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Photon Recoil in Light Scattering by a Bose–Einstein Condensate of a Dilute Gas

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Abstract—Photon recoil upon light scattering by a Bose–Einstein condensate (BEC) of a dilute atomic gas is analyzed theoretically accounting for a weak interatomic interaction. Our approach is based on the Gross–Pitaevskii equation for the condensate, which is coupled to the Maxwell equation for the field. The dispersion relations of recoil energy and momentum are calculated, and the effect of weak nonideality of the condensate on the photon recoil is ubraveled. A good agreement between the theory and experiment [7] on the measurement of the photon recoil momentum in a dispersive medium is demonstrated.

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1. INTRODUCTION

High-precision measurements of the photon momentum in a dispersive medium is not only of fundamental, but also of practical importance. Studies of such kind are used, in particular, in quantum metrology to refine the fundamental constants [1–6] and to manipulate individual atoms.

The Ketterle's group measured the photon recoil momentum in the process of light scattering by a Bose–Einstein condensate (BEC) of a dilute gas [7]. The scheme of experiment was as follows. A BEC of rubidium atoms (^{87}Rb) in the state $|5^2S_{1/2}, F = 1; m_F = -1\rangle$, elongated in one direction and located in the Ioffe–Pritchard magnetic trap [8, 9], was irradiated in the perpendicular direction by two identical counterpropagating laser pulses with a carrier frequency quasi-resonant to two transitions between the components of the hyperfine structure:

$$|5^2S_{1/2}, F = 1; m_F = -1\rangle \to |5^2P_{3/2}, F = 1; m_F = -1\rangle,$$

$$|5^2S_{1/2}, F = 1; m_F = -1\rangle \to |5^2P_{3/2}, F = 2; m_F = -1\rangle.$$

The laser radiation was linearly polarized in the direction of elongation of the condensate, so that the superradiant Rayleigh scattering of light in this direction was suppressed [10, 11]. As a result of multiple scattering events, due to the photon recoil, two series of coherent atomic clouds were excited in the condensate. They moved with different velocities in opposite directions (along the wave vectors of counterpropagating pump pulses). After some delay time, the system was exposed to the second pair of counterpropagating laser pulses. As a concequence, two new series of atomic clouds appeared, which interfered with the previously produced ones. The phase progression of the wave functions of atoms in the primary clouds at this moment led to the interference dependence of their net density on the delay time. This, in turn, caused a change in the density of atoms in the main (static) cloud of the condensate, because the total number of atoms in the BEC is conserved. Measuring the atomic density in the main cloud as a function of the delay time allowed one to estimate the phase progression of moving atomic clouds and, thus, to determine the recoil energy acquired by the atoms.

In the recent article [12], we performed a computer simulation of the interference experiment [7], considering atoms as two-level systems and a BEC as an ideal gas. In the present work, we propose a description of the recoil effect for conditions that are more consistent with the experimental ones, that is, we consider a three-level model of the BEC atom and take into account the weak nonideality of the BEC in the Gross–Pitaevskii approximation [13–16]. Our approach allows us to calculate the energy and recoil momentum acquired by the atoms in the process of light scattering, restricting the analysis to only a single excitation of the BEC and without resorting to the simulation of the interference experiment [7]. A comparison of the results obtained in two ways for the same condensate model makes it possible to estimate the accuracy of the interference method in evaluating the average values of the photon recoil energy and momentum.

2. FORMALISM

In accordance with the geometry of the experiment [7], we restrict ourselves to the one-dimensional model of interaction of a BEC with an electromagnetic field and describe the evolution of the BEC state by means of the Gross–Pitaevskii equation [13–16]:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + U(x) - \hat{d}E(x,t) + G|\Psi(x,t)|^2\right]\Psi(x,t). \tag{1}$$

Here, the first two terms on the right-hand side describe the motion of an atom with mass M in a trap with potential $U(x)$, the third term is the operator of interaction of the atom with the electromagnetic field, $E(x, t)$ is the electric field strength, and $\hat{d}$ is the atomic dipole moment operator. The nonlinear term $G|\Psi(x, t)|^2$ describes the interatomic interaction (the weak nonideality of a dilute gas) in the mean field approximation, where G is the interatomic interaction constant. This approach is a generalization of the previously applied method for studying the superradiant light scattering, which is based on solving the Maxwell–Schrödinger or Maxwell–Bloch system of equations [17–33].

