Passivity-Based Lag-Compensators with Input Saturation for Mechanical Port-Hamiltonian Systems Without Velocity Measurements

Hamada, Kiyoshi; Borja, Pablo; Scherpen, Jacquelien M. A.; Fujimoto, Kenji; Maruta, Ichiro

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Passivity-Based Lag-Compensators With Input Saturation for Mechanical Port-Hamiltonian Systems Without Velocity Measurements

Kiyoshi Hamada, Pablo Borja, *Member, IEEE*, Jacquelien M. A. Scherpen, *Senior Member, IEEE*, Kenji Fujimoto, *Member, IEEE*, and Ichiro Maruta, *Member, IEEE*

***Abstract*—In this letter, we propose a passivity-based control technique, where the resulting controllers can be interpreted as lag-compensators for nonlinear mechanical systems described in the port-Hamiltonian framework. The proposed methodology considers a dynamic controller such that the relationship between the control input and the error signal of interest can be expressed in terms of a transfer function. Accordingly, the control gains can be tuned through a frequency analysis approach. Additionally, two practical advantages of the resulting controllers are that they do not require velocity measurements, and they can cope with input saturation. We illustrate the applicability of the proposed methodology through the stabilization of a planar manipulator, where the experimental results corroborate the effectiveness of the technique.**

***Index Terms*—Control applications, Lyapunov methods, stability of nonlinear systems.**

I. Introduction

THE PORT-HAMILTONIAN (pH) framework has proven to be suitable to represent a broad class of mechanical systems [1], [2]. An advantage of the pH approach is the explicit representation of physical phenomena and concepts such as energy, interconnection patterns, and dissipation, which may provide some intuition to ease the analysis of the system and the control design process. Due to the energy-based nature of the pH models, passivity-based control (PBC) techniques arise as a natural option to devise controllers to stabilize these systems [3], where the control design process consists of two steps: energy-shaping and damping injection.

Concerning the stabilization of mechanical systems via PBC techniques, the literature is vast, e.g., [4]–[10]. However, most of PBC methods focus only on stabilizing the system under study. Therefore, there is no clear guidelines on how to tune the controller gains to ensure desired responses of the closed-loop system. To address this problem, in [11], the authors propose a dynamic extension, where the dynamics of the new state are designed such that it is possible to find a transfer function that relates the control input with an error signal of interest. Accordingly, the control gains can be chosen by performing a frequency analysis. In this particular approach, the resulting controllers admit a lead-compensator interpretation. Moreover, in [11], the authors provide some guidelines on how to select the controller gains to remove oscillations in the closed-loop system response. It is noteworthy that, while this control approach can improve the responsiveness of the closed-loop system, it cannot reduce the steady-state error since these compensators cannot change the characteristics of low-frequency signals. An alternative to overcome this issue is given by the so-called lag-compensators, which amplify the input signals at low frequencies. This property makes it possible to reduce the steady-state error without changing the responsiveness property [12].

In this letter, we propose a PBC approach to stabilize nonlinear mechanical systems, where the controllers can be interpreted as lag-compensators. Therefore, the resulting controllers can effectively reduce the steady-state error while mitigating the windup phenomenon often exhibited by integral control [13]. To this end, we propose a dynamic extension such that the pH structure is preserved for the closed-loop system, which eases the stability proof. Moreover, the proposed control methodology does not require velocity measurements and can deal with input constraints by naturally saturating the control signals.

The rest of this letter is organized as follows. In Section II, we introduce the pH representation of mechanical systems, the problem formulation, and briefly revisit some previous results regarding PBC techniques with dynamic extension. Next, in Section III we propose a passivity-based lag-compensator and a modified passivity-based lag-compensator where the controller is saturated. In Section IV, we illustrate experimental results of the implementation of the lag-compensators in a two degrees-of-freedom (DoF) planar manipulator with flexible joints. We summarize this letter in Section V.

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Kiyoshi Hamada, Kenji Fujimoto, and Ichiro Maruta are with the Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Kyoto 615-8540, Japan (e-mail: hamada.kiyoshi.22s@st.kyoto-u.ac.jp).

Pablo Borja and Jacquelien M. A. Scherpen are with the Jan C. Willems Center for Systems and Control, ENTEG-DTPA, Faculty of Science and Engineering, University of Groningen, 9747 AG Groningen, The Netherlands (e-mail: l.p.borja.rosales@rug.nl; j.m.a.scherpen@rug.nl).

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Notation: The symbol I_n represents the $n \times n$ identity matrix, and $0_{n\times m}$ is the $n \times m$ matrix of zeros. The Euclidean weighted-norm is expressed as $\|x\|_A^2 := x^\top Ax$. The differential operator is defined as $\nabla_x f := \frac{\partial f}{\partial x}$. The symbol A_i represents the (i, i)-th element of the matrix A or the i-th element of the vector A. A diagonal matrix A is expressed as $\mathrm{diag}(A_i)_{i=1}^n$, where $(A_i)_{i=1}^n = (A_1, \ldots, A_n)$. $\mathcal{L}[a(t)]$ denotes the Laplace transformation of $a(t)$.

