

University of Groningen

## Nonsingular and ghost-free infinite derivative gravity with torsion

de la Cruz-Dombriz, Alvaro; Torralba, Francisco Jose Maldonado; Mazumdar, Anupam

*Published in:*  
Physical Review D

*DOI:*  
[10.1103/PhysRevD.99.104021](https://doi.org/10.1103/PhysRevD.99.104021)

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2019

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

de la Cruz-Dombriz, A., Torralba, F. J. M., & Mazumdar, A. (2019). Nonsingular and ghost-free infinite derivative gravity with torsion. *Physical Review D*, 99(10), [104021].  
<https://doi.org/10.1103/PhysRevD.99.104021>

### Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

### Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

## Nonsingular and ghost-free infinite derivative gravity with torsion

Álvaro de la Cruz-Dombriz,<sup>1</sup> Francisco José Maldonado Torralba,<sup>1,2</sup> and Anupam Mazumdar<sup>2</sup>

<sup>1</sup>*Cosmology and Gravity Group, Department of Mathematics and Applied Mathematics,  
University of Cape Town, Rondebosch 7701, Cape Town, South Africa*

<sup>2</sup>*Van Swinderen Institute, University of Groningen, 9747 AG Groningen, Netherlands*



(Received 22 January 2019; published 13 May 2019)

We present the most general quadratic curvature action with torsion including *infinite covariant derivatives* and study its implications around the Minkowski background via the Palatini approach. Provided the torsion is *solely* given by the background axial field, the metric and torsion are shown to decouple, and both of them can be made ghost and singularity free for a fermionic source.

DOI: [10.1103/PhysRevD.99.104021](https://doi.org/10.1103/PhysRevD.99.104021)

### I. INTRODUCTION

The theory of general relativity (GR), which respects diffeomorphism invariance, can be modified to incorporate the gauge structure of the Poincaré group, provided a torsion field is added [1]. This is known as Poincaré gauge gravity (cf. Ref. [2]). Both GR and Poincaré gravity suffer from the short distance behavior at a classical level in terms of black hole and cosmological singularities, known as the UV problem. In this paper, our aim will be to construct an action which recovers GR and Poincaré theory of gravity in the IR, while ameliorating the UV behavior of both metric and torsion fields. Infinite derivative gravity (IDG) can potentially ameliorate the classical UV behavior of a metric theory of gravity [3–6]. Different approaches of IDG have been made in the context of *teleparallel* gravity [7] and symmetric teleparallel gravity [8]. However, constructing a Poincaré gravity possessing a better UV behavior at a classical level remains very challenging. To the best of our knowledge, the systematic study of infinite derivative extensions of the Poincaré gravity has never been done before. The aim of this paper will be to construct an action including metric and torsion fields up to quadratic in curvature with infinite covariant derivatives.

### II. MOST GENERAL QUADRATIC ACTION WITH TORSION

We start with the most general covariant action of gravity with no prior assumptions on the connection. In order to obtain the *quadratic* action, let us consider perturbations around the Minkowski metric  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , where  $\mu, \nu = 0, 1, 2, 3$  and we work with the  $(-, +, +, +)$  signature. We stick to the terms up to quadratic in curvature  $\mathcal{O}(h^2)$ . Regarding the connection, the only requirement is that the metricity condition, i.e.,  $\nabla_\rho g_{\mu\nu} = 0$ , is fulfilled. With that, one can find a relation between the Levi-Civita connection  $\Gamma$  and the general one  $\tilde{\Gamma}$ , namely  $\tilde{\Gamma}^\rho_{\mu\nu} \doteq \Gamma^\rho_{\mu\nu} + K^\rho_{\mu\nu}$ , where  $K$  is

the so-called *contorsion* tensor,  $K^\rho_{\mu\nu} \doteq T^\rho_{\mu\nu} + T^\rho_{\nu\mu} + T^\rho_{\nu\mu}$ , which is defined in terms of the *torsion* tensor,  $T^\mu_{\nu\rho} = \tilde{\Gamma}^\mu_{[\nu\rho]}$ , where the symbol  $[\cdot]$  means antisymmetrization of the indices. Note that if the effects of torsion in the action are to be considered the deviation from the Levi-Civita connection must be  $\mathcal{O}(h)$ , i.e.,  $K \sim \mathcal{O}(h)$ , which is compatible with experiment [9]. The most general action for metric and torsion, quadratic in both, which generalizes both the metric [6,10,11] and the teleparallel actions [7], and generalization of second order curvature invariants [12] will be of the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{\tilde{R}}{2} + \tilde{R}_{\mu_1\nu_1\rho_1\sigma_1} \mathcal{O}_{\mu_2\nu_2\rho_2\sigma_2}^{\mu_1\nu_1\rho_1\sigma_1} \tilde{R}^{\mu_2\nu_2\rho_2\sigma_2} + \tilde{R}_{\mu_1\nu_1\rho_1\sigma_1} \mathcal{O}_{\mu_2\nu_2\rho_2}^{\mu_1\nu_1\rho_1\sigma_1} K^{\mu_2\nu_2\rho_2} + K_{\mu_1\nu_1\rho_1} \mathcal{O}_{\mu_2\nu_2\rho_2}^{\mu_1\nu_1\rho_1} K^{\mu_2\nu_2\rho_2} \right], \quad (1)$$

where  $\mathcal{O}$  denote differential operators containing covariant derivatives and the Minkowski metric  $\eta_{\mu\nu}$ . Also, the tilde ( $\tilde{\cdot}$ ) represents the quantities calculated with respect to the total connection  $\tilde{\Gamma}$ . Indeed, Eq. (1) is the most general action satisfying the aforementioned requirements, since operators acting on the left can always be integrated by parts to provide operators acting on the right plus total derivatives. Action (1) is captured by 46 functions containing infinite covariant derivatives. Such functions reduce to 19 when imposing the Bianchi identities, and the total derivatives are taken into account; see the Appendix. Further note that the usual Poincaré gauge gravity (including Einstein-Cartan gravity) can be recovered from (1), provided one takes the local limit. The linearized version of the action (1) around the Minkowski background can be written as

$$S_q = - \int d^4x \sqrt{-g} (\mathcal{L}_M + \mathcal{L}_{MT} + \mathcal{L}_T) = S_M + S_{MT} + S_T, \quad (2)$$

