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Published in:
Theoretical and Applied Climatology

DOI:
10.1007/s00704-019-03070-w

Theoretical and Applied Climatology (2020) 140:25–36
https://doi.org/10.1007/s00704-019-03070-w

ORIGINAL PAPER

Characterization of regional hydrological drought using improved precipitation records under multi-auxiliary information

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Received: 24 February 2019 / Accepted: 5 December 2019 / Published online: 20 December 2019

Abstract

Drought is a complex natural hazard that has been recurrently occurred in many regions across the globe. Therefore, precise drought characterization and its regional monitoring are key challenges for advanced water management and hydrological research. In this research, we provided a novel method to improve annual average time series data for the Standardized Drought Index (SDI)–type drought monitoring tools. We proposed multi-auxiliary information–based estimation strategy that improves annual moving average/total precipitation time series records. Therefore, we incorporated a minimum and maximum temperature as auxiliary variables under multi-auxiliary regression estimator. In summary, this study propagates a new drought index named: the Precision-Weighted Standardized Precipitation Index (PWSDI). We evaluated the performance of PWSDI for 10 meteorological stations in Pakistan. We found that improved estimates of temporal precipitation time series are good candidates for modelling and monitoring hydrological drought at the regional settings under SDI procedure.

1 Introduction

Due to the prevailing situation of climate change and global warming, the risk of recurrent occurrences of drought has been increased in most part of the world (Dai 2013). Drought and its recurrence occurrences have adverse effects on society in many ways (Dracup et al. 1980; Ding et al. 2011; Van Loon 2015). Its impact can be observed in various ways. The most notable are as follows: (1) economic losses in terms of bad impact on a wide range of agricultural and its related sectors, transportation, energy and banking and (2) environmental impacts: severe damages in wildlife habitat, forest, soil erosion, degradation of landscape quality, plant and animal species.

Drought is a complex natural hazard (Wilhite et al. 2007). There are several drought types in terms of monitoring: agricultural drought, hydrological drought socioeconomic drought and meteorological drought (Hayes et al. 2011). These drought types are based on decreased trends in precipitation amount and its episodes. However, the incidence of hydrological drought is complex (Mishra and Singh 2010). In addition, other spatial, climatic and socioeconomic factors play a significant role in creating the drought situation in a particular region. Therefore, it makes precise drought monitoring and drought characterization at the regional level challenging for advanced water resources management.

There are several drought monitoring tools and indices. Svoboda et al. (2016) provided a detailed description of the most commonly used drought indices. Recently, several studies reported various drought monitoring tools for different regions. Yu et al. (2019) proposed a new drought index—the Modified Palmer Drought Severity Index (MPDSI). In MPDSI, the PDSI (Palmer 1965) is improved by adding irrigation quotas and soil water deficit. Shen et al. (2019)

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developed the Integrated Drought Condition Index (IDCI) by combining precipitation, soil moisture potential evaporation, temperature and vegetation conditions.

Among all the types of drought indices, Standardized Drought Indices (SDI) has significant importance. Depending upon the regional climate and available data, several indicators have proposed for the SDI (Ali et al. 2017). Some of them are McKee et al. (1993), Tsakiris and Vangelis (2005), Vicente-Serrano et al. (2010) and Ali et al. (2017). These standardized procedures are helpful in making effective drought monitoring and mitigation policies by adopting early warning strategies. Several studies used these indices for drought monitoring and forecasting using various statistical and machine learning methods under temporal and spatial settings (Mishra and Desai 2005; Demisse et al. 2015).

Moreover, other climatic variables (i.e. humidity, wind speed and temperature) and their spatio-temporal characteristics have a substantial importance in drought analysis and monitoring. However, the spatial characteristics of rainfall differ from other variables. For example, the distributions of rainfall variability in short distance and time as well. These characteristics of rainfall require optimized meteorological network for its recording and estimation (Einfalt et al. 1990; Scarsoglio et al. 2013). There are several standard guidelines and optimized meteorological networks, which provide a compact distribution of meteorological stations (Van Dijk and Rientjes 1994; Fridzon and Ermoshenko 2009; Lagouvardos et al. 2017).

However, the implications of each meteorological network are difficult for all countries. Particularly, in developing countries, the lack of resources and technology lessens the scope and utilization of these optimized networks. Further, geographical facts such as desserts, hills, forest and political situation of a region are key factors for the distribution of meteorological stations (Fig. 1). Consequently, rainfall measurements are usually observed at some sampled realization (monitoring station) from a continuum (region) (Webster and Oliver 2007). Hence, from a statistical point of view, the observed rainfall measurements should be unbiased and regionally representative. But the rainfall being the most spatial variable has substantial variation in its observed amount (Rafiuddin et al. 2010). Hence, it requires some type of calibration before using the observed rainfall time series data in the drought monitoring phase. This study proposes to improve rainfall records by taking auxiliary information under the spatial sampling structure.

