Students' perceptions of errors in mathematics learning in Tanzanian secondary schools

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ADDING TWO FRACTIONS: WHY DON'T WE JUST ADD THE TOPS AND THEN ADD THE BOTTOMS? ROLE OF MANIPULATIVES

Yasmine Abtahi

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Adding fractions is among the most challenging concepts in elementary school mathematics. I explore the learning of fractions by focusing on the role of feedback from manipulatives. I describe feedback from the physical regularities and design of a mathematical tool and/or the interrelationships among its various components that make the mathematical affordances of the tool more apparent. In fraction circles (specially designed manipulatives), the relationships among the sizes of the pieces constitute the feedback. The theoretical framework of Martin and Schwartz (2005) was used (and further elaborated) to examine students' mathematical reasoning as they attempted to adapt environments, interpret their actions and construct meaning for the addition of fractions. The participants were five 10-year-old children from a fifth-grade class in a private school in Ottawa, who had already learnt the concept of adding two fractions in their school. I asked them to interact with Fraction Circles and Cuisenaire rods to add 3 sets of fractions. My findings show a systematic relation between the physical regularities of the manipulative and its feedback which in turn creates a recursive adaptation and interpretation between the child and the tool. Working with the fraction circles pupils had clear idea of the stable environment of the fraction circles, and of the addition of fractions. Stability and clear feedback from the environment and the clarity of the mathematical interpretation helped students to increase their efficiency by off-loading the cognitive task of the addition of fractions to the environment. In working with Cuisenaire rods, there was no clear feedback related to adding of fractions, which made it challenging to interpret the environment, which was initially a physically distributed learning environment. Although not defined in the theoretical framework, there was throughout the process of working with the tool, cases where pupils created their own useful feedback to then be able to solve the tasks, making the feedback (built-in to the environment or made by the user) an elemental feature of a learning environment.

Ideas		
Adaptable	Induction	Physically Distributed Learning (PDL)
Stable	Off-loading	Repurposing
	Stable	*Environment* Adaptable

Figure 1

References

Martin, T., & Schwartz, D. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science*, *29*, 587–625

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ACCESS TO POWERFUL MATHEMATICS KNOWLEDGE: AN ANALYSIS OF THE TEACHING OF NUMBER SENSE IN FOUNDATION PHASE TEACHER EDUCATION

Jogymol Kalariparampil Alex[1], Lise Westaway[2]

[1]Walter Sisulu University, [2]Rhodes University

Number sense development is a key component of the Curriculum and Assessment Policy Statement (CAPS) in the Foundation Phase (Grades R-3). The concept 'number sense' is contested; the CAPS document however states that number sense is knowledge of the relative size and meaning of different numbers, the relationships between numbers, and how to represent and calculate with numbers in different ways (Department of Basic Education, 2011). Initial Teacher Education (ITE) programmes are expected to equip prospective teachers with the content and pedagogical knowledge required to develop Foundation Phase learners' number sense.

Concerns about the extent to which ITE programmes are supporting preservice teachers' competence to develop learners' number sense, has led to the establishment of the Primary Teacher Education Number Sense Project. The purpose of this project is to develop a common set of standards for all ITE institutions that focus on equipping preservice teachers with the knowledge and skills to develop learners' number sense. One of the outputs of the project was to analyse the number sense component of the mathematics education curricula of five higher education institutions.

Document analysis of the curricula of the five institutions, particularly the assessment tasks and examinations showed that even though some Higher Education Institutions teach the CAPS related content at a much deeper level than expected at school, there is no uniformity in the emphasis placed on the different mathematics content and the assessment thereof. This variability in the content, and level of the content taught, has implications for the access Foundation Phase learners have to powerful mathematics knowledge, that is, number sense. It is recommended that teacher education institutions develop a common set of standards, and a curriculum framework, to guide the development of the number sense component of their mathematics education courses so that the preservice teachers are enabled to teach their learners to be competent and confident with number and calculations.

References

South Africa. Department of Basic Education. (SA.DBE). (2011). *Curriculum and Assessment Policy Statement for Mathematics (Grades 1-3)*. Pretoria: Government Printer.

DOMINANT DISCOURSE PRACTICES IN NIGERIAN MULTILINGUAL MATHEMATICS CLASSROOMS

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This study contributes to the debate on exploring ways to address the challenges teachers grapple with while teaching mathematics in a language in which students are still learning to understand. According to Akinoso (2014), one of the main factors contributing to students' poor performance in multilingual mathematics classrooms in Nigeria, is the problem of teaching mathematics to students who are not proficient in English, the language of learning and teaching (LoLT). There is, however, limited research conducted in Nigeria to interrogate the extent of this problem. In the present study, Gee's (2005) Discourse analysis theory and method provided a theoretical lens for the investigation of the challenges faced by the teachers in Northern Nigerian mathematics classrooms. The dominant Discourse practices of teachers in SS2 (Grade 11) classrooms were explored in order to understand the extent and exact nature of this problem. The study adopted a qualitative research design. Data gathering techniques included videoed lesson observations in four classrooms and face-to-face interviews with the four Secondary school Mathematics teachers who teach in these classrooms. Findings in this study revealed that mathematical code-switching and mathematical symbolising were the two dominant practices, and were used differently from their purposes in the international literature. Code-switching, for example, was used for the purpose of translation only, instead of using it as a resource to harness the teaching and learning of mathematics in the multilingual classrooms. The teachers' used of translation suggests that the home languages possess the great potential of being used in the teaching and learning of mathematics in Nigerian multilingual classrooms. The teachers also attempted the use of symbolising practice productively in teaching students to understand the knowledge of trigonometry during their lessons.

References

Akinoso, S. O. (2014). Causes and remedies of students' mathematics learning difficulties in Nigerian secondary schools. *The ABACUS, Journal of the Mathematical Association of Nigeria, 39*(1), 219-233.

Gee, J. P. (2005). *An Introduction to Discourse Analysis: Theory and Method* (2nd Edition). London: Routledge Taylor & Francis Group.

EXPLORING METACOGNITIVE PROCESSES OF A HIGH AND A LOW ACHIEVING GRADE 9 STUDENT WHEN SOLVING NON-ROUTINE PROBLEMS

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Problem solving is at the centre of most school curricula. Research evidence (Stendall, 2009) shows that students' low problem-solving performance is due to lack of metacognitive behaviour, such as awareness and control of one's own cognitive processes. This case study explores patterns of the metacognitive processes of both a high and a low achieving grade 9 student when solving two non-routine problems. The participants were selected based on convenience. Two different non-routine problems were chosen such that they demanded strategy flexibility, thinking flexibility, and provided the participants with opportunities to engage in metacognitive activities. One problem involved the distance travelled by a bouncing ball and the other involved locating the best position for an airport equidistant to three towns. Data were collected using audio recordings of think aloud sessions followed by interviews. The two participants were exposed to a preliminary interview protocol to elicit their background characteristics. The procedure allowed them adequate practice to understand and develop confidence before using the technique with the research problems. The transcripts were analysed using Kuzle's (2013) cognitive-metacognitive framework, which involved identifying seven types of macroscopic episodes (read, understand, analyse, plan, explore, implement, verify), as well as the transitions between the episodes. Observations and analyses of the preliminary exposures of the participants to non-routine problems showed that the high achiever was relatively more engaged in metacognitive behaviours. However, it was also observed that the use of metacognitive activity does not necessarily involve productive problem-solving processes. The study is ongoing and further results will be discussed during the presentation.

References

Kuzle, A. (2013). Patterns of metacognitive behavior during mathematics problem-solving in a dynamic geometry environment. *International Electronic Journal of Mathematics Education, 8*(1), 20-40.

Marchis, I. (2012). How primary school teachers develop their pupils' mathematical word problem solving skills. *Studia Universitatis Babes-Bolyai Psychologia-Paedagogia, LVII/1,* 97-107.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

RESULTS OF A SURVEY WITH FIRST-YEAR MATHEMATICS STUDENTS ABOUT PHENOMENA AT THE TRANSITION TO UNIVERSITY

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High abandonment (dropout) rates of university students in mathematics are nationally well known and the differences between mathematics in school and university, especially regarding proving, are delineated in theoretical studies (summary in Rach & Heinze, 2017). The digital survey (with several measurement time points) was piloted in 2016 (Schreiber, 2017), and was implemented in 2017/18 with first-year mathematics students. The intention was to investigate the relationship between the dimensions, performance in school, mathematical self-concept, self-reported performance in university (in real analysis), and self-reported experience with proving in school, with different sub-items (e.g. performance in the final examination, performance in homework, performance in the attendance course as sub-items of performance in university). In the data we found expected correlations among different sub-items of the dimensions (e.g. between the performance in the exam and the performance in the attendance course) and between the dimensions (e.g. between the self-concept regarding math in the university and the performance in university and between the final examination mark in math in school and the performance in university). Remarkably, no correlation could be found between the performance in university and the final school examination. To investigate that finding further, we considered students with an excellent performance in school. They could be divided into two groups, students with high performance in university and students with low performance. A qualitative comparison showed that there is no distinction between experience in proving (first measuring time point) and proof validation tasks. Differences that are connected to the variation of performance can be found regarding self-concept, satisfaction with performance in university and reported difficulties in university. In the data we can also observe a difference in the experience in proving (second measurement time point), which can be an indication for a possible reason for the variation of performance or a consequence. Further qualitative research on this special group would be interesting.

References

Rach, S. & Heinze, A. (2017). The Transition from School to University in Mathematics: Which Influence Do School-Related Variables Have? *International Journal of Science and Mathematics Education*, *15*(7), 1343-1363.

Schreiber, I. (2017). Difficulties of first-semester mathematics students. In B. Kaur, W.K. Ho, T.L. Toh & B.H. Choy (Eds.), *Proc. 41th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 76). Singapore: PME.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TEACHER–STUDENT RELATIONSHIP IN GHANAIAN SECONDARY SCHOOLS: IS CRITICAL FRIENDSHIP POSSIBLE?

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Positive teacher-student relationships are associated with positive academic and social outcomes (e.g. McCormick, O'Connor, Cappella, & McClowry, 2013). Drawing on critical friendship (Hamre et al., 2012) as a theoretical base, the current study investigated how senior high school students and teachers perceive the teacher-student relationship in the mathematics classroom. The research question was: What are students' and teachers' views on the teacher-student relationship in the mathematics classroom?

The study used a sequential explanatory research design. First, a sample of ten schools (3,342 students and 49 teachers) from Cape Coast metropolitan area in Ghana was randomly chosen to respond to questionnaires measuring teacher-student relationship. The instrument for students had 33, and that of the teachers had 59 Likert style items. For the qualitative sequence of the study, I conducted 20 student focus group interviews (240 students) and conducted one-to-one interviews with 11 teachers.

A principal component analysis identified six factors from the student questionnaire and the responses indicate that the students perceive their learning environment to be characterised by fear, disrespect for their' learning, and lack of support. The interviews further revealed that students expected friendship, support, healthy context, reciprocal respect, and trust from their teachers. The young adults want their teachers to treat them like colleagues to enable them relax while learning. In addition, the students expect the teacher-student relationship to include some elements of care for their welfare beyond the mathematics taught. These attributes, they claim, will enable them to bring out their learning difficulties. However, teachers expected submissiveness and absolute obedience from their students. Teachers' expectations stem from their understanding of teaching as transmission of knowledge to students. Hence, their expectation that student should be docile with respect to their promptings in the classroom.

References

Hamre, B. K., Pianta, R. C., Burchinal, M., Field, S., Crouch, J. L., Downer, J. T., Little, C. S. (2012). A course on effective teacher-child interactions: Effects on teacher beliefs, knowledge, and observed practice. *American Educational Research Journal,, 49*(1), 88-123. doi:10.3102/0002831211434596

McCormick, M. P., O'Connor, E. E., Cappella, E., & McClowry, S. G. (2013). Teacher–child relationships and academic achievement: A multilevel propensity score model approach. *Journal of School Psychology, 51*, 611-624.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

AFTER-SCHOOL MATHEMATICS CLUB LEARNERS' SHIFTS IN NUMBER SENSE AND PROCEDURAL FLUENCY

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Much research points to learners having extremely weak basic number sense in later primary school, resulting in the dominance of inefficient strategies for calculations with the four operations. There are also particular errors and misconceptions that dominate learners' computations, which are often attributed to the use of either tallies or incorrectly applied mathematical procedures. Five teachers expressed an interest in running after-school mathematics clubs to address these concerns with their learners. They ran fifteen structured after-school club sessions with grade six learners. This research aimed to investigate the nature of learners' evolving number sense, procedural fluency and teachers' experiences of working with learners in the extra-curricular club space.

A social constructivist perspective of learning guides this study, notably Vygotsky's (1978) notion that cognitive development stems from social interactions and guided learning within the Zone of Proximal Development of children and guided by more knowledgeable others. Furthermore, Kilpatrick, Swafford and Swindell's (2001) strands of mathematical proficiency provide the conceptual frame with a particular focus on procedural fluency and number sense. Quantitative data was drawn from learner's scores on pre- and post-assessments on the four basic operations, and qualitative narratives were drawn from learner progression data, as well as from teacher post club experience questionnaires and teacher interviews.

The findings of this study contribute to an understanding of how to support learners to progress towards using more flexible methods of calculation, particularly for poor performing learners. The club space proved effective in enabling such shifts, and improvements were evident in learner flexibility, fluency and performance as reflected in learner methods and scores in the assessments. The teachers' observations about the relaxed atmosphere in the club space, the small groups and learning through play may have enabled the shifts in procedural fluency and number sense in club learners.

References

Kilpatrick, J., Swafford, J. & Findell, B. (2001) *Adding it up: Helping children learn to learn mathematics*. Washington DC: National Academy Press.

Vygotsky, L.S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

SENSEMAKING OF NON-REHEARSING TEACHERS IN REHEARSAL DEBRIEFS

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Research on rehearsals (e.g., Lampert et al., 2013) has focused on the learning of participants who take on the role of teacher, but we know much less about the experience of those in the role of student, or non-rehearsing teachers (NRTs). We examine how NRTs use debrief discussions to make sense of the experience. We draw on a mathematics teacher noticing framework (Jacobs, Lamb, & Philipp, 2010) which is comprised of *attending* to specific details, *interpreting* those details, and, in this context, we posit, *implicating* teaching. NRTs spend rehearsals acting as students, and then they are asked to apply what they learn from rehearsals to their work as teachers, shifting their positionality as they do so. We ask the following research question: In what ways is positioning related to NRT noticing in debriefs?

Data consisted of 8 rehearsals and debriefs with 22 secondary mathematics teachers. NRT talk turns (n = 454) were segmented and coded to reflect instances of *attending* to the rehearsal, *interpreting* the rehearsal, and *implicating* practice; and also the speaker's positionality: student, teacher, or teacher educator (n = 780 segments). We used a chi-square analysis to test for independence of noticing and positionality.

We found that, predominantly, NRTs attended to and interpreted their experiences. Of the 570 segments, 34.6% were attending, 46.0% were interpreting, and 19.5% were implicating. Additionally, NRTs most frequently took the position of students (81% of noticing segments), particularly when they attended to or interpreted their experience, and NRTs' positionality often switched to teacher when implicating practice. The chi-square test indicated that the relationship between the noticing component and positionality was significant, χ^2 (4, n = 570) = 183.23, $p < 0.0001$. NRTs were more likely to attend and interpret as students and more likely to implicate as teachers.

These findings suggest that rehearsal debriefs are venues for NRTs to make sense by attending to, interpreting, and implicating their experiences. Debriefs offered an opportunity for NRTs to cognitively bridge the roles of teacher and student when making sense of practice.

References

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., … Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INTERPRETATIONS OF NUMBER AND IMPLICATIONS FOR MULTIPLICATIVE REASONING

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Why are multiplication, measurement, fractions, and variables such hard topics for students? Much research has investigated students' difficulties in these topics but has not considered how students interpret number-noun phrases. There are (at least) two ways that people could interpret the meaning of number-noun phrases such as "5 paperclips" or "5 Liters": a *measurement* interpretation, in which the 5 indicates how many of 1 paperclip or it takes 1 Liter to make the stuff in question, and what we call a *trope* interpretation, in which "paperclips" and "Liters" say what kind of things the stuff in question consists of but do not imply units of measurement. Although a measurement or magnitude approach to numbers is widely recognized as important (e.g., Bass, 2018), researchers have not considered the possibility that a measurement view of numbers may be difficult for linguistic reasons, because of a competing interpretation of number-noun phrases.

Although the measurement and trope interpretations can be compatible for whole numbers, they diverge when extended to fractions, such as "2/3 Liters," or to multiplication expressions, such as "$3X$" or "$1/8X$". In these extensions, the measurement interpretation can be productive: 2/3 Liters can be understood as 2/3 of 1 Liter; the variable X can be understood as a measurement unit, so that $3X$ means 3 iterates of X, and $1/8X$ means 1/8 of X. However, a trope interpretation can lead to predictable errors. With a trope interpretation of "2/3 Liters," the fraction 2/3 and the noun Liters both describe the stuff in question, but the fraction 2/3 may not be understood as depending on 1 Liter as a referent. Given an X cm strip that is partitioned into 3 parts, a trope interpretation could lead to formulating $3X$ because the 3 and the X are two independent ways of describing the same strip. A trope interpretation of $1/8X$ could lead to identifying 1/8 with X so that both indicate one part of an 8-part strip. We have repeatedly seen these and other kinds of non-normative interpretations that could come from a trope view, in fact with virtually every research participant in our multi-year research project in which we are studying the multiplicative reasoning of future teachers. Thus, it may be important to recognize and study how a trope view could impact reasoning with fractions and multiplication.

Reference

Bass, H. (2018). Quantities, numbers, number names and the real number line. In M.G. Bartolini Bussi, X.H. Sun (Eds.), *Building the foundation: Whole numbers in the primary grades*, New ICMI Study Series. (pp. 465 – 475). Cham, Switzerland: Springer.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TESTING THE TEST: RETHINKING THE LEVELS OF UNDERSTANDING GEOMETRY BASED ON MATHS STUDENTS' VAN HIELE TEST RESULTS

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The van Hiele theory and the Usiskin test built on it are widely known and recognized all around the world. The theory distinguishes five levels of geometric understanding. It also says that one can only pass through the levels "step by step" which means a student cannot achieve one level of understanding without having mastered all the previous levels. These levels classify students into disjoint classes based on their geometric knowledge and approach.

The topic of this presentation is an experiment where we tested both the Usiskin test and the validity of the van Hiele theory at university level. The experiment was carried out at Eötvös Loránd University, Budapest. Subjects of our experiment were 65 first year math major students and 42 first year pre-service maths teachers. They filled in the test twice, in their second semester of their studies, 2018 January and June, before their first geometry course and after they passed the exam of the course. In January the students achieved either level 3 or level 5 in both groups. Surprisingly there was no student at level 4. Moreover lot of them completed the 3rd and 5th levels of the test correctly but failed on the questions of the 4th level. At the second filling, - (in June) every student remained on the exact same level where they were in January. This means that finishing the geometry course did not develop their geometric understanding. The reason might be that at a university course students are assumed to be on level 4. It is also a part of the van Hiele theory, that a student can make a next level only if he is taught from that level, in the "language" of that level. To clarify the question we interviewed about 60% of the students. Those who filled the test on level 3, they were on level 3, and those filled the test on level 5, they were at least on level 4. We couldn't test level 5, because none of them was aware of different axiomatic systems.

On the basis of our university experiment we found that the theory has not led to contradiction, however, we have proved, that the 5th level of the test may not measure being on the 5th level. Furthermore, it has not been proven that this leveling is complete and exclusive. In order to examine what kind of leveling structures can be imagined at all, we tried to map and to ascertain what is meant by geometry these days. Based on these aspects, knowledge system and geometric culture, we started to create a new test.

References

Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry*. Chicago: University of Chicago.

NINE-YEARS-OLD STUDENTS' UNDERSTANDING OF 2D-SHAPES

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Students' understanding of 2D-shapes is a key aspect in the development of geometrical thinking. The young children's (age 3 to 6) understanding of 2D-shapes is influenced by the way in which they recognise different attributes in a polygon and reason with them (Clements, Swaminathan, Hannibal, & Sarama, 1999). This research aims to identify how 9-years-old students recognise attributes of a figure and reason with them. For that, we regard Duval's (2017) cognitive approach to geometrical learning. Duval highlights the role played by the coordination of three types of apprehensions (perceptual, discursive and operative) and the coordination between them through the dimensional deconstruction, which allows the perceptual recognition of a figure in a configuration of figurative units of smaller dimensions (from 2D to 1D). The participants were 59 third-grade students of primary school (9 years old), that is those who answered to four tasks demanding answers in which student must use discursive and non-discursive registers. Task 1 consists on turning a non-example of a polygon into a polygon and, tasks 2, 3 and 4 asked to identify a common attribute in a set of polygons. We analysed the students' answers by the computer software called C.H.I.C. to obtain hierarchical similarity connections and implicative relations between variables. Results show that (i) recognizing relevant attributes of polygons and identifying the common attributes in a set of polygons depend on the attribute considered and, (ii) the difference in the use of discursive and non-discursive registers influenced how students reason with the attributes. The first result could be explained by the different development of dimensional deconstruction. The second finding indicates that the capacity to generate an explanation about the relation among attributes is a sufficient condition to draw a figure with some specific characteristics, which seems to indicate a hierarchical relation between verbal declarative knowledge and perceptual apprehension in this age.

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References

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*, 192–212.

Duval, R. (2017). *Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations.* London: Springer.

PRE-SERVICE TEACHERS' PERCEPTIONS AS A PART OF LESSON STUDY TEAM

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Even though the pre-service teachers attend many courses in the university, it is however, very difficult for them to learn and think from the perspective of teachers (Isoda, 2007). Lesson Study is accepted as a main method of professional development for Japanese teachers (Fujii, 2016; Inprasitha, 2006; Lewis, 2016; Shimizu & Chino, 2015; Takahashi & McDougal, 2016).

The objective of this study is to investigate the perceptions of pre-service teachers as a part of lesson study team. Participants were 150 pre-service teachers in the 2017 academic year in the Mathematics Education Program, Faculty of Education, Khon Kaen University, Thailand. These teachers had taught and participated in the lesson study team for one year in the lesson study schools project which was conducted by the Center for Research in Mathematics Education, Khon Kaen University. Data were collected from firstly,participant observations in a workshop for internship students prior to teaching and during the academic year, and from observations in the pre-service teachers' classrooms at school, as well as from six reflection meetings over one academic year. Then secondly, data were collected from a documentary review of the reflections of pre-service teachers. In addition, the text from the observation notes, and document reviews were considered together and coded through the process of a weekly cycle of Open Approach Lesson Study (Inprasitha, 2017).

The results revealed that the pre-service teachers perceived as a part of lesson study team according the process of weekly cycle; 1) collaboratively design research lesson (Plan), the pre-service teachers familiarized with students' ideas from sharing with the members in lesson study team, that helped them to prepare the appropriate problem situation as well as to anticipate students' ideas, 2) collaboratively observing research lesson (Do), the pre-service teachers worked with their peer and in-service teachers to talked less and more observed and more patient for waiting for student's response and tried to form the behaviour of students to listen to each other in the classroom, 3) collaboratively doing post-discussion or reflection on teaching practice (See), the pre-service teachers learned to receive the critical reflection in order to improve the classroom. Moreover, they got closer to students' ideas from various perspectives from lesson study team.

References

Inprasitha, M. (2017). *Open Approach Lesson Study: An Innovation for Teaching Mathematics to support STEM*. Paper presented at the International STEM Education Conference, Chiang Mai, Thailand.

FROM TREES AND LISTS TO NUMERICAL EXPRESSIONS: SYMBOLIC INTERMEDIATION IN YOUNG CHILDREN'S LEARNING OF COMBINATORICS

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We present the analysis of Primary School textbooks and of children's use of representations in combinatorial problem solving. Duval (1995) emphasises the importance of semiotic transformations: *treatments* and *conversions*. Depending on the level of congruency between representations, students may present more/less difficulty in conversions – which leads to the need of intermediate auxiliary transitional registers.

Transformations of representations are necessary in pupil's conceptual development and must be addressed in textbooks and in classroom learning. In Combinatorics, conversions allow the recognition of relations present in different problem types.

Textbooks directed to children (6 to 10 years) were analysed in respect of combinatorial situations presented and representation conversions required. All problems involved at least one conversion – usually converting natural language and drawing into numerical expression. Conversions were a positive aspect in the textbooks, but these resources were not always useful in aiding children in understanding numerical expressions.

In the use of intermediate representations, the analysis showed that children were more successful in identifying conversions from natural language (NL) to lists (L) and to trees of probabilities (TP) (more than 50% success), than identifying conversions from L and TP to numerical expressions (NE) (25% success). This finding corroborates that L and TP have a high level of congruency, but both have low congruency with NE.

When taught in situations that used these registers, the 10-year-old children progressed in converting representations, specially using TP as intermediate symbolisms between NL and NE in problems that resulted in larger numbers. This finding corroborates that there is more congruency between TP and NE, than between L and NE.

The set of results, in the analysis of textbooks and in children's problem solving, leads to the recommendation that intermediate representations, especially trees of probabilities should be used in the process of teaching Combinatorics to young children.

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Reference

Duval, R. (1995). *Sémiosis et pensée humaine*. Berna: Peter Lang.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

FOSTERING PROPORTIONAL REASONING USING AN ICT BLENDED MODULE

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The position of proportional reasoning (PR) in school mathematics curriculum bears significance because of its wide applicability within different subject domains. The development of PR progresses through understanding relations between quantities and shifting between additive and multiplicative reasoning by analysing quantities and identifying and understanding relationships. As is often seen in practice, learners appear to have an over-reliance on procedural knowledge over conceptual understanding particularly in the context of routine multiplicative tasks through which PR concepts are developed and tested in school and which may not lead to an advantageous situation (Hallett, Nunes & Bryant, 2010). Exploring ways to leverage technology for developing learners' PR by designing simulative tasks is a challenge. This presentation discusses *a digital games enabled blended module* to facilitate students' proportional reasoning and scaffold their problem solving strategies.

Data is taken from a current project, the *Connected Learning Initiative (CLIx)* aimed at transforming students' learning experiences by providing a resource rich environment with access to technology-enabled open educational resources developed under CLIx. This project is currently being implemented in the four Indian states of Mizoram, Telangana, Rajasthan and Chhattisgarh with a population of around 48,000 Grade 8 and 9 students and 3,000 teachers of science, mathematics and English using 17 ICT integrated modules and courses for in-service teachers to facilitate use of these modules and to facilitate reflection on the use of ICT in classroom pedagogy. Proposing PR progressions through four units, the module begins with consolidating additive reasoning using a food sharing tool that simulates the everyday situation of fractions, their equivalent representations and operations over them; the digital game of pattern tool strengthens the scale concept and scale ratio in linear and two dimensions while the variation tool simulates direct and inverse variation situations. The focus is on creating a discursive classroom practice which creates a safe space for learning from each other by valuing, asking questions, challenging each others' contentions, seeking and providing justifications and constructing proofs. Such mathematical practice helps in justifying and reconciling students' different strategies and misconceptions, and calls for revisiting the use of ICT and classroom discussion for fostering meaningful learning.

References

Hallett, Nunes, T. & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology, 102*(2), 395-406.

ENABLING SHIFTS IN CLASSROOM NORMS TO INTEGRATE OUT-OF-SCHOOL AND SCHOOL MATHEMATICS

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A challenge for a teacher or instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful math learning. What should the goals be of such a pedagogical intervention? What forms of participation can be seen in a classroom implementing these goals? Charting a pedagogical approach building such connections remains a challenge in the domain of out-of-school mathematics with few studies reported in the South Asian developing world context such as India's. We analyse a design experiment conducted in a middle grade mathematics classroom aimed at integrating mathematical knowledge gained from everyday and work-contexts with school learning, focused on these research questions:

- How can mathematical knowledge gained from everyday and work-contexts be integrated with school learning to enhance students' conceptual understanding? Challenges in designing instructions for such integration?

The pedagogic approach connected out-of-school and school mathematics learning by enabling a series of shifts in classroom norms. These shifts encouraged merging of students' identities as knowers, doers and learners which facilitated drawing on what they knew from their exposure and experience in work practices and used such knowledge in school mathematics learning. The lesson focused on strengthening and extending students' binary fractions understanding gained from everyday contexts and connecting binary and decimal fractions. Analysis draws on the goals and enacted episodes focusing on students' agency and negotiation of identities (as used in Cobb, et. al, 2009) about how clarifications were sought and queries addressed, how assistance and support were extended, contentions were challenged or explained. Examples show that enabling of shifts mediated connection of prior knowledge with the lesson and facilitated knowledge transfer. These shifts accorded power and legitimacy to students' agency as knowledgeable persons contributing to pedagogical processes during classroom learning.

References

Bose, A. & Subramaniam, K. (2011). Exploring school children's out of school mathematics. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 177-184). Ankara: PME.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education, 40*(1), 40-68.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

DEVELOPING A PROBLEM SOLVING MEASURE FOR GRADE 4

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Problem solving is central to mathematics learning (NCTM, 2014). Assessments are needed that appropriately measure students' problem-solving performance. More importantly, assessments must be grounded in robust validity evidence that justifies their interpretations and outcomes (AERA et al., 2014). Thus, measures that are grounded in validity evidence are warranted for use by practitioners and scholars. The purpose of this presentation is to convey validity evidence for a new measure titled Problem-Solving Measure for grade four (PSM4). The research question is: What validity evidence supports PSM4 administration? The PSM4 is one assessment within the previously published PSM series designed for elementary and middle grades students. Problems are grounded in Schoenfeld's (2011) framework and rely upon the perspective of Verschaffel et al. (1999) that word problems be open, complex, and realistic. The mathematics in the problems is tied to USA grade-level content and practice standards (CCSSI, 2010).

Method, Results, and Future Research

This study uses the *Standards* (AERA et al., 2014) as a basis for describing validity evidence for the PSM4. The five sources of validity evidence described in the *Standards* are test content, response processes, relations to other variables, internal structure, and consequences from testing. A design science approach guided assessment development and data collection. This study used quantitative and qualitative data collection and analysis techniques to explore evidence for the PSM4. Taken collectively, evidence supports use of the PSM4 among practitioners and scholars.

References

AERA, APA, & NCME. (2014). *Standards for educational and psychological testing*. Washington, DC: American Educational Research Association.

Common Core State Standards Initiative. (2010). *Common core standards for mathematics*. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Schoenfeld, A. (2011). *How we think: A theory of goal-oriented decision making and its educational applications.* New York, NY: Routledge.

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking and Learning, 1*, 195-229.

MATHEMATICAL IDENTITY AND INTEREST IN THE MIDDLE GRADES: RETHINKING INTERVENTION PRACTICES

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Extended learning time in mathematics is a strategy often proposed as an intervention for struggling students in the middle grades. In the United States, this extra time is often created by enrolling students in an additional period of mathematics. However, broad-based strategies that fail to account for the needs and social realities of children can be problematic (Martin, 2011) and these intervention courses are often mere replications of the original course. Personal interest has been shown as a predictor for persistence in mathematics, achievement, and identity formation (Middleton, Jansen, & Goldin, 2016). I developed a partnership with a local museum to provide students currently enrolled in a “double-block” mathematics intervention course with opportunities to use their additional time to stimulate interest and develop positive mathematical identities through active participation in out-of-school STEM workshops. The research questions are:

1. In what ways do the incorporation of these out-of-school STEM based models support the development of student mathematical interest and identity in a classroom setting?
2. What are the challenges in implementing this curriculum?

In this self-study, I critically reflected upon my experiences teaching the redesigned course. These personal reflections, recorded in personal journals, formed a source of data for this study along with analytic memos.

My initial perceptions of student dialogue and participation data suggest that these out-of-school STEM based tasks provided students with a greater sense of utility with respect to mathematics. This encouraged classroom persistence with challenging problems as students began to self-identify themselves as “engineers”. Regular work in a “laboratory”, which admittedly diverted time from other portions of the school day, also supported positive dispositions towards careers requiring mathematics.

References

Martin, D. B. (2010). Liberating the production of knowledge about African-American children and mathematics. In D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation in the lives of Black children* (pp. 3-38). New York, NY: Routledge.

Middleton, J. A., Jansen, A., & Goldin, G. A. (2017). The complexities of mathematical engagement: Motivation, affect, and social interactions. In Jinfa Cai (Ed.), *Compendium for research in mathematics education* (pp. 667-699). Reston, VA: National Council of Teachers of Mathematics.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INTERDISCIPLINARY TEACHER TRAINING: ENACTIVE CONTEXT AND BLENDED LEARNING

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Starting from suggestions of the principles of vicariance and simplexity (Berthoz, 2000), our teaching research is heading towards certain trajectories that enhance interdisciplinary paths, capable of interconnecting disciplinary skills and deepening epistemological and methodological issues of great interest. The research involved a group of 92 Italian high school teachers. The in-service teachers attended a training course in a multidisciplinary context. In Italy, involving teachers of different disciplines in the same training path is not very frequent for high school. Our experiment is designed on the base of enactivism (Hutto, 2005). In this framework, the aim of the study was to develop interdisciplinary activities working collaboratively in small groups by blended learning modality on a social platform. The teachers designed interdisciplinary activities based on a mathematical impulse: all disciplines acted on the same theme. The current paper focuses only on the qualitative data, which were subjected to an inductive content analysis. Results provided evidence of the processes involved during the collaborative activities. Difficulties of students in mathematics often depend on aspects that are transversal to mathematics and, in particular, the incidence of linguistic aspects in the process of learning mathematics is increasingly emerging. We adopt Stephen Toulmin's (1975) definition of argument, as it offers a model that interprets most of the types of argumentation usually used in mathematics and establishes connections with many types of argumentation used in other fields and in everyday life. The results expected from the research are an increase in the specific disciplinary competences of the students and, in a collateral way, an increase in the competences of the teachers on transversal themes. The competences of students and teachers will be investigated through classroom observations. The qualitative analysis of the whole training process will be carried out through questionnaires and interviews with all the actors of the project. The initial results were so encouraging that the test is going to be repeated on a wider number of classes and on different subjects.

References

Berthoz, A. (2000). *The brain's sense of movement (Vol. 10)*. Boston: Harvard University Press.

Hutto, D. D. (2005). Knowing what? Radical versus conservative enactivism. *Phenomenology and the Cognitive Sciences, 4*(4), 389-405.

Toulmin, S. E. (2003). *The uses of argument*. Cambridge: Cambridge University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

IDENTIFYING A LEARNING PROGRESSION FOR THE RESOLUTION OF A SYSTEM OF LINEAR EQUATIONS

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Systems of linear equations (SLE) are considered important mainly for two reasons: their diverse applications both in the field of engineering and social sciences and their importance as mathematical content for further studies in mathematics, especially in Linear Algebra. Despite this, SLE have not been studied extensively by mathematics educators (Oktaç, 2018). Thus, in this paper, we analyze the reasoning of first-year university students when they determine the solution set of a SLE has infinite elements. Participants in this research were 45 university students of the bachelor of engineering degree. These students participated in a teaching experiment in which the teacher used a hypothetical learning trajectory composed by 4 tasks about solving SLE that was designed taking into consideration the instructional design heuristic of emergent models (EM). The collected and transcribed data were the responses of the students to the 4 tasks together with audio and video recordings. Each of the statements (oral or written) of the students was considered as the unit of analysis. Then, the levels of activity of the EM were identified in the analysis units. To ensure the validity and reliability of the analysis, two researchers coded the data separately and discussed the discrepancies. This process allowed us to analyze the reasoning of the students when they determine that a SLE has an infinite solution set using the ranges of the augmented matrix and the coefficient matrix, and the number of unknowns of the SLE.

The main results of this research shown that selected students transitioned from their model-of informal mathematical activity to a model-for formal mathematical reasoning. Therefore, we can point out that the EM provided them the necessary support to establish when a SLE has as an infinite solution set. On the other hand, connecting the activities with the mechanism of reflection on the activity-effect relationship allowed to characterize the transition between the different levels of activity enriching the analyses of teaching-learning processes (Tzur & Simon 2004).

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References

Tzur, R., & Simon, M. (2004). Distinguishing two stages of mathematics conceptual learning. *International Journal of Science and Mathematics Education*, *2*(2), 287–304.

Oktaç A. (2018). Conceptions About System of Linear Equations and Solution. In S. Stewart, C. Andrews-Larson, A. Berman, & M. Zandieh (Eds.), *Challenges and Strategies in Teaching Linear Algebra. ICME-13 Monographs* (pp. 71-101). Cham: Springer.

HYPOTHETICAL LEARNING TRAJECTORIES IN UNIVERSITY MATHEMATICS: THE CASE OF LINEAR ALGEBRA

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This exploratory study documents key characteristics of research on hypothetical learning trajectories (HLTs) in a university level Linear Algebra course, as found in journal articles and conference papers we reviewed. Because most HLTs have been developed and used in K-12 settings, our intention is to provide an image of how HLTs can be designed for teaching mathematics at the university level. We depict the study context (e.g., country), research objective, theoretical perspectives, methodological approaches, HLT perspective used, and components of the particular HLT. This exploratory study review is limited by our methodological choice and by the budding research experiences that focus on HLTs at the university level.

We found that five of the nine selected papers focused only on the first two components of HLT, namely, the learning goal and a set of learning tasks. However, the third component (hypothetical learning process), which underlies the HLT meaning, is mentioned only in four of these papers. Moreover, the hypothetical learning process described in three of these papers seems to be about a set of markers in the development of the intended mathematics – not about the transition from one marker to the next (Tzur, 2019). On the other hand, all nine papers reviewed indicated that, for our conceptual goal, the objective of using the HLT is to support students in the construction of new concepts in Linear Algebra. To continue contributing to this, we believe it behooves that future research must focus on explanation of the third component of HLT (the hypothetical learning process). For example, a future study could help teachers understand the transition in students' conceptualization, from linear dependence to linear independence, by detailing the cognitive processes involved so that teachers can adapt such HLTs to their contexts.

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References

Tzur, R. (2019). HLT: A Lens on conceptual transition between mathematical "markers". In D. Siemon, T. Barkatsas, & R. Seah (Eds.), *Researching and learning progressions (trajectories) in mathematics education* (pp. 56-74). Boston, United States: Brill Sense.

HYPOTHESIS FORMULATION, DATA ANALYSIS AND CONCLUSIONS: LEARNING OF ELEMENTARY STUDENTS

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Research should be considered the guiding theme in statistical education. Developing the Statistical Literacy of individuals also implies requiring them to reflect on data conveyed by the media, thereby allowing a critical formation of the population (Gal, 2002). In this sense, the aim of this study was to analyze the learning of 5th and 7th grade students (10 and 12 years old) about hypotheses formulation, analysis with real data in bar charts, comparing hypotheses and data, evaluating conclusions and, finally, use probabilistic language during predictions. For this, a pre-test, followed by three meetings of pedagogical intervention, and post-test were administered to students of public schools in Recife / Brazil. A total of 114 students from 6 (six) classes participated, at the beginning of each year of schooling. In both years, one of the classes was constituted as a control group, which did not undergo any pedagogical intervention with the researcher. The other groups participated in teaching processes starting from the context of each class, as core of interpersonal relations and with knowledge. In each test, students answered 9 (nine) questions that dealt with the skills listed in the study objective, which were presented from two activities with contexts familiar to the students, one with a univariate distribution and the other with a bivariate (correlation).

As a result, it was observed that there were significant advances in the learning of 5th graders ($t(31) = 3.908$, $p < 0.001$) and also in the 7th grade ($t(46) = 2{,}718$, $p = 0.009$) for the groups with intervention teaching. The same was not observed for the control groups: 5th year ($t(14) = 0.135$, $p = 0.894$) and 7th year ($t(19) = 0.863$, $p = 0.399$). The results show that teaching organized in order to emphasize the evidence of the real data represented, with analysis and reanalysis of the graphs as confirmation for arguments, were no longer based on their beliefs and started to be based on the information presented in the graph. Thus, the results show that these students are able to modify behaviors in relation to data and decision making with appropriate interventions. Responses to activities normally used in the school environment claim certainties. Rethinking the educational practice for development from the earliest years of schooling, aiming at changes in the culture in relation to statistical knowledge, tends to favor the constitution of citizens.

References

Gal, I. (2002). Adults Statistical Literacy: meanings, components, responsibilities. *International Statistical Review, 70*(1), p. 1-25.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

ARE WE PAINTING A COMPLETE PICTURE? CONSIDERATIONS OF EXPLAINING A PROCEDURE

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Community colleges (CC), well known for being *teaching institutions*, institutions that focus on and care about teaching, play an essential role in the tertiary education experience of many students in the United States. Because procedures heavily dominate mathematics instruction in CCs, it is important to begin to understand the way that procedures are taught during instruction. This paper seeks to analyse and describe how one developmental mathematics instructor explains the procedure for solving absolute value equations in her intermediate algebra course. I investigate the following question: What elements are present during the explanation of a procedure in a developmental mathematics class?

Grossman et al. (2009) have argued that the practice of teaching can be decomposed; instructors can learn to improve their practice though the activity of decomposing their practice, which "involves breaking down practice into its constituent parts for the purposes of teaching and learning" (Grossman et al., 2009, p. 2069). Herbst (2013) created a decomposition of practice for explaining a procedure, which does not only relate to a series of steps, but also helps students make sense of the purposes, conditions, advantages, and limitations of the procedure.

In this study, I observed 12 class meetings of an intermediate algebra course at a CC. I transcribed moments when the instructor explained procedures to students and analysed the explanations by features of Herbst's decomposition of practice. This paper looks at the explanation of the procedure of solving absolute value equations, an explanation which demonstrated the most elements of the decomposition (four of the seven elements). By decomposing the explanation of solving absolute value equations, we can begin to see what features are more emphasized than others. The instructor was able to highlight some essential features of explaining the procedure, however, other important aspects, such as focusing the procedure, connecting the procedure to prior knowledge, and identifying tools or notations as students saw fit, were regularly less explicitly discussed. Findings suggest that students are not exposed to a complete picture of mathematics, as important features of procedures are not being addressed.

References

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9), 2055-2100.

Herbst, P. G. (2013). *Explaining procedures: A decomposition of practice*. University of Michigan, Ann Arbor, MI. Retrieved from http://hdl.handle.net/2027.42/113192

ONE SHORT YEAR: REFLECTIONS ON THE FOSTERING INQUIRY IN MATHEMATICS PROJECT

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Blair (2014) found that inquiry-based learning has not found its way into daily teaching practice in Early Years mathematics classrooms. The *Fostering Inquiry in Mathematics* (FIiM) project was undertaken in three Australian schools to investigate ways to enable teachers to take a problem solving approach to mathematics. Incorporating new elements into existing teaching is always a complex process. The project adopted an empirically grounded theory of action for instructional improvement in mathematics (Jackson et al, 2015). It used supportive curriculum materials, teacher professional learning days out of the classroom, collaborative meetings, and school-based support for learning. The research question was: what are the effects of attempts to stimulate ambitious teaching involving problem solving and inquiry with young children?

Teachers' views of the effects of the year-long project are reported here. At the start and end of the FIiM year 22 teachers of young children (aged 5 -7 years in the first two years of formal school) responded to an online survey. Six questions, common to both surveys, were designed for response on a Likert scale. The pre- and post- responses to each question were compared. The teachers' self-reported findings showed: significantly increased commitment to inquiry/problem solving; significantly increased confidence in teaching mathematics; significantly higher levels of belief in the curiosity of young children about mathematics; increased pedagogical skills in teaching mathematics; greater knowledge for teaching mathematics with young children; and increased enthusiasm for teaching mathematics.

Outcomes of the FIiM project showed that with supportive inquiry-based problem solving curriculum materials and opportunities for professional learning, in-service teachers of young children are stimulated to experiment with their teaching practice. It is over-stating the results to claim that teachers' greater commitment to inquiry-based problem solving led to the long-term change in their practice. Follow-up research is needed to investigate this matter. Nevertheless, the current findings show that teachers' belief in children's mathematical curiosity, together with their increased pedagogical skills and knowledge, re-kindled their enthusiasm for teaching mathematics.

References

Blair, A. (2014). Inquiry maths: An idea whose time has come. *atm, May*, 32-35.

Jackson, K., Cobb, P., Wilson, J., Webster, M., Dunlap, C., & Appelgate, M. (2015). Investigating the development of mathematics leaders' capacity to support teachers' learning on a large scale. *ZDM, 47*(1), 93-104.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

VISUALISATION PROCESSES IN MATHEMATICS CLASSROOMS - THE CASE OF GESTURES

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This study was guided by the observation that when teachers talk and teach, they very often gesture, which can reveal information that may not be apparent in their verbal speech. This qualitative interpretive study, framed by an enactivist perspective, aimed to investigate the nature and role of gestures that three purposefully selected junior primary phase (Grades 0-3) teachers used in the teaching of mathematics. The study also aimed at understanding the selected mathematics teachers' views on the roles of their gestures as visualisation tools in the teaching of mathematics. Data was collected through classroom video-recorded observation and stimulated recall interviews. Findings were based on 30 lesson observations across an entire school term involving three foundation phase teachers. An analytical framework, which was imposed on the video-recorded lessons, was grounded in McNeill's (1992) classification of gestures, which included pointing (deictic) gestures, iconic (illustrative) gestures, metaphoric gestures, beat (motor) gestures and symbolic (emblem) gestures. This study aligned itself with the argument that gestures can be important visual resources that can play a valuable role in the teaching-learning process of mathematics. Very pertinently, they can be used as an important bridge between imagery and speech. They may be seen as a nexus bringing together action, memory, speech, imagery and mathematical problem solving. The analysis was two pronged; firstly, a frequency analysis was done across all the participating teachers for all the 30 lessons; secondly, a qualitative analysis of the interviews conducted with each of the selected teachers.

In general, the participating teachers felt that the use of gestures provided for a learning environment that was dynamic and rich. Gestures enabled the teachers to reinforce, support, illustrate and strengthen concepts they were trying to teach. Gestures facilitated both instructional and conceptual communication, the former being communication that refers to procedures of what needs to be done, and the latter being communication that is mathematical referring to concepts and content. The study concluded by acknowledging that gestures are recognized as legitimate teaching resources and strategies, provided they are used strategically and meaningfully.

References

McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. Chicago: University of Chicago Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INVESTIGATING THE MATHEMATICAL CONTENT KNOWLEDGE OF STUDENT TEACHERS IN A TEACHER EDUCATION COURSE.