where

$$\begin{aligned} \mathcal{L}_M = & \frac{1}{2} h_{\mu\nu} \square a(\square) h^{\mu\nu} + h_{\mu}^{\alpha} b(\square) \partial_{\alpha} \partial_{\sigma} h^{\sigma\mu} + hc(\square) \partial_{\mu} \partial_{\nu} h^{\mu\nu} \\ & + \frac{1}{2} h \square d(\square) h + h^{\lambda\sigma} \frac{f(\square)}{\square} \partial_{\sigma} \partial_{\lambda} \partial_{\mu} \partial_{\nu} h^{\mu\nu}, \end{aligned} \quad (3)$$

$$\begin{aligned} \mathcal{L}_{MT} = & h \square u(\square) \partial_{\rho} K^{\rho\sigma}{}_{\sigma} + h_{\mu\nu} v_1(\square) \partial^{\mu} \partial^{\nu} \partial_{\rho} K^{\rho\sigma}{}_{\sigma} \\ & + h_{\mu\nu} v_2(\square) \partial^{\nu} \partial_{\sigma} \partial_{\rho} K^{\mu\sigma\rho} + h_{\mu\nu} \square w(\square) \partial_{\rho} K^{\rho\mu\nu}, \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{L}_T = & K^{\mu\sigma\lambda} p_1(\square) K_{\mu\sigma\lambda} + K^{\mu\sigma\lambda} p_2(\square) K_{\mu\lambda\sigma} + K_{\mu}^{\rho} p_3(\square) K^{\mu\sigma}{}_{\sigma} \\ & + K_{\nu\rho}^{\mu} q_1(\square) \partial_{\mu} \partial_{\sigma} K^{\sigma\nu\rho} + K_{\nu\rho}^{\mu} q_2(\square) \partial_{\mu} \partial_{\sigma} K^{\sigma\rho\nu} \\ & + K_{\mu\nu}^{\rho} q_3(\square) \partial_{\rho} \partial_{\sigma} K^{\mu\nu\sigma} + K_{\mu\nu}^{\rho} q_4(\square) \partial_{\rho} \partial_{\sigma} K^{\mu\sigma\nu} \\ & + K^{\mu\rho}{}_{\rho} q_5(\square) \partial_{\mu} \partial_{\nu} K^{\nu\sigma}{}_{\sigma} + K^{\lambda}{}_{\lambda\sigma} q_6(\square) \partial_{\mu} \partial_{\alpha} K^{\sigma\mu\alpha} \\ & + K_{\mu}^{\nu\rho} s(\square) \partial_{\nu} \partial_{\rho} \partial_{\alpha} \partial_{\sigma} K^{\mu\alpha\sigma}, \end{aligned} \quad (5)$$

where  $a, b, c, d, f, u, v_{1,2}, w, p_{1,2,3}, q_{1,2,3,4,5}$ , and  $s$  are functions of covariant *infinite derivatives* of  $\square = g^{\mu\nu} \nabla_{\mu} \nabla_{\nu}$  (see the Appendix for a detailed discussion). Note that  $\square$  is a dimensionful quantity, since strictly speaking there is a scale  $\square_s = \square/M_s^2$ , where  $M_s$  is the new scale at which gravity is modified in four dimensions with  $M_s < M_{\text{Planck}} = 1.2 \times 10^{19}$  GeV. In order not to clutter our formulas, we shall suppress writing  $M_s$ . Furthermore, note that  $\mathcal{L}_M$  in (3) has only metric terms and coincides with the nontorsion case Lagrangian [6], as expected. On the other hand,  $\mathcal{L}_{MT}$  in (4) represents the mixed terms between metric and torsion, and  $\mathcal{L}_T$  in (5) only contains torsion terms. Thus, Eq. (2) is the new most generalized linearized action of gravity without making any assumption about the choice of connection. To the best of our knowledge, this is the first such generalization within Poincaré theory of gravity. Furthermore, when  $\square/M_s^2 \rightarrow 0$ , then the terms involving the operators  $u$  and  $w$  tend to 0, recovering the local action, therefore yielding a Poincaré gauge gravity in the IR. In general, these two terms in (4) break the Poincaré invariance.

### III. FIELD EQUATIONS

We apply the Palatini formalism [13] to obtain the field equations, finding differences with respect to the torsion-free case in both the Einstein and Cartan equations. We vary action (2) with respect to the metric, i.e.,  $\delta_g/\delta g^{\mu\nu}$ , to find the Einstein equations and with respect to contorsion  $K_{\nu\rho}^{\mu}$  to yield the Cartan equations, i.e.,  $\delta_K/\delta K_{\nu\rho}^{\mu}$ . Equations of motion derived from  $\mathcal{L}_M$  when varying with respect to metric tensor have already been calculated in Refs. [6,14], which serves as a consistency check for our calculations. It is worth noting that this is the first time the Palatini approach has been used in the context of IDG, since the existing literature has always assumed the Levi-Civita as the underlying connection, hence only variations with

respect to the metric are performed to obtain the field equations.

#### A. Einstein equations

Variations with respect to the metric in  $\mathcal{L}_M$  as presented in (4) yield

$$\begin{aligned} \frac{\delta_g S_M}{\delta g^{\mu\nu}} = & \square a(\square) h_{\mu\nu} + b(\square) \partial_{\sigma} \partial_{\nu} h_{\mu}^{\sigma} \\ & + c(\square) [\partial_{\mu} \partial_{\nu} h + \eta_{\mu\nu} \partial_{\rho} \partial_{\sigma} h^{\rho\sigma}] + \eta_{\mu\nu} \square d(\square) h \\ & + \frac{2f(\square)}{\square} \partial_{\mu} \partial_{\nu} \partial_{\rho} \partial_{\sigma} h^{\rho\sigma}, \end{aligned} \quad (6)$$

an expression which is compatible with the results in Ref. [6].  $f(\square)$  can be proven to have a polynomial form in  $\square$ , so there are no inverse, nonanalytic  $1/\square$  operators involved in (6). From the explicit expression of the functions in (6), the following relations can be obtained [6],

$$\begin{aligned} a(\square) + b(\square) = 0, \quad c(\square) + d(\square) = 0, \\ b(\square) + c(\square) + f(\square) = 0, \end{aligned} \quad (7)$$

which are a consequence of the Bianchi identities and the conservation of the energy-momentum tensor. For  $\mathcal{L}_{MT}$ , we have

$$\begin{aligned} \frac{\delta_g S_{MT}}{\delta g^{\mu\nu}} = & \eta_{\mu\nu} \square u(\square) \partial_{\rho} K^{\rho\sigma}{}_{\sigma} + v_1(\square) \partial_{\mu} \partial_{\nu} \partial_{\rho} K^{\rho\sigma}{}_{\sigma} \\ & + v_2(\square) \partial_{\sigma} \partial_{\rho} \partial_{\nu} K_{\mu}^{\sigma\rho} + \square w(\square) \partial_{\rho} K^{\rho}{}_{(\mu\nu)}. \end{aligned} \quad (8)$$

Again, from the explicit decomposition of the functions in (8) (see Appendix), one obtains

$$u(\square) + v_1(\square) = 0, \quad v_2(\square) - w(\square) = 0, \quad (9)$$

where the second constraint above arises from the conservation of the energy-momentum tensor. Interestingly, when  $u(\square) = v_1(\square) = v_2(\square) = w(\square) = 0$ , the mixed term  $\mathcal{L}_{MT}$  vanishes. Note that the contributions of the contorsion tensor in (8) are purely *symmetric*. In the next section, this property will allow us to obtain solutions able to ameliorate the classical UV behavior.