In literature, it can be validated that the sampling theory and methods play a vital role in the estimation and prediction of any random phenomena (Cochran 2007). In sampling techniques, most of the advanced procedures of estimation utilize

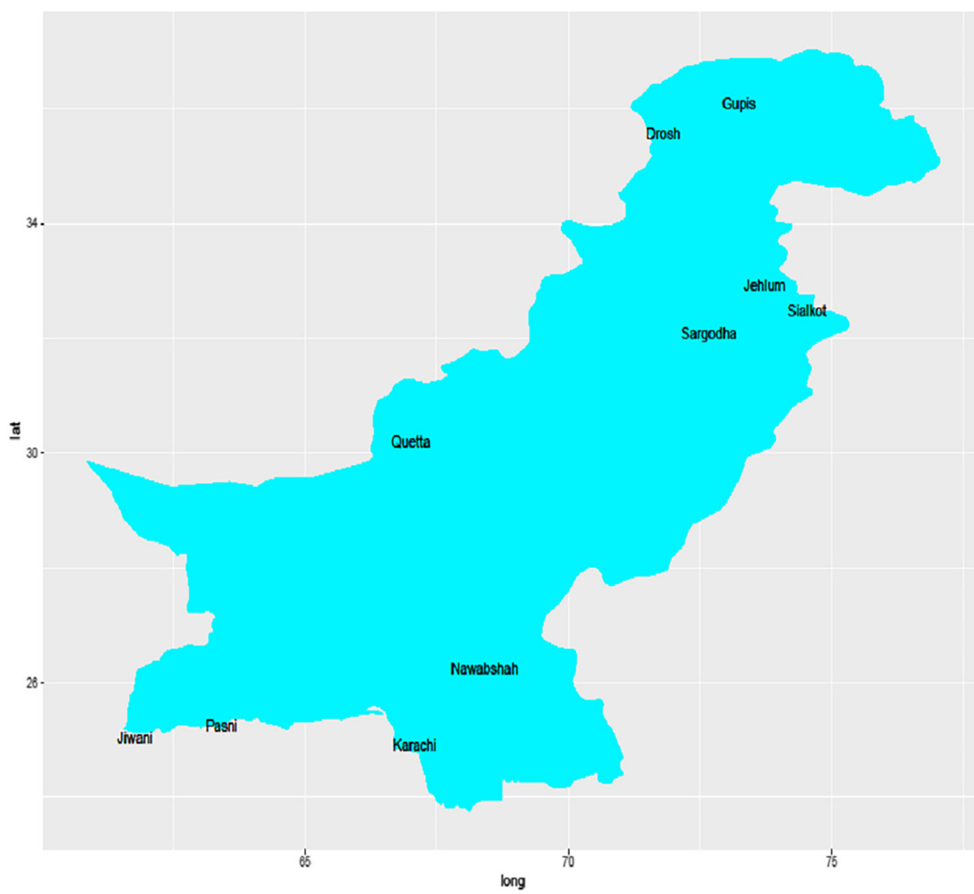

Fig. 1 Geographical representations of selected ten meteorological stations as a study area

the information of auxiliary variables. We suggest utilizing auxiliary information for accurate drought monitoring and its precise characterization. Therefore, we used two auxiliary information–based estimators proposed by Mukerjee et al. (1987) for improving time series characteristics of precipitation data.

In this study, we aimed to develop a new drought indicator for SDI procedure which incorporates regionally improved monthly precipitation estimates. Here, we integrate minimum and maximum temporal records of temperature in the estimation of precipitation records. Consequently, the research provides a newly established time series comprising an improved monthly precipitation amount. The study assumed that comparative with precipitation, the spatial distribution of temperature is more homogeneous. Therefore, it is simple, straightforward and rationally valid that improved precipitation records provide more valid and regionally acceptable results for accurate monitoring of drought.

2 Methodology

There are several estimation techniques that incorporate supplementary information of multiple auxiliary variables such as ratio, regression and product estimators (Cochran 2007). Integration of auxiliary variables depends on the rationale of the proposed estimator. For instance, if the auxiliary variable is negatively correlated with the study variable, the product estimator produces precise estimates of population characteristics. On the other hand, a perfect and positive correlation suggests the ratio estimator. In 1973, auxiliary information–based estimation was used by incorporating the correlation effect (Pugachev 1973; Tarima and Pavlov 2006). Later studies incorporated auxiliary information at the estimation stage (Raj 1965; Kuk and Mak 1989; Rao et al. 1990; Estevao and Sarndal 1999). Kanwai et al. (2016) proposed two auxiliary information–based estimators for two-phase sampling techniques. Vishwakarma and Kumar (2015) proposed a class of estimators for estimating the population means in two-phase sampling based on several auxiliary variables. Likewise auxiliary information, the sampling strategy also significantly contributes to the precision of the estimates (Kowalewski et al. 2015).

Estimation of a sample data using regression type estimator is common in many applications and fields. Regression estimator gives an efficient estimate by using additional information about the auxiliary variable. There are several types of regression estimators. However, each estimator has certain constraints and settings. Usually, if the study variable positively relates with the auxiliary variable then regression estimator provides more representative estimates.

