# Electromagnetic Form Factors of the Proton in the "Unphysical" Region from gamma p->pe^{+}e^{-} Reaction

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# Electromagnetic form factors of the proton in the "unphysical" region from the $\gamma p \to pe^+e^-$ reaction

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**Abstract**

Investigation of the proton form factor $F_1(q^2, W^2)$ in the time-like region $4m_e^2 < q^2 < 4M^2$ by measuring the $e^+e^-$-asymmetry in the $\gamma(p,p)e^+e^-$ reaction is proposed. Selection of "longitudinal" photons ($q^2 \ll q^2 \ll 4M^2$) suppresses contributions of $F_2$ and nucleon resonances. © 1997 Elsevier Science B.V.

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## 1. Introduction

A lot of information about the electromagnetic (EM) structure of the proton in the space-like region ($q^2 < 0$) was collected during the last decades. The experimentally determined on-mass-shell form factors (FF) at $q^2 < 0$ are in excellent agreement with the well known "dipole-fit" [1]. The information about FF in the time-like ($q^2 > 0$) region is much scantier [2]. Since on-shell time-like FF may be obtained only from $p\bar{p} \to l^+l^-$ experiments they exist only at $q^2 \geq 4M^2$. Therefore, the region $4m_e^2 < q^2 < 4M^2$ is referred to as "unphysical". Only *off-shell* FF which depend on $q^2$ and $W^2 - M^2$ virtualities ($W$ is the invariant mass of the off-shell nucleon) can be defined there. They may be investigated, for example, in the processes [3,4] $pp \to ppe^+e^-$, $pd \to {}^3\text{He}\ e^+e^-$, or $\pi p \to pe^+e^-$.

To avoid ambiguities, connected with hadronic vertices, a purely EM reaction $\gamma p \to pe^+e^-$ has been recently considered [5] under *symmetric* conditions: $\{\theta_+, E_+\} \equiv \{\theta_-, E_-\}$, where $\theta_\pm$ and $E_\pm$ are the angles and energies of the positrons/electrons. It was shown that extraction of the nucleon FF from the cross section is difficult, since at small angles the Bethe-Heitler (BH) mechanism dominates over the Virtual Compton Scattering (VCS), while at large angles contributions from nucleons and their resonances are comparable.

In general the VCS amplitude is much smaller than the BH one [5–7] and their interference may be used for separation [6,8]. Since the $e^+e^-$ pair from VCS (BH) has negative (positive) C-parity [6] only BH-VCS interference will change sign under $e^+e^-$-permutation. At *symmetric* (or inclusive) kinematics the BH-VCS interference vanishes [6].

In a special case of "almost real" ("transverse") photons, when the $e^+e^-$-invariant mass is very close

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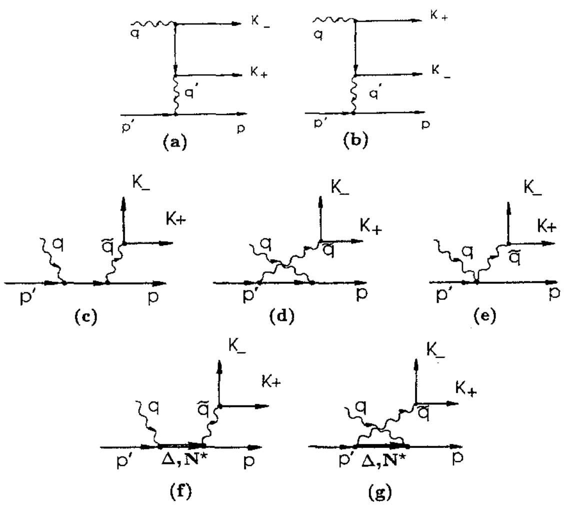


Fig. 1. Gauge invariant sets of covariant diagrams for the $\gamma(p,p')e^+e^-$ reaction.

to zero, the BH-VCS interference was used [8] to investigate the "forward Compton amplitude". Recently, it was extended to polarized hadrons [9].