#### B. Cartan equations

Variations now with respect to the contorsion in (4) and (5) yield

$$\begin{aligned} \frac{\delta \mathcal{L}_{MT}}{\delta K_{\nu\rho}^{\mu}} = & -\square u(\square) \partial^{[\nu} \eta_{\mu]}^{\rho} h - v_1(\square) \partial^{\alpha} \partial^{\beta} \partial^{[\nu} \eta_{\mu]}^{\rho} h_{\alpha\beta} \\ & + v_2(\square) \partial^{\beta} \partial^{\rho} \partial^{[\nu} h_{\mu]\beta} + \square w(\square) \partial_{[\mu} h^{\nu]\rho}, \end{aligned} \quad (10)$$

$$\begin{aligned}
 \frac{\delta \mathcal{L}_T}{\delta K^\mu{}_{\nu\rho}} &= 2p_1(\square)K_\mu{}^{\nu\rho} + 2p_2(\square)K_{[\mu}{}^{\rho]\nu} + 2p_3(\square)\eta^{\nu[\rho}K_{\mu]}{}^{\sigma}{}_{\sigma} - 2q_1(\square)\partial_\sigma\partial_{[\mu}K^{\rho]\nu\sigma} + 2q_2(\square)\partial_\sigma\partial_{[\mu}K^{\sigma\rho]\nu} \\
 &+ q_3(\square)(\partial^\nu\partial_\sigma K_{[\mu}{}^{\rho]\sigma} + \partial_\sigma\partial^{[\rho}K_{\mu]}{}^{\sigma\nu}) + 2q_4(\square)\partial^\nu\partial_\sigma K_{\mu}{}^{\sigma\rho} + 2q_5(\square)\eta^{\nu[\rho}\partial_{\mu]}\partial_\lambda K^{\lambda\sigma}{}_{\sigma} \\
 &+ q_6(\square)(\partial_\lambda\partial_\alpha\eta^\nu{}_{[\mu}K^{\rho]\lambda\alpha} - \partial^\nu\partial^{[\rho}K_{\mu\lambda]}{}^{\lambda}) + 2s(\square)\partial^\sigma\partial^\lambda\partial^\rho\partial^\nu K_{\mu\sigma\lambda}.
 \end{aligned} \tag{11}$$

Note that in the standard Poincaré gravity there are no mixed terms. However, in our case, the presence of differential operators causes the emergence of mixed terms between the metric and the torsion. While applying the Palatini formalism, there will be terms in the Cartan equations which are nonzero even if the torsion is set null. If we set the torsion to zero, the Cartan equations do not play any role in the dynamics of the system, and variations with respect to the contorsion cannot be performed. Hence, the sole remaining equations would be the usual Einstein equations take the same form as in Ref. [6].

#### IV. SOLUTIONS

In order to obtain solutions for the classes of theories provided by the action (2), let us recall the torsion tensor property thanks to which such a tensor can be uniquely decomposed as [2]

$$T_{\mu\nu\rho} = \frac{1}{3}(T_\nu g_{\mu\rho} - T_\mu g_{\nu\rho}) - \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma}S^\sigma + q_{\mu\nu\rho}, \tag{12}$$

where the components above are given by the *trace* vector  $T_\mu = T^\nu{}_{\mu\nu}$ , the *axial* vector  $S^\mu = \varepsilon^{\rho\sigma\nu\mu}T_{\rho\sigma\nu}$ , and a tensor,  $q^\mu{}_{\nu\rho}$ , such that  $q^\nu{}_{\mu\nu} = 0$  with  $\varepsilon^{\rho\sigma\nu\mu}q_{\rho\sigma\nu} = 0$ , where  $\varepsilon$  denotes the totally antisymmetric tensor in four dimensions. Making use of this decomposition, we find that the only components contributing in the Einstein equations turn out to be the trace and the tensor parts. Therefore, if we assume that the torsion field only possesses a nonvanishing axial component, the Einstein equations would reduced to those in Ref. [6]. This means that the metric solutions would be the same ones as in standard IDG. Nevertheless, Cartan equations still need to be solved for those metrics in order to obtain the torsion field solutions. One possible solution would then be

$$u(\square) = v_1(\square) = v_2(\square) = w(\square) = 0, \tag{13}$$

which yields  $\mathcal{L}_{MT} = 0$ ; i.e., the metric and torsion fields are decoupled. As a consequence, in this scenario (13), the degrees of freedom (d.o.f.) can be studied for the metric and the Cartan theories separately. Consequently, the theory space reduces to that of the Poincaré gauge gravity. Now, let us study the conditions for which the field equations do not host extra dynamical d.o.f.

#### V. GHOST-FREE CONDITIONS FOR METRIC AND TORSION

The equations of motion for the pure metric theory are given by (6). Since there are infinite covariant derivatives present, new dynamical d.o.f. arise, which may include ghosts. It has been shown in Ref. [6] that, in order to ensure the metric part of the theory is ghost free with the same on-/off-shell d.o.f. as that of the massless graviton in four dimensions, we would require [6]

$$a(\square) = c(\square) = e^{\gamma(\square)}, \tag{14}$$

where  $\gamma(\square)$  is an *entire function*, which has no poles, suggesting that  $a(\square)$  does not introduce any new dynamical d.o.f. [6]. We can also show that the form factor  $\tilde{F}_3(\square)$  in (A1) becomes redundant due to the fact that the Weyl part does not contribute at the background level around the Minkowski spacetime. The simplest choice would be to consider  $\gamma(\square) = \square/M_s^2$ . Also, the above expression (14) for  $a(\square)$  appears in the graviton propagator. The gauge-independent part of the graviton propagator can be recast in terms of the spin projection operators, i.e., spin-2,  $P^{(2)}$ , and spin-0,  $P^{(0)}$  [6],

