Teacher Questioning in Problem Solving in Community College Algebra Classrooms

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Chapter 17
Teacher Questioning in Problem Solving in Community College Algebra Classrooms

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Abstract In this chapter, we focus on the ways two community college instructors worked with students to demonstrate the solution of contextualized algebra problems in their college algebra lessons. We use two classroom episodes to illustrate how they sought to elicit students' mathematical ideas of algebraic topics, attending primarily to teachers' questioning approaches. We found that the instructors mostly asked questions of lower cognitive demand and used a variety of approaches to elicit the mathematical ideas of the problems, such as using examples relevant to the students and dividing the problems into smaller tasks, that together help identify a solution. We conclude by offering considerations for instruction at community colleges and potential areas for professional development.

Keywords Questioning practices · Algebra · Community colleges

17.1 Introduction

About 43% of all undergraduate students in the United States enroll in community colleges to further their school education (Blair, Kirkman, & Maxwell, 2018). Community colleges are public postsecondary institutions that offer the first 2 years of a higher education degree, attracting a body of racially diverse students who are also older, working, or with family responsibilities. Class schedules are flexible, and tuition is low compared to universities. Community colleges also offer remediation, vocational, or continuing education and the option to transfer to a 4-year university to complete an undergraduate degree.

At community colleges courses range from those designed to prepare students for college-level mathematics to those required in the first 2 years of an undergraduate

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degree. Unfortunately, failure rates in mathematics courses, especially in algebra, range from 30% to 70% (Bahr, 2010), which can be detrimental for many students as they need to pass the algebra courses to either transfer to a 4-year university or earn a degree credential or certificate from their community college to join the workforce. Nonetheless, community colleges have not gained much attention from the research community in recent years (Mesa, 2017), and important questions about mathematics instruction remain: What instructional practices could facilitate the elicitation of mathematical ideas? Could problem solving help reverse failure rates? What is the nature of problem solving at community colleges?

Mathematical problem solving has been an important aspect of mathematics and its teaching and learning since our field started (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). Even though teaching and learning problem solving have been scarcely studied in the context of community college mathematics classrooms, we think that teaching problem solving is crucial for community college mathematics students because students may relate the mathematics to their experiences. Promoting and practicing problem solving is even more vital for students in algebra courses, as this might be the last mathematics course they take before joining the workforce, attending credential programs, or transferring to 4-year institutions, all of which they need problem-solving skills to excel.

Within the context of the large federally funded project, *Algebra Instruction at Community Colleges* (AI@CC, Watkins et al., 2016), we searched for explicit instances of whole-class problem solving, where the instructor and students engaged with each other to solve non-procedural problems involving uncertainty. Our consideration of a problem involves a situation for which solvers do not have a ready answer based on known procedures (e.g., Perkins, 2000; Resnick & Glaser, 1976; Schoenfeld, 1992). Such a situation can be described in Schoenfeld's (1992) terms of problem solving: solvers of any level of expertise experience uncertainty and may explore the problem at hand using tentative approaches and related mathematical examples. A first exploration of our large data set of videos collected from 261 algebra courses taught by 88 instructors in three states, Arizona, Michigan, and Minnesota, revealed that the instructor was the one who presented the material on the board, with the students taking notes. In this context, the instructors involved the students in the mathematics of the lesson predominantly by asking mathematical questions and occasionally by problem solving. In this chapter, we present an analysis of two problem-solving episodes that we found in two community college algebra courses to acknowledge the importance of problem solving at community colleges. The two problems that worked out during these lessons are as follows:

> If Joe can paint a house in three hours and Sam can paint the same house in five hours, how long does it take them to paint the house together?
>
> When making lemonade for a lemonade stand, a child pays $10 for the water, lemons, and sugar, and 10 cents for each plastic cup. How is the cost of producing the lemonade shared among the number of cups sold?

For these problems, the problem-solving process started with the instructor and the students exploring approaches to understand the underlying mathematical ideas of rate and average and proposing a rational expression modeling the situation. We noticed the key role of the teachers' questioning in engaging the students in the mathematics and eliciting their mathematical ideas relevant to the problems at hand. We specifically asked:

1. What is the cognitive demand of the mathematical questions posed by teachers when solving contextualized problems?
2. How are mathematical ideas elicited through these mathematical questions?

We intentionally chose problem-solving episodes from our larger data set that involve contextualized problems. We assume that problems with context from everyday life are more inviting for students as they are close to most students' lives and may involve skills and knowledge that students bring to the classroom. Real-world problems are intended to prepare students for life beyond the boundaries of mathematics classrooms (American Mathematical Association of Two-Year Colleges, 2018; National Council of Teachers of Mathematics, 2000). Some scholars (e.g., Brown, Collins, & Duguid, 1989) have argued that real-world mathematics problems, also known as word problems, are inauthentic or "make-believe" (Boaler, 1993, p. 14). The problems presented in this chapter showcase honest attempts by community college instructors to engage their students with problem-solving situations that are closer to students' everyday lives than decontextualized problems (e.g., "solve for *x*"). We want to highlight the instructors' effort in inviting more contextualized mathematics problems in their teaching; the context of those problems nevertheless is the one currently used at community colleges and may not reflect "real-world" problems. Furthermore, the problems presented fit Schoenfeld's (1992) terms of problem solving as they were not of a procedural nature (students were not expected to go through a set of steps to solve them), the students seemed to experience some level of uncertainty, and they could explore the problem using tentative approaches.