II. Problem Setting and Previous Results

Let us consider mechanical systems whose behavior is represented by

$$\begin{pmatrix}\dot q\\ \dot p\end{pmatrix} = \begin{pmatrix}0_{n\times n} & I_n\\ -I_n & -D(q,p)\end{pmatrix}\begin{pmatrix}\nabla_q H(q,p)\\ \nabla_p H(q,p)\end{pmatrix} + \begin{pmatrix}0_{n\times m}\\ G\end{pmatrix}u,$$
$$H(q,p) = \frac{1}{2}p^\top M(q)^{-1}p + V(q), \tag{1}$$

where $q, p \in \mathbb{R}^n$ are the generalized positions and momenta, respectively, $u \in \mathbb{R}^m$ is the input vector, with $n < m$, $D : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the positive definite symmetric damping matrix, $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ is the Hamiltonian function of the system, where $V : \mathbb{R}^n \to \mathbb{R}_+$ is the potential energy of the system and M:$\mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the positive definite inertia matrix, and the input gain matrix G is defined as

$$G := \begin{pmatrix}0_{\ell\times m}\\ I_m\end{pmatrix}; \quad \ell := n - m. \tag{2}$$

Hence, we can split the state vector as follows

$$q = \begin{pmatrix}q_{\mathrm{u}}\\ q_{\mathrm{a}}\end{pmatrix}, \quad p = \begin{pmatrix}p_{\mathrm{u}}\\ p_{\mathrm{a}}\end{pmatrix}, \tag{3}$$

where

$$q_{\mathrm{u}} := G^\perp q,\ q_{\mathrm{a}} := G^\top q, \quad p_{\mathrm{u}} := G^\perp p,\ p_{\mathrm{a}} := G^\top p, \tag{4}$$

with $G^\perp = \begin{pmatrix}I_\ell & 0_{\ell\times m}\end{pmatrix}$.

To formulate the problem under study, we first define the set of assignable equilibria for (1), which is given by

$$\mathcal{E} = \{q \in \mathbb{R}^n \mid \nabla_{q_u} V(q) = 0_\ell\}, \tag{5}$$

and we define the error $q^{\mathrm{e}} = q - q^*$ and $q_{\mathrm{a}}^{\mathrm{e}} = G^\top q^{\mathrm{e}}$, where $q^* \in \mathcal{E}$. Then, the problem under study can be formulated as follows.

Problem Setting: Given the mechanical system (1) and the desired equilibrium point $(q^*, 0_n)$, find a controller u that renders asymptotically stable $(q^*, 0_n)$ while ensuring that:

- No velocity measurements are required to achieve the control task.
- There is a systematic method to select the control gains to reduce the steady-state error caused by modeling errors of nonlinear friction.

A. Some Previous Results on PBC With Dynamic Extension

In this section, we briefly revisit the results reported in [11]–[14], where the reported controllers are suitable to suppress oscillations or reject disturbances. The main idea of these methods is to propose a dynamic extension $x_{\mathrm{c}} \in \mathbb{R}^m$ and a dynamic control law of the form

$$u = f^u(q, p, x_{\mathrm{c}}), \tag{6}$$
$$\dot x_{\mathrm{c}} = f^{x_{\mathrm{c}}}(q, p, x_{\mathrm{c}}), \tag{7}$$

such that the closed-loop system takes the form

$$\dot\xi = \begin{pmatrix}0_{n\times n} & I_n & F_{13}\\ -I_n & -D(q,p) & F_{23}\\ -F_{13}^\top & -F_{23}^\top & F_{33}\end{pmatrix}\nabla_\xi H_{\mathrm{d}}(\xi), \tag{8}$$
$$H_{\mathrm{d}}(\xi) = H(q,p) + \bar H(\xi), \tag{9}$$

where $\xi = (q^\top, p^\top, x_{\mathrm{c}}^\top)^\top$, $F_{13} \in \mathbb{R}^{n\times m}$, $F_{23} \in \mathbb{R}^{n\times m}$, $F_{33} \in \mathbb{R}^{m\times m}$. Following this approach, passivity-based controllers that can be interpreted as lead-compensators are reported in [11], while in [15], a kind of integrator is proposed for removing matched disturbances. Additionally, in [11], [14], [16], the dynamic extension removes the necessity of velocity measurements to inject damping into the closed-loop system and ensure the asymptotic convergence towards the desired equilibrium. Inspired by these results, in the following section, we propose a new PBC methodology where a controllers can be interpreted as a lag-compensators.

III. Proposed Method

The lead-compensator in [11] is effective for removing oscillations without measuring velocities, but cannot deal with steady-state errors. On the other hand, the integrator in [15] ensures that the steady-state error equals zero and is suitable to reject some disturbances. Alas, this controller requires velocity measurements. To address these issues, in this section, we present the main contribution of this letter, namely, a passivity-based lag-compensator that can reduce the steady-state error without measuring velocities. To this end, we implement a dynamic extension that leads to a pH system different from (8).