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The problem of poor learner performance in mathematics is prevalent in South Africa. Studies that investigated the crisis found that the majority of mathematics teachers lack adequate knowledge of both mathematics content and pedagogy. The South African Department of Basic Education (DBE) (2009) have questioned the way teachers are prepared for teaching leading to focused pre-service teacher education efforts to address the problem of inadequate teacher knowledge. This study investigated the mathematical content knowledge of preservice teachers (PSTs), registered for a Post Graduate Certificate in Education (PGCE) in Foundation Phase (FP) at a university in South Africa. The aim was to identify possible misconceptions or gaps in their content knowledge so as to strengthen this knowledge during the course. We sought to answer the question: What primary school mathematical content knowledge do PSTs registered for a PGCE have at the beginning of their teacher training?

The study was informed by Ball, Thames and Phelps's (2008) Mathematical Knowledge for Teaching (MKfT) framework that propose that there is knowledge that is specific to the work of mathematics teaching. This paper focuses on the common content knowledge PSTs need for effective teaching. Content knowledge is central to development of MKfT and without MKfT, teachers cannot teach mathematics effectively (Ball et al., 2008).

Following a case study approach, thirty FP PGCE PSTs were requested to write the most recent 2014 Grade 6 mathematics Annual National Assessment. The results showed that six PSTs scored less than 50%, 18 got 50-79% and only six achieved above 80%. These results indicated that the majority of PSTs had limited content knowledge. An error analysis of the PSTs' work established that the majority of the PSTs had challenges with multiplication and division problems. From this study, we concluded that PSTs need additional support with the mathematics content knowledge, particularly multiplication and division, in order to teach effectively.

References

Ball, D., Thames, M.H., Phelps, G. (2008). Content knowledge for teaching. What makes it special? *Journal of Teacher Education,* 59(5), 389-407.

Department of Basic Education. (2009). *National Curriculum Statement for Mathematics. Grade 10-12*. Pretoria: Government Printer.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

IMPACT OF A HEURISTIC METHOD OF TEACHING ON SOUTH AFRICAN MATHEMATICS TEACHERS' PEDAGOGY

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In order to solve a given problem, individuals need to engage heuristics. Heuristics are problem-solving strategies that learners use to solve mathematics problems rather than using algorithms (Polya, 1957). For effective mathematical problem-solving learners need to have access to an organised and retrievable set of problem-solving strategies which are known as heuristics. The process by which teachers support learners to use heuristics to solve mathematics problems is known as the heuristic method of teaching (HMT). South African teachers are known to adopt a 'traditional' method of teaching mathematics and rarely engage a HMT. The traditional method of teaching views the learner as a passive absorber of algorithms and learning through repeated practice. This paper discusses the impact of the HMT approach on mathematics teachers' pedagogy.

This study is part of a large two-cycle classroom-based design project which endeavoured to design a professional development (PD) intervention that supports teachers in teaching problem-solving. Before the study, we conducted a baseline investigation which revealed that participant teachers were teaching using 'traditional' approaches (Chirinda & Barmby, 2018). Four grade 9 mathematics teachers participated in a 6-month PD intervention in which they were trained on how to implement the HMT approach. Teachers attended three, 3-hour PD workshops which resulted in a total of nine hours of training on the HMT. After each PD workshop, we encouraged teachers to go and implement the HMT approach in their lessons for two months whilst we supported, observed and carried out semi-structured interviews with them. During classroom observations, we recorded information on an Observation Comment Card (OCC). The OCC was a valid tool for the study because it incorporated all the heuristics that we used in the study. Data were analysed using inductive data analysis. The findings were that the participant teachers began to incorporate the HMT approach and helped learners to use the various heuristics to solve mathematics problems. After solving problems, teachers were encouraging learners to look back on the answer and the solution process. The findings give insight into how the HMT can be implemented in the teaching of mathematics in South African schools.

References

Polya, G. (1957). *How to Solve It*. Princeton, NJ: Princeton University Press.

Chirinda, B. & Barmby, P. (2018). South African Grade 9 mathematics teachers' views on the teaching of problem-solving. *African Journal of Research in MSTE, 22*(1), 114–124.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TEACHING ACTIVITY IN MATHEMATICS AND ART CLASSROOMS: EXPLORING CONNECTIONS FOR ENHANCING MATHEMATICS LEARNING

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Many studies assert various connections between mathematics and art (Grzegorczyk & Stylianou, 2006). However, most research refers to out of school contexts. Aiming at creative ways to incorporate into regular mathematics teaching, building bridges between art and mathematics in teaching could be of educational benefit. Using an activity theory perspective (Engeström, 2001), we examine the interplay between mathematics and art teaching activity systems, focusing on the nature of the mathematical activity enacted, intending to explore possible congruencies or tensions. In particular, the research questions pursued were: a) What features of mathematical activity developed in a visual art class can be identified? b) In what ways do these features operate in a visual art class compared to the math/s class?

Situated in two art-based schools (grades 7-12), the study adopts an ethnographic approach; the participant researcher visited each school twice per week, 8 hours per day over a period of 3 months, keeping field notes (observation of mathematics and art classrooms, informal discussions, open-ended interviews and self- reflections). Using Grounded Theory techniques, selected episodes were coded based on a) the features of mathematical activity taking place (concepts, processes, procedures); b) the emerging tensions in relation to tools, goals and rules of the two activity systems.

Several mathematical practices common to art and mathematics classrooms were identified, but also tensions between the two systems. For instance, in constructing a ballerina tutu, two students used mathematical concepts (e.g. geometric shapes), procedures (e.g. geometrical constructions) and processes (e.g. problem-solving). However, tensions related to tools (e.g. thread/point as compass), goals (time-limited art-creation) and rules of the art community (precision-level flexibility) emerged. Students' active involvement with successfully drawing a bisector, a task with inherent mathematical authenticity, indicate that visual art lessons may provide meaningful contexts for embracing rich engagement in mathematics teaching and learning.

References

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of education and work*, *14*(1), 133-156.

Grzegorczyk, I., & Stylianou, D. A. (2006). Development of Abstract Mathematical Thinking Through Artistic Patterns. *Proc. 30th Conf. of the IGPME* (Vol. 3, pp. 217-224). PME30.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TEACHERS' IDEAS ABOUT ARGUMENTATION IN PRESCHOOL MATHEMATICS

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Kindergarten teachers (KT) have an important role as guides and mediators in the classroom, and what they think about the mathematics they teach has an impact on their teaching and on children's learning (Hedges, 2005). Drawing on the premise that argumentation is key to the teaching and learning of mathematics, this Oral Communication aims at characterizing KT’s ideas about students’ argumentation. This research is part of an ampler doctoral study seeking to characterize students’ arguments in kindergarten mathematics and how they evolve during a year.

We maintained two individual, semi-structured interviews with two KTs (T1 and T2) in order to characterize their ideas about argumentation in preschool mathematics in the context of the construction of the notion of number. The interviews consisted of a total of 17 questions, organized in three areas of interest: sophistication of arguments, planning and orchestration (in relation to fostering argumentative competency) and the roles of argumentation. Interviews were analysed according to categories in two complementary dimensions: function of arguments, adapted from De Villiers (1990), and developmental trend, according to Krummheuer (2013).

Our main result is that T1 identifies the explanatory and verification functions of argumentation and T2 identifies the explanatory and communicative functions but neither of them identifies other functions of arguments. Both teachers notice the pre-eminence of diagrammatic characteristics (associated to the use of concrete resources) over narrative characteristics (characterized by sequentiality and a mixture between the real and the imaginary) in kindergarten arguments related to counting tasks. Nevertheless, they do not recognize the narrative characteristics of arguments as an aspect to develop in kindergarten. Our results suggest that for these KTs, argumentation is characterized by being predominantly diagrammatic and explanatory. In the presentation, analysis and results will be discussed in detail.

References

De Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, *24*, 17-24.

Hedges, H., & Cullen, J. (2005). Subject knowledge in early childhood curriculum and pedagogy: Beliefs and practices. *Contemporary issues in early childhood*, *6*(1), 66-79.

Krummheuer, G. (2013). The relationship between diagrammatic argumentation and narrative argumentation in the context of the development of mathematical thinking in the early years. *Educational Studies in Mathematics*, *84*, 249-265.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

DEVELOPMENT OF AN ANALYTICAL TOOL TO BETTER UNDERSTAND TEACHER LEARNING IN THE CLASSROOM

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The international research project *Learning from Lessons** (2017–2019) investigates mathematics teacher professional learning during lesson planning, teaching, and reflection. The non-linear ‘interconnected model of teacher professional growth’, initially developed by Clarke and Peter (1993), provides the theoretical background. A key element in the research design is the provision of lesson plans which participating teachers are asked to adapt and then deliver the lesson to their usual class.

The main research interest of the overall project is concerned with the question how participating teachers from Australia, China and Germany structure and construct their teaching by adapting lesson plans that are provided by the research team. Related sub-questions that are addressed in this oral communication seek to determine what adaptions to the lesson plan teachers make, we ask what they find important to teach those *successfully* and can one distinguish *patterns*.

The international data collection involves quantitative and qualitative data with respect to two sub-studies: *Case studies* with teachers of Grade levels 5, 6 and 7 aim to illicit the mechanisms connecting attention to learning. *Online surveys* (about 40 teachers per grade in each country) seek to formulate and test hypotheses regarding teacher attention and consequent learning.

One aim of the ongoing project – and the topic of this paper – is to analyse the adaptations teachers make and find criteria on what teachers focus on. Hence, the focus of this oral communication is to identify different categories of teacher adaptations using qualitative content analysis of the lesson plans and the transcripts of subsequent teacher interviews. The aim is to develop an analytical tool to be applied to the data of the online study. Our findings will be used to analyse teachers’ learning during lesson planning. First results indicate that German teachers tend to focus the most on time frame and scope of tasks, context (of mathematical word problems), and presentation.

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References

Clarke, D. J., & Peter, A. (1993). Modelling teacher change. In W. Atweh, C. Kanes, M. Carss, & G. Booker (Eds.), *Contexts in mathematics education - Proceedings of the 16th conference of the Mathematics Education Research Group of Australasia* (pp. 167–175). Brisbane, Australia: MERGA.

COMPREHENSION AND TRANSFORMATION: INSIGHTS INTO THE PEDAGOGICAL REASONING OF AN EXPERIENCED TEACHER

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Shulman (1987) argued that teaching is an act of reasoning, and proposed a model of pedagogical reasoning and action, comprising of *comprehension*, *transformation*, *instruction*, *evaluation*, *reflection*, leading to *new comprehension*. In his model, teaching begins with an act of comprehension of the content and goals. This is followed by a process of transformation, which requires one or more of the following processes:

> (1) preparation (of the given text materials) including the process of critical interpretation, (2) representation of the ideas in the form of new analogies, metaphors, and so forth, (3) instructional selections from among an array of teaching methods and models, and (4) adaptation of these representations to the general characteristics of the children to be taught, as well as (5) tailoring the adaptations to the specific youngsters in the classroom (p. 16).

In this paper, we focus on the factors that an experienced teacher of mathematics at the secondary level considers in his comprehension and transformation of the content he has to teach. Specifically, we consider the following research question: Which factors influence the pedagogical reasoning of an experienced teacher in his planning for the teaching of the mathematical content? While the documented lesson plan shows very few details, this experienced teacher reports an important set of factors that influence his pedagogical reasoning. These factors include: his sequential focus on procedural and then conceptual knowledge, connections to the real world, extrapolation of the content beyond the prescribed syllabus, questions set for homework, and how he catered the materials to his students' background. Some possible implications and suggestions for future research will be discussed.

References

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TEMPLATES, SIGNIFIED-SIGNIFIER PAIRS, AND KNOWLEDGE OF LINEAR INDEPENDENCE

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Recent advancements in computational tools put matrices at the center stage of linear algebra curriculums, where students are required to consider matrices within highly symbolic multi-representational environments. Then, there arises a question of the nature of knowledge formed in these environments. We set out to document features of such knowledge through the classification of cognitive structures. Data came from interview responses of a group of first-year, college-level, linear algebra students who went through traditional, definition-theorem based, matrix oriented instructions with the use of computational examples. The following research questions guided us: 1) what cognitive structures (if any) are included in interview responses? 2) what are their identifiable features? and, 3) what are their similarities and differences within/between participant responses?

We borrowed from Sfard's ideas on discursive learning and Presmeg's ideas on semiotic chaining. Presmeg (1998) views metaphors and metonymies as cognitive structures where the act of using one object to stand for another (a signified-signifier pair) is considered as functioning with a metaphor or a metonymy. Sfard (2001) characterizes the progression of a discursive process in two stages: 1) Template-driven use of new signifiers; and 2) Objectified use of symbols. Following the two perspectives, our analysis focused on the identification of features of templates, signified-signifier pairs, and the nature of metaphoric/metonymic reasoning involved in these pairs.

Our findings showed signified-signifier pairs facilitating semiotic spaces from which new signifiers (with new meanings) were adopted. Furthermore, our analysis revealed similarities between features of templates and signified-signifier pairs within a single participant response, and differences between participants' responses.

In our presentation, we will discuss findings from the analysis of two participants' responses to a single question, the methodology, and outline the relevant research as well as the instructional activities through which participants formed their knowledge.

References

Presmeg, N.C. (1998). Metaphoric and Metonymic signification in mathematics. *Journal of Mathematical Behavior*. 17 (1), 25-32.

Sfard, A. (2001). Learning mathematics as developing a discourse. In R. Speiser, C. Maher, C. Walter (Eds), *Proceedings of 21st Conference of PME-NA* (pp. 23-44). Columbus, Ohio: Clearing House for science, mathematics, and Environmental Education.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

AN ANALYSIS OF THE KNOWEDGE LEGITIMATED IN A PRIMARY MATHEMATICS EDUCATION EXAMINATION PAPER

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There are multiple factors contributing to learner underperformance in primary school mathematics in South Africa. One such explanation is that primary school teachers have insufficient content and pedagogical knowledge. In response, research suggests that teacher education programmes have failed to equip pre-service teachers with the required content and skills to teach mathematics.

The paper forms part of a broader PhD study that seeks to identify the content and pedagogical knowledge legitimated in assessment tasks and examinations in the mathematics education courses at five higher education institutions offering Bachelor of Education (Foundation and Intermediate Phases) qualifications in South Africa.

The data in this paper consists of a single examination paper from one of the five higher education institutions. The reason for a single examination paper is to explore the use of the methodological tools employed to analyse the data in order to identify the knowledge legitimated.

Two analytic and explanatory tools are employed: Evaluative Event (Davis, 2001) and Mathematics Knowledge for Teaching (Ball et al., 2005). The Evaluative Event is used to identify the objects of acquisition in the examination paper. The MKfT framework is used to denote the domain of knowledge legitimated in each examination question.

The weighting of the objects of acquisition for this paper is ascertained through the contribution or total marks of mathematical or pedagogical objects in the mathematics education examination paper. The majority of the examination paper focuses on mathematical objects. In other words, most of the paper asks questions that focus explicitly on content knowledge.

The questions in the examination paper were categorized in relation to the Mathematics Knowledge for Teaching domains. The majority of the questions focused on the pre-service teachers' Subject Matter Knowledge, in particular Common Content Knowledge. This examination paper thus privileges Subject Matter Knowledge. Given the concern, in South Africa, with learners' and teachers' content knowledge this is not surprising.

References

Ball, D. L., Thames, M.H., Phelps, G. (2008). Content Knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Davis, Z. (2001). Measure for measure: Evaluative judgement in school mathematics pedagogic texts. *Pythagoras, 56*, 2–11.

THE PEDAGOGICAL CONTENT KNOWLEDGE OF PRE-SERVICE MATHEMATICS TEACHERS

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This contribution presents a research study conducted with pre-service teachers of the Faculty of Education from two Italian Universities; we outline a task designed to investigate Pedagogical Content Knowledge (PCK) as defined by Shulman (1986). Ball, Thames and Phelps (2008) describe the various subfields that characterize PCK. They include Knowledge of Content and Curriculum (KCC) within PCK together with two other areas: Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) (Hill et al., 2008). Each of these subfields requires continuous interaction between mathematics as a discipline and pedagogical knowledge about students' learning. We therefore designed a task to observe the PCK mobilized by the pre-service teachers. We administered a task composed of a stimulus (two primary pupils' solutions to an exercise) and three questions about the involved mathematics, the students' difficulties and possible development of classroom activities on the topic. The task was administered to a sample of pre-service teachers (N=204) and the protocols were analysed referring to MKT. We have evidence of all three subfields, but KCC (Ball et al., 2008) is less evident in our protocols. There is the need of a review of the task to make explicit the expected reference to the National Guidelines. More evidence emerges about the KCS and the KCT. Although only a minority of the pre-service teachers in our sample have actual teaching experience, it is interesting to notice their ability to design, in a rational way, didactic situations and the implementation of specific methodologies based on analysis of students' difficulties. In the analysis, we can note that the lack of a disciplinary knowledge does not necessarily preclude the evaluation of PCK. KCS and KCT are observable even though the mathematical terms used were not always fully correct. These results can offer food for thought for assessment design, aimed at pre-service mathematics primary school teachers.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EMPIRICAL VALIDATION OF A DEVELOPMENTAL MODEL OF ARITHMETIC CONCEPTS

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Young children's early numerical knowledge is a strong predictor for their later mathematical achievement, while pre-school performance predicts the mathematical performance during primary school (Aunola et al., 2004). A profound theory of children's arithmetical development helps with the design successful intervention.

Fritz et al. (2013) designed a five-level developmental model of arithmetic concepts that describes the successive development of precise arithmetic competencies in the sense of conceptual knowledge for the age range from four to eight years within the number range up to 20. The model hierarchy was supported in a Rasch analysis with more than 1200 learners in Germany (Fritz et al., 2013).

To test if the model describes learners' individual development, N=26 German children (17 girls, M_{age}=71.5 months, SD_{age}=10.4 months) were followed for 18 months at four measurement time points. 11 children were kindergarteners, 15 first-graders. At each measurement time point children's arithmetical concepts were assessed. A non-parametrical Page's trend test conceptual level ($L = 728.52$, $chi^2(1) = 970.61$, $p<0,001$, rho=.604) show significant monotonous progress. All children individually developed conceptual knowledge as described by the model.

These results show that the developmental model can describe individual learners' empirical developmental trajectories of early arithmetic concepts. This underpins the models validity as the theory-based levels match children's empirical development. The model can thus help structuring assessment and teaching.

References

Aunola, K., Leskinen, E., Lerkkanen, M-K. & Nurmi, J.-E. (2004). Developmental Dynamics of Math Performance from Preschool to Grade 2. *Journal of Educational Psychology, 96*(4), 699-713.

Fritz, A., Ehlert, A. & Balzer, L. (2013). Development of mathematical concepts as the basis for an elaborated mathematical understanding. *South African Journal of Childhood Education, 3*(1), 38-67.

Siegler R.S. & Alibali, M.W. (2005). *Children's thinking*. Englewood Cliffs, NJ: Prentice Hall.

USING ARTIFACTS AND INSCRIPTIONS TO MEDIATE ADDITION OF NUMBERS IN GRADE 2

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The way a teacher introduces the concept of addition in the early years has a significant effect on the development of additive reasoning in learners (Ekdahl, Venkat, Runesson, & Askew, 2018). Additive reasoning is very important because it forms the basis of other concepts in primary school arithmetic. For instance, multiplication is often introduced as repeated addition. This paper is aimed at answering the question: How do teachers use artifacts and inscriptions to represent addition of numbers in early years? It is based on a pilot study under an ongoing doctoral research focusing on teachers' mediation of mathematics in the early years of primary school in Malawi.

The pilot study focused on a single case of a grade 2 teacher covering a unit on addition of two numbers with a sum of not more than 20. The unit was completed in 5 lessons which were video recorded and analysed using the Mediating Primary Mathematics (MPM) framework developed by Venkat and Askew (2018). This framework is based on Vygotskian concept of mediation and adopted the sociocultural view of the teacher as the sole mediating agent in the classroom. The MPM framework was used to analyse the extent to which a teacher works with example spaces, artifacts, inscriptions, and talk during lesson enactment to assess the quality of mediation.

It was found that the teacher mediated addition using multiple artifacts and inscriptions for the same task. For instance, the teacher used counters and sticks while giving the learners liberty to use fingers. On the chalkboard, the teacher used both roman numerals and tally marks to represent the same numbers. By varying the inscriptions and artifacts for the same task, the teacher made it possible for the learners to discern that addition is not only associated with the standard expressions seen in textbooks.

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References

Ekdahl, A.L., Venkat, H., Runesson, U., & Askew, M. (2018). Weaving in connections: Studying changes in early grades additive relations teaching. *South African Journal of Childhood Education, 8*(1), 1–9.

Venkat, H., & Askew, M. (2018). Mediating primary mathematics: theory, concepts, and a framework for studying practice. *Educational Studies in Mathematics*, *97*(1), 71–92.

COMPARING FRACTIONS AND COMPARING DIVISIONS: THE SAME COGNITIVE PROCESS?

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A growing body of research investigates the cognitive foundations of rational number knowledge, and more specifically of the fraction comparison task (e.g. Gómez & Dartnell, forthcoming; Vamvakoussi, Van Dooren, & Verschaffel, 2012). Rational numbers have many interpretations (Kieren, 1976), one of them being that the fraction p/q is the result of p divided by q. Here, we present preliminary results from a study contrasting with a similar methodology the tasks of fraction and division comparison. 200 Chilean undergraduates answered a fraction comparison task (n=150) or a division comparison task (n=50). For this report we only considered those participants who answered 80% or more of the task correctly, leading to final numbers of n=103 and n=48. Moreover, we considered only items in which the two elements to be compared shared a common component, distinguishing between *congruent* items (where choosing the element with the larger components leads to the correct answer, e.g. 2/7 < 4/7, 48:8 > 32:8) and *incongruent* items (where the same reasoning leads to the incorrect answer, e.g. 5/8 < 5/6, 48:6 > 48:12). Participants in the fraction group showed, as expected, longer response times to answer incongruent items although this difference did not reach statistical significance (2954 ms vs. 3030 ms). Participants in the division group showed, in contrast, statistically significantly shorter response times to the same type of items (2912 ms vs. 2695 ms). This difference between groups was statistically significant as well. These results, as well as those of looking at the evolution over time of these differences, suggest that participants engage in different cognitive processes for comparing fractions and comparing divisions, in the latter case even pointing against a natural number bias as conceptualized by Vamvakoussi et al. (2012) and others. We explore possible explanations for this pattern.

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References

Gómez, D. M., & Dartnell, P. (forthcoming). Middle schoolers' biases and strategies in a fraction comparison task. *International Journal of Science and Mathematics Education*.

Kieren, T. E. (1976). In R. A. Lesh & D. A. Bradbard (Eds.), *Number and measurement: Papers from a research workshop* (pp. 101-144). Columbus, OH: ERIC/SMEAC.

Vamvakoussi, X., Van Dooren, W. & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior, 31*, 344-355.

PRE-SERVICE TEACHERS' ABILITIES TO NOTICE STUDENTS' ANSWERS TO GEOMETRIC PATTERN PROBLEMS

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A part of teachers' professional practice is evaluating students' outcomes, in particular *noticing of students' thinking*. Students may be introduced to algebra by solving *geometric pattern* (gp) *problems*. Several authors have defined levels of competence in noticing of students' thinking when solving gp problems and have described teachers' abilities to analyse students' solutions. Our objective was to analyse and classify secondary ps-teachers' acts of noticing of students' answers to gp problems. *Professional noticing* focuses making sense of students' answers. We focused on strategies (Arbona, 2016), using graphical patterns (*visual*, *numerical*), calculating values of terms (*counting*, *recursive*, *functional*, *proportional*), and calculating positions of terms (*use of equations*, *correct/wrong inversion*, *trial and error*). A framework (Van Es & Sherin, 2008) was adapted, identifying categories: *ways of using graphical patterns*, *calculating values* of terms and *calculating positions of terms*. The study was based on a teaching experiment with 58 secondary ps-teachers without prior knowledge about gp problems. Before, we posed four gp problems (Figure 1) to secondary students in grades 7 and 8. The teaching experiment had three parts: a) solve the four gp problems; b) present to the ps-teachers the structure of gp problems and the criteria to analyse answers; c) analyse students' answers to the four gp problems.

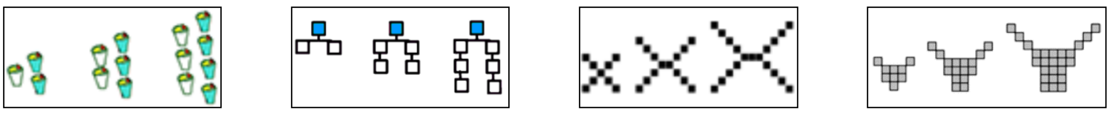

Figure 1: The three first terms as presented in the gp problems used

Data show that the ps-teachers had little difficulty to identify students' ways of using the graphical patterns (76% correct) but had more difficulties to identify the types of calculations of values of terms (47% correct) and positions of terms (47% correct). These results are in the line of results obtained by other researchers.

The results reported are part of the research projects EDU2017-84377-R (AEI/ERDF, EU) and GVPROMETEO2019-090 (Generalitat Valenciana).

References

Arbona, E. (2016). *Introducción al álgebra mediante problemas de patrones geométricos a un estudiante de educación primaria con altas capacidades matemáticas*. University of Valencia, Valencia (Spain). Retrieved from roderic.uv.es/handle/10550/56731

Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*(2), 244-276.

THE OBSTACLE OF INTUITION TO TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE

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Intuitive conceptions of arithmetic content taught in elementary school have a wide range of influence on children's and adults' mathematical reasoning (Fischbein, 1987). Teachers' knowledge of students' (mis)conceptions is an essential component of teachers' pedagogical content knowledge (PCK), which helps them anticipate the errors that the (mis)conceptions may lead to (Hill, Ball, & Schilling, 2008).

The present study investigates how intuitive conceptions influence teachers' PCK, notably their ability to understand students' thought processes on problem solving. Since intuitive conceptions are widespread in the population, whereas PCK is characteristic of teachers. Thirty six (36) elementary school teachers and 36 adults (not teachers) were presented with pairs of word problems that were either within or outside the scope of the intuitive conception of subtraction as "taking away". Each item contained a problem empirically shown to be much easier for students than the other one and for which the solving strategies are well documented (Brissiaud & Sander, 2010). Participants were asked to identify which problem is easier for second grade students and describe the strategies students use. The analysis revealed that teachers successfully identified the strategies students use on items where the harder one of the paired problems was incongruent with the intuitive conception. Yet, when the harder one of the paired problems was congruent with the intuitive conception, teachers were not more successful than non-teaching adults in their descriptions of students' strategies.

These findings suggest that when a task falls within the scope of the intuitive conception, teachers can overlook the difficulties underlying the thought processes students engage in when solving problems. This provides important directions for strengthening teachers' PCK in mathematics, by stressing the importance of bringing attention to teachers' intuitive conceptions in teacher education programs.

References

Brissiaud, R., & Sander, E. (2010). Arithmetic word problem solving: A situation strategy first framework. *Developmental Science*, *13*(1), 92–107.

Fischbein, E. (1987). *Intuition in science and mathematics. An educational approach.* Dordrecht: Kluwer Academic Publishers.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, *39*(4), 372-400.

DO LEARNERS REALLY HAVE DIFFICULTY WITH THE DIDACTIC CUT WHEN SOLVING LINEAR EQUATIONS?

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A key difficulty in introductory algebra lies in the transition from arithmetic to algebraic thinking. In the context of equations the transition involves difficulties in solving equations of the form, $ax + c = bx + d$. Filloy and Rojano (1989) called this the *didactic cut.* More recently, others e.g. Lima & Healey (2010), did not find evidence of the didactic cut. This study worked from the assumption that the didactic cut may not be a barrier for all learners and may rather depend on the strategies employed by learners in solving linear equations.

We investigated the progress of six grade 9 learners in solving linear equations over a six month period, using four task-based interviews each. The learners were selected because they had shown evidence, in a test, of being able to solve equations with letters on one side but not on both sides of the equation. The main interview instrument was a task-matrix, which allowed the interviewer to move flexibly between different linear equations to explore shifts in learners' abilities to solve the equations and in their reasoning over time. The data analysis focussed on describing learners' progress, their approaches in solving linear equations, the difficulties they encountered and the resulting errors. Four learners showed evidence of overcoming the didactic cut at different stages in the six-month period albeit using different forms of reasoning. An interview item with two terms with letters on the same side (e.g. $4x - 2x = 14$) appeared to prompt one learner in overcoming her difficulties with letters on both sides. The remaining two learners were able to solve this item but not items with letters on both sides.

The findings of the study support Filloy and Rojano's (1989) claim that the transition from arithmetic to algebraic approaches lies at the heart of learners' difficulties in solving linear equations. In particular, some learners' difficulties appeared to stem from the use of inappropriate arithmetic approaches involving inverse operations on letters.

References

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics*, 9, 19–25.

Lima, R. D., & Healy, L. (2010). The didactic cut in equation solving or a gap between the embodied and the symbolic mathematical worlds. In M. Pinto & T. Kawasaki (Eds.), *Proc. 34th Conf. of the Int. Group for Psychology of Mathematics Education* (Vol. 3, pp. 353-360). Belo Horizonte, Brazil: PME.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

"IT IS SO HARD AND DIFFICULT; IT MAKES ME FAIL": VOICES OF NAMIBIAN HIGH SCHOOL STUDENTS WHO EXPERIENCE DIFFICULTY LEARNING MATHEMATICS

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The problems associated with difficulty in learning mathematics often begin in elementary school. This implies that students experiencing Mathematics Learning Difficulties (MLD) may have poor mathematics foundations. This paper, considering the relevance of personal beliefs to student learning and academic success, builds upon the ideas of Perry (1970) and Hofer and Pintrich (2002). Personal beliefs are realized through thoughts, actions, or motivation of the individual, and generate behaviour, attitudes, and habits. The paper examines the voices of 28 eleventh-grade (ages 16-18) Namibian students (10 males and 18 females) from four northern regions of Namibia. The students all voluntarily identified themselves as experiencing MLDs, and were from different socio-economic backgrounds. In the Namibian context, students are not streamed in their mathematics classrooms by ability. Instead, Namibia mandates that all post-secondary mathematics classrooms be inclusive. Within this context the research questions are: What are the beliefs about mathematics of Namibian high school students experiencing mathematics difficulties? And, How do the students perceive the impact of their beliefs on their own learning of mathematics?

Through semi-structured interviews, students shared their personal beliefs and histories about learning mathematics such as personal use and reasons why they experience difficulty within an inclusive setting. A coding framework was used to facilitate a thematic analysis of the transcripts. Analysis revealed four themes: (i) the impact of learning mathematics on future career prospects, (ii) fear of failure, (iii) whether mathematical achievement is dependent on innate ability, and (iv), students' utility and interest in mathematics. For future research, longitudinal studies are needed to see how personal beliefs influence academic performance of students with MLD and examine the consequence of belief changes.

References

Hofer, B. K. & Pintrich, P. R. (2002). *Personal epistemology: The psychology of beliefs about knowledge and knowing.* Mahwah, NJ: Lawrence Erlbaum Associates.

Perry W. G. (1970). *Forms of intellectual and ethical development in the college years*. New York: Holt, Rinehart and Winston.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

IDENTIFYING MULTIPLICATIVE REASONING FLUENCY AND REASONING IN THE PRIMARY GRADES

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Prior work in the Wits Maths Connect-Primary study in South Africa, while showing potential for learning gains from structured short-term interventions, has also highlighted gaps in our knowledge of how learners' Multiplicative Reasoning (MR) fluencies and reasoning develop in the middle grades. Askew (2016) describes fluency as being able to choose and carry out procedures, knowing recalled facts as well as reasoning with and understanding numbers, whilst reasoning includes thinking about the relationships between numbers in different multiplicative situations. This paper reports on the initial findings from data collected in a doctoral study seeking to explore these aspects of MR in Grades 4 and 6.

Two versions of a written assessment were completed by 75 learners (37 Grade 4s and 42 Grade 6s) in one Johannesburg school. The assessment was designed to identify learners' multiplicative proficiencies in three strands – fluency, reasoning and problem-solving. In this paper, I report on only the first two strands. Within the fluency strand, there were 20 items which included multiplication and division by single-digit numbers up to 12. Under reasoning, there were 16 items which required learners to use the relationships between the numbers to find the solutions (for example $21 \times 18 = 21 \times 17 + \_\_$.)

Findings revealed that within the fluency strand, the mean facility was 13.8% in Grade 4 and 25.2% in Grade 6. On the reasoning strand, the mean facility stood at 1.9% in Grade 4 and 13.5% in Grade 6. Whilst the Grade 6 results were better than Grade 4, these averages were still low with learners in both grades rarely working with recalled facts and/or noticing the relationships between numbers to find solutions. A lack of awareness of multiplicative relationships such as commutativity, inverse, doubles and halves was also noted as learners failed to make sense of the numbers in the calculations, for example: $7 \times 60 = 420$, $420 \div 60 = \underline{424}$ and $420 \div 7 = \underline{204}$ were among the responses seen. Lastly, whilst learners diligently attempted to answer all the questions, associated solutions were often randomly guessed with little sense made. These low levels of MR fluency and reasoning raise questions for the level and range of tasks to be used in follow-up task-based interviews with a cross-attainment learner sample.

References

Askew, M. (2016). *A Practical Guide to Transforming Primary Mathematics*. London & New York: Routledge

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

CORPORATE TEACHING OF MATHEMATICS

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The starting point of the “MATHElino” project is a generally held view that kindergarten and elementary school represent two stations of a lifelong educational chain and therefore must not be viewed in isolation (Heinze & Grüßing, 2009). With this assumption, the “MATHElino” project aims at a cooperation of kindergarten educators and school teachers who, as a tandem throughout the kindergarten / primary school transition, jointly initiate learning processes and in doing so, getting into conversation about one’s own approaches to mathematics and the design of mathematical learning environments, about the learning levels of children as well as expectations of the respective institution in terms of teaching mathematics. The aim of this cooperation is to develop a common language to know the positions of each other’s profession and to respect differences for, in terms of a successful transition for children, both creating continuities and purposefully designing discontinuities. For the qualitative part of the study, six tandem teachers of the “MATHElino” project, who had taken part for at least five years, were interviewed using semi-structured conductive expert interviews. These interviews were videotaped and transcribed. Particular episodes were analyzed by coding according to the qualitative content analyses (see Mayring, 2012). For the quantitative analysis, a questionnaire consisting of closed and open items was conducted to assess the attitude of the kindergarten educator and elementary school teacher in terms of the following variables: cooperation between the institutions, expectations of the other institution, support of mathematics learning processes, and views on learning mathematics.

First results taken from both guided interviews and the use of questionnaires show that the project has an impact on the cooperation between kindergarten educator and school teacher as well as on the management level of the institutions. Essential for these developments are common training courses in which the further development of professionalism and understanding of each other’s institution must be one central point. In addition, the joint preparation and follow-up of the “MATHElino” classes had an impact on mathematical beliefs. During the weekly cooperation meetings with the children, it could be ensured that both professionals had effectively learned from and with each other as well as that an intense verbal exchange of mathematical theories and concepts took place.

References

Heinze, A & Grüßing, M (2009). *Mathematiklernen vom Kindergarten bis zum Studium.* Münster: Waxmann.

Mayring, P. (2012). Qualitative Inhaltsanalyse - ein Beispiel für Mixed Methods. In M. Gläser-Zikuda, T. Seidel, C. Rohlfs, A. Gröscher & S. Ziegelbauer (Hrsg.), *Mixed Methods in der empirischen Bildungsforschung* (S. 27-36). Münster: Waxmann.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INFLUENCE OF THE INTERACTIVITY OF DYADS' COMMUNICATION ON THEIR LEARNING OUTCOME

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Spoken language and associated communication processes play an important role in learning mathematics. The interactive-constructive-active-passive framework (ICAP) introduced by Chi and Wylie (2014) provides the means to analyse occurring communication processes between students in a collaborative learning environment with regard to their interactivity. The ICAP-hypothesis claims that pairs of students behaving more interactively should show a greater learning benefit.

Using the ICAP framework we developed a theoretical instrument suitable to analyse video recordings of first year university students (N = 112 in 56 dyads) from three different fields of study (psychology, (primary) school teacher and engineering students), who were asked to learn descriptive statistics with three different digital instructional media in a classical (pre-test | intervention | post-test) design.

A thorough discussion of our methodology was presented at last year's PME and details can be found in Hattermann et al. (2018). In short, for each dyad we obtain a *dialogue pattern score (dps)* that quantifies the interactivity of the communication within a dyad. Dyads' are subsequently grouped into high- and low interacting dyads based on the median of all observed *dps* (light resp. dark coloured in figure 1). To gauge the influence of interactivity on the learning outcome, we calculate the *normalised gain score* as a measure of learning benefit from the students' pre- and post-test results: $normalised\ gain\ score := \frac{posttest\ \% - pretest\ \%}{100\ \% - pretest\ \%}$.

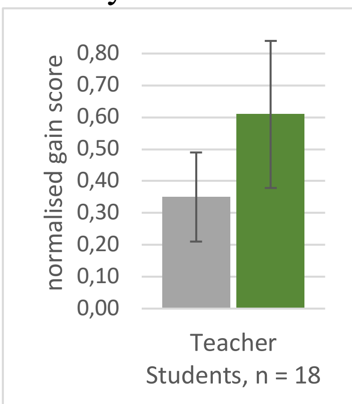

Figure 1

We were able to find at least significant ($p < 0.02$) differences in learning benefit between high- and low-dps dyads within all three cohorts. Results from the teacher students are illustrated in figure 1. We now aim to identify features of the digital media which promote interaction between learners.

References

Chi, M.T.H, & Wylie, R. (2014). The ICAP Framework: Linking Cognitive Engagement to Active Learning Outcomes. *Educational Psychologist*, *49*(4), 219-243.

Hattermann, M., Heinrich, D., Salle, A., & Schumacher, S. (2018): Instrument to analyse dyads' communication at tertiary level. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proc. 42nd Conf. of the Int. Grp. for the PME* (Vol. 3, pp. 27-34). Umeå, Sweden: PME.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

SUPPORTING GENERALIST ELEMENTARY SCHOOL TEACHERS TO INTEGRATE MATHEMATICS AND MUSIC

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The idea of integrating subjects in the curriculum is not new to the English education system. When reflecting on the power of integrating curriculum subjects, it is necessary to reflect on the nature of the links between the disciplines. Research has identified the link between mathematics and music (e.g. Hofstadter, 1999). Within mathematics education, there have also been studies that have explored the possibility of integrating mathematics and music teaching (e.g. An et al., 2013). Engagement with the Comenius project 'European Music Portfolio-Maths (EMP-M)' prompted us to devise a 2-day course with a gap task for elementary school teachers.

This professional development programme is not concerned with exploring the use of music as a tool to support the learning of mathematics, but rather, our approach has been one in which patterns, sequences and relationships in music and mathematics form the basis of a truly integrated approach. Our training is based on the premise that both mathematical and musical thinking rely on pattern recognition, iteration and repetition. The research project aimed to establish whether our model of professional development would support generalist elementary school teachers to establish integrated teaching approaches into their practice.

Following the first workshop, teachers experimented with the activities in their own classrooms. During the second workshop (2 months later), they shared their experiences and further learning. Our initial findings are that:

- Teachers developed a greater awareness of 'where the Maths is' in an activity, for example through explicit use of patterns to support composition activities
- Teachers gained a better understanding of how integrating Maths and Music teaching can be used to create more inclusive learning environments
- Integrated approaches provide a shared language that begins to bridge the musical and mathematical learning in the curriculum
- Teachers worked collaboratively with colleagues with different strengths to plan and deliver interdisciplinary activities.

References

Hofstadter, D. (1979). *Gödel, Escher, Bach: An Eternal Golden Braid.* New York: Basic Books.

An, S., Capraro, M. M., & Tillman, D. A. (2013). Elementary Teachers Integrate Music Activities into Regular Mathematics Lessons: Effects on Students' Mathematical Abilities. *Journal for Learning through the Arts*, *9*(1).

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

UNIVERSITY MATHEMATICS TEACHERS' PEDAGOGICAL DISCOURSE - A COMMOGNITIVE LENS

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There is a large body of research on teaching practices at the K-12 level, but very little research on the teaching practices of university teachers exists (Speer et al., 2010). Examining the teaching practices used by university mathematics teachers when lecturing, a topic within university mathematics education research is gaining an increasing interest. This paper reports on a discursive analysis of mathematical discourse on the derivative concept through the lens of the commognitive framework (Sfard, 2008).

The empirical data in this study consists mainly of videotaped lectures given by three teachers in calculus classrooms at a university in Taiwan. It took all teachers 35 lessons to teach the concept of derivatives. The transcribed lectures were then analysed, using Sfard's Commognitive Framework (2008) with its four components of mathematical discourse (words, visual mediators, narratives and routines) to try to distinguish the discursive patterns characterizing the teachers' respective discourses of derivative.

The findings indicate that the discourses of the teachers are similar with respect to words, visual mediators, and to a large extent also narratives. There are both similarities and differences, where both construction and substantiation routines occur in the discourses of all the teachers. A categorization of routines was found in the pedagogical discourses of the teachers. The construction routines include stipulation, naming, motivation, saming, and exploration routines. The substantiation routines include proof, auxiliary proof, and making contradiction routines. There are significant differences in the way the three teachers' pedagogical discourse is used in their lectures. These differences present themselves on the level and kind of discursive routines. Moreover, attention to mathematics discourses is not only useful for analysis across a range of lessons for teachers; it also connects discursively with discourse in ways that speak more directly to practice. In the presentation, further results will be discussed in detail.

References

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. New York, NY: Cambridge University Press.

Speer, N., Smith, J., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *Journal of Mathematical Behavior, 29*(2), 99–114.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

FOURTH GRADE STUDENTS' LEARNING OF AREA IN OPEN APPROACH LESSON STUDY CONTEXT

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The study of measurement is crucial in the school mathematics curriculum because of its practicality and pervasiveness in so many aspects of life (NCTM, 2000). When trying to understand initial measurement concepts, students need extensive experiences with several fundamental ideas prior to introduction of the use of rulers and measurement formulas such as comparison; students need to compare objects on the basis of a designated attribute without using numbers (Bergeson et al., 2000). The move from one-dimensional units involves additional complexity, so not surprisingly, research indicates that students have poor understanding of units of area and their spatial characteristics (Owens & Outhred, 2006). This research aimed to analyse how students learned area. Target group included 35 fourth grade students in 3 mathematics classrooms. Data collection was based on Open Approach Lesson Study cycle: video recording and lesson plan format used collaboratively in plan; LessonNote, video recording and students' worksheet used collaboratively in observation; video recording used collaboratively in reflection after class. Open Approach as a teaching approach was employed in the mathematics classroom as the following processes; posing open-ended problem situation, students solve the problem by themselves and the teacher takes notes, students presented their idea and discussion, summary based on students' ideas (Inprasitha, 2011). Pattern Blocks were use as semi-concrete aids to grasp students' ideas.

The results indicated that students learned area in the following ways 1) students used their hands as a direct measure to compare area. These students' ideas were found in the first mathematics classroom. 2) Students counted pieces of Pattern Blocks to compare one by one. These students' ideas were found in all mathematics classroom. 3) Students attempted to use some shape of pattern blocks as an indirect measure to compare area. These ideas were found only in four groups of students from two mathematics classrooms.

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References

Inprasitha, M. (2011). One Feature of Adaptive Lesson Study in Thailand: Designing a Leaning Unit. *Journal of Science and Mathematics Education in Southeast Asia*, *34*(1), 47-66.

Owens, K. & Outhred, L. (2006). The Complexity of Learning Geometry and Measurement. A. Gutierrez, P. Boero (eds.*), Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future,* 83-115. Rotterdam: Sense Publishers.

FUTURE TEACHERS' USE OF VARIABLE PARTS TO FORMULATE LINEAR EQUATIONS

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Numerous reports have documented that students and teachers have trouble modelling problem situations with linear equations. We have introduced a promising new path into this difficult material based on the *variable-parts perspective* for proportional relationships (Beckmann & Izsák, 2015). In prior work, we have examined future teachers' reasoning in content courses that intentionally built toward variable-parts reasoning (e.g., Izsák, Kulow, Stevenson, Ölmez, & Beckmann, 2019). In the present study we examine future teachers' performance with variable-parts reasoning *before* any instruction in proportional relationships to see what they can do without support.

As part of a longer pre-test given in the first week of two content courses, we gave a task designed to encourage variable-parts reasoning to 56 future middle-grades and secondary-grades mathematics teachers at the beginning of their preparation programs at two institutions in the USA. The task asked future teachers to consider: 9 trucks each carrying the same weight where 4 trucks carry a total of X tons sand and 5 trucks carry a total of Y tons gravel. The trucks serve as a fixed number of parts, where the weight of each of the 9 equal parts is variable (i.e., variable parts). The task asked future teachers to make a math drawing "to show the relationship between the quantities of sand and gravel," to "indicate where X and Y are in your drawing," and to "write a few sentences to explain how to develop an equation relating X and Y by reasoning about the quantities of sand and gravel."

Twenty-one future teachers coordinated a drawing, an equation that related X and Y, and a sound explanation for their equation in terms of weight. Twelve of these future teachers wrote a version of $X/4 = Y/5$, which equates the weight of one sand truck and one gravel truck. Of the remaining teachers, 11 omitted either a drawing or an equation and another 11 did not express X tons distributed over 4 trucks and 5 tons distributed over 5 trucks consistently in their drawings and equations. We will present more details.

References

Beckmann, S., & Izsák, A. (2015). Two perspectives on proportional relationships: Extending complementary origins of multiplication in terms of quantities. *Journal for Research in Mathematics Education*, *46*(1), pp. 17–38.

Izsák, A., Kulow, T., Stevenson, D., Ölmez, İ. B., & Beckmann, S. (2019). *Using a measurement meaning of multiplication to connect topics in a content course for future teachers.* Manuscript accepted.

PEDAGOGIC EVALUATION IN CONTEXTS DIFFERENTIATED WITH RESPECT TO SOCIAL CLASS

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Difference in performance in mathematics along social-class lines is well documented in the literature. Bernstein (2000) claims that explicit evaluative criteria are critical to the academic success of learners from low economic status communities. The contention of this study is that insufficient fine-grained analyses have been undertaken to surface the computational specificity of what it is that constitutes evaluative criteria in mathematics education studies of pedagogy.

This study set out to examine the functioning of evaluation in two secondary schools differentiated with respect to learners' social class membership. At each school, two mathematics teachers and their Grade 10 mathematics class of learners constituted the research participants of the study. The research question pursued is as follows: How does pedagogic evaluation function in four Grade 10 pedagogic contexts in schools that differ with respect to social class membership?