In the next sections, we start by a discussion of relevant literature that informed this study, followed by its theoretical underpinnings and the analytical process used to address the two research questions. After presenting the findings, we offer considerations for instruction at community colleges and implications for future research.

17.2 Questioning in Postsecondary Mathematics Classrooms

Studying problem-solving practices with an emphasis on teacher questioning is an area worthy of investigation because it opens opportunities to understand how to support instructors in using practices that focus on elicitation of students' mathematical ideas by encouraging student participation in classroom interaction. Unfortunately, we were not successful in finding studies that simultaneously analyze problem solving and teacher questioning. This lack of attention to two highly relevant areas of teaching practice was even more noticeable in research in postsecondary mathematics education. Nonetheless, we present literature that has contributed to teacher questioning in mathematics classrooms in postsecondary settings as they use the theoretical lenses that we are interested in, such as cognitive demand. We found six studies, three regarding universities and three regarding US community colleges, that analyzed teacher questioning by categorizing the questions based on their level of cognitive demand or based on the type of mathematical ideas that the questions elicited from the students. In this section, we first review the three studies concerned with teacher questioning in universities, followed by a review of the studies concerned with teacher questioning in community colleges.

In the context of universities, Viirman (2015) categorized teacher questions of calculus, algebra, and abstract algebra of seven mathematics teachers in three Swedish universities. He found that while "rhetorical" and "control or comprehension" questions—such as "Do you follow?"—were common, "genuine questions," which require higher cognitive demand, were not. Viirman (2015) explained that these questions failed to examine student knowledge with the purpose of "modelling general patterns of mathematical discourse" (p. 1178), depriving students the opportunity to observe how mathematics is done and what questions are asked while solving a mathematical problem. Similarly, Paoletti et al. (2018) studied teacher questioning in advanced mathematics courses (e.g., number theory, linear algebra, real analysis) in 11 American universities and found analogous results as Viirman (2015), even though teachers' questions varied in frequency and type from teacher to teacher, "the majority of questions asked students to recall information or procedures (Fact) or provide the next step in a computation or proof (Next step)" (p. 9) which requires lower levels of cognitive demand. Paoletti et al. (2018) concluded that teachers ask easier questions because they tend to believe that students are most capable of answering those. They also sought to investigate the way in which questions supported the elicitation of mathematical ideas. They classified students' contributions into *mathematical content* (e.g., properties, definitions, a step of a procedure) and *ways of reasoning* (e.g., how to approach a mathematical problem). In another study, Jaworski, Mali, and Petropoulou (2017) focused on a corpus of observational studies that characterize university mathematics teaching in lectures and tutorials to a small group of students. In one of the studies discussed, a tutor collaborated with a researcher to develop ways of eliciting students' mathematical meaning-making in first year mathematics classrooms; a teacher questioning approach emerged as one of those ways (the tutor is considered as the instructor in this study). Instead of presenting the mathematics to the student directly, the tutor developed a questioning approach using two major types of questions: "meaning questions" which asked students to articulate mathematical meaning (e.g., "Can you say why?") and "inviting questions" that asked the whole class or specific students to offer a mathematical response.

Three studies have explicitly investigated the use of teacher questions in mathematics classrooms in community colleges in the United States. In an exploratory study, Mesa (2010) observed seven, mostly developmental mathematics, community college instructors and categorized the teacher questions based on the cognitive demand of the teachers' questions. Her analysis indicated that "the students in these classes were engaged at low levels of lexical and cognitive complexity" (p. 81). In another study, Mesa and Lande (2014) investigated teacher questions in community college trigonometry courses. Using the lens of cognitive demand, they defined novel questions as questions that were asked without expecting the students to know the answer or the procedure to find the answer. They found that the instructors either answered their posed novel questions or restated them to reduce the level of cognitive demand posed by their original questions. In a similar study of six algebra instructors from three different community colleges, Mesa, Ullah, Mali, and Diaz (2018) used a revised taxonomy of teacher questions, based on whether the question elicited student response: realized mathematical questions (answered by the students or had a wait time of at least 5 seconds), unrealized mathematical questions (did not have a student response), and non-mathematical questions. The questions were also coded by cognitive demand: authentic (open-ended questions that require information that students have not been exposed to), quasi-authentic (questions that require the application of a known or partially known procedure), and inauthentic (questions that require recalling information that is familiar to the students). Using these categorizations, Mesa et al. (2018) found that 62% of the instructor questions were realized mathematical questions, while 14% were unrealized mathematical questions. Interestingly, a statistically significant association was found between the level of cognitive demand and whether the question was realized or not, suggesting that these instructors provided more opportunities for students to answer inauthentic questions rather than authentic or quasi-authentic questions.