With the context of global warming, the purpose of this research is to assess drought using improved time series data of precipitation. To improve the data, this research recommends the use of minimum and maximum temperature as auxiliary information. One can observe that the distribution of temperature is more homogenous than the precipitation occurrences. Hence, the use of temperature as an auxiliary variable is rationally valid and straightforward. In past research, many authors have worked on the assessment of the relationship between precipitation and temperature. Most of the research has shown that temperature is positively correlated with the rainfall amount (Zhao and Khalil 1993; Mueller and Seneviratne 2012; Rajeevan et al. 1998; Jain et al. 2013; Chen et al. 2013).

Therefore, this research employed regression estimator under two auxiliary variable. The general expression of the estimator is as follows.

$$\overline{y}_r = \overline{y} + b_1\left(X_1 - \overline{x}_1\right) + b_2\left(X_2 - \overline{x}_2\right) \quad (1)$$

In Eq. 1, $\overline{y}_r$ is the sample mean of improved data under regression setting, $\overline{y}$ is the usual mean, X_1 and X_2 are the grand total of auxiliary variables, $\overline{x}_1$ and $\overline{x}_2$ are the sample mean of auxiliary variables and b_1 and b_2 are the regression slopes between study variables and auxiliary variables.

Based on the available time series data and auxiliary variables, the above equation is employed for improving time series data of precipitation for annual characterization of drought. Consequently, we suggest the following expression for data improvement under auxiliary information.

$$\mathrm{PW}_r = \overline{p} + b_1\left(T_{\min} - \overline{t}_{\min}\right) + b_2\left(T_{\max} - \overline{t}_{\max}\right) \quad (2)$$

where PW is the improved precipitation estimate of the mean and simple precipitation records are weighted by the dependence characteristics of extreme (minimum/maximum) temperature with precipitation. $\overline{p}$ is the annually moving mean of simple precipitation records. $T_{\min}$ and $T_{\max}$ are the overall mean minimum and maximum temperature observed at the single observatory, respectively. Here, $\overline{t}_{\min}$ and $\overline{t}_{\max}$ has usual interpretation mapped in Eq. 1.

2.1 Proposed method

This research suggests precision-weighted (PW) (see Eq. 2) estimates with the placement of simple precipitation records in SDI procedure. The use of PW estimates is straightforward, as they are precise and more representative in the domain. In addition, PW estimates have the ability to characterize drought by using more than one variable. In this research, we modelled PW estimates using varying probability functions concept of Stagge et al. (2015) and the graphical technique provided by Hao and AghaKouchak (2014). The former technique is usually called parametric estimation approach, while the latter is known as non-parametric method. A short description of each approach is presented as follows.

2.1.1 Varying probability function approach

The use of varying probability distributions approach is due to Stagge et al. (2015). Therefore, the study recommends various multi-parameter probability distributions, where the appropriateness of probability function is decided by the minimum value of Bayesian information criteria (BIC).

In the experiments and computational analysis, propagate (Spiess 2018) R package is employed. The search of appropriate probability function is made for both Standardized Precipitation Index (SPI)-12 and Precision-Weighted Standardized Precipitation Index (PWSDI). The study includes 32 multi-parameter distributions, such as generalized normal distribution, Gumbel distribution and generalized extreme value distribution, where the parameters of each selected distribution are estimated by employing a Levenberg-Marquardt algorithm using *minpack.lm* (Elzhov et al. 2013) R package.

The cumulative distribution function (CDF) of those probability functions which have minimum BIC values is modified by the following equation.

$$G(x) = q + (1-q)F(x) \tag{3}$$

In the above equation, x denotes the time series data on either simple precipitation records (P) or improved estimates (PW) and q denotes the ratio of undefined values (outside the range). This modification is made to adjust those values that lie outside the range of the probability function.

2.1.2 Non-parametric estimation approach–based drought indices

Parallel to probability function–based standardization, non-parametric approach (probability plotting (PP) position formulas) is another technique of getting standardized values. Initially, Hao and AghaKouchak (2014) have provided the concept of non-parametric-based standardization. They integrated the Gingorten probability position formula under the same standardization module of multi-scalar type drought indices (i.e. SPI, SPEI and SPTI). In the most recent work, Zhang et al. (2018) have included parallel analysis of non-parametric approach in usual hydrological research. In our research, instead of using the Gingorten formula, the compatibility of other PP formula with selected probability distributions is also examined to compute PWSDI (Table 1).

Standardization Finally, the CDF of appropriate probability function and the numerical vector of probability PP formula are referred to as compute standardized values. To do this, following McKee et al. (1993) and Ali et al. (2017), we employed the approximate transformation provided by Abramowitz and Stegun (1965).

$$\text{PWSDI} = -\left(z + \frac{c_o + c_1 z + c_c z^2}{1 + d_1 z + d_2 z^2 + d_3 z^3}\right) \tag{4}$$

for

$$z = \sqrt{\ln\left(\frac{1}{\{T(x)\}^2}\right)}$$

when

$$0 < T(x) \le 0.5 \tag{5}$$

$$\text{PWSDI} = +\left(z - \frac{c_o + c_1 z + c_c z^2}{1 + d_1 z + d_2 z^2 + d_3 z^3}\right) \tag{6}$$

and for

$$z = \sqrt{ln\left(\frac{1}{\{1-T(x)\}^2}\right)}$$

when

$$0.5 < T(x) \le 1 \tag{7}$$

where $c_o = 2.515517$, $c_1 = 0.802853$, $c_2 = 0.010328$, $d_1 = 1.432788$, $d_2 = 0.985269$ and $d_3 = 0.001308$ are constants.