A sequence of three consecutive Grade 10 mathematics lessons taught by each teacher, was observed and video-recorded. Methodological resources for describing the functioning of pedagogic evaluation in terms of the computational activity of teachers and learners derive from the work of Davis (2013) which draws on a computational theory of mind. The analysis reveals the following: (1) the commonly used descriptions of evaluative criteria as explicit/implicit are analytically blunt and consequently mask the complexity of criteria operative in pedagogic contexts; (2) differences as well as strong similarities in the functioning of evaluation and, therefore, differences and similarities in what is constituted as mathematics are evident in pedagogic situations differentiated with respect to social class.

The study contributes to the mathematics education field concerned with equity in schooling. It recruits and develops more mathematically-attuned methodological resources for examining the functioning of pedagogic evaluation in contexts differentiated with respect to social class.

References

Bernstein, B. (2000). *Pedagogy, Symbolic Control, and Identity: Theory, Research, Critique. Revised Edition*. Oxford: Rowman & Littlefield.

Davis, Z. (2013). Constructing descriptions and analyses of mathematics constituted in pedagogic situations, with particular reference to an instance of addition over the reals. (Plenary address to the 7th Mathematics and Society Conference). In M. Berger, K. Brodie, V. Frith, & K. le Roux (Eds.), *Proceedings of the Seventh Mathematics and Society Conference* (pp. 31-59). Cape Town.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

UNDERSTANDING STUDENT PROCESSES AND COMMUNICATIVE FUNCTIONS IN PROBLEM SOLVING WITH TECHNOLOGY

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Mathematical problem solving with technology (MPST) has been analysed separately using different lenses. One of which is investigating the processes students go through (Jacinto & Carreira, 2017) and the other considered the nature of student discourse when the task is performed in a collaborative setting (Kumpulainen & Kaartinen, 2003). It is of great value if the two frameworks can be used together to describe and analyse student ability in MPST.

This study seeks to determine the relationship between the processes in mathematical problem solving with technology (MPST) and the communicative functions of students. The study sample comprised six (6) dyads of Grade 10 students, divided into three ability groups. While solving the problems, each dyad was videotaped and their screen activity was recorded and supplemented with the researcher's field notes. Each session lasted for 40 to 60 minutes. The peer interaction between the students was transcribed together with their GeoGebra activity. The findings show that students normally read aloud during their initial efforts in comprehending the problem (grasp, analysis). The MPST processes analysis, exploration and verification are characterized by verbal utterances that were classified as informative, argumentative, reasoning, evaluative, interrogative, responsive, agreement, and affective. Students normally organize behavior during exploration, planning and verification phases. Verbal communication classified as disagreement and dictation, is usually observed during exploration stage. Further results will be discussed in detail during the presentation.

References

Jacinto, H., & Carreira, S. (2017). Mathematical Problem Solving with Technology: the Techno-Mathematical Fluency of a Student-with-GeoGebra. *International Journal of Science and Mathematics Education*, 1115–1136.

Kumpulainen, K., & Kaartinen, S. (2003). The Interpersonal Dynamics of Collaborative Reasoning in Peer Interactive Dyads. *The Journal of Experimental Education, 4*(71), 333-370.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

WEB-BASED LEARNING: EFFECTS OF FEEDBACK ON THE COMPUTER SELF-EFFICACY OF HIGH SCHOOL STUDENTS

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Computer- and web-based learning plays an increasingly important role in mathematics education. However, usability-studies indicate that the use of computers in mathematics education is actually not very popular in Germany (Fraillon et al., 2014). Computer self-efficacy as well as feedback are regarded as important factors influencing the learning with web-based learning environments (e.g. Wang, & Wu, 2008). Besides, feedback has been described frequently as an aspect linked to computer self-efficacy. For example, *elaborated peer* feedback is positively related to computer self-efficacy (e.g. Wang, & Wu, 2008), whereas no significant effect of *knowledge of the correct response* (KCR) feedback could be identified.

Our aim is to investigate to what extent the implementation of *elaborated* feedback (EF) in a web-based learning environment improves the computer self-efficacy of high school students, although it is not given by peers. The sample comprises 254 students of grades 8 and 9 (11 classes). The intervention took place in six school lessons (6 x 45 min) and was framed by a pre-post-test questionnaire about computer self-efficacy. These data were collected by identical questionnaires with 17 items and a four-level Likert scale. During the intervention students worked either in a web-based learning environment with EF or in the same environment with KCR feedback.

The findings indicate a significant decrease of computer self-efficacy from pre- to post-test ($F(1, 252) = 145.408$, $p < .001$, *partial* $\eta^2 = .366$). A trend becomes apparent towards a smaller decrease in computer self-efficacy for EF compared to KCR feedback ($\Delta M_{KCR} = -.42$, $\Delta M_{EF} = -.34$). Here, no statistically significant interaction between time and group was identified ($F(1, 252) = 1.710$, $p = .192$, *partial* $\eta^2 = .007$). Based on these results, we assume that students' working processes on web-based learning need further research, since it is not obvious which factors caused the contradictory results. We consider the short period of time for the intervention as well as an unusual way of working as potential factors of influence.

References

Fraillon, J., Ainley, J., Schulz, W., Friedmann, T., & Gebhardt E. (2014). *Preparing for Life in a Digital Age. The IEA International Computer and Information Literacy Study International Report.* SpringerLink, doi:10.1007/978-3-319-14222-7.

Wang, S.-L., & Wu, P.-Y. (2008). The Role of Feedback and Self-Efficacy on Web-Based Learning: The Social Cognitive Perspective. *Computers & Education, 51*(4), 1589-1598.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

GRADE 6 TEACHERS' OBJECTIFICATION OF THE ALGEBRA DISCOURSE

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In tackling the well-known challenges of learning algebra, Sfard (2008) highlights the need for objectification of mathematical discourse. Teachers can support students in this objectification by modelling the discourse they want students to develop (Sfard, 2016). Our study uses Sfard's (2008) definition of objectification in order to analyse grade 6 teachers' algebra discourse. The purpose is to understand if and how objectification occurs in the discourse when algebra is introduced, and how this might influence students' understanding of algebra. Data consist of video recordings from three Swedish teachers' introductory lessons in algebra with 12-year old students, with four consecutive introductory algebra lessons for each teacher. The recordings are from a larger international video study. Teachers' discourse about algebraic entities was analysed concerning *word-use*, *visual mediators*, *routines*, and *endorsed narratives* (Sfard, 2008).

Preliminary results show that there were mainly three algebraic entities that were in focus in the lessons, described as equations, expressions and variables. Generally, we note that as more formal algebraic symbols were introduced and used by two of the teachers, the more objectified became their discourse. This objectification did not occur in the third teacher's discourse, which lacked an early introduction of formal symbols. This could imply that symbols reinforce objectification. Particularly, the progression of the objectification process concerning expressions was very prominent over the course of one teacher's four lessons. The word use clearly changed from treating the introduced symbols as processes, where an expression was "to describe something with the help of a variable"; to objects, where an expression was "a system of symbols" that "could be used in an equation". A more comprehensive discussion of the results, including routines and endorsed narratives, will be included in our presentation.

References

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing.* New York, NY: Cambridge University Press.

Sfard, A. (2016). Ritual for ritual, exploration for exploration: Or, what learners are offered is what you get from them in return. In J. Adler and A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 41–63). New York: Routledge.

E-FEEDBACK TO OVERCOME MISCONCEPTIONS

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The PhD-Project EoM (E-Feedback to overcome Misconceptions) is connected to different projects and studies (eg., project CODI of Nitsch, 2015) and researches the possibilities to optimise computer-based feedback in digital learning environments. The goal of the project is to develop feedback elements that enable pupils to overcome their diagnosed misconceptions in the field of functional relations. Therefore, digital instruments are being used with the aim to activate Conceptual-Change-processes and by considering the personal learning preferences. At the end of the project it should be possible for teachers to use a tool to diagnose mathematical misconceptions and simultaneously stimulate the learning processes of the pupils.

Feedback on the performances of tasks is, next to diagnosis before and a supportive framework afterwards, one main part of learning environments. Feedback elements can influence learners' cognitive, meta-cognitive and motivational factors and also can influence on the self-image and self-efficacy beliefs (Bangert-Drowns et al, 1991; Butler & Winne, 1995). From practical experience and available literature (eg., Shute, 2008), it is known that there are phenomena of individual feedback reception. The concepts of learning styles that are based on personality traits, are used to take into account potentially existing differences in learning behaviour. This view from the perspective of the learners can serve as the basis for optimizing individual feedback elements. Consequently, feedback can effectively activate conceptual change processes if it fits the learning style. Case studies and developed feedback elements will be presented at the conference.

References

Bangert-Drowns, R. L., Kulik, C.-L. C., Kulik, J. A., & Morgan, M. (1991). The Instructional Effect of Feedback in Test-like Events. *Review of Educational Research, 61*(2), 213.

Butler, D. L., & Winne, P. H. (1995). Feedback and Self-Regulated Learning: A Theoretical Synthesis. *Review of Educational Research, 65*(3), 245.

Nitsch, R. (2015). Diagnose von Lernschwierigkeiten im Bereich funktionaler Zusammenhänge: Eine Studie zu typischen Fehlermustern bei Darstellungswechseln. Wiesbaden: Springer Spektrum.

Shute, V. J. (2008). Focus on Formative Feedback. *Review of Educational Research, 78*(1), 153–189.

TEACHERS' AND STUDENT-TEACHERS' CHALLENGES WITH ADDITION AND MULTIPLICATION STRATEGY PROBLEMS

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The Swedish National Agency for Education emphasizes that students in grades 1-3 should be able to develop addition and multiplication strategies based on the properties of these operations before focusing on standard algorithms. This means that teachers need to understand student perceptions of these concepts in order to be able to develop instructions for promoting such strategies. According to Ball, Thames and Phelps (2008) teachers need mathematical knowledge to be able to meet such a demand. Other researchers recommend carrying out qualitative studies investigating a field experience of pre-service and in-service teachers. Since the in-service teachers' knowledge depends on what they were taught in school themselves, we studied the teaching process in grade 3 in order to learn how they were initially introduced to addition and multiplication (Van Dooren, De Bock, & Verschaffel, 2010).

The teaching process and teachers' understanding of problems with multiplication content was studied in 3rd grade mathematics classrooms. This study investigated challenges faced by teachers when they went from interpretation to formulation of the multiplication problems. Data is drawn from three main sources: observation, interviews, field notes and video recordings. The data was analysed in a qualitative way, using variation theory (Marton, 2015). The analysis reveals that the participating teachers were more successful in using addition strategies than multiplication ones in problem development. For example, most teachers identified multiplication as a repeated addition. Consequently, they often missed important multiplication structures and their link to division. The results explain some reasons for difficulties in elementary mathematics experienced by in-service teachers and the consequences they lead to in their teaching. Teachers' understanding of mathematical concepts is crucial for their interpretation and setting of the example problems.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Marton, F. (2015). *Necessary conditions of learning.* London: Routledge.

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication…and back. The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*, 360-381.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TURBULENCE IN THE NATURE OF MATHEMATICS: WAY AHEAD FOR MATH EDUCATION RESEARCH

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It is a generally acknowledged and often assumed that the mathematical objects (like numbers, equations etc.) are abstract. Platonic idealism has been a dominant part of our understanding of the nature of the mathematical objects and mathematical practices. However, the recent upsurge of biologically rooted cognitive theories (like situated and embodied) ground the mental processes in material interactions resulting in embodied accounts of the genesis of mathematical knowledge (Lakoff & Núñez, 2000). Further studies in learning contexts, characterise mathematical thinking in terms of bodily actions (Nemirovsky et al., 2004) and interactions with artefacts including digital media. Symbols are attributed concreteness when Presmeg (2006) calls them as *inscriptions* (concrete shapes created using paper and pencil/pen or chalk and board or any other medium) making mathematical objects apprehensible and abstract symbolic reasoning is understood as a special kind of embodied reasoning (Landy, Allen, & Zednik, 2014). These embodied accounts, physicality of symbols result in arguments strengthening the material ontology of the mathematics (de Freitas & Sinclair, 2013). These can disturb the idealistic abstractions that mathematical objects are understood as and related practices. These create *turbulence* (an embodied term instead of a visual term like turbidity for disturbance or a lack of clarity) in our current understanding about the nature of mathematical knowledge and hence the practices and education.

Examining processes while students learn math and mathematicians do math could be a productive empirical endeavour in addressing this turbulence. Nuances related to this examination of the processes, while students learn mathematics, concern math education researchers and shall be explored and discussed in the presentation.

References

de Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: the body in/of mathematics. *Educational Studies in Mathematics, 83*(3), 453–470.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic books.

Landy, D., Allen, C., & Zednik, C. (2014). A perceptual account of symbolic reasoning. *Frontiers in Psychology, 5*, 275.

Nemirovsky, R., Borba, M., Dimattia, C., Arzarello, F., Robutti, O., Schnepp, M., Scheffer, N. F. (2004). PME Special Issue: Bodily Activity and Imagination in Mathematics Learning. *Educational Studies in Mathematics, 57*(3), 303–321.

Presmeg, N. C. (2006). A semiotic view of the role of imagery and inscriptions in mathematics teaching and learning. *PME 30* (Vol. 1, pp. 19–34).

THE INSTRUCTIONAL CORE OF MATHEMATICS LESSONS IN SINGAPORE SECONDARY SCHOOLS

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The Enactment Project (Kaur et al., 2018) in Singapore is a national study of classroom practices of mathematics teachers in Singapore secondary schools. In the first phase of the study classroom practices of 30 experienced and competent teachers were documented using the renowned complementary accounts methodology (Clarke, 1998). Through an in-depth analysis of the practices, common characteristics were found. These characteristics illuminated an instructional core, the Development, Seatwork and Review (DSR) cycle (Kaur, 2017) that pervaded the lessons of these teachers. Based on these findings, a survey was constructed to explore how widespread the practices of the competent and experienced teachers were in classrooms of secondary school mathematics teachers in Singapore schools.

In the second phase the survey was administered to 689 teachers. Sixty items of the survey were related to the instructional core. Participants responded on a Likert scale of 1 (Never/Rarely) to 4 (Mostly/Always). Analysis of the data firstly established the reliability and validity of the survey. All the six subscales of the instrument demonstrated satisfactory internal consistency with Cronbach's $\alpha > 0.55$. A Principal Component Analysis with Promax rotation was performed. Four factors, namely: student-centred in-class learning, teaching for practice and fluency, teacher-led conceptual learning and teacher-guided student self-directed learning, were identified. These factors help us clarify how the DSR cycle plays out as an instructional core in Singapore classrooms.

References

Clarke, D. J. (1998). Studying the classroom negotiation of meaning: Complementary accounts methodology. In A. Teppo (Ed.), Qualitative research methods in mathematics education, monograph number 9 of the *Journal for Research in Mathematics Education* (pp. 98-111). Reston, VA: NCTM.

Kaur, B. (2017). Mathematics classroom studies: Multiple lenses and perspectives. In, K. Gabriele (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 45-61). Springer Open.

Kaur, B., Tay, E.G., Toh, T.L, Leong, Y.H & Lee, N.H. (2018). A study of school mathematics curriculum enacted by competent teachers in Singapore secondary schools. *Mathematics Education Research Journal, 30,* 103-116. DOI 10.1007/s13394-017-0205-7.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

MATHEMATICS TEACHERS' STRATEGIES AND THEIR IMPACT ON SOCIETY THROUGH STUDENT LEARNING

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Effective teachers' teaching strategies and students' learning strategies in mathematics affect student achievements fostering good relation among mathematics teachers, students and parents. However, the mismatch between teachers' teaching strategies and students' learning strategies lead to frustration and lack of continued growth of achievement (Dunn, 1995). Homans (1958) states that social relation is created and people are integrated based on the individual's behavior which is centered on present achievement. In this study, research questions were: What are the roles of teaching strategies to promote students' learning strategies? How do teaching strategies help to promote students' learning strategies? What is the impact of teaching strategies on making better relations in society through students' learning?

Two schools of Kathmandu were selected through a convenience sampling procedure. Observation and interview guidelines were used to collect the information. Thirty lessons of two mathematics teachers were observed, and six students and their parents were purposively selected and interviewed. The information was transcribed, encoded and categorized thematically and analysed relating with theory.

The results show that there was a mismatch between teachers' teaching strategies and students' learning strategies as teachers were self-centered and authoritative whereas students' preferred peer learning, help seeking and rehearsal learning strategies. Teachers with student friendly strategy had good reputation in the society. However, teachers' strategies remained vital to promote students' learning strategies resulting their academic success and parental satisfaction. Therefore, teachers are expected to understand students' learning strategies and design the teaching strategies accordingly so that their pedagogy is more context responsive and student friendly.

References

Dunn, R. (1995). *Strategies for educating diverse learners.* Bloomington, IN: Phi Delta Kappan.

Homans, G. C. (1958). Social behavior as exchange. *American Journal of Sociology, 63*, 597-606.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INTERPERSONAL DISCOURSE IN SMALL GROUP DISCUSSIONS TO DEVELOP MATHEMATICAL ARGUMENTS

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K-12 student development of mathematical arguments remains a priority in mathematics education. Scholars agree that social interactions and discussions develop the mathematical practice of developing arguments and critiquing others' reasoning (Stylianou, 2013). Using a situated learning lens, this research studies small group interpersonal discourse to advance mathematical arguments.

Previous literature has established that argumentation is largely inaccessible in current teaching practices (Karunakaran, Freeburn, Konuk, & Arbaugh, 2014). Further, Brown (2017) emphasized the need for student reasoning to develop through the use of communication with one another. This study had forty-four 8th grade Pre-Algebra students first communally consider criteria for a valid mathematical argument (Yee, Boyle, Ko, & Bleiler-Baxter, 2017), then write an argument for one mathematical task each day. The students worked individually, then in small groups (Brown, 2017), and finally as a whole class to validate each other's work. Preliminary results show student validation of each other by discussing the communal criteria and overall improvements in mathematical arguments according to the Stylianides' (2008) framework. Implications for future research include understanding student discourse that leads to advanced arguments and K-12 teacher pedagogical practices to support argumentation.

References

Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. *Mathematics Education Research Journal, 29*, 183-199.

Karunakaran, S., Freeburn, B., Konuk, N., & Arbaugh, F. (2014). Improving preservice secondary mathematics teachers' capability with generic example proofs. *Mathematics Teacher Educator, 2*(2), 158-170.

Stylianides, G. (2008). An analytic framework of reasoning-and-proving. *For the Learning of Mathematics, 28*(1), 9-16.

Stylianou, D. (2013). An examination of connections in mathematical processes in students' problem solving: Connections between representing and justifying. *Journal of Education and Learning, 2*(2), 23-35.

Yee, S. P., Boyle, J. D., Ko, Y., & Bleiler-Baxter, S. K. (2017). Effects of constructing critiquing, and revising arguments within university classrooms. *Journal of Mathematical Behavior*, *49*, 145-162. doi:10.1016/j.jmathb.2017.11.009

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EXAMINING MATHEMATICS TEACHERS' ATTITUDES TOWARD ARGUMENT-BASED TEACHING

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Argument-based teaching helps students to better understand relevant concepts, builds cognitive knowledge and social context. It enables students to listen, to think, to propose claims and to respond directly to their classmates' claims. The teachers' role in this process is central and only after training in argument-based teaching can teachers deeply understand what is required to make the process successful (McNeill & Knight, 2013). Teachers' attitudes and beliefs guide the choice of instructional strategies they take in classrooms, but studies concerning teachers' attitudes toward argument-based teaching have not yet been reported. This research addresses this need.

The participants of this study were 176 mathematics teachers. The research tool contained three parts. Part I asked participants for their demographic information. Part II provided an example explaining the meaning of argument-based teaching in mathematics based on the Toulmin model of argument. Part III, the questionnaire, was composed of 29 items on a Likert 1-5 scale, with items divided into three categories: a) student benefits from argument-based learning-7 items, b) teachers' confidence in using argument-based teaching-11 items, c) technical issues relating to argument-based teaching-11 items. Three experts reviewed the instrument to establish content and face validity. Cronbach's Alpha for the whole instrument was 0.87; 0.70 for category a), 0.78 for b), and 0.75 for c). The means and standard deviations of the categories: a) 3.64, 0.50 b) 3.85, 0.43 c) 3.56, 0.63, indicated that teachers hold both positive and negative views simultaneously while most hold more positive attitudes than negative ones. Comparison of male and female teachers using the t-test for independent samples showed no significant differences in the means of most items and higher female attitudes in five items vs. two higher male attitudes ($p<0.05$ in each of them), meaning that women are less conservative to argument-based teaching. The mean attitudes of teachers participating in training were higher in 7/29 items ($p<0.05$ in each of them), i.e. training helps the teachers to understand the advantage of this method. More experienced teachers were less conservative toward argument-based teaching having significantly higher attitudes in 5 of 29 items ($p<0.05$ in each of them).

References

McNeill, K. L. & Knight, A. M. (2013). Teachers' pedagogical content knowledge of scientific argumentation: The impact of professional development on k-12 teachers. *Science Education, 97*(6), 936-972.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

PRESCHOOL CHILDREN'S UNDERSTANDING OF NUMBERS SHOWN IN A PARTITIONING TASK

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Children's ways of handling numbers in arithmetic tasks has been studied extensively, providing us with insights about strategies for solving tasks and development of arithmetic skills (Carpenter & Moser, 1982). Children's ability to decompose numbers is one important part of development, since the ability allows children to apply different strategies when solving tasks (Hunting, 2003). Children's different ways of encountering numbers in simple tasks may give a comprehensive understanding of the challenges in learning to use numbers in proficient ways. When experiencing part-part-whole relations of numbers, the child needs to consider the parts and the whole simultaneously. This paper reports on an analysis of 103 individual interviews with 5-year-old children on two occasions during their last year in preschool in Sweden. We report on the analysis of one particular task illustrating children's experience of numbers when partitioning seven hidden marbles into two parts. The specific research question was: What different ways of experiencing numbers by 5-year-old children were exposed in a partitioning task?

Variation theory (Marton, 2015) was used to analyse children's ways of experiencing numbers and what aspects were critical to discern in order to solve the task. Variation theory emanates from more than thirty years of phenomenographic research, investigating different ways in which the same phenomena can be experienced. We found that children experienced numbers in six different ways: *as number words, as names, as extents, as countables, as structure,* or *as known number facts*. Our study shows that those ways of experiencing numbers that are foregrounding the cardinal, ordinal and the parts and whole *simultaneously* end up in plausible answers and the children initiate ways to handle the task in powerful ways. Consequently, if the children experience either the cardinal (e.g. numbers as extent) or the ordinal (e.g. numbers as names) they are not able to decompose the whole and thereby solve the task.

References

Carpenter, T. P., & Moser, J. M. (1982). The development of addition and subtraction problem-solving skills. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective.* Hillsdale, NY: Lawrence Erlbaum

Hunting, R. P. (2003). Part-whole number knowledge in preschool children. *Journal of Mathematical Behavior, 22*(3), 217-235.

Marton, F. (2015). *Necessary conditions of learning*. New York: Routledge.

STUDENTS’ PERCEPTIONS OF ERRORS IN MATHEMATICS LEARNING IN TANZANIAN SECONDARY SCHOOLS

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Using errors in mathematics is a powerful instructional practice. Heemsoth and Heinze (2016) showed that student reflection on their own errors improved procedural and conceptual mathematics knowledge. Despite potential benefits of errors in promoting learning, errors are often perceived negatively by students and teachers. Studies using the Oser and Spychiger (2005) error perceptions questionnaire reported positive impact of error handling training on mathematics teachers’ affective and cognitive student support in error situations on students’ affect (Rach, Ufer, & Heinze, 2012), but rarely on students’ use of errors. The Rach *et al.* study used a two-stage “train-the-trainer” approach, leaving open whether teacher support and student use of errors for learning can be developed by more direct professional training. This study examined the role of teacher error handling training on students’ use of errors in learning.

A quasi-experimental pre-test, intervention, post-test design with waiting control group was used to investigate the effects of mathematics teacher professional error handling training. The sample consisted of eight Dar es Salaam region secondary school teachers and their respective grade 11 students (N = 251). Using validated questionnaires to measure students’ perceptions of errors and of teacher support in error situations, latent means analysis showed that students’ perceptions of teacher support in error situations significantly improved for teachers in the experimental group but not in the control group. However, students’ perceptions of anxiety in error situations and using errors for learning were not affected. Exploratory analyses of video-recorded plenary discussions illustrate that experimental group mathematics teachers appeared to be error friendly after the intervention.

References

Heemsoth, T., & Heinze, A. (2016). Secondary school students learning from reflections on the rationale behind self-made errors: A field experiment. *Journal of Experimental Education, 84*(1), 98-118. doi: 10.1080/00220973.2014.963215

Oser, F., & Spychiger, M. (2005). *Lernen ist schmerzhaft. Zur Theorie des negativen Wissens und zur Praxis der Fehlerkultur [Learning is painful. On the theory of negative knowledge and the practice of error culture*]. Weinheim, Germany: Beltz.

Rach, S., Ufer, S., & Heinze, A. (2012). Learning from Errors: Effects of a teacher training on students' attitudes towards and their individual use of errors. In T. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 329-336). Taipei, Taiwan: PME.

REIMAGINING MATHEMATICS TEACHER KNOWLEDGE: RESOURCE KNOWLEDGE AND ITS IMPLICATIONS

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Developments in educational theory and philosophy highlight the complex nature of teachers' work and the dynamic knowledge necessary for highly effective teaching. Literature on teachers' mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) and technological pedagogical content knowledge (Koehler & Mishra, 2009) provide frameworks to understand/develop teacher learning. However, these frameworks highlight teachers' pedagogical content knowledge and use of material resources, while placing context and sociocultural resources on the periphery. Through our work on a four-year K-8 mathematics teacher professional development project we began reimagining teacher knowledge to encompass *resource knowledge*. Resource knowledge is defined as teachers' knowledge of available material and immaterial resources to accomplish a variety of goals and tasks. The idea of resource knowledge arose as we used a *decentring* approach to professional development (Hodge et al., 2018) that positioned teachers as experts making sense of the professional development materials in-practice-in-context. We propose a vision of teacher knowledge that considers teachers' *resource pedagogical content knowledge, RPACK.* This brings focus to context, culture, and material and immaterial resources and our theoretical understandings of mathematics teaching to the realities of the classroom.

In our presentation, we begin with our ongoing attempt to make sense of teachers' resource knowledge and resources in and for mathematics education. We proceed to share our developing framework for conceptualizing teachers' RPACK. Finally, we discuss implications for resource knowledge and the RPACK framework on the design of teacher education, professional development, curriculum materials and for research on teacher learning/knowledge, and teachers' moment-to-moment decision making.

References

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Hodge, L., Cheng, K., King, S., & Kim, N. (2018). Elementary teachers' take-aways from summer math Institutes in a three-year professional development project in rural Appalachia. *Proceedings of the 42nd Annual Meeting of the International Group for the Psychology of Mathematics Education* (p. 250). Umeå, Sweden: University of Umeå.

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education, 9*(1), 60-70.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EYE TRACKING RESEARCH IN MATHEMATICS EDUCATION: A PME LITERATURE REVIEW

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A human eye provides high-resolution only in a small region. It must scan what is of interest in detail. Hence, gaze sequences allow inferences about attention and cognitive processes. ET is promising for mathematics education research (MER) and, fueled by recent hardware and methodological developments, is increasingly studied.

We analyzed PME proceedings, assuming that, as one of the primary conferences dedicated to MER with a fast publication process, PME reflects current trends well. We read all PME publications of the last ten years that contain eye tracking (ET) related keywords.

Our literature review asked first (RQ1): How did the number of ET related papers evolve over time? Interest is evident. We found 33 contributions, including 13 research reports and 12 oral communications. There are no publications before 2013 (PME35) in the period considered. Yet, we did not find a clear ongoing rising trend. PME40 with most ET contributions and PME42 with most ET research reports could be outliers.

We then asked (RQ2): What ET equipment is used in the MER community? Video-based systems dominate, the majority (21) in remote eye trackers, which are attached to a screen showing visual stimuli. We see a recent trend towards ET glasses (11 papers, 8 in the last two years) and a possible trend towards synchronized systems of two or more ET devices that allow studying interactions and social learning situations.

Interpretation of gaze patterns is crucial. We thus finally asked (RQ3): What analysis methods are used in MER? For 26 ET studies identification of the analysis method was possible—yet not for all. We found quantitative analyses of derived features (15) and manual (laborious) qualitative analysis of ET videos (9). Six recent methodological papers address analysis explicitly, one of them computerized automatic analysis. For all references, further details (methodology, topics studied, ET result presentation), and a discussion of possible future developments see Lilienthal and Schindler (2019).

References

Lilienthal, A.J., & Schindler, M. (2019). *Current trends in the use of eye tracking in mathematics education research: A PME survey*. https://bit.ly/2XOE8tA.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

THE TROUBLE WITH WORDS IN MATHEMATICS PROBLEMS: AN EXPLORATORY STUDY

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Solving mathematics word problems often involves reading and interpreting the words and phrases in mathematics problems in real-world contexts. Students have to learn to handle the language of mathematics and connect it to mathematical concepts. The language of mathematics has been shown to have specific features and be quite different from the everyday language students are familiar with (Middleton, Llamas-Flores & Guerra-Lombardi, 2013; Laborde, 1990).

The paper reports the preliminary findings from a study that provided insights into the linguistic difficulties students at Primary 1 (age 7 years old), Primary 3 and Primary 5 face when solving mathematics word problems. In Singapore, students are bilingual as the medium of instruction is English and they also learn another language in school. In this study, only about half the students speak predominately English at home. About 200 students were surveyed and interviewed in focus groups to explore their perceptions and attitudes to learning mathematics. Observations of twenty-four lessons and interviews with mathematics teachers contextualised the students' views.

In this presentation, findings from lesson observations and students' interviews will be discussed in detail to provide preliminary evidence about the linguistic difficulties students face in solving word problems. The results point towards teaching for greater metalingual awareness as well as metacognitive awareness even though the Primary Mathematics Syllabus (MOE, 2012) has emphasised the importance of the use of mathematical language in the classroom.

References

Laborde, C. (1990). Language and mathematics. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: a research synthesis by the international group for the psychology of mathematics education*, 53-69. Cambridge: Cambridge University Press.

Middleton, J.A., Llamas-Flores, S., and Guerra-Lombardi, P. (2013). English language learning and learning academic language in mathematics. In M. B. Arias & C. J. Fatis (Eds), *Academic language in second language learning*, 201-224. Charlotte, North Carolina, IAP Information Age Publishing.

Ministry of Education. (2012). *Primary Mathematics Teaching And Learning Syllabus*. Singapore: Curriculum Planning And Development Division, Ministry Of Education.

MERGING ANALYTICAL FRAMEWORKS FOR GRADE R TEACHER IDENTITY STORIES

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This paper, and the supporting presentation, draws on the mathematics teacher identity and professional development (PD) literature, and focuses specifically on the development of an analytical framework used to illuminate the identity stories of pre-school (Grade R) teachers engaged in a Numeracy focused PD programme. In this study, the four facets of Gee's (2000) analytic frame – Nature, Institution, Discourse and Affinity Identity - are supplemented by Wenger-Trayner et. al.'s (2014) conceptualisation of trajectories through landscapes of practice. The broader study researches five participating Grade R teachers' identity stories, utilising data gathered through interviews, questionnaires and classroom observations.

The merging of Gee's (2000) four facets of identity, with Wenger-Trayner et. al.'s (2014) trajectories, into one analytical framework has allowed for a richer understanding of these teachers' identity stories *across identities* and *through time*. As an example, the institutional identity of Maria, a participant in this research, could be expressed and analysed as shown in Table 1 below. The oral presentation will further elaborate on what was learned from Maria's case through this analytic frame.

MARIA	Past	Present	Future
Institutional Identity	"I passed with a Cum Laude"	"But I am lucky because I am head of the department as well" "Also cluster leader for Maths"	"help other teachers like you people do"

Table 1: Maria's institutional identity story

The use of this framework for understanding teacher identity stories has enabled analytic depth of the data and has allowed for a deeper understanding of how each facet of the participants identities (nature, institution, discourse and affinity) connect the past and the future in the negotiation of the present through their landscapes of practice.

References

Gee, J. P. (2000). Chapter 3: Identity as an analytic lens for research in education. *Review of Research in Education*, *25*(1), 99-125.

Wenger-Trayner, E., Fenton-O'creevy, M., Hutchinson, S., Kubiak, C., & Wenger-Trayner, B. (Eds.), (2014). *Learning in Landscapes of Practice: Boundaries, Identity, and Knowledgeability in Practice-Based Learning*. London: Routledge

AN EXPLORATION OF THE NATURE OF STUDENT PARTICIPATION IN A PRIMARY MATHEMATICS TEACHER EDUCATION CLASSROOM

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In any mathematics class, meaningful learner participation is fundamental to the kind of mathematics made available to learn, as it exposes learners to learn to articulate and justify their mathematical ideas, thereby, developing a deep understanding of mathematics. Little is known about how pre-service teachers are taught to encourage meaningful participation during their pre-service education. Studies on mathematics teacher education have mostly focused on pre-service teachers' practices, not on teacher educators. I argue that pre-service teachers' ability to encourage meaningful participation does not happen automatically; it is a function of how they were taught during their teacher education.

This paper employs the Mathematics Discourse in Instruction (MDI) framework (Adler & Ronda, 2015) to answer the question: What is the nature of student participation in a mathematics teacher education classroom in Malawi? MDI focuses on the mathematics made available to learn and has four interacting elements: object of learning, exemplification, explanatory talk and learner participation. This paper focuses on the learner participation element to explore how mathematics teacher educators encourage meaningful participation when teaching pre-service teachers. It is part of an ongoing PhD project exploring how primary mathematics teacher education prepares pre-service teachers to teach mathematics in the early primary years.

Pilot data was collected on one mathematics teacher education class. I observed and video-recorded three lessons on number concepts and operations. Findings from the data analysed show that student teachers were involved in individual, group and whole class participations through verbal talk, writing, and the use of representations, such as manipulatives. During presentation, I will explain how these kinds of participation offered student teachers opportunities to learn.

This study is kindly funded by the Norwegian Programme for Capacity Building in Higher Education and Research for Development (NORHED) through Strengthening Numeracy Project (Ref: QZA-0498 MWI 16/0020).

References

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education, 19*(3), 237-254.

INSTRUCTIONAL PRACTICES FOR LEARNERS WITH LEARNING DIFFICULTIES IN MATHEMATICS

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Since 1994, the South African government has shifted its focus to having inclusive schools (DoE, South Africa, 2001). That implies that teachers must be able to use effective instructional strategies when teaching learners with learning difficulties. We argue that, before planning to have inclusive schools, it is necessary to investigate instructional practices at schools for learners with special educational needs (LSEN), in particular, for specialised subjects like mathematics, which are challenging. In that way, we will know what to concentrate on as teacher trainers in designing relevant training for teachers who will be capable to teach at inclusive schools. Currently many learners in mainstream schools and Technical and Vocational and Training (TVET) Colleges have intrinsic barriers to learning. Their teachers need to be able to adapt their teaching to accommodate these learners. We suggest that if inclusive education is to be successful in South Africa, then all teachers should be equipped with the necessary special needs education training, with the main focus being on effective instructional strategies for learners with intrinsic barriers to learning.

Four teachers (Grade 4, 5, 6 and 7) were observed on how they use five dominant instructional strategies (identified from literature). Our study revealed that all teachers used scaffolding more than the other strategies, but failed to use problem-solving. It was also evident that teachers with special needs training used concrete objects when introducing lessons. We proposed further research on whether learning was taking place on not. Scherer, Beswick, DeBlois, Healy and Opitz (2016) argue for inclusive education system for all, the focus should not be on trying to assist students with mathematics learning disabilities, but rather focus on how to build a mathematics education system that no longer disables our mathematics students.

References

Department of Education (DoE), South Africa. (2001). *Special needs education: building an inclusive education and training system. Education White Paper 6*. Pretoria: DoE.

Scherer, P., Beswick, K., DeBlois, L., Healy, L., & Opitz, M.E. (2016). Assistance of students with mathematical learning difficulties: how can research support practice? *ZDM Mathematics Education, 48*, 633–649.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TOWARDS A THEORY OF POST-SECONDARY GENERAL EDUCATION MATHEMATICS

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General education (GE) is founded on the belief that all people who who receive a formal education benefit, in terms of personal development and understanding and enacting their role in a society, from a breadth of educational experiences. These individual benefits are, or ought to be, felt broadly, as GE is claimed to be essential for a functioning democracy.

Most universities in many countries—for example, in Canada and the United States of America—mandate GE courses as a part of every degree program: an engineering student may take a philosophy course; a dance major may take statistics. Mathematics, in some form, is typically included as one of the primary subjects in a general education. GE students bring different learning conditions to the classroom than math majors, e.g. their motivation, mathematical background knowledge, and the perceived usefulness of mathematics for their program of study. It is therefore important to adjust the design and teaching of these courses to these specific circumstances. To do this, three questions must be addressed:

1. What should be included in a post-secondary GE mathematics curriculum?
2. Who might be post-secondary GE mathematics students?
3. Who should teach post-secondary GE mathematics?

In this presentation, I offer a perspective on these questions from my own institution (Maciejewski, Tortora, & Bragelman, under review) in which students who attend lower-division GE math courses tend to: struggle with confidence, view solutions to math problems as being singularly right or wrong, and necessarily numeric; and to not see the connection between math and the real world. I then present a lesson from a course designed to address these student success factors.

Research should focus on GE mathematics because most students taking mathematics courses do not pursue mathematically-intensive degree programs. Focusing on their education will not only benefit under-served students, but ought to have the greatest impact on the public perception of, and fluency with mathematics.

References

Maciejewski, W., Tortora, C., & Bragelman, J. (under review). Beyond skill: a comparative analysis between developmental and non-developmental mathematics students' affects and dispositions.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

PROOFS WITHOUT WORDS AS ARTIFACTS FOR SUPPORTING THE LEARNING OF MATHEMATICAL PROOFS

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Proofs Without Words (PWWs) are mathematical texts in which diagrams or graphs allude to the proving of a certain mathematical proposition or theorem (Nelsen, 1993). The diagram might contain mathematical symbols, characters or even calculations, but with no words. Hanna and Sidoli (2007) questioned the rigor and validity of PWWs but praised their didactical potential as explanatory objects that encapsulate insights and beauty. Hence, our research aim is to verify and to exploit the didactical potential of PWWs as vehicles for the learning of proofs. We seek to understand how students approach PWWs and characterize the process of reading them.

To address these questions, we propose a hypothesized model to be used as a theoretical framework for discourse analysis of peer groups co-reading of PWWs. This model consists of 7 *epistemic actions,* essential for the reading of PWWs: (1) *Data extraction.* In PWWs, data is shown implicitly in the diagram and should be properly extracted. (2) *Decomposition and reconstruction.* The diagram is decomposed into its components, each of them being analyzed and categorized and then reconstructing the diagram in a certain order. (3) *Movement imagination*. Some PWWs require dynamic transformation. Imagination is required to see movements in a static diagram. (4) *Gaps discovery and bridging*. In PWWs, the diagram provides only key clues of how to prove a theorem but considerable gaps are left for the reader to complete. (5) *Data organization by epistemic status*. One needs to discern which information is given, which is due to construction and which is a proposition. (6) *Ordering*. While diagrams present all the data simultaneously, findings must be arranged in a precise order. (7) *Generalization*. Seeing how the concrete diagram shows a general way to prove a larger set of cases.

To verify and refine our model, we conducted a collective case study, giving six small groups of advanced 10th graders to read 3 sequences of PWWs from 3 different domains of school mathematics: geometry, trigonometry and number theory. The reading occurred in 3 sessions of 90 minutes each with considerable time gaps between them. The data from the case study is still being analyzed but preliminary results show that our model is reasonable.

References

Hanna, G., & Sidoli, N. (2007). Visualisation and proof: a brief survey of philosophical perspectives. *ZDM, 39*, 73-79.

Nelsen, R. B. (1993). *Proofs without words: Exercises in visual thinking* (No. 1). MAA.

TO WHAT EXTENT DO TEACHERS HAVE SHARED VIEWS OF LEARNING AND TEACHING MATHEMATICS?

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Research in mathematics education (e.g. Christiansen, 2007) argues that teachers' views on learning mathematics and mathematical activities conflict with some mandated teaching practices. This paper explores views of secondary mathematics teachers working in highly prescriptive teaching contexts in South Africa where scripted lesson plans are common.

I developed a closed questionnaire consisting of 36 statements from three constructs: (1) understanding of mathematics content; (2) confidence in the learning and teaching of mathematics; and (3) accountability towards the teaching practices. The statements were orientated towards five interconnected components of learning: meaning, community, practice, confidence and identity (Graven, 2003; Wenger, 1998).

Forty secondary teachers who had participated in a professional development course responded with their levels of agreement with the statements. Exploratory factor analysis, using SPSS, led to two factors being retained after parallel analysis. The factors, named Factor 1 and Factor 2, had reliability of 0.82 and 0.76 respectively.

Factor 1 had 15 statements with two significant constructs: confidence and identity. In the first group of statements, teachers illustrated the importance of confidence in the learning and teaching of mathematics after participating in the course. For example, because of confidence, teachers were "no longer stressing about knowing everything" and were "not afraid to grapple with mathematics tasks". The second group of statements highlighted teachers' sense of being "more valued by other teachers". In addition, teachers had a sense of becoming the "best mathematics teachers in the future" (identity). Factor 2 had five statements. In this factor, teachers attributed their increased understanding of mathematics content to the course. Furthermore, teachers indicated that they were now teaching learners for understanding (meaning).

References

Christiansen, I. M. (2007). Some tensions in mathematics education for democracy. *The Montana Mathematics Enthusiast*, *3*(3), 49-62.

Graven, M. (2003). Teacher learning as changing meaning, practice, community, identity and confidence: The story of Ivan. *For the Learning of Mathematics, 23*(2), 28-36.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

ANALYSING COHERENCE IN DIVISION TEACHING IN SOUTH AFRICA USING SIGNIFICATION PATHWAYS

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In earlier work using semiotic theory, differing degrees of coherence was seen in the teaching of division in the primary grades in South Africa led to the development of a 'signification pathway' framework (Mathews, Venkat & Askew, 2018). This framework included four categories: Fully coherent (C) teaching episodes in which a sequence of signifiers are mathematically justifiable at all points in the episode, coherent with limitations (C-L) episodes where the signification pathway was mathematically justifiable in a localised way without being generally mathematically applicable; coherent with ambiguity (C-A) episodes in which the signification pathway involved instances of ambiguity; incoherent (I) episodes where signification pathways that are mathematically incoherent.

In this paper, I present the balance of the empirical episodes in my empirical dataset across these four categories. The analysis was based on a dataset compromised of six teachers from three primary schools in Johannesburg. In one year (2011-2012) all their lessons on division were observed to study how teachers explained and solved division problems, with lessons broken down into episodes demarcated on the basis of work with single division tasks.

Across the six teachers, 115 episodes were transcribed and analysed to look at the extent to which signifiers appearing within pathways could be mathematically linked to previous signifiers in the signification pathway.

The results indicated that just over half of these episodes (66/115) showed coherent work with signifiers in instruction. The remaining episodes (C-L: 22/115; C-A: 23/115; I 4/115) fell short of the coherence criterion in a range of ways. In the presentation, I share exemplars of instruction across these categories.

References

Mathews, C., Venkat, H. & Askew, M. (2018). Primary Teachers' Semiotic Praxis: Windows into the Handling of Division Tasks. In N. Presmeg, L. Radford, W. Roth, & G. Kadunz (Eds.), *Signs of Signification*. Switzerland: Springer.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

PROFESSIONAL DEVELOPMENT LIFE STORIES AND TEACHERS' IDENTITIES

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This paper reports part of a wider study in which we use teachers' narratives to describe their identities. Human beings live and tell stories about their everyday lives, and through these, they reveal their individual identities. Drake (2006) has shown that teachers' narratives can illuminate their reform experiences. We have thus turned to narrative research to illuminate teachers' professional development identities. We draw first on Sfard and Prusak (2005)'s notions of actual identity and designated identity to study shifts in teachers' identities. Like Sfard and Prusak (2005), we define identity as a collection of stories about individuals which are reifying, endorsable and significant. Our second framework uses McAdams's (1993) notions of high point, low point, and turning point to describe a mathematics teacher's identity embedded in her narratives. We use them to elaborate Sfard and Prusak (2005)'s theory of identity. The research question for this paper is: *What is the nature of the teacher's professional development identity, and how can it be characterised?* Our data is derived from one 90 minute narrative interview with a teacher participant in our wider study. Initial analysis determined whether the teacher's stories contained instances of high points, low points and turning points. Results revealed that turning points were dominant in the teacher's stories. These stories had negative beginnings and positive endings, suggesting that turning points were positive. Our second analysis interpreting the dominance of positive turning points revealed: (i) a shift from a negative actual identity to a positive designated identity; (ii) that designated identity was reified, evidenced by repetitive behaviour of turning points; (iii) that designated identity was endorsable because it was authored by the teacher. (iv) that designated identity was significant because it implied membership into teaching mathematics at higher Grades.

References

Drake, C. (2006). Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education, 9*(6), 579-608.

McAdams, D. P. (1993). *The stories we live by: Personal myths and the making of self.* New York: The Guilford Press.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14-22.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

INDIVIDUAL AND GROUP MATHEMATICS CREATIVITY

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Mathematics educators agree that promoting mathematical creativity is an important aim of mathematics education, and may be promoted by engaging students with open-ended tasks. An open-ended problem does not have a single exact answer but rather a range of solutions, which in turn can lead to lead to fluency, flexibility, and originality (Silver, 1997). Most studies of students' engagement with open-ended tasks investigated the creativity of students working individually (e.g., Kwon, Park, & Park, 2006). This study asks: How might the mathematical creativity of students working as individuals compare with the mathematical creativity of students working in groups?

Participants were 92 post high-school students, separated into two similarly-mixed heterogeneous classes. Both classes engaged with the same three geometric open-ended tasks. For the first two tasks, one class worked individually, while the second worked in small groups of 5-6 students. For the third task, all students worked individually. Results were analysed in terms of fluency, flexibility, and originality. Independent samples t-tests were used to assess differences between classes.

For all three tasks, no significant differences between the groups were found regarding originality. As seen in Table 1, no significant differences were found between classes for fluency and flexibility on Task 1. However, for the second and third tasks, there was greater fluency and flexibility among those that worked or had worked in groups.

		Individuals: M (SD)	Groups: M (SD)	p	$\square^2$
Fluency	Task 1	7.56 (3.08)	8.42 (1.78)	.359	.015
	Task 2	9.40 (5.11)	19.08 (13.02)	<.001	.228
	Task 3	1.90 (1.43)	3.00 (1.39)	.001	.135
Flexibility	Task 1	4.78 (1.41)	5.50 (.067)	.093	.051
	Task 2	4.91 (1.88)	7.08 (1.44)	<.001	.200
	Task 3	1.60 (1.22)	2.82 (1.14)	<.001	.215

Table 1: Individual and group fluency and flexibility per task

The results indicate that group work may have an impact on mathematical creativity.