In this letter we propose to investigate the Dirac form factor of the proton $F_1(q^2, W^2)$ in the "unphysical" region through a measurement of the cross section asymmetry $A_{e^+e^-}$. Selection of the "longitudinal" photons with a mass much larger than its 3-momentum suppresses contributions from the lowest nucleon resonances and the Pauli form factor $F_2$.

## 2. Structure of the VCS amplitude

We calculate the matrix element for $\gamma(p,p)e^+e^-$ reaction on the basis of the BH mechanism [6] (Figs. 1a,b) and the VCS amplitude, including: i) the Born current (Figs. 1c,d); ii) the contact current (Fig. 1e), providing nucleon current conservation; and iii) the $\Delta$(Roper)-resonance contribution (Figs. 1f,g). The Born current is $(p+\tilde{q}=p'+q)$:

$$J^B_{\mu\nu} = \Gamma_\mu(p,p+\tilde{q})S(p+\tilde{q})\Gamma_\nu(p+\tilde{q},p') + \Gamma_\nu(p,p-q)S(p-q)\Gamma_\mu(p-q,p'). \quad (1)$$

Here the initial (final) Dirac spinor $U(p')$ ($\bar{U}(p)$) is omitted, $S(p)$ is a spin 1/2 propagator, and $\Gamma_\mu(p,p')$ is $\gamma NN$-vertex, satisfying the Ward-Takachashi identity (WTI):

$$q_\mu\Gamma_\mu(p,p') = e\{S^{-1}(p) - S^{-1}(p')\}. \quad (2)$$

Note that in general the Born term (1) is not gauge invariant:

$$\tilde{q}_\mu J^B_{\mu\nu} = e\bar{U}(p)\{\Gamma_\nu(p,p-q) - \Gamma_\nu(p+\tilde{q},p')\}U(p') \neq 0. \quad (3)$$

$J^B_{\mu\nu}$ becomes a conserved current [10] only for on-shell vertices $\Gamma^{\rm on}_\mu(p,p') \equiv \Gamma_\mu(p-p')$. If $\gamma NN$-vertex is off-shell, a contact current (Fig. 1e) should be added [11]:

$$J^C_{\mu\nu} = e\int_0^1 \frac{d\lambda}{\lambda}\left\{\frac{d}{d\tilde{q}_\mu}\Gamma_\nu(p+\lambda\tilde{q},p'-(1-\lambda)\tilde{q}) + \frac{d}{dq_\nu}\Gamma_\mu(p-\lambda q,p'+(1-\lambda)q)\right\}. \quad (4)$$

The derivative of $J^C_{\mu\nu}$ (4) cancels the same derivative (3) of $J^B_{\mu\nu}$ and as a result, the total nucleon current is conserved for any on/off-shell $\gamma NN$-vertex.

There is another "recipe" [7,10] to preserve nucleon current like gauge invariant Born current, transferring off-shell effects in the $\gamma NN$-vertices to the non-Born part of the VCS amplitude. This is a well defined procedure for $q^2 \leq 0$, where *on-shell* FF exist. But in "unphysical" region there are no *on-shell* FF at all, and it makes sense to work directly with *off-shell* nucleon currents, since their $q^2$- and $W^2$-dependences are equivalent.

## 3. Structure of the vertices and "longitudinal" photons

To estimate the sensitivity of the asymmetry to the nucleon FF we will use the simplest form of the half-off-shell "reducible" $\gamma NN$-vertex [12], satisfying the WTI (2):

$$\Gamma_\mu(p,p') = F_1(q^2,W^2)\gamma_\mu + [1-F_1(q^2,W^2)]\frac{\hat{q}q_\mu}{q^2} - \frac{\sigma_{\mu\nu}q_\nu}{2M}F_2(q^2,W^2). \quad (5)$$

At the "photon point" gauge invariance demands [11]:

$$F_1(q^2=0,W^2)\equiv 1. \qquad (6)$$

As a result, its model-independent expansion at small virtualities has the form:

$$F_1(q^2,W^2)=F_1(q^2)\{1+a\frac{q^2}{M^2}\frac{W^2-M^2}{M^2} + \text{"higher-order"}\}, \qquad (7)$$

