
4.2 The IIA algebra	10
<b>5. M-theory</b>	<b>13</b>
<b>6. Conclusions</b>	<b>13</b>

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## 1. Introduction

Ten-dimensional type-IIB superstring theory is conjectured to possess an  $SL(2, \mathbb{Z})$  self-duality [1]. This non-perturbative symmetry, which is a discrete subgroup of the  $SL(2, \mathbb{R})$  symmetry of the low-energy effective action [2], transforms the various BPS branes in the theory. While the D1-brane and D5-brane solutions belong to a doublet, and the D3-brane to a singlet, the D7-brane solution of [3] transforms non-linearly with respect to  $Sl(2, \mathbb{Z})$ . Its charge matrix has vanishing determinant [4], while half-supersymmetric 7-brane solutions in other conjugacy classes [5] can be obtained as bound states of the (anti-) D7-brane and its S-dual. It turns out that the 7-branes transform as a nonlinear doublet under  $SL(2, \mathbb{Z})$ . Recently, the leading terms of a kappa-symmetric action for these 7-branes, involving their tensions, have been derived [6].

Type-IIB string theory also possesses D9-branes, that identify the open sector of the type-I theory, obtained from a projection of IIB [7] known as *orientifold projection*. In the closed sector, this projection corresponds to the insertion of O9-planes, whose charge has to cancel that of the D9-branes. Although the presence of D9-branes is not consistent if the overall charge is not canceled, it is possible to write a kappa-symmetric effective action for these objects, whose Wess-Zumino term contains a coupling to a RR 10-form. The gauge and supersymmetry transformations for this form were derived in [8]. From a careful analysis of the supersymmetry algebra, it was shown in [9] that this 10-form belongs to a quadruplet of  $SL(2, \mathbb{R})$ . Requiring the leading terms in the corresponding effective action to be invariant under 16 linear supersymmetries leads to a constraint on the charges, so that only a *non-linear* doublet of 9-branes remains. Furthermore, the theory contains an additional *linear* doublet of 10-forms. The 10-forms in this linear doublet give rise to supersymmetric effective actions without the need to impose constraints [6].

One of the aims of this paper is to make for IIA supergravity the same analysis that was done for IIB in [9]. In [10] a *democratic* formulation for IIA was given, in which all the RR forms were considered together with their magnetic duals. The resulting supersymmetry algebra has the feature of describing both the ‘massless’ IIA supergravity [11] and Romans’ massive theory [12], whose cosmological constant is treated as the dual of a 10-form field strength, whose 9-form RR potential couples to D8-branes [13, 14]. This 9-form does not carry propagating degrees of freedom, and is therefore not dual to propagating supergravity fields. A further dualization, now also including the dilaton and the NSNS 2-form, was performed in [15] (see also [16]). In [17] it was shown that the IIA supergravity theory can be extended in order to include a 10-form. The corresponding spacetime-filling brane has a tension scaling like  $g_S^{-2}$  in the string frame, and it is the T-dual [17] of a similar solitonic IIB 9-brane in the linear doublet. In order to determine all the possible 10-forms in IIA supergravity, we perform an analysis analogous to the one in [9]. In particular, we construct a completely democratic formulation, in which all the fields, and not only the RR ones, are introduced together with their magnetic duals. The outcome of this analysis will be that there are two independent 10-forms in the IIA theory. We also analyze which of these 10-forms can give rise to a kappa-symmetric 9-brane action.

It turns out that the 10-forms implied by supersymmetry in both IIA and IIB supergravity are the ones that are predicted by  $E_{11}$  [18, 19], a conjectured infinite-dimensional symmetry underlying string and M-theory [20–22]. Related approaches with extended symmetry algebras have been discussed in [23–25]. In this paper we wish to further discuss the intriguing relationship between the  $E_{11}$  approach and results derived from supersymmetry. In particular, we will show that, after a suitable (field-dependent) redefinition of the gauge fields and the gauge parameters, the gauge transformations of all the forms become *linear* in the gauge fields, while the resulting bosonic gauge algebra is *non-Abelian* [15]. We perform this analysis for both IIA and IIB supergravity.

The structure of the paper is as follows. In section 2 we write IIA supergravity in a completely democratic formulation, and we show that the IIA theory allows two independent 10-forms. In section 3 we show that the two 10-forms can give rise to a single kappa-symmetric IIA 9-brane. Section 4 is devoted to an analysis of the bosonic gauge algebras of both IIA and IIB supergravity. In particular, we point out an intriguing relationship between the commutation rules for the gauge transformations and certain predictions from  $E_{11}$ . In section 5 we discuss  $E_{11}$  and M-theory. Finally, section 6 contains the conclusions.

## 2. IIA supergravity and ten-form potentials

In this section we show that IIA supergravity allows two independent 10-form potentials. We perform the same analysis as was done in [9] for the IIB case. We use the notations and conventions of [10], so that we will work in string frame, with mostly plus signature. In this formulation, all RR fields and their magnetic duals are included, together with the RR 9-form, whose field strength is dual to the cosmological constant. This formulation

describes both the ‘massless’ theory [11] and Romans’ theory [12]. We will generalise this by including the fields dual to the dilaton and the NSNS 2-form, that we call  $B_{(8)}$  and  $B_{(6)}$  respectively, and two 10-forms.

The propagating fields in the theory are the graviton  $g_{\mu\nu}$ , the dilaton  $\phi$ , the NSNS 2-form  $B_{\mu\nu}$ , the RR 1-form  $C_\mu$  and the RR 3-form  $C_{\mu\nu\rho}$ , together with a Majorana non-chiral gravitino  $\psi_\mu$  and a Majorana non-chiral dilatino  $\lambda$  in the fermionic sector. One then introduces the 7-form and 5-form duals of the RR forms, together with a RR 9-form, whose 10-form field strength is the dual of Romans’ cosmological constant. Finally, supersymmetry allows the introduction of at least one 10-form to this set of fields [17]. In the rest of this paper, we will often denote n-forms  $F_{\mu_1\dots\mu_n}$  by  $F_{(n)}$ . Furthermore, antisymmetrization (with weight one) of the indices is always understood. For instance, the expression  $F_{(n)}G_{(m)}$  means  $F_{[\mu_1\dots\mu_n}G_{\mu_{n+1}\dots\mu_{n+m}]}$ . The same notation will be used for gamma matrices, while the vielbein will be denoted by  $e^a$ , the gravitino by  $\psi$  and the partial derivative by  $\partial$ . With  $\Gamma_{11}$  we denote the chirality matrix, defined as

$$\Gamma_{\mu_1\dots\mu_{10}} = -\epsilon_{\mu_1\dots\mu_{10}}\Gamma_{11} \quad . \quad (2.1)$$