$$\Pi(k^2) = \frac{1}{a(k^2)} \left[ \frac{P^{(2)}}{k^2} - \frac{P^{(0)}}{2k^2} \right] = \frac{1}{e^{\gamma(k^2)}} \Pi(k^2)^{(GR)}, \tag{15}$$

where spacetime indices have been suppressed. As a consequence, provided  $a(k^2)$  is given by an exponential of an entire function, then it does not introduce any new pole nor any new dynamical d.o.f., and therefore the true dynamical d.o.f. remain that of the massless GR. In order to obtain the ghost-free conditions for the torsion axial vector, the relevant Lagrangian  $\mathcal{L}_T$  in (2) can be rewritten as

$$\mathcal{L}_T = S_\mu \square \Lambda(\square) S^\mu - S_\mu \Sigma(\square) \partial^\mu \partial_\nu S^\nu, \tag{16}$$

where  $\Sigma(\square) = q_1(\square) - q_2(\square) - q_3(\square) + q_4(\square)$  and  $\Lambda(\square) = 3(p_1(\square) + p_2(\square)) + \Sigma(\square)$ . Hence, the Cartan equations in vacuum become

$$\square \Lambda(\square) S^\mu - \Sigma(\square) \partial^\mu \partial_\nu S^\nu = 0. \tag{17}$$

Consequently, now the torsion propagator  $\mathcal{O}$  can be recast in terms of its corresponding d.o.f., namely the spin-0 and spin-1 modes, as follows:

$$\mathcal{O}(k^2) = \frac{P^{(0)}}{\Lambda(-k^2) - \Sigma(-k^2)} + \frac{P^{(1)}}{-k^2\Lambda(-k^2)}. \quad (18)$$

In order to have a ghost-free axial vector field, we need to impose that the scalar mode does not propagate, so only the spin-1 component propagates, and both  $\Lambda$  and  $\Sigma$  in (16) must be of the form of an exponential of an entire function in order to not introduce any new d.o.f. Therefore, we have

$$\Lambda(-k^2) = \Sigma(-k^2) = e^{\beta(k^2)}, \quad (19)$$

where  $\beta$  is an entire function, which introduces neither new poles nor new d.o.f.

## VI. AVOIDANCE OF POINT-SOURCE SINGULARITY

The simplest choice of both entire functions  $\beta$  and  $\gamma$  would be  $\beta(k^2) \sim \gamma(k^2) = k^2/M_s^2$ . Other choices for these entire functions can also be made [15], but neither the UV nor the IR part is overtly sensitive enough to such choices. Let us first deal with the metric theory of gravity, where it has already been shown that, in the presence of a massive static point source, the 00 component of the energy-momentum tensor can be written as:  $\tau_{00} = m\delta^{(3)}(r)$ . Accordingly, the Einstein equations (6) along with (7) yield a nonsingular solution in isotropic coordinates,

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{r}^2, \quad (20)$$

$$\Phi(r) = \Psi(r) = -\frac{Gm}{r} \text{Erf}(rM_s/2), \quad (21)$$

where  $G$  is Newton's constant and one can see that when  $r \gg 2/M_s$  we recover the correct IR limit of a metric theory of gravity, i.e., the correct Newtonian limit of massless gravity, while in the UV counterpart, i.e.,  $r < 2/M_s$ , the metric potentials approach being constant,  $\Phi = \Psi = GmM_s/\sqrt{\pi}$ . In fact, it has been shown that in the UV limit the Weyl tensor vanishes linearly in  $r$ , and both the Ricci scalar and the Ricci tensor approach constant values. Effectively, gravity becomes conformally flat in the UV limit [16]. The time-dependent dynamical equations, such as those in the matter collapse case, have also been solved and shown to be nonsingular [17]. Analogous nonsingular solutions exist in three [18] and higher dimensions [19]. The complete nonlinear equations of motion do not permit singular behavior  $r^{-\alpha}$ , ( $\alpha > 0$ ) [20], and it is possible that astrophysical objects can be made devoid of singularity as well as horizons [16,21].

Now, let us study the evolution of the axial torsion field  $S^\mu$ , which couples to fermionic sources only. Since

fermions have an intrinsic spin, instead of having a Dirac-delta point source for a fermion, we would need to consider a singular source endowed with angular momentum. Indeed, for the sake of simplicity, we can fix the angular momentum to be in the  $z$  direction, without any loss of generality. However, a mere Dirac delta at  $r = 0$  will not be able to capture the spin of the fermion; instead, we would need a rotating singular Dirac-delta ring. We will use again isotropic coordinates (20), in which the singular source can be expressed as  $A^\mu \delta(z) \delta(x^2 + y^2 - R^2)$ .

Then, adding this source to the Cartan equations in (17), and substituting the ghost-free conditions in the torsion sector (19), one finds that

$$\square e^{\beta(\square)} S^\mu = A^\mu \delta(z) \delta(x^2 + y^2 - R^2), \quad (22)$$

where  $A^\mu$  holds for a source vector and  $R$  holds for the constant Cartan radius of a singular rotating ring, where effectively the singularity is located. For illustrative purposes, we may assume  $\beta(\square) = -\square/M_s^2$ . In order to solve Eq. (22), we need to calculate the Fourier transform  $\mathcal{F}$  of the source, as follows,

$$\mathcal{F}[\delta(z) \delta(x^2 + y^2 - R^2)] = \pi J_0\left(-R\sqrt{k_x^2 + k_y^2}\right), \quad (23)$$

where  $J_0$  represents the Bessel function of first kind ( $n = 0$ ). Thus, applying the Fourier transform to Eq. (22), one obtains

$$\begin{aligned} \mathcal{F}[\square e^{-\square/M_s^2} S^\mu(\vec{x})] &= \mathcal{F}[A^\mu \delta(z) \delta(x^2 + y^2 - R^2)] \Rightarrow \\ -k^2 e^{k^2/M_s^2} S^\mu(\vec{k}) &= \pi A^\mu J_0\left(-R\sqrt{k_x^2 + k_y^2}\right) \Rightarrow \\ S^\mu(\vec{k}) &= -\pi A^\mu \frac{e^{-k^2/M_s^2}}{k^2} J_0\left(-R\sqrt{k_x^2 + k_y^2}\right), \end{aligned} \quad (24)$$

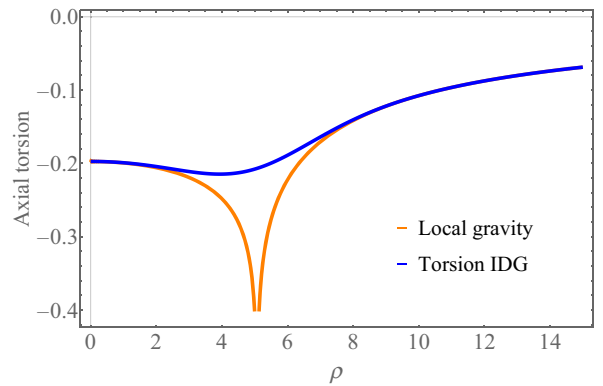


FIG. 1. Results of the numerical computation of (26) for the case of local theories of gravity (limit when  $M_s \rightarrow \infty$  or  $\square/M_s \rightarrow 0$ ) and IDG theories with torsion. We have chosen  $A^\mu = 4$ ,  $\mu = \{1, \dots, 4\}$ ,  $R = 5.06$ , and  $M_s = 1$ .