So, $q^2$- and $W^2$-dependences of $F_1$ cannot be factorized, while for $F_2(q^2,W^2)$ a separable form may be adopted. We will use an extrapolation of the "dipole-fit" to the time-like region for $F_{1,2}(q^2)$, taking $F_2$ at $W^2=M^2$. Under condition (6), the simplest contact current is proportional to a time-like vector $b_\mu=(b_0,\mathbf{0})$ and does not influence observables:

$$J^C_{\mu\nu}=e\frac{b_\nu}{bq}\{\Gamma_\mu(p-q,p')-\Gamma_\mu(p,p'+q)\},$$
$$\epsilon^{(\pm)}_\nu J^C_{\mu\nu}\sim\epsilon^{(\pm)}_\nu b_\nu\equiv 0, \qquad (8)$$

where $\epsilon^{(\pm)}_\nu=(0,\boldsymbol{\epsilon}^\pm)$ is polarization vector of the real photon. Using Eqs. (2),(3), (5)-(8) we get: $\tilde q_\mu J^B_{\mu\nu}=\tilde q_\mu J^C_{\mu\nu}=0$ and $(J^B_{\mu\nu}+J^C_{\mu\nu})q_\nu=0$.

To estimate the contribution from resonances (Figs. 1f,g) we took into account only the lowest ones: the $\Delta$-$P^{33}(1230)$ and Roper-$P^{11}(1440)$. The currents corresponding to diagrams 1f,g, which are gauge invariant themselves, were calculated using the same model as for the VCS amplitudes [7] in the space-like region. We excluded the admixture of $J=1/2$-states from the Rarita-Schwinger propagator for off-shell $J=3/2$-particles [13] in order to avoid "double counting" of effective off-shell nucleon states, neglected $E2$-transitions in $\gamma N\Delta$-vertex, and took corresponding couplings at $q^2=0$.

The conventional way to separate contributions from "longitudinal" ($\epsilon^L$) and "transverse" ($\epsilon^T$) photons is to decompose the cross section into different structure functions [14], which are independent of the lepton variables. But in our case the total cross section cannot be presented in terms of such structure functions, since the BH- and VCS-amplitudes involve different leptonic tensors. To estimate the role of the $\epsilon^L$- and $\epsilon^T$-photons, let us consider the structure of their contributions in the c.m. system (Fig. 2):

$$\epsilon^T J^{\gamma^* NN(N^*)}\sim\chi^+\boldsymbol{\epsilon}^T[\boldsymbol{\sigma}\times\tilde{\mathbf{q}}]\chi,$$

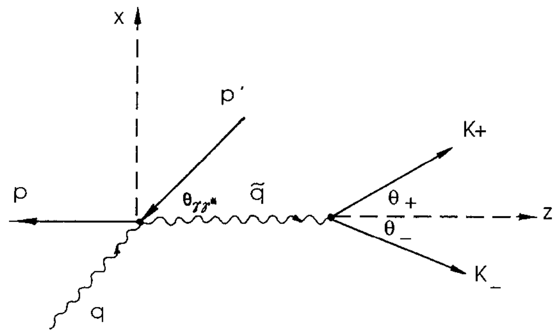


Fig. 2. Kinematics of di-lepton photoproduction on the proton.

$$\epsilon^T J^{\gamma^* N\Delta}\sim\chi^+[\boldsymbol{\epsilon}^T\times\tilde{\mathbf{q}}]\boldsymbol{\phi},$$
$$\epsilon^L J^N\sim\{F_1+(\tilde q^2/4M^2)F_2\}\sqrt{\tilde q^2}\chi^+\chi, \qquad (9)$$

Since the contributions from the $\epsilon^L$- and $\epsilon^T$-photons are proportional (9) to $\tilde q^2$ and $\tilde{\mathbf{q}}^2$, respectively, the "longitudinal" one can be enhanced under condition:

$$\tilde{\mathbf{q}}^2\ll\tilde q^2\ll 4M^2. \qquad (10)$$

The first inequality in (10) suppresses resonances, exited mainly by "transvers" photons, while the second one suppresses $F_2$ and "monopole" transitions in the $\gamma NN^*$-vertex.