References

Kwon, O. N., Park, J. S., & Park, J. H. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review, 7*(1), 51-61.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM Mathematics Education*, *3,* 75-80.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

TEACHERS' LEARNING IN THE CONTEXT OF LESSON STUDY: A CASE STUDY

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Lesson Study *seems* to provide an ideal platform for teacher learning to take place. It is a teacher-led professional development model, which involves teachers' collaborative planning and evaluation of research lessons that focus particularly on student learning (Vrikki et al, 2017). Teachers need to learn new ideas and knowledge to improve teaching and learning. At the same time teachers also need to have opportunities to try those ideas and use new knowledge in their classroom and reflect upon how the ideas and knowledge (Takahashi, 2015).

The purpose was to study the teachers' learning in the context of lesson study. The target group of this study consisted of seven teachers who participated in lesson study process for 1 year and reflected every Wednesday about what they observed in the class. Data were collected by interview and using of questionnaires. Data were analysed by content analysis based on the process of lesson study (Inprasitha, 2016).

The results shown that teaching practices with 3 phases of Lesson Study support teachers' learning about how to improve teaching and understanding students' ideas as following; (1) Collaboratively planning, supported member of lesson study team learnt about the way of learning management in the aspect of content. Learning activity was designed appropriate for characteristics of learners and analyzed textbook in order to design learning activity, (2) Collaboratively doing, collaborated observe makes the teacher completely realizes the problem in the class, enable to identify the problem, strength, weakness which is beneficial for learning and teaching development respectively. Teachers collaborated to observe students' ideas and they needed to improve and simultaneously makes the teachers themselves understand more about the students. (3) Collaboratively reflecting, recognized an aptitude and behavior of students and enhanced their learning, improved their teaching, and found the way to improve teaching and learning.

References

Takahashi, A. (2015). Lesson Study: An Essential Process for Improving Mathematics Teaching and Learning. In M. Inprasitha, M. Isoda, P. Wang- Iverson, & B.H. Yeap (Eds.), *Lesson Study: Challenges in Mathematics Education* (pp. 51 – 58). Toh Tuck Link, Singapore: World Scientific Publishing.

Inprasitha, M. (2018). *How is Lesson Study enhancing new School Mathematics*? Khon Kaen: Khon Kaen University.

MATHEMATICAL PHRASES IN INDIGENOUS LANGUAGES: THE CASE OF ISIXHOSA AND 'MORE THAN'

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This paper offers an analysis of the construction and use of 'comparison phrases' in early grade mathematics (e.g. Thuli has three more eggs than Thandi) in one African language, isiXhosa, as an example of research into the implications that linguistic features can have for learning and teaching mathematics.

Comparison is a concept that is both mathematically important and linguistically complex. Comparison of quantities forms the basis of the notion of difference and measurement. While humans are able to compare sets by discriminating between 'more' and 'fewer' even before language acquisition (Xu & Spelke, 2000), the use of quantifiers such as 'more' and 'fewer' is an example of a later-developing language skill that children typically only master after the age of five (Berman, 2004).

For the analysis reported on in this paper, the English and isiXhosa versions of four early grade South African mathematics texts were examined. In the initial analysis of the English texts, a total of 16 different phrases containing the words 'more' or 'more than' were identified and classified. In the subsequent analysis of the isiXhosa translation of the comparison phrases, a number of linguistic differences between the English and the isiXhosa identified. For example, in isiXhosa word order is not as rigid as in English and so 'Sive has three more eggs than Sbu' can be phrased as 'Sive has more eggs, by three, than Sbu' or 'Sive has more eggs than Sbu, by three'.

These findings have specific implications for the teaching of isiXhosa such as the need to emphasize that prefixes rather than word order indicate the relationship between quantities. The findings suggest that the construction and use of comparison phrases in early grade mathematics is an important and potentially fruitful area of research into languages with linguistic features that are different from the Indo-European languages that have traditionally been used to develop mathematics (Barton, 2008).

References

Barton, B. (2008). *The language of mathematics: Telling mathematical tales.* New York: Springer.

Berman, R. A. (2004). Between emergence and mastery: The long developmental route of language acquisition. In R. A. Berman (Ed.), *Language development across childhood and adolescence* (pp. 9–34). Amsterdam: John Benjamins Publishing Company.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74,* 1–11.

HOW LEARNERS HOME LANGUAGE AND ENGLISH ARE USED IN THREE HIGH-SCHOOL MATHEMATICS CLASSROOMS

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Research in the field of multilingualism in mathematics has mostly positioned itself with learners' home language as a resource that facilitates meaningful mathematics discourse (e.g. Moschovich 2012). Research in neighbouring South Africa also shows that teachers, learners and parents prefer English as a medium of instruction (Setati 2008). Lesotho's proximity to South Africa brings into question how high-school mathematics teachers use each language in their mathematics classrooms. The research reported explores the purposes for which each language was used by three mathematics teachers from two schools in Grade 11. It uses Kilpatrick *et al*'s (2001) strands of mathematical proficiency as a lens to categorise the function of each language in mathematics instruction. An observation schedule was used to code teacher utterances as serving classroom management, teacher explanation, response to learners' contribution or teacher questioning functions. Lesson transcripts were segmented and coded for mathematical proficiency strands, and teachers were interviewed.

The findings revealed that Sesotho was, for all three teachers, the language of classroom management while English was the language of procedures. In two of the three lessons both Sesotho and English were used for explanation and clarification. In these lessons more of the five strands of mathematical proficiency were evident whereas in the lesson where English only was used for explanation only procedural fluency was present. The findings also suggest that in the instances where learners' home language is leveraged more comprehensive mathematics teaching happens. That is, in addition to procedural fluency, conceptual understanding, strategic competence and adaptive reasoning were fostered in the teacher's utterances.

References

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Moschkovich, J. (2012). Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core. *Proceedings of "Understanding Language" conference*. Palo Alto, CA: Stanford University.

Setati, M. (2008). Access to mathematics versus access to the language of power: the struggle in multilingual mathematics classrooms. *South African Journal of Education, 28*(1), 103-116.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

USING REHEARSAL DEBRIEFS TO DEVELOP EMOTIONAL UNDERSTANDING OF PEDAGOGY

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Research on rehearsals (e.g., Lampert et al., 2013) has focused on the learning of participants who take on the role of teacher, but we know less about the experience of those in student or non-rehearsing teacher (NRT) roles. Debriefs create a space for sensemaking about the rehearsal and the pedagogy enacted. Emotions are intertwined with cognition/motivation and influence how people interpret and respond (Sutton & Wheatley, 2003). Asking teachers to position themselves as students creates the potential for emotional interpretations. In this study we ask, how do NRTs surface emotional understanding of students' experience of pedagogy during rehearsal debriefs, and how is this connected to NRT sensemaking?

Data consisted of 8 rehearsals and debriefs with 22 secondary mathematics teachers. NRT talk turns (n=454) were segmented and coded to isolate instances of *interpreting* the rehearsal (n=259), and further coded (double-coded where warranted) to reflect the interpreting stance used: emotive, descriptive, evaluative, mathematical, inquiry, or positional. Each was coded for the speaker's positionality: teacher/student. Emotive interpretations were sub-coded emergently for the emotion described.

NRTs most often used a descriptive stance to interpretations (52%), followed by evaluative (30%), emotive (29%), mathematical (7%), inquiry (7%), and positional (6%) stances. When NRTs used an emotive stance (n=76), they overwhelmingly did so from the student position (96%). NRTs, as students, cited a range of emotions, the most prevalent of which were: anxiety (15%), appreciation (14%), security (12%), excitement (11%), confusion (11%), and awkwardness (10%).

The findings suggest that rehearsals create opportunities for NRTs to interpret pedagogy through emotional understanding of students' experiences of it. Debriefs allow NRTs to make sense of pedagogy publicly by considering how students might feel and using this as a lens for the risks and benefits of new pedagogy.

References

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., … Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243.

Sutton, R. E., & Wheatley, K. F. (2003). Teachers' emotions and teaching: A review of the literature and directions for future research. *Educational Psychology Review*, *15*(4), 327–358. https://doi.org/10.1023/A:1026131715856

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

AN EXAMINATION OF GRADE 2 MATHEMATICS TEXTBOOKS' CONTENT ON NUMBER CONCEPT IN COMPARISON WITH OUTCOME BASED EDUCATION GOALS

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Malawi shifted from objective based education model to outcome based education (OBE) model in 2008 with an aim of improving education quality in primary and secondary education. OBE model focuses on promotion of learners' achievement, active learner participation and independent learning. The Ministry of Education, Science and Technology (MoEST) reviewed the primary education curriculum and its accompanying textbooks to align them to OBE goals in 2008. There is 1 teachers' guide and 1 learners' textbook for each subject in each grade as commissioned by MoEST. Although textbooks are the only readily available teaching and learning resource due to economic constraints in Malawi, no research has compared the content of the textbooks to the OBE goals. This oral communication presents part of findings from an ongoing study on analysis of Malawian early grade primary mathematics textbooks content in relation to OBE goals. The specific focus of this presentation is on the nature of tasks/examples presented in grade 2 mathematics teachers' guide and the learners' textbook on the topic of number and operation. The intended learning outcomes on this topic are counting, identifying and writing numbers up to 20. The analysis was conducted using mediating primary mathematics (MPM) framework by Venkat and Askew (2017). The results showed that the grade 2 mathematics teachers' guide has presented different types of tasks/examples which might enable learners to be involved in active learning activities as required in OBE. However, both textbooks have suggested the use of low level artifacts in a low level manner, hence not suitable for promoting learners' deeper understanding of some number concepts like number relationships. The learners' textbooks contain very few tasks/examples on each number, hence giving little opportunities for learners to practice counting and writing the numbers independently as required in OBE. This implies that the tasks/examples and artifacts in both the teachers' guide and the learners' textbook might not foster some OBE goals like learner achievement and independent learning.

References

Venkat, H. & Askew, M. (2017). Mediating primary mathematics: theory, concepts, and a framework for studying practice. *Education Studies in Mathematics*, *97*, 71–92. DOI 10.1007/s10649-017-9776-1.

AN INVESTIGATION OF MALAWIAN TEACHERS' EXPLANATORY TALK WHEN INTRODUCING MEASUREMENT

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Standard units of measuring length, capacity and mass are first introduced in standard 4 in the Malawian Primary School Mathematics curriculum. In this class the learners are introduced to *cm* and *m* for measuring length, *ml* and *l* for capacity and *g* and *kg* for mass. In the first three years (standard 1 to 3), learners use non-standard units (MoEST, 2005).

This paper reports on an on-going study that is investigating standard 4 teachers' explanatory talk when introducing standard units of measuring length, capacity and mass. The study is using Mathematics Discourse in Instruction (MDI) as its theoretical framework to analyse teachers' explanatory talk. Explanatory talk can simply be regarded as explanations that teachers offer as they elaborate mathematics in their classrooms (Adler, 2017). Data was collected from two teachers using video recording and was transcribed. Preliminary findings of this study showed teachers' struggles when explaining the concept of standard units of measurement in a local language (Chichewa). The teachers often used code switching which did not help learners much to understand the concepts. The implications of this finding is that learners have little or no access to the concept of standard units of measuring length, mass and capacity.

References

Adler, J. (2017). Mathematics Discourse in Instruction: A Discursive Resource as Boundary. *Proceedings of the 13th International Congress on Mathematical Education* (pp. 125-143). Berlin: Springer.

Ministry of Education Science and Technology. (2005). *Primary Curriculum and Assessment Reform: Standard 4 Numeracy and Mathematics syllabus*. Lilongwe: MoEST.

MATHEMATICS TEACHERS LEARN: AN EXAMINATION OF DISCOURSE DURING A PEDAGOGY OF ENACTMENT

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Studies on existing pedagogies of enactment, such as microteaching, lesson study and rehearsal, have shown *what* preservice teachers (PSTs) can learn through these pedagogies and explored the opportunities that the pedagogies provide for that learning (e.g., Sims & Walsh, 2009). However, little is known about *how* PST learning takes place through the implementation of these pedagogies.

This research is part of a larger study (Ochieng, 2018) that examined PST learning through the Bell Ringer Sequence (BRS), a pedagogy of enactment that draws on the affordances of microteaching, lesson study and rehearsal. The BRS is centred on the bellringer—a brief mathematical task implemented as students arrive for class. The aim of this study is to provide insight into PST learning by examining the conversations that took place during implementation of the BRS.

Participants were 11 PSTs enrolled in a secondary mathematics methods course. Data sources included audio records of bellringer preparation conversations, video records of the methods class sessions, interviews with PSTs, and PSTs' written reflections. Data analysis involved identification of learning prompts—verbal interactions that prompted learning—and characterization of the conversations in the prompts to gain insight into how the ideas learned emerged and how PSTs interacted with those ideas.

Analysis of the learning prompts led to the identification of three learning stages: *initiation, precisification* and *equilibration*. The three stages suggest collaborative negotiation of ideas that requires a repertoire of knowledge for PSTs to draw from, class norms that allow PSTs to freely share their ideas, and instructor guidance of the process to ensure learning of appropriate content; all which underscore the role of course context in PST learning. Additional results that will be discussed in detail during the presentation include the projection of dual identities by PSTs in the learning process, as both learners and teacher-evaluators.

References

Ochieng, M.A. (2018). *The bellringer sequence: Investigating what and how preservice mathematics teachers learn through pedagogies of enactment*. Unpublished doctoral dissertation, Western Michigan University, Kalamazoo, MI.

Sims, L., & Walsh, D. (2009). Lesson study with preservice teachers: Lessons from lessons. *Teaching and Teacher Education, 25*, 724-733.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EXPLORING AN EXAMINER'S COMMENTS TO A THESIS WORK IN MATHEMATICS EDUCATION

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Thesis manuscripts are perceived as a means for discovering students grasp of subject matters as well as an indicator of their professional orientation (Maaranen 2009). In the present study an examiner's (the second author) written comments to students' theses are interrogated *with the purpose of revealing the intervention process facilitating the writing of a thesis meeting the goals of mathematics education research*. Student consent was obtained. Based on Peirce semiotics as a point of departure, the undergraduate thesis manuscripts in focus and the corresponding comments from the examiner are perceived as signs. "A sign, or representamen, is something which stands to somebody for something in some respect or capacity" (Peirce, CP 2.228). Thus, the examiner's written texts are analyzed based on a cultural semiotic framework (Olande 2014) that characterized the nature of students' interaction with semiotic means of objectification.

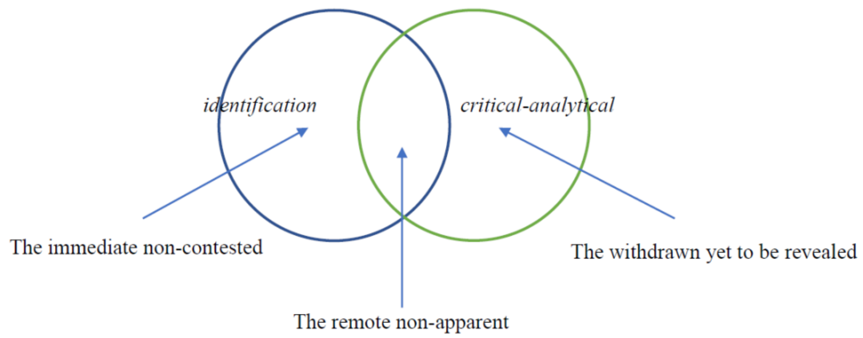

Figure 1: A model for analysing examiners written comments

Using this model, it is shown that the intervention process utilizes aspects that are familiar (*immediate non-contested*) to successively make accessible aspects that are not apparent (*withdrawn yet to be revealed*). Here is revealed a region, *the remote non-apparent* which allows for negotiation of meaning and where the tension between mathematics subject matter and mathematics education concerns are brought to fore in the assessment process.

References

Maaranen, K. (2009). Practitioner research as part of professional development in initial teacher education. *Teacher Development*, *13*(3), 219-237.

Olande, O. (2014). Graphical artefacts: Taxonomy of students' response to items. *Educational Studies in Mathematics*, *85*, 53-74.

Peirce, C.S. (1931-1958). *Collected Papers, vol. I-VIII.* Cambridge, Mass: Harvard University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

PROGRAMMING AND ANGLES - PATTERNS OF VARIATION

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In several countries including Sweden programming is introduced to secondary students mainly as a part of the mathematics curriculum. “Scratch” is used as a tool in programming and this has become a common trend in many countries. However, teaching mathematics using “Scratch” is not appropriate for in-service teachers.

Several research findings show that angle, as a mathematical concept, is difficult to teach and learn (Mitchelmore & White, 2004). We conducted a study in which researchers worked with 32 in-service mathematics teachers in grades 7-9. Because angles are essential characteristics of two- and three-dimensional geometrical objects, the aim of this study was to give in-service teachers the opportunity to improve their experience of using “Scratch” in order to enhance students understanding of angles. Observation, tasks used, video samples and field notes were used as data collecting tools.

The data gathered were analysed in a qualitative way using concepts of the variation theory (Marton, 2015). The variation theory was also used to focus teachers’ attention on aspects that were possible to discern by using “Scratch”. Angle is a geometric concept but it possesses properties that can be found in various other contexts. The variation theory of learning stems from the concept of phenomenography (e. g., Marton, 2015). It emphasizes variation as a necessary condition for learners to be able to discern new aspects of a subject of study.

Based on a preliminary analysis, we could discern three concepts of angle: (1) angle as a movement (in rotation or sweep), (2) angle as a geometric shape (a partitioning of space by two intersecting beams), and (3) angle as a unit for measure. There appears to be an alignment between in-service teachers using “Scratch” and their instructional practices that will be discussed in this presentation in detail. The use of patterns of variation in programming will also be presented.

References

Marton, F. (2015). *Necessary conditions of learning*. London: Routledge.

Mitchelmore, M.C., & White, P. (2004). Abstraction in mathematics and mathematics learning. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 329-339).

INCREASING PARTICIPATION IN ADVANCED MATHEMATICS: EVALUATING A POSSIBLE APPROACH

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Improving participation in STEM, and especially mathematics, at upper secondary level has been a focus for much research (Hodgen et al., 2013; Honey et al., 2014). In Ireland, the Department of Education and Skills (DES) has set a goal that the country will become a leader in Europe with regards to developing STEM talent (DES, 2017). This presentation considers the potential impact of one measure taken by the DES to increase enrolments in advanced mathematics.

In Ireland, students must sit a summative examination - the Leaving Certificate (LC), at the end of upper secondary school. The LC acts as a gatekeeper to tertiary education with students awarded points based on their six best subjects. Prior to 2012, the maximum points that could be awarded for the top grade in a subject studied in its most advanced form (Higher Level [HL]) was 100. Since 2012, mathematics has been assigned a special status with the introduction of the Bonus Points Initiative (BPI), with students now awarded 25 additional points if they achieve above 40% at HL. While, the DES (2017) is considering expanding this initiative to other subjects, little research has been carried out on the impact of the BPI. Thus, this study investigates teachers' perspectives on the BPI and its impact on the teaching and learning of mathematics.

In 2018 a questionnaire designed for this study was distributed to a representative sample of 800 LC mathematics teachers, with 266 teachers responding. While a large proportion of teachers (46%) appeared to favour the BPI, more than half (56%) wanted it to be maintained but with alterations. When asked about the impact of the BPI on the student profile in their mathematics classes, 31% of teachers indicated that unsuitable candidates were now studying HL mathematics and this led to challenges in relation to higher student-teacher ratios (n=33); greater diversity in the classroom (n=61) and maintaining an appropriate learning environment for the most able students (n=23). These findings suggest a need to consider the wider implications of the BPI.

References

DES (2017). *STEM Education Policy Statement 2017-2026*. Dublin: DES.

Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). *Evidence for review of mathematics teaching: Improving mathematics in Key Stages Two and Three: Evidence Review*. London: Education Endowment Foundation.

Honey, M., Pearson, G. and Schweingruber, H.A. eds., 2014. *STEM integration in K-12 education: Status, prospects, and an agenda for research*. Washington, DC: NAP

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

SELF-EFFICACY BELIEFS INSTRUMENT: USE OF HISTORY OF MATHEMATICS IN MATHEMATICS TEACHING

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This study is related to the use of a self-efficacy beliefs tool in mathematics teaching (MT) through the use of history of mathematics (HM), based on the Self-efficacy (SE) Theory of Bandura (1977). Moreover, the instrument is used as a valid and reliable tool for investigating pre-service teachers' (PTs) self-efficacy beliefs about the use of HM in MT. The use of HM in MT is considered as an alternative approach that utilizes primary and secondary sources of HM (Tzanakis & Arcavi, 2000). The instrument is adapted from Enoch, Smith and Huinker's (2000) instrument which contains 21 statements grouped into two dimensions of self-efficacy beliefs: personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). There have been substantial studies in the field of self-efficacy beliefs however; there is a scarce of empirical research related to PTs' self-efficacy beliefs about the use of HM as well Self-Efficacy Beliefs Instrument (SEBI) employing SE Theory (Bandura, 1977) for the assessment of the use of HM does not exist so far. Exploratory factor analysis was employed to maintain construct validity; and reliability was maintained by Cronbach alpha ($\alpha = .829$, $N = 305$) to assess internal consistency of the items. Principal components analysis with oblique rotation was performed on SEBI to determine factors. The first factor PMTE with 6.160 factor loadings consisting of 13 items explained 19.810 % of variance; and the second factor MTOE with 4.242 factor loadings consisting of 8 items explained 14.485%. The values of Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy obtained in SEBI was high (.767) and the results of Barlett's test were significant ($p < 0.000$), which indicated that the data were appropriate for analysis. (Approx. Chi-Square was 3313.591 with degree of freedom 210). Further, the correlation coefficient between PMTE and MTOE was located .607. Hence from this study, it is concluded that PTs' self-efficacy beliefs can be increased through the use of this SEBI tool. It is recommended that further studies can be conducted to confirm the reliability and validity of this instrument across subjects.

References

Bandura, A. (1977). Self-efficacy: toward a unifying theory of behavioural change. *Psychological Review*, *84*(2), 191–215.

Enochs, L. G., Smith, P. L. & Huinker, De A. (2000). Mathematics teaching efficacy beliefs instrument. *School Science and Mathematics*, *100*(4), 194-202.

Tzanakis, C. & Arcavi, A. (2000). Integrating history of mathematics in the classroom In, J. Fauvel & J. van Maanen (Eds.), *History in mathematics education* (pp. 201 – 240). Dordrecht: Kluwer Academic Publishers.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

DESIGNING MATHEMATICAL TASKS INCLUDING SUSTAINABLE DEVELOPMENT GOALS

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Recently, educational reform discourses emphasize sustainable development of future society. In order to nurture future generation who can contribute to establishing sustainable society, educational reform discourses seek creative and rational solutions regarding problems faced in future society through globalization, complexity, and knowledge and information-based society based on mathematical knowledge. For the purpose, this study is developing materials related to Sustainable Development Goals (SDGs) and tasks connected to mathematics based on issues related to life to cultivate key concepts of mathematics and mathematical competencies.

In this study, preservice teachers designed mathematical tasks in 17 sustainable development goals context including economic inequality, environmental sustainability, innovation, peace, and justice (UNESCO, 2016). As a result, preservice teachers were able to experience diverse social issues which they were unaware of in daily life. A variety of controversial topics such as nuclear energy development and disposal matter, reduction of marine product output due to overfishing young fishes, economic inequality due to income growth-led outcomes, social inequality, and plastic island due to use of disposable plastic and unable to discard them effectively appeared in mathematical tasks. However, overly difficult questions were made since mathematics was connected to complicated social phenomena.

Furthermore, preservice teachers realized diverse social phenomena and mathematical connectedness through developing mathematical tasks in SDGs contexts. In addition, mathematical ability and creative and converging thinking of preservice teachers were cultivated by associating mathematical concepts used in item development and activities related to mathematics which converged knowledge of other subjects related to SDGs. Moreover, 'mathematical tasks in SDGs context' will contribute to promoting mathematical knowledge and knowledge of future world citizen.

Acknowledgement

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References

UNESCO (2016). *Education 2030: Incheon Declaration and framework for action for the implementation of Sustainable Development Goal 4*. Paris: UNESCO.

TEACHERS' KNOWLEDGE AND PRACTICE ON MATHEMATICAL MODELLING AND PROBLEM-POSING

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Modelling is the process of structuring, generating real world facts and data, mathematizing, working mathematically, interpreting, validating and evaluating (Blum et al., 2007). In particular, an integral part of the modelling cycle is problem-posing. Modelling and problem-posing are powerful tools to increase students' reasoning, to develop mathematical concepts and to recognize the power of mathematics as a critical instrument to interpret reality (Bonotto, 2013). The aim of this study is to investigate teachers' knowledge and practice on mathematical modelling and problem-posing. The research questions are: i) Do teachers implement modelling activities in their school practice? ii) Do teachers know and use problem-posing in their classroom activities? In which situations do teachers implement problem-posing activities? The sample comprises 107 primary schools and 72 secondary school in-service teachers from the North of Italy. Data was collected by a questionnaire with closed and open questions and Likert scales. The approach for the data analysis is mixed quantitative and qualitative and univariate and bivariate analysis has been performed.

The findings indicate that teachers implement regularly modelling activities. Nevertheless, when teachers were asked suggestions to improve their teaching, they called for more materials based on realistic contexts. Problem-posing activities are performed only by 39,6 % of the sample. A significant correlation could be observed between the use of artifacts and the implementation of problem-posing activities ($\phi^2 = .13$, $p < .01$). Moreover, almost every teacher (95,8%) who implements problem-posing activities implements also problem-solving ones ($\phi^2 = .44$, $p < .001$). Based on these results, we believe that mathematical activities need to be changed with more realistic problem situations, in order to give students mathematical competencies and instruments to interpret the world they live in. In the specific, teachers need more materials to support their preparation and practice on mathematical modelling. Moreover, problem-posing must become an integral part of classroom activities. In the future we would like to develop practical resources based on modelling and problem-posing available for teachers of every school level.

References

Blum, W., Galbraith, P. L., Henn, H.W. & Niss, M. (2007). *Modelling and applications in mathematics education.* The 14th ICMI Study. New York: Springer.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics, 83*(1), 37-55.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EVALUATIVE CRITERIA AND MODES OF REPRESENTATION IN SOUTH AFRICAN GRADE 3 MATHEMATICS TEACHING

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Bernstein (1997) uses the notion of “evaluative criteria” to refer to messages that are communicated about what learners are expected to do in order to produce the legitimate text. Studying evaluative criteria is particularly important in South African classrooms given evidence of the absence of communication of evaluative criteria in instruction within early grades’ classrooms (Hoadley, 2006). In her work, Hoadley extended Bernstein’s notion of strong and weak evaluative criteria (F^+ and F^- respectively) to include F^0 – the absence of evaluation. In a broader study comparing evaluative criteria in four Grade 3 classrooms during the teaching of early number and additive relations across English and Sepedi-medium of instruction classrooms, I found that teachers transmitted a range of evaluative criteria. In this paper, I share details of an analytical framework that integrates attention to both evaluative criteria (EC) and moves between representational modes (MR). Lesson transcripts from the empirical data were broken down into excerpts based on the goal or theme of each task. Ninety two excerpts were analysed for the extent of work with evaluative criteria and modes of representation within each excerpt. Constant comparison of excerpts led to the creation of the EC-MR Framework with four broad levels of transmitting the evaluative criteria. In the first level EC0, no evaluative criteria were offered. At the close of each excerpt in this level, learners were left unaware of whether offers they made were correct or incorrect. The second level is the EC1 which comprised of excerpts where the evaluative criteria were offered. At the close of each excerpt in this category learners were made aware of whether their offers were correct or incorrect. EC2 was separated into EC2A and a subcategory referred to as EC2B. EC2A comprised of excerpts where teachers transmitted localised procedural evaluative criteria. At the close of each excerpt offers based on calculating by counting strategies were accepted across additive and multiplicative tasks. EC2B comprised of excerpts where teachers offered or accepted generalised mathematical procedures. At the close of each excerpt, offers based on calculating by structuring or formal calculations were accepted or offered across additive and multiplication tasks. The last level, EC3 comprised of excerpts where the use of known number facts to produce answers was encouraged. In this category of excerpts learners were asked to explain and justify their solutions to tasks.

References

Hoadley, U. (2006). Analysing pedagogy: The problem of framing. *Journal of Education, 40*, 15-34.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

MAKING SCHOOL MATHEMATICS REAL AND MEANINGFUL FOR DIVERSE LEARNERS

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Existing mathematics classroom practices lack connections to students' everyday lives, especially for students who have traditionally been underrepresented in STEM fields. In New Zealand, Māori and Pāsifika students' cultural ways of being and home languages are often neglected or devalued, which constrain students' opportunities to demonstrate multiple mathematical competencies. New Zealand teachers require support in learning how to implement culturally sustaining mathematics classroom practices (Averill et al., 2009). Learning to facilitate meaningful classroom activities is a global challenge because of the amount of knowledge required of both everyday experiences and mathematical concepts. We subscribe to the *funds of knowledge* framework, stating that families and students should collaborate with teachers to connect mathematics lessons with community resources. Moll and Gonzalez (1994) introduced the idea of *funds of knowledge* as a way to build upon students' understandings using non-traditional methods. The third grade teachers in their study incorporated students' multilingual assets and perspectives into the learning processes. This corresponds to d'Ambrosio's (1985) call for *ethnomathematics*, in which mathematics educators value the mathematics that exists within cultural activities. Following a funds of knowledge approach, we focused our study on examining ways to utilise students' lived experiences as entry points into the mathematics. We implemented a Community Project at a culturally and socioeconomically diverse primary/intermediate school in New Zealand. Sixteen students between the ages of nine and thirteen received and used cameras to document where they noticed mathematics in their lives. We conducted weekly, unstructured interviews with students to inquire about the situations involved in the photographs and the mathematics students noticed. We used students' perceptions of mathematics to create problematic tasks related to the mathematical strands taught in class. In this paper, we illustrate the array of places students observed mathematics and the mathematical concepts they described in interviews. We highlight ways teachers can utilise students' assets to make mathematics more meaningful.

References

Averill, R., Anderson, D., Easton, H., Maro, P. T., Smith, D., & Hynds, A. (2009). Culturally responsive teaching of mathematics: Three models from linked studies. *Journal for Research in Mathematics Education, 40*(2), 157-186.

d'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics, 5*(1), 44-48.

Moll, L. C., & González, N. (1994). Lessons from research with language-minority children. *Journal of Reading Behavior, 26*(4), 439-456.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EXPERIMENTAL WORK IN MATHEMATICAL TEACHING

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This research focuses on *two central questions:* 1. Which experimental processes of natural science can be distinguished for learning mathematics? 2. What are the opportunities and limitations of this approach in classes? To answer the first question a distinction is made here between experimental method, exploratory experiment and an attempt (e.g., Medawar, 1969). These three procedures vary with regard to the connection between theory and empiricism as well as between the knowledge of the corresponding relations. While, for example, the experimental method can be characterized as a structured procedure for generating and verifying newly mathematic coherences, only previously known coherences are regarded in an attempt. *Related to mathematics teaching,* mathematical connections can be elaborated in different ways. Therefore, the procedures from the natural sciences are examined with philosophical logic (Meyer, 2010) – not only to assign the inferential processes during the experiment (Leuders & Philipp, 2013), but especially to analyze the interplay between theory and empiricism in mathematical actions. This gives a benefit for mathematics teaching, which allows an outlook on the second research question. In order to observe the interplay between theory and empiricism in experimental work in mathematics, an *empirical study* was carried out with 24 university students and 24 high school students, who worked in groups on mathematical questions. The processes were videotaped, transcribed and analyzed by an interpretative research paradigm (Voigt 1985). Beside the complex interplay of theory and empiricism it has been shown that experimental work in mathematics lessons can be used for the discovery and justification of mathematical connections. Different types of experimental work in mathematics will be elaborated according to different levels of use of empiricism and theory. Various experimental procedures, their potential and first results of the underlying empirical study will be shared.

References

Leuders, T., & Philipp, K. (2013). Preparing students for discovery learning. In A. Heinze (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 241-248). Kiel, Germany: PME.

Medawar, P. B. (1969). *Induction and intuition in scientific thought*. Philadelphia: American Philosophical Society.

Meyer, M. (2010). Abduction – A logical view of processes of discovering and verifying knowledge in mathematics. Educational Studies in Mathematics*, 74*, 185-205.

Voigt, J. (1985). Pattern and routines in classroom interaction. *Recherches en Didactique des Mathématiques*, *6*(1), 69-118.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

FROM COUNTING IN ONES TO CONDENSED COLUMN ADDITION: A BRIDGE TOO FAR FOR THE EARLY GRADES

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It remains a major concern that South African children continue to use ‘counting in ones’ strategies for mathematical calculations for much of primary school. The newly released ‘The Framework for Teaching and Learning Mathematics in South Africa: Mathematics with understanding’ (the framework) attempts to address this concern but appears to contradict guidance to that which is offered in ‘Curriculum and Assessment Policy Standards’ (CAPS). The research questions being reflected on for this paper are theoretical. First, focusing on addition calculations, what is the same and what is different about the teacher guidelines in ‘the framework’ and in CAPS? And second, what is a possible teaching trajectory from ‘counting in ones’ to efficient written calculation methods for addition that is likely to be recognisable to South African teachers?

In pondering these questions I drew on mathematics education literature pertaining to counting pathways into number, structured representations and alternatives to the condensed column method to frame the paper and inform my document analysis. I conducted a detailed content analysis of the addition methods offered in two canonical documents to identify ‘visual-quantitative learning supports and written-numeric aspects’ (Fuson and Li 2009). The fundamental argument advanced in this paper is that the shift from counting ones to the condensed column addition in Foundation Phase, is a bridge too far. An alternative proposed learning trajectory for multi-digit addition is presented. This delineates four main calculation strategies. The work of Fuson and Li (2009) was drawn upon as these researchers bridge ‘western’ and eastern’ mathematic traditions and play explicit attention to progression towards column calculations; and they build on counting in ones markings (commonly evident amongst South African children). Alternatives to the condensed column method: expanded notation, write all totals, and new groups below are included. These are expected to be accessible to South African teachers for whom ritualized condensed column addition is their primary calculation strategy.

References

Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement Grades 1-3, and Grades 4-6 Mathematics*. Government of South Africa: Pretoria

Department of Basic Education (2018). *The Framework for Teaching and Learning Mathematics in South Africa: Mathematics with understanding*. Pretoria:DBE

Fuson, K. C., & Li, Y. (2009). Cross-cultural issues in linguistic, visual-quantitative, and written-numeric supports for mathematical thinking. *ZDM Mathematics Education, 41*, 793–808.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

HOME LANGUAGE AS A RESOURCE FOR ACCESSING THE POWER OF MATHEMATICS

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Mismatches between students' classroom and home languages frequently impede mathematical sense-making. This is a situation that impacts upon the majority of South African students and that has been directly implicated in the country's poor TIMSS mathematics performance (Reddy et al., 2016). English - the home language for fewer than one in ten students - is the dominant medium for most South African mathematics classrooms. The country's Language in Education Policy does, however, support the principle of additive bilingualism, thereby offering legitimate space for student's home language as an important additional resource for sense-making.

We report here on the classroom language practices of a grade 4 mathematics teacher (Ms P). We locate our discussion within a broadly sociocultural framework in terms of which cognitive development is socially mediated and historically and culturally situated with language constituting the principal means both of thought and of communicating such thought (Vygotsky, 1986). Ms P was one of two teachers contributing to a small-scale qualitative case study, the goal of which was to explore teachers' use of classroom talk in mediating their students' mathematical sense-making. Data were generated through lesson observation and interviews in two public schools serving isiXhosa home language students. Observations from the broader study revealed that whereas the teacher at the other school adhered strictly to her school's English-only policy, Ms P allowed extensive use of her students' home language. While Ms P's classroom language practices may have reduced her students' overall exposure to English, and their opportunities to practise interacting in it, assessment data from a larger teacher development project in which both case study teachers participated suggest that the extensive use of isiXhosa in Ms P's lessons aided students' mathematical sense-making.

References

Reddy, V., Isdale, K., Juan, A., Visser, M., Winnaar, L., & Arends, F. (2016). TIMSS 2015: *Highlights of mathematics achievement of Grade 5 South African learners*. Pretoria: Department of Basic Education/ Human Sciences Research Council.

Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin. Edited translation). Cambridge, MA: Massachusetts Institute of Technology Press. (Original work published in Russian in 1934.)

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

THE INFLUENCE OF STUDENT ENGAGEMENT ON THE PERFORMANCE OF FIRST-YEAR MATHEMATICS STUDENTS

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Student engagement is an important field of research. Studies have indicated a meaningful relationship between student engagement and academic performance (Gunuc, 2014). However, not much has been done regarding student engagement in mathematics, specifically in the higher education context. Student engagement is a complex concept and is defined in various ways, consisting of different components.

The purpose of this paper is to report on a few facets of student engagement in a first year mathematics module. The aim of this research is to determine the influence of student engagement on the performance of first-year mathematics students in their first semester in a Faculty of Natural Sciences. Although the National Survey of Student Engagement (NSSE) divides student engagement into five facets, this paper will only focus on level of academic challenge and active and collaborative learning.

A sequential explanatory mixed method design was used, but only quantitative research will be reported on in this paper. The sample comprises 107 Natural Sciences students. Data were collected through a modified version of the NSSE. Confirmatory factor analyses, linear regressions and Cohen's effect sizes were used in order to analyse the data. According to the effect sizes for perseverance, the Grade 12 Mathematics mark and philosophical views of the students, had a large effect on predicting students' Mathematics module mark,

Level of academic challenge emerged as the most prominent facet of student engagement. Perseverance crystallized as the most prominent construct within this facet and the lack of motivation by students to do academic work, was also evident in the results of this study. *Active and collaborative learning* also appear to have a meaningful influence on student performance. This is meaningful in the area of mathematics education at tertiary level since it illustrates that the complexity of mathematics directly impacts the students' engagement on a multitude of levels.

References

Gunuc, A.P.D.S. (2014). The relationships between student engagement and their academic achievement. *International journal on new trends in education and their implications, 5*(4), 216-231.

NSSE (National Survey of Student Engagement). (2015). *Engagement indicators*. Retrieved from http://nsse.indiana.edu/html/engagement_indicators.cfm#a7

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

PEER OBSERVATION FOR TUTOR DEVELOPMENT

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Peer observation of teaching may be regarded as a supportive and developmental process for improving the quality of teaching (Hendry & Oliver, 2012). It can assist educators improve their teaching practice, develop confidence in the classroom and aid with the integration of tutors into the department (Bell, 2005). However, peer observation may also be viewed as intrusive and challenging to academic freedom.

In 2018, a departmental-based tutor programme at a South African university implemented a peer observation model project on tutor development. This paper reports on tutors' views and experiences as participants in this project. The study aims to research sustainable and practical approaches to tutoring higher education courses and support tutors in cultivating mathematically rich learning environments for students by focussing on normative ways tutors engage in classroom practices. Hence feedback from participants regarding peer observation will provide invaluable data to the study.

The sample comprised of 5 tutors who were either Masters or PhD students with a strong mathematics background. Observers were paired according to the availability of tutors at the time of a given tutorial period. The observations ran over the semester giving observers the opportunity to sit in on 11 different tutorials with groups of 15 – 20 students. Data for analysis was obtained from written observations and reflections by the observers and the presenting tutors (submitted weekly) and discussions at the weekly meetings.

Results revealed the strengths and the areas in which tutors need to develop their teaching practices and mathematical content knowledge. All participants found the exercise valuable. The observers indicated that they learnt new teaching and class control strategies that they implemented in their own classes which surmounted to respect for the approaches of their colleagues. The presenting tutors also asked for feedback on their teaching as they felt that it would enhance their performance in the classroom. The data identified areas that needed to be improved. These areas have since been given additional prominence in the tutor development programme.

References

Bell, M. & Mladenovic, R. (2008). *The benefits of peer observation for tutor development.* http:// link.springer.com/article/10.1007%2Fs10734-007-9093-1

Hendry, G.D. & Oliver, G.R. (2012). Seeing is Believing: The Benefits of Peer Observation, *Journal of University Teaching& Learning Practice, 9*(1).

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

CONCEPTUAL ANALYSIS OF AN ERROR PATTERN IN CHINESE ELEMENTARY STUDENTS

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We examine two conceptions of multiplicative reasoning (Tzur et al., 2013) in Chinese students. First, *Multiplicative Double Counting* (mDC), involves distributing items of one composite unit (Steffe, 1992) (e.g., 3 floors per pagoda) over items of another composite unit (e.g., 4 pagodas), and then figuring out the accrual of 1s in all of them (e.g., 1-is-3, etc.). Second, *Same-Unit Coordination* (SUC), entails being cognizant of sets of equal-size numbers as being composed of 1s while operating additively on the composite units (9 pagodas ± 4 pagodas =13 pagodas, or 5 pagodas). We articulate conceptual roots of an error pattern, found in Chinese elementary students' work on SUC tasks.

This study arose as a secondary analysis of findings from two studies, the quantitative study on Chinese students' responses to SUC tasks and another, a series of qualitative teaching experiments, focused on Chinese students' multiplicative conceptions. For the first study, all students (N=511) in grades 3, 4, and 5 at two schools in China were invited to participate. For the qualitative (conceptual) analysis, we chose a 4th grader and a 5th grader, based on initial analyses of student erroneous solutions to SUC tasks, which revealed the two plausible conceptions.

The results of the first study show that only 54-68% of all participants answered SUC-sum and SUC-difference tasks correctly. A grade-reversal is seen in the decrease on the SUC-sum task (χ^2=7.0, df=2, p=.031). These results are intriguing: Why would 32-46% of students give an incorrect answer and why would 3rd graders outperform 5th graders in solving the SUC-sum task? To examine this, we searched for *patterns in incorrect answers* to the SUC-sum task. Based on the qualitative study, we attribute the error pattern to students' use of a previously established way of operating, either a *Totaling All 1s* or mDC conception. We explain, from an observer's point of view, the difference between these conceptions, including (a) why mDC is more supportive of SUC and (b) why the errors are reasonable to the students, as they arise from instructional emphasis on multiplying to find totals of 1s in equal-size sets.

References

Steffe, L. P. (1992). Schemes of action and operation involving composite units. *Learning and Individual Differences, 4*(3), 259-309.

Tzur, R., Johnson, H. L., McClintock, E., Kenney, R. H., Xin, Y. P., Si, L., . . . Jin, X. (2013). Distinguishing schemes and tasks in children's development of multiplicative reasoning. *PNA, 7*(3), 85-101.

RETHINKING THE NOTION OF CONJOINING

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The notion of conjoining has been widely documented across the world. Typically, it is described as the tendency to reduce an expression such as $5a + 3$ to $8a$ (Falle, 2007; Stacey & MacGregor, 1994). However, there does not appear to be a formal, universally accepted definition of conjoining. Based on work with learners in South Africa, an item such as $x + 5$ produces responses such as $5x; 6x; 5$ and 6. We have also noticed other errors involving like terms such as: $3a - a \rightarrow 3$ and $2a + 3a \rightarrow 5a^2$. The research literature does not differentiate between these different forms of conjoining and hence this differentiation is the first aim of the research. The second aim is to study the extent to which conjoining errors persist over time as learners are increasingly exposed to algebra.

We investigated the type and prevalence of conjoining in test items focusing on solving linear equations and simplifying algebraic expressions. Test responses were gathered from 285 Grade 9 learners in 10 secondary schools in South Africa in the first and last quarters of the academic year. Responses were coded according to a) whether the item contained like or unlike terms, b) whether the letter was dropped or kept in the response and c) whether the operation on the numbers was correct.

Overall there was a decrease in the number of conjoining errors (Equations: 30%; Expressions: 2%). In disaggregating the findings further, there was a decrease in conjoining errors when operating on unlike terms (Equations: 20%; Expressions 10%) but there was an increase in conjoining errors involving like terms (Equations: 21%; Expressions: 11%). These findings would not have been visible had conjoining errors not been disaggregated. This suggests a more nuanced view of conjoining is necessary and that greater attention be paid to the types of conjoining errors that learners make.

References

Falle, J. (2007). Students' tendency to conjoin terms: An inhibition to their development of algebra. Mathematics: Essential research, essential practice, 285-294.

Stacey, K., & MacGregor, M. (1994). Algebraic sums and products: Students' concepts and symbolism. *Paper presented at the Proceedings of the 18th International Conference for the Psychology of Mathematics Education.*

WHAT TEACHERS LEARN THROUGH IMPLEMENTING MATHEMATICS TEXTBOOK WITH LESSON STUDY AND OPEN APPROACH

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In Thailand, most teachers' learning depends on professional development by disseminating new information or technique to teachers by various kinds of short course training (Inprasitha, 2015). But it is not enough, it also requires learning through planning, doing, and reflecting (Takahashi, 2015). Especially the process of reflection is the key to teacher learning and development (Shulman. & Shulman, 2004). Inprasitha and his team introduced the Lesson Study and Open Approach in Thailand since 2002. Through the approach, Thai teachers have opportunities to use Japanese mathematics textbooks as a guiding tool for designing the lesson (Inprasitha, 2015).

This study aimed to analyze teachers' learning on their reflection about subject matter, pedagogy, and learners during implementing the mathematics textbook with Lesson Study and Open Approach based on Inprasitha's framework and Shulman, L. & Shulman, J's framework. The mathematics textbooks for this study were Japanese mathematics textbooks (Thai version), consisting of mathematics activities, sequencing of teaching, and students' mathematical ideas. The participants in this study were two teachers who used the textbook and taught in their classrooms using Lesson Study and Open Approach since 2013. The data were collected in the 2017 school year by interview and classroom observation with video recording on four first-grade lessons. The lessons were on shapes that the teachers collaboratively interpreted to a problem situation, sequenced the teaching, designed the material and anticipated students' ideas from the textbook. Teaching through the four steps of the Open Approach and students' ideas were observed and then they collaboratively reflect on the lesson focusing on students' ideas in their classroom.

The results showed that implementing the mathematics textbook with Lesson Study and Open Approach can be an effective professional development tool as 1) they learned the mathematics content during designing and reflecting on the lessons, 2) they learned shared views about how to teach mathematics content among their team during designing and reflecting the lessons, and 3) they learned students' ideas that they found in the textbook and understood more when they observed and reflected on it among their team.

References

Inprasitha, M. (2015). *Lesson Study: Challenges in Mathematics Education*. Toh Tuck Link, Singapore: World Scientific Publishing.

Shulman, L. S., & Shulman, J. H. (2004). How and What Teachers Learn: A Shifting Perspective. *Journal Of Curriculum Studies, 36*(2), 257-271.