According to the experimental conditions [7], we will consider an atom as a three-level Bose particle with the ground state $|a\rangle$ and two excited states $|b\rangle$ and $|c\rangle$. The electromagnetic field which the atoms interact with is a superposition of the exciting laser field

$$E_0(x,t) = E_0^+(t)\exp\left[-i\omega_0\left(t-\frac{x}{c}\right)\right] + E_0^-(t)\exp\left[-i\omega_0\left(t+\frac{x}{c}\right)\right] \tag{2}$$

of frequency ω_0 and the field produced by the polarization $P(x, t)$ of the atomic medium [17],

$$E(x,t) = E_0(x,t) - \frac{2\pi}{c}\int_0^L dx'\frac{\partial}{\partial t} \times P\left(x', t - \frac{|x-x'|}{c}\right), \tag{3}$$

$$P(x,t) = n_0\langle\Psi(x,t)|\hat{d}|\Psi(x,t)\rangle. \tag{4}$$

Here, c is the speed of light in vacuum, L is the BEC size in the direction of propagation of the pump pulses, and n_0 is the number concentration of atoms in the condensate. The averaging in (4), denoted by angle brackets, is carried out only over the electronic degrees of freedom of the atom.

The wave function of the atom will be tried in the form

$$\Psi(x,t) = \sum_{j=0,\pm2,\ldots}\{a_j(x,t)\phi_j(x,t)|a\rangle + \exp(-i\omega_0 t) \times[b_{j+1}(x,t)\phi_{j+1}(x)|b\rangle + c_{j+1}(x,t)\phi_{j+1}(x)|c\rangle]\}, \tag{5}$$

where $\phi_r(x) = L^{-1/2}\exp(irk_0x)$ is the wave function describing the translational motion of the atom with momentum $r\hbar k_0$ ($r = 0, \pm1, \pm2, \ldots$), which is a multiple of the photon momentum $k_0 = \omega_0/c$ of the laser field.

In the approximation of the slowly varying amplitudes of the field and atomic wave function, the system of Maxwell–Gross–Pitaevskii equations has the form

$$\frac{\partial a_j(x,t)}{\partial t} + v_j\frac{\partial a_j(x,t)}{\partial x} = -iw_ja_j(x,t) + E^{+*}(x,t)\times[b_{j+1}(x,t) + \eta c_{j+1}(x,t)] + E^{-*}(x,t)[b_{j-1}(x,t) + \eta c_{j-1}(x,t)] - ig\sum_{m,l=0,\pm2,\ldots}a_{j+m-l}(x,t)a_m^*(x,t)a_l(x,t), \tag{6}$$

$$\frac{\partial b_{j+1}(x,t)}{\partial t} + v_{j+1}\frac{\partial b_{j+1}(x,t)}{\partial x} = i\left(\Delta_{ba} - w_{j+1} + i\frac{\gamma}{2}\right)b_{j+1}(x,t) - E^+(x,t)a_j(x,t) - E^-(x,t)a_{j+2}(x,t), \tag{7}$$

$$\frac{\partial c_{j+1}(x,t)}{\partial t} + v_{j+1}\frac{\partial c_{j+1}(x,t)}{\partial x} = i\left(\Delta_{ca} - w_{j+1} + i\frac{\gamma}{2}\right)c_{j+1}(x,t) - \eta E^+(x,t)a_j(x,t) - \eta E^-(x,t)a_{j+2}(x,t), \tag{8}$$

where $j = 0, \pm2, \pm4, \ldots$ and the field amplitudes $E^+(x, t)$ and $E^-(x, t)$ satisfy the equations

$$E^+(x,t) = E_0^+(t) + 2\int_0^x\left\{\sum_{j=0,\pm2,\ldots}[b_{j+1}(x',t)] + \eta c_{j+1}(x',t)]a_j^*(x',t)\right\}dx',$$
$$E^-(x,t) = E_0^-(t) + 2\int_x^1\left\{\sum_{j=0,\pm2,\ldots}[b_{j-1}(x',t)] + \eta c_{j-1}(x',t)]a_j^*(x',t)\right\}dx'. \tag{9}$$