## 4. $A_{e^+e^-}$-asymmetry and numerical results

The c.m.s. cross section has the structure ($K^{-1}_{-+}=E_-+E_+\cos(\theta)+E_p$):

$$\frac{d^5\sigma^{\gamma p\to pe^+e^-}_{-+}}{dE_-d\Omega_-d\Omega_+}=\frac{\alpha^3}{2\pi^2}\frac{E_+E_-}{s-M^2}\times K_{-+}\{T^{\rm BH}_{-+}+T^{\rm int}_{-+}+T^{\rm VCS}_{-+}\},$$

where $s=(p'+q)^2=(p+\tilde q)^2$, $E_p$-energy of the final proton, and $T^i_{-+}$ correspond to the square of BH/VCS amplitudes, which are $C$-even under permutation of $e^+e^-$ ($T^{\rm BH,VCS}_{-+}=T^{\rm BH,VCS}_{+-}$), and their interference, which is $C$-odd ($T^{\rm int}_{-+}=-T^{\rm int}_{+-}$). Therefore the asymmetry will be proportional to the interference of the BH-VCS amplitudes:

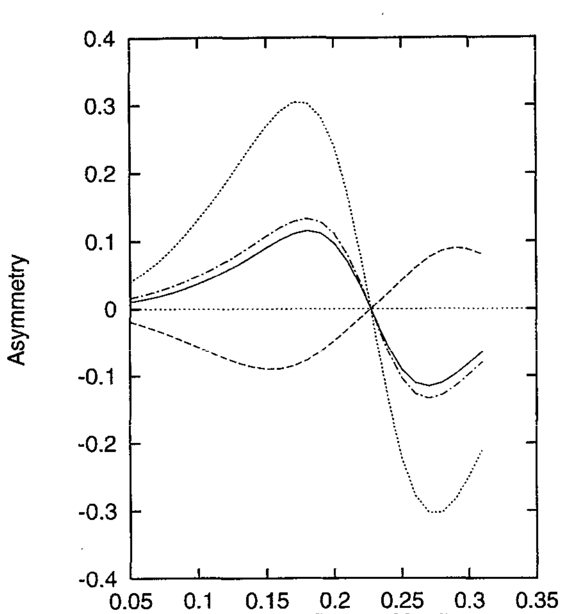


Fig. 3. $A_{e^+e^-}$-asymmetry as a function of the electron energy. The solid (dotted/dashed) curve corresponds to $a = 0$ ($\pm 1$) in Eq. (7) and was obtained without resonance contributions. The dashed-dotted line includes resonance contributions (for $a = 0$).

$$A_{e^+e^-} = \frac{K_{+-}^{-1}\sigma_{+-} - K_{-+}^{-1}\sigma_{-+}}{K_{+-}^{-1}\sigma_{+-} + K_{-+}^{-1}\sigma_{-+}}\,,$$

$$\equiv \frac{T_{-+}^{\rm int}}{T_{-+}^{\rm BH} + T_{-+}^{\rm VCS}} \approx T_{-+}^{\rm int}/T_{-+}^{\rm BH}. \qquad (11)$$

The simplest kinematics for the $\gamma p \to pe^+e^-$ reaction is given in Fig. 2 (3-momentum of the virtual photon $\vec{q}$ is fixed along the Z-axis). The set of independent variables is: $\theta_{\gamma\gamma^*}$, $\theta = \theta_+ + \theta_-$, $E_-$, $E_\gamma$ and the $\phi$-angle between the electron and hadron planes.