A. Passivity-Based Lag-Compensator

The following theorem introduces a dynamic extension and a control law such that the closed-loop system admits a pH representation. Additionally, it provides conditions to ensure the stability of the desired equilibrium.

Theorem 1: Consider system (1), the virtual state $x_{\mathrm{c}} \in \mathbb{R}^m$ with nonlinear dynamics

$$\dot x_{\mathrm{c}} = -\tilde D \nabla_{x_{\mathrm{c}}} \bar H(q_{\mathrm{a}}, x_{\mathrm{c}}), \tag{10}$$

and the nonlinear control law

$$u = -\nabla_{q_{\mathrm{a}}} \bar H(q_{\mathrm{a}}, x_{\mathrm{c}}) - 2\nabla_{x_{\mathrm{c}}} \bar H(q_{\mathrm{a}}, x_{\mathrm{c}}). \tag{11}$$

Then, the closed-loop system takes the form of a pH system

$$\dot\xi = (\mathcal{J} - \mathcal{D})\nabla_\xi H_{\mathrm{d}}(\xi), \tag{12}$$
$$\mathcal{J} = \begin{pmatrix}0_{n\times n} & I_n & 0_{n\times m}\\ -I_n & 0_{n\times n} & -G\\ 0_{m\times n} & G^\top & 0_{m\times m}\end{pmatrix}, \tag{13}$$
$$\mathcal{D} = \begin{pmatrix}0_{n\times n} & 0_{n\times n} & 0_{n\times m}\\ 0_{n\times n} & D & G\\ 0_{m\times n} & G^\top & \tilde D\end{pmatrix}, \tag{14}$$

if the following condition holds.

$$\begin{pmatrix}D(q,p) & G\\ G^\top & \tilde D\end{pmatrix} \succeq 0, \tag{15}$$

where $\tilde D \in \mathbb{R}^{m\times m}$ is a positive definite symmetric matrix, and $H_{\mathrm{d}}(\xi) := H(q,p) + \bar H(\xi)$, with $\bar H(\xi)$ to be defined. Furthermore, the desired equilibrium point $\xi^* =$

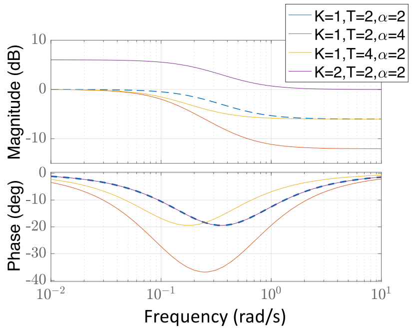

Fig. 1. Bode plot of the lag-compensator (24).

$(q^{*\top}, 0_{n\times 1}^\top, 0_{m\times 1}^\top)^\top$ is *asymptotically stable* if the following conditions hold.

C1. $\begin{pmatrix} D(q,p) & G \\ G^\top & \tilde{D} \end{pmatrix} \succ 0.$

C2. $H_d(\xi)$ has an isolated minimum at $\xi = \xi^*$.

C3. $\nabla_p H_d(\xi) = 0_{n\times 1}, \nabla_{x_c} H_d(\xi) = 0_{m\times 1} \Rightarrow q = q^*, x_c = 0_{m\times 1}$. (16)

Proof: Note that

$$\nabla_q H_d(\xi) = \nabla_q H(q,p) + \nabla_q \bar{H}(q_a, x_c), \tag{17}$$
$$\nabla_p H_d(\xi) = \nabla_p H(q,p),\ \nabla_{x_c} H_d(\xi) = \nabla_{x_c} \bar{H}(q_a, x_c). \tag{18}$$

By substituting (11) in (1), we have

$$\dot{q} = \nabla_p H(q,p) = \nabla_p H_d(\xi), \tag{19}$$
$$\begin{aligned}\dot{p} &= -\nabla_q H(q,p) - D(q,p)\nabla_p H(q,p) + Gu \\ &= -\nabla_q H(q,p) - D(q,p)\nabla_p H(q,p) \\ &\quad + G\big(-\nabla_{q_a}\bar{H}(q_a,x_c) - 2\nabla_{x_c}\bar{H}(q_a,x_c)\big) \\ &= -\nabla_q H_d(\xi) - D(q,p)\nabla_p H_d(\xi) - 2G\nabla_{x_c} H_d(\xi),\end{aligned} \tag{20}$$

and (10) leads to

$$\dot{x}_c = -\tilde{D}\nabla_{x_c}\bar{H}(q_a,x_c) = -\tilde{D}\nabla_{x_c} H_d(\xi). \tag{21}$$

Hence the dynamic extension (11) and (10) transforms (1) into (12), and if (15) holds, $\mathcal{D} \succeq 0$ holds and this shows that (12) is a pH system from the fact that $\mathcal{J}^\top = -\mathcal{J}$. Hereafter, we omit the arguments q, p in D in this proof for simplicity. It follows from (13) and (14) that

$$\dot{H}_d = -(\nabla_\zeta H_d(\xi))^\top \begin{pmatrix} D & G \\ G^\top & \tilde{D} \end{pmatrix} \nabla_\zeta H_d(\xi) \le 0.$$

Moreover, if C1 holds, $\dot{H}_d = 0$ if and only if $\nabla_p H_d(\xi) = 0_{n\times 1}$ and $\nabla_{x_c} H_d(\xi) = 0_{m\times 1}$, where $\zeta = (p^\top, x_c^\top)^\top$. Hence, it follows from the assumptions that Krasovskii-Barbashin theorem [17] proves asymptotic stability. ■

The following theorem establishes a linear relationship between the control input and the error in positions such that the controller (11) with (10) admits a lag-compensator interpretation.