The majority of the studies reviewed in this section mainly attended to categorizing teacher questions—either based on cognitive demand or based on ways of elicitation of mathematical ideas, comparing the relevant frequencies of each category, and offering various explanations for why teachers ask these mathematical questions the way they do. These studies show that teachers usually engage in asking questions about mathematical content, such as facts and next step of procedures, rather than questions with higher cognitive demand that elicit ways of doing and reasoning mathematics. In this chapter, we explore the level of cognitive demand of teacher questions in a contextualized problem-solving situation to offer insights into whether a context relevant to student experiences can potentially help teachers ask realized questions that elicit students' mathematical ideas, going beyond facts and next step of procedures.

17.3 Theoretical Underpinnings

We assume that classroom teaching and learning are social phenomena between the people involved—the college instructor and the student—who enact two *roles*, that of teacher or that of student. These two roles are markedly different; for the most part, people in their role of teachers are the ones responsible for organizing activities that will create opportunities to learn the content, whereas people in their role of students are responsible for engaging in the activities that teachers propose.

According to the theory of didactic situations (TDS, Brousseau, 1997), the "three-way interactions between the teacher, the student, and the subject of studies" are regulated by "implicit norms that operate like a contract" (Herbst, 2003, p. 204). These norms establish basic rules of engagement among teachers, student, and content during class (Herbst & Chazan, 2011). We locate the investigation on the cases of instruction associated with teachers *demonstrating examples* (i.e., showing students a process that could be followed to find a solution to a textbook problem or showing students how a theorem or a definition works) to students in community college mathematics (Mesa & Herbst, 2011). Mesa and Herbst have postulated several norms that regulate this work:

- No one is responsible for justifying steps on an example or for explaining why an answer makes sense.
- Teachers engage the students by asking questions about how to apply known procedures rather than asking them to decide what procedure to apply.
- Teachers offer as examples problems that contain all the information needed to produce only one solution.
- Students need to participate in order to show their engagement with the lesson, and their participation is restricted to executing [given] steps (pp. 45–46).

While the framing of norms helps in understanding the division of labor in teacher-student work in community college classes, it falls short in accounting for the cognitive work that students are expected to pursue when they are asked to fulfill their role as students in this instructional situation. To address this shortage, we attend to the cognitive demand implicit in the questions that teachers pose in the process of arriving at the solution of a problem such as those described in the introduction. Using the revised version of Bloom's taxonomy of objectives (Anderson et al., 2001), we have proposed a categorization of mathematical questions, based on Mesa and colleagues' work (Mesa, 2010; Mesa, Celis, & Lande, 2014; Mesa & Lande, 2014) either as authentic, quasi-authentic, or inauthentic, that are asked in community college classrooms and that help describe the level of cognitive demand offered to students. We describe this framework in the next section.

17.4 Methods

This study is part of the AI@CC project (Watkins et al., 2016) that investigates the relationship between the quality of teacher-student interaction and student performance in community college algebra courses. We selected two instructors (out of about 88 participants) based on the presence of contextualized problems in their lessons. The two full-time instructors, A and B, teach at two different suburban community colleges in the United States (the institution where A teaches has approximately 20,000 students; the institution where B teaches has about 10,000 students). The instructors reported that they have participated in professional development opportunities ranging from reading articles to engaging in social interactions such as face-to-face meetings, attending professional meetings, and online discussions. Instructor A has a bachelor's degree in history and a master's in educational supervision. Instructor B has both his bachelor's and master's degrees in mathematics. They both have more than 20 years of math teaching experience. There were 25 students enrolled in Instructor A's class (9 female) and 30 students in Instructor B's class (13 female). Also, 60% of the students in both courses indicated a goal of transferring to a 4-year institution to complete their bachelor's degree. About 72% of Instructor A's and 53% of Instructor B's students reported holding a paid job for 10 or more hours a week.

Each instructor in the study was video-recorded teaching all the lessons on two of three possible topics: linear equations, rational equations, or exponential equations. For the purpose of this study, we used partial transcripts of video recordings of one lesson each taught by instructors A and B to classify the questions they asked according to the taxonomy of questions used in the study (see Fig. 17.1).

To answer the first research question, we first identified and transcribed all questions (mathematical and non-mathematical) in the lessons taught by both instructors. We then applied a modified version of Mesa (2010)'s question coding system, attending to whether the question was mathematical or not and when the questions were mathematical, their level of authenticity as inauthentic, quasi-authentic, and authentic. Inauthentic questions elicit from student's known facts or steps of procedures that were taught in a previous class or course. Their cognitive demand is low and usually asks for remembering facts. Questions in which the instructor asks students to complete an arithmetic computation (e.g., "What is 4 divided by 2?") or to provide a fact or procedure that have been just presented ("What did we say is the formula for slope?") or are assumed to have been memorized ("What is the formula for computing the roots of a quadratic function?") are coded as inauthentic.