The results for $A_{e^+e^-}$ are presented in Fig. 3 as a function of the electron energy at $E_\gamma = 0.575$ GeV, $\theta = 150°$, and $\theta_{\gamma\gamma^*} = 0°$ ($\tilde{q}^2 \simeq 0.2$ GeV$^2$/c$^2$, $W^2 \simeq 1.6$ GeV$^2$). The solid curve shows the calculation without off-shell effects ($a = 0$ in (7)), while the dotted (dashed) curve corresponds to $a = 1$ ($a = -1$). The strong sensitivity of the asymmetry to the sign of the "slope" of the off-shell form factor $F_1(q^2, W^2)$ (its decrease (increase) at $W < M$ ($W > M$)) is connected with the "competition" of the s- and u-channel diagrams (Figs. 1c,d), which contain $F_1(q^2, W^2)$ with $W^2 > M^2$ and $W^2 < M^2$, respectively. The difference between the dashed-dotted and solid lines reflects the insignificant role of resonances ($\sim$ 2–3%). The $F_2$-contribution is too small ($\sim 1\%$) to be shown. The VCS is also about 1–2% of the BH and approximation (11) for $A_{e^+e^-}$ is reliable. Note, conditions (10), existing only at $\tilde{q}^2 > 0$, are satisfied in our case: $\vec{q}^2/\tilde{q}^2 \simeq 0.1$; $\tilde{q}^2/4M^2 \simeq 0.05$, and the zero in the asymmetry (Fig. 3) corresponds to the *symmetric* kinematics.

As for the $\tilde{q}^2$-dependence, we compared two models for $F_1(\tilde{q}^2)$, namely a "dipole-fit" and $F_1 = 1$, which have shown a difference about 5–6%. Such a not very strong sensitivity of the asymmetry to the $\tilde{q}^2$-virtuality may be ascribed to two facts: i) s- and u-channels depend on $F_1(\tilde{q}^2, W^2)$ at the same $\tilde{q}^2$, and ii) the use of the relatively "light" photons $\tilde{q}^2/M^2 \sim 0.2 \ll 1$. Indeed, at such small photon masses the $\rho$-meson pole is not visible [5].

The vertex (5), considered above, corresponds to the assumption: $F_i^{++} = F_i^{+-} = F_i$ in the general structure of the half-off-shell $\gamma NN$-current [15]. Of course, this is a good approximation only for $F_1$, since condition (7) exists for $F_1^{+-}$ also. However, it is not so evident [15,16] for $F_2$, but its contribution is negligible under conditions (10).

Finally we note that since $T^{\rm VCS}/T^{\rm BH} \sim$ 1–2%, *Real Radiative Corrections* (RRC $\sim \alpha^2$) and *Virtual* (VRC $\sim \alpha^{5/2}$) ones should be considered [17] only to BH amplitude ($\sim \alpha^{3/2}$). The RRC does not interfere [17] with BH or VCS and will not contribute to the numerator (11) of $A_{e^+e^-}$. The C-even VRC-BH interference also does not contribute, and the C-odd VRC-VCS interference is negligible in comparison with BH-VCS one. Since, $|{\rm BG}|^2 \sim \alpha^3$ and $|{\rm RRC}|^2 \sim$ VRC-BH $\sim \alpha^4$, they are of the same order of magnitude as $|{\rm VCS}|^2$ and may also be neglected in the denominator (11). Therefore, if VCS is only few percent of BH, we do not expect strong effects in the asymmetry from radiative corrections.

## 5. Conclusions

To summarize, the $e^+e^-$-asymmetry in the $\gamma p \to pe^+e^-$ reaction appeared to be sensitive to the off-shell proton form factor $F_1(q^2, W^2)$ in the time-like region: $4m_e^2 < q^2 < 4M^2$. Special selection of the kinematic conditions (10) suppresses resonance contributions and $F_2$. So, measurements of $A_{e^-e^+}$ for "longitudinal" photons may give practically model-independent information on the Dirac form factor in "unphysical" region. Note that the definition of the off-shell FF is not unique: it depends on the representation of the full amplitude. In principle, off-shell effects may be distributed over the FF and the contact currents in different way, or even moved completely from the FF to the contact current, which also depends on the "trajectory" [11]. This will be considered in a following paper.

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