Theorem 2: Design the function $\bar{H}(q_a, x_c)$ as

$$\bar{H}(q_a, x_c) = \frac{1}{2}\|q_a^e\|_{K_P}^2 + \frac{1}{2}\|x_c - q_a^e\|_{K_I}^2, \tag{22}$$

and $\tilde{D}$ as $\tilde{D} = R_c$, where $K_P, K_I, R_c \in \mathbb{R}^{m\times m}$ are diagonal positive definite matrices. When $K_{P,i} - K_{I,i} > 0$ $(i = 1, 2, \ldots, m)$ holds, the controller (11) with (10) represents a lag-compensator, where the relation between q_a^e and u

$$\mathscr{U}(s) = \mathrm{diag}(G_i(s))_{i=1}^m \mathscr{Q}_a^e(s), \tag{23}$$

is given by

$$\begin{aligned} G_i(s) &= K_i\frac{T_i s + 1}{\alpha_i T_i s + 1}, \\ K_i &= K_{P,i},\ T_i = \frac{K_{P,i} - K_{I,i}}{K_{P,i}K_{I,i}R_{c,i}},\ \alpha_i = \frac{K_{P,i}}{K_{P,i} - K_{I,i}}, \end{aligned} \tag{24}$$

where $\mathscr{Q}_a^e(s) = \mathcal{L}[q_a^e(t)]$, $\mathscr{U}(s) = \mathcal{L}[u(t)]$.

Proof: The dynamic extension (11) and (10) with (22) is calculated as

$$\begin{aligned} u &= -K_P q_a^e - K_I(x_c - q_a^e), \\ \dot{x}_c &= -R_c K_I(x_c - q_a^e). \end{aligned} \tag{25}$$

Since the matrices K_P, K_I, R_c are diagonal, for each element,

$$\mathscr{U}_i = -K_{P,i}\mathscr{Q}_{a,i}^e - K_{I,i}(\mathscr{X}_{c,i} - \mathscr{Q}_{a,i}^e), \tag{26}$$
$$s\mathscr{X}_{c,i} = -R_{c,i}K_{I,i}(\mathscr{X}_{c,i} - \mathscr{Q}_{a,i}^e), \tag{27}$$

hold, where $\mathscr{U}_i(s) = \mathcal{L}[u_i(t)]$, $\mathscr{X}_{c,i}(s) = \mathcal{L}[x_{c,i}(t)]$, $\mathscr{Q}_{a,i}^e(s) = \mathcal{L}[q_{a,i}^e(t)]$. Hence we have the following relation

$$\begin{aligned} \mathscr{U}_i &= -K_{P,i}\mathscr{Q}_{a,i}^e - K_{I,i}\left(\frac{R_{c,i}K_{I,i}}{s + R_{c,i}K_{I,i}}\mathscr{Q}_{a,i}^e - \mathscr{Q}_{a,i}^e\right) \\ &= -\frac{(K_{P,i} - K_{I,i})s + K_{P,i}R_{c,i}K_{I,i}}{s + R_{c,i}K_{I,i}}\mathscr{Q}_{a,i}^e. \end{aligned} \tag{28}$$

It follows from (28) that (23) with (24) holds. ■

As Theorem 2 claims, the value of α_i in (24) takes more than one if $K_{P,i} - K_{I,i} > 0$ $(i = 1, 2, \ldots, m)$ holds, which implies that the controller (11) with (10) works as a lag-compensation. Figure 1 shows the bode plot of the transfer function (24), where the values of K_i, T_i, α_i are varied as in the legends of the figure. As in the figure, the lag-compensator keeps the gain high at low frequencies and low at high frequencies. Hence, this compensator can improve the steady-state characteristics. The tuning of the controller can also be done intuitively. It follows from (28) that

$$K_{P,i} = K_i,\ K_{I,i} = \frac{\alpha_i - 1}{\alpha_i}K_i,\ R_{c,i} = \frac{1}{(\alpha_i - 1)T_i K_i}, \tag{29}$$

hold, so the parameters in (22) and R_c are decided by specifying K, α, T. When tuning the gains, one can choose K, α, T appropriately, referring the bode plot of the lag-compensator. In practical applications, inputs are often restricted. In the next subsection, we propose another passivity-based controller that represents a passivity-based lag-compensator dealing with input saturation.