In the above procedure,

$$T(x) = \begin{pmatrix} G(x) & \text{if standardization is done under parametric approach} \\ P(x) & \text{if standardization is done undernon-parametric approach} \end{pmatrix}$$

Here, $G(x)$ and $P(x)$ are the probability vectors defined in Eq. 3 and Table 1, respectively.

Resulting standardized values of PWSDI will have zero mean and unit variance. Further, following McKee et al. (1993) and Ali et al. (2017), quantitative values of PWSDI can be categorized according to drought severity. The range of PWSDI values is presented in Table 2 with respect to drought categories.

Table 1 List of non-parametric probability plotting position functions

List	Proponent (authors)	Expression
PP-1	Hazzan (Allen 1914)	$p(x_i) = \frac{2i-1}{2n}$
PP-2	Gringorten (Gringorten 1963)	$p(x_i) = \frac{i-0.44}{n+12}$
PP-3	Tukey (Tukey 1962)	$p(x_i) = \frac{i-0.333}{b+0.333}$
PP-4	Weibull (Weibull 1939)	$p(x_i) = \frac{i}{n+1}$
PP-5	Laplace (Lund et al. 1995)	$p(x_i) = \frac{i-1}{2+n}$

Table 2 Drought classifications under SDI procedure by following Ali et al. (2017)

Range of SDI	Drought classes
≥2	Extremely wet
0.5 to 1.99	Very wet
to 0.99	Moderate wet
0.99 to − 0.99	Near Normal
− 1 to − 1.49	Moderate drought
− 1.5 to 1.99	Severe drought
≤ − 2	Extreme drought

3 Application

3.1 Data and study area

We applied our proposed method on 10 meteorological stations in Pakistan. Due to political instability, the country has a poor natural disaster management system. There is a variety of natural hazards, such as landslides, floods, droughts, earthquakes and cyclones. Pakistan is listed in the three most water-stressed countries of the globe (Farooqi et al. 2005). In Pakistan, the annual average amount of precipitation falls below 250 mm. Recent research on climate change shows the increasing trend in the frequency and severity of natural hazards. However, natural disasters, including reoccurring drought and floods, are the major environmental and financial challenge for sustainable development of the country (Shaw et al. 2014). For this research, time series data of precipitation and temperature are collected from Karachi Data Processing Center (KDPC http://www.pmd.gov.pk/rmc/RMCK/Services_Climatology.html). The data sets fulfil standard requirements of the World Meteorological Organization (WMO). Before dispatching data to us, issues related to the tabulation, removal of errors, adjusting outliers and missing values are done by KDPC their self. This dataset has been cited in our recent publications (see Ali et al. 2017, 2019a, b). Table 1 shows the summary statistic of the selected stations.