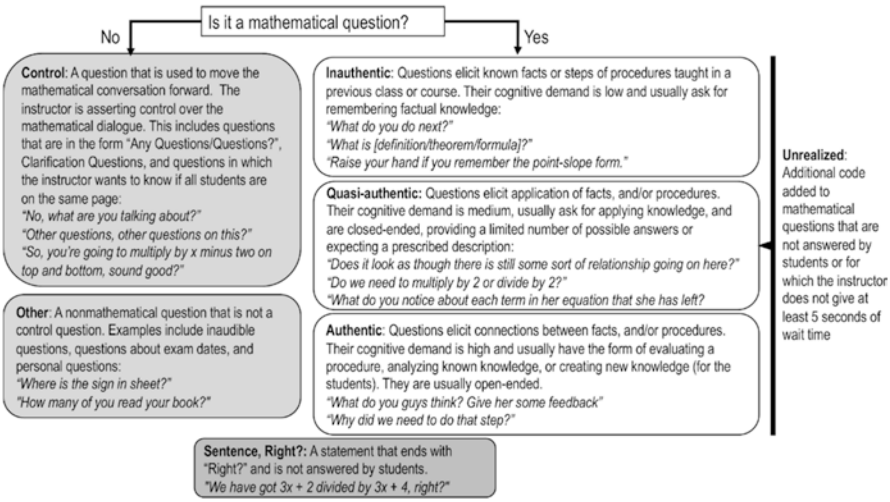

Fig. 17.1 Teacher questions coding diagram

Quasi-authentic questions elicit from students the application of facts or procedures. Their cognitive demand is medium, usually asking for the application of prior knowledge; they are usually closed-ended because they provide a limited number of possible answers or they are phrased in ways that a prescribed description is expected (e.g., "in this case, do we multiply or divide by 2?"). Authentic questions elicit connections between facts or procedures or both. Their cognitive demand is high and usually has the form of evaluating a procedure, analyzing known knowledge, or creating knowledge that is new for the students. They are usually open-ended. Non-mathematical questions were classified into two categories, control and other. Control questions are those that seek to maintain the discourse going or that seek to check on students in general (e.g., "No, what are you talking about?" "Is he telling you what he's doing or is he just doing?"). Other questions include classroom business questions, personal questions, or those that were not possible to hear (e.g., "Is there some early Friday morning Halloween party I don't know about?"). An additional code, *Sentence Right*, was given to any statement that ended with the word "Right?" and was not answered by students (e.g., *"We have got 3x + 2 divided by 3x + 4, right?"*). These statements were treated as questions. All mathematical questions were tagged as *unrealized*, if there were no responses from students or if the instructor had not given a wait time of 5 seconds or more after starting the question. The intention of this tag is to underscore the importance of giving students opportunity to answer the questions.

In order to answer the second question, we identified 15 episodes of problem solving in the classes of two instructors and examined two in which they solved a problem that had a real-life context. In this paper, we present excerpts of the 30-minute-long solution processes. After transcribing the episodes, we wrote narratives describing how the teacher questions elicited mathematical ideas. First, we identified the underlying mathematical ideas embedded in the problems (e.g., unit rate, average). Next, we identified instances in which instructors sought to elicit two types of mathematical contributions, *mathematical content* and *ways of reasoning*, following Paoletti et al. (2018). The narratives identify how the instructors engaged students in contributing to the mathematical ideas of the problems (e.g., by questioning, inviting students into the class dialogue using their names, devising examples relevant to students) and who articulated the mathematical contributions (instructor or students).

There are some limitations of this study. First, it focuses on only two of the many problems that instructors solved; the episodes were chosen because their context is most applicable and proximal to life outside of mathematics classrooms. As such they can't be taken as representative of what the instructors would have done if they were solving problems set in real-life context. However, the episodes showcase how instructors used their problems to include students in mathematical activities that are akin to creating situations where arriving at solutions cannot be readily made, thus providing evidence that problem-solving work is possible in community college mathematics. Second, the equipment to record the lessons was not very sensitive to students' utterances, so student inaudible responses may have increased the number of unrealized questions in the analysis.

17.5 Findings

In the next sections, we present episodes where the two community college instructors engage their students in problem solving through contextualized problems. In the episodes, we analyze teacher questioning as one of the ways that elicited mathematical ideas in the process of solving the problems. Our focus is on types of questioning, associated cognitive demand of questions posed, and whether, how, and from whom mathematical ideas are elicited.

17.5.1 Problem 1: Rate of Work

We identified 322 questions in the 53-minute-long lesson, 85 inauthentic, 68 quasi-authentic, and 16 authentic. The lesson started with exercises on the process of finding the least common denominator (LCD) for rational expressions with unequal denominators. The students faced each other in groups of four; the instructor stood by the whiteboard and talked about the mathematics. Toward the middle of the class session, the instructor introduced the mathematical problem using a 3-minute clip taken from the 1994 movie *Little Big League* to illustrate application of rational expressions. The problem motivated by the movie was about two people working together to paint a house:

> If Joe can paint a house in three hours and Sam can paint the same house in five hours, how long does it take them to paint the house together?