As units of length and time in Eqs. (6)–(9), we take the transverse dimension L of the condensate and the superradiant time $\tau_R = \hbar/(\pi|d_{ba}|^2k_0n_0L)$ [17], where $d_{ba} = \langle b|\hat{d}|a\rangle$ is the matrix element of the dipole moment transition operator. The slowly varying amplitudes of the waves of the induced field propagating in the positive and negative directions, $E^{+}(x, t)$ and $E^{-}(x, t)$, as well as the amplitudes $E_0^{\pm}$ of the laser field, are represented in the scale of $i\hbar/dba\tau_R$. The quantities $w_j = \hbar j^2 k_0^2 \tau_R/2M$ and $v_j = \hbar j k_0 \tau_R/ML$ are the kinetic energy (in frequency units) and the atomic velocity, respectively. Further, we restrict ourselves to the analysis of BEC atoms in the ground electronic state with indices "j" running over even values 0, ±2, ±4, ... The quantities $\Delta_{ba} = (\omega_0 - \omega_{ba})\tau_R$ and $\Delta_{ca} = (\omega_0 - \omega_{ca})\tau_R$ are the detunings of the frequency ω_0 of the external field from the atomic resonance frequencies ω_{ba} and ω_{ca}; $\gamma = \Gamma\tau_R$, where Γ is the radiation constant of the excited states of the atom (which is the same for both states); and $\eta = d_{ca}/d_{ba}$ is the ratio of the dipole moments of the transitions $a \leftrightarrow c$ and $a \leftrightarrow b$. The dimensionless interatomic interaction constant is $g = G\tau_R n_0/\hbar$; moreover, in what follows, we restrict ourselves to the interatomic interaction only between atoms in the ground electronic state. We also do not take into account the retardation in Eqs. (6)–(9), because the transit time of a photon through the system, L/c, is the smallest of all characteristic times of the model. The only nonzero initial condition for solving the system of equations (6)–(9) is the amplitude of the initial state of the atom, $a_0(x, t = 0) = 1$.

When solving the system of equations (6)–(9), we used conditions close to the experimental ones [7]: the transverse size of the BEC was $L = 16$ μm; the concentration of condensate atoms was $n_0 = 4.15 \times 10^{13}$ cm^{-3}; the laser radiation frequency varied near the value $\omega_0 = 2.4 \times 10^{15}$ s^{-1}; the radiation constant of the transition $a \leftrightarrow b$ ($|5^2S_{1/2}, F = 1; m_F = -1\rangle \leftrightarrow |5^2P_{3/2}, F = 1; m_F = -1\rangle$) was $\Gamma = 0.37 \times 10^8$ s^{-1}; the wavelength and the dipole moment of this transition were, respectively, $\lambda = 780$ nm and $d_{ba} = 2.07 \times 10^{-29}$ K m; and $\eta = d_{ca}/d_{ba} = (3/5)^{1/2}$ [34]. Under these conditions, the superradiant time is estimated as $\tau_R \approx 1.75 \times 10^{-9}$ s. Then, for the parameters in Eqs. (6)–(9), we obtain

$$w_j = 5\times 10^{-5} j^2, \quad v_j = 7.8\times 10^{-7} j,$$

$$\gamma = 6\times 10^{-2}, \quad g = 3.5\times 10^{-6}.$$

The detuning from the resonance of the $a \leftrightarrow b$ transition was varied within the interval -1.1 GHz $\le \Delta_{ba}/2\pi \le 1.1$ GHz (in dimensionless units, $-12 \le \Delta_{ba} \le 12$). To excite the condensate, we used rectangular pulses of duration $\delta t \approx 5$ μs (in dimensionless units, $\delta t \approx 3 \times 10^3$). The delay time between pulses, τ was varied within the interval $[\delta t, 50\delta t]$. The amplitude E_0 of the laser pulse was adjusted (depending on the detuning from the resonance) so that the fraction of atoms in the static cloud of the condensate would be at a level of 0.9 during the excitation time.