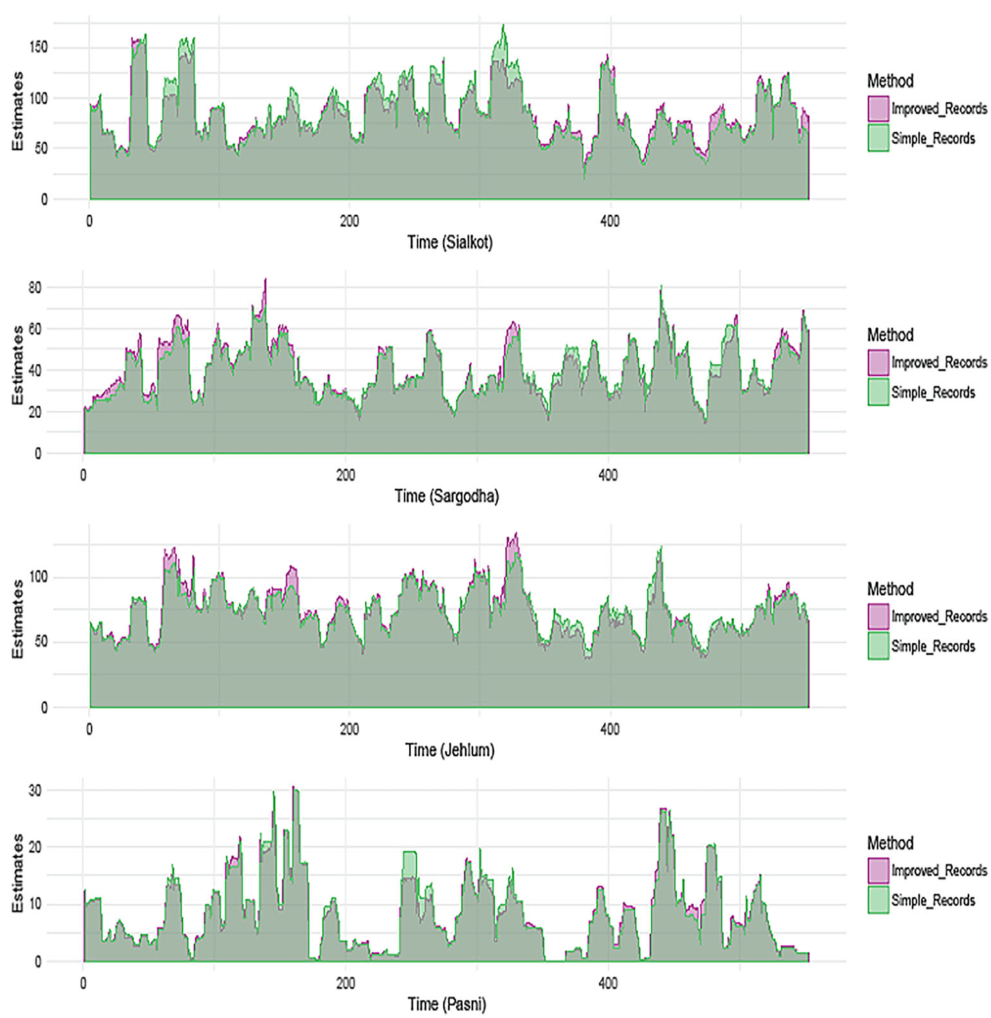

Fig. 2 Temporal deviations between simple and improved records of precipitation (mm)

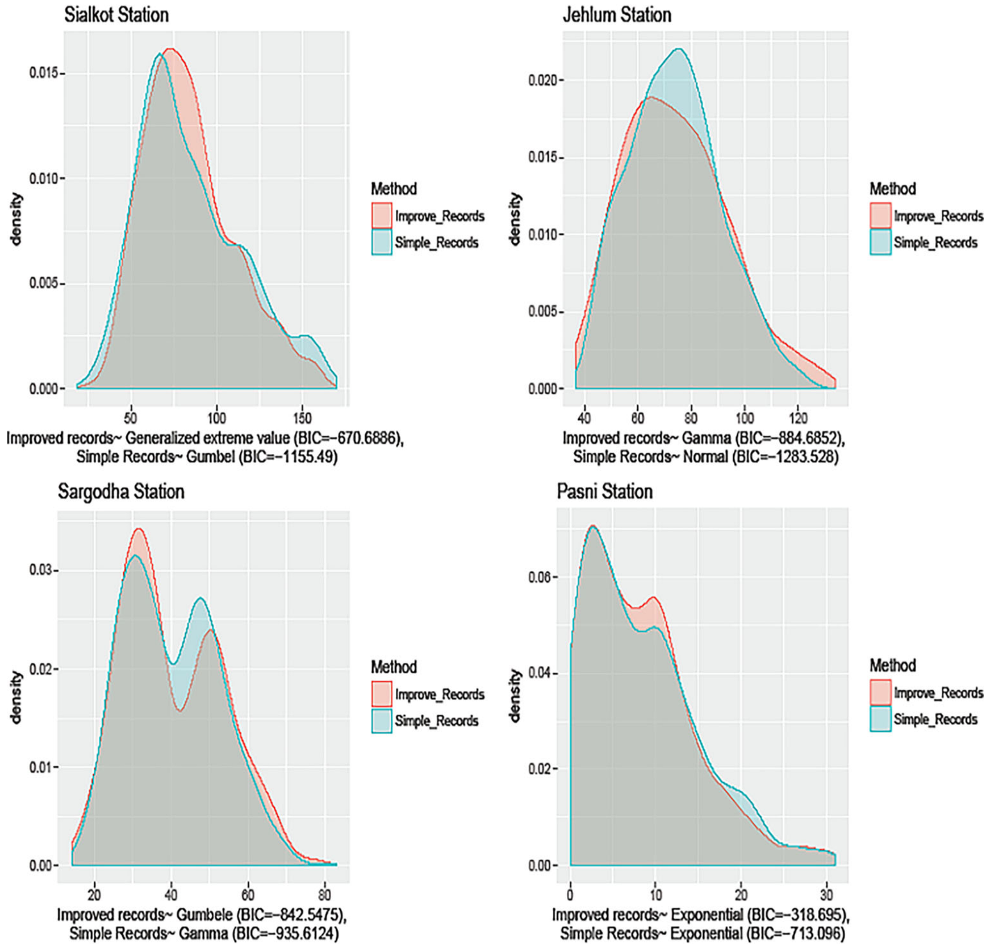

Fig. 3 Candidate probability distribution models for PWSDI and SPI-12

3.2 Quality and comparative measures

We assess the quality of the proposed improvement method of data by comparing the time series data of PWSDI with SPI-12 time-scale drought index. The comparison is made using a simple Pearson correlation statistics. In previous research, several authors used the Pearson correlation for comparison. Some of them are Vicente-Serrano et al. (2010), Ali et al. (2017) and Tsakiris and Vangelis (2005). The reason for choosing SPI-12 is very simple and rationally valid, as the existence of several drought indices restricts us to choose the optimal and most relevant drought index.