B. Passivity-Based Lag-Compensator With Input Saturation

In [16], Wesselink *et al.* propose a lead-compensator considering input saturation. Inspired by this method, we propose a passivity-based lag-compensator that takes into account input saturation.

Theorem 3: Select the function $\bar{H}(q_a, x_c)$ as

$$\bar{H}(q_a, x_c) = \phi^1_{(K_P)}(q_a^e) + \phi^2_{(K_I)}(x_c - q_a^e), \tag{30}$$

and $\tilde{D}$ as $\tilde{D} = R_{\rm c}$, where $K_{\rm P}, K_{\rm I}, R_{\rm c} \in \mathbb{R}^{m\times m}$ are diagonal positive definite matrices and $\phi^{l}_{(\cdot)}(\cdot)$ $(l = 1, 2)$ are given as

$$\phi^{l}_{(C)}(z) = \sum_{i}^{m} C_i \frac{\alpha_{l,i}}{\beta_{l,i}} \log(\cosh(\beta_{l,i} z_i)), \tag{31}$$

with design parameters $\alpha_{l,i} > 0$, and $\beta_{l,i} > 0$. Then, the input (11) always satisfies

$$|u_i| \le K_{{\rm P},i}\alpha_{1,i} + K_{{\rm I},i}\alpha_{2,i}. \tag{32}$$

In addition, the linear approximation of the controller (11) with (10) represents a lag-compensator under the condition

$$K_{{\rm P},i}\alpha_{1,i}\beta_{1,i} - K_{{\rm I},i}\alpha_{2,i}\beta_{2,i} > 0.$$

Proof: The input (11) is calculated as

$$\begin{aligned} u_i &= -\nabla_{q_{{\rm a},i}} \phi^{1}_{(K_{\rm P})}(q^{\rm e}_{\rm a}) - \nabla_{q_{{\rm a},i}} \phi^{2}_{(K_{\rm I})}(x_{\rm c} - q^{\rm e}_{\rm a}) \\ &\quad - 2\nabla_{x_{\rm c}} \phi^{2}_{(K_{\rm I})}(x_{\rm c} - q^{\rm e}_{\rm a}) \\ &= -K_{{\rm P},i}\alpha_{1,i} \tanh(\beta_{1,i} q^{\rm e}_{{\rm a},i}) \\ &\quad - K_{{\rm I},i}\alpha_{2,i} \tanh(\beta_{2,i}(x_{{\rm c},i} - q^{\rm e}_{{\rm a},i})). \end{aligned} \tag{33}$$

Since $|\tanh(\cdot)| \le 1$, it follows from (33) that

$$|u_i| \le K_{{\rm P},i}\alpha_{1,i} + K_{{\rm I},i}\alpha_{2,i}.$$

Maclaurin series of $\tanh(z)$ is $\tanh(z) = z + o(\|z\|)$ as $z \to 0$, hence, if $\beta_{1,i} q^{\rm e}_{\rm a}$ and $\beta_{2,i}(x_{{\rm c},i} - q^{\rm e}_{{\rm a},i})$ are small enough that $\tanh(\cdot)$ can be linearly approximated, the input (11) and the dynamics (10) are given as

$$\begin{aligned} u_i &= -K_{{\rm P},i}\alpha_{1,i}\beta_{1,i} q^{\rm e}_{{\rm a},i} - K_{{\rm I},i}\alpha_{2,i}\beta_{2,i}(x_{{\rm c},i} - q^{\rm e}_{{\rm a},i}), \\ \dot{x}_{{\rm c},i} &= -R_{{\rm c},i} K_{{\rm I},i}\alpha_{2,i}\beta_{2,i}(x_{{\rm c},i} - q^{\rm e}_{{\rm a},i}). \end{aligned} \tag{34}$$

Replacing $K_{{\rm P},i}\alpha_{1,i}\beta_{1,i}$ and $K_{{\rm I},i}\alpha_{2,i}\beta_{2,i}$ with $\tilde{K}_{{\rm P},i}$ and $\tilde{K}_{{\rm I},i}$ immediately confirms that (34) represents a lag-compensator, and this completes the proof. ■

The parameters of the controller (11)-(10) with (30) are designed in the same way as the proposed lag-compensator by specifying K, T, and α of (24). If the input is saturated as $|u_i| \le \mathcal{U}_{\rm max_i}$, the parameters $\alpha_{1,i}$ and $\alpha_{2,i}$ are chosen so that $(K_{{\rm P},i}\alpha_{1,i} + K_{{\rm I},i}\alpha_{2,i}) \le \mathcal{U}_{\rm max_i}$ is satisfied. The parameters $\beta_{1,i}$ and $\beta_{2,i}$, that affect the region where the controller can be linearly approximated, can be freely chosen.

IV. Practical Implementation of the Passivity-Based Lag-Compensator

To confirm the effectiveness of the proposed controllers in Section III, this section shows experimental results of the implementation of the controllers in the 2 DoF manipulator by Quanser depicted in Fig. 2. The first experiment consists in applying the passivity-based lag-compensator to the manipulator and corroborate its suitability to deal with steady-state errors by choosing appropriate gains. The second experiment compares the performance of a PID controller and the passivity-based lag compensator, where the inputs are saturated.