The actors included a coach and the baseball players. In the plot, the coach stated the problem and the baseball players gave four different answers to that problem. While the clip was playing, the instructor wrote on the board key information about the problem including the four solutions (see Fig. 17.2).

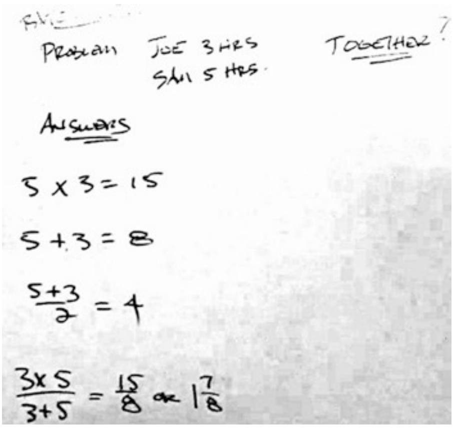

Fig. 17.2 Information about the problem on the whiteboard

The instructor told the students that the problem was about “rate of work.” The students’ task was to make sure that the fourth answer given by the actors made sense mathematically and that the previous three did not. We present parts of this discussion in the episode below.

Episode 17.1: Rate of Work (33:44-39:10)

Turn			Question code
1	I:	How can we help the others see that not only because of the supreme confidence [of the actor] this [fourth one] has to be the answer? How can we rule out these three [the first three answers]?	Unrealized authentic
2	I:	How could we say hey, it can’t be 8, it can’t be 15?	Authentic
3	S:	Well, for the second one it can’t be right because they can’t paint the house together and take more time than if they worked separately.	
4	I:	Okay. So you’re saying what?	Control
5	S:	The second one would be showing that it takes more time to paint the house when they work together than separately.	
6	I:	OK, go a little deeper. I agree with you. But there is a reason we can rule out all three of them. Go here George.	
7	S:	Shouldn’t it take longer to do it together than just one person? It should take 2 hours what would take 4 hours for one of them.	
8	I:	Brian, what do you think about that?	Quasi-authentic
9	I:	Yeah, right? So Joe by himself can do three hours, right?	Sentence right
10	I:	If he has some help, the time should be … ?	Quasi-authentic
11	S:	Shorter	
12	I:	Less than three, right?	Sentence right
13	I:	Unlike a project I do at home where I start drinking beer with my buddies; that takes us all day, alright? My wife knows, but assuming these guys all work hard, right? If Joe can do it in 3 hours and he has any help, right? I expect an answer to be less than 3. So that immediately rules out those three answers.	Sentence right
14	I:	They just don’t make real-world sense, based off of what we have, alright?	Sentence right
15	I:	The guy with supreme confidence, what’s nice about his answer?	Quasi-authentic
16	S:	It is supposed to be less than--	
17	I:	It is less than 3. Is that okay?	Control
18	I:	So now we got to figure out: is the guy right?	Unrealized inauthentic
19	I:	What is this supreme formula that he came up with, alright?	Unrealized inauthentic

Turn			Question code
20	I:	And so to give that, let's go ahead and take a look at this problem and what is gonna come down to, we are talking hours so to understand or to set up the formula, we are gonna kind of consider the job done in one hour, alright?	Sentence right
21	I:	That's what we're going to consider here. How much of the work is done, if I just work for one hour?	Unrealized authentic
22	I:	It takes Joe 3 hours to complete the job. So in one hour, how much of the house is going to be done? How much of the job is going to be completed?	Authentic
23	S:	One third	
24	I:	A third! Does that make sense to everybody in the room?	Control
25	S:	Yes.	
26	I:	Makes sense?	Control
27	I:	Right, yeah so an hour, two hours--	Sentence right
28	I:	Two hours, two thirds, right? Three hours three thirds, a full painted house.	Sentence right
29	I:	We [are] good?	Control
30	S:	Yes	
31	I:	Sam, we're told, takes five hours, alright?	Sentence right
32	I:	In an hour, how much of the house will be done?	Quasi-authentic
33	S:	One fifth	
34	I:	Does everybody agree with Lyn there?	Control
35	S	Yes	
36	I:	And for the same reason, right? If it takes 5 hours, in one hour I have a fifth of it. 2 hours 2 fifths, three hours three fifths.	Sentence right
37	I:	We all good? S: Mhhhmm	Control
38	I:	Now, one thing that is really important and it's gonna come out several times in the new hand out I gave you. We have to identify our unknown. But what do we usually use to identify our unknown?	Inauthentic
39	S:	x	
40	I:	x, the variable, right? I am going to call it t just because my variable represents a time value. Unknown right now: t is the time that it takes to do it together.	Sentence right
41	I:	If it's taking t hours to do something together, we don't know what that t is, how much of the job is done in one hour? S: [inaudible]	Authentic
42	I:	So from there, right, how much is done? So it takes me t hours to get the work done, in 3 hours I have one third done, in 5 hours I have got one fifth done, so in t hours, I have got?	Unrealized authentic
43	S:	1	
44	I:	No, no. Just for t hours, how about we do this? Ryan, how fast can you work?	Inauthentic
45	S:	Faster	

Turn			Question code
46	I:	So it takes Ryan two hours, right? So, how much of the job would he have done in an hour?	Authentic
47	S:	Half.	
48	I:	One half. OK, so, together they got *t* hours. Right, we don't know what that is. So, in one hour how much work is gonna be done?	Quasi-authentic
49	S:	One over *t*.	