However, for defining hydrological drought, SPI at 12-month time-scale procedure utilizes 12-month moving average data of monthly recorded precipitation. Therefore, due to analogous structure and data nature, we made a comparative

Table 3 Summary statistics of selected ten meteorological stations as a study area

Stations	Latitude	Longitude	Precipitation (mm)			Temperature (C°)		
			Mean	S.D	C.V	Mean	S.D	C.V
Sargodha	72.671	32.084	40.05	55.61	138.84	24.452	7.783	31.831
Gupis	73.24	36.1	15.87	29.982	188.97	12.638	8.722	69.01
Nawabshah	68.41	26.248	12.21	37.117	304.12	26.891	7.065	26.273
Jehlum	73.726	32.933	73.84	94.872	128.48	23.715	7.158	30.185
Karachi	67.082	24.906	15.92	41.54	260.99	26.612	4.443	16.694
Pasni	63.415	25.251	8.5	23.371	276.18	25.793	4.288	16.626
Quetta	67.01	30.199	21.7	35.673	164.29	16.557	8.523	51.474
Sialkot	74.531	32.493	84.04	128.98	153.47	22.943	7.195	31.36
Drosh	71.804	35.568	47.25	46.099	97.561	17.631	9.066	51.418
Jiwani	61.771	25.054	8.8	26.38	300.73	25.775	4.023	15.609

Table 4 Optimal choices of probability distribution functions based BIC values for selected stations

Stations	Statistics	PWSDI	SPI
Sialkot	Function	Generalized extreme value	Gumbel
	Parameters	$\mu = 71.04$, σ 24.2, $\varsigma = 0.2202$	$\mu = 863.26$, $\beta = 264.76$
	BIC	− 670.68	− 1155.49
Jhelum	Function	Gamma	Normal
	Parameters	$\kappa = 12.47$, $\theta = 0.168$	$\mu = 866.411$, $\sigma = 217.95$
	BIC	− 884	0.32525
Sargodha	Function	Gumbel	Gamma
	Parameters	$\mu = 33.96$, $\beta = 12.01$	$K = 8.10$, $\theta = 0.01$
	BIC	− 842.5475	− 935.6124
Pasni	Function	Exponential	Exponential
	Parameters	$\lambda = 0.09952$	$\lambda = 0.00824$
	BIC	− 318.695	− 713.096
Nawab Shah	Function	Exponential	Exponential
	Parameters	$\lambda = 0.07612$	$\lambda = 0.00629$
	BIC	− 0.07654	− 1398.404
Karachi	Function	Exponential	Gamma
	Parameters	$\lambda = 0.05065$	$\kappa = 1.23$, $\beta = 0.0057$
	BIC	− 633.7871	− 1184.320
Quetta	Function	Normal	Logistic
	Parameters	$\mu = 19.17$, $\sigma = 8.97$	$\mu = 234.01$, $\sigma = 62.50$
	BIC	− 7,957,896	− 1067.321
Gupis	Function	Laplace	Cauchy
	Parameters	$\mu = 9.28$, $\beta = 9.57$	$(x = 116.82, y = 54.90)$
	BIC	− 847.2828	− 933.833
Drosh	Function	Gamma	Normal
	Parameters	$\kappa = 12.372$, $\theta = 0.254$	$\mu = 561.02$, $\sigma = 153.68$
	BIC	− 683.5554	− 1536.87
Jiwani	Function	Exponential	Gamma
	Parameters	$\lambda = 0.09883$	$\kappa = .08$, $\theta = 0.0094$
	BIC	− 319.0582	− 735.703

analysis of PWSDI with SPI at 12-month time scale. SPI-12 is efficient and extensively used drought indicator for defining the hydrological drought (Habibi et al. 2018). Hence, comparing SPI-12 with the proposed index is straightforward.

4 Results and discussion

4.1 Temporal behaviour and deviations

In this section, we assessed and compared the temporal behaviour of improved records with those which were used in SPI-12.

Figure 2 explores the temporal deviations between simple and improved precipitation records for Sialkot, Jhelum, Sargodha and Pasni stations. In all these stations, significant deviations have been observed in both records. Moreover, the probability distributions of these records vary greatly (see Fig. 3). In Sialkot station, we found generalized extreme value distribution with BIC = − 670.6886 and Gumbel distribution with BIC = − 1155.49 on simple and improved precipitation records, respectively. Further, gamma distribution with BIC = − 884.6856 is observed in simple records, whereas normal distribution with BIC = − 1283.528 is observed on improved records at Jhelum station. Similarly, probability distributions in Sargodha station differ from each other. However, in Pasni station, both records have the same distribution but have different values of parameters. Although both data sets follow an exponential distribution, significant variation can be seen in the model selection criterion.

These results show how the discrepancies between simple and improved records may lead to inaccurate determination of drought condition, especially when the data set have extreme values.