3. PHOTON RECOIL MOMENTUM

Before proceeding to the simulation of the interference experiment [7], which is based on the double excitation of the condensate, we address the case of a single-pulse excitation. To this end, we consider the Fourier transform (with respect to the coordinate) of the amplitudes $a_j(x, t)$ of the atom's ground electronic state:

$$F_j(k,t) = \int_0^1 e^{-ikx} a_j(x,t)dx. \tag{10}$$

It should be specially noted that the Fourier variable k in (10) represents the deviation of the atomic wave vector from its vacuum value jk_0 (in what follows, we identify the wave vector with the momentum). Then the normalized probability density distribution of the deviation has the form

$$W_j(k,t) = \frac{|F_j(k,t)|^2}{\int_{-\infty}^{\infty} |F_j(k',t)|^2 dk'}. \tag{11}$$

By calculating $W_j(k, t)$, we can find the average deviation $\bar{k}_j$ and its variance D_j:

$$\bar{k}_j = \int_{-\infty}^{\infty} kW_j(k,t)dk, \tag{12}$$

$$D_j = \int_{-\infty}^{\infty} (k - \bar{k}_j)^2 W_j(k,t)dk. \tag{13}$$

Since $|\bar{k}_j| \ll k_0$, the average value of the recoil kinetic energy (in units of frequency) can be represented as

$$\varepsilon_j = \overline{\frac{\hbar(k_0 j + k_j)^2}{2M}} \approx \frac{\hbar k_0^2 j^2}{2M} + \frac{\hbar k_0 j \bar{k}_j}{M}. \tag{14}$$

Figure 1 shows examples of the probability density functions of the deviation of the wave vector k from its vacuum value $\pm 2k_0$ in the clouds $a_{\pm 2}$ immediately after the excitation (at $t = \delta t$). For the chosen initial conditions, these distributions are mirror symmetric.

The average value $\bar{k}^2$ and its standard deviation $D_2^{1/2}$ as a function of detuning Δ_{ba} from resonance for an atom in the cloud a_2 are shown in Fig. 2. Note that the relatively large value of the standard deviation, which is practically independent of the detuning Δ_{ba}, is due to the finite size of the trap and a spatial inhomogeneity of the atomic density of the condensate assotiated with this finiteness. In view of relation (14), the dispersion dependence of the deviation of the average value of the recoil kinetic energy ε_j from the value $\hbar k_0^2 j^2/2M$ is practically determined by the dispersion curve for the average deviation of the recoil momen-