The supersymmetry transformations of all fields to lowest order in the fermions are

$$\delta e^a = \bar{\epsilon}\Gamma^a\psi, \quad (2.2)$$

$$\delta B_{(2)} = 2\bar{\epsilon}\Gamma_{11}\Gamma_{(1)}\psi, \quad (2.3)$$

$$\delta\phi = \frac{1}{2}\bar{\epsilon}\lambda, \quad (2.4)$$

$$\delta C_{(1)} = -e^{-\phi}\bar{\epsilon}\Gamma_{11}\psi + \frac{1}{2}e^{-\phi}\bar{\epsilon}\Gamma_{11}\Gamma_{(1)}\lambda, \quad (2.5)$$

$$\delta C_{(3)} = -3e^{-\phi}\bar{\epsilon}\Gamma_{(2)}\psi + \frac{1}{2}e^{-\phi}\bar{\epsilon}\Gamma_{(3)}\lambda + 3C_{(1)}\delta B_{(2)}, \quad (2.6)$$

$$\delta C_{(5)} = -5e^{-\phi}\bar{\epsilon}\Gamma_{11}\Gamma_{(4)}\psi + \frac{1}{2}e^{-\phi}\bar{\epsilon}\Gamma_{11}\Gamma_{(5)}\lambda + 10C_{(3)}\delta B_{(2)}, \quad (2.7)$$

$$\delta C_{(7)} = -7e^{-\phi}\bar{\epsilon}\Gamma_{(6)}\psi + \frac{1}{2}e^{-\phi}\bar{\epsilon}\Gamma_{(7)}\lambda + 21C_{(5)}\delta B_{(2)}, \quad (2.8)$$

$$\delta C_{(9)} = -9e^{-\phi}\bar{\epsilon}\Gamma_{11}\Gamma_{(8)}\psi + \frac{1}{2}e^{-\phi}\bar{\epsilon}\Gamma_{11}\Gamma_{(9)}\lambda + 36C_{(7)}\delta B_{(2)}, \quad (2.9)$$

$$\delta\mathcal{D}_{(10)} = e^{-2\phi}(-10\bar{\epsilon}\Gamma_{(9)}\psi + \bar{\epsilon}\Gamma_{(10)}\lambda), \quad (2.10)$$

for the bosons, while the fermions transform according to<sup>1</sup>

$$\begin{aligned} \delta\psi_\mu &= D_\mu\epsilon + \frac{1}{8}H_{\mu\nu\rho}\Gamma^{\nu\rho}\Gamma_{11}\epsilon + \frac{1}{8}e^\phi G^{(0)}\Gamma_\mu\epsilon \\ &\quad + \frac{1}{16}e^\phi G_{\nu\rho}\Gamma^{\nu\rho}\Gamma_\mu\Gamma_{11}\epsilon + \frac{1}{8\cdot 4!}e^\phi G_{\mu_1\dots\mu_4}\Gamma^{\mu_1\dots\mu_4}\Gamma_\mu\epsilon, \end{aligned} \quad (2.11)$$

$$\begin{aligned} \delta\lambda &= \partial_\mu\phi\Gamma^\mu\epsilon - \frac{1}{12}H_{\mu\nu\rho}\Gamma_{11}\Gamma^{\mu\nu\rho}\epsilon + \frac{5}{4}e^\phi G^{(0)}\epsilon \\ &\quad + \frac{3}{8}e^\phi G_{\mu\nu}\Gamma_{11}\Gamma^{\mu\nu}\epsilon + \frac{1}{4\cdot 4!}e^\phi G_{\mu_1\dots\mu_4}\Gamma^{\mu_1\dots\mu_4}\epsilon \quad . \end{aligned} \quad (2.12)$$

The bosonic gauge transformations are

$$\delta B_{(2)} = 2\partial\Sigma_{(1)}, \quad (2.13)$$

$$\delta C_{(1)} = \partial\Lambda - G^{(0)}\Sigma_{(1)}, \quad (2.14)$$

$$\delta C_{(3)} = 3\partial\Lambda_{(2)} - H_{(3)}\Lambda - 3G^{(0)}B_{(2)}\Sigma_{(1)}, \quad (2.15)$$

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<sup>1</sup>In the case of the fermions, we leave the index structure explicit since contractions are involved.

$$\delta C_{(5)} = 5\partial\Lambda_{(4)} - 10H_{(3)}\Lambda_{(2)} - 15G^{(0)}B_{(2)}^2\Sigma_{(1)}, \quad (2.16)$$

$$\delta C_{(7)} = 7\partial\Lambda_{(6)} - 35H_{(3)}\Lambda_{(4)} - 105G^{(0)}B_{(2)}^3\Sigma_{(1)}, \quad (2.17)$$

$$\delta C_{(9)} = 9\partial\Lambda_{(9)} - 84H_{(3)}\Lambda_{(6)} - 945G^{(0)}B_{(2)}^4\Sigma_{(1)}, \quad (2.18)$$

$$\delta\mathcal{D}_{(10)} = 10\partial\Sigma_{(9)}, \quad (2.19)$$

and with respect to these, the field strengths

$$G_{(2)} = 2\partial C_{(1)} + G^{(0)}B_{(2)}, \quad (2.20)$$

$$H_{(3)} = 3\partial B_{(2)}, \quad (2.21)$$

$$G_{(4)} = 4\partial C_{(3)} - 4H_{(3)}C_{(1)} + 3G^{(0)}B_{(2)}^2, \quad (2.22)$$

$$G_{(6)} = 6\partial C_{(5)} - 20H_{(3)}C_{(3)} + 15G^{(0)}B_{(2)}^3, \quad (2.23)$$

$$G_{(8)} = 8\partial C_{(7)} - 56H_{(3)}C_{(5)} + 105G^{(0)}B_{(2)}^4, \quad (2.24)$$

$$G_{(10)} = 10\partial C_{(9)} - 120H_{(3)}C_{(7)} + 945G^{(0)}B_{(2)}^5 \quad (2.25)$$

are invariant. The various RR field strengths satisfy the duality relations

$$G_{\mu_1\dots\mu_{2n}}^{(2n)} = (-1)^n \frac{1}{(10-2n)!} \epsilon_{\mu_1\dots\mu_{2n}}^{\mu_{2n+1}\dots\mu_{10}} G_{\mu_{2n+1}\dots\mu_{10}}^{(10-2n)}. \quad (2.26)$$

In particular, the 10-form field strength is related to Romans' cosmological constant  $G^{(0)}$ . For vanishing  $G^{(0)}$ , one recovers the massless IIA supergravity theory.