Then, performing the inverse of the transform of (24), we find that the solution of Eq. (22) can be expressed as

$$S^\mu = -\pi A^\mu \int \frac{d^3k}{(2\pi)^3} \frac{e^{-k^2/M_s^2}}{k^2} J_0\left(-R\sqrt{k_x^2 + k_y^2}\right) \times e^{i(k_x x + k_y y + k_z z)}, \quad (25)$$

where  $d^3k = dk_x dk_y dk_z$  and  $k^2 = k_x^2 + k_y^2 + k_z^2$ . In order to see how the axial vector behaves at the singularity  $r = R$ , we can restrict the study of the integral in (25) to the  $z = 0$  plane, assuming that the ring rotation axis lies along the  $z$  direction. By using cylindrical coordinates,  $k_x = \xi \cos(\varphi)$ ,  $k_y = \xi \sin(\varphi)$ ,  $k_z = k_z$ , we obtain

$$\begin{aligned} S^\mu(\rho) &= -\pi A^\mu \int_{\xi=0}^{\xi=\infty} \int_{\varphi=0}^{\varphi=2\pi} \int_{k_z=0}^{k_z=\infty} \frac{\xi d\varphi d\xi dk_z}{(2\pi)^3} \frac{e^{-(\xi^2+k_z^2)/M_s^2}}{\xi^2+k_z^2} J_0(-R\xi) e^{i\xi x \cos(\varphi)} e^{i\xi y \sin(\varphi)} \\ &= -\frac{\pi A^\mu}{(2\pi)^3} \int_{\xi=0}^{\xi=\infty} \xi d\xi J_0(-R\xi) \left( \int_{k_z=0}^{k_z=\infty} dk_z \frac{e^{-(\xi^2+k_z^2)/M_s^2}}{\xi^2+k_z^2} \right) \left( \int_{\varphi=0}^{\varphi=2\pi} d\varphi e^{i\xi x \cos(\varphi)} e^{i\xi y \sin(\varphi)} \right) \\ &= -\frac{1}{4} A^\mu \int_0^\infty d\xi J_0(-R\xi) J_0(-\xi\rho) \text{Erfc}(\xi/M_s), \end{aligned} \quad (26)$$

where  $\text{Erfc}(z) = 1 - \text{Erf}(z)$  is the complementary error function. Since finding the analytically closed form is not possible, the integral in (26) can be solved numerically for physically relevant values, as can be seen in Fig. 1. In the case of stable local Poincaré gauge theories of gravity, in the limit  $M_s \rightarrow \infty$ , the singularity at  $r = R$  is unavoidable; see Ref. [22]. Within the infinite derivative theory of Poincaré gravity, which has no ghosts, the ring singularity can be smeared out. In both cases in the figure, the axial torsion presents a  $1/r$  behavior in the IR limit. Therefore, for IDG theories, we conclude that the axial torsion is regular everywhere in the presence of a Dirac-delta fermionic source with spin. This result is similar to the Kerr-like singularity, which is cured in the infinite derivative metric theory of gravity [23].

## VII. CONCLUSIONS

In this paper, we have presented the most general action for infinite derivative gravity with torsion. We have applied the Palatini formalism for the first time in this context to obtain the linearized field equations around the Minkowski background. Under the assumption that the torsion component is solely given by an axial field, we have shown that the coupling between the metric and the torsion vanishes, therefore leading to two separate infinite derivative theories of gravity along with an axial torsion field. For both sectors,

we were able to show that the ghost-free conditions are able to smear out pointlike and ringlike singularities. The solution where the axial torsion couples to a fermionic source with a spin is absolutely novel. Consequently, our results may have seminal consequences for building quantum theory of gravity with spins and understanding the UV aspects of Poincaré gauge theories of gravity.

## ACKNOWLEDGMENTS

A. M.'s research is funded by the Netherlands Organization for Scientific Research (NWO) Grant No. 680-91-119. A. d. I. C. D. and F. J. M. T. acknowledge financial support from University of Cape Town (UCT) Launching Grants Programme and National Research Foundation (NRF) Grants No. 99077 2016-2018, Reference No. CSUR150628121624, 110966 Ref. No. BS170509230233, and the NRF IPRR, Ref. No. IFR170131220846. F. J. M. T. acknowledges financial support from the Erasmus+ KA107 Alliance4Universities program and from the Van Swinderen Institute at the University of Groningen.

## APPENDIX

The full nonlinear quadratic Lagrangian in both curvature and torsion (around Minkowski) is given by