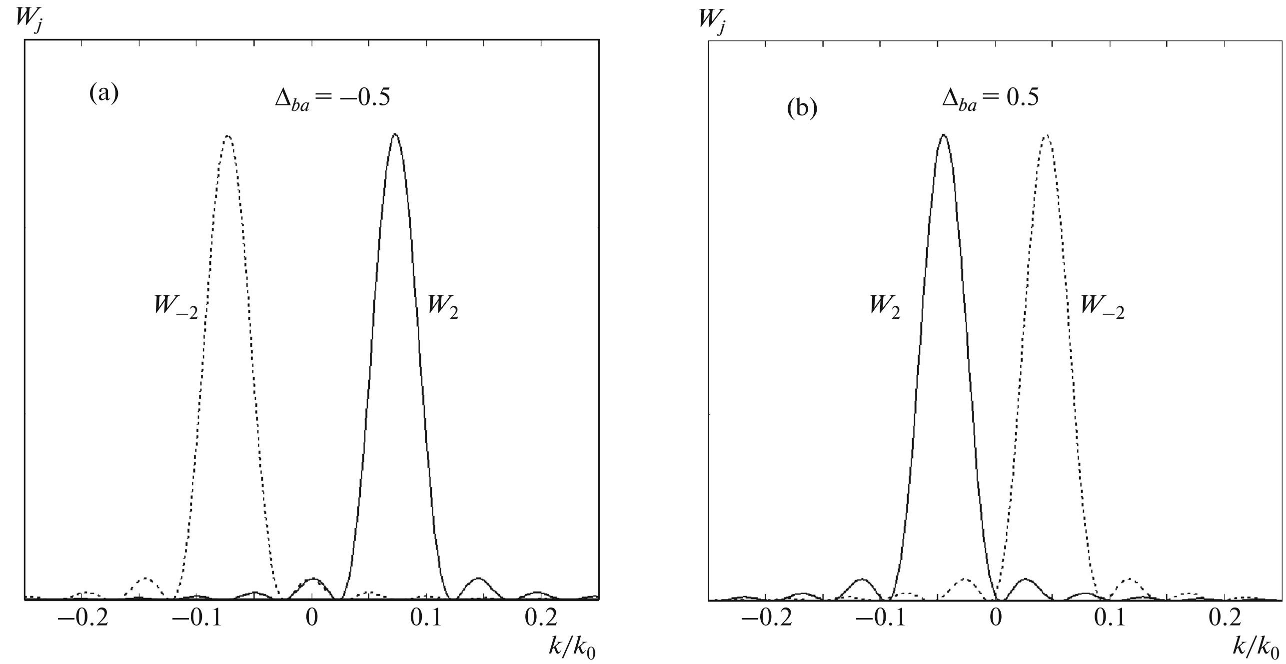

Fig. 1. Probability density distributions of the deviation of the atomic wave vector from its vacuum value $\pm 2k_0$ in the clouds a_{-2} and a_2 (dotted and solid curves, respectively) immediately after the first laser pulse of duration δt has passed, calculated for two values of the detuning from the resonance: (a) $\Delta_{ba} = -0.5$ and (b) $\Delta_{ba} = 0.5$. The difference in the relative position of the curves when varying the sign of the detuning is due to the presence of the second level in the excited state.

tum. If the quantity $\pm 2k_0 + \bar{k}_{\pm 2}$ is interpreted as the recoil momentum acquired by an atom during the field scattering in a dispersive medium, we can set $\pm 2k_0 + \bar{k}_{\pm 2} = \pm 2k_0 n$, where n is the refractive index.

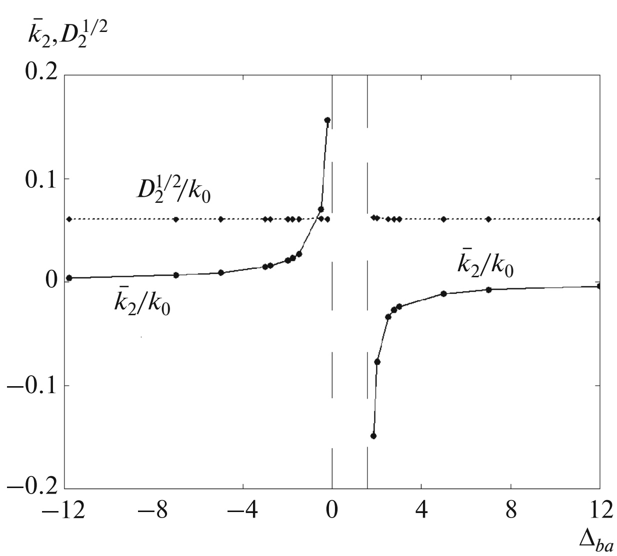

Fig. 2. The mean deviation $\bar{k}_2$ of the recoil momentum from its vacuum value $2k_0$ and its standard deviation $D_2^{1/2}$ in units of k_0 as a function of the detuning Δ_{ba} from the resonance. The points represent the results of calculations.