Imposing the closure of the supersymmetry algebra, one can then determine the supersymmetry transformation for the 6-form  $B_{(6)}$ , whose field strength is related to  $H_{(3)}$  by means of

$$H_{\mu_1\dots\mu_7} = \frac{1}{6} e^{-2\phi} \epsilon_{\mu_1\dots\mu_7\mu\nu\rho} H^{\mu\nu\rho}. \quad (2.27)$$

The result is

$$\begin{aligned} \delta B_{(6)} = & 6e^{-2\phi}\bar{\epsilon}\Gamma_{(5)}\psi - e^{-2\phi}\bar{\epsilon}\Gamma_{(6)}\lambda + 6C_{(5)}\delta C_{(1)} - 10C_{(3)}\delta C_{(3)} \\ & - 30C_{(3)}B_{(2)}\delta C_{(1)} + 30C_{(3)}C_{(1)}\delta B_{(2)} + 30B_{(2)}C_{(1)}\delta C_{(3)}. \end{aligned} \quad (2.28)$$

The 7-form field strength reads

$$\begin{aligned} H_{(7)} = & 7\partial B_{(6)} + G^{(0)}[-C_{(7)} + \frac{105}{2}C_{(3)}B_{(2)}^2 - \frac{105}{2}C_{(1)}B_{(2)}^3] \\ & + G_{(2)}[21C_{(5)} - 105C_{(3)}B_{(2)}] + G_{(4)}[-\frac{35}{2}C_{(3)} + \frac{105}{2}C_{(1)}B_{(2)}], \end{aligned} \quad (2.29)$$

and gauge invariance implies that  $B_{(6)}$  transforms according to

$$\begin{aligned} \delta B_{(6)} = & 6\partial\Sigma_{(5)} + G^{(0)}[\Lambda_{(6)} - \frac{45}{2}\Lambda_{(2)}B_{(2)}^2 + \frac{15}{2}\Lambda B_{(2)}^3 + 30C_{(3)}\Sigma_{(1)}B_{(2)} \\ & - 45C_{(1)}\Sigma_{(1)}B_{(2)}^2] + G_{(2)}[-15\Lambda_{(4)} + 45\Lambda_{(2)}B_{(2)} - 30C_{(3)}\Sigma_{(1)}] \\ & + G_{(4)}[\frac{15}{2}\Lambda_{(2)} - \frac{15}{2}\Lambda B_{(2)} + 15C_{(1)}\Sigma_{(1)}]. \end{aligned} \quad (2.30)$$

Observe that if  $G^{(0)}$  is non-zero, i.e. in the massive theory, one can use  $\Lambda_{(6)}$  to gauge away  $B_{(6)}$ . This is consistent with the fact that the 2-form becomes massive, because in 10 dimensions the dual of a massive 2-form is a massive 7-form, of which  $B_{(6)}$  describes the longitudinal components [26].

Following the same strategy, we now determine the gauge and supersymmetry transformations for the 8-form  $B_{(8)}$  dual to the dilaton. The duality relation is

$$H_{\mu_1 \dots \mu_9} = e^{-2\phi} \epsilon_{\mu_1 \dots \mu_9 \rho} \partial^\rho \phi, \quad (2.31)$$

while the supersymmetry transformation turns out to be

$$\begin{aligned} \delta B_{(8)} = & \frac{1}{2} e^{-2\phi} \bar{\epsilon} \Gamma_{(8)} \Gamma_{11} \lambda - 6C_{(7)} \delta C_{(1)} + 14B_{(2)} \delta B_{(6)} + 14C_{(5)} \delta C_{(3)} \\ & - 210B_{(2)}^2 C_{(1)} \delta C_{(3)} + 210B_{(2)}^2 C_{(3)} \delta C_{(1)} - 42C_{(5)} C_{(1)} \delta B_{(2)}. \end{aligned} \quad (2.32)$$

The 9-form field strength we find is

$$\begin{aligned} H_{(9)} = & 9\delta B_{(8)} + G^{(0)} \left[ \frac{5}{4} C_{(9)} - 18C_{(7)} B_{(2)} + 315C_{(3)} B_{(2)}^3 - \frac{945}{4} C_{(1)} B_{(2)}^4 \right] \\ & + G_{(2)} \left[ -27C_{(7)} + 378C_{(5)} B_{(2)} - 945C_{(3)} B_{(2)}^2 \right] \\ & + G_{(4)} \left[ \frac{63}{2} C_{(5)} - 315C_{(3)} B_{(2)} + \frac{945}{2} C_{(1)} B_{(2)}^2 \right] - 18H_{(7)} B_{(2)}, \end{aligned} \quad (2.33)$$

and the 8-form gauge transformation is

$$\begin{aligned} \delta B_{(8)} = & 8\delta \Sigma_{(7)} + G^{(0)} \left[ -\frac{5}{4} \Lambda_{(8)} + 14\Lambda_{(6)} B_{(2)} - 105\Lambda_{(2)} B_{(2)}^3 + \frac{105}{4} \Lambda B_{(2)}^4 \right. \\ & \left. - 4C_{(7)} \Sigma_{(1)} + 210C_{(3)} \Sigma_{(1)} B_{(2)}^2 - 210C_{(1)} \Sigma_{(1)} B_{(2)}^3 \right] \\ & + G_{(2)} \left[ 21\Lambda_{(6)} - 210\Lambda_{(4)} B_{(2)} + 315\Lambda_{(2)} B_{(2)}^2 + 84C_{(5)} \Sigma_{(1)} \right. \\ & \left. - 420C_{(3)} \Sigma_{(1)} B_{(2)} \right] + G_{(4)} \left[ -\frac{35}{2} \Lambda_{(4)} + 105\Lambda_{(2)} B_{(2)} - \frac{105}{2} \Lambda B_{(2)}^2 \right. \\ & \left. - 70C_{(3)} \Sigma_{(1)} + 210C_{(1)} \Sigma_{(1)} B_{(2)} \right] - 4H_{(7)} \Sigma_{(1)}. \end{aligned} \quad (2.34)$$

As for the 6-form, in the massive theory, in which  $G^{(0)}$  is non-vanishing, this 8-form can be gauged away by means of  $\Lambda_{(8)}$ . In this case this is related to the fact that in ten dimensions the dual of a massive scalar is a massive 9-form potential.