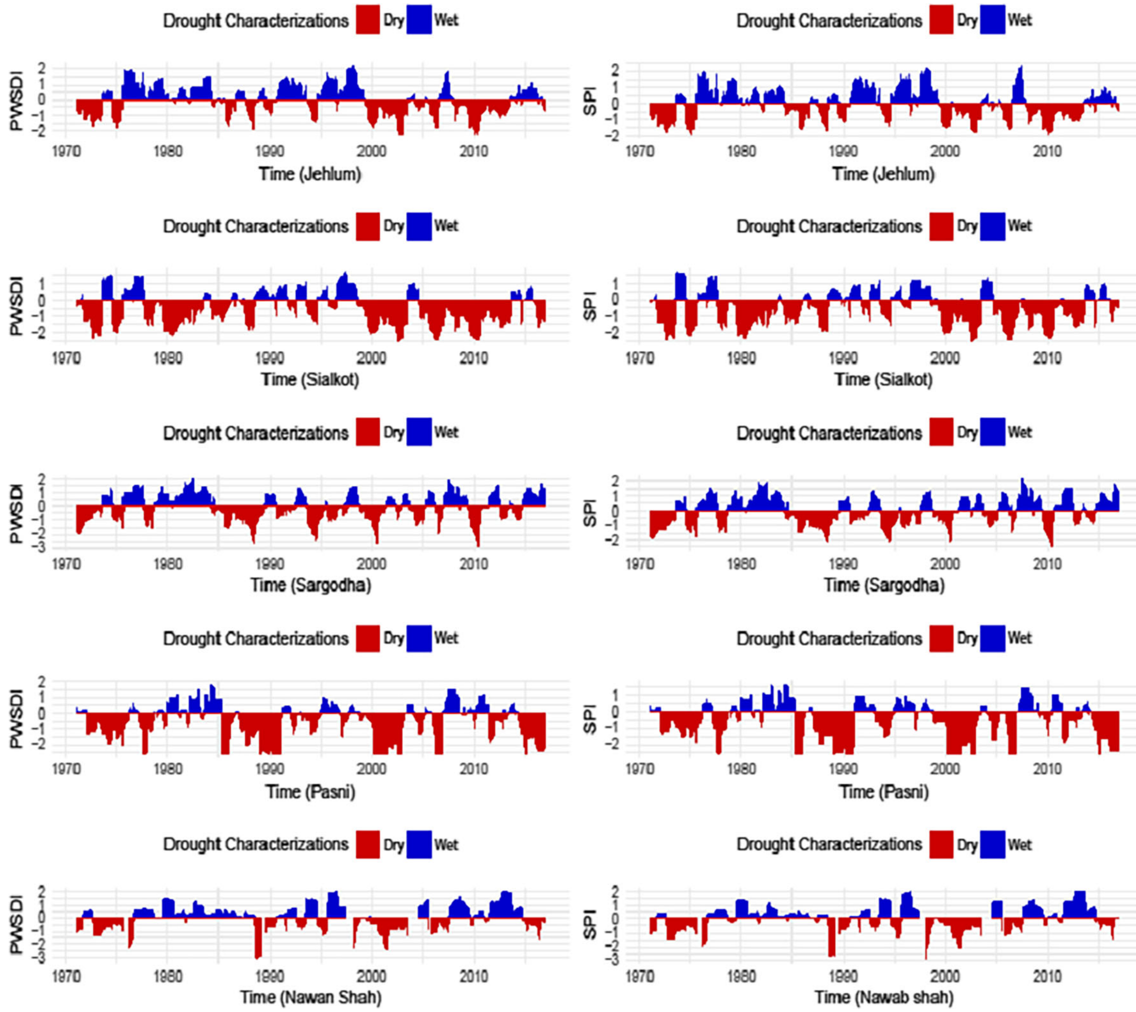

Fig. 4 Temporal representation of PWSDI and SPI under an optimal choice of probability distribution functions

4.2 Computation and comparisons of drought indices

To evaluate and compare the competence of PWSDI with SPI-12, time series numerical vectors on standardized indices are prepared using simple and improved precipitation records under the parametric and non-parametric scenario. Here, we used monthly time series secondary data of total precipitation, mean minimum temperature and mean maximum temperature ranging from 1971 to 2017 at ten meteorological stations of Pakistan (see Fig. 1 and Table 2).

In the parametric approach, following the guidelines of Stagge et al. (2015), Table 3 provides the summary statistics of the selected probability functions in both types of records. Here, the parameters of each probability distribution are estimated using (weighted) residual sum-of-squares as the minimization criterion based on the Levenberg-Marquardt algorithm.

Outcomes associated with this research show that the distribution of both records varies greatly in all stations, except at Pasni and Nawab Shah. In these two stations, although both records have the same distribution functions, the behaviour and fitness criterion are varied. At Pasni station, BIC of exponential distribution in improved estimates is comparatively less than in simple records. This shows that the exponential distribution is a more suitable choice for improved records. Therefore, the choice of using improved estimates overlaps the simple precipitation records. That is, improved precipitation records may lead to lessen the uncertainty associated with extreme drought condition such as severe drought and severe wet. Further, in Sargodha, Karachi and Jiwani stations, the gamma distribution is a more suitable choice for improved records. Details on other stations with respect to observed optimal probability function are given in Table 4. Overall, we found a significant variation in the selection of optimal probability distributions for modelling the temporal behaviour of both records.