$$\begin{aligned} \mathcal{L}_q &= \tilde{R}\tilde{F}_1(\square)\tilde{R} + \tilde{R}\tilde{F}_2(\square)\partial_\mu\partial_\nu\tilde{R}^{\mu\nu} + \tilde{R}_{\mu\nu}\tilde{F}_3(\square)\tilde{R}^{(\mu\nu)} + \tilde{R}_{\mu\nu}\tilde{F}_4(\square)\tilde{R}^{[\mu\nu]} + \tilde{R}_{(\mu}{}^\nu\tilde{F}_5(\square)\partial_\nu\partial_\lambda\tilde{R}^{\mu\lambda} + \tilde{R}_{[\mu}{}^\nu]\tilde{F}_6(\square)\partial_\nu\partial_\lambda\tilde{R}^{\mu\lambda} \\ &+ \tilde{R}_{\mu}{}^\nu\tilde{F}_7(\square)\partial_\nu\partial_\lambda\tilde{R}^{\mu\lambda} + \tilde{R}_{\mu}{}^\nu\tilde{F}_8(\square)\partial_\nu\partial_\lambda\tilde{R}^{[\mu\lambda]} + \tilde{R}^{\lambda\sigma}\tilde{F}_9(\square)\partial_\mu\partial_\sigma\partial_\nu\partial_\lambda\tilde{R}^{\mu\nu} + \tilde{R}_{(\mu\lambda)}\tilde{F}_{10}(\square)\partial_\nu\partial_\sigma\tilde{R}^{\mu\nu\lambda\sigma} \\ &+ \tilde{R}_{[\mu\lambda]}\tilde{F}_{11}(\square)\partial_\nu\partial_\sigma\tilde{R}^{\mu\nu\lambda\sigma} + \tilde{R}_{\mu\lambda}\tilde{F}_{12}(\square)\partial_\nu\partial_\sigma\tilde{R}^{(\mu\nu|\lambda\sigma)} + \tilde{R}_{\mu\lambda}\tilde{F}_{13}(\square)\partial_\nu\partial_\sigma\tilde{R}^{[\mu\nu|\lambda\sigma]} + \tilde{R}_{\mu\nu\lambda\sigma}\tilde{F}_{14}(\square)\tilde{R}^{(\mu\nu|\lambda\sigma)} \\ &+ \tilde{R}_{\mu\nu\lambda\sigma}\tilde{F}_{15}(\square)\tilde{R}^{[\mu\nu|\lambda\sigma]} + \tilde{R}_{(\rho\mu|\nu\lambda)}\tilde{F}_{16}(\square)\partial^\rho\partial_\sigma\tilde{R}^{\mu\nu\lambda\sigma} + \tilde{R}_{[\rho\mu|\nu\lambda]}\tilde{F}_{17}(\square)\partial^\rho\partial_\sigma\tilde{R}^{\mu\nu\lambda\sigma} + \tilde{R}_{\rho\mu\nu\lambda}\tilde{F}_{18}(\square)\partial^\rho\partial_\sigma\tilde{R}^{(\mu\nu|\lambda\sigma)} \\ &+ \tilde{R}_{\rho\mu\nu\lambda}\tilde{F}_{19}(\square)\partial^\rho\partial_\sigma\tilde{R}^{[\mu\nu|\lambda\sigma]} + \tilde{R}_{(\mu\nu|\rho\sigma)}\tilde{F}_{20}(\square)\partial^\nu\partial^\sigma\partial_\alpha\partial_\beta\tilde{R}^{\mu\alpha\rho\beta} + \tilde{R}_{[\mu\nu|\rho\sigma]}\tilde{F}_{21}(\square)\partial^\nu\partial^\sigma\partial_\alpha\partial_\beta\tilde{R}^{\mu\alpha\rho\beta} \end{aligned}$$

$$\begin{aligned}
 & + \tilde{R}_{\mu\nu\rho\sigma}\tilde{F}_{22}(\square)\partial^\nu\partial^\sigma\partial_\alpha\partial_\beta\tilde{R}^{(\mu\alpha|\rho\beta)} + \tilde{R}_{\mu\nu\rho\sigma}\tilde{F}_{23}(\square)\partial^\nu\partial^\sigma\partial_\alpha\partial_\beta\tilde{R}^{[\mu\alpha|\rho\beta]} + K_{\mu\nu\rho}\tilde{F}_{24}(\square)K^{\mu\nu\rho} + K_{\mu\nu\rho}\tilde{F}_{25}(\square)K^{\mu\rho\nu} \\
 & + K_{\mu}{}^\rho{}_\nu\tilde{F}_{26}(\square)K^{\mu\sigma}{}_\sigma + K_{\mu}{}^\nu{}_\rho\tilde{F}_{27}(\square)\partial_\mu\partial_\sigma K^{\sigma\nu\rho} + K_{\mu}{}^\nu{}_\rho\tilde{F}_{28}(\square)\partial_\mu\partial_\sigma K^{\sigma\rho\nu} + K_{\mu}{}^\rho{}_\nu\tilde{F}_{29}(\square)\partial_\rho\partial_\sigma K^{\mu\nu\sigma} \\
 & + K_{\mu}{}^\rho{}_\nu\tilde{F}_{30}(\square)\partial_\rho\partial_\sigma K^{\mu\sigma\nu} + K^{\mu\rho}{}_\rho\tilde{F}_{31}(\square)\partial_\mu\partial_\nu K^{\nu\sigma}{}_\sigma + K_{\mu}{}^{\nu\rho}\tilde{F}_{32}(\square)\partial_\nu\partial_\rho\partial_\alpha\partial_\sigma K^{\mu\alpha\sigma} + K^\lambda{}_{\lambda\sigma}\tilde{F}_{33}(\square)\partial_\rho\partial_\nu K^{\nu\rho\sigma} \\
 & + \tilde{R}_{\nu\rho\sigma}\tilde{F}_{34}(\square)\partial_\mu K^{\nu\rho\sigma} + \tilde{R}_{\mu\nu}{}^\rho{}_\sigma\tilde{F}_{35}(\square)\partial_\rho K^{\mu\nu\sigma} + \tilde{R}_{(\rho\sigma)}\tilde{F}_{36}(\square)\partial_\nu K^{\nu\rho\sigma} + \tilde{R}_{[\rho\sigma]}\tilde{F}_{37}(\square)\partial_\nu K^{\nu\rho\sigma} + \tilde{R}_{\rho\sigma}\tilde{F}_{38}(\square)\partial_\nu K^{\rho\nu\sigma} \\
 & + \tilde{R}_{(\rho\sigma)}\tilde{F}_{39}(\square)\partial^\sigma K^{\rho\mu}{}_\mu + \tilde{R}_{[\rho\sigma]}\tilde{F}_{40}(\square)\partial^\sigma K^{\rho\mu}{}_\mu + \tilde{R}\tilde{F}_{41}(\square)\partial_\rho K^{\rho\mu}{}_\mu + \tilde{R}^\mu{}_{\alpha}{}^\rho{}_\sigma\tilde{F}_{42}(\square)\partial_\mu\partial_\rho\partial_\nu K^{\nu(\alpha\sigma)} \\
 & + \tilde{R}^\mu{}_{\alpha}{}^\rho{}_\sigma\tilde{F}_{43}(\square)\partial_\mu\partial_\rho\partial_\nu K^{\nu[\alpha\sigma]} + \tilde{R}^\mu{}_{\alpha}{}^\rho{}_\sigma\tilde{F}_{44}(\square)\partial_\mu\partial_\rho\partial_\nu K^{\alpha\omega\sigma} + \tilde{R}^{(\mu}{}_{\sigma)}\tilde{F}_{45}(\square)\partial_\mu\partial_\nu\partial_\alpha K^{\sigma\nu\alpha} \\
 & + \tilde{R}^{[\mu}{}_{\sigma]}\tilde{F}_{46}(\square)\partial_\mu\partial_\nu\partial_\alpha K^{\sigma\nu\alpha},
 \end{aligned} \tag{A1}$$