4. SIMULATION OF THE INTERFERENCE EXPERIMENT

Consider the double excitation scheme of the BEC corresponding to the experiment [7]. We are primarily interested in the fraction of atoms $S_0(t)$ in the static cloud a_0 at time instant $t = \tau + \delta t$, i.e., immediately after the action of the second pulse (recall that δt is the duration of the laser pulse and τ is the delay time of the second pulse, i.e., the difference between the switching times of the second and first pulses). This value is defined as

$$S_0(\tau) = \int_0^1 |a_0(x, \tau + \delta t)|^2 dx. \qquad (15)$$

It seems interesting to compare the value of $S_0(t)$ with the fraction of atoms in the moving clouds $a_{j \neq 0}$:

$$S_j(\tau) = \int_0^1 |a_j(x, \tau + \delta t)|^2 dx, \quad j = \pm 2, \pm 4, \ldots \qquad (16)$$

Figure 3 demonstrates such a comparison for the clouds a_0 and $a_{\pm 2}$. One can see that S_0 and $S_{\pm 2}$, as functions of the delay time τ, exhibit oscillations similar to those observed in the experiment [7]. Note that the oscillations of the populations S_0 and $S_{\pm 2}$ show a strong correlation in frequency, phase, and amplitude. This reflects the fact that the total number of BEC atoms is conserved.

Let us estimate the effect of interatomic interaction on the magnitude of the recoil energy. Taking into account in the Gross–Pitaevskii equation (1) only the interatomic interaction, we obtain

$$\frac{\partial a_j(x,t)}{\partial t} = -ig\sum_{m,l=0,\pm 2,\ldots} a_{j+m-l}(x,t)a_m^*(x,t)a_l(x,t). \quad (17)$$

As we assume that the depletion of a fixed cloud of the condensate is weak, i.e., $a_0 \approx 1$, the equation for a_0 reads

$$\frac{\partial a_0(x,t)}{\partial t} \approx -iga_0(x,t)a_0^*(x,t)a_0(x,t) \approx -iga_0(x,t), \quad (18)$$

while, for $j \neq 0$,

$$\begin{aligned}\frac{\partial a_j(x,t)}{\partial t} &\approx -ig\{a_j(x,t)a_0^*(x,t)a_0(x,t) \\ &\quad + a_0(x,t)a_0^*(x,t)a_j(x,t)\} \quad (19) \\ &= -2iga_j(x,t)a_0^*(x,t)a_0(x,t) \approx -2iga_j(x,t).\end{aligned}$$

This implies that the adopted model of interatomic interaction gives rise to the energy level shifts approximately by amounts g and $2g$ for the states a_0 and $a_{j\neq 0}$, respectively.

Thus, in the interference experiment, the atomic energy levels in clouds $a_{j\neq 0}^{(1)}$, arising after the action of the first pulse, experience a shift by an amount of $2g$. Therefore, the wave functions after the delay time τ acquire a phase factor $\exp[-i(\varepsilon_j + 2g)\tau]$. By the time the second pulse acts on the condensate, the static cloud a_0 already acquired the phase factor $\exp(-ig\tau)$. In order to get, after the secondary excitation, clouds $a_j^{(2)}$ with the same phase dependence, leading to positive (constructive) interference, it is necessary that the delay time would be a multiple of the oscillation period of the static cloud, $2\pi/g$. Accordingly, the frequency of interference fringes will be equal to $w_j + g$, in contrast to the eigenfrequency of the clouds, $\varepsilon_j + 2g$, which determines the recoil energy.

Figure 4 demonstrates a comparison of the dispersion curves of the photon recoil energy computed after single excitation (*1*, *3*) and by the simulation of the interference method (*2*). Curves *1* are obtained by calculating the oscillation frequency of the real (or imaginary) part of the complex amplitude $a_2(x = L/2, t)$ at the center of the trap, curves *2* are the result of simulation of the interference experiment, and curves *3* describe the dispersion of the recoil kinetic energy ε_2.