Note: *I* Instructor, *S* Student

After this discussion, the instructor said that the fractions represented how much of the painting was done in an hour and asked students to create an equation out of the fractions. The students suggested the equal sign between the right-hand side of the equation which is "1/t" and the left-hand side of the equation which is "1/3 + 1/5" with both sides signaling the amount of time that it takes Joe and Sam to complete the house painting together per hour.

What is the cognitive demand of the mathematical questions posed by instructors when solving contextualized problems?

We identified 16 mathematical questions asked by the instructor in this episode; of these 12 were either authentic or quasi-authentic and 9 of these were realized. Thus, in this short episode of 49 turns, an important number of questions sought to probe students into doing more challenging work beyond reproducing known facts or procedures. An important number of teacher questions (17) were control or sentence right that sought to manage the conversation or ascertain that everybody was on the same page.

How are mathematical ideas elicited via mathematical questioning?

In this episode, various instructor attempts, via questioning, are toward moving the students from exploring the specific context of the problem to eliciting an abstract situation with the variable *t*. The instructor divided the mathematical problem into two different tasks: in the first task, the interactions lead students to identify an answer that they believe solves the situation; in the second task, they corroborate the solution.

The first part of the episode (Turns 1–18) comprises instructor prompts so students eliminate three of four answers to the problem. In order for the instructor to eliminate the three first answers, a single argument is needed: If Joe needs 3 hours to paint the house alone, then certainly with mindful help from Sam the time needed has to be less than 3 hours. The instructor employs various ways to elicit this argument from students relative to the mathematical idea of "rate of work." He asks one realized authentic and three realized quasi-authentic questions (Turns 2, 8, 10, and 15), brings different students into the discussion calling them by their names (Turns 6 and 8), and uses student responses of Turns 3 and 5 which are local to the problem at hand to promote more abstract argumentation "with more people working, less hours are needed" (Turn 11). However, the instructor is the one who offers the mathematical content of the argument relative to mathematical symbolization "less than" and to the number of hours needed "three" (Turns 12 and 13).

The second part of the episode (Turns 19–49) includes the evaluation of the validity of the fourth answer in the clip. To get to the second part, where the mathematical idea is "unit rate," the instructor is again the one who provides the mathematical idea (Turn 20) and the next step of labeling and defining the unknown (Turns 38 and 40). He nevertheless elicits the arithmetic that produces the unit rates for each of the two painters with a realized authentic and a realized quasi-authentic question (Turns 22 and 32) and emphasizes what each of them means (Turns 28 and 36). Although the students offer correct input for the concrete unit rates (Turns 23 and 33), they are not in a position to offer the abstract unit rate 1/*t*. The instructor attempts to elicit abstraction by offering an analogy (Turn 42), an authentic, and a quasi-authentic question embedded in an example relevant to a student of the class (Turns 44, 46, and 48). Following the example, a student offers the mathematical content 1/*t* of the abstract unit rate.

17.5.2 Problem 2: Average Cost

In his 72-minute lesson, Instructor B asked 168 questions, 83 inauthentic, 23 quasi-authentic, and 7 authentic questions. The following episode lasts for 4 minutes of his first lesson on rational equations in the 13th week of the semester. At the beginning of the lesson, the instructor introduced a problem with the real-life scenario of his daughter setting up a lemonade stand. He would give her money to buy cups and the ingredients for the lemonade, and she should pay him back earning profit. In solving the problem, the instructor discussed with the students the production of the rational expression:

$$A(x) = \text{cost of } x \text{ cups} / x$$

where the numerator is the total cost for *x* cups of lemonade.

Episode 17.2: Average Cost (00:50-04:49)

Turn			Question code
1	I:	Here is the story. My daughter wanted to do a lemonade stand. Every kid wants to do a lemonade stand. She asked: Daddy, how much money am I going to make?"	
2	I:	What does it depend on? How much money is she going to make?	Authentic
3	S:	How much she sells.	
4	I:	How many cups of lemonade she is going to sell. But it also depends on the cost per cup of lemonade, right?	Sentence right
5	I:	Because what if each cup of lemonade [costs] for 50 cents—that is, what it costs to make it, right?—and she sells each cup for 25 cents?	Sentence right, inauthentic