Additionally, after standardization, the study includes a temporal plot of PWSDI and SPI-12 (see Figs. 4 and 5). At all stations, the temporal behaviour of PWSDI strongly correlated with SPI-12. Although different probability functions are

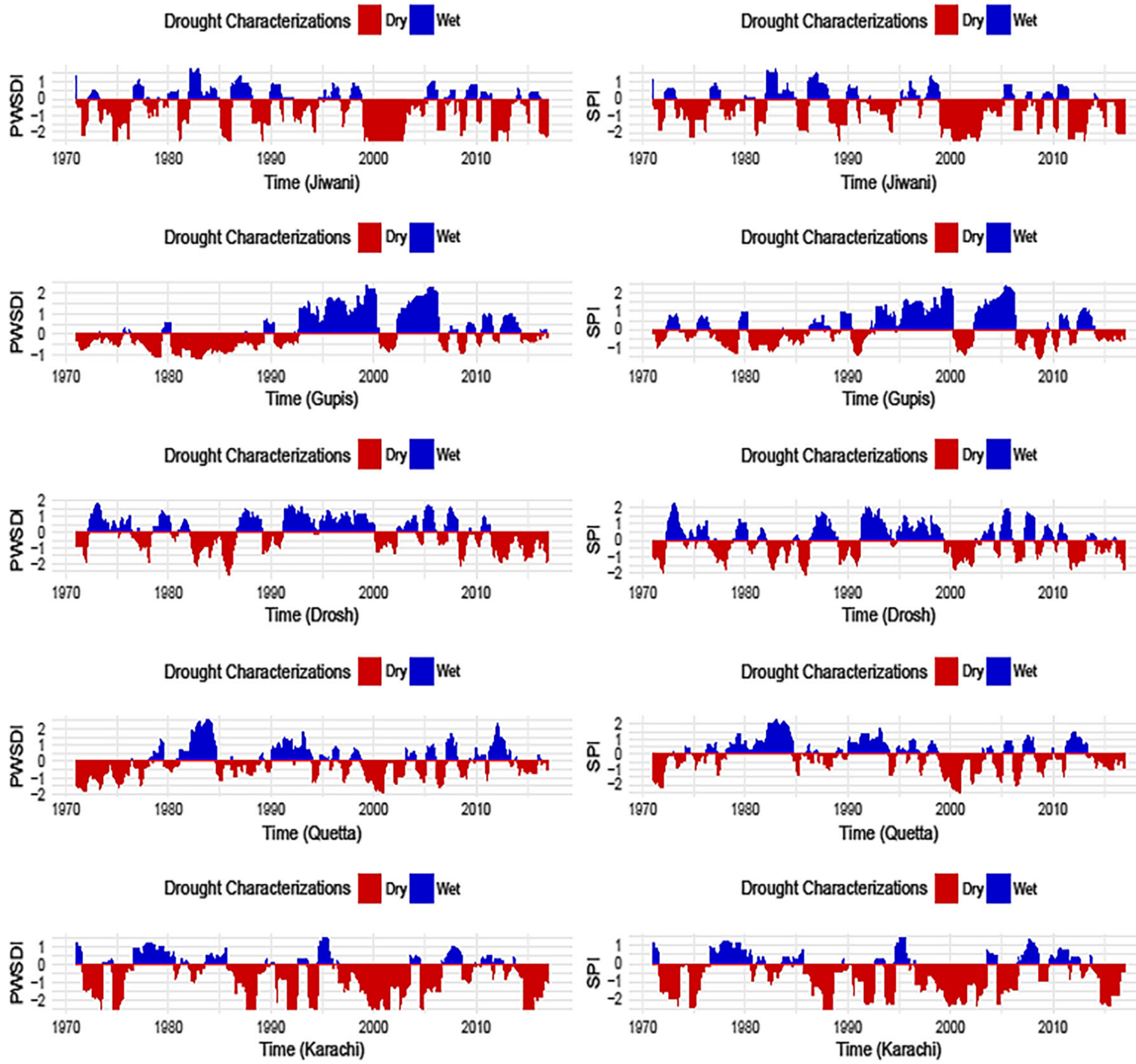

Fig. 5 Temporal representation of PWSDI and SPI-12 under an optimal choice of probability distribution functions

involve in both methods (see Table 4), both indices are temporally consistent. However, some deviations are observed at Gupis and Nawab Shah, where temporal differences are not negligible.

Further, Table 5 provides the correlation between PWSDI and SPI observed. Here, we found a significantly high correlation in both methods. This stability and consistency of PWSDI with SPI show that improved estimates are good

Table 5 Parametric evaluation: correlation between PWSDI and SPI-12

Stations	*r*
Gupis	0.919517
Drosh	0.916372
Pasni	0.975975
Jiwani	0.969969
Quetta	0.919924
Nawab Shah	0.99327
Karachi	0.985527
Sialkot	0.974538
Jehlam	0.971497
Sargodha	0.97286

Table 6 Non-parametric evaluation: correlation of