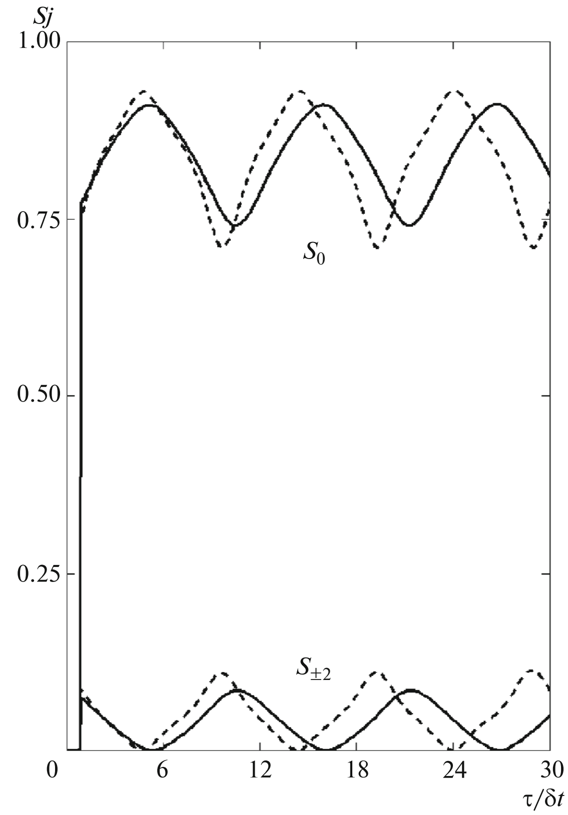

Fig. 3. Results of modeling of the interference experiment: the populations S_0 and $S_{\pm 2}$ of atomic clouds as a function of the delay time τ. The solid (dashed) curves are obtained for the detuning $\Delta_{ba} = 0.5$ ($\Delta_{ba} = -0.5$).

5. CONCLUSIONS

Within the framework of a microscopic approach, we have developed a theory that allows one to analyze the photon recoil energy and momentum in a dispersive medium, in particular in a BEC of a dilute atomic gas accounting for a weak interatomic interaction. The theory is based on the system of coupled Maxwell–Gross–Pitaevskii equations. These equations have been elaborated by making use of the slowly varying amplitude approximation. The calculations performed for a single and a double excitation of the BEC (the second one is the interference method for evaluating the photon recoil [7]) have allowed us to unravel the effect of the weak nonideality of the BEC on the photon recoil in a dispersive medium, as well as to obtain consistent dispersion relations for the average values of the kinetic and total recoil energies in scattering events. The theory also correctly describes the oscillation frequencies of the number of atoms in the main (immobile) cloud of the condensate, which have been observed in the experiment [7].

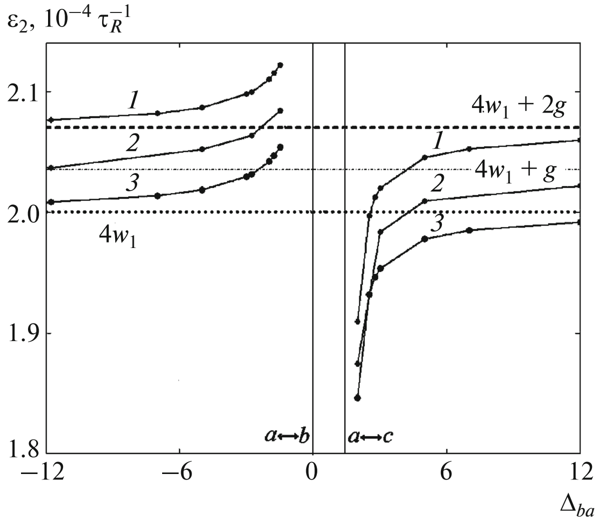

Fig. 4. Dispersion curves of the recoil energy ε_2 (in units of $10^{-4}\tau_R^{-1}$) for atoms in the cloud a_2. The horizontal asymptotes correspond to the energy values $4w_1$, $4w_1 + g$, and $4w_1 + 2g$, while the vertical ones are compliant with the frequencies of the transitions $a \leftrightarrow b$ and $a \leftrightarrow c$. The points are the results of calculations. The numbering of curves is explained in the text.

In addition to the average values of the recoil momentum and energy, we have also calculated the variances D_j of these quantities. The values of the standard deviation $D_j^{1/2}$ determine the error which the average values can be considered with as the characteristics of an individual atom. At the same time, the simulation of the interference method confirms the fundamental possibility of evaluating the average values of the recoil momentum and energy, using this method, with a sufficiently high degree of accuracy.

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