CONFLICTUAL NATURE OF SOCIAL PRACTICE: A TEACHER'S REFLECTION ON GENERATING EXAMPLES FOR GENERALISATION

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The purpose of the study is to explore, post-professional development (PD), a teacher's interpretation of the practice of exemplification. The PD adapted a research analytic framework, the Mathematics Discourse in Instruction (Adler & Ronda, 2015), to create a framework for teaching referred to as the Mathematics Teaching Framework (MTF). In the PD the MTF, conceptualised as a cultural artefact, embodies a theory for teaching and learning secondary mathematics. Its function is to guide teachers to reflect on the core practices of teaching. One of these practices, exemplification, aims to help teachers generate examples that in turn help learners generalise key mathematical concepts and procedures. This is achieved through purposefully selecting examples in relation to a lesson's goals. A key consideration in the selection is the identification of critical features that must be present across an example set, to help learners generalise a particular mathematics concept. In the context of simplifying exponential expressions, this could be in terms of the interaction between the arithmetic and algebraic properties that bear on simplifying exponential expressions. Reported on here are a teacher's reflections, obtained through semi-structured interviews that elicited the rationale for selecting an example set from an episode of teaching. Taking into account the complex nature of social practice and role of cultural artefacts in enabling the appropriation of knowledge of a particular culture, the study draws from notions of mediating artefacts and inner contradictions (Cole & Engeström, 2007) to investigate the teacher's interpretation of exemplification in his context. The analysis of post-teaching reflections highlights that while some aspect of the exemplification were appropriated, contextual circumstances arising from conflicting discourses limited how the teacher works with it. Further, the teacher's interpretation is not only a function of his interpretation of the principles of exemplification in the MTF but also his perceived autonomy to reconfigure his practice in order to adapt or reject the diverse and often conflicting objects in his context.

References

Adler, J., & Ronda, E. (2015). A Framework for Describing Mathematics Discourse in Instruction and Interpreting Differences in Teaching. *African Journal of Research in Mathematics, Science and Technology Education*, *8457*(October), 1–18.

Cole, M., & Engeström, Y. (2007). Cultural-historical approaches to designing for development. In J. Valsiner & A. Rosa (Eds.), *The Cambridge handbook of sociocultural psychology* (pp. 484-507). New York: Cambridge University Press.

CHARACTERISTICS OF NEGOTIATION IN THE COMPOSITION OF MATHEMATICAL KNOWLEDGE

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The importance of discussion in mathematics has previously been pointed out (e.g., Lampert, 1990). However, consideration from the perspective of what interactions are necessary to compose mathematical knowledge remains insufficient. This study examined "negotiations between children" as necessary interactions for structuring mathematical knowledge. As such, the following two points were addressed.

・The relationship between the composition of mathematical knowledge and negotiation between children was clarified.

・The characteristics of the negotiation required for the composition of mathematical knowledge was clarified.

A descriptive framework for capturing negotiations in mathematics classes based on the "speech acts theory" proposed by Searle (1969) was considered. As a result, the labels "request," "question," "assert," "advice," "agreement," and "permit" were set in the descriptive framework to capture the speech intention. Furthermore, to reveal the relation between mathematical knowledge and negotiation and the characteristics of the observed negotiation, an experimental class in which students learned to eliminate fractions in G5 was planned. Data was taken from two class groups (Class A and Class B). The data obtained was divided into episodes at each group of utterances and classified quantitatively and qualitatively based on the above mentioned descriptive framework. To appropriately construct the intention behind the children's speech, these were interpreted by three researchers.

The results of the survey confirmed a difference in class episodes between Class A, where negotiation between children was widely confirmed, and Class B, where such scenes were not widely confirmed. Therefore, the mathematical knowledge students gained also differed between the two classes. In addition, the important role of speech as a "request" in negotiation between children was clarified. In the presentation, further results will be elaborated.

References

Searle, J, R. (1969). *Speech Acts: An Essay in the Philosophy of Language.* Cambridge: Cambridge University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

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TECHNOLOGY-RICH CLASSROOM PRACTICE: A SECONDARY MATHEMATICS TEACHER'S USE OF DYNAMIC DIGITAL TECHNOLOGY TO TEACH GEOMETRIC SIMILARITY

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Mathematics teachers' exploitation of the potential of digital technologies in the classroom to promote students' learning has been limited (Clark-Wilson & Hoyles, 2017). Teachers have struggled to effectively integrate digital resources in everyday classroom practice as they need to have specific expertise underpinning a successful teaching process with digital tools that facilitate students' understanding of the mathematics. Therefore, researchers have recently focused on teachers' integration of technology into real classroom settings to elucidate the complex process of integration and to identify the associated knowledge and skills required.

This paper reports on an ongoing doctoral study that aims to investigate the ordinary practices of three English lower secondary mathematics teachers using dynamic digital technology to teach geometric similarity (GS). The Structuring Features of Classroom Practice framework (Ruthven, 2009) was employed as a main conceptual tool as it offers a set of constructs that shapes classroom practices with technology, namely *working environment, resource system, activity structure, curriculum script, time economy*. The data were collected through video-recorded lesson observations and post-lesson interviews. The participants were selected among the Cornerstone Maths (CM) project teachers as they have begun to integrate the technology, CM software, into their practices to teach GS. This paper particularly focuses on the case of one of the three participant teachers, Jack (pseudonym). The paper presents the initial findings from Jack's two lessons, indicating how he adapted and developed expertise to manage his teaching of GS with technology in terms of five structuring features of classroom practice. For example, Jack exploited the dynamic and visual nature of the CM software during whole-class discussion and students' independent work that involved the technology to promote their use of precise mathematical and technological vocabulary. I will discuss further findings in detail at the conference.

References

Clark-Wilson, A. and Hoyles, C. (2017). *Technical Report: Dynamic digital technologies for dynamic mathematics*. London: UCL Institute of Education.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education & Didactique, 3*(1), 131–159.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

A CROSS-CULTURAL STUDY OF PRIMARY STUDENTS' UNDERSTANDING OF MATHEMATICAL EQUIVALENCE

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A good understanding of mathematical equivalence is regarded as a prerequisite for arithmetic and algebra learning. However, much research has shown that many primary students in some countries have difficulties understanding mathematical equivalence. Difficulties involve defining the equals sign operationally, and encoding and solving equations, e.g. $a + b = \_ + c$. In this study, we cross-culturally examine primary students' understanding of equivalence. Our main aim is to investigate whether difficulties with equivalence reported in the literature are widespread across the participating countries. We believe that such examination could help us reflect on the potential underpinnings of why students' conceptual difficulties with equivalence arise. Students (N = 3214, aged 8-12) from seven countries, namely China, England, New Zealand, South Africa, South Korea, Turkey, and the US, completed a mathematical equivalence assessment (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). We also assessed their strategy use in only equation-solving items.

A significant positive correlation was observed between definition scores and equation item scores ($r = .399$, $p < .001$). A one-way ANOVA showed significant differences between how students from different countries performed on the mathematical equivalence assessment (F (5, 2670) = 88.785, $p < .001$). Pairwise comparisons demonstrated that all countries differed between them (ps < .001) except for the following pairs: New Zealand and South Africa; Turkey and England; US and South Korea. Furthermore, a Kruskal-Wallis H test showed that students' strategy choice differed across countries, χ^2 (6) = 725.89, $p < .001$. Non-parametric pairwise multiple comparisons illustrated that Chinese students significantly differed from the rest of the sample; they were the most frequent users of compensation strategies. Conversely, Turkish students were the least frequent users of these compensation strategies. These results indicate that there are differences between how students from different countries performed on the equivalence assessment. The results have important theoretical and methodological implications for understanding students' mathematical development.

References

Rittle-Johnson, B., Matthews, P. G., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence: A construct-modeling approach. *Journal of Educational Psychology, 103*(1), 85–104. https://doi.org/10.1037/a0021334

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

ARTICULATION OF MATHEMATICAL MODELING AND ARGUMENTATION IN THE MATH CLASSROOM

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Mathematical modeling and argumentation are two competencies that are widely present in curriculums of many countries. Different authors conceive of modeling as a cornerstone of mathematical activity, mainly because modeling tasks emphasize students' processes over produced models (Blomhøj, 2004). Argumentation, on the other hand, is considered in many curricular proposals to be a key feature of dialogic, inquiry-oriented classrooms where students engage in and take responsibility for the collaborative construction of mathematical knowledge (Krummheuer, 1995).

Although both competencies have been investigated from a variety of theoretical lenses and methodological perspectives, the arguably fertile ground where both competencies meet remains under researched. Therefore, our project aims at studying students' mathematical learning when mathematical activity in the classroom is organized around the articulation of modeling and argumentation.

Twenty-five primary teachers were part of an in-service training process focused on mathematical modeling and argumentation in the math classroom, in Concepción and Santiago (Chile) between August and December 2018. Nine of them were selected to implement teaching units in their classrooms focused on developing modeling and argumentation in an articulated manner, between April and June 2019. We are designing an analytical approach to describe and comprehend how modeling and argumentation take place and foster particular forms of mathematical learning.

During the conference we will present preliminary results of these analyses. We aim at characterizing the type of mathematical learning that takes place during primary math lessons in which the articulated development of modeling and argumentative competencies is fostered.

References

Blomhoj, M. (2004). Mathematical modelling—a theory for practice. In B. Clarke et al. (Eds.), *International perspectives on learning and teaching mathematics* (pp. 145-160). Göteborg University: National Center for Mathematics Education.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229-269). Hillsdale, NJ: Lawrence Erlbaum.

APPRAISALS FOR DIFFERENT TYPES OF PROOF

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Argumentation and proof are seen as characteristic for mathematics and are part of standards worldwide (e.g., CCSSI, 2010). When dealing with proof in secondary school, teachers have a certain freedom to choose different types of proof. These range from experimental arguments using examples, to narrative or iconic operative arguments based on informal knowledge and representations, to more formal-deductive arguments (Brunner & Reusser, 2019). Selecting a specific proof can be assumed to depend on educational goals and local socio-mathematical norms (Sommerhoff & Ufer, 2019), but also on the teacher's appraisal of the proof.

Although observational studies have described teachers' choice of types of proof (Brunner & Reusser, 2019), there is little knowledge of how teachers appraise different types of proof. The presented study thus examines, how future teachers judge different types of proof regarding their acceptability as mathematical proofs, the soundness of their underlying arguments, how accessible they are for low- and high-achieving students, and the student teachers' own comprehension of the proof.

Student teachers from secondary education degree programs ($N = 183$) in their 3rd semester participated in a questionnaire study. They received two mathematical statements with four proofs of different type each, which they each rated on 4-point Likert scales regarding each of the abovementioned criteria. Generalized Mixed Effects Models were used to analyze the ordinal data descriptively and statistically.

Students' appraisals show minor differences between both statements yet differ considerably between different types of proof. In particular narrative operative proofs were rated as more acceptable as proofs, compared to iconic operative proofs, although these entailed the same underlying arguments, which were rated as similarly sound by the students. The results indicate that student teachers do appraise different types of proof in different ways. Further research will have to clarify to which extent these appraisals determine the selection of a proof for instruction.

References

Brunner, E., & Reusser, K. (2019). Type of mathematical proof: personal preference or adaptive teaching behavior? *ZDM*. https://doi.org/10.1007/s11858-019-01026-y

CCSSI (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Assoc. Center for Best Practices & Council of Chief State School Officers.

Sommerhoff, D., & Ufer, S. (2019). Acceptance Criteria for Validating Mathematical Proofs Used by Pupils, University Students, and Mathematicians in the Context of Teaching. *ZDM*. https://doi.org/10.1007/s11858-019-01039-7

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

ELEMENTS OF MATHEMATICS ANXIETY AND THE GENDER OF PRE-SERVICE MATHEMATICS TEACHERS

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Mathematics anxiety (MA) is incontrovertibly linked to the teaching of mathematics (Gresham, 2007). If teachers are agents in the transfer of MA, then there needs to be a focus on pre-service mathematics teachers (PMTs) in terms of the MA.

This study aims to establish a relationship between elements of MA and the gender of pre-service mathematics teachers (PMTs), which has not been researched as fully as the relationship between gender and mathematics performance (Recber, Isiksal, & Koc, 2018). MA is a multisided construct intertwining attitudinal, cognitive and emotional elements. If a person experiences MA momentarily during mathematics activities (state MA) or views mathematics-related circumstances as a risk to his/her self-concept (trait MA) (Gresham, 2007), he/she becomes frustrated or afraid (*emotional elements*), thus anxious. This fear translates into worry, panic or mental disorganization (*cognitive elements*) toward mathematics. This cognitive discomfort results in an uneasiness or dislike toward mathematics (*attitudinal elements*), which affects mathematics learning.

A quantitative survey using the Mathematics Anxiety Scale (MAS) (Fennema & Sherman, 1976) was administered to 125 PMTs. The results reveal that females experience less of the emotional ($p = .001 < .05$, $r = -.31$), attitudinal ($p = .02 < .05$, $r = -.20$) and cognitive ($p = .000 < .05$, $r = -.7.1$) elements of MA than males do. As a result of social pressure to empower females, female students may have less MA than in the past. Male PMTs should be provided with more emotional support, assistance in the development of positive attitudes toward mathematics teaching and reassurance. The study points to a shift in the direction of MA levels of females entering the high school mathematics teaching profession.

References

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman Mathematics Attitudes Scales: Instruments Designed to Measure Attitudes toward the Learning of Mathematics by Females and Males. *Journal for Research in Mathematics Education*, *7*(5), 324–326.

Gresham, G. (2007). A Study Of Mathematics Anxiety in Pre-Service Teachers. *Early Childhood Education Journal*, *35*(2), 181–188.

Recber, S., Isiksal, M., & Koc, Y. (2018). Investigating Self-Efficacy, Anxiety, Attitudes and Mathematics Achievement Regarding Gender and School Type. *Anales de Psicología*, *34*(1), 41–51.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

"IS IT POSSIBLE OR IMPOSSIBLE?" CHILDREN´S IDEAS ABOUT CHANCE AND COMBINATORIAL REASONING

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The notion of possibility is a subject of interest to cognitive development researchers (Muthivhi, 2010; Piaget, 1987; Spinillo, 1997) that underlies complex forms of mathematical reasoning such as those requiring the concept of chance and combinatorial reasoning. Thinking about the chance of obtaining a given element (e.g. blue marbles) from a set of elements (e.g. blue and white marbles) and thinking about the possible combinations between elements belonging to distinct sets (e.g. skirts and blouses) are hypothetical situations that can represent possible realities. This study examines whether the notion of possibility is manifested in the same way in situations that involve the concept of chance and combinatorial reasoning. One hundred and eighty Brazilian children from kindergarten to the 5th grade of elementary school were individually asked to judge whether a particular situation (chance and Cartesian product) would be possible or impossible to occur, and asked to justify their responses. The justifications varied from vague responses to those expressing appropriate notions of possibility and impossibility. Children from 3rd to 5th grade did better in the chance situation than in the combinatorial one. Appropriate justifications were provided more often in the chance situation than in the combinatorial one, and this was observed in all grades. The conclusion was that although the notion of possibility is related to both forms of reasoning, this notion is more easily understood when associated with the concept of chance rather than with combinatorial reasoning. Educational implications are discussed with regard to the teaching of the notion of possibility in elementary school.

References

Muthivhi, A. E. (2010). Piaget, Vygotsky, and the cultural development of the notions of necessity: an experimental study among rural South African learners. *South African Journal of Psychology, 40*(2), 139-148.

Piaget, J. (1987). *Possibility and necessity, vol. 1: the role of possibility in cognitive development*. Minneapolis: University of Minnesota Press.

Spinillo, A. G. (1997). Chance estimates by young children: strategies used in an ordering chance task. In E. Pehkonen (Ed.), *Proc. 21st Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 296-302). Lahti: PME.

VALIDITY IN A DIFFERENT CONTEXT: EXPLORING RELATIONSHIP TO OTHER VARIABLES VALIDITY EVIDENCE

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Multiple forms of validity evidence should be reviewed to produce assessments with valid and reliable results (AERA, APA, NCME, 2014). Most mathematics validation studies do not, however, investigate beyond content and internal structure (Bostic, Krupa, Carney, & Shih, in press). The purpose of this study is to examine the less commonly reviewed validity evidence of "relationships to other variables" (RTOV) using mathematics problem-solving assessments (PSM3-5) as an example. RTOV explores how test scores may be related to other variables. When RTOV has been examined in mathematics validation studies, it was at the overall test level (see Bostic, Sondergeld, Folger, & Kruse, 2017 for an example). As such, the research question guiding our study is: What information is present when examining RTOV at both the overall test and individual item-levels?

PSM assessment items were hypothesized to be unrelated to the variables of gender and race/ethnicity. To test this hypothesis, student outcome measures were compared, using independent samples *t*-tests. Item level differences were then evaluated using Rasch differential functioning (DIF) analysis. While findings revealed few *test-level PSM* differences by gender or race/ethnicity, *specific item level differences* were found across all *PSM*s.

When *PSM* assessments demonstrated test-level differences in scores by gender and/or race/ethnicity, item level DIF better informed researchers on how to specifically modify the assessments to address biases. Without investigating RTOV in multiple ways, results from mathematics assessments risk being unintentionally biased, and may produce spurious results of problem-solvers' abilities.

References

AERA, APA, & NCME. (2014). *Standards for educational and psychological testing.* Washington, DC: American Educational Research Association.

Bostic, J., Krupa, E., Carney, M., & Shih, J. (2019). Reflecting on the past and thinking ahead in the measurement of students' outcomes. In J. Bostic, E. Krupa, & J. Shih (Eds.), *Quantitative measures of mathematical knowledge: Researching instruments and perspectives.* New York, NY: Routledge.

Bostic, J. D., Sondergeld, T. A., Folger, T., & Kruse, L. (2017). PSM7 and PSM8: Validating two problem-solving measures. *Journal of Applied Measurement, 18*(2), 1-12.

CHILDRENS'S NUMERICAL AND PROBABILISTIC REASONING ABILITY: COUNTING WITH OR AGAINST?

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Basic numerical skills are often introduced in kindergarten, but the concept of probability is usually introduced much later. Still, research suggests that much younger children have some understanding of probability. Little is known on the relation between basic numerical skills and probabilistic reasoning in young children (Ruggeri, Vagharchakian, & Xu, 2018). Ruggeri et al. (2018) found a positive association in 7- to 10-year olds. In the current study, we investigate whether such associations exist in 5- to 6-year old children, and whether the relation differs for specific probability tasks.

377 Flemish children (192 boys) at the end of kindergarten solved nine numerical skills tasks and four probability tasks. In the first three tasks children had to select the 'best' of two boxes to blindly pick a winning element. In Task 1 (5 items), one of two boxes certainly would provide a win (measuring the ability to distinguish certain from uncertain events). In Task 2 and 3, both boxes contained a mixture winning and losing elements (measuring the ability to compare probabilities). In Task 2 (12 items) the 'best box' contained more winning elements (congruent); in Task 3 (12 items) it contained less winning elements (incongruent). In Task 4 (9 items), children received a box with winning and losing elements and a box with only losing elements, in which they had to add winning elements to make probabilities equal.

A canonical correlation analysis revealed a significant positive linear relation between the canonical score on the numerical tasks and the canonical score on the probability tasks, $F(45, 1622.417)=3.674$, $p<.001$, with an R of .542. While numerical skills were not associated with the ability to distinguish uncertain from certain events (Task 1), they were positively associated with scores on congruent (Task 2) and incongruent items (Task 3). No association was found with the score on Task 4. However, children with better numerical skills tended to add as many winning elements to their box as there were in the box that was offered, thereby ignoring the number of losing elements in that box. This suggests that at this young age, good early numerical skills might promote the use of erroneous strategies in probabilistic situations. Future research could investigate whether these erroneous strategies can be seen as the first step in reasoning about probabilities or whether it deters proper probabilistic reasoning.

References

Ruggeri, A., Vagharchakian, L., & Xu, F. (2018). Icon arrays help younger children's proportional reasoning. *British Journal of Developmental Psychology, 36*(2), 313-333.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

EFFECTS OF DISTRIBUTED PRACTICE ON TWO GRADE 10 MATHEMATICS CLASSES

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Distributed practice is a learning strategy which allows students to revisit prior content over longer periods of time than massed practice, where content is only revisited shortly before a test. Effects of prolonged distributed practice on learning and test performance in mathematics education are elusive. This study contributes to literature because it applies this practice to several topics in mathematics building up over several months. Specifically, this study investigated two research questions:

- What are the effects, if any, of distributed practice of several topics building up over a prolonged period of time in two grade 10 mathematics classes?
- What are the effects, if any, of distributed practice on summer learning loss?

Two teachers at a South African high school each taught one treatment class and one control class of about 30 students each. The treatment groups wrote a short test or "starter", containing two to five items, at the start of every class. The starter had a set of items from one day ago, two days ago, four days ago, etc. (1-2-4-8-16 spacing). The study was carried out during the second semester and the final exam was readministered in January as a post-test to investigate summer learning loss. There were ultimately just under 10,000 data points for the 60 treatment students.

There were no significant correlations between students' performance on starters and their respective overall performances in the first or second semester. Further analyses included grouping students into three groups (high-, middle-, and low performance on tests), and no significant interaction was found between these groups and first- or second-semester scores ($p = 0.832$, and $p = 0.880$). Comparison between the control and treatment classes was inconclusive on test performance and non-significant in terms of summer learning loss ($p = 0.057$). However, interview data from one teacher suggested enhanced strategic competence (Kilpatrick, 2001) on certain exam items. On these items, treatment students were able to coordinate concepts from algebra, trigonometry, and geometry whereas this coordination was completely absent from the control groups' responses. This study found no statistically significant effects of distributed practice on exam performance or summer learning loss; however, qualitative data suggest improved strategic competence.

References

Kilpatrick, J., Swafford, J., Findell, B., & Committee, N. R. C. (U. S.). M. L. S. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington: National Academies Press.

5-6-YEAR OLD CHILDREN'S MATHEMATICAL THINKING IN BUDAPEST AND IN NEW YORK – COMPARATIVE ANALYSIS

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Starting school is an important period in the development of children's thinking (Duncan & Magnuson, 2011). In this comparative study, children from two different countries solved playful tasks embedded in their teacher-initiated daily activities. Starting school is organized in a different way in Hungary and in New York State. In Hungary the *363/2012 Decree sets the mathematical education in the active acquaintance with the world. This provides the kindergarten teacher the opportunity to decide about the method and the depth of the mathematical development of the children. In New York State the mathematical development is in pre-school groups, following the instructions of a detailed framework* (Clements et al., 2003). Departing from this situation it becomes interesting how the difference between the two systems effects children's thinking. We conducted a wide-scope, activity-based analysis in the area of the number concept and of logical competences. The study comprised 332 children in Hungary and 80 in New York State.

The focus of the questions is twofold: (1) What are the elements in the interpretation of natural numbers where the difference is considerable? (2) What problem-solving methods appear during the activities carried out by logical sets?

Our findings show that children's thinking seem to be very similar. This justifies that the testing instruments were not country-specific. We have found significant differences in the interpretation of numbers, if numbers were represented by fingers, in motivation, and in the way of solution of logical problems. By analysing these differences we can point out areas where more attention and improvement are needed.

References

Clements, D. J., Sarama, J., DiBiase A-M. (2003). Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education. New York: Routledge.

Duncan, G. J., & Magnuson, K. (2011). The nature and impact of early achievement skills, attention skills, and behaviour problems. In G. J. Duncan, & R. J. Murnane (Eds.), *Whither opportunity: rising inequality, schools, and children's life chances* (pp. 47-69). New York: Russell Sage.

*363/*2012 *Decree of the Government of 17 December 2012 [Kormányrendelet az óvodai nevelés országos alapprogramjáról]*

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

ON CONFUSING DIAGONALS WITH LINES OF SYMMETRY

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Many geometry curricula focus on the technical aspects of the subject, such as on computing angles and areas as well as proofs, rather than on building students' understanding of basic concepts. For example, most teachers regarded the concept of diagonals of a polygon as a straightforward concept to learn for students. The instruction is usually operational with the teacher demonstrating how to join two non-adjacent vertices with a ruler but without much explanation. We find, however, evidence from TIMSS and public exams in Taiwan that the concept can be confusing as students mixed up the concepts of diagonals with lines of symmetry.

In order to help students learn basic geometrical concepts via a hands-on inquiry approach, we have developed learning materials that covered traditional topics using paper folding. The activities are organized according to Huzita's axioms of paper folding (Huzita, 1989). The focus of this study is to report on how students perceived diagonals and lines of symmetry through analysis of their performances.

The new materials were developed under design research and implemented in a twenty-hour trial course during the summer of 2017. Nine eighth graders from a remote area of Taiwan participated in the study. The class was taught by a teacher with previous experience in paper folding. All the lessons were videotaped.

All students found paper folding very interesting and engaged themselves in exploring how to use it to solve problems. It was observed during the classes and in the post-test that paper folding had revealed students' mistaking diagonals with lines of symmetry. The confusion was partly due to their overlap in some configurations and partly to their similarity in Chinese terminology. The findings reflected that the simple instruction of a diagonal as being the line joining a vertex to another vertex is not enough.

Diagonals can be mistaken by some students as lines of symmetry, and vice versa. Paper folding provides an opportunity to reveal students' misconceptions that may not be possible under traditional paper-and-pencil tests.

References

Huzita, H. (1989). Axiomatic development of origami geometry, *Proceedings of the First International Meeting of Origami Science and Technology*, 143-158.

PURPOSES OF MATHEMATICS TEACHER ARGUMENTATION DURING THE DISCUSSION OF TASKS IN THE CLASSROOM

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Current research in mathematics education recognizes the importance of argumentation in the classroom. Regarding features associated with the teacher, the role of the teacher in the orientation and interpretation of class discussions has been studied or on the structure and quality of the teacher's argumentation (i.e. Metaxas, Potari & Zachariades, 2016). However, there are no studies that inquire on the purposes of teacher's argumentation when she or he discusses mathematics tasks in the classroom. In this report, we address this issue.

We consider that argumentation is "a communicative and interactional act complex aimed at resolving a difference of opinion" (van Eemeren *et al.*, 2014, p. 6). We position the argumentation in a social environment, incorporated as part of classroom discourse, where the justification or the refutation of statements is essential.

The data were obtained from a ten-grade course (16-year-old students) in a public school in Medellin (Colombia) during five lessons, where the teacher discussed tasks about geometry, probability, and trigonometry. For the data analysis, we used the 'types of teacher reaction' proposed by Ruthven & Hoffman (2016) and the features of our stance regarding argumentation.

We have recognized different purposes in teacher's argumentation: to solve students' questions, to clarify task solutions' processes, to describe properties and uses of mathematical objects, to justify or to refute students' controversial standpoints, to show different ways to solve a task, to specify mathematical language, to convince students of the task's answer, and to validate the students' answers.

References

van Eemeren, F., Grassen, B., Krabbe, E., Snoeck Henkemans, F., Verheij, B., & Wagemans, J. (2014). *Handbook of Argumentation Theory*. Dordrecht, Netherlands: Springer

Metaxas, N., Potari, D., & Zachariades, T. (2016). Analysis of a teacher's pedagogical arguments using Toulmin's model and argumentation schemes. *Educational Studies in Mathematics, 93,* 383–397. doi: 10.1007/s10649-016-9701-z

Ruthven, K., & Hofmann, R. (2016). A case study of epistemic order in mathematics classroom dialogue. *PNA, 11*(1), 5-33.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

COMPARISON AND CRITIQUE OF EARLY ALGEBRA IN THE CURRICULA OF SOUTH AFRICA, GERMANY AND IRELAND

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Expressing general rules for functions facilitates children in generalizing from the particular, and in articulating this generalization, with or without abstract symbols (Kieran, Pang, Schifter, & Ng, 2016). Geometric patterns may be implemented as a particularly accessible presentation of the function concept in elementary school. In this oral communication we interrogate the opportunities presented by curricula in Germany, Ireland and South Africa to support children's emerging algebraic thinking, with a specific focus on geometric patterning.

The National Curriculum Statements for Grade R – 9 in South Africa (SA) and the Primary School Mathematics Curriculum (PSMC) in Ireland include specific content areas related to algebra. In SA, algebra is defined as the language for investigating and communicating most of mathematics and can be extended to the study of functions and other relationships between variables. In contrast, the national standards for primary school mathematics in Germany avoid the use of the term algebra in any form but include a content area ‘pattern and structures’. Functional thinking is not included in the Irish PSMC, and while rules for number patterns are included as curriculum content objectives, the examples included are all recursive, thereby precluding a functional approach. Beyond the repeating patterns of the earliest grades, patterning in the Irish PSMC is limited to number sequences, and as an aid to the memorisation of number facts. Pattern activities are seen as a support for number concept development in both Ireland and SA and while there is an emphasis on the use of number and geometric patterns to develop algebraic thinking skills in SA, there is a lack of guidance on the selection and sequencing of activities for teachers. Similarly, the expected competencies for 4th graders in Germany are roughly sketched and include the abilities to recognise, describe, and represent regularities of arithmetical and geometrical patterns as well as functional relations. In this oral communication, we will elaborate upon the content of the three curricula, exploring in what ways their interpretation by teachers may support children’s algebraic thinking, and identifying limitations and omissions in light of the expanding field of research in early algebra.

References

Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). Early Algebra: Research into its Nature, its Learning, its Teaching. In G. Kaiser (Ed.), *ICME-13 Topical Surveys*. Cham, Switzerland: Springer Open.

HOW DO THE LEARNERS HAVE CONSCIOUSNESS ABOUT THE COMPONENTS OF INDIRECT PROOF?

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Learning indirect proof has been expected to foster the learners' ability of thinking logically. According to the findings of previous studies on the teaching and learning of proof in mathematics education research (e.g., Miyazaki et al., 2017), the explicit teaching of the structure of proofs, enhances the quality of teaching and learning in itself. This means that the teaching and learning of a proof as a structural object plays a central long-term role. We can assert the above things, not only for learning direct proof but also for learning indirect proof as well. However, in previous studies about indirect proof, there has been little research on properly associating the structure of indirect proof with learners' understanding despite its importance. In Japan, our recent research has been addressing two issues; what are components of indirect proof that learners need to become conscious of, and how do learners interpret them?, constructing a theoretical framework that is to describe learners' understanding the structure of indirect proof.

There were three main theoretical frameworks in this research: a model of indirect proof (Antonini & Mariotti, 2008), deduction system of classical logic, NK, (Gentzen, 1969) and a framework of understanding the structure of deductive proofs (Miyazaki et al., 2017). As a result of Japanese mathematics textbook analysis using NK, we identified the concrete components of indirect proof (i.e., embodied meta-theory specific to indirect proof) and then, revealed that learners need to interpret the components (i.e., inference rules) using the set theory symbols and diagrams. Based on these results, each level in a framework of understanding the structure of deductive proofs was elaborated in order to make it applicable to indirect proof. To fill the gap in the research, we should address how the learners turn their consciousness towards the components. From the viewpoints of reflective, structural and reflectural learners (Scheiner, 2016), we address the issue and elucidate characteristics of learners who are positioned at **I**ndirect-**P**artial-**R**elational level.

References

Antonini, S., & Mariotti, M. A. (2008). Indirect proof: what is specific to this way of proving? *ZDM, 40*(3), 401-412.

Gentzen, G., & Szabo, M. E. (Eds.). (1969). *The collected papers of Gerhard Gentzen*. North-Holland.

Miyazaki, M., Fujita, T., & Jones, K. (2017). Students' understanding of the structure of deductive proof. *ESM, 94*(2), 223-239.

Scheiner, T. (2016). New light on old horizon: Constructing mathematical concepts, underlying abstraction processes, and sense making strategies. *ESM, 91*(2), 165-183.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

SOUTH AFRICAN TEACHERS' ENACTMENT OF A NEW APPROACH TO THE TEACHING OF FRACTIONS

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The dominant image of fractions in many South African mathematics classrooms is that of a whole that is 'fractured' into equal parts. Cortina, Visňovská and Zúñiga (2014) argue that this approach can become a didactical obstacle when teaching fractions and propose an instructional sequence that rather uses length measurement activities to introduce and build the concept of the relative sizes of fractions. The focus of this presentation is on the first four parts of this sequence, and the story book and learning resources designed to accompany these lessons. Cortina et al. (2014) have worked to refine this sequence in a long-running design research project over more than a decade. The initial work has been done in Mexico and Australia, but most recently the instructional sequence has been introduced to a group of South African teachers through the South African Numeracy Chair Project's collaboration with these researchers.

After facilitating the lesson sequence (by the first author), the test results of three classes of South African Grade 3 learners convincingly showed the ability of the sequence to powerfully convey the concept of fractions, which is consistent with the success of its implementation with Australian and Mexican learners. It has been established that the design works, but in the South African context we are yet to understand how it will be enacted by teachers, particularly given that the classrooms in which we are seeking to implement this design are in less advantaged contexts where resources, including time, are severely constrained. The research question we are seeking to answer is: How do South African teachers enact the teaching of the 'Fractions as Measures' sequence with the designed resources in their own contexts?

Since the initial South African trial, the sequence has now been introduced to a group of Grade 4 – 7 teachers, of whom three are actively working with the sequence with groups of learners in their school. In this presentation, we will tell the stories of how these three teachers have enacted the sequence as told in individual semi-structured interviews and a group interview with the first author, and from observations of these lessons. From these stories, we will reflect on the considerations that need to be made when seeking to implement new teaching approaches in such contexts.

References

Cortina, J.L., Višňovská, J. & Zúñiga, C. (2014). Equipartition as a didactical obstacle in fraction instruction. *Acta Didactica Universitatis Comenianae Mathematics, 14*, 1-18.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

THE ANSWER PATTERN OF JAPANESE G4 CHILDREN IN TIMSS2015 MATHEMATICS SURVEY

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Suzukawa et al. (2008) found that Japan has unusual answer pattern among 13 countries and areas (namely, Australia, Canada, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Korea, Netherland, New Zealand and the United States) by making secondary analysis of PISA2003 data. Following this research, Watanabe (in press) revealed that Japan still has unusual answer pattern among same 13 countries and area by analysing PISA2012 data. PISA is targeted for 15 years old children. This means that the answer pattern of secondary school children become clear. However, the answer pattern of primary school children is not determined yet.

The aim of this study is to reveal the answer patterns of Japanese primary school children by making cross-national comparison with above 13 countries and area. In order to achieve the aim, TIMSS2015 data is analysed. Because, although PISA is not conducted for primary school children, TIMSS is carried out in primary school level, especially for grade 4 children. In this analysis, the method of multiple group item response theory is applied with similar uses as Suzukawa et al. (2008).

Analysis revealed that Japanese children in grade 4 have higher level of mathematics achievement among 13 countries. However, the answer pattern of Japanese primary school children is unusual among 13 counties and area as well as Korea and Hong Kong. This suggests that the East Asia countries may have similar answer patterns in whole. Looking at Japanese case in more detail, Japanese children can have the correct answer of items related to simple calculation such as addition of decimal numbers etc. This characteristic is not applicable to Korea and Hong Kong as well as other 10 countries. This means that the remarkable feature of answer pattern that Japanese primary school children have is determined in this study by analysing TIMSS2015 data.

References

Suzukawa, Y., Toyota, H., & Kawahashi, I. (2008). Do Students in Japan Learn How to Use Mathematics in Real-Life Situation? An Analysis of Items in the 2003 Programme for International Student Assessment (PISA2003). *The Japanese Journal of Educational Psychology, 56*(2), 206-217.

Watanabe, K. (in press). A Study on the Secular Changes of Japanese Students' Mathematical Literacy: Focusing on the Pattern of Item Difficulty between PISA2003 and PISA2012. *The Journal of Japan Academic Society of Mathematics Education*.

AN EMPIRICAL STUDY FOR IDENTIFYING BASIC MENTAL MODELS FOR THE DERIVATIVE AND THE INTEGRAL

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In Weigand et al. (2017) the theoretical basis of the concept of *Aspects* and *Basic Mental Models* (*BMMs)* was explained and integrated into the present state of research. An *Aspect* of a mathematical concept is a subdomain of the concept that can be used to characterize it on the basis of mathematical content. A *BMM* (or *Grundvorstellung)* of a mathematical concept is a conceptual interpretation that gives it meaning. Concerning the derivative, we have two *Aspects* and the 4 *BMMs*: "*Tangent slope", "Local rate of change", "Local linearity and "Amplification factor.* Regarding the definite integral, we have three Aspects and the 4 *BMMs*: "*Area*", "*(Re)construction*", "*Accumulation*" and "*Average value*".

An empirical investigation–as a multiple-choice test–focuses on assessing the existence of *BMMs* in certain types of problems. A total of 599 students took part. The goal of this test was finding out which *BMMs* first semester freshmen in the first week of lectures found suitable or not suitable for certain problem contexts.

Every question of the test consists of one mathematically correct statement and four correct explanations respectively, out of which each emphasizes one BMM. The test subjects were asked to mark on a pentavalent Likert-scale to what extend these explanations matched their own thinking.

This test was able to answer to some research questions. The students' choice of *BMM*s is not specific to a person but occur respective to the problem type, with a higher range of variation with the choice of BMMs. The test also provides clues for a reasonable categorization of the problems in terms of training certain BMMs. Thus, special cases of functions are suitable for reflecting *BMMs* critically; verbally or symbolically worded problems are suitable to emphasize the *BMM Local rate of change*. Concerning the integral, the test results showed the importance of the *BMM Area* and they provide approaches to the development of the *BMM Accumulation,* which occurs most succinctly when functions are viewed on a symbolic level.

References

Weigand, H.-G., Greefrath, G., Oldenburg, R., Siller, S., Ulm, V. (2017): Aspects and Basic Mental Models ("Grundvorstellungen") of basic Concepts of Calculus. In: B. Kaur, W.K. Ho, T.L. Toh, & B.H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 313-320). Singapore: PME.

HOW TO SUPPORT TEACHER NOTICING: PERSPECTIVES ON DIFFERENT MEDIA REPRESENTATIONS

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Different media representations (e.g., video-clips, animations, written cases) have been used in teacher education as means to demonstrate effective teaching practices and support teachers' noticing skills. Each medium provides various opportunities for teachers to develop their noticing skills. There is currently a gap in the literature regarding teachers' noticing skills and their views about the learning facilitated within different environments (van Es & Sherin, 2014). This research investigates elementary pre-service teachers' (PSTs) noticing skills and their perspectives on the representation of teaching practices with different media. We framed PSTs' professional noticing in our study as two connected skills: *interpreting* elementary students' multiplicative reasoning and *deciding how to respond* to support students' mathematical understanding (Jacobs, Lamb, & Philipp, 2010). Twenty-two PSTs were asked to interpret and decide how to respond to mathematically significant details of elementary students' thinking. Elementary students' mathematical thinking was presented in three different forms: 1) Written Student Work, 2) Interaction-focused Clip ($\leq$ 7 minutes), and 3) Instruction-focused Video ($\geq$ 25 minutes). PSTs were then asked to fill out a four-item survey regarding their experiences with noticing students' mathematical thinking in different media representations. We present preliminary results from PSTs' responses to the survey items. Statistically more PSTs (16 PSTs, 73%) viewed *Interaction-focused Clip* as the easiest medium to interpret elementary students' multiplicative reasoning, $z = 3.357$, $p < 0.01$. In deciding how to respond, 11 (50%) and 10 PSTs (41%) respectively selected *Written Student Work* and *Interaction-focused Clip* as the easiest medium. Statistically more PSTs considered the *Instruction-focused Video* the most challenging means both in order to interpret (15 PSTs, 68%; $z = 2.501$, $p < 0.01$) and decide how to respond to student thinking (17 PSTs, 77%; $z = 4.158$, $p < 0.01$). We additionally open-coded PSTs' views about different media representations to support their professional noticing skills and found 8 themes informing their comparisons. Our results provide the prospect of PSTs' perspectives regarding their noticing skills in different media representations. We will discuss possible implications of our results for mathematics teacher training.

References

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169-202.

van Es, E. A., & Sherin, M. G. (2006). How different video club designs support teachers in "learning to notice". *Journal of Computing in Teacher Education*, *22*(4), 125-135.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Pretoria, South Africa: PME

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POSTER PRESENTATIONS

"ALPHAMENTE PROJECT": AN EDUCATIONAL PROPOSAL TO IMPROVE MATHEMATICAL COMPETENCIES

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In this paper, we present an ongoing research project of teaching and learning about improving mathematical competencies that will last two years. A lot of international and national studies show that some learning difficulties in mathematics can be ascribed to poor linguistic competence (Hughes, 1986) and to the difficulties in managing and coordinating different semiotic systems (Duval, 2006). Mathematical literacy can be analyzed in terms of processes, capabilities that underlie those processes, contents and contexts in which the assessment items are located (OECD, 2015). In the present work, we confined to communication, argument and representing capabilities (Turner et al., 2013). Within this theoretical framework, we wonder if activities focused on the management of different semiotic systems and on argumentation could influence the acquisition of these capabilities. In order to assess mathematical abilities among secondary school students, we developed a framework for constructing test items, based on international studies such as TIMSS and OCSE-PISA and national educational studies such as INVALSI. The sample comprises about 14000 fifteen-years-old students. About 160 teachers will be involved in the project. Methodologically, the project is subdivided into six phases. The 1st one is the analysis of training needs, starting from previous projects results. The 2nd phase is a teacher training by researchers and academic experts. Subsequently, there is a co-planning phase, where teachers and academic experts plan all together some Interdisciplinary Learning Units, including problem solving and cooperative learning activities. Trained teachers, in turn, will be trainers in their schools in the fourth and fifth phases: educational paths and e-learning workshops to simulate INVALSI and OCSE-PISA tests. Finally, researchers will analyse the obtained results and disseminate best practices. First results will be shown in detail.

References

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1-2), 103-131.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics*. Oxford: Blackwell Publishing.

OECD. (2015). *PISA 2015, Draft Mathematics Framework.*

Turner, R., Dossey, J., Blum, W., & Niss, M. (2013). Using mathematical competencies to predict item difficulty in PISA: A MEG study. In *Research on PISA* (pp. 23-37). Dordrecht: Springer.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

4–7 = ? – HOW PRIMARY SCHOOL STUDENTS SOLVE A SUBTRACTION TASK WITH A NEGATIVE RESULT

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Teaching integers in school is connected with manifold challenges (e.g. Schindler et al., 2017). Students very often encounter negative numbers first in the context of subtraction tasks where the subtrahend is larger than the minuend. Therefore, it seems important to explore students' intuitive understanding of such tasks in more detail.

In a case study, we investigated how third and fourth graders prior to instruction answer to the open number sentence 4–7= and especially how they justify their result. In order to stimulate argumentation and to make it easier for students to go beyond the "official classroom knowledge" if necessary, they were given "minus three", "zero" and "unsolvable" as possible answers to choose from. Until now, 183 students from nine classes were asked to write a letter in response to a fictitious classmate. So we had the possibility of detailed analysis of students' responses on the one hand. On the other hand, students had more time to formulate a detailed and thoughtful answer independently of their classmates. Both in the survey phase and in the student products, there were indications that most students were very motivated in writing the letter and able to easily cope with the requirement.

Students' answers were content analyzed by the authors. In the following, we present some important results: 100 students (55%) chose 4–7 = -3, 25 (14%) stated 3 as solution (although it was not available for selection), 22 (12%) chose 0 and 20 (11%) considered the task as unsolvable. Other answers were given by 16 students (about 9%).

52 of 100 students answering correctly used a developable mathematical argumentation by referring to the existence of the negative numbers (19 students) or even by deducing the exact result (33 students). Also other results than -3 were partly justified by referring to the existence of negative numbers.

Results indicate that many primary students already have viable concepts for subtraction situations with negative results prior to instruction. Further details also underline that some children consider the answer "unsolvable" as a transitional stage. These are good basics for teaching integers at a later stage in school. Other aspects such as self-designed "rules" seem problematic as they can become entrenched and hamper the development of students' understanding.

References

Schindler, M., et al. (2017). Sixth-grade students' reasoning on the order relation of integers as influenced by prior experience. *Mathematics Education Research Journal, 29*(4), 471–492.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

LEARNING TO RESPOND TO ERRORS IN APPROXIMATIONS

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The use of approximations of practice (Grossman et al., 2009) in teacher education offers a promising way for teacher candidates (TCs) to develop skill with key aspects of the work of teaching. There exists the need to document TCs' skill from their enactments and to identify the way in which TCs' skill develops through such designs. We connect analyses of two types of approximations—coached rehearsals and scripting tasks—to investigate the relationships between learning opportunities and evidence of learning captured across time. We use two cases to illustrate what can be learned about TC development through these multiple data sources.

We present the cases of Greg and Travis as they participated in an initial scripting task, a coached rehearsal of a sorting task, and a follow-up scripting task. In each scripting task, they wrote dialogues in response to a student's error made while sorting shapes into examples and non-examples of polygons. In the coached rehearsals, they responded to a "planted error" where a student made an error while sorting cards into examples and non-examples of linear functions. To analyse these data, we looked holistically across each case for evidence of learning, and examined features such as the types of teaching moves used and the representation of student voice.

Through this analysis, we found that Greg's developing practices, made evident through differences in his dialogues, had direct connections to the moves he experimented with during his rehearsal. While these practices were continually developing throughout the rehearsal, their use in response to errors seemed to shape his vision of how discussions around errors could unfold without the need for immediate resolution. Travis also demonstrated through his dialogues how his practice of using orienting moves continued to develop as he refined the purpose for using these practices. Travis's rehearsal provided opportunities to experiment with moves and experience discussions around errors that play out in novel ways. While this contributed to his developing vision that included a valuing of students' ideas, that was being negotiated with a developing focus on how discussions are moving toward a goal. This resulted in Travis's focus on involving students who contribute ideas in the continued discussion of that idea (particularly involving resolving errors). A main takeaway from these two cases is how the variety of data sources—across multiple approximations and time—offer a more complete picture of TCs' developing practice and the way a vision of teaching informs that practice.

References

Grossman, P. L., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. W. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9), 2055–2100.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

LEARNING FROM LESSONS: STRUCTURE AND CONSTRUCTION OF MATHEMATICS TEACHER KNOWLEDGE – FIRST INSIGHTS OF THE GERMAN CASE STUDY

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"Learning from Lessons" is a study focusing on the structure of mathematics teacher knowledge to "understand teacher in situ learning" (Chan et al., 2017) in Australia, China and Germany. To foster teacher development it is important to acknowledge and better understand teacher learning in the classroom. Hence, the theoretical framework is based on the non-linear "interconnected model of teacher professional growth" (Clarke & Peter, 1993) focussing on domains of enactment and reflection. The project addresses the following research questions:

To what classroom objects, actions and events do teachers in different countries/cultures attend to and with what consequence for their learning?

How is the teacher's selective attention influenced by knowledge and beliefs, the lesson's content and structure, and contextual characteristics of school and classroom?

The international data collection involves quantitative and qualitative data with respect to two sub-studies: case studies with teachers of Grade levels 5, 6 and 7 which aim to illicit the mechanisms connecting attention to learning. Online surveys (about 40 teachers per Grade) seek to highlight patterns in teacher attention and consequent learning. Participants in both sub-studies commit to teach lessons according to their own adaptions of a provided lesson plan, teach a follow-up lesson, and respond to questionnaires about their beliefs, pedagogical and mathematical content knowledge drawn from the TEDS-M study. This poster will present data from one German Grade 5 teacher, highlighting his professional learning over 6 lessons with respect to the instances the teacher identifies when reflecting the lesson supported by video recall.

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References

Chan, M. C. E., Clarke, D. J., Clarke, D. M., Roche, A., Cao, Y., & Peter-Koop, A. (2017). Learning from Lessons: Studying the structure and construction of mathematics teacher knowledge in Australia, China and Germany. *Mathematics Education Research Journal, 30*(1), 89-102. doi:10.1007/s13394-017-0214-6

Clarke, D. J., & Peter, A. (1993). Modelling teacher change. In W. Atweh, C. Kanes, M. Carss, & G. Booker (Eds.), *Contexts in mathematics education. Proceedings of the 16th conference of the Mathematics Education Research Group of Australasia* (pp. 167-175). Brisbane, Australia: MERGA.

A STUDY OF AN ARTEFACT ON PUPILS' ERRORS IN THE ALGORITHM OF MULTIPLICATION

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During a pre-service of primary school teachers in mathematics in Switzerland, a research team developed an artefact using marbles, boards and, forms in order to let prospective teachers make better links between numeration and the classical algorithm of the multidigit multiplication. Teacher's knowledge underlying in this algorithm is the distributive and associative laws, the place value and, the definition of the multiplication (Clivaz & Deruaz, 2013; Ma, 1999). These authors highlighted errors due to "move over" the number on the second line of the algorithm. Moreover, the main source of errors is due to the presence of carries (Brousseau, 2010).

This study questions the potential effects of the use of an artefact using marbles, boards and, forms on students' errors in the algorithm of multidigit multiplication.

In a first experimentation, two prospective teachers used the artefact in order to help six pupils of 4th that have already learned, but that have difficulties to realise the algorithm of multiplication. We collect pupils' productions of a same test before and after using the artefact. In the second experimentation, two educators used the artefact in a class of 4th in order to teach the algorithm of multidigit multiplication for the first time. Data are the video of the lesson, pupils' productions and, the test, and an equivalent test in another class of 4th in the same school without using the artefact. In the first experimentation, the analysis of pupils' errors highlights a positive effect of the artefact on errors for four pupils on six. The second experimentation shows that with or without using the artefact, pupils do in average two errors to carry numbers and one pupil per class does errors in "moving over" the numbers. The analyse of pupils' productions also shows that pupils make links between the use of this artefact and the algorithm of multidigit multiplications. We conclude that the use of this artefact can have a positive effect on pupils' errors and on the comprehension of this algorithm.

References

Brousseau, G. (2010). Le calcul humain des multiplications et des divisions de nombres naturels. *Grand N, 85*, 13-41.

Clivaz, S. & Deruaz, M. (2013). Des mathématiques à leur enseignement, l'algorithme de la multiplication. *Grand N, 92*, 15-23.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

TEST-ENHANCED LEARNING IN STUDYING MATHEMATICS AT SECONDARY SCHOOL AND UNIVERSITY

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Retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material. Test-enhanced learning is a method which uses active recall of the information during the learning process (Dunlosky *et al*., 2013). It has been proved to be efficient concerning texts or foreign words, these experiments were principally carried out in laboratory-environment. We carried out two experiments on the efficacy of test-enhanced learning in mathematics.

In the first one, the experimental group were students of a 9^{th} grade class in an underprivileged vocational school. For five weeks they studied elementary geometry by test-enhanced learning. For the testing effect retrieval has to be done soon after the learning phase. In our experiment this was achieved by writing a 5-8-minute test on 2 problems on the topic of the actual lesson. There were two control groups studying the same topic on the standard way; one in the same school and the other one from an elite high school. Preliminary results were presented at PME 42 (Szabó & Szeibert, 2018).

We carried out our second experiment for four semesters on 103 pre-service math teacher students in the Algebra and Number Theory 1,2,3,4 courses at a university in order to examine the efficacy and long-term effect of test-enhanced learning in higher-level mathematics. Each semester half of the students – the experimental group – learned the subject using the testing effect and the other half – the control group – were taught in the usual manner. The last three semesters included more abstract material, polynomials, groups, rings, linear algebra. All the students wrote three midterms on each topic. The first one was in the usual time, during the semester. The second one was written six weeks and third one four months after the first test. There was no significant difference between the midterm scores of the control and experimental groups during these semesters. However, the two after-tests showed a big difference for the experimental group that was increasing with the time. Our results prove the long-term efficacy of testing effect in university-level mathematics.

References

Dunlosky, J., Rawson, K. A., Marsh, E. J., Nathan, M. J. & Willingham, D T. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest*, *14*(1), 4-58.

Szabo, C., & Szeiber, J. (2018). Efficiency of test-enhanced learning in teaching elementary geometry 2018. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, p. 296). Umeå, Sweden: PME.

CAUSAL RESEARCH FOR WEAK RESULTS IN CENTRAL EXAMINATIONS IN SECONDARY SCHOOLS WITH ANALYSES OF CLASS TESTS - PROJECT ELMA

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There are large performance differences in the central written exam mathematics for the completion of secondary school in different regions of a German federal state, which have been relatively stable for many years. The aim of the ELMA project was to find possible causes for this phenomenon in the school environment and at the level of teaching. To this end, 15 schools with widely scattering examination results were examined. A central element of the analysis were the class tests of the respective graduation classes 2017/2018, from which "valid conclusions can be drawn about the focal points of the lessons, since the practice of examination-relevant task types [represents] a special focus of mathematics lessons in Germany" (Kunter et al., 2006).

In the evaluation, qualitative and quantitative approaches were combined in order to obtain, within the framework of a reconstructive descriptive paradigm, as holistic a picture as possible of the complex problem area of "influencing factors on performance success".

In order to check the fit of the 4 to 5 class tests per class with the curricular standards and the final central examination, the tasks were examined with regard to the content-related and process-related competences required therein. The task quality of the class tests was described by an analysis of the task structure and the cognitive requirements. This is a prognosis of the expected empirical task difficulty due to the presumed solution approaches. The degree of formalisation F, the degree of complexity K and the execution effort A of the individual tasks are included in the weighted assessment of the empirical task difficulty.

The analyses show that a broader spectrum of competences is addressed in the class tests of the classes that pass the 2018 examination more successfully than in the classes that tend to score worse. In particular, various phenomena of a "Teaching to the Test" can be observed, which are documented in detail.

References

Kunter, M., Baumert, T., Blum, W., et al. (2006). Mathematics teaching in the PISA classes 2004: Framework conditions, forms and teaching-learning processes. In *PISA 2003: Studies on competence development in the course of a school year* (pp. 161-194). Münster: Waxmann.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

THE POTENTIAL OF METACOGNITION TO FAST-TRACK MATHS CATCH-UP INTERVENTIONS IN SOUTH AFRICA

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Research has shown that by Grade 9, 60% of South African Maths learners are functioning four grades behind their actual grade (Spaull & Kotze, 2015). This prevents most learners from studying Maths further, and limits the achievement of those that do. This study looks at the potential of metacognition to fast-track the catching-up of learning gaps among high school learners. Metacognition was measured by comparing learners' self-predictions with their actual performance using Kruger and Dunning's predictions linking competence, metacognitive ability and self-assessment (1999). Maths achievement was compared on the same pre- and post-intervention assessments for three small samples of high school learners (46, 15 and 23) who participated in different catch-up interventions. All three interventions were based on equivalent personalised formative feedback from the diagnostic assessment. Metacognitive skills were measured before and after using a scale based on the estimation patterns noted by Kruger and Dunning. Intervention 1, with an intentional focus on active teaching of metacognitive skills through the content, showed an average catch-up success of 41.7 percentage points (pp) over 12 months. Intervention 2, with a content focus and no structured metacognitive component, showed an average catch-up success of 11.7 pp over 12 months. Intervention 3, using personalised, printed content with integrated metacognitive development activities, showed an average catch-up success of 17.3 pp in only 4 months. Interventions 1 and 3 showed similarly larger improvements in metacognitive abilities when compared with Intervention 2. All learners showed a shift from over-estimating through an observable point of metacognitive activation to under-estimating before any closer estimation. This is an ongoing study with pre-intervention measures of Maths learning gaps, achievement and metacognitive ability currently collected for more than 5 000 high school learners. The hope is to leverage metacognitive development to fast-track catch-up Maths programmes in South Africa and simultaneously to develop independent, self-sustainable learners.

References

Kruger, J., & Dunning, D. (1999). Unskilled and unaware of it: How difficulties in recognizing one's own incompetence lead to inflated self-assessments, *Journal of Personality and Social Psychology. 77*(6), 1121-1134.

Spaull, N., & Kotze, J. (2015). Starting behind and staying behind in South Africa: The case of insurmountable learning deficits in mathematics, *International Journal of Educational Development, 41,* 13–24.

STAGES OF STUDENTS' INSTRUMENTAL GENESIS OF PROGRAMMING FOR MATHEMATICAL INVESTIGATIONS

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With the growing integration of programming in our mathematics classrooms, we see a crucial need to understand how (undergraduate) students may come to appropriate programming as an instrument (Rabardel, 1995/2002) for 'authentic' mathematical investigations, i.e., complete programming-based mathematical investigations "as mathematicians would do" (Weintrop et al., 2016). We are proposing to extend the instrumental integration model (Assude, 2007) to describe four stages of students' appropriation process (i.e., instrumental genesis):

- Stage 1. Instrumental initiation—student engages only in learning how to use the technology, i.e. *mainly* develops usage schemes of programming type, *e.g. scheme of writing a loop*;
- Stage 2. Instrumental exploration—math problems motivate the student to further learn to use the technology, i.e. *mainly* develops instrumented action schemes of programming type, *e.g. scheme of code remixing*;
- Stage 3. Instrumental reinforcement—student solves math problems with the technology, but must extend his/her technology skills, i.e., *mainly* develops instrumented action schemes of both intertwined programming/math and math types, *e.g. scheme of validating the programmed math* or *scheme of conjecturing*;
- Stage 4. Instrumental symbiosis—students' fluency with technology scaffolds the mathematical task, i.e., can with ease mobilize and further develop schemes needed for his/her mathematical work.

We illustrate our approach by analysing a student's instrumental genesis as he engaged in diverse activities as part of an undergraduate programming-based math course.

References

Assude, T. (2007). Teachers' practices and degree of ICT integration. In D. Pitta-Pantazi & G. N. Philippou (Eds.), *Proc. 5th Cong. of the European Society for Research in Mathematics Education* (pp. 1339-1348). Larnaka, Cyprus: University of Cyprus.

Rabardel, P. (1995/2002). *Les hommes et les technologies; approche cognitives des instruments contemporains*. Paris, France: Armand Colin.

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal for Science Education and Technology, 25*, 127–147.

ON RELATIONSHIP BETWEEN STUDENTS' LOGIC BASE ABILITIES AND CALCULUS STUDYING

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The present work reviews the relationship between students' proficiency in the basic logical apparatus, and their mastering of the calculus key concepts, in frame of the Math for Economists A1 course. We examine here particularly the influence on student's calculus concepts perception, from their comprehension of universality ($\forall$) and existence ($\exists$) logical categories, use of which in argumentation indicates (even for students completed the school math of 5 units) serious omissions (see Sriraman, 2012). In this framework, a group of 35 students was offered to solve several problems of the course exercise devoted to the function limits. The disputed problem required the proof or denial of several statements from the limits area, wherein:

- all propositions are defined as logical implications,
- preconditions represent several properties of generalized limits,
- conclusions represent some universal or an existential predicate associated with several altered limits, as illustrated (partly) at Fig. 1:

5. Prove or disprove the following propositios. If a proposition is true, prove it, if it is false, supply a counter-example:

A. If $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = \infty$, then it is possible that $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \infty$;

B. If $\lim\limits_{x\to\infty} f(x) = -\infty$, $\lim\limits_{x\to\infty} g(x) = \infty$, then $\lim\limits_{x\to\infty} f(x)\cdot g(x) = -\infty$;

Figure 1: The problem formulation (partly)

As the study showed, the students managed this logic-based problem significantly weaker than the calculative tasks (6 in total) of this exercise. The main reason was their inability to apply the logical categories of universality and existence (up to 50%), when trying to prove the truth or falsity of A as a general proposition, instead of finding a supporting (or disproving, respectively) example, and vice versa for the B sentence. After focusing students' attention and exercising in the qualitative difference between categories of universality and existence, as in logical predicates for all and exists:

- the comprehension increased sharply (fallibility declined from a half to 25%)
- the improved approach induced best calculus studying advances in general.

References

Sriraman, B. (2012). Crossroads in the History of Mathematics and Mathematics Education. *Monograph Series in Mathematics Education*, 12, IAP.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

USING DESIGN-BASED RESEARCH TO DEVELOP A PROFESSIONAL DEVELOPMENT FRAMEWORK FOR SOUTH AFRICAN MATHEMATICS TEACHERS

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This poster illustrates how I used a mixed methods design-based research (DBR) methodological approach to develop a professional development (PD) framework that can be used to support grade 9 South African mathematics teachers in the teaching of mathematical problem-solving (MPS). The first phase of the study involved a review of relevant literature, context analysis and a baseline investigation with 31 grade 9 mathematics teachers at 20 public secondary schools within a certain district in Gauteng, South Africa. During the baseline investigation, participant teachers completed an open-ended questionnaire. The purpose of the baseline study was to investigate participant teachers' views, teaching strategies and the support they required in their teaching of MPS. The baseline investigation revealed that teachers in the district of study had a partial understanding of the teaching of MPS and were teaching it using 'traditional' approaches that did not seem to support problem-solving processes in learners (Chirinda & Barmby, 2018).

Based on the findings from the baseline study, I developed the PD framework in phase two of the study through two design-enactment-evaluation-redesign cycles. These cycles took place in two successive years with a total of four grade 9 mathematics teachers and 211 learners (cycle 1: two female teachers and 115 grade 9 learners; cycle 2: one female and one male teacher and 96 grade 9 learners). I collected data through mixed methods data collection procedures. Each cycle was followed by data analysis and evaluation and refinement of the prototype. I analysed qualitative data through inductive data analysis techniques and quantitative data using SPSS.

Although limited to a particular context, the study developed a PD framework that can be modified and used by the South African Department of Basic Education. Furthermore, the study generated design principles that can be used by other researchers, mathematics education practitioners and teachers developing PD interventions to support grade 9 teachers in the teaching of MPS. As a result, the design principles that emerged from this study include design principles for PD in general and design principles specifically for PD for teaching MPS.

References

Chirinda, B. & Barmby, P. (2018). South African Grade 9 mathematics teachers' views on the teaching of problem-solving. *African Journal of Research in Mathematics, Science and Technology Education*, *22*(1), 114–124.

DISCIPLINE DIFFERENCES IN SPATIAL TRAINING: A COMPARISON OF RESULTS IN CALCULUS AND CHEMISTRY

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Encouraging evidence exists suggesting that one can train to improve spatial ability, using spatial training and explicit teaching of spatial skills, and that such training may help close the gender gap in spatial cognition. The research questions in this study were: Are differences present in calculus or chemistry ability and spatial ability between students who participated in spatial training and those who did not?

Data measuring calculus or chemistry ability and spatial ability (both pre and post) were collected from a treatment and comparison group of undergraduate students enrolled in a second-term calculus course and an organic chemistry course. Participants in the treatment group received spatial training during the term consisting of exercises from a spatial training workbook developed by Sorby et al. (2013) was used.

Findings in the calculus student data revealed no significant increase in calculus ability or spatial ability coupled with the spatial training. In both the control and treatment group, male participants' average spatial ability scores were higher than women's on both pre and post, however, they were not significantly higher. Results from the chemistry student data found that students, males had significantly higher spatial skills than females at the beginning of the term. However, at the end of the term, the gender differences on the test of spatial skills were no longer significant. Additionally, on average, students in both groups significantly increased their spatial ability during the course. These results are consistent with the majority of studies showing that spatial training can eliminate the gender gap, but inconsistent with those in our calculus study.

The spatial training effects were markedly different between the mathematics students and the chemistry students. The chemistry results show that students as a whole significantly increased their spatial skills, but that there were no significant differences in spatial skills between groups. This suggests that participation in the organic chemistry course may have been enough to decrease the gender gap in spatial skills. Future research may be needed to determine if discipline-specific spatial training may need to be developed to see large gains in spatial skills in a particular discipline.

References

Sorby, S., Casey, B., Veurink, N., & Dulaney, A. (2013). The role of spatial training in improving spatial and calculus performance in engineering students. *Learning and Individual Differences, 26*, 20–29.

SOMBRERO VUELTIAO – ENGAGING WITH MATHEMATICS THROUGH TRADITIONAL WEAVING ARTWORKS

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Following the theme of the conference "Improving access to the power of mathematics", I propose a poster about the weaving technique and traditional patterns of the *Zenú,* a Colombian indigenous tribe, and the mathematics behind it. The Zenú use this technique to create beautiful hats called *sombrero vueltiao,* which are one of the main handicraft products of Colombia, and very popular throughout the country and beyond (Puche Villadiego, 1984). Similar weaving artworks can be found in several cultures. Paulus Gerdes, a Dutch mathematician famous for being one of the founders of ethnomathematics research, wrote for instance about "interweaving arts and mathematics in Mozambique" (Gerdes, 2011).

The main questions I am interested in are if and how weaving techniques and patterns of indigenous tribes involve mathematics and in what way can that be used to engage people with mathematics?

The process of making a sombrero vueltiao, as well as the patterns and symbols that are woven are indeed mathematically very interesting and provide several learning opportunities. Concepts from different mathematical branches, such as geometrical concepts like angles, geometric shapes, patterns, tilings, and symmetries (see for example, Gerdes, 2014), combinatorical thoughts, such as counting patterns, and the concept of modelling and representations are involved.

The poster presents the weaving technique and patterns the Zenú use as well as related mathematical questions. Furthermore, it shows that the patterns and the technique are not at all random but are due to mathematical properties.

In conclusion, we will see that weaving arts is a wonderful example on how people can be engaged with mathematics in an ethnomathematical way and with that provide access to its beauty and power.

References

Gerdes, P. (2011). African Basketry: Interweaving Art and Mathematics in Mozambique. *Bridges Conference Proceedings*, (pp. 9-16). Coimbra: Portugal.

Gerdes, P. (2014). Ethnomathematics and Education in Africa. *ISTEG Belo Horizonte Boane Mozambique*, (pp. 45-56).

Puche Villadiego, B. (1984). El sombrero vueltiao zenú. *Universidad Nacional de Colombia:* http://bdigital.unal.edu.co/69843/1/El%20sombrero.pdf.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

MATH EDUCATION DEALING WITH MATHEMATICAL LEARNING DISABILITIES: A LITERATURE REVIEW

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In recent times, research interest in *mathematical learning disabilities* (MLD) has increased around the globe, but their definition and diagnosis does not enjoy a clear scientific consensus (Lewis & Fisher, 2016). Research regarding MLD is carried out in different fields, with various theoretical backgrounds and aims (Lewis & Fisher, 2016): cognitive sciences, neuroscience, psychology, mathematics education. We conducted a systematic literature review of the last 10 years of PME and CERME proceedings. We followed the main lines of the methodology used by Joklitschke, Rott, and Schindler (2018) for our study which is structured in 3 main steps: the identification of keywords about MLD, the selection of the founded articles following these keywords and a synthesis of the main features of these papers. Our research question is: How has the field of mathematics education dealt with MLD these last 10 years? According to our literature review, the keywords that identify MLD in math education are: *disab*, dyscalcul*, disord*, difficult*, inclus**. We used them to search articles for our review by focusing on titles, abstracts and keywords in proceedings of PME and CERME (2009-2018). We obtained 348 articles. After reading the articles, we retained 13 pertinent papers for our review about MLD. Our review shows that there is no consensus about the definition of MLD in mathematics education. In fact, the 13 articles can be subdivided into three classes according to the specificities of the participants of the experiments evoked in the articles: with diagnosis of MLD (often referred as dyscalculia, 4 articles), with learning disabilities with or without comorbidity (8 articles), with specific difficulties in learning mathematics but without diagnosis of MLD (6 articles). To conclude, it would be interesting to have a better understanding of research questions, definitions, theoretical frameworks and methods in research about MLD in math education.

References

Lewis, K. E., & Fisher, M. B. (2016). Taking Stock of 40 Years of Research on Mathematical Learning Disability: Methodological Issues and Future Directions. *Journal for Research in Mathematics Education*, *47*(4), 338–371.

Joklitschke, J., Rott, B. and Schindler, M., (2018). Theories about mathematical creativity in contemporary research: a literature review. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 171-178). Umeå, Sweden: PME.

EXPERT VS. NOVICE STRATEGY USE IN MULTI-VARIABLE INTEGRATION PROBLEMS

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Research examining students' success in introductory mathematics courses consistently shows that students are not learning the intended material (Apkarian & Kirin, 2017). Ongoing efforts to reform calculus instruction arise from concerns that students are learning calculus as simply a series of algorithms without conceptual understanding (Dawkins & Epperson, 2014). Such learning is problematic for students in multivariate calculus, where students need to be able to recognize and convert to appropriate coordinate systems to complete many multivariate integral problems.

This basic qualitative interview study (Merriam, 1998) was conducted using embodied cognition as a theoretical framework. The research questions for this study were: (1) What are the strategies used by expert and novice multivariate calculus students when solving multiple integration problems? (2) To what extent do experts and novices employ similar strategies?

The expert participants for this study were three tenured faculty members who had all taught multivariate calculus at least five times. The novice participants were six undergraduate sophomores and juniors who had recently completed a multivariate calculus course. All participants completed the same task-based interview of representative multiple integration problems. Analysis of the interviews revealed that experts work from a geometric interpretation to derive limits of integration, establishing a coordinate system first, while students work roughly in reverse: deriving limits and working on calculators first and only after encountering difficulties considering a picture or alternative coordinates, suggesting that experts and novices have fundamentally different integration strategies and that more time for instruction on the geometric interpretation of integration may be warranted.

References

Apkarian, N. and D. Kirin. 2017. Progress through calculus: Census survey technical report. https://www.maa.org/programs/faculty-and-departments/curriculum-development-resources/national-studies-college-calculus/ptc-publications.

Dawkins, P. C., & Epperson, J. A. M. (2014). The development and nature of problem-solving among first-semester calculus students. *International Journal of Mathematical Education in Science and Technology*, *45*(6), 839-862.

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education. Revised and Expanded from "Case Study Research in Education"*. San Francisco, CA: Jossey-Bass Publishers.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

EMPOWERING PRESERVICE TEACHERS TO MAKE SENSE OF SOLUTIONS IN PROBLEM SOLVING

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To enhance problem solving skills, an 8-step process was explicitly explained to 100 sophomore pre-service teachers taking probability and statistics. SPSS software was trained to plot different statistical graphs. A test involving non-routine, critical thinking, and multi-step questions was given to investigate their acquired problem solving skills. They were also asked to produce statistical graphs such as histogram, pie chart, frequency polygon, and boxplot using SPSS and to comment on the graphs obtained and to give conclusion about the findings. The result indicated that 90% could understand the problem, 80% devise a plan, 80% carryout the plan effectively, and 85% could use SPSS software to calculate the measure of variability and plot different statistical graphs. However, only 5% could check the solution and give the right interpretation of the results and the graphs.

Based on the findings of the test, further instruction focused on looking back at solution, checking the solution and the interpretation of the results from the graphs. A second test focussing on interpretation and drawing conclusion was given to the same participants. The results indicated that 75% could provide the correct interpretation while only 55% of the participants could make the right conclusion. Scores of participant's final exam administered at the end of the trimester were also analysed. Overall, the mean score was 72 and the standard deviation was 8.56. Participants who incorporated the 8-steps of problem solving scored higher in the final exam than those who did not

The study shows that empowering pre-service teachers with problem solving skills enhances their sense of making in mathematics (Ali, Hukamdad, Akhter, & Khan, 2010). Pre-service teachers should be trained to design their own investigation, formulate research questions, collect data, and make conclusions based on data. They should also be trained to communicate the findings.

References

Ali, R., Hukamdad, Akhter, A., & Khan, A. (2010). Effect of using problem solving method in teaching mathematics on the achievement of mathematics students. *Asian Social Science* (Vol. 6, pp. 67-72). Retrieved from: www. ccsenet.org/ass.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

PD FACILITATORS: ROLES, BELIEFS AND CHALLENGES

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Design principles of content-related professional development (PD) courses for in-service mathematics teachers have become a vivid research area. Although facilitators play an important role in scaling up PD programs (Jackson et al., 2015), research on the relevant processes is rare. In particular there is a lack of research into competences and strategies that facilitators need to provide rich learning opportunities for other teachers (Borko et al., 2014). Thus, this study focuses on the facilitators' role in the implementation of a PD program on differentiation in heterogeneous classrooms. We examine how PD facilitators perceive their own role as educators and facilitators in connection to their beliefs and goals. The main research questions addressed here are:

- How do facilitators see their specific role as educators and do they notice any challenges for improving their professional work?

The sample for this study consists of ten mathematics teacher facilitators who all have several years of experience in teaching and teacher training. Semi-structured interviews were coded using qualitative methods based on grounded theory. PD facilitators were asked how they design content-specific learning pathways for their PD sessions in relation to their beliefs concerning the teacher PD level. They were also asked about their views of their own role as educators and their goals in PD sessions, and about their perception of their own competences.

The results show that all facilitators considered appropriate learning activities and settings to be of major importance in prioritizing various adaptions of PD content resources. Facilitators mentioned several issues taking into account holistic challenges and multi-facet complexities of their PD sessions (e.g. community and context). Using research based textbooks and hands-on material was a major and central facilitator goal and a challenge to support in-service teachers with different forms of knowledge. Participants see themselves as mentors providing impulses for innovations rather than being problem solvers or process helpers. This is consistent with their beliefs on their role as facilitators. Overall, the results suggest that further connections between facilitators' competences, their role in adult education, their self-perception, beliefs and efficacy should be identified in future research.

References

Borko, H., Koellner, K. & Jacobs, J. (2014). Examining novice teacher leaders´ facilitation of mathematics professional development. *Journal of Mathematical Behavior*, *33*, 149-167.

Jackson, K., Cobb, P., Wilson, J., Webster, M., Dunlap, C., & Appelgate, M. (2015). Investigating the development of mathematics leaders' capacity to support teachers' learning on a large scale. *ZDM Mathematics Education, 47*(1), 93-104.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

DIFFICULTIES IN UNDERSTANDING OF DERIVATIVE CONCEPT: A LOOK FROM THE APOS THEORY

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The Calculus is an area of mathematics that accumulates the elementary knowledge of both Algebra and Geometry and corresponds to the starting point for the development of university mathematics and its applications. In this course, the derivative is one of the fundamental concepts and is considered as an essential tool in the study and understanding of phenomena that involve the change or variation of magnitudes in different areas of science (Vrancken & Engler, 2014).

In this study, we focus on the analysis of student errors in order to identify possible difficulties. To do this, we have considered the framework proposed by APOS theory (Arnon et al., 2014), through the use and logical connections that university students established between mathematical elements and representation modes when solving tasks about the derivative concept (Sánchez-Matamoros, García & Llinares, 2006). The participants of this study were 103 university students of a course of Calculus of the undergraduate program of engineering. The data of this work was collected during the first semester of the year 2018 and corresponded to the students' productions obtained through the application of a questionnaire. This questionnaire was composed of three tasks that address different aspects associated with the derivative concept. The application of the questionnaire took approximately 90 minutes.

The analysis and classification of the information provided by the questionnaire, under the framework of the APOS theory, allowed to identify some difficulties associated with the understanding of the derivative concept which mainly related to the construction of processes reversal.

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References

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S. R., Trigueros, M., y Weller, K. (2014). *APOS theory: A framework for research and curriculum development in mathematics education*. Berlin: Springer.

Sánchez-Matamoros, G., García, M., & Llinares, S. (2006). El desarrollo del esquema de derivada. *Enseñanza de las Ciencias*, *24*(1), 85–98.

Vrancken, S., & Engler, A. (2014). Una introducción a la derivada desde la variación y el cambio: resultados de una investigación con estudiantes de primer año de la universidad. *Bolema*, *28*(48), 449-468.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

MAINTAINING MATHEMATICS TEACHING PRACTICES IN ONLINE TEACHER EDUCATION: AN EXAMPLE USING BLACKBOARD COLLABORATE, LOGARITHMS AND PH

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Online mathematics teacher education is becoming a more popular option due to the many affordances it allows teachers and teacher education. However, research is needed to demonstrate how online mathematics teacher education can be conducted in ways that uphold research-based practices. For example, the National Council of Teachers of Mathematics (NCTM) describe eight research-based essential teaching skills, called Mathematics Teaching Practices (MTPs), that can be difficult to enact in online settings (e.g. facilitating meaningful mathematical discourse). Additionally, teachers need to experience MTPs as learners (NCTM, 2014), especially when learning historically difficult mathematical concepts such as logarithms and connecting logarithms to other concepts and disciplines (Park & Choi, 2013).

This poster details a lesson designed to develop middle and secondary teachers' mathematics content knowledge of logarithms and science content knowledge concept of pH. The theoretical background of the MTPs was used to redesign a face-to-face lesson and implement it in an online format using the platform Blackboard Collaborate. The lesson was implemented with 21 teachers in an online graduate content course meeting twice a week for six weeks on Blackboard Collaborate. The research question for this study was: in what was did the integrated lesson maintain MTPs in both planning and implementation?

The poster answers the research question using descriptions and screenshots of the lesson and the implementation on Blackboard Collaborate as well as verbal and written data from teachers in the course. For example, a laptop placed by the poster will show a short video of Blackboard Collaborate demonstrating how meaningful mathematical discourse was facilitated between the teachers using synchronous audio and video in breakout groups. Finally, the poster includes recommendations for other teacher educators wishing to engage teachers in online learning that supports mathematics education as well as how modeling the MTPs in online settings for teachers might help them maintain MTPs when working with K12 students in online or hybrid settings.

References

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

Park, E. J., & Choi, K. (2013). Analysis of student understanding of science concepts including mathematical representations: pH values and the relative differences of pH values. *International Journal of Science and Mathematics Education*, *11*(3), 683-706.

MATHEMATICAL LEARNING DISABILITIES IN EARLY ALGEBRA

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Mathematical learning disabilities (MLD) is a research topic which relates to different fields, such as cognitive science and mathematics education. These two research domains have complementary standpoints.

A shared definition of MLD does not exist. The main definitions with their different points of view will be presented in the poster presentation, and a way to overcome the inconsistencies will be discussed. In cognitive science, MLD are defined as *"a biologically based difference in the brain, which results in significant difficulties with mathematics"* (Lewis & Fisher, 2016, p.338). On the other side, math education often includes students with specific and persistent difficulties in maths, that is, those which are identified as being the worst in maths within a certain group the category of MLD students (Pfister, Opitz & Pauli, 2015). With the aim to link the two fields, my study considers both definitions: students have MLD if they possess a diagnosis or are identified by the school as having specific and persistent difficulty in maths.

87% of the research on MLD are about arithmetic, although MLD are heterogeneous (Lewis & Fisher, 2016). Research on MLD in other mathematical domains is nowadays necessary. The study presented in this poster investigates the difficulties of MLD students in early algebra and the possible remediation. With this objective, activities of patterns generalisation are used (Radford, 2010). Research questions are: *Which kind of difficulties in early algebra do pupils with MLD have? How to discriminate between MLD and "regular" difficulties in algebra?* The hypothesis is that the nature of the difficulties is the same for MLD and not MLD students. What will discriminate between them is the durability of the difficulties: pupils with MLD need more time to acquire algebraic skills. If typical MLD difficulties do not exist, neither would typical MLD solutions.

References

Lewis, K. E., & Fisher, M. B. (2016). Taking Stock of 40 Years of Research on Mathematical Learning Disability: Methodological Issues and Future Directions. *Journal for Research in Mathematics Education*, *47*(4), 338–371.

Pfister, M., Opitz, E. M., & Pauli, C. (2015). Scaffolding for mathematics teaching in inclusive primary classrooms: a video study. *ZDM*, *47*(7), 1079–1092.

Radford, L. (2010). Algebraic thinking from a cultural semiotic perspective. *Research in Mathematics Education*, *12*(1), 1–19.

FROM FIGURAL PATTERN TO DRAWING PATTERN

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In this study, a group of first and second grade elementary school students were observed while working with pattern blocks, flagging tape, equilateral triangles, wooden cubes, dice and muggle stones. The children worked in pairs formed according to mixed ages and abilities and collectively performed a variety of tasks related to repetitive and growing pattern sequences. Previous investigations in this area show that even young children at the age of seven or eight are able to create and document repetitive patterns and sometimes they're even capable of describing their structures (Warren et al., 2005).

An empirical study was conducted. The data of this study relied on qualitative analysis of students' documentations collected by using videography. The associated leading question was: Are 7 to 8-year-old children able to document patterns they previously made from pattern blocks or other learning material and if so, what strategies do they choose? The interpretation of the video data was supported by using a selection of individual images clearly and visually emphasizing central moments of interaction (Moritz, 2010). For further analysis, the patterns were photographed and then compared with the students' documentations. The analysis was supported by a self-developed categorical system to enable a comparison of the results (Oswald, 1997).

The results of the study show that both kindergarten and elementary school children are able to document their patterns. Especially the pattern blocks, flagging tape, muggle stones and equilateral triangles seemed to be well suited for documentation. Wooden cubes and dice, however, proved to be more difficult for children to use. Freehand drawing is the most frequently chosen documentation style out of all three alternatives. It is important to know that the children start off by tracing the materials before actually drawing freehandedly. Interestingly enough, the documentation of students' work in the area of mathematics seemed to have an impact on other school subjects in primary school as a side effect.

References

Moritz, C. (2010). *Dialogische Prozesse in der Instrumentalpädagogik: Eine Grounded Theory Studie*. Essen, Ruhr: Die blaue Eule.

Oswald, H. (1997). Was heißt qualitativ forschen? Eine Einführung in Zugänge und Verfahren. In Friebertshäuser, B. & Prengel, A. (Hrsg), *Handbuch qualitativer Forschungsmethoden in der Erziehungswissenschaft* (S. 71-78). Weinheim: Juventa.

Warren, E. (2005). Young children's ability to generalize the pattern rule for growing patterns. In H. Chick & J.L. Vincent (Eds). *Proceedings of 29th PME* (Vol. 4, 305-312). Melbourne: PME.

WHAT CAN BE LEARNED ABOUT STUDENTS' COMMUNICATION FROM GAZE-TRACKING?

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Increasingly, studies aim to understand how students' identities are constructed communicationally within the mathematical activity (Heyd-Metzuyanim & Sfard, 2012). We present the communicational information that can be gained about an episode of mathematical problem solving from multiple advanced recorders, including multiple video cameras, Smartpen recorders and gaze tracking mobile glasses. Gaze tracking provides a window to what the student is attending to during interaction (Haataja, Garcia Moreno-Esteva, Toivanen, & Hannula, 2018). We ask – what do each of the recording devices add to our understanding of the interaction between subjectifying (communication about participants) and mathematizing (communication about mathematical objects)?

Data included stationary videos from three perspectives as well as Smartpen, and gaze recordings on a collaborative geometrical problem solving session of four Finnish 9th grade students, taking place in the students' authentic mathematics classroom.

Findings show that understanding the episode was crucially based on each of the different recording mechanisms. Subjectifying communication was largely achieved by facial expressions, accessed by cameras pointing at students' faces. Through these, we gained access to social conflicts which were hardly evident in other channels. Mathematizing could be split into inter-communicational and intra-communicational activity (thinking). Inter-communicational activity was achieved in this geometrical activity mostly by pointing and by deictic markers (e.g "here", "this"), for which the pen and gaze data were crucial. Gaze data gave indications of intra-communicational activity. Through it, we were able to see the detrimental effects that the subjectifying activity had on the mathematizing activity in terms of missed opportunities for advancing students' reasoning about the task. The different recording channels thus gave access to unique mathematizing and subjectifying information.

References

Haataja, E., Garcia Moreno-Esteva, E., Toivanen, M., & Hannula, M. S. (2018). Teacher's gaze behavior when scaffolding peer interaction and mathematical thinking during collaborative problem-solving activity. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 475-482). Umeå, Sweden: PME.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51*, 128–145.

MATHEMATIZING, VISUALIZING, AND POWER (MVP): STUDENTS CREATING STATISTICAL LITERACIES THROUGH POPULAR REPRESENTATIONS

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Mathematics, and statistics in particular, is often taught as a set of definitions without a deep understanding of the application of the concepts (McGatha, Cobb, & McClain, 2002; Bajak, 2014). Real world scenarios are presented to students. However, they often lack the connection to students' cultural and linguistic resources. In this study, we conduct exploratory research toward the development of a model learning environment that promotes statistical literacies for student and their communities. In MVP, students from historically underrepresented groups in mathematics create data visualizations designed to engage members of the community in discussions around topics of interest. The research focuses on two questions:1) Within an integrated statistical design arts pedagogy, what concepts and practices do participants learn? 2) What do community members learn from interacting with the visualizations? The project began in January of 2019 is currently ongoing. Each week, the project team engages ten-year-old students in activities that involve data collection, data analysis, and creating data visualizations. These 45-minute sessions include the authors, the art teacher, and the fourteen student participants. The sessions take place in the art classroom of an elementary school located in an urban setting in the Southeast United States. The overarching goal of the project is to support students in summarizing data through their own visualizations. Preliminary findings show that students are able to explain the rationale behind their visualizations, and most of these explanations are grounded in their data analysis. Students' visualizations reflect additive and multiplicative thinking. In addition, students' visualizations consider size, shape, and color primarily while considerations of density are more challenging. Data related to the learning of the community will be collected and preliminary findings shared during the poster session.

References

Bajak, A. (2014) *Lectures aren't just boring, they're Ineffective, too, study finds*. Retrieved fromhttp://www.sciencemag.org/news/2014/05/lectures-arentjust-boring-theyre-ineffective -too-study-finds.

McGatha, M., Cobb, P., & McClain, K. (2002). An analysis of students' initial statistical understanding: Developing a conjectured learning trajectory. *Journal of Mathematical Behavior, 21*, 339-355.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

MATHEMATICAL LEARNER IDENTITIES OF LEARNERS WHO PARTICIPATED IN AFTER-SCHOOL MATHEMATICS CLUBS

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This research explores the mathematical learner identities of high school learners who participated in after-school mathematics clubs during their primary schooling. In response to a range of challenges in mathematics education in South Africa, the South African Numeracy Chair Project established a number of these clubs for Grade 3 learners in 2012. The clubs prioritised enabling learners to think independently and to develop the enjoyment of a mathematical challenge and sought to shift the passive learning identities that were found to be highly prevalent in local schools (Graven & Heyd-Metzuyanim, 2014).

The goal of the research to be presented in this poster is to understand the mathematical learner identities of high school learners who share the common experience of having attended an after-school mathematics club as younger learners. For the purposes of this study, identity is defined as “the set of stories people tell about themselves and others tell about them” (Sfard & Prusak, 2005, p.16). Aligned to this, the data gathered will be the learners’ own stories of their mathematical learning trajectories, as well as the stories told about the learners by their club facilitator and their current mathematics teachers (aligned to the triple BAC model proposed by Sfard and Prusak, 2005).

The stories will be analysed with reference to Wenger’s (1998) modes of belonging: imagination, engagement and alignment. These learners have passed through numerous mathematical learning communities over their schooling years, of which the mathematics club represents only one. As “the most significant stories are often those that imply one’s memberships in, or exclusions from, various communities” (Sfard & Prusak, 2005, p.44), these three modes of belonging are expected tol be reflected in the stories the learners tell.

References

Graven, M. & Heyd-Metzuyanim, E. (2014). Exploring the limitations and possibilities of researching mathematical dispositions of learners with low literacy levels. *Scientia in Educatione, 5*(1), 20-35.

Sfard, A. & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14-22.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

COGNITIVE DOMAINS OF PROBLEMS IN KOREAN MATHEMATICS TEXTBOOKS

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Materials that are commonly used in schools are textbooks. Most of the lessons focus on textbooks, so the sequence of textbooks is the order of the lessons presented to students. Textbooks for students can be considered to have duality as well as students' concept formation and misconception. In addition, the problem that the teacher provides in the textbook becomes a tool to observe the situation of the student by observing the process of solving the problem.

Recently, mathematics education emphasizes mutual consistency in the three-way relationship of mathematics curriculum, mathematics textbook, and mathematical evaluation problem (Ministry of Education, 2015). Therefore, it would be of great significance to examine the problem of the textbook reflecting the newly revised curriculum.

In this study, ten kinds of mathematics textbooks reflecting the newly revised curriculum were selected and the problems presented in the texts and 'Character and the expressions' section of this textbook were analysed based on Bloom’s taxonomy. Bloom (1954) has classified educational learning objectives into levels of complexity and specificity which are the cognitive domain, affective domain, and sensory domain. The cognitive domain has six stages (knowledge, comprehension, application, analysis, synthesis, evaluation) assuming that the cognitive ability of the learner is hierarchical. As a result of the study, it was found that the comprehension stage is the most significant part in the textbook, followed by knowledge. On the poster, further results will be presented in detail.

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References

Bloom, B. S. (1954). *Taxonomy of Educational Objectives, Handbook 1: Cognitive Domain* (2nd edition). New York: Longman.

Ministry of Education. (2015). *The 2nd Mathematics Education Comprehensive Plan*. Ministry of Education Press Release.

THE WAY IN ARGUMENTATIVE MATHEMATICAL DISCUSSIONS

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Argumentative discussions in classrooms are the purpose for learning mathematics. And we ought to judge the quality of the lesson by it. This contributes to the improvement of the lesson study indirectly. Because lesson study aims at the improvement of the lesson and suitable evaluation of the lesson is indispensable to it. Argumentative discussion leads to the interaction in the classroom. In this paper, it is the purpose that we present the actual condition of suitable evaluation of mathematics lessons.

Saxe et al. (2009) paid attention to "travel of ideas; the interaction between each idea of the student and the whole class community". Surely, mathematical communication is convenient method to describe the mathematical discussion. On the other hand, Ofri and Tabach (2017) developed a methodological approach to document collective activity (DCA) and DCA methodology uses Toulmin's model of argumentation as an empirical tool. Toulmin (1964) pointed claim(C), data(D), warrant(W), qualifier(Q), rebuttal(R), and backing(B).

We analysed the episode of lessons. The episode which I describe here took part in a mathematics lesson in the second grade of junior high school in Japan. The topic of this lesson was the sum of the interior angles of the polygon. The teacher’s aim was for students to develop the fact that the sum of the interior angles of an n-cornered polygon is 180(n-2). Individual solutions and preparation of the argumentation are summarized in it. We drew the following conclusions through analysis. It analysed using the model about the discussion in this lesson. As a result, it was shown that gap of students' meaning actualizes in discussion. And therefore, it was shown that it is effective to hide a part of figure expression which shows a view.

References

Saxe, G. B., Gearhart, M., Shaughnessy, M., Earnest, D., Cremer, S., Sitabkhan, Y., Platas, L., & Young, A. (2009). A methodological framework and empirical techniques for studying the travel of ideas in classroom communities. In B. B. Schwarz, T. Dreyfus & R. Hershkowitz (Eds.), *Tranformation of knowledge through classroom interaction* (pp. 203-222). London, UK: Routledge.

Ofri. O. & Tabach. M. (2017). The spread of mathematical ideas in argumentative classroom discussions: Overt and convert participation. In Kaur. B., Ho, W.K., Toh, T.L., & Choy, B. H. (Eds), *Proc.41th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol, 3, pp. 329-336). Singapore: PME.

Toulmin, S. E. (1969). *The uses of argument*. Cambridge: Cambridge University.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

PEER LEARNING OF GEOMETRIC CONCEPTS BY PRIMARY SCHOOL LEARNERS IN A HOMEWORK CLUB

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According to Freudenthal (1973) Geometry is grasping that space in which the child lives, breathes and moves. Furthermore, Clements and Sarama (2011) contend that the quantitative, spatial, and logical reasoning competencies of mathematics may form a cognitive foundation for thinking and learning across all other subjects. In particular, this paper responds to the question: how can peer learning in a homework club assist primary school learners to acquire geometric concept knowledge? Peer learning refers to productive learning that connects behaviours with foundation phase curriculum content by actively applying relevant strategies during the peer cooperative learning session in the homework club. The study draws from both the variation theory that captured learners' ways of seeing or experiencing a particular object/concept, and the van Hieles' theory of geometric thought. A convenient sample of 24 grade 1-3 learners, in groups of four, participated in a qualitative study in which structured worksheets covering mathematics content for grade 1-3 were given to the learners to practice after school in a homework club. The worksheets covered content on shifts in learners' understanding of geometric shapes. Data were collected through classroom observations, semi-structured interviews together with structured worksheets.

The observed practice after the participants had been issued with different paper cut shapes, revealed that some peers assumed the instructors' role. Learners who operated below the visualization level could only sort the given shapes according to colour. Classroom discourse through argument that assisted learners to make appropriate connections between the definitions of a geometric concept and its representation was observed. During interviews some learners indicated that they learnt properties of the different shapes from their peers. Through further peer-assisted learning and engagement learners can contrast, discern and separate geometric figures and further identify them in their immediate environment. Based on the results, peer-assisted learning in a supportive classroom environment enables learners to support and challenge each other's strategic thinking. During interviews some learners indicated that they were free to ask and challenge each other as peers while their knowledge of geometric concepts was enhanced.

References

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education, 14*, 133–148.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Reidel.

STUDENT TEACHERS' CORE VALUES: A CASE OF THE MATHEMATICS EDUCATION PROGRAM

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Initial teacher education also faces issues associated with transforming the identity of student teachers from that of student to that of teacher (Ponte & Chapman, 2008). In Thailand, during the last 3 decades, the initial teacher education program has undergone many changes. However, the most crucial problem among every model of the program is the spirit of the teaching profession. Khon Kaen University, Thailand, initiated a new model of teacher education based on Inprasitha's ideas (2013) which were driven by four core values (Inprasitha, 2013). This 5 year Mathematics Education Program aims to build the spirit of the teaching profession through participating in core value activities from year 1 to year 5. These core value activities have been designed by weaving together 3 steps of Lesson Study. In the final year, student teachers practice teaching with lesson study and the open approach in weekly cycle. The research focuses on determining the student teachers' perceptions about the 4 core values according to Inprasitha (2013). The participants were 124 student teachers in the mathematics education program, at Khon Kaen University during the 2017 school year. Data was collected by questionnaires after their completion of practicum. Data was analyed by descriptive statistics and content analysis.

The results revealed 100% of student teachers perceived each value. The four values are: 1) Building Collaboration: they valued students' working together and recognized the need to compromise opinions arising from mutual discussions while planning and reflecting together; 2) Open-minded Attitudes: they valued listening to comments from their peers, and they valued waiting for students to understand the problem/situation by themselves and gave students the opportunity to think by themselves and explain their ideas; 3) Public Concern: they valued supporting student learning in the class and taking care of all students, and they could help with school activities and; 4) Emphasis on product-processes approach: they valued thoroughly planning, anticipated students' ideas and patiently waited for students to solve their own problems.

References

Inprasitha, M. (2013). Innovations and Exemplary Practices in Teacher Education Program in Thailand. In M. Inprasitha (Ed.), *Proc. 6th East Asian Regional Conference on Mathematics Education EARCOME 6*: (Vol. 1, pp. 32-43). Phuket, Thailand: EARCOME.

Ponte, J. P., & Chapman, O. (2008). Preservice mathematics teachers' knowledge and development. In L. D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century* (2nd ed., pp. 225–263). New York: Routledge.

THE EXTENT TO WHICH MATHEMATICS TEACHERS ENABLE OR CONSTRAIN DEEP CONCEPTUAL UNDERSTANDING

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This research explores the extent to which FET mathematics teachers who participated in a Mathematics Teacher Professional Development Programme (MTPDP) scaffolded their questioning to promote deep conceptual understanding. This study emanates from my PhD study which explored how MTPDP participation encouraged teachers to promote conceptual teaching. Conceptual teaching is defined as teaching aimed at promoting conceptual understanding, productive maths talk, visualization, effective use of manipulatives and demonstrates positive self-efficacy. The results of the PhD study showed that these teachers grew professionally and they partially embraced and implemented a conceptual teaching approach. Through this study I would like to explore the impact of questioning in scaffolding deep conceptual understanding.

Key features that define deep conceptual understanding involve richly connected knowledge, the generality and breadth of knowledge application (Baroody, Feil, & Johnson, 2007). Data used is drawn from interviews and classroom observations of five purposefully selected teachers over a three-year period. Vygotsky's theory of Zone of Proximal Development (ZPD) will be used as a theoretical and analytical tool to understand how questioning was used to scaffold deep conceptual understanding. Effective scaffolding is able to extend the upper limit of the ZPD, making it possible for learners to reach beyond what they are thought to be capable of (Hammond & Gibbons, 2005). The findings suggest that using questioning to scaffold goes beyond asking the what, how and why questions and instead involves managing learner response to extend their current level of understanding to mutually constructed deep understanding. Scaffold should be removed when learner's responses provide evidence that they have clearly understood maths concepts taught. Teacher's intervention through questioning should extend beyond supporting learners with what to think into how to manage thinking and deduce meaning for themselves. Implications of the research are that MTPDP's, pre-and in-service need to support teachers to challenge learner's thought construction process through focused and intentional questioning that develops deep conceptual understanding.

References

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education,* 115-131.

Hammond, J., & Gibbons, P. (2005). Putting scaffolding to work: The contribution of scaffolding in articulating ESL education. *Prospect, 20*(1), 6-30.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

TOUCHY FEELY VECTORS - MATERIAL EXPERIENCES OF GEOMETRICAL REPRESENTATIONS OF VECTORS

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Studies of student learning of vectors report students' struggle with graphical methods and their comfort with ijk components. Given the embodied nature of human cognition, there is a possible link between this conceptual behaviour and the interactions supported by different materials. We explored the nature of mathematical experiences produced by different materialities, such as paper-pencil media and a dynamic geometric environment called Touchy Feely Vectors (TFV), an interactive system designed by us. TFV makes geometric representations of vectors tangible similar to Touchcounts (Sinclair & de Freitas, 2014), allowing active manipulations on geometrical representations to add and resolve 2D vectors, which is normally not easily supported by paper-based media. Interactions in TFV-1 (bit.ly/tfv-1) were based on mouse clicks and key presses, while TFV-2 (bit.ly/tfv-2) was based on touch.

In pilot phase, 6 students grade-11 performed tasks using TFV-1 and paper-pencil for ~90 minutes. Later, we observed 3 grade-11 classrooms (129 students), where 3 teachers executed lesson plans (designed collaboratively with the TFV development team) using TFV-2. Students (in groups of 2-3) did tasks using tablets and paper-based worksheets. All sessions were video recorded. Student approaches to a task to find 2 vectors, whose resultant is a target vector (magnitude 60 at 40° from the x axis) were:

- Trial & error (the most common approach, where students manipulate the magnitude and direction of vectors randomly).
- Calling x and y components estimated using paper-pencil as 2 vectors. These students struggled when asked to create another set of vectors without TFV.
- Creating two vectors with magnitude 30 at 20° from the x-axis, and then adding to find resultant of magnitude 60 at 20° instead of at 40°. This challenges direct algebraic addition of magnitude and direction.
- Creating a vector of magnitude 60 at 40° from the x-axis, and arguing that the other vector is a zero vector. One student pair tried this for a long time using trial and error, and eventually applied the idea of zero vectors.

A student (who used both TFV-1 and paper-pencil) remarked "this (pointing at paper-pencil) is theory and this (pointing at TFV-1) is experiment". Such explications and the above diverse approaches indicate how DGEs like TFV can provide different material experiences of geometric representations of mathematical entities, further changing student conceptual behaviour, details of which shall be part of the poster.

References

Sinclair, N., & de Freitas, E. (2014). The haptic nature of gesture: Rethinking gesture with new multitouch digital technologies. *Gesture*, *14*(3), 351–374.

ELEMENTARY TEACHERS' LEARNING IN THE CONTEXT OF A PROFESSIONAL DEVELOPMENT PROJECT IN RURAL APPALACHIA

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Research on teacher learning in mathematics emphasizes the importance of engaging teachers in discourse communities that dive deeply into issues of content and pedagogy (Lampert, 2001). Through the presentation of open tasks, we sought to support elementary teachers' development of content understanding and discourse practices in a professional development project. In addition to prompting teachers' engagement with math-focused tasks, we engaged teachers in sessions focusing on discourse practices in which they were positioned as experts of their students. This *decentering* approach involved asking teachers how their students would react to the focus discourse practices and how they would modify these practices with their students in mind. Regarding this approach, we had one central question: Did teachers view these discourse practices as important aspects of teaching mathematics?

A two-week Summer Math Institute took place during each year of the three-year project. Each Institute focused on the notion of *Number Talks* (Parrish, 2014) in supporting teachers' learning about discourse-in-practice. The project team collected surveys during each Institute. These preliminary findings are based on the year three survey data and offer a glimpse of participants' ideas related to the central research question. The majority of the participants noted that they implemented discourse practices or number talks on a regular basis. Specifically, 52.3% said they implemented number talks one to two times a week, 13.3% said three to four times a week, and 6.7% said five or more times a week. The remaining teacher did not implement number talks on a regular basis. Importantly, 63.3% of the teachers indicated that they were not implementing number talks as often as they would like. Based on these preliminary findings, we propose that this decentering approach toward discourse practices is productive in supporting teachers in developing an appreciation and understanding of discourse in mathematics lessons while considering students' challenges and assets in the implementation of whole-class and small-group discussions.

References

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Parrish, S. (2014). *Number Talks*. Sausalito, CA: Math Solutions.

THE CHARACTERISTICS OF THE PUPILS' DRAWINGS FOR SOLVING WORD PROBLEMS

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The purpose of this study is to show the characteristics of pupils' drawings for solving word problems. We consider drawings on questionnaires by pupils from grade 1 to 3 in elementary school, based on the theory of mathematical expressions by Nakahara (1995) and the process of solving word problems by Mayer (1992) as a theoretical framework.

THEORETICAL FRAMEWORK

Nakahara (1995) organized illustrative representations used in mathematics education including drawings in mathematics textbook for elementary school. And, Mayer (1992) showed the process of solving word problems consisted of two steps: Problem Representation and Problem Solution. Based on these, we discuss diagram categories in the mathematics textbook for elementary school and a relation between Mayer's study (1992) and a flow of teaching/learning calculations.

CONCLUSION

We show characteristics of the pupils' drawings for solving word problems as next. First, many pupils who can answer correctly tend to represent the mathematical scene diagram or the structure diagram. They can pass correct steps of Mayer's process of solving word problems because they can utilize these diagrams which are useful to grasp quantitative relations and formularize algebraic expression. Next, we classify pupils who cannot answer correctly into two cases; (1) they cannot represent the problem scene by drawings, and (2) although they can represent the mathematical scene diagram or the structure diagram, they cannot formularize correct algebraic expression. In (1), there are further two cases that pupils cannot grasp the problem scene, or they cannot grasp quantitative relations. They have the difficulty in the "Problem representation". In (2), they cannot combine their drawings with algebraic expression. They have difficulty in going back and forth between "Problem representation" and "Problem solution". In other words, each pupil who cannot answer correctly has different difficulties.

References

Mayer, R., E. (1992). *Thinking, problem solving, cognition*, New York: W. H. Freeman and Company.

Nakahara, T. (1995). *The Study of Constructive Approach in Mathematics Education*. Japan: Seibunsha Publishing, in Japanese.

VARIATIONS OF UPTAKE RELATED TO TEACHERS' PARTICIPATION IN PROFESSIONAL DEVELOPMENT

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Studies have shown small changes in teacher knowledge and practice after attending professional development workshops, but researchers are puzzled as to why the growth over time is not greater. Clarity may rest in mixed method studies of teacher learning. The research questions for this study include: What is the nature of what teachers take up and use after participating in professional development workshops? What factors influence what teachers take up and use? This study is conceptually grounded by the theoretical perspectives of professional vision (van Es, 2011) and the knowledge of mathematics for teaching.

Our poster is based on an efficacy study of the Learning and Teaching Geometry PD (NSF 1503399). Participants consisted of 103 teachers (49 treatment and 54 control). Treatment teachers took post PD surveys that elicited their perceptions and self-reported uptake of the content and pedagogy. Teachers were videotaped on three occasions to collect pre - post - post video of their teaching. Nine teachers agreed to be case study teachers for additional data collections. Our qualitative analysis triangulated the ratings of video lessons, reflections and surveys.

Repeated measures ANOVA identified shifts from pre to post PD in teachers' practice. Teachers who attended the LTG PD intervention made significantly greater improvements in their instruction relative to the comparison teachers on two of the broad constructs (richness of the mathematics and engaging students in mathematical practices).

We present three case studies of teachers that showed differing levels of uptake of content and pedagogy. The following aspects appear to be associated with uptake: 1) ability to make connections to their curriculum, 2) school contexts, and 3) teachers' beliefs about teaching and learning.

References

van Es, E. A. (2011). A framework for learning to notice student thinking. In M. Sherin, V. Jacobs, & R. Philipp (Eds.), *Mathematics teacher noticing: seeing through teachers' eyes*. (pp. 164-181). Routledge.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

EMPIRICAL FRAMEWORK TO ASSESS STUDENTS' MATHEMATICAL EXPLANATIONS: FORMING A MATRIX FOR MATHEMATICAL PROOF SKILLS

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In Japanese junior high schools, students begin learning mathematical "proof" in Grade 8 (aged 13–14). Of course, students have many opportunities to "explain" the reasons why things are correct in elementary school mathematics before then. However, the current situation is that these experiences of student explanations before 8th Grade do not effectively connect to the matrix formation for mathematical proof skills. This is partly due to the lack of a clearly defined framework appropriate for assessing students' explanations. Consequently, such assessments are made based on the subjectivity and individual values of each teacher. Hence, the following point is the research question.

> How can we create a framework to assess students' explanations to form a matrix for mathematical proof skills?

I considered the assessment framework from the perspective of the "Deductive-Nomological explanation" (D-N explanation) by Hempel (1965), which was advocated as one of the basic concepts of scientific explanations. Accordingly, I created a framework that assesses students' explanations by dividing them into a total of 5 types: 4 types comprised of the 2x2 of the presence or absence of "α axis: objectivity of the explanation's components," and the presence or absence of "β axis: connections with the law that determine the conclusion of the proposition," and one non-response. I then assessed accounts of 5th to 9th graders' explanations (1,121 students) under this framework and identified the distribution of students by type in each grade. Additionally, statistical analysis based on the concept of the "Common Cognitive Path" by Vinner & Hershkowitz (1980) enabled me to identify the existence of a Common Cognitive Path from growth along the α axis to growth along the β axis, common to all 5th to 7th graders who have yet to learn mathematical proof.

References

Hempel, C. (1965). *Aspects of science explanation and other essays in the philosophy of science*. G, Nagasaka, trans. (1973). Tokyo: Iwanami Publishers.

Vinner, S. & Hershkowitz, R. (1980). Concept images and some common cognitive paths in the development of some simple geometric concepts. *Proceedings of the 4th PME Conference*, 177-184.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

EXAMINING SECONDARY STUDENTS' ACHIEVEMENT IN LINEAR EQUATIONS IN MATHEMATICS AND PHYSICS IN MALAWI

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Mathematics and physics are taught separately in many countries, including Malawi. The topic of "function" is shared by both subjects. Empirical findings show that many students were able to solve a problem concerning a line graph slope in a mathematical context but failed to solve a parallel problem in the physics context (Planinic, Milin-Sipus, Katic, Susac & Ivanjek, 2012). This is known as context-dependency. Although context-dependency between mathematics and physics has been recognised, little is known about the types of questions that cause context-dependency in Malawi. The objective of this study was to examine the types of questions that cause context-dependency in function, focusing on the graph, table, and equation.

Ninety-one Form-4 (Grade-12) students from three different secondary schools in Malawi were selected for this study. The research was conducted using two tests for solving linear equations focusing on the graph, table, and equation. Each pair of questions referred to the same understanding in the different contexts of mathematics and physics. The tests were developed based on Malawian textbooks on mathematics and physics. The mathematics test was administered in September 2018, while the physics test was administered in February 2019. This was done to avoid the influence of the first test on the second one. Context-dependency was studied using the chi-square test. The types of questions that cause context-dependency were identified by examining the methods students used when answering the questions.

The results showed that the total percentage of students who provided correct answers was 43.8% in the mathematics test (Cronbach's alpha =.76) and 45.6% in the physics test (Cronbach's alpha =.71). Seven out of the 16 questions caused context-dependency. More students provided correct answers in the mathematical context and failed in the physics context in four out of seven questions. The other three questions presented opposite results. More students noticed the vertical (horizontal) relationship in the table in the mathematical (physics) context. Further results will be discussed in detail in the presentation.

References

Planinic, M., Milin-Sipus, Z., Katic, H., Susac, A., & Ivanjek, L. (2012). Comparison of student understanding of line graph slope in physics and mathematics. *International Journal of Science and Mathematics Education, 10*, 1393-1414.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

THREE LEVELS OF SENSE IN INTERPRETATION OF CUMULATIVE GRAPHS

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Faced with an abundance of graphic representations in research articles, newspapers and internet, today's students are expected not only to build graphs correctly, but also to interpret them. Yet the current studies show that students tend to perform poorly on graph interpretation. Moreover, despite the perceived importance of cumulative graphs, most of the existing research focuses on bar graphs, histograms and box plots. The present study aims to tend to this void by considering three levels of graph perception such as: (a) reading the data-questions that are supposedly answered on the graph, (b) reading between the data- interpolating and finding relationship in the data presented in a graph and (c) reading beyond the data-extrapolating or inferring from the graph in order to solve complicated questions (Friel et. al, 2001). 98 first year college students studying business administration participated in the study (59 females and 39 males).We employed a questionnaire of 8 true-false items: 4 of them examined the (a) level, 2 of them examined the (b) level and 2 of them examined the (c) level of understanding. All of the items referred to the same cumulative frequency distribution graph. The questionnaire was a part of the semester exam, so the students studied these topics and their motivation to answer as good as possible was high. The following percentages (%) of right answers were found: (a) level 85, 79, 87, 89; (b) level 75, 71; (c) level 60, 62. The true-false questions may create a justified guessing concern, so the hypothesis of guessing was checked and rejected ($p>0.05$). The ANOVA revealed significant differences between the mean percentage of right answers for the three levels ($p<0.01$). No gender differences were found for each of the three levels ($p>0.05$).The most difficult questions required building a frequency distribution table from the graph in order to calculate the average or to determine the shape of the distribution.

The results revealed that more difficulties were observed in the higher level questions where students had to convert one representation into another several times. Working within a multi-representational learning environment may pose a difficult challenge for learners in linking representations and moving flexibly between them. Only deep understanding of each of the representations and the link between them can enable students to perform successfully. Educators have to take this into account in designing better and more efficient learning frameworks.

References

Friel, S., Curcio, F., and Bright, G. (2001). Making sense of graphs: critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education, 32* (2), 24-158.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

TEACHERS' AND STUDENTS' CONSTRUCTIONS USING TURTLE LOGO

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In this poster, we will display and describe the types of constructions made by teachers and students using Turtle Logo software. Building on Papert's (1980) idea of constructionism, Turtle Logo is a software which allows students to learn mathematics through programming a Turtle to move using commands. The teachers learnt this software during a face to face workshop as a part of a geometric reasoning module for 8^{th} and 9^{th} grade students which integrated ICT based materials to address content knowledge of geometry and pedagogical objectives of learning through errors, collaboration and promoting authentic learning. The students made these constructions when teachers used this module in their own schools with their students and instructed them to use Turtle Logo to make different shapes. This study is situated in a large scale project called Connected Learning Initiative (CLIx) which aims at leveraging ICT for providing quality education to underserved population across India in 478 schools, 1767 teachers and about 46420 students in four different states in three languages - English, Hindi and Telugu.

The poster will provide examples and types of constructions made by teachers as well as students through the pictures collected during workshop and classroom observation and those shared by teachers in a chat based community established using Telegram app. The pictures of the constructions by teachers and students have been categorised on the basis of shapes being geometrical or drawings, the extent of thought given to coding and the exhibition of agency in making different types of shapes. We found that teachers immediately try making regular and geometrical shapes while students liked exploring innovative and challenging shapes like house or a ship.

Findings from analysis of the interactions of the teachers during the face to face workshops indicate that teachers were able to explore different types of shapes including a circle and were able to make a conjecture that "as the number of sides of a polygon increases the polygon tends to look like a circle". Teachers expressed that this will be an interesting tool to make students aware of the properties of shapes like the equal or parallel sides and the angles. The classroom observations revealed the pedagogical challenges in leading a discussion on properties of shapes including the need to communicate how the turtle interprets the commands, the angle with which turtle turns and providing the space for students to explore making diverse shapes.

References

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. NY: Basic Books, Inc.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

CHARACTERISTICS OF BRAIN ACTIVITY IN STUDENTS ALTERNATING BETWEEN TEACHING AND LEARNING ROLES

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The aim of this study is to discover the characteristics of brain activity when alternating between teaching and learning roles by focusing on students teaching and being taught by one another. A "tangram experiment task" in which subjects alternate between teaching and learning roles was developed, and brain activity was simultaneously measured in six groups of university student subjects (12 students in total). The results showed that switching from a teaching role to a learning one produces different brain activity in the same subject.

Applications in neuroscience education have reached the stage of brain activity measurement experiments in environments that are close to actual learning environments in school settings. The aim of this study is to discover, from the perspective of brain physiology, the educational effects of alternating between teaching and learning roles by focusing on the process of students teaching and being taught by one another, which has long been used theoretically and experientially in actual classroom settings.

The subjects were six groups of university students, totalling 12 people (average age: 21.1 years). The experimental task set for subjects was four trials of a tangram experiment task in which subjects used all seven pieces provided to form a specified shape. The experiment was conducted with two people per group under the condition that subjects would switch roles with one another after a set amount of time (every 15 seconds). One tangram set was prepared for each pair of subjects (A, B), and subject A (the task solver) worked on the task for the first 15 seconds while subject B (the task observer) observed the progress.

The results of brain activity measurement showed that the oxyHb of the two subjects tends to increase more in experimental tasks that are more difficult and time-consuming. Additionally, changes in brain activity were observed when the same subject alternated between teaching (task solver) and learning (task observer). Specifically, it was clear that oxyHb tended to increase more in the periods when a subject was teaching (task solver) than when he or she was learning (task observer). Meanwhile, results also showed that brain activity dropped or went into equilibrium when a subject was learning (task observer).

OVERCOMING COUNTING IN ZAMBIA: A PRELIMINARY ANALYSIS OF CHILDREN'S NUMBER RECOGNITION

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Many Zambian children in Grades 1–9 count on fingers during calculations, which is a challenge in mathematics education. However, prior research has not yet revealed how children utilise the place value system in formal calculations with concrete objects. This study takes the constructive point of view to investigate how Zambian children approach formal addition as well as recognise numbers and patterns with concrete objects. Our research question is as follows: How can children perform simple addition, see and count concrete objects? Data were collected from a primary school in Lusaka, Zambia, in 2018. We conducted formal interviews with each child. Firstly, 96 Grade 4 children were asked to solve 3 simple addition problems (e.g., 18+13) and explained the way. Secondly, we chose 18 children in a mix of academically good and poor pupils from Grades 1–4 at random and asked them to make simple calculations using bottle tops, referring from Mulligan, Mitchelmore and Stephanou (2015). Each child's action was recorded, transcribed, and summarised. In the first phase, all children counted but none recognised a group of 10. In the second phase, 5 children were able to recognise certain groups of concrete numbers and a Grade 4 boy could explain how they had identified these groups. However, 17 pupils seemed able to recognise groups of numbers when shown the structural arrangement of bottle tops on a frame of 5×2 squared boxes, but very few could explain their action verbally. These results show that, first, Grade 4 children counted to solve addition problems without using the group of 10, and, second, 5 pupils (Grades 1–4) were able to recognize certain groups of numbers (including groups of 5 and 10). In conclusion, first, in formal addition, all children counted and did not see any groups of 10; second, with concrete materials, some of them recognised numbers in a pattern and saw the group of 10. The second result shows that primary school pupils may be able to develop a way to see numbers using groups of 5 and 10 even in formal calculations if they practice enough with concrete objects and blank squares. We will develop a questionnaire and assessment tool, based on Australia's Pattern and Structure Assessment (PASA), and collect further data to identify the learning stages of pupils in different grades.

References

Mulligan, J., Mitchelmore, M., & Stephanou, A. (2015). *PASA (Pattern and Structure Assessment) an assessment program for early mathematics (F–2) teacher guide*. Victoria: ACER Press.

NAVIGATING STRUCTURAL DIFFERENCES BETWEEN ABDUCTIVE AND DEDUCTIVE REASONING

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Formal deductive proofs are the final products of a long process that often involves following a nonlinear path including exploratory, inductive and deductive reasoning (Boero, 1999). However, novices in proving are reluctant to abandon the linear deductive reasoning exemplified in formal proofs (Karunakaran, 2018). Pedemonte (2007) argued that there is a structural difference between abductive reasoning usually required in exploratory parts of the proving process and deductive proofs, resulting in a disconnect between proving and documenting their proof. It is therefore important that math educators create learning environments in which students can engage in mathematical argumentation and non-linear construction of proofs, while providing supports to make the transition to the writing of a formal deductive proof.

I designed and piloted a puzzle-like proving activity for the Pythagorean theorem that uses small paper cards with annotated figures, definitions, and theorems as proof-construction resources. Preliminary analysis of interview video data (6 students, ages 11-17) suggests that despite the different ways in which students engaged with this activity, they were able to utilize both abductive and deductive reasoning and they successfully constructed a linear documentation of the proof. The organization of mathematical knowledge in pieces of papers afforded social and physical interactions with cognitive resources that tend to remain hidden in most proof-related classroom activities. These interactions shed light both on tacit social facets and diverse forms of reasoning occasioned in constructing and communicating proofs. This study illustrates design insights for activities that support students in utilizing abductive and deductive reasoning to construct and document a mathematical proof.

References

Boero, P. (1999). Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof.* Retrieved from www.lettredelapreuve.it/OldPreuve/Newsletter/990708Theme/990708ThemeUK.html

Karunakaran, S.S. (2018). The need for "linearity" of deductive logic: An examination of expert and novice proving processes. In A.J. Stylianides & G. Harel (Eds.), *Advances in Mathematics Education Research on Proof and Proving* (pp. 171-183). Dordrecht: Springer International Publishing.

Pedemonte, B. (2007). How can the relationship between argumentation and proof be analysed? *Educational Studies in Mathematics, 66*(1), 23-41.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

WHAT MULTIPLICATIVE SITUATIONS ARE PROPOSED IN BRAZILIAN TEXTBOOKS FOR THE FIRST YEARS OF PRIMARY SCHOOL?

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Textbooks in Brazil, especially in the first years of Primary School, have played an important role in mathematical development. In most schools, the curriculum in action (experienced in the classroom) is shaped by the guidelines given in textbook teacher manuals. Vergnaud (1983) points out that a psychological understanding of mathematical concepts requires considering situations, invariants, action schemes and symbolic representations. In this sense, the diversity and complexity of multiplicative relations can be illustrated by different situations – problems that require the use of multiplication and/or division as solution. This study aimed to analyse multiplicative problems of two Brazilian textbook collections. These collections were chosen because they are the most used in the first years of Primary School in Brazil.

The results were analysed considering the investigations carried out by Vergnaud (1983) and Gitirana, Campos, Magina and Spinillo (2014). Most of the activities focus on multiplicative comparison, simple one-to-many proportion and partition division. More problems of combinatorial reasoning were present, in both collections, than rectangular configuration situations and no problems of area were observed. Knowing that textbooks play a fundamental role in curriculum (as also attested by Son, 2005), it is pertinent to promote a greater teacher understanding about the nature of the problems that are presented to their students, since problem variety may favour concept use in different situations, enabling students' conceptual development.

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References

Gitirana, V., Campos, T., Magina, S., & Spinillo, A. G . (2014). *Repensando Multiplicação e Divisão: Contribuições da Teoria dos Campos Conceituais*. São Paulo: PROEM.

Son, J. (2005). A comparison of how textbooks teach multiplication of fractions and division of fractions in Korea and in U.S. In H. Chick & J. Vincent, (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 201-208). Melbourne, Australia: University of Melbourne.

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics: Concepts and process* (pp. 127-174). London: Academic Press.

TEACHING MATHEMATICS IN RURAL APPALACHIA: DILEMMAS AND RESOURCES

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As the field continues to better understand mathematics teachers' decision making and knowledge for teaching, it is important to consider the role of resources. This study continues this work by aiming to further understand resources and dilemmas *in-practice-in-context* (Lampert, 2001; Adler, 2000). Additionally, rural communities in the United States are numerous and diverse, leading to a "pragmatic necessity and empirical imperative to understand the individuals who teach in these districts" (Burton et al., 2013). The need to further understand this context leads to the purpose of this study to understand how teachers in rural Appalachia conceptualize and make use of resources to manage dilemmas of practice. Thus, this study takes a situated perspective to study the following research questions: (1) From the teachers' perspective, what are the dilemmas of practice that secondary mathematics teachers, in a rural setting, experience? (2) How do mathematics teachers in rural settings conceptualize resources for the teaching and learning of mathematics?

To explore the research questions, an instrumental case study design was used to explore a sample of five secondary mathematics teachers from rural Appalachia. For each participant, data were collected over a two-month period and include two interviews, one classroom observation, and teachers' completion of three resource diaries (structured reflections about planning and teaching a mathematics lesson).

Preliminary analysis indicates the need for a re-conceptualization of resources in and for the teaching and learning of mathematics, with an expanded focus on contextual and sociocultural resources. Additionally, it is observed that experience teaching, access to resources, and understanding of resources has implications for the array of dilemmas teacher's experience and how teacher's go about managing these dilemmas; in addition to having implications for how teachers make decisions about resources. Analyses are also showing implications for the design of curriculum materials and design of mathematics teacher education and professional development.

References

Adler, J. (2000). Conceptualizing resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205-224.

Burton, M., Brown, K., & Johnson, A. (2013). Storylines about rural teachers in the United States: A narrative analysis of the literature. *Journal of Research in Rural Education, 28*(12), 1-18.

Lampert, M. (2001). *Teaching Problems and the Problems of Teaching.* New Haven, CT: Yale University Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

PEDAGOGICAL DESIGN CAPACITY AND PRESCRIPTION

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Teachers' pedagogical design capacity or PDC (Brown, 2009) defines their capacity to perceive and mobilise existing curricular resources for quality mediation in the classroom. Persistent failure rates and poor curriculum coverage in South Africa have resulted in the implementation of mandated daily scripted lesson plans (SLPs) in under-performing schools. Prior to this, prescribed textbooks had been the major resource for teaching mathematics. This change begs the question: does the nature of the resource influence teachers' PDC, and how? More so when earlier research had indicated to low PDC with respect to the textbook. This is the rationale for the current ongoing case study; socio-culturally grounded, and targeting selected secondary school teachers who have participated in a professional development course. The course exposes teachers to, among other things, a focus on: a) selecting and sequencing examples such that generalising is possible; b) classroom explanations, which mediate formal mathematics discourse, and distinguish between mathematical versus non-mathematical justifications. The current poster focuses particularly on the explanations in one SLP lesson on factoring trinomials at grade 9, and by one teacher in the same lesson and asks: what does teachers' PDC look like in the context of high prescription. The analysis entails determining the affordances and constraints of the SLP explanations to the teacher's practice with respect to mathematical coherence and justifications on the one hand; and the teacher's explanations on the other, with a specific focus on what the teacher omits from and inserts to the SLP lesson.

Initial results point to a disruption of the intended flow of a mathematically coherent SLP approach to factorising, by the teacher; the approach focussed on the factoring/expansion relationship, but the teacher replaced the initial tasks in the SLP, disrupting this flow. Secondly, the SLP's explanation of the procedure for factorising reflected a set of steps that were not mathematically derived, but the teacher offered a similar explanation without mathematical justification too. So, what then of the teacher's PDC? Despite these, the teacher inserted additional worked examples in the classroom which varied signs and variables used, thus offering more clarity to the steps, and enhancing the lesson. Thus, the teacher adapts the SLP in ways that enhance, but at the same time, detract from mediation. We suggest hence that teacher's rationale for her decisions needs to complement her PDC in action.

References

Brown, M. (2009). The teacher-tool relationship: theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics Teachers at Work: Connecting Curriculum Materials and Classroom Instruction* (pp. 17–36). New York and London: Routledge.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

COLLABORATION WITH *DIRECTED ACTIONS* FOR PROOF

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We examine mediators-for-communication within collaborative learning instructional designs, from a grounded and embodied mathematical cognition (Nathan & Walkington, 2017) theoretical framework, which argues coordinating learner's cognitive systems for action with those for language, facilitates grounding conceptual understanding in mathematics. Concrete *tangible objects* provide learners the affordance of direct manipulation during reasoning yet instantiate mathematical concepts with rigid magnitudes from fixed shapes. Semi-concrete *directed actions*, which are "physical movements that learners are instructed to formulate" (Nathan & Walkington, 2017, p. 3), provide the unique affordance of instantiating mathematical concepts with malleable and flexible magnitudes through embodiment. Our question:

Within a collaborative learning instructional design, what are the effects that different mediators-for-communication have, specifically utilizing *tangible objects* compared to *directed actions*, on learning and transfer, defined conceptually as the demonstration of *deeper conceptual understanding* in STEM's geometry, and defined operationally as *deductive justifications* formulated in transformational or axiomatic proof schemes?

This study is planned for 2019-2020 and represents a companion experiment to Nathan and Walkington's (2017) research on embedding *directed actions* into motion-capture video games. Those findings reveal that an instructional design utilizing *directed actions* coupled with activating learner's language system improved mathematical insight (odds ratio = 3.1, $d = 0.6$, $p < 0.001$) and improved valid informal proof (odds ratio = 4.7, $d = 0.9$, $p < 0.001$). We hypothesize *directed actions* will be the more successful mediator-for-communication. Engaging scholars at PME-43 with our current set of *directed actions* will initiate dialogue and feedback.

Experimental Design: N=352 undergraduates, randomly assigned to 2 conditions, subdivided into 2 further sections, yielding 1A (n=88), 1B (n=88), 2A (n=88), 2B (n=88). Between-subjects design, three implementation phases: proof training, communication training, jigsaw collaboration. Proof training: Condition/Section 1A learns transformational proof, 1B learns axiomatic proof, and jigsaw collaboration pairs within-Condition but different Sections. Same for 2A and 2B. Communication training: Condition 1 learns communicating with *tangible objects*, Condition 2 learns with *directed actions*. Jigsaw collaboration: mediators-for-communication activate learner's action cognitive systems and collaboration activates learner's language cognitive systems. Condition 1's *tangible objects* vs. to Condition 2's *directed actions*.

References

Nathan, M. J., & Walkington, C. (2017). Grounded and Embodied Mathematical Cognition. *Cognitive Research: Principles and Implications*, *2*(9), 1-20.

INFORMAL MATHEMATICAL EDUCATION CONTEXTS TO PREVENT THE SCHOOL DROP-OUT

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This work stems from an experience of intervention against the school drop-out, in the district of Naples of Scampia where in June 2018 the educational project "Proud of You" began at the Comprehensive School “Virgilio IV”. This project, still underway, was born in response to the alarming phenomenon of school drop-out, which in this area reaches very high numbers and it manifests already in the delicate transition between primary and secondary schools. One of the key features of the project is to link the development of pupils’ mathematics and language skills with the chance to offer them the possibility to go out of the area of Scampia, for most of the pupils perceived as the unique known reality, in other words a “ghetto”, and to explore other parts of the city of Naples. In particular, in the activities designed for the first part of the project the development of disciplinary skills is always intertwined with the discovery of the artistic and cultural heritage of the city. For this reason, for the design of the project activities we refer to the emerging informal mathematical education framework. Its core ideas are the socio-cultural nature of mathematics, the care to the diversity of strategies generated by students and the search for a "connection" with everyday life. According to Nasir, Hand and Taylor (2008):

> “These models of teaching and learning view math not simply as cognitive activity but also a social and political activity – activities that who do with one another as we seek to improve our world and push for social justice.” (p. 220)

Informal mathematics education environments are created with explicit pedagogical objectives and are intentionally designed to support mathematical learning, therefore they are structured through programs with regular schedules and specially trained educators. In the poster, we present the experimental project activities, characterized by sensory and motor experiences, thus based both on perception and action according to the enactivist view on cognition (Varela, Thompson & Rosch, 1991).

References

Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between “cultural” and “domain” knowledge in the mathematics classroom and beyond. *Review of Research in Education, 32,* 187-240.

Varela, F., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience.* Cambridge, MA: MIT Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

CREATIVE MATH ACTIVITIES FOR PRIMARY SCHOOL PUPILS

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Offering primary school pupils the opportunity to be creative while doing mathematics is very often a challenge for their teachers. The pilot study reported here aimed to offer a proposal for overcoming such a challenge. It involved the design and implementation in a Greek primary school of an event labelled as ‘Maths Day’. This was a normal school day, during which all the pupils of the school were not taught their normal curriculum. Instead, they were involved in fun mathematical activities, which were designed with the aim to provoke ‘inquiry based learning’.

Some of the activities were essentially “games”: in order to win them, the pupils were involved in serious (but fun!) problem solving (see for example, Misailidou & Keijzer, 2019). Others, required creativity and imagination in order to be completed. For example, groups of 8-10 year old pupils were asked to design their own “Amusement Park”. All the children worked with enthusiasm and cooperated effectively. The result was amusing games with mathematical content attached to them such as the “Kitchen”. This included a kitchen counter full of pots. Every pot had a number on it and the goal of the game was to score some balls inside the pots and get as many points as the numbers written on them.

The analysis of the results indicated that in general, the children used their imagination and their mathematical knowledge and created original activities accompanied by elaborated descriptions and drawings.

Acknowledgement

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References

Misailidou, C. & Keijzer, R. (2019). Making Mathematics fun: The ‘Fear Room’ game. *Proceedings of the 11th CERME Conference.* (To be published).

STUDY ON "FRINGE" OF MATHEMATICAL KNOWLEDGE: FOCUSING ON MATHEMATICAL INSTRUMENTALITY

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Ichiei Hirabayashi, who is called as a father of Japanese mathematics education, regards mathematical knowledge as the instrumental concept for problem solving and points out that students should acquire mathematical knowledge as "instruments" that can be used freely like our own hands and legs; which is conceptualized as "activated knowledge" (Hirabayashi, 2001). He also points out that what makes the knowledge activated is "fringes of knowledge". Hirabayashi (2001) describes the "fringe" of the knowledge as "the table on which the knowledge is placed, or the context in which the knowledge is placed". Even though the role of "fringe" seems very important, the exact meaning of "fringe" is yet unclear and there are still little teaching practices with this concept. The aim of this study is clarifying the concept of "fringe" of knowledge with some concrete examples.

One of the concrete examples of "fringe", which is shown in Hirabayashi (2001), is in the case of simple addition. It is said that the hands-on problem solving with marbles should be the "fringe" of the knowledge of addition and subtraction. In this case, children will learn the concept of addition, such as 2+3=5, within the concrete situation or context of problem by using marbles. Problem solving with marbles is the situation or context which makes the formal knowledge, e.g. 2+3=5, activate. On the other hand, considering the relationship from the formal knowledge to the situation, the activated knowledge of addition will be used in the real context or situation. "Fringe" in learning mathematics is the situation or context in which the knowledge is used, and the activated knowledge is needed to be understood with "fringe". There are reciprocal interactions between mathematical knowledge and the situation or context. Because the activated knowledge can be always used in situation or context, the situation or context in which students learn mathematical knowledge is very important for them to activate their knowledge. Mathematical knowledge should be learnt with its situation or context, or "fringe". "Fringe" should be defined as "the context or situation in which knowledge is used as instruments for problem solving". Further concrete examples will be proposed in my poster.

References

Hirabayashi, I. (2001). What is Lesson? Epistemological Theory of Lesson in Mathematics Education. *Journal of Kinki Society of Mathematics Education, 14*, 34-41 (in Japanese).

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

ONE TEACHER'S TENSIONS IN THE CONTRASTING INTRODUCTIONS TO TWO ALGEBRA TOPICS

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Liljedahl, Andra, Rouleau, and Di Martino (2018) suggest that teacher tensions are caused by dilemmas between what they want to do and what they have time for. Findings from this study suggest that teacher tensions also include learner-related issues like learner performance, confusion and resistance. The study tracks 4 secondary maths teachers who completed a year-long professional development (PD) course, exploring their take-up of elements of the course. A teacher, Neo, was chosen because he is mathematically strong and displayed high levels of engagement and commitment to the course. Three of his grade 10 lessons on number patterns and 2 on quadratic equations were observed and video-recorded. These were followed by an interview 3 months after the observations, analysed thematically.

His introduction to number patterns reflected tensions related to learner confusion and learner performance, and not to the meaning of the formula used. The recommended formula for the nth term in grade 10 is $T_n = an + q$,but he taught the formula $T_n = a + (n-1)d$, normally used at the grade 12 level. He focused on substituting values into the formula with no attention to the origins of the formula. In the interview he justified his choice of formula because (1) it proved successful in the past;(2) he had faced resistance from learners when using the recommended formula as they said it was not examined in tests;(3) learners found the recommended formula confusing and asked questions which took up too much time. His introduction to quadratic equations though focussed on the zero-product concept before introducing the procedure. He first discussed the value of a in $a \times 0 = 0$, values of a and b if $a \times b = 0$;then $(x-1)(x+2) = 0$. His justification was that in the past learners had not understood his introduction when he started with $(x-1)(x+2) = 0$.His change in the introduction to quadratic equations was due to a similar task being offered in the PD course.

Neo's tensions relating to time, seems to be the most frequent source of tension. While the introduction to patterns reflects this, the introduction to quadratic equations backgrounds this and prioritizes the development of conceptual understanding of the zero-product concept because the means to do so was provided in the PD course.

References

Liljedahl, P. G., Andra, C., Annette, R., & Di Martino, P. (2018). Teacher tensions as a lens to understand teachers' resistance to change. In K. Beswick, T. Muir, & J. Wells (Eds.), *Proc. 39th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 193-200). Hobart, Australia: PME.

SOLVING WORD PROBLEMS USING VISUALISATION

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This empirical, qualitative study explored twenty eight grade 9 learners' use of visualisation when solving word problems. After all the due ethical clearance protocols were observed, one class of learners completed a task based exercise and five learners were interviewed. The results showed that when learners drew a diagram that represented the problem, they were more likely to solve it. Those learners who drew no diagram, or drew an incorrect one, failed to arrive at a suitable solution. An important finding is that the visualisation technique is a powerful tool when working with the solving of problems in mathematics.

Solving word problems is a fundamental aspect of mathematics teaching. Despite the amount of time dedicated to this part of mathematics teaching, there is a tendency for learners to struggle with finding solutions that are correct. South African learners in particular have many additional barriers, one of which is related to the many official languages that have been legislated. According to Teahen (2015, p.1), the issues of mathematical proficiency or computation skills are not the only major factors that contribute to students' poor performance in mathematical problem solving; but the language used in word problems and the inability to comprehend those problems have had a significant impact on learners' achievement in mathematical word problems. According to Debrenti (2015, p.22), visualisation is an effective strategy that allows learners to make connections between the text in the mathematics problem and their prior knowledge or experiences to create relevant images in their minds which can aid the overall comprehension of the word problem. Visualisation is a reading technique that encourages learners to draw what they have pictured in their mind after reading the mathematics problem. By producing a picture that relates to the problem, students may get a deeper insight into what the problem is asking as well as vital information the problem is providing (Debrenti, 2015, p.22).

References

Debrenti, E. (2015). Visual representation in mathematics teaching: An experiment with students. *Acta Didactica Napocensia*, *8*(1), pp. 21-26.

Teahen, R. J. (2015). *Exploring Visualisation as a Strategy for Improving Year 4 & 5 Student Achievement on Mathematics Word Problems.* Master's thesis, Victoria University of Wellington, Wellington, New Zealand.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

ACTIVITIES THAT ENABLE ACCESS TO SCHOOL MATHEMATICS: AN EXPOSITION FROM A MENTORSHIP STUDY

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Typically, activities that are used to introduce mathematical concepts appear to be far-removed from real-life. That is, they are too close to mathematics. This has the consequence of limiting access to the very mathematics which these activities intend to make accessible to learners. On the other hand, there are activities that appear to be far-removed from mathematics but close to real-life and do not seem to have much prevalence in most mathematical classroom practices including school textbooks. In this article, we focus on a research study involving the second type of activities when used in teaching concepts that are linked to number patterns. Data was collected from grades 10-11 learners from two rural secondary schools in Limpopo Province in South Africa that participated in the study. Sfard's (2008) commognitive perspective was used to examine participants' experiences of mathematics as they participate and engage in the activities during the mentoring program. The commognition framework foregrounds thinking as a special activity of communication in which participants of a discourse such as mathematics engage.

Our analysis of the learners' written responses to the tasks that were given in the mentoring program demonstrates that the activities that appear to be far-removed from mathematics have the potential to enable participation from all learners. This is consistent to Clausen (1992) assertions that working directly from a visual context is often preferable to an algebraic treatment derived from purely numerical patterns. Hence, we argue that these activities lead to experiences that are more essential for enabling access to mathematics as a discourse.

References

Clausen, T. (1992). Say it with Pictures. *Mathematics in School, 21*(4), 17-19.

Sfard, A. (2008). *Thinking as Communicating. Human Development, the growth of discourses and Mathematizing*. Cambridge: Cambridge University Press.

THE RELATIONSHIP BETWEEN DISCOURSE MOVES AND AUTHORITY STRUCTURES IN MATHEMATICS CLASSROOMS

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Wagner and Herbel-Eisenmann (2014) argue that language practices of participants play a key role in structuring authority in mathematics classrooms. In line with research development in mathematics classroom discourse, this case study examines how one teacher's dialogic discourse moves influenced the patterns of authority structures she established in a mathematics classroom. We collected and coded textual data, in the form of four sets of lesson transcripts, with nine "discourse moves" codes: *ask for expression*, *re-voice*, *press for reasoning, say more*, encourage to *add on* or *explain other* students' ideas, prompt to *agree/disagree, evaluate* students' responses and request for *choral response* (Ni et al., 2017). In addition, we provide qualitative analyses of the teacher's discourse patterns in terms of four authority structures proposed by Wagner and Herbel-Eisenmann (2014): *personal authority, discourse as authority, discursive inevitability,* and *personal latitude*. By focussing on the authority structures as reflected in various classroom discourse moves practiced by Ms. L, the primary mathematics teacher participant, our analysis shows that certain patterns embedded in classroom discourse can constrain or enable certain kinds of authority. For example, we found evidence that Ms. L often *asked for expressions* in the form of collaborative completion questions and requested her students to provide non-linguistic (gestural) responses, and both of these discourse patterns were related to high teacher control and low student agency in the form of closed questions, hence limiting *personal latitude* in the responses. By contrast, Ms. L also used an interactive, or dialogic, classroom discourse, marked by her *press for reasoning* and providing more opportunities for students to discuss in pairs. Previous research has examined teachers' discourse behaviours and authority structure in mathematics classrooms but yet to address the connection between the two. This study contributes to understanding the ways in which dialogic discourse may influence how authority is structured, including how mathematics teaching and learning are enacted through authority styles, and how the discipline of mathematics is positioned from an authority perspective.

References

Ni, Y., Ho, G., Cai, J., Cheung, A., Chen, G., & Ng, O. (2017). Research Protocol: Teacher Interventions in Engaging Students with Dialogic Classroom Discourse for Rich Learning Opportunity in Mathematics Classrooms. *International Journal of Educational Research, 86, 23-35.*

Wagner, D. & Herbel-Eisenmann, B. (2014). Identifying authority structures in mathematics classroom discourse: a case of a teacher's early experience in a new context. *ZDM Mathematics Education, 46*(6), 871-882.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

TEACHING NUMERACY ACROSS THE CURRICULUM – WHAT KNOWLEDGE DO PRESERVICE TEACHERS NEED?

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Ireland's poor performance in PISA 2009 and the decline in students' mathematical literacy abilities has led to the introduction of a new emphasis on numeracy across the curriculum. In an effort to improve student's numeracy skills, prospective and practising teachers are required to incorporate numeracy teaching and learning in all subjects across the curriculum (DES, 2011). Nevertheless, teachers are given very little guidance on how to implement numeracy teaching in their lessons and teacher educators are similarly unaware of numeracy teaching strategies. Thus the aim of this study is to develop a framework for numeracy teaching that can be utilised by pre-service teachers to support them in embedding numeracy across the curriculum. The poster will present the development and key characteristics of the framework.

The framework is informed by an extensive review of literature in teacher knowledge and numeracy. Teacher knowledge and numeracy are both complex and multi-faceted. A number of frameworks for teacher knowledge exist; however while these frameworks address subject specific knowledge (SSK), and pedagogical content knowledge (PCK), none were found to address numeracy knowledge (NK). We argue that being an effective teacher of numeracy within any subject area requires all three components of knowledge. We have therefore developed a new three-component model for teacher knowledge that we refer to as the "N" framework, aligning with the three parts of the letter "N". The framework's first pillar is NK, the ability to make informed decisions using mathematical knowledge, tools and positive dispositions in a range of different contexts. The second pillar, SSK, describes the content knowledge needed for teaching a specific subject. The third pillar, PCK, connects the other pillars, and represents knowledge of how to embed numeracy within the subject.

Building on the work of Goos et al. (2011), this study elaborates on the nature of numeracy knowledge as an additional component of teachers' professional knowledge. The "N" framework is an attempt to capture the interrelatedness of key knowledge categories needed for developing numeracy across the curriculum.

References

Department of Education and Skills. (2011). *Literacy and numeracy learning for life: The national strategy to improve literacy and numeracy among children and young people 2011-2020.* Dublin

Goos, M., Geiger, V., & Dole, S. (2011). Teachers' personal conceptions of numeracy. In B. Ubuz (Ed.), *Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 457-464). Ankara, Turkey: PME.

REASONING COVARIATIONALLY TO DEVELOP PRODUCTIVE MEANINGS OF SYSTEMS OF RELATIONSHIPS AND INEQUALITIES

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Although systems of equations are an important topic in both U.S. and international mathematics curricula, there is limited research examining productive ways of supporting students' understandings of systems of equations. In this poster, we explore ways middle-grades students can reason quantitatively and covariationally (e.g., Carlson et al., 2002; Thompson, 1995) about systems of relationships, including how two quantities are graphically represented when one is greater than, less than, or equal to the other with respect to a third quantity. We conducted a design study (Cobb, et al., 2003) consisting of four small-group (one to three students) teaching experiments with middle-grades students (age 10-13) who did not have experience graphing systems of equations. To analyse the data, we used open (generative) and axial (convergent) approaches to build models of students' mathematics that viably explained their words and actions. We engaged the students in a task entailing a dynamic cone and cylinder with varying heights. We asked students to graphically represent the surface area (or volume) of each 3D shape with respect to height on its own coordinate system. We also asked students to determine height values such that the surface area (or volume) of the cone was less than, equal to, or greater than the surface area (or volume) of the cylinder. Finally, we asked them to graph the relationships between surface area (or volume) and height of the cone and cylinder on the same coordinate system. We found that the students reasoned about magnitudes (Thompson, 1995) when coordinating the direction and amounts of change (Carlson et al., 2002) of surface area with respect to height for each shape when describing the changing quantities situationally and graphically. We highlight two different ways of reasoning students engaged in as they compared the relative magnitudes of two quantities with respect to a third quantity and leveraged this reasoning to graphically represent two relationships on the same coordinate system.

References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for research in mathematics education, 33*(5), 352-378.

Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

Thompson, P. W. (1995). Notation, convention, and quantity in elementary mathematics. In J. Sowder & B. Schapelle (Eds.), *Providing a foundation for teaching middle school mathematics* (pp. 199-221). Albany, NY: SUNY Press.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

DESIGNING MATHEMATICAL MODELING TASKS TO IMPLEMENT SUSTAINABLE DEVELOPMENT GOALS

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There has been a gradually increasing focus on adopting mathematical modeling techniques into school curricula and classrooms as a method to promote students' mathematical problem solving abilities. However, this approach is not commonly realized in today's classrooms due to the difficulty in developing appropriate mathematical modeling problems. This research focuses on designing mathematical modeling tasks to implement Sustainable Development Goals (SDGs). One of the important perspectives of real-life problems is to reflect the SDGs context. The problem contexts in 17 SDGs cover people, prosperity, planet, peace, and partnership (UNESCO, 2016).

For the purpose, this study is conducted as a part of development research in a mathematics teacher education course. Based on a comprehensive review of literature concerning mathematical modelling and Education for Sustainable Development (ESD), this study identified the guiding principles and the methods on how to develop mathematics preservice teachers' competencies of context levels to practice the SDGs.

Specifically, this study presents how to introduce ESD to expedite active reflective learning activities of preservice teachers and develop mathematical tasks and materials based on SDGs context. The SDGs related to social and economic development issues including poverty, hunger, health, education, global warming, gender equality, water, sanitation, and energy. Implementing teaching strategies, setting up educational environments, and developing leadership to achieve the goals of Education for sustainable development in mathematics class.

The SDGs is an emerging issue within worldwide reform discourse. In this context, this study will contribute to the development of guiding principles and methods of how to prepare mathematics preservice teachers for ESD. In addition, this study will offer instructional approaches in which students are encouraged to be active producers of mathematics by thinking about mathematical modeling based on issues related to life.

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References

UNESCO (2016). *Education 2030: Incheon Declaration and framework for action for the implementation of Sustainable Development Goal 4.* Paris: UNESCO.

COLLABORATION BETWEEN SPECIAL EDUCATION TEACHER AND MATH TEACHER TO PROMOTE ARGUMENTATION IN THE MATHEMATICS CLASSROOM

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Collaborative work among teachers has been identified as a key element the development of quality learning (Villa, Thousand & Nevin, 2008). However, there is not enough empirical research about what teaching practices promote equity in the math class. A practice that can be considered inclusive is argumentation, where students engage in and take responsibility for the collaborative construction of mathematical knowledge (Krummheuer, 1995). The goal of this study is to characterize collaboration between a special education teacher and a mathematics teacher when promoting argumentation in the mathematics classroom. The context of the study is an intervention consisting of a professional learning course, which purpose is to study and implement argumentation practices in mathematics instruction. The sample includes 57 public schools in Santiago of Chile (with a high percentage of immigrant, indigenous and socially vulnerable students), 61 mathematics teachers teaching 7th grade (12 and 13 years old), 50 teachers of special education and 2,048 students. We studied two cases of two teacher partners (a mathematics teacher and a special education teacher) that through the intervention worked collaboratively in promoting argumentation in the respective partners' classroom. Lessons designed to promote argumentation in the mathematics classroom were videotaped. Afterwards, each couple of partner teachers were interviewed to identify the elements that facilitate and those that hinder collaboration to promote argumentation, and to identify the teachers' perceptions about their respective roles in the classroom. The analysis approach included identifying categories related to argumentation and collaboration between teachers.

In the conference we will share the preliminary results of this study that characterize the collaborative work developed by the special education and the mathematics teachers in promoting argumentation in the mathematics classroom. We look to uncover the collaborative practices that promote equality in the mathematics.

References

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229-269). Hillsdale, NJ: Lawrence Erlbaum.

Villa, R., Thousand, J. & Nevin, A. (2008). *A Guide to Co-teaching. Practical Tips for Facilitating Student Learning*. Thousand Oaks: Corwin Press

FOSTERING ELEMENTARY PRE-SERVICE TEACHERS' UNDERSTANDING OF TEACHING AND LEARNING ALGEBRA

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Fostering algebraic thinking in elementary school has been suggested as a way to provide all students with access to algebra. However, elementary school teachers often need support in developing their understanding of algebraic concepts, as well as how to best introduce them to students (Carpenter, Franke, & Levi, 2003; Kaput, Carraher, & Blanton, 2008). The details on how to best provide this support is still unclear. This project set out to study the following question: How does algebra-focused professional development (PD) foster elementary pre-service teachers' content knowledge, attitudes, and self-efficacy?

Fifteen pre-service elementary school teachers participated in a five-month PD intervention. They met once a month for three hours and studied: 1) expressions, 2) variables, 3) equality, 4) functions, and 5) equations. In each of the five sessions, they 1) completed algebraic tasks, 2) analysed aligned videos, 3) discussed aligned student work, and 4) rehearsed pedagogical strategies focused on one of the topics above.

Three valid and reliable instruments were used: The Learning Mathematics for Teaching Test (content knowledge), The Mathematics Attitudes and Perceptions Survey (attitudes), and a Teaching Efficacy Belief Instrument (self-efficacy). All three were given at the start and end of the PD. In addition, reflections were collected at the end of each of the five sessions and students were interviewed to better understand their perceptions of what they took up from the PD experience.

The analysis will consist first of descriptive statistics and regression analyses using ANOVA independently on the math content and beliefs instrument. We will use a principal factor analysis with VARIMAX rotation. Subsequently, we will use exploratory statistical and qualitative analyses.

Initial results from qualitative analyses on reflections suggest that students learned content and pedagogy focused on the five algebraic topics but experienced difficulty applying to curriculum and implementing strategies to support their students' development in their classrooms. Statistical analyses and additional findings will be shared on our poster along with limitations and implications.

Reference

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating algebra in the elementary school.* Portsmouth, NH: Heinemann.

Kaput, J. J., Carraher, D.W., & Blanton, M. L. (Eds.). (2008). *Algebra in the Early Grades*. Mahwah, NJ: Lawrence Erlbaum Associates.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

FOUR STEPS FORWARD, ONE STEP BACK: TOWARD DESIGN PRINCIPLES FOR DEVELOPING MATH MATERIALS SERVING RURAL EARLY GRADE CLASSROOMS

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The goal of education design research is to develop effective interventions by studying successive intervention prototypes in their target context (Plomp, 2007). This paper reflects on a ten year education design research programme bringing together a collective of teacher educators, researchers and rural foundation phase teachers to build and sustain quality education in primary schooling, accountable to the educational context of rural isiXhosa speaking children and teachers. Central to the programme are a set of workbooks designed as instructional resources used daily by teachers to scaffold teaching and learning progression.

If "one big step forward" is metaphorically taken as a 10% gain in Grade 3 mathematics results, then by 2014 the collective managed to take four steps forward. By 2017, the collective took one-step back. After investigating a range of data reflecting potential contributing factors, the investigation of the reasons for the "step back" focused on the differences in the design of the workbooks in the two period. Each workbook page was coded by conceptual focus, the quality and quantity of representation, the number range, mathematical procedure and conceptual signalling. This intensive coding exercise allowed a comparative analysis to identify the significant design differences between the two streams of workbooks.

The findings point to a number of significant differences between the streams of workbooks including conceptual density and progression, conceptual signalling, and the quantity and quality of mathematical representation. The poster presentation will provide a visual story of the experience, the impact data, and the comparative analyses that establishes a number of productive hypotheses about design principles for productive early grade math materials in the context of rural schools.

The significance of the study is that it begins to provide evidence upon which to base design principles for early grade mathematics workbooks that have the generative potential to improve teaching and learning in poor and low performing classrooms.

References

Plomp, T. (2007). *An Introduction to Educational Design Research.* Netherlands: Institute for Curriculum Development.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

A SURPRISING SOURCE OF DIFFICULTY IN SOLVING LINEAR EQUATIONS: THE CASE OF $ax = bx$

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It is well-known that learners have difficulty in solving equations of the form $ax + c = bx + d$ as they navigate the transition from arithmetic to algebra (Filloy & Rojano, 1989; Linchevski & Herscovics, 1996). We report on two studies that found evidence to support the notion of the *didactic cut*. In addition, both studies found learners had difficulty with the form $ax = bx$.

Sanders (2007) studied the test responses of 106 Grade 10 learners on three linear equations, two of which had letters on both sides. She then conducted a detailed error analysis on 46 scripts and found six learners who made conjoining errors in solving equations of the form $ax + c = bx + d$, leading to the form $ax = bx$. They then manipulated the equation to obtain a familiar-looking solution, e.g. a learner obtained $3x = x$, then divided both sides by x but then gave the solution as $x = 3$.

Halley (2019) investigated the progress of six Grade 9 learners in solving linear equations over a six-month period, using task-based interviews. The main interview instrument was a task-matrix, which allowed the interviewer to move flexibly between different linear equations to explore shifts in learners' abilities to solve the equations. Aware of Sanders's earlier finding, Halley deliberately inserted an item $ax = bx$ into this matrix. All six learners demonstrated ability to solve equations with letters on both sides by the end of the six-month period. However, not all learners could solve the item $ax = bx$, and all six learners experienced difficulty with it. They all appeared to treat this form as entirely different from $ax + c = bx + d$. A common concern of learners was that if one subtracts "everything" from one side then there is "nothing left" to use in finding the value of x. The poster will provide further details of learners' responses.

References

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics, 9*, 19–25.

Halley, A. (2019). *Linear equations: Grade 9 learners' progress and their difficulties over time.* Unpublished Masters Dissertation, University of the Witwatersrand, Johannesburg.

Linchevski, L., & Herscovics, N. (1996). Crossing the cognitive gap between arithmetic and algebra: Operating on the unknown in the context of equations. *Educational Studies in Mathematics, 30*(1), 39-65.

Sanders, Y. (2017). *Learners' performance in arithmetic equivalences and linear equations*. Unpublished M.Sc Research Report, University of the Witwatersrand, Johannesburg.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

INTEGRATING VIDEO TUTORIALS IN MATHEMATICS TEACHING: QUALITY CRITERIA TO CHOOSE GOOD VIDEOS

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The use of multimedia in the mathematics classroom is increasing day-by-day. Among different digital technology, such as mathematical software and handheld devices, the most recent media introduced into practice are video tutorials. There are millions of videos freely available on the internet in a variety of different subject areas. Integrating these videos in teaching is becoming widely spread and hence creating new research areas (Ljubojevic, Vaskovic, Stankovic & Vaskovic, 2014). Research indicates that the use of video tutorials has potential to support students' learning (e.g, Jones & Cuthrell, 2011).

Integrating videos into mathematics lessons can have different purposes depending on the setting. For example, the video tutorial could be closely related to the lesson or it could form part of the lesson. When it is a part of the lesson, the teacher needs to decide when to integrate the video into his/her lesson. However, the content of the video needs to be closely matched with the goal/s of the lesson. Thus, it is important to have quality criteria to choose a good video. These criteria also depend on the purpose of using the video, the goals of integrating it with the lesson and the goals of the lesson. With this background, the aim of this study is to support teachers in choosing good videos and demonstrating how a video can be integrated into their teaching plan to enhance students' mathematical understanding. Design based research methodology is used in two cycles to achieve these goals. Firstly, the quality criteria presented in this poster were designed based on available literature on teaching with multimedia, the researchers' own experience in using technology in teaching and research results regarding quality of e-learning environments. Then, the researchers and a group of secondary mathematics teachers worked collaboratively in modifying the quality criteria based on the teachers' experience in using video tutorials in their teaching.

References

Ljubojevic, M., Vaskovic, V., Stankovic, S., & Vaskovic, J. (2014). Using supplementary video in multimedia instruction as a teaching tool to increase efficiency of learning and quality of experience. *The International Review of Research in Open and Distributed Learning, 15*(3). https://doi.org/10.19173/irrodl.v15i3.1825.

Jones, T., & Cuthrell, K. (2011). YouTube: Educational potentials and pitfalls. *Computers in the Schools, 28*, 75-85. doi: 10.1080/07380569.2011.553149

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

HYBRID LESSON STUDY AND CULTURAL TRANSPOSITION: INTERTWINED REQUIRMENTS IN AND FOR IMPROVING TEACHERS' KNOWLEDGE AND PRACTICES

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In Mathematics, Lesson Study has been developed/implemented with a variety of realizations and associated perspectives – necessarily related with the views, aims and understandings of the nature of Lesson Study and the role of context and culture in which such approach has been developed and is being implemented. One of such realizations, being developed in Brazil, considers a perspective of what we have named as a Hybrid Lesson Study – HLS (Ribeiro, Fiorentini, Losano, & Crecci, 2018). It considers the intertwined nature and specificities of the culture in which it is embedded with the education practices coming from other cultural contexts – as condition for decentralizing the assumptions rooted in specific cultural paradigms and give a ground for supporting improving teachers' knowledge and mathematical practices. Such cultural approach is considered in the perspective of the so-called Cultural Transposition (CT) framework (Mellone, Ramploud, Di Paola & Martignone, 2018), and it aims at allowing a focused discussion on the educational practices coming from different cultural contexts as condition for decentralizing the assumptions rooted in specific cultural paradigms.

In this poster we will give an overview of the implementation of a HLS in Brazil with a CT perspective discussing some of the preliminary results enhancing particular features and assumptions needed to be taken into account when focusing on the context (in and) for improving teachers knowledge and mathematical practices.

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References

Mellone, M., Ramploud, A., Di Paola B., & Martignone, F. (2018). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM, Mathematics Education.* Retrieved from https://doi.org/10.1007/s11858-018-0992-7.

Ribeiro, M., Fiorentini, D., Losano, L., & Crecci, V. (2018). *Teachers' professional practices and knowledge development in a hybrid lesson study in the Brazilian context.*

TEXTBOOKS AND THE MISUSE OF CROSS MULTIPLICATION

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Cross multiplication is an algorithm used for solving proportions. A common and worldwide phenomenon relating to this algorithm is its application by students to any type of problem involving two fractions, regardless of the algorithm's appropriateness (e.g. Ncube, 2016). To understand a possible contributing factor to the widespread misuse of cross multiplication, this study asks the question, "How are proportions initially introduced to students in the USA?" In the USA, textbooks are the de facto national curriculum (Wu, 2011), so to answer this question nine reasonably popular textbooks from the largest USA publishers were selected for this study. A content analysis using: page, problem, and solution strategy counts was performed on the sections of these textbooks that cover proportions. The level of cognitive demand of the problems in these sections was assessed using Smith and Stein's (2018) framework.

The reviewed textbooks introduced only two methods for solving proportions: cross multiplication and equal ratios–a method using common denominators. Two textbooks introduced only cross multiplication, three focused primarily on cross multiplication and only briefly mentioned equal ratios, three others covered cross multiplication and equal ratios equally, and one gave slightly greater coverage to equal ratios. Of the 722 problems in the proportion sections of these textbooks, 74% specifically asked for cross multiplication as a solution strategy, while only 20% specifically asked for equal ratios. Nearly all problems were of low cognitive demand relating to proportions, the major reasons for this included: 65% gave a proportion and specifically asked for the application of an algorithm, 17% could be more easily solved using other methods, 6% were not proportion problems, and 2% asked only for a proportion to be set up.

The findings of this study, only briefly described here, indicate that textbooks in the USA focus on cross multiplication as *the strategy* to use when solving proportions and do so using low cognitive demand tasks. Since prior work has indicated that high cognitive demand tasks are needed for students to learn mathematics (Smith & Stein, 2018), this emphasis on the memorization of a single algorithm without learning the mathematics behind it may contribute to the widespread misuse of cross multiplication.

References

Ncube, M. (2016). *Analysis of errors made by learners in simplifying algebraic expressions at grade 9 level*. Unpublished dissertation, University of South Africa, Pretoria.

Smith, M. S. & Stein, M. K. (2018). *5 practices for orchestrating productive mathematics discussion*. Reston, VA: National Council of Teachers of Mathematics.

Wu, H. (2011). Phoenix rising: Bringing the common core state mathematics standards to life. *American Educator, 35*(3), 3-13.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

LENGTH MEASUREMENT AND ESTIMATION IN PRIMARY SCHOOL

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Due to the importance of length measurement learning (e.g. Barrett, Clements, & Sarama, 2017), we want to deepen the understanding of similarities and differences on it in different countries. Starting with the comparative analysis of Lee & Smith III (2011) between the US and Singapore, we analyzed the similarities and differences in Taiwanese and German elementary written textbooks concerning the treatment of length understanding, measurement, and estimation.

The length measurement units of four German elementary textbook series and three from Taiwan form the corpus of analysis. In applying the coding scheme of Lee and Smith III (2011) to our data, we added BL (benchmark learning) and CS (curvy vs. straight line) as conceptual categories, and RJ (reasoning and justification) as a category linking between conception, procedure, metacognition, and language.

Nearly the same time for teaching length is intended in both countries. As in other countries, procedural affordances dominate the conceptual learning situations in both countries, a tendency which was stronger for the German than the Taiwanese textbook series.

Although *units can be converted*, *reasoning and justification* and *benchmark learning* were the most common conceptual elements in both countries, their importance differed. The Taiwanese textbooks mainly stressed the idea of conversion, whereas the German textbooks focused more on benchmark learning. The comparison of procedural elements showed that both countries very often focused on unit conversion, measuring with a ruler, and addition and subtraction of length. In the Taiwanese textbooks more tasks asked for direct comparison and measurement with nonstandard units, whereas the German textbooks stressed visual estimation.

References

Barrett, J. E., Sarama, J. & Clements, D. H. (Eds.). (2017). *Children's measurement: A longitudinal study of children's knowledge and learning of length, area, and volume*. Reston, VA: NCTM.

Lee, K. & Smith III, J. P. (2011). What is different across an ocean? How Singapore and US elementary mathematics curricula introduce and develop length measurement. *ZDM Mathematics Education 43*(5), 681–696. [DOI 10.1007/s11858-011-0339-0]

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

A CONSIDERATION OF DIDACTICAL PHENOMENOLOGY

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In Freudenthal's theory, didactical phenomenology is used for analyzing teaching materials in order to make learning process based on method of "re-invention" to be concrete. However, there is no mention about how to analyze teaching materials in Freudenthal's work. Yamaguchi (1996) proposed a framework of analysis, which has three categories: 1) finding mental object in mathematical concept which is object of analysis, 2) considering how phenomenon is organized by the mental object, and 3) based on previous two categories, describing the mathematical concept which adjust learning process. However, it is not clear enough that how mathematical concept is described after analyzing in Yamaguchi's framework, since concrete example of teaching materials has not mentioned.

The aim in this paper is to explore how mathematical concepts are described by the analysis in Yamaguchi's framework. In short, didactical phenomenology means describing noumenon which is mathematical concept, mathematical structure, mathematical idea, in its relation to the phenomenon which is organized by noumenon, with a focus on how to acquire by leaner in leaning-teaching process. Noumenon will be phenomenon in the next level, and the phenomenon is organized by higher noumenon. It seems that phenomena and noumena have hierarchical structure. Freudenthal (1983) described that "The reader of this didactical phenomenology should keep in mind that we view the noumena primarily as mental objects and only secondarily as concepts" (p.33). Noumenon may have two aspects which are mathematical object (mathematical concept, etc.) and mental object. Noumenon as mental object is what construct as object of consciousness with reality, in the process of construct noumenon as mathematical object. In this sense, mathematical concept which is analyzed with Yamaguchi's framework, is to be described in hierarchical structure with its phenomenon and mental object. The model of hierarchical structure and some case analysis with High school teaching materials will be presented in my poster.

References

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures.* Dordrecht-Holland: D. Reidel.

Yamaguchi, J. (1996). Content Oriented Study on H. Freudenthal's Didactical Phenomenology (2). *Journal of the Chugoku-Shikoku Society of Education, 42*(2), 269-274 (in Japanese).

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

MATHEMATICS LEARNING OPPORTUNITIES FOR STUDENTS WITH LEARNING DISABILITIES AT SPECIAL SCHOOLS

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An inclusive education system is one that generates and promotes access, participation and learning opportunities for all students. Specifically, for students with intellectual disabilities (ID) who historically have never had access to high quality mathematics programs, the Ministry of Education of Chile has endorsed since 2013 a Functional Mathematics Program (FMP) based on Brousseau's approaches. However, there is not enough research on what learning opportunities (LO) in mathematics (McDonnell, 1995) are promoted at special schools and if these LO are improved with the implementation of the FMP. In this context, an investigation was conducted to analyse the LO provided to students with ID in courses that implement the FMP and courses that do not, exploring the relation between those LO, school context elements and teacher's characteristics.

This is a multiple case study (Yin, 2000) with each of the 8 cases corresponding an elementary grade math course in a special school, 4 courses implement the FMP and 4 do not. The information was collected by: i) Students registers (assessments, notebooks); ii) Classroom observation (video, audio); iii) Teacher's interviews. Three main categories were considered: mathematical skills, curricular content and level cognitive demand involved in the tasks (Stein et al., 1996). Finally, an integrated analysis of the information was carried out. Results show LO created in these special schools are focused almost exclusively on learning "numbers and operations" and tasks mainly promote the development of skills with low levels of cognitive demand. However, the LO that are generated in the cases that have implemented the FMP have a higher level of skill and cognitive demand. Based on these results, the development of future LO research for students with ID in more educational contexts and the need to train special education teachers for the teaching of mathematics is projected. In the conference, further results will be discussed in detail.

References

McDonell, L. M. (1995). Opportunity to learn as a research concept and policy instrument. *Educational Evaluation and Policy Analysis, 17,* 305-322.

Stein, M. K, Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American educational research journal, 33*(2), 455-488.

IMPROVING THE CLARIFICATION OF ERRORS IN DIGITAL DIAGNOSTIC TESTING TASKS (DDTA) INSERTED IN MOODLE

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Especially at the beginning of a preparation course it is important to show the students which individual priorities they should set for a successful participation in this kind of course. Our aim is to be able to explain students' errors and to create automatically individual supportive feedback for each learner. So, based on the concept of Digital Diagnostic Testing Tasks we combined automatic analysis of answers with STACK and adaptive test-items with a content-related elementarisation of the task to improve the clarification of the errors (Bruder et al., 2017). In order to find out if both methods are necessary for a high clarification of errors we examined the proportion of clarification of errors p_{rs} that means $p_{rs} \coloneqq \frac{N_R}{N_W}$ where N_W is the number of all wrong answers and N_R is the number of all answers that at least partly explained. The higher the proportion of the clarification of all given errors is higher the quality of a DDTA. While STACK tests the quality of a given answer, the adaptive part tests the isolated steps of the complex task by giving new elementary tasks. We want to know if each of the two methods can clear up errors differently and thus both should be used. To show this we separated and examined the proportion of the p_{rs} using only STACK and using only the elementarisation in our study of the last winter semester. For this we used four DDTA-tasks in the diagnostic quiz in moodle which was held at the beginning of the VEMINT-preparation course for students who were starting their studies in a STEM subject in Darmstadt. There were 306 students who participated in the test. The value of general p_{rs} in each of the four DDTA-tasks was high with at least 69,9%. For example, in one task we wanted to know how the term $\sqrt{\left(\frac{a}{3}\right)^2 + a^2}$, when $a \geq 0$, can be simplified. The value of the general p_{rs} in this task was 87% among them 34,3% were explained only by using STACK, another 34,3% only by elementarisation and the remaining 18,4% by using both methods. However, when we consider all answers for all four tasks there were answers explained only by STACK ($\geq$ 4,8%) and only by elementarisation ($\geq$ 34,3%). Thus, this study shows that the use of both methods is very useful for increasing the clarification of error (p_{rs}).

References

Bruder, R., Feldt-Caesar, N., Kallweit, M., Krusekamp, S., Neugebauer, C., Schaub, M., & Winter, K. (2017). Digital Diagnostic Testing Tasks (DDTA) - Theoretical Design and interactive example. In B. Kaur, W. K. Ho, & T. L. Toh (Eds.), *Proc. 41th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2). Singapore: PME.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

ABOUT THE DEVELOPMENT OF CONCEPTS OF SETS AND NUMBERS – A QUALITATIVE CASE STUDY WITH 3- TO 4-YEAR OLDS

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It is the target of my research project to reconstruct children's mathematical knowledge to gain an insight in the development of children's concepts of sets and numbers. Therefore, children's behavior in situations where they are confronted with mathematical games (Schwank & Schwank, 2015) is observed. To analyse their concepts, the children's knowledge is reconstructed with the help of the approach of the Theory Theory (Gopnik, 2012; Gopnik & Meltzoff, 1997): Children behave as if they have a certain theory on the phenomenon area.

The spiral stairs calculation (SSC) used in the extract of the transcript offers space for 0 to 9 balls that are piled up on which players (e.g. a ladybird) can be placed (cf. Schwank & Schwank, 2015). These players can move on the SSC by jumping (moving forward or backwards on the stairs). Therefore, the basis for talks are the number of the balls at one place (statistical perspective) as well as the number of jumps (dynamical perspective).

Analyzing the scenes, key positions could have been found in which it was necessary that playing adults stimulate the child (Schlicht, 2016). With regard to a first theory on numbers of objects (balls, ladybirds or jumps), zero does not seem to be a number. Without intervention, the space with zero balls was not regarded equally as the other spaces. Only due to the subsidence of the interviewer, it is taken into consideration. On this poster, a crucial scene of a play with the mathematical playworld SSC is introduced and analyzed with the help of the outlined background theory.

References

Gopnik, A. (2012). Scientific Thinking in Young Children: Theoretical Advances, Empirical Research, and Policy Implications. *Science, 337*, 1623-1627.

Gopnik, A. & Meltzoff, A. (1997). *Words, Thoughts, and Theories*. Cambridge, MA: MIT Press.

Schlicht, S. (2016). *Zur Entwicklung des Mengen- und Zahlbegriffs.* Wiesbaden: Springer Spektrum.

Schwank, I. & Schwank, E. (2015). Development of mathematical concepts during early childhood across cultures. In J. D. Wright (Ed.), *The International Encyclopedia of the Social and Behavioral Sciences* (2. Ed., pp. 772-784). Amsterdam: Elsevier.

IMPACT OF TASK AND PERSON CHARACTERISTICS ON PERFORMANCE IN THE HOSPITAL PROBLEM

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The hospital problem by Kahneman and Tversky (1972, p. 443) and variants thereof are often used to analyze students' understanding of the empirical law of large numbers (e.g., "What is more likely: That at least 7 out of 10 births in a hospital are boys, or that at least 70 out of 100 births in a hospital are boys."). However, a review of prior studies (Lem, van Dooren, Gillard, & Verschaffel, 2011) reports mixed and conflicting empirical results of students' performance in this kind of tasks. Mixed results are hypothesized to originate from varying characteristics of the tasks used in different studies, which influence the salience of relevant task features such as the involved sample sizes and frequencies of the events.

The present study (Weixler, Sommerhoff, & Ufer, 2019) investigated the impact of multiple task characteristics, including problem context, sample sizes, and frequencies on students' performance in tasks based on the hospital problem. Also, the impact of the person characteristics gender and grade was examined. To find variants with high educational potential, effects of single task variants on subsequent tasks were analyzed.

Teacher education students ($N = 242$) specializing in mathematics each answered 16 variants of the hospital problem in randomized sequences. Responses were scored dichotomously and analyzed by using Generalized Linear Mixed Models.

Results support the hypothesis that differences in students' performance are caused by the salience of relevant task features. In particular, larger deviations from the expected relative frequency, a bigger ratio between large and small sample size, and a verbal presentation of a 100% frequency increased students' performance. Moreover, tasks including 100% frequencies appeared to trigger a better performance on subsequent tasks. These task variants thus might be educationally beneficial to trigger students' intuitions of the empirical law of large numbers in other hospital problem variants.

References

Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgement of representativeness. *Cognitive Psychology*, *3*, 430–454.

Lem, S., Van Dooren, W., Gillard, E., & Verschaffel, L. (2011). Sample size neglect problems: A critical analysis. *Studia Psychologica*, *53*(2), 123–135.

Weixler, S., Sommerhoff, D., Ufer, S. (2019). The empirical law of large numbers and the hospital problem: systematic investigation of the impact of multiple task and person characteristics. *Educational Studies in Mathematics, 100*(1), 61–82.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

TOWARD INSTRUCTIONAL IMPROVEMENT IN MATHEMATICS AT SCALE

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In this poster, we propose a model for school math instructional improvement that is adaptable to local settings and the organizations and practitioners in them. Different school districts have different problems of practice, and thus adaptive integration of interventions is important as they go to scale—as Penuel et al. (2011) find, successful "scaling up" depends on local actors who make continual, coherent adjustments to interventions as they make their way through various levels of an organization. Indeed, school- and district-level infrastructures that are not optimally designed to support instructional improvement can constrain professional development (PD) efforts to improve the effectiveness of the existing teaching force (Spillane & Hopkins, 2013). Similarly, school districts have been shown to influence the ways in which schools and school leaders implement a wide range of improvement efforts at the school level, thus helping or hindering such implementation (Honig & Rainey, 2014).

The model we propose is particularly designed to improve teachers', teacher leaders', and administrators' understanding of effective math teaching and learning, and to enhance the organizational capacities of schools and districts to support such improvements in math. The model is grounded in a Design-Based Implementation Research process involving collaboration between researchers, and district and school personnel to co-develop PD from district through teacher levels. The components are: (1) gathering information about problems of practice collaboratively identified by districts, schools, and the research team, and developing related goals; (2) designing and implementing coherent PD that is aligned with identified problems of practice; and (3) engaging in iterative cycles of development, implementation, and revision to productively adapt the model to changing conditions. The iterative redesign process enhances the productive adaptation of the model, allowing it to be effective at scale.

References

Honig, M., & Rainey, L. (2014). Central office leadership in principal professional learning communities: The practice beneath the policy. *Teachers College Record, 116*, 1-48.

Penuel, B., Fishman, B., Cheng, B., & Sabello, N. (2011). Organizing research and development at the intersection of learning, implementation and design. *Education Researcher, 40*(7), 331-337.

Spillane, J., & Hopkins, M. (2013). Organizing for instruction in education systems and school organizations: How the subject matters. *Journal of Curriculum Studies, 45*(6), 721-747.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

DIVIDING EQUALLY – RESULTS OF AN EXPLORATORY STUDY ON PRECONCEPTIONS OF FIRST GRADERS IN UGANDA

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Preconceptions, i.e. ideas held before formal instruction, have been the focus of research in science and mathematics education ever since Piaget's (1929) work. Current approaches to teaching and learning primary school mathematics all over the world frequently acknowledge that children bring important out-of-school knowledge to the classroom that influences their school-based learning.

Dividing equally is a basic concept for the understanding of division and fractions. Kieren (1993) found that being able to divide equally is central for the development of fractional number knowledge. International research in this area also suggests that children can solve sharing problems before the related mathematical operation and procedures are taught in school (e.g. see Pepper & Hunting, 1998).

The international research project underlying this poster presentation aims to explore young children's preconceptions of sharing/dividing equally on all five continents, analyzing data from Australia, Chile, China, Germany and Uganda. Four tasks were presented to groups of children from these countries asking them to equally divide in situations that imply actions of quotitioning and partitioning with and without remainders. The problems were presented in reading and writing and were also supported by a drawing that the children could choose to elaborate on for their solution. For all participants division had not been formally taught in school. The Ugandan data sample comprises 101 Grade 1 students from a private school in a major city in Uganda following the Ugandan as well an international curriculum.

The data analysis using SPSS shows that the majority of the children could solve the two tasks without remainder successfully (67 % for quotition, 64 % for partition). Even when having to deal with remainders, the success rate remained surprisingly high (59 for quotition, 49 % for partition). Overall, children using a drawing for their solution were more successful than the ones who did not. In our poster the four problems as well as typical solutions will be presented and more detailed results will be discussed.

References

Kieren, T.E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational numbers: An integration of the research* (pp. 49-84). Hillsdale, NJ: Lawrence Erlbaum Associates.

Pepper, K., & Hunting, R. (1998). Preschoolers' counting and sharing. *Journal for Research in Mathematics Education, 29*(2), 164-183.

Piaget, J. (1929). *The child's conception of the world*. New York: Harcourt Brace.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

MICRO-ETHNOGRAPHY OF COLLABORATION: SHARING PERSPECTIVES AND BROADENING LENSES WITHIN A COMMUNITY OF PRACTICE

Aisling Twohill, Siún NicMhuirí, Mary Kingston

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The community of practice that is the focus of this poster presentation is part of a larger project, tasked with developing a website of resources to support the teaching of mathematics. The collaborators are academic staff from Dublin City University (DCU) and an experienced practicing teacher who is studying at postgraduate level in DCU. As part of the website project we individually wrote lesson plans for Kindergarten to 3rd grade, which we then collaboratively evaluated. We drew on the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2013) to facilitate the articulation and negotiation of shared intentions for teaching mathematics.

Wenger describes a community of practice as a "social learning system" (2010, p. 179). Wenger stresses the duality of participation and reification and contends that meaning is negotiated in their interplay. Participation here is taken to refer to the process of taking part in social practice, as well as the relationships with others arising from the process. In our case, participation involved individual as well as collective elements. We planned, researched and reflected individually as well as participating collectively in team meetings. Reification can be understood as both process and product and is concerned with abstractions that reify something of the practice of a community. Meeting notes, agreed lesson plans, and this poster presentation can be considered a reification around which the negotiation of meaning was organised. In this poster we interrogate our collaboration, focusing on the participation, reification, and negotiation of meaning. We explore how our engagement in the community of practice informed our understandings, and how the TRU Maths framework supported us in navigating our differing lenses in order to gain access to broader perspectives. Findings will include reflections on our successes and challenges, along with research based recommendations for establishing a purposeful community of practice.

References

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM: The International Journal on Mathematics Education, 45*(4), 607-621.

Wenger, E. (2010). Communities of practice and social learning systems: the career of a concept. In C. Blackmore, (Ed.), *Social learning systems and communities of practice* (pp. 179-198). London: Springer.

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THE IMPACT OF INTERACTIVE MATHEMATICS (IM) CONTENT SOFTWARE ON STUDENTS' BASIC EDUCATION MATHEMATICS PERFORMANCE

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The availability of technological aid in mathematics education is regarded as important to improve quality mathematics learning and students' performance (Khun-inkeeree, 2016). However, mathematics teaching and learning in Rwanda is still dominated by chalk and talk teaching approaches resulting in rote learning, poor performance and a disconnection of the outcomes of teaching and learning mathematics and the community utilities in the long run (Uworwabayeho, 2009). With the current adoption of the Competence Based Curriculum (CBC) in Rwanda education system, it is necessary to investigate the influence of ICT supported mathematics classroom on the quality of mathematics education outcomes by focusing on performance. The aim of this study is to investigate the impact of using Interactive Mathematics (IM) content software for Rwanda on students' mathematics performance.

The sample consisted of 57 students from Primary 4, Primary 5 and senior 1 with their respective mathematics teachers. Results from pre-test and post-test indicate that, considering all 57 students, the overall mathematics score increased by 22.8%. Using the best class as an example, the average correct answers in the pre-test was 36.7% and become 78.9% in the post- test. Besides, students 'engagement, interest and motivation to learn were remarkable during the period of treatment. From the findings, the IM implementation has proven its impact in increasing students' performance. However, effective strategies must be adopted for the IM to be a teaching and learning tool and not an object to students' distraction. If a similar study can be conducted in a real classroom situation with an improved methodology and a focus on particular content this may bring more reliable results.

References

Khun-inkeeree, H. (2016). The Mathematics Performance of Primary School Students in Southern Thailand, (June 2017). https://doi.org/10.6007/IJARPED/v5-i3/2208

Uworwabayeho, A. (2009). Teachers' innovative change within countrywide reform: a case study in Rwanda. *Journal of Mathematics Teacher Education, 12*(5), 315-324.

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GOING BEYOND NUMBERS AND COUNTING: THE DEVELOPMENT OF EARLY MATHEMATICAL COMPETENCIES

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Mathematical competencies in early childhood, both in research as in practice, are often reduced to basic number and arithmetic skills. The current project starts from the assumption that it is possible to assess and stimulate the development of more complex mathematical competencies at younger ages than is currently the case. More specifically, this project looks into the emergence and development of young children's patterning, computational estimation, proportional reasoning and probabilistic reasoning competencies. These four mathematical competencies, as well as basic number competence, math achievement, general cognitive skills, and home and classroom environment are assessed in a longitudinal study following children from 4 to 9 years old (N=409). We aim to look into the development of these complex mathematical competencies, but also into the association between them, and the way a certain competency may facilitate the development of another competency.

We exemplarily look into whether early patterning predicts the later development of proportional reasoning. Some authors have described how repeating patterns can be used to introduce ratio and proportion (e.g., Warren & Cooper, 2007), but it remains unclear whether patterning and proportional reasoning competencies are also longitudinally related. In this study we examined the extent to which 4-year olds' patterning performance explained individual differences in proportional reasoning 2 years later. A medium positive correlation (r=.363, p<0.01) was found between patterning and proportional reasoning competence. A hierarchical linear regression analysis showed that patterning ability at age four uniquely explained 2.0% of the variance in proportional reasoning at age six, over and above sex, age, numerical, and cognitive competencies. The total model explained 41.0% of the variance. This study provides evidence for an association between early patterning and proportional reasoning competencies. Our results suggest that focusing on these more complex mathematical competencies is valuable for future studies, given that they can be assessed at a young age and that they are predictive for later mathematical development. Research may also investigate whether these competencies can be stimulated, and whether this impacts the development of other competencies.

References

Warren, E., & Cooper, T. (2007). Repeating patterns and multiplicative thinking: analysis of classroom interactions with 9-year-old students that support the transition from the known to the novel. *Journal of Classroom Interaction, 41*(2), 7-17.

THE ROLE A MALE TEACHER PLAYS IN THE RELATIONSHIP FEMALE LEARNERS BUILD WITH MATHEMATICS IN EARLY ELEMENTARY YEARS

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Numerous studies show the gender gap in math achievement has nearly closed. However, there is a gender gap in the relationship formed with math favouring males (Erturan & Jansen, 2015), which should not be disregarded as it significantly influences decisions to embrace or avoid math. The effects of this gap can be observed in the percentage of females in STEM degrees and careers. Female elementary teachers with a poor math relationship have been shown to pass on their relationship to female students (Beilock, Gunderson, Ramirez, Levine, & Smith, 2010). There are calls for more male elementary teachers with little justifying data. The purpose of this research study was to understand the role a male teacher plays in the relationship female learners build with mathematics in the early elementary years.

A qualitative methodological approach through means of exploratory case study was used. The study was guided by a constructivist framework that acknowledges social and individual cognitive aspects of learning. Ten early elementary female students from two schools were engaged in one-on-one semi-structured interviews allowing the researcher to gain insight into their understanding of their relationship with math while having a male teacher. Data analysis was conducted with pattern coding using NVIVO software. Triangulation amongst the interviews was used as validation.

Five strong themes emerged from the data including: participants want to be good at math; believe that their teacher likes math; think that math is fun; think fast equals good in math; had negative experiences with math. Teacher's belief in a student's learning ability appeared to be key in the relationship the student formed with math. This study suggests male teachers can have a positive role in the relationship female learners build with math in the early elementary years by believing in female math learning ability, being knowledgeable and having fun with math; while can negatively affect the relationship by allowing too much competition and focus on speed in math.

References

Erturan, S., & Jansen, B. (2015). An investigation of boys' and girls' emotional experience of math, their math performance, and the relation between these variables. *European Journal of Psychology of Education, 30*(4), 421-435.

Beilock, S. L., Gunderson, E. A., Ramirez, G., Levine, S. C., & Smith, E. E. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences of the United States of America, 107*(5), 1860-1863.

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1). Pretoria, South Africa: PME

INTERVENTION TO DEVELOP SPATIAL REASONING STRATEGIES FOR PRIMARY SCHOOL STUDENTS THROUGH MENTAL FOLDING OPERATIONS

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The ability to mentally rotate objects in space has been singled out by cognitive scientists as a central metric of spatial reasoning (Jansen, Schmelter, Quaiser-Pohl, Neuburger & Heil 2013). However, there are few studies on spatial reasoning strategies used by students to develop their spatial reasoning skills and thus I hypothesized if students would be able to find a particular spatial reasoning strategy when doing a mental folding operation, such as to choose side of an imaginary unfolded cube, and then move or rotate it in their minds to be able to see its other faces, then they would be able to see its other faces, then they would and notice the usefulness of these spatial reasoning skills when used in other tasks while also developing their spatial reasoning skills by themselves. Accordingly, the question arises as to how students' spatial reasoning skills can develop though classroom-based interventions involving mental folding operations. In view of this, the research questions is:

- Is it appropriate to choose the bottom side of an imaginary unfolded cube as a spatial reasoning strategy for students to imagine mental folding operations?

Moreover, I designed lesson involving assessments that were intended to develop students' spatial reasoning skills using mental folding operations. A total of 33 primary school students in the fourth grade (aged 10 years) participated in the lesson; The lesson consisted of determining where to place the missing face of an imaginary unfolded cube to assemble it correctly; at this point, I observed a group of students who found a spatial reasoning strategy that involved choosing the imaginary unfolded cube's bottom side and manipulating its sides, so as to identify the corresponding upper face. Furthermore, these successful students also explained this strategy to the others. In total, six of the right students, who had low score in the pre-test assessment improved in the post-test assessment. Based on that, it was possible to infer that they had acquired experience in applying a learned spatial reasoning strategy to other problems, and understood the usefulness of the mental folding operations and the spatial reasoning strategies that comes with it.

References

Jansen, P., Schmelter, A, Quaiser-Pohl, C., Neuburger, & Heil, M. (2013). Mental rotation performance in primary school age children: are there gender differences in chonometric test? *Cognitive Development, 28,* 51-62.

REIMAGINING PROFESSIONAL DEVELOPMENT FOR IRISH MATHEMATICS TEACHERS

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Upon completion of initial teacher education, continuous professional development (CPD) opportunities are often the only formal education teachers receive in how to teach. The current format of half-day, in-school, generic workshops offered to teachers in Ireland, although common internationally, has been shown to be ineffective. Recent reform of the Irish mathematics curriculum has placed an emphasis on mathematical problem solving, and while much research has been conducted on the teaching and learning of problem solving, research-based knowledge rarely makes its way into classrooms. This furthers the problem of a disconnect between theory and practice.

This study aims to address the theory-practice disconnect by designing a school-based CPD programme for Irish secondary school mathematics teachers that engages them in a reflective community of practice. The objectives are: (i) to examine the classroom realities of a group of teachers and analyse how they are currently teaching mathematical problem solving; (ii) to design a longitudinal, evidence-based, support-centred CPD programme based on the work of Loucks-Horsley and Matsumoto (1999), among others; and (iii) to monitor the programme's impact on mathematics teaching practice and student learning.

Initially a nationwide survey was sent to all mathematics teachers in Ireland to gain insight into both their current teaching practices and prior experience with CPD. While the 115 respondents spent 15.5 hours, on average, in mathematics specific CPD in the past 12 months, they report that over 70% of their student contact time is still used for the traditional *explain then practice* style of teaching. Group and individual problem solving is given little attention, being allocated the same proportion of class time as examinations and administration tasks.

The novel CPD programme designed as part of this research study has the potential to impact the teaching and learning of mathematical problem solving, in Ireland and other countries. This poster presents preliminary survey findings together with the design and rationale for the planned intervention programme.

References

Loucks-Horsley, S. & Matsumoto, C. (1999). Research on professional development for teachers of mathematics and science: The state of the scene. *School Science and Mathematics, 99*, 258-271.

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