where  $\tilde{R}_{(\alpha\beta|\gamma\rho)} = \frac{1}{2}(\tilde{R}_{\alpha\beta\gamma\rho} + \tilde{R}_{\gamma\rho\alpha\beta})$  and  $\tilde{R}_{[\alpha\beta|\gamma\rho]} = \frac{1}{2}(\tilde{R}_{\alpha\beta\gamma\rho} - \tilde{R}_{\gamma\rho\alpha\beta})$ . In order to be consistent with the action in Ref. [6] when the torsion is zero, we need to satisfy the following relations:

$$\begin{aligned}
 \tilde{F}_5(\square) + \tilde{F}_7(\square) &= F_4(\square), & \tilde{F}_{10}(\square) + \tilde{F}_{12}(\square) &= F_7(\square), \\
 \tilde{F}_{16}(\square) + \tilde{F}_{18}(\square) &= F_{11}(\square), & \tilde{F}_{20}(\square) + \tilde{F}_{22}(\square) &= F_{12}(\square).
 \end{aligned} \tag{A2}$$

After some computations, we arrive at Eq. (2), in which the coefficients are given by

$$\begin{aligned}
 u(\square) &= -4\tilde{F}_1(\square) - \tilde{F}_5(\square)\square - \tilde{F}_7(\square)\square - \tilde{F}_9(\square)\square^2 + \frac{1}{2}\tilde{F}_{39}(\square) + \tilde{F}_{41}(\square), \\
 v_1(\square) &= 4\tilde{F}_1(\square) + \tilde{F}_5(\square)\square + \tilde{F}_7(\square)\square + \tilde{F}_9(\square)\square^2 - \frac{1}{2}\tilde{F}_{39}(\square) - \tilde{F}_{41}(\square), \\
 v_2(\square) &= -\frac{1}{2}\tilde{F}_3(\square) - \tilde{F}_{10}(\square)\square - \tilde{F}_{12}(\square)\square + \tilde{F}_9(\square)\square^2 - 4\tilde{F}_{14}(\square) - \tilde{F}_{16}(\square)\square - \tilde{F}_{18}(\square)\square - \tilde{F}_{20}(\square)\square^2 - \tilde{F}_{22}(\square)\square^2 \\
 & \quad + \frac{1}{2}\tilde{F}_{34}(\square) + \frac{1}{2}\tilde{F}_{35}(\square) + \frac{1}{2}\tilde{F}_{36}(\square) + \frac{1}{2}\tilde{F}_{42}(\square), \\
 w(\square) &= -\frac{1}{2}\tilde{F}_3(\square) - \tilde{F}_{10}(\square)\square - \tilde{F}_{12}(\square)\square + \tilde{F}_9(\square)\square^2 - 4\tilde{F}_{14}(\square) - \tilde{F}_{16}(\square)\square - \tilde{F}_{18}(\square)\square - \tilde{F}_{20}(\square)\square^2 - \tilde{F}_{22}(\square)\square^2 \\
 & \quad + \frac{1}{2}\tilde{F}_{34}(\square) + \frac{1}{2}\tilde{F}_{35}(\square) + \frac{1}{2}\tilde{F}_{36}(\square) + \frac{1}{2}\tilde{F}_{42}(\square), \\
 q_1(\square) &= \frac{1}{2}\tilde{F}_3(\square) + \frac{1}{2}\tilde{F}_4(\square) + \frac{1}{2}\tilde{F}_{10}(\square)\square + \frac{1}{2}\tilde{F}_{11}(\square)\square + \frac{1}{2}\tilde{F}_{12}(\square)\square + \frac{1}{2}\tilde{F}_{13}(\square)\square + \frac{1}{2}\tilde{F}_{16}(\square)\square + \frac{1}{2}\tilde{F}_{18}(\square)\square \\
 & \quad + \frac{1}{2}\tilde{F}_{19}(\square)\square + \frac{1}{2}\tilde{F}_{20}(\square)\square + \frac{1}{2}\tilde{F}_{21}(\square)\square + \frac{1}{2}\tilde{F}_{22}(\square)\square + \frac{1}{2}\tilde{F}_{23}(\square)\square + \tilde{F}_{27}(\square) - \frac{1}{2}\tilde{F}_{36}(\square) - \frac{1}{2}\tilde{F}_{37}(\square) \\
 & \quad - \frac{1}{2}\tilde{F}_{42}(\square)\square - \frac{1}{2}\tilde{F}_{43}(\square)\square, \\
 q_2(\square) &= \frac{1}{2}\tilde{F}_3(\square) - \frac{1}{2}\tilde{F}_4(\square) + \frac{1}{2}\tilde{F}_{10}(\square)\square - \frac{1}{2}\tilde{F}_{11}(\square)\square + \frac{1}{2}\tilde{F}_{12}(\square)\square - \frac{1}{2}\tilde{F}_{13}(\square)\square + 2\tilde{F}_{14}(\square) - 2\tilde{F}_{15}(\square) \\
 & \quad + \frac{1}{2}\tilde{F}_{16}(\square)\square + \frac{1}{2}\tilde{F}_{20}(\square)\square - \frac{1}{2}\tilde{F}_{21}(\square)\square + \frac{1}{2}\tilde{F}_{22}(\square)\square - \frac{1}{2}\tilde{F}_{23}(\square)\square + \tilde{F}_{28}(\square) - \frac{1}{2}\tilde{F}_{36}(\square) + \frac{1}{2}\tilde{F}_{37}(\square) \\
 & \quad - \frac{1}{2}\tilde{F}_{42}(\square)\square + \frac{1}{2}\tilde{F}_{43}(\square)\square, \\
 q_3(\square) &= -\tilde{F}_{17}(\square)\square - \tilde{F}_{18}(\square)\square + \tilde{F}_{19}(\square)\square + \tilde{F}_{29}(\square) + \tilde{F}_{34}(\square) - \tilde{F}_{35}(\square) - \tilde{F}_{38}(\square) - \tilde{F}_{44}(\square)\square, \\
 q_4(\square) &= -\tilde{F}_{14}(\square) - \tilde{F}_{15}(\square) + \tilde{F}_{30}(\square), \\
 q_5(\square) &= 4\tilde{F}_1(\square) + 2\tilde{F}_2(\square)\square + \frac{1}{2}\tilde{F}_3(\square) - \frac{1}{2}\tilde{F}_4(\square) + \tilde{F}_5(\square)\square + \tilde{F}_7(\square)\square + \tilde{F}_9(\square)\square^2 + \tilde{F}_{31}(\square) - \frac{1}{2}\tilde{F}_{39}(\square) \\
 & \quad - \frac{1}{2}\tilde{F}_{40}(\square) - 2\tilde{F}_{41}(\square), \\
 q_6(\square) &= \tilde{F}_3(\square) + \tilde{F}_4(\square) + \tilde{F}_{32}(\square) + \frac{1}{2}\tilde{F}_{36}(\square) + \frac{1}{2}\tilde{F}_{37}(\square) - \tilde{F}_{38}(\square) - \frac{1}{2}\tilde{F}_{39}(\square) + \frac{1}{2}\tilde{F}_{40}(\square) + \frac{1}{2}\tilde{F}_{45}(\square)\square + \frac{1}{2}\tilde{F}_{46}(\square)\square,
 \end{aligned}$$