Finally, we consider the inclusion of 10-forms. Since these objects are not related by duality to lower-rank fields, we can only use the closure of the supersymmetry algebra to determine their gauge and supersymmetry transformations. The final result is that, besides the 10-form in eq. (2.10), another 10-form  $D_{(10)}$  can be included in the algebra. Its supersymmetry transformation reads

$$\begin{aligned} \delta D_{(10)} = & \frac{1}{2} e^{-2\phi} \bar{\epsilon} \Gamma_{(10)} \lambda - \frac{15}{2} C_{(9)} \delta C_{(1)} - 45B_{(2)} \delta B_{(8)} + 315B_{(2)}^2 \delta B_{(6)} \\ & + \frac{63}{2} C_{(5)} \delta C_{(5)} - 315C_{(3)} B_{(2)} \delta C_{(5)} + 315C_{(5)} B_{(2)} \delta C_{(3)} \\ & - 315C_{(5)} C_{(3)} \delta B_{(2)} + \frac{945}{2} C_{(1)} B_{(2)}^2 \delta C_{(5)} - \frac{945}{2} C_{(5)} B_{(2)}^2 \delta C_{(1)} \\ & - 945C_{(5)} C_{(1)} B_{(2)} \delta B_{(2)} - 4725C_{(1)} C_{(3)} B_{(2)}^2 \delta B_{(2)} \\ & - 4725C_{(1)} B_{(2)}^3 \delta C_{(3)} + 4725C_{(1)} B_{(2)}^4 \delta C_{(1)}, \end{aligned} \quad (2.35)$$

while the gauge transformation is

$$\begin{aligned} \delta D_{(10)} = & 10\delta \Sigma'_{(9)} + G^{(0)} \left[ -2\Lambda_{(10)} + \frac{225}{4} B_{(2)} \Lambda_{(8)} - 315B_{(2)}^2 \Lambda_{(6)} - \frac{1575}{4} B_{(2)}^3 \Lambda_{(4)} \right. \\ & \left. + \frac{4725}{2} B_{(2)}^4 \Lambda_{(2)} - \frac{945}{2} B_{(2)}^5 \Lambda - \frac{25}{2} C_{(9)} \Sigma_{(1)} + 180B_{(2)} C_{(7)} \Sigma_{(1)} \right. \\ & \left. + \frac{945}{2} B_{(2)}^2 C_{(5)} \Sigma_{(1)} - 6300B_{(2)}^3 C_{(3)} \Sigma_{(1)} + 4725B_{(2)}^4 C_{(1)} \Sigma_{(1)} \right] \end{aligned}$$

$$\begin{aligned}
& +G_{(2)}\left[\frac{135}{4}\Lambda_{(8)} - 945B_{(2)}\Lambda_{(6)} + \frac{23625}{4}B_{(2)}^2\Lambda_{(4)} - \frac{14175}{2}B_{(2)}^3\Lambda_{(2)}\right] \\
& +270C_{(7)}\Sigma_{(1)} - 4725B_{(2)}C_{(5)}\Sigma_{(1)} + 14175B_{(2)}^2C_{(3)}\Sigma_{(1)} \\
& +G_{(4)}\left[\frac{1575}{4}B_{(2)}\Lambda_{(4)} - \frac{4725}{2}B_{(2)}^2\Lambda_{(2)} + \frac{4725}{4}B_{(2)}^3\Lambda\right] \\
& -\frac{315}{2}C_{(5)}\Sigma_{(1)} + 3150B_{(2)}C_{(3)}\Sigma_{(1)} - \frac{14175}{2}B_{(2)}^2C_{(1)}\Sigma_{(1)} \\
& +G_{(6)}\left[-\frac{105}{4}\Lambda_{(4)} + \frac{315}{2}B_{(2)}\Lambda_{(2)} - \frac{315}{4}B_{(2)}^2\Lambda - 105C_{(3)}\Sigma_{(1)}\right] \\
& +315B_{(2)}C_{(1)}\Sigma_{(1)} + 180H_{(7)}B_{(2)}\Sigma_{(1)} + 10H_{(9)}\Sigma_{(1)} \quad . \quad (2.36)
\end{aligned}$$

The gauge parameter  $\Lambda_{(10)}$  (see the second term in the first line<sup>2</sup>) plays a crucial role in closing the algebra, and can be interpreted as the gauge parameter of an 11-form. If we allowed the dimension of spacetime to change from  $d = 10$  to  $d > 10$  the 10-form  $D_{(10)}$  generically would describe propagating degrees of freedom which would convert the 11-form into a *massive* 11-form analogous to the massive 7-form and 9-form we obtained above. Since an 11-form is trivial in ten-dimensions, we are only left with the 10-form potential  $D_{(10)}$ . A similar phenomenon occurs in the IIB case [9]: the field-strength of the (quadruplet of) 10-forms, considered formally in  $d > 10$  dimensions, contains non-trivial information about the gauge transformations of potentials with rank higher than ten. These observations hint at an underlying algebraic structure which might be independent of the dimensionality of space-time. We will discuss this structure in section 4.

A natural question to ask is whether the supersymmetry algebra allows for the inclusion of additional 10-forms. The only freedom we have in the transformations to  $\psi_\mu$  and  $\lambda$  is to change the dilaton factor, or to include an additional  $\Gamma_{11}$  in the transformation rule. The last possibility leads to 10-forms for which the dilaton factor is not restricted by the supersymmetry algebra: these are all proportional to the ten-dimensional volume form and therefore not independent. The possibility of changing the power of  $e^{-\phi}$  without including  $\Gamma_{11}$  in the transformation rules is ruled out by checking closure of the algebra. This analysis shows that there are indeed only two independent 10-forms in the IIA supergravity multiplet.

### 3. Nine-branes of IIA

In [6] we discussed the relation between the 1/2 BPS  $p$ -branes and  $p + 1$ -form potentials in IIB supergravity. In particular, we obtained the tensions as well as the operator which projects onto the unbroken linear supersymmetry. In this section we will do a similar analysis for IIA supergravity, using the results of section 2.

As an example we can work out the case of a 10-brane. We start from the action (see [6])

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{-g} + x \epsilon^{\mu_1 \dots \mu_{10}} \mathcal{D}_{\mu_1 \dots \mu_{10}} \quad . \quad (3.1)$$

Here we have assumed the existence of a (gauge-fixed) kappa-symmetric action, in static gauge. Since  $\tau_{\text{brane}}$  will depend on the dilaton, the background fields present in the Nambu-Goto term are metric and dilaton, the Wess-Zumino term depends on the background

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<sup>2</sup>Observe that due to the presence of this term, the 10-form  $D_{(10)}$ , like the 6-form and the 8-form, can be gauged away in Romans' theory.



potential. World-volume fields play no role in this analysis. The action should exhibit 16 linearly realized supersymmetries. Therefore, if we perform an  $N = 2$  supersymmetry transformation of the background fields in (3.1), we should find that half of the supersymmetry parameters are projected out. It is sufficient to consider the transformation from the bosonic fields to the gravitino and dilatino. If  $\tau_{\text{brane}}$  is chosen correctly, the variation to the gravitino will give a projection operator if the relative constant between Nambu-Goto and Wess-Zumino terms is appropriately chosen. The variation to the dilatino is the consistency check of this procedure.