In these experiments, only the positions q are measured, and the inputs are the currents supplied to the motors. Note that, strictly speaking, the control inputs we analytically devise should be torques. However, there exists a static relationship between the torque of each motor and the corresponding current. Such relationships are considered during the practical implementation of the controllers. We refer the reader to [18] for further details.

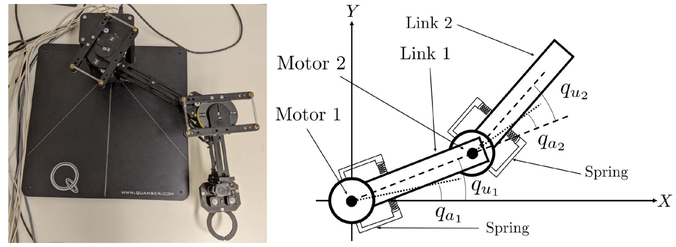

Fig. 2. 2 DoF serial flexible joint by Quanser and its corresponding schematic.

TABLE I
System Parameters

$d_{\rm u_1}$	0.38 $[N\cdot m\cdot s/\text{rad}]$	$d_{\rm u_2}$	0.30 $[N\cdot m\cdot s/\text{rad}]$
$d_{\rm a_1}$	0.30 $[N\cdot m\cdot s/\text{rad}]$	$d_{\rm a_2}$	0.14 $[N\cdot m\cdot s/\text{rad}]$
a_1	0.068 $[kg\cdot m^2]$	a_2	0.013 $[kg\cdot m^2]$
b	0.018 $[kg\cdot m^2]$		
$\mathcal{I}_1$	0.042 $[kg\cdot m^2]$	$\mathcal{I}_2$	0.0070 $[kg\cdot m^2]$
$K_{\rm s_1}$	9.4 $[N\cdot m/\text{rad}]$	$K_{\rm s_2}$	4.2 $[N\cdot m/\text{rad}]$

A. Control Design

The 2 DoF planar robot with flexible joints in Fig. 2 admits a pH representation of the form (1) where

$$\begin{aligned} D &= \begin{pmatrix} D_{\rm u} & 0_{2\times 2} \\ 0_{2\times 2} & D_{\rm a} \end{pmatrix}, \\ D_{\rm u} &= \mathrm{diag}\big(d_{\rm u_1}, d_{\rm u_2}\big), D_{\rm a} = \mathrm{diag}\big(d_{\rm a_1}, d_{\rm a_2}\big), \\ M(q) &= \begin{pmatrix} M_{\rm u}(q) & 0_{2\times 2} \\ 0_{2\times 2} & M_{\rm a} \end{pmatrix}, M_{\rm a} = \mathrm{diag}(\mathcal{I}_1, \mathcal{I}_2), \\ M_{\rm u}(q) &= \begin{pmatrix} a_1 + a_2 + 2b\cos\big(q_{\rm u_2}\big) & a_2 + b\cos\big(q_{\rm u_2}\big) \\ a_2 + b\cos\big(q_{\rm u_2}\big) & a_2 \end{pmatrix}, \\ V(q) &= \frac{1}{2}\|q_{\rm u} - q_{\rm a}\|^2_{K_{\rm s}}, K_{\rm s} = \mathrm{diag}\big(K_{\rm s_1}, K_{\rm s_2}\big). \end{aligned}$$

For this system, $n = 4$ and $m = 2$. Furthermore, $q_{\rm a_1}$ and $q_{\rm a_2}$ denote the angle of the first and second motor, $q_{\rm u_1}$ and $q_{\rm u_2}$ denote the angle of the first and second link, respectively, where each link is connected to a motor through springs. The parameters of this system are provided in Table I.

Note that the assignable equilibria for this system are characterized by the constraint $q_{\rm a} = q_{\rm u}$. Accordingly, the control objective is to stabilize the manipulator at the desired configuration

$$q_{\rm a} = q_{\rm u} = q^*_{\rm a}, \tag{35}$$

where $q^*_{\rm a} \in \mathbb{R}^2$. To this end, the following corollary proves that the passivity-lag compensator proposed in Section III solves the control problem.

Corollary 1: The desired equilibrium positions of the system defined in (35) are asymptotically stabilized by the controller (11)-(10) with (22) or (30) if $R_{{\rm c},i} > 1/D_{{\rm a},i}$ holds.