Turn			Question code
6	S:	Losing money.	
7	I:	Yes, she is going to lose money. So, isn't it important to figure out what the average cost of a cup of lemonade is going to be for her?	Unrealized quasi-authentic
8	I:	So here is the story. I have to go and get lemons, sugar, assume water is free, and the cups. I told her I was going to charge her 10 cents for each cup she used. Because she gotta pay me back, right?	Sentence right
9	I:	I am not going to give her this money for free. She has to learn about profit, right? Free money is dad's money, so she has to make enough to pay me back.	Sentence right
10	I:	So it costs me 10 dollars to buy the lemons, sugar, and I already had cups. It must be 10 cents per cup. So I want to know what her average cost per cup of lemonade is. How do I figure that out?	Unrealized quasi-authentic
11	I:	What's an average?	Quasi-authentic
12	S:	Average of what?	
13	I:	Any average. If I say, what's the average weight of the people in this room? [Many students respond at the same time.]	Quasi-authentic
14	I:	Say it again. You said something, what did you say? Something interesting. I heard it!	Control
15	S:	Add all the numbers together and divide.	
16	I:	Add them all together and divide by the number of things. That's the algorithm for finding the average, right?	Sentence right
17	I:	So if I said the average weight of the people in this room, if we add all our weights together and divide by the number of things, then that would be the average weight. If everybody weighed the same, we would all weight whatever that average is, right?	Sentence right
18	I:	If I say, "hey take all the money out of your pockets", and you pile it up here on the desk. I am going to give everybody the average of all the money we have in our pockets. We equally distribute among the people in the room, right?	Sentence right
19	S:	I have zero.	
20	I:	Yeah, I have zero also. So, we pile up all the money that you have in your pockets, and then we equally distribute it around the people in the room, right? So that everybody would have the same amount, the average.	Sentence right
21	I:	So we need to figure out her average cost for an individual cup of lemonade. How am I going to do that?	Authentic
22	S:	The 10 cents.	
23	I:	It has to do with the 10 cents, yeah. So, 10 cents for each cup, right? But we have something else involved.	Sentence right
24	S:	It's the lemons.	
25	I:	Yeah, it's the ten dollars for the lemons and the sugar, we have to include that. So, how do we do an average again?	Inauthentic
26	S:	Add them all up, and divide by the number of things	
27	I:	Add them all up, and divide by the number of things. So, we are going to add up all the cost, and divide by what?	Inauthentic
28	S:	However many things.	

Turn			Question code
29	I:	However many things there are. What are we counting up? What did it ask for, the average cost of what?	Inauthentic
30	S:	One cup.	
31	I:	The average cost per cup. So what am I going to divide by?	Inauthentic
32	S:	The number of cups.	

Note: *I* Instructor, *S* Student

What is the cognitive demand of the mathematical questions posed by instructors when solving contextualized problems?
We identified 11 mathematical questions asked by the instructor in this episode; of these six were either authentic or quasi-authentic and four of these were realized. Thus, in this episode of 32 turns, about half of the questions sought to probe students into doing more challenging work beyond recalling known facts or procedures. The ten control or sentence-right questions that the instructor asked sought to gain insights into what the students were doing—whether they were on the same page—and manage the conversation.

How are mathematical ideas elicited via mathematical questioning?
In this episode, the instructor starts discussing with the students the mathematical idea of "average" by creating the rational equation of the cost per number of cups sold. In order for the instructor to get to the idea of average, he is the one who offers the argument that the money his daughter is going to make depends on the cost per cup of lemonade (Turns 2 and 4). He reinforces this argument with a concrete example of his daughter spending 50 cents to produce a cup of lemonade and selling it for 25 cents (Turn 5), thereby promoting student meaning at the local level of the problem; the student indeed confirms that the girl is going to lose money (Turn 6). The ways with which the instructor attempts to elicit from students the rational equation needed for the solution of the problem include devising examples relevant to students—one is about body weight and the other is about equal distribution of a pile of money among the students (Turns 13 and 18)—and asking follow-up authentic and inauthentic questions about the problem at hand (Turns 21, 25–31). Following their responses to the questions asked in the context of the examples (e.g., Turn 15), the students are the ones who offer the mathematical content for the average cost per number of cups sold: the numerator is the total cost for x cups of lemonade and the denominator is the total number of cups (Turns 26 and 32).

17.6 Discussion

This study explored how questioning approaches that cognitively engages students can be used to explore opportunities for eliciting mathematical ideas when situations interpreted as inclusive of problem solving occur in community college intermediate and college algebra classes. We found that largely in this setting, the instructor does the work for the students by offering the mathematical ideas and suggesting next steps, guiding, via questioning, the whole discussion about the mathematics. Indeed, in Episode 17.1 the instructor offered the ideas of “unit rate” and setting up an unknown, and in Episode 17.2 the instructor highlighted the importance of finding the “average” cost. This dominant role of the instructor in class work is not surprising considering that Mesa and Herbst (2011) described community college norms according to which instructors give students problems in a way that a single solution is produced while student participation is restricted to executing steps within solutions of problems.