In the present case the complete supersymmetry variation of  $\mathcal{D}_{(10)}$  is (2.10):

$$\delta\mathcal{D}_{\mu_1\dots\mu_{10}} = e^{-2\phi} \left( -10\bar{\epsilon}\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}} + \bar{\epsilon}\gamma_{[\mu_1\dots\mu_{10}}\lambda \right) . \tag{3.2}$$

This determines the tension to be  $e^{-2\phi}$ . The variation to the gravitino then fixes  $x = 1/10!$ , while we find for the projection operator  $P = \frac{1}{2}(\mathbb{1} + \Gamma_{11})$ . Using the same value for  $x$  the variation of  $\mathcal{D}_{(10)}$  and dilaton to  $\lambda$  produces the same projection operator.

The case of the 10-forms is particularly interesting because there is a second 10-form,  $D_{(10)}$ , whose supersymmetry variation is (2.35)

$$\delta D_{\mu_1\dots\mu_{10}} = \frac{1}{2}e^{-2\phi} (\bar{\epsilon}\gamma_{\mu_1\dots\mu_{10}}\lambda + \text{gauge} - \text{field dependent terms}) . \tag{3.3}$$

$D_{(10)}$  by itself cannot couple supersymmetrically to a 9-brane, because there is no gravitino contribution to match the variation of  $\sqrt{-g}$  in (3.1). The result is therefore that it is precisely  $\mathcal{D}$ , the only combination which does not transform to gauge-field dependent terms, that might correspond to a kappa-symmetric 9-brane.

In the IIB case we have a similar 10-form potential, which supersymmetrically couples to a solitonic  $(1/g_S)^2$  brane, and also does not transform to gauge-field dependent terms [6]. The absence of gauge fields in the supersymmetry transformation implies that these potentials have trivial bosonic gauge transformations. This implies in turn that the Wess-Zumino term in (3.1) is gauge-invariant as it stands.

For completeness and further reference we present in table 1 a list of all the BPS branes, their tension, potential and projection operator. Note that also the NSNS-form  $B_{(8)}$  is absent from the table, the reason being that like  $D_{(10)}$  it does not transform linearly to the gravitino.

#### 4. The bosonic gauge algebras

In this section we will analyse the algebra of bosonic gauge transformations which is contained in the supersymmetry algebra. We will first do this for IIB supergravity, then for IIA supergravity.

Our analysis will reveal a surprising structure and a relation to results from the  $E_{11}$  approach [20–22, 18]. It may also be seen as an extension and derivation from supersymmetry of the results of [15].

potential	brane	tension	projection operator
$C_{(1)}$	D0	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_0)$
$B_{(2)}$	F1	1	$\frac{1}{2}(\mathbb{1} + \gamma_{01}\Gamma_{11})$
$C_{(3)}$	D2	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_{012})$
$C_{(5)}$	D4	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_{01\dots 4}\Gamma_{11})$
$B_{(6)}$	NS5	$e^{-2\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_{01\dots 5})$
$C_{(7)}$	D6	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_{01\dots 6})$
$C_{(9)}$	D8	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \gamma_{01\dots 8}\Gamma_{11})$
$\mathcal{D}_{(10)}$	NS9	$e^{-2\phi}$	$\frac{1}{2}(\mathbb{1} + \Gamma_{11})$

**Table 1:** Potentials, branes, tensions and projection operators for all IIA supersymmetric branes.

### 4.1 The IIB algebra

Our starting point is the set of bosonic gauge transformations of IIB supergravity in Einstein frame we obtained in [9] where we used a mostly minus signature:

$$\begin{aligned}
 \delta A_{(2)}^\alpha &= 2\partial\Lambda_{(1)}^\alpha, \\
 \delta A_{(4)} &= 4\partial\Lambda_{(3)} - \frac{i}{4}\epsilon_{\gamma\delta}\Lambda_{(1)}^\gamma F_{(3)}^\delta, \\
 \delta A_{(6)}^\alpha &= 6\partial\Lambda_{(5)}^\alpha - 8\Lambda_{(1)}^\alpha F_{(5)} - \frac{160}{3}F_{(3)}^\alpha\Lambda_{(3)}, \\
 \delta A_{(8)}^{\alpha\beta} &= 8\partial\Lambda_{(7)}^{(\alpha\beta)} + \frac{1}{2}F_{(7)}^{(\alpha\beta)}\Lambda_{(1)} - \frac{21}{2}F_{(3)}^{(\alpha\beta)}\Lambda_{(5)}, \\
 \delta A_{(10)}^\alpha &= 10\partial\Lambda_{(9)}^\alpha, \\
 \delta A_{(10)}^{\alpha\beta\gamma} &= 10\partial\Lambda_{(9)}^{(\alpha\beta\gamma)} - \frac{2}{3}F_{(9)}^{(\alpha\beta)}\Lambda_{(1)}^\gamma + 32F_{(3)}^{(\alpha\beta)}\Lambda_{(7)}^{\beta\gamma}. \quad (4.1)
 \end{aligned}$$

The field-strengths, which are invariant under the bosonic gauge transformations, are given by:

$$\begin{aligned}
 F_{(3)}^\alpha &= 3\partial A_{(2)}^\alpha, \\
 F_{(5)} &= 5\partial A_{(4)} + \frac{5i}{8}\epsilon_{\alpha\beta}A_{(2)}^\alpha F_{(3)}^\beta, \\
 F_{(7)}^\alpha &= 7\partial A_{(6)}^\alpha + 28A_{(2)}^\alpha F_{(5)} - \frac{280}{3}F_{(3)}^\alpha A_{(4)}, \\
 F_{(9)}^{\alpha\beta} &= 9\partial A_{(8)}^{\alpha\beta} + \frac{9}{4}F_{(7)}^{(\alpha\beta)}A_{(2)} - \frac{63}{4}F_{(3)}^{(\alpha\beta)}A_{(6)}, \\
 F_{(11)}^\alpha &= 11\partial A_{(10)}^\alpha = 0, \\
 F_{(11)}^{\alpha\beta\gamma} &= 11(\partial A_{(10)}^{\alpha\beta\gamma} - \frac{1}{3}F_{(9)}^{(\alpha\beta)}A_{(2)}^\gamma + 4F_{(3)}^{(\alpha\beta)}A_{(8)}^{\beta\gamma}) = 0. \quad (4.2)
 \end{aligned}$$

It is clear that the bosonic transformations commute, because the transformations  $\delta A_{(2n)}$  contain only parameters and gauge invariant curvatures. In other words, we have nonlinear transformation rules and an Abelian gauge algebra. Following [15] we write out the curvatures in (4.1), using (4.2). Next, we redefine the parameters  $\Lambda$  and  $\Sigma$  of the gauge transformations such that the transformations only depend on  $d\Lambda, d\Sigma$ , and not on  $\Lambda, \Sigma$ .