Proof: We only prove the case of (22) due to space constraints. Since the pH system (1) is transformed into the new pH system (12) by the controller, if an isolated minimum of $H_{\rm d}(\xi)$ is the equilibrium point $q^* := (q^*_{\rm a}, q^*_{\rm a})^\top$ and if (16) holds, the desired positions (35) are asymptotically stable. We first check whether (16) holds. Define

$$\hat{D} = \begin{pmatrix} D_{\rm u} & 0_{\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & D_{\rm a} & I_2 \\ 0_{2\times 2} & I_2 & R_{\rm c} \end{pmatrix}. \tag{36}$$

Since $D_u \succ 0$ hold, the condition (16), that can be written as $\hat{D} \succ 0$, holds if and only if

$$\begin{pmatrix} D_a & I_2 \\ I_2 & R_c \end{pmatrix} \succ 0. \tag{37}$$

From the Schur complement condition, (37) holds if and only if $D_a \succ 0$ and $R_c - I_2^\top D_a^{-1} I_2 \succ 0$ hold. Noting that R_c, D_a, I_2 are all positive diagonal matrices, this condition can be rewritten as $R_{c,i} > 1/D_{a,i}$, hence (16) holds.

Since the time derivative of $H_d(\xi)$ is

$$\dot{H}_d(\xi) = -(\nabla_\zeta H_d(\xi))^\top \hat{D} \nabla_\zeta H_d(\xi) \le 0, \tag{38}$$

the equilibrium point q^* is asymptotically stabilized if both

$$\nabla_p H_d(\xi) = 0_{4\times 1}, \ \nabla_{x_c} H_d(\xi) = 0_{2\times 1}, \tag{39}$$

hold only at the desired point. It follows from (39) that $p = 0_{4\times 1}$ hold since $M(q)$ has full rank. In addition, since $\dot{p}$ is also zero at the equilibrium point, we have

$$\begin{aligned} \dot{p} &= 0_{4\times 1} = -\nabla_q H_d(\xi) - D\nabla_p H_d(\xi) \\ &= -\nabla_q H_d(\xi) - 0_{4\times 1} \\ &= -\frac{\partial}{\partial q}\left(\frac{1}{2} p^\top M(q)^{-1} p\right) + \begin{pmatrix} 0_{2\times 1} \\ K_I(x_c - q_a^e) \end{pmatrix} \\ &\quad - \begin{pmatrix} K_s(q_u - q_a) \\ -K_s(q_u - q_a) + K_P(q_a - q_a^*) \end{pmatrix}. \end{aligned} \tag{40}$$

It follows from (39) that the first term and the second term of the bottom row of (40) become zero. Hence, $q_u - q_a = 0_{2\times 1}, \ q_a - q_a^* = q_a^e = 0_{2\times 1}, x_c = 0_{2\times 1}$ always hold under the condition (39) and this completes the proof. The proof of the case (30) is the same as the above. ■

The following subsections are devoted to the experimental results.

B. Experiment 1: Reduction of the Steady State Error

The objective of this experiment is to confirm that the proposed passivity-based lag-compensator (11)-(10) with (22) is effective for reducing steady-state errors. Towards this end, we perform two experiments with different gains. In the first case, the response of the closed-loop system exhibits steady-state errors, which are probably the result of non-modeled phenomena, e.g., dry friction. In the second experiment, we successfully reduce these errors by modifying the control gains.

For the experiments, we consider $q_a^* = (1, -1)^\top$. Figs. 3 and 4 show the response of q and u respectively, where the blue lines are the results of applying the controller designed with $K = \mathrm{diag}(0.2, 0.4)$, and the red lines are the case that $K = \mathrm{diag}(0.4, 0.6)$. For both cases, we select $T = \mathrm{diag}(0.4, 0.2)$, $\alpha = \mathrm{diag}(1.7, 1.01)$. As mentioned before, the steady-state error present in the first experiment–blue case–may be caused by nonlinear friction that is neglected in the model. On the other hand, in the red case with a greater gain K, the steady-state error is zero. This result shows that the proposed controller actually works as a lag-compensator, where the deviations are reduced by amplifying the low frequency signals. Note that the removal of oscillations is outside the scope of our control objectives.

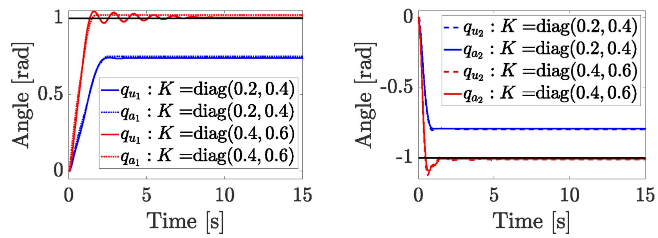

Fig. 3. The resulting responses of $q(t)$ with the proposed compensator.

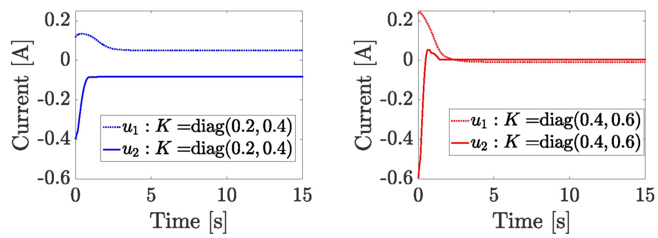

Fig. 4. The resulting responses of $u(t)$ with the proposed compensator.