In both episodes, we found several questions of all categories of cognitive demand that were left unrealized with no response, or no opportunity for a response, from the students. The majority of unrealized instructor questions occurred when the instructor waited for less than 5 seconds after asking a question. Larson and Lovelace (2013) argued that a wait time that equals or exceeds 5 seconds resulted in students sometimes answering the questions. Mesa et al. (2018) found that across 16 community college algebra lessons, the proportion of authentic and quasi-authentic questions would rise from 14% to about 20% in case instructors waited for students to answer those questions; in other words, a fifth of all questions asked did not adhere to an optimal 5-second wait time. We think that the norm that in a community college class no one is responsible for justifying steps or for explaining why an answer makes sense (Mesa & Herbst, 2011) may play a role in forcing instructors making the decision to frequently use unrealized mathematical questions.

We believe that altering the norms would be beneficial for students in this context. This is because by having only a certain way to do the mathematics of a problem, the students are not given the agency of making many of the decisions about the problem, thus hindering their engagement in practicing and understanding the mathematics. Jaworski et al. (2017) offered a questioning approach that delves into bringing out student mathematical meaning and participation in the class work; in community colleges, instructors could start engaging students in the underlying mathematical ideas of the real-life context of problems by directly inviting students to respond (see Turns 6, 8, 34, and 44 in Episode 17.1) and by listening to their responses to infer students’ cognitive needs. Also, using related problems (see Turns 13 and 17 in Episode 17.2) is a problem-solving technique suggested by various scholars (e.g., Mayer, 2003; Polya, 1971; Schoenfeld, 1992). In the episodes of this chapter, both instructors drew on related examples to the problem at hand (“how fast Ryan works in an hour” in Episode 17.1; “the average weight of the students in the classroom” in Episode 17.2), in this way facilitating student responses; the proximity of these related problems to real-world situations allowed students to make relevant contributions. Because community college students in the United States tend to be employed (see the demographics of the students of this study), it is likely that they are confronted with real-world situations daily. Capitalizing on this knowledge (e.g., sales, time management) and staying away from fabricated real-world problem solving can support the students as they make connections between mathematics and their lives.

We found a low frequency of authentic and quasi-authentic questions asked in spite of the problem-solving context: 26% of questions for Instructor A and 18% for Instructor B. This is consistent to the norm that governs how community college instructors teaching trigonometry engage students in demonstrating solutions to problems (Mesa & Herbst, 2011): the questions are about how to apply known procedures rather than about deciding what procedure to apply; thus authentic and quasi-authentic questions are less likely to occur. Viirman (2015) and Paoletti et al. (2018) set their studies in different contexts than problem solving, but they also found infrequent occurrence of questions corresponding to those levels of cognitive demand in lower and upper division courses at universities. While the proportions might seem dismal, a better interpretation is that instructors need to use questions of various levels of cognitive demand as they teach. Temple and Doerr (2012) suggested that low cognitive demand questions have an important role during the lessons; it is useful, for example, to make sure that students can recall a definition that they will be using in a lesson, or that they can apply a very well-known procedure. Also, Mesa and Lande (2014) noted that different class sessions will demand different types of questions. During a review session, for example, inauthentic questions may be more frequent because the instructor may aim at making sure that students are ready for an examination.

17.7 Conclusion

Mathematical problems set in a real-life context have the potential to provide community college students with important connections between mathematical knowledge and their personal experiences. This chapter suggests that such contextualized problems can provide instructors with opportunities to ask questions that engage students with more cognitively demanding work with mathematics. Even if the curriculum does not offer problems that have validity outside of classroom life, instructors may modify the problems in order to draw on students' experiences in order to facilitate the elicitation of the mathematical ideas while solving a problem. More works need to be done for supporting instructors in becoming proficient in capitalizing on these resources. For example, we noticed that instructors do not tend to give opportunities to students to answer authentic and quasi-authentic questions when solving contextualized problems; thus, programs of professional development can support instructors in this direction.

This chapter offers various ways instructors may employ to elicit mathematical ideas. Making sure students have time to think about an answer by giving wait time of at least 5 seconds may increase the opportunities for students to participate in the solution of a problem. Inviting students in the class dialogue by using the students' names (see Jaworski et al., 2017) may prompt the students, who do not usually speak, to feel included in the class work and to offer their thoughts. In addition, modifying mathematical examples so that they are relevant to students' experiences may enable them to connect their prior knowledge to what is asked of them in new situations. Lastly, separating the mathematical problem at hand into smaller, more manageable, tasks may break down the complexity of the problem and aid students in understanding the problem and coming up with new ways of solving it.

Mathematics courses at community colleges tend to have low passing rates (Blair et al., 2018). We think that the ways instructors work with students to demonstrate the mathematics and retain students' interest in mathematical ideas merit attention. The design and use of problems with context from real life has the potential to engage more students in their classes. While we found a low frequency of authentic and quasi-authentic questions asked within the context of problem solving, including questions of various levels of cognitive demand in teaching is important, as different types of questions serve different purposes during instruction (see Mesa & Lande, 2014; Temple & Doerr, 2012). The students' perspective on their work with contextualized problems also needs attention; exploring students' recollections of class situations of problem solving and their perceived opportunities to access mathematical ideas via their instructors' questioning approaches may offer insights into ways in which instructors can reach out to students and engage them in the mathematical ideas at stake.

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