After that, we redefine the bosonic gauge fields to make the bosonic gauge transformations linear in the gauge fields. Finally, we suitably rescale the fields and parameters to simplify the form of the transformations. This leads to the following form for the gauge transformations:

$$\begin{aligned}
 \delta A_{(2)}^\alpha &= \Lambda_{(2)}^\alpha, \\
 \delta A_{(4)} &= \Lambda_{(4)} + i\epsilon_{\gamma\delta}\Lambda_{(2)}^\gamma A_{(2)}^\delta, \\
 \delta A_{(6)}^\alpha &= \Lambda_{(6)}^\alpha + \Lambda_{(4)} A_{(2)}^\alpha + \gamma\Lambda_{(2)}^\alpha A_{(4)}, \\
 \delta A_{(8)}^{\alpha\beta} &= \Lambda_{(8)}^{\alpha\beta} + \Lambda_{(6)}^{(\alpha} A_{(2)}^{\beta)} + \epsilon\Lambda_{(2)}^{(\alpha} A_{(6)}^{\beta)}, \\
 \delta A_{(10)}^{\alpha\beta\gamma} &= \Lambda_{(10)}^{\alpha\beta\gamma} + \Lambda_{(8)}^{(\alpha\beta} A_{(2)}^{\gamma)} + \mu\Lambda_{(2)}^{(\alpha} A_{(8)}^{\beta\gamma)}, \\
 \delta A_{(10)}^\alpha &= \Lambda_{(10)}^\alpha
 \end{aligned} \tag{4.3}$$

with

$$\gamma = -2, \quad \epsilon = -3, \quad \mu = -4. \tag{4.4}$$

Note that, even though we have rescaled both fields and gauge parameters we use the same notation as for the original fields in (4.1), to avoid an excess of complicated notation. Also, in this subsection we use the notation  $\Lambda_{(2n)} \equiv \partial\Lambda_{(2n-1)}$ , following [15]. The three coefficients  $\gamma, \epsilon, \mu$  can either be derived directly from the supersymmetry algebra as explained, or be obtained by closure of the bosonic gauge algebra. In either case we find the values given in (4.4).

So the structure we find is very rigid and, with our requirements (parameters appear only with derivatives, linearity in fields, non-trivial transformations), unique. The bosonic gauge algebra in this form is given by the following commutation relations:

$$\begin{aligned}
 [\delta_{\tilde{\Lambda}_{(2)}}, \delta_{\Lambda_{(2)}}] &= \delta_{\Lambda_{(4)}} (\Lambda_{(4)} = -2\epsilon_{\gamma\delta}\tilde{\Lambda}_{(2)}^\gamma \Lambda_{(2)}^\delta), \\
 [\delta_{\tilde{\Lambda}_{(2n)}}, \delta_{\Lambda_{(2)}}] &= \delta_{\Lambda_{(2n+2)}} (\Lambda_{(2n+2)} = -(n+1)\tilde{\Lambda}_{(2n)} \Lambda_{(2)}) \text{ for } n > 1.
 \end{aligned} \tag{4.5}$$

In this formula we have suppressed the  $SU(1,1)$  indices.

Note that, again, we use the same notation as for the original fields in 4.1, to avoid an excess of complicated notation. Thus the bosonic gauge algebra is rather special. In a sense, the starting point is also rather special, because it is commutative. In [15] it is suggested that nonabelian algebras, such as the one we obtained above are always related to commutative algebras, as in our starting point.

The above results suggest that it might be possible to find a basis for the fields of IIB supergravity, in which the supersymmetry transformations of the gauge fields are linear in the gauge fields. This is indeed the case. In fact, the supersymmetry transformations, as presented in [9], already are in this form. To show this it is convenient to denote the terms in the supersymmetry transformations of the bosonic gauge fields that explicitly contains the gravitino or dilatino with  $\delta_F$ . Using this notation we can write out the supersymmetry rules given in formulae (5.1) to (5.11) in [9] as:

$$\delta A_{(2)}^\alpha = \delta_F A_{(2)}^\alpha,$$

$$\begin{aligned}
\delta A_{(4)} &= \delta_F A_{(4)} - \frac{3i}{8} \epsilon_{\gamma\delta} A_{(2)}^\gamma \delta_F A_{(2)}^\delta, \\
\delta A_{(6)}^\alpha &= \delta_F A_{(6)}^\alpha + 40 A_{(4)} \delta_F A_{(2)}^\alpha - 20 \delta_F A_{(4)} A_{(2)}^\alpha, \\
\delta A_{(8)}^{\alpha\beta} &= \delta_F A_{(8)}^{\alpha\beta} + \frac{21}{4} A_{(6)}^{(\alpha} \delta_F A_{(2)}^{\beta)} - \frac{7}{4} A_{(2)}^{(\alpha} \delta_F A_{(6)}^{\beta)}, \\
\delta A_{(10)}^{\alpha\beta\gamma} &= \delta_F A_{(10)}^{\alpha\beta\gamma} - 12 A_{(8)}^{(\alpha\beta} \delta_F A_{(2)}^{\gamma)} + 3 A_{(2)}^{(\alpha} \delta_F A_{(8)}^{\beta\gamma)},
\end{aligned} \tag{4.6}$$

so, there are no terms nonlinear in the gauge fields.

Note that the relative coefficients in (4.6), i.e., between  $A_{(4)} \delta_F A_{(2)}^\alpha$  and  $\delta_F A_{(4)} A_{(2)}^\alpha$  etc., are  $-2$ ,  $-3$  and  $-4$ . The same coefficients occur in the corresponding curvatures (4.2), if the  $F_{(n)}$  on the right-hand side are replaced by  $n \partial A_{(n-1)}$ .

The absence of terms of higher order in the gauge fields in (4.6), and the numerical correspondence with (4.2), can be understood from the requirement that (4.2) can be extended to a set of supercovariant curvatures. The appearance of the same coefficients in (4.3) is not surprising considering the close correspondence between (4.3) and (4.2).

## 4.2 The IIA algebra

Our starting point for the IIA algebra are the bosonic gauge transformations<sup>3</sup> and field-strengths given in section 2. As in the IIB case, we write out the curvatures in the variations of the potentials explicitly, and redefine the parameters  $\Lambda$  and  $\Sigma$  of the gauge transformations such that the transformations depend on  $\partial\Lambda$  and  $\partial\Sigma$ ,<sup>4</sup> but not on  $\Lambda$  and  $\Sigma$ . The second step is to redefine the bosonic gauge fields to make the bosonic gauge transformations linear in the gauge fields. The last step is to suitably rescale fields and parameters to simplify the form of the transformations. We find:

$$\begin{aligned}
\delta B_2 &= \partial\Sigma_{(1)}, \\
\delta C_{(1)} &= \partial\Lambda_{(0)} - G_{(0)}\Sigma_{(1)}, \\
\delta C_{(3)} &= \partial\Lambda_{(2)} - C_{(1)}\partial\Sigma_{(1)}, \\
\delta C_{(5)} &= \partial\Lambda_{(4)} - C_{(3)}\partial\Sigma_{(1)}, \\
\delta C_{(7)} &= \partial\Lambda_{(6)} - C_{(5)}\partial\Sigma_{(1)}, \\
\delta C_{(9)} &= \partial\Lambda_{(8)} - C_{(7)}\partial\Sigma_{(1)}, \\
\delta B_{(6)} &= \partial\Sigma_{(5)} + G_{(0)}\Lambda_{(6)} + \frac{1}{2}(-C_{(1)}\partial\Lambda_{(4)} + C_{(3)}\partial\Lambda_{(2)} - C_{(5)}\partial\Lambda_{(0)} + G_{(0)}C_{(5)}\Sigma_{(1)}), \\
\delta B_{(8)} &= \partial\Sigma_{(7)} - G_{(0)}\Lambda_{(8)} + \frac{2}{5}B_{(6)}\partial\Sigma_{(1)} \\
&\quad + \frac{1}{5}(2C_{(1)}\partial\Lambda_{(6)} - C_{(3)}\partial\Lambda_{(4)} + C_{(7)}\partial\Lambda_{(0)} - G_{(0)}C_{(7)}\Sigma_{(1)}), \\
\delta D_{(10)} &= \partial\Sigma_{(9)} + G_{(0)}\Lambda_{(10)} + \frac{5}{8}B_{(8)}\partial\Sigma_{(1)} + \frac{1}{16}(-5C_{(1)}\partial\Lambda_{(8)} + C_{(3)}\partial\Lambda_{(6)} \\
&\quad + C_{(5)}\partial\Lambda_{(4)} - C_{(7)}\partial\Lambda_{(2)} - C_{(9)}\partial\Lambda_{(0)} + G_{(0)}C_{(9)}\Sigma_{(1)}).
\end{aligned}$$

Note that, even though for simplicity our notation does not indicate it, the fields and gauge parameters have been redefined and are not the same as those of section 2. This leads to

<sup>3</sup>Note that in the IIA case we work in string frame and use a mostly plus metric.

<sup>4</sup>We treat the mass parameter  $G^{(0)}$  like a derivative for this purpose, so  $G^{(0)}\Lambda$  would, for example, also be of the desired form. However, this makes the notation  $\Lambda_{(2n)} \equiv \partial\Lambda_{(2n-1)}$  which we used in the previous subsection unpractical.

the following algebra. On the RR forms we find only:

$$[\delta_{\tilde{\Sigma}_{(1)}}, \delta_{\Sigma_{(1)}}] = \delta_{\Lambda_{(2)}} (\Lambda_{(2)} = -G_{(0)} \tilde{\Sigma}_{(1)} \Sigma_{(1)}), \quad (4.7)$$

$$[\delta_{\Lambda_{(2k)}}, \delta_{\Sigma_{(1)}}] = \delta_{\Lambda_{(2k+2)}} (\Lambda_{(2k+2)} = -\Lambda_{(2k)} \partial \Sigma_{(1)}), \quad (4.8)$$

$$[\delta_{\Lambda_{(2k)}}, \delta_{\Lambda_{(2l)}}] = 0 \quad . \quad (4.9)$$

This algebra is extended once we consider the action on the NSNS forms. For example, the commutator (4.7) must also be realized on the NSNS fields. This is indeed the case. The commutator (4.8) is extended with a  $\delta_{\Sigma_{(2k+1)}}$  transformation:

$$\begin{aligned} [\delta_{\Lambda_{(2k)}}, \delta_{\Sigma_{(1)}}] &= \delta_{\Lambda_{(2k+2)}} (\Lambda_{(2k+2)} = -\Lambda_{(2k)} \partial \Sigma_{(1)}) \\ &\quad + \delta_{\Sigma_{(2k+1)}} (\bar{\Sigma}_{(2k+1)} = x_{2k} G_{(0)} \Lambda_{(2k)} \Sigma_{(1)}), \end{aligned} \quad (4.10)$$

with  $x_6 = 1$ ,  $x_8 = -3/5$ , and  $x_{10} = 6/16$ . We also have:

$$[\delta_{\Sigma_{(2k+1)}}, \delta_{\Sigma_{(1)}}] = \delta_{\Sigma_{(2k+3)}} (\bar{\Sigma}_{(2k+3)} = y_{2k+1} \Sigma_{(2k+1)} \partial \Sigma_{(1)}), \quad (4.11)$$

with  $y_5 = 2/5$ ,  $y_7 = 5/8$ . Finally, many of the commutators between two  $\Lambda$  transformations become nonzero and give a  $\Sigma_{(2k+1)}$  transformation for  $k > 1$ . We write these as

$$[\delta_{\Lambda_{(2k)}}, \delta_{\Lambda_{(2l)}}] = \delta_{\Sigma_{(2k+2l+1)}} (\bar{\Sigma}_{(2k+2l+1)} = z_{2k,2l} (\Lambda_{(2k)} \partial \Lambda_{(2l)} - \Lambda_{(2l)} \partial \Lambda_{(2k)}), \quad (4.12)$$

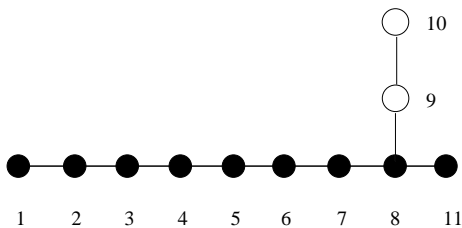
with  $z_{4,0} = -1/2$ ,  $z_{6,0} = 3/10$ ,  $z_{8,0} = -3/16$ ,  $z_{4,2} = 1/10$ ,  $z_{2,2} = 1/4$ ,  $z_{4,4} = 1/32$ . Other combinations vanish.

Having established the form of the IIA and IIB bosonic gauge algebras we are now in a position to discuss an intriguing relation between these algebras and the Kac-Moody algebra  $E_8^{+++}$ , which is also called  $E_{11}$ . We first consider the IIB algebra, see (4.5). Using an obvious notation this algebra has the following schematic form:

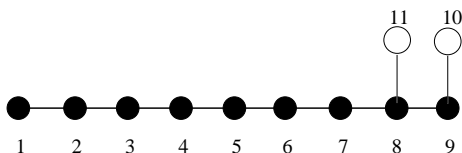
$$\text{IIB : } \boxed{[\mathbf{2}, \mathbf{2}] = 4, \quad [\mathbf{2}, \mathbf{4}] = 6, \quad [\mathbf{2}, \mathbf{6}] = 8, \quad \dots} \quad (4.13)$$

We thus see that the gauge transformation  $\Lambda_{(2)}^\alpha$  of the 2-form, indicated by  $\mathbf{2}$  above, acts like a raising operator in the sense that all the  $2n$ -form gauge transformations  $\Lambda_{(2n)}$  with  $n > 1$  can be obtained as multiple commutators of the  $\mathbf{2}$  transformation. This is reminiscent to a similar structure that occurs in  $E_{11}$  in a rather different context. For instance, in [18] the algebra  $E_{11}$  was decomposed in a particular way with respect to  $SL(10)$ , which should be thought of as the spacetime symmetry group. This leads to the Dynkin diagram in figure 1.