C. Experiment 2: Suppressing the Windup Phenomenon

The objective of this experiment is to confirm that the proposed passivity-based lag-compensator mitigates the windup phenomenon under input restrictions. Consider the case that the system is physically constrained for a certain amount of time such that the state cannot reach the desired values (35) during this interval. Consequently, applying a PID will cause that the internal variables of the integrator to continue increasing while constrained, producing an overshoot in the response after the constraints are removed if the inputs are saturated. Such a problem does not occur when the lag-compensator is applied. In the experiment, first we just applied the lag-compensator and a PID controller to the system, and verify that the control objective is achieved by both approaches under normal operation conditions. Next, we fix the links so that all the angles remain 0 while $t \le 2$ [s]. Then, we release the links. The desired values are set to $q_a^* = (1, -1)^\top$. The saturated lag-compensator (11)-(10) with (30) is designed by specifying the parameters as $K = \mathrm{diag}(0.15, 1.2)$, $T = \mathrm{diag}(2, 1)$, $\alpha = \mathrm{diag}(1.7, 1.01)$, $\alpha_1 = (1.7, 0.29)$, $\alpha_2 = (4, 0.7)$, $\beta_1 = (0.8, 2.8)$, $\beta_2 = (0.8, 2.8)$, and the PID controller is designed as

$$u(t) = -G_P q_a^e(t) - G_D \dot{q}_a^e(t) - G_I \int_0^t q_a^e(t) \mathrm{d}t,$$

with $G_P = \mathrm{diag}(1.5, 4)$, $G_D = \mathrm{diag}(1, 2)$, and $G_I = \mathrm{diag}(0.4, 1)$, where the magnitude of each input is restricted as $|u_1(t)| \le 0.5$, $|u_2(t)| \le 0.35$. The velocities $\dot{q}_a^e$ are estimated from q_a^e by using a derivative filter provided by Quanser. Note that such a filter is not necessary for the lag-compensator since we use a dynamic extension. Figs. 5 and 6 show the result of the experiments. Fig. 5 shows the case when the system is not constrained, and Fig. 6 depicts the case when the system is constrained for 2 seconds, where the first row figures show the response of the angle $q(t)$, the second row figures show the response of $u_1(t)$, and the third row figures show the response of $u_2(t)$, the blue lines are the result of the PID controller and the red lines are the result of the lag-compensator. In the right figures, the dashed black line and the black solid line show the saturation values of u_1 and u_2, respectively. Figs. 5 and 6 show that, although the steady-state error is

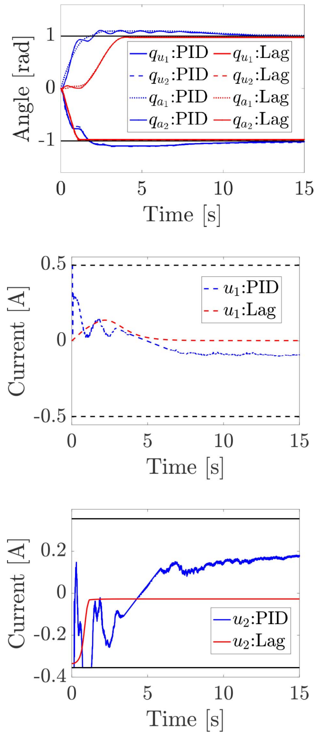

Fig. 5. The case that the physical constraint is not imposed.

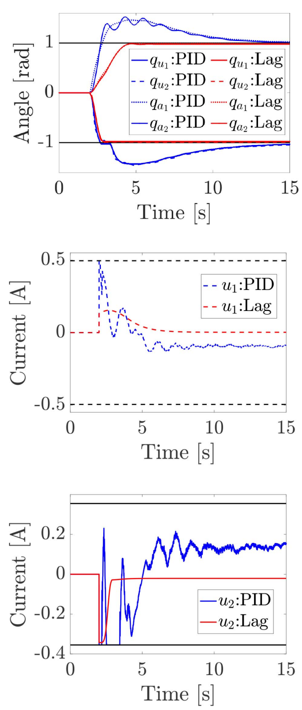

Fig. 6. The case that the system is constrained for 2 seconds.

almost zero in both cases, there is overshoot in the PID case, while the lag-compensator does not evoke such an overshoot. This result proves that the passivity-based lag-compensator is also effective for mitigating the windup phenomena.

V. Conclusion and Future Work

In this letter, we have proposed a PBC methodology suitable to stabilize a class of nonlinear mechanical systems, where the control law admits a lag-compensator interpretation. Some additional properties of the resulting controllers are that they do not require velocity measurements and can be designed to deal with input constraints via the saturation of their signals. The proposed methodology has two main advantages: first, the pH preservation simplifies the stability analysis of the closed-loop system. Second, the lag-compensator interpretation provides clear insight, via a frequency analysis, into the performance of the closed-loop system. These advantages have been illustrated through the implementation of the proposed methodology to stabilize a planar robot, where the frequency analysis provided the guidelines to select control gains that ensures the reduction of the steady-state error in the closed-loop system. As future work we propose the development of a passivity-based lead-lag compensator for removing both oscillations and steady-state errors.

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