$$\begin{aligned}
 p_1(\square) &= \tilde{F}_{14}(\square)\square + \tilde{F}_{15}(\square)\square + \tilde{F}_{24}(\square), & p_2(\square) &= \frac{1}{2}\tilde{F}_{18}(\square)\square^2 - \frac{1}{2}\tilde{F}_{19}(\square)\square^2 + \tilde{F}_{25}(\square) + \tilde{F}_{34}(\square), \\
 p_3(\square) &= \frac{1}{2}\tilde{F}_3(\square)\square + \frac{1}{2}\tilde{F}_4(\square)\square + \tilde{F}_{26}(\square) - \frac{1}{2}\tilde{F}_{39}(\square)\square + \frac{1}{2}\tilde{F}_{40}(\square)\square, \\
 s(\square) &= -\frac{1}{2}\tilde{F}_{10}(\square) - \frac{1}{2}\tilde{F}_{11}(\square) - \frac{1}{2}\tilde{F}_{12}(\square) - \frac{1}{2}\tilde{F}_{13}(\square) - \frac{1}{2}\tilde{F}_{16}(\square) + \tilde{F}_{17}(\square) - \frac{1}{2}\tilde{F}_{20}(\square) - \frac{1}{2}\tilde{F}_{21}(\square) - \frac{1}{2}\tilde{F}_{22}(\square) \\
 &\quad - \frac{1}{2}\tilde{F}_{23}(\square) + \tilde{F}_{33}(\square) + \frac{1}{2}\tilde{F}_{42}(\square) - \frac{1}{2}\tilde{F}_{43}(\square) - \frac{1}{2}\tilde{F}_{44}(\square) - \frac{1}{2}\tilde{F}_{45}(\square) - \frac{1}{2}\tilde{F}_{46}(\square).
 \end{aligned}$$

Explicit expressions for the purely metric functions  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $f$  are given in Ref. [6].

- 
- [1] T. W. B. Kibble, *J. Math. Phys. (N.Y.)* **2**, 212 (1961).
  - [2] I. L. Shapiro, *Phys. Rep.* **357**, 113 (2002); *Gauge Theories of Gravitation. A Reader with Commentaries*, edited by M. Blagojevic and F.W. Hehl (Imperial College, London, 2013).
  - [3] E. T. Tomboulis, [arXiv:hep-th/9702146](https://arxiv.org/abs/hep-th/9702146).
  - [4] L. Modesto, *Phys. Rev. D* **86**, 044005 (2012).
  - [5] T. Biswas, A. Mazumdar, and W. Siegel, *J. Cosmol. Astropart. Phys.* **03** (2006) 009; A. A. Tseytlin, *Phys. Lett. B* **363**, 223 (1995); W. Siegel, [arXiv:hep-th/0309093](https://arxiv.org/abs/hep-th/0309093).
  - [6] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, *Phys. Rev. Lett.* **108**, 031101 (2012).
  - [7] T. Koivisto and G. Tsimperis, [arXiv:1810.11847](https://arxiv.org/abs/1810.11847).
  - [8] A. Conroy and T. Koivisto, *Eur. Phys. J. C* **78**, 923 (2018).
  - [9] C. Lämmerzahl, *Phys. Lett. A* **228**, 223 (1997).
  - [10] T. Biswas, A. S. Koshelev, and A. Mazumdar, *Fundam. Theor. Phys.* **183**, 97 (2016); *Phys. Rev. D* **95**, 043533 (2017).
  - [11] P. Van Nieuwenhuizen, *Nucl. Phys.* **B60**, 478 (1973); T. Biswas, T. Koivisto, and A. Mazumdar, [arXiv:1302.0532](https://arxiv.org/abs/1302.0532); L. Buoninfante, [arXiv:1610.08744](https://arxiv.org/abs/1610.08744).
  - [12] S. M. Christensen, *J. Phys. A* **13**, 3001 (1980).
  - [13] M. Ferraris, M. Francaviglia, and C. Reina, *Gen. Relativ. Gravit.* **14**, 243 (1982).
  - [14] T. Biswas, A. Conroy, A. S. Koshelev, and A. Mazumdar, *Classical Quantum Gravity* **31**, 015022 (2014); **31**, 159501 (2014).
  - [15] J. Edholm, A. S. Koshelev, and A. Mazumdar, *Phys. Rev. D* **94**, 104033 (2016).
  - [16] L. Buoninfante, A. S. Koshelev, G. Lambiase, and A. Mazumdar, *J. Cosmol. Astropart. Phys.* **09** (2018) 034; L. Buoninfante, A. S. Koshelev, G. Lambiase, J. Marto, and A. Mazumdar, *J. Cosmol. Astropart. Phys.* **06** (2018) 014.
  - [17] V. P. Frolov, A. Zelnikov, and T. de Paula Netto, *J. High Energy Phys.* **06** (2015) 107; V. P. Frolov, *Phys. Rev. Lett.* **115**, 051102 (2015); V. P. Frolov and A. Zelnikov, *Phys. Rev. D* **93**, 064048 (2016).
  - [18] A. Mazumdar and G. Stettinger, [arXiv:1811.00885](https://arxiv.org/abs/1811.00885).
  - [19] J. Boos, V. P. Frolov, and A. Zelnikov, *Phys. Rev. D* **97**, 084021 (2018).
  - [20] A. S. Koshelev, J. Marto, and A. Mazumdar, *Phys. Rev. D* **98**, 064023 (2018).
  - [21] A. S. Koshelev and A. Mazumdar, *Phys. Rev. D* **96**, 084069 (2017).
  - [22] A. de la Cruz-Dombriz and F. J. M. Torralba, *J. Cosmol. Astropart. Phys.* **03** (2019) 002.
  - [23] L. Buoninfante, A. S. Cornell, G. Harmsen, A. S. Koshelev, G. Lambiase, J. Marto, and A. Mazumdar, *Phys. Rev. D* **98**, 084041 (2018).