The nine black dots represent the  $SL(10)$  sub-algebra on which the gravity sector is embedded. It can be shown that the "lowest" irreducible representations arising for this decomposition coincide with the fields of IIB supergravity including the 10-form potentials, see table 3 in appendix A.1 of [18]. The way this works is that the two white dots act as two raising operators and the number of times they act corresponds to the "level" of the representation. In this way all representations can be obtained. In our supergravity approach a similar thing happens in the bosonic IIB gauge algebra where the two white



**Figure 1:** The IIB decomposition of the  $E_{11}$  Dynkin diagram.



**Figure 2:** The IIA decomposition of the  $E_{11}$  Dynkin diagram.

dots should be identified with the  $\Lambda_{(2)}^\alpha$  transformations. The fact that  $\Lambda_{(2)}^\alpha$  is a 2-form follows in the Dynkin diagram from the presence of the two black dots 8 and 11. The fact that the 2-form gauge transformations transform as a doublet under  $SL(2, \mathbb{R})$  follows from the presence of the two white dots 9 and 10. The analogy is that in the same way as all relevant  $SL(10)$  representations can be obtained by a multiple action of the two raising operators all bosonic gauge transformations can be obtained as a multiple commutator of the basic  $\Lambda_{(2)}^\alpha$  transformation.

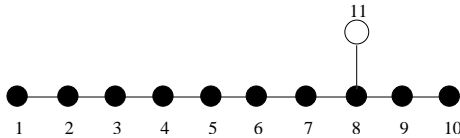
We next consider the IIA algebra, see subsection 3.2, where a similar thing happens. Schematically the IIA bosonic gauge algebra is given by

$$\text{IIA : } \boxed{[\mathbf{1}, \mathbf{1}] = 0, [\mathbf{1}, \mathbf{2}] = 3, [\mathbf{1}, \mathbf{3}] = 0, [\mathbf{2}, \mathbf{3}] = 5, [\mathbf{1}, \mathbf{5}] = 6, \dots} \quad (4.14)$$

We thus see that in this case the gauge transformations  $\Lambda_{(0)}$  and  $\Sigma_{(1)}$ , indicated by **1** and **2** above, act as two raising operators in the sense that all other gauge transformations can be obtained as multiple commutators of **1** and **2**. This corresponds to the level structure in another decomposition of the  $E_{11}$  Dynkin diagram with respect to  $SL(10)$  (see figure 2) [18].

Like in the IIB case the two white dots indicate the two raising operators. However, in this case, they correspond to a 2-form  $\Sigma_{(2)}$  (the 11 white dot) and a 1-form  $\Lambda_{(1)}$  (the 10 white dot). The calculation of the  $SL(10)$  representations leading to the fields of IIA supergravity including the 10-form potentials can be found in table 2 and the corresponding table in appendix A.1 of [18]. An explicit construction of the gauge algebra from the  $E_{11}$  point of view has been given in [19] and earlier works (see references in [19]).

Although the similarities between the IIA and IIB bosonic gauge algebras and the predictions by  $E_{11}$  are intriguing there are also striking differences. The most important one is that the  $E_{11}$  symmetries predict many more  $SL(10)$  representations whose interpretation from the supergravity point of view are unclear at the moment. Nevertheless we consider it



**Figure 3:** The M-theory decomposition of the  $E_{11}$  Dynkin diagram.

remarkable that there is so much overlap between the predictions of IIA/IIB supersymmetry and the bosonic  $E_{11}$  symmetry.

## 5. M-theory

It is natural to consider our results from an M-theory perspective. It turns out that none of the two IIA 10-form potentials has a  $d = 11$  origin. It is well-known that the same is true for the RR 9-form potential. This is related to the fact that massive IIA supergravity has no known  $d = 11$  origin at the field theory level. We have independently verified that the  $d = 11$  superalgebra does *not* allow the inclusion of an 11-form potential.

It is interesting to see what happens with the bosonic gauge algebra of M-theory which was investigated in [15]. The fields of the  $d = 11$  supergravity multiplet consist of a graviton  $g_{\mu\nu}$ , a 3-form potential  $C_{(3)}$  and a dual potential  $C_{(6)}$ . Using the same notation as above the bosonic gauge algebra has the following schematic form:

$$\boxed{[\mathbf{3}, \mathbf{3}] = 6, \quad [\mathbf{3}, 6] = 0.} \quad (5.1)$$

In order to produce the  $E_{11}$  structure we would like a rank 9 symmetry to occur at the right-hand side of the  $[\mathbf{3}, 6]$  commutator. However, there is no 9-form potential available in  $d = 11$  supergravity. Instead, the  $[\mathbf{3}, 6]$  commutator can also give rise to a  $(8, 1)$ -form which one could identify at the linearized level with the  $d = 11$  dual graviton [27, 20, 28]. This is in fact predicted by  $E_{11}$  [20].<sup>5</sup> These representations follow from yet another decomposition of the  $E_{11}$  Dynkin diagram in terms of an  $SL(11)$  bosonic subalgebra.

The ten black dots correspond to the  $SL(11)$  subalgebra and the white dot indicates a single raising operator. The fact that the gauge transformation is a 3-form follows from the three black dots 8, 9, and 10. The specific representations predicted by  $E_{11}$  can be found in [18, table 1].

Extending dual gravitons to the nonlinear level seems to be problematic [29]. It would be interesting to reconsider this issue in the context of (linearised)  $d = 11$  supergravity and the underlying  $E_{11}$  structure.

## 6. Conclusions

We have presented the supersymmetry and gauge transformations of a completely democratic IIA supergravity theory. This has led to the insight that IIA supergravity admits

<sup>5</sup>Actually, similar dual gravitons are predicted to occur in the IIA and IIB case [18].

two distinct 10-form potentials. In the massive version of the theory, which is naturally included in our completely democratic formulation, one of the 10-forms, as well as the 6- and 8-forms can be gauged away. The natural role of the 10-forms is to couple to 9-branes. We have shown that the IIA theory may contain a kappa-symmetric 9-brane. The consistency of such a 9-brane would require the presence of a corresponding orientifold plane, along the lines of [30].

The second part of this paper was concerned with the bosonic gauge algebras, which are contained in the IIA and IIB theories. We have presented a formulation in which the transformation rules are linear in the gauge fields and the bosonic gauge algebras are Non-Abelian. These algebras turn out to be the bosonic algebras of [15], extended with 10-forms. These algebras also play a role in the conjectured  $E_{11}$  symmetry, which might underly M-theory [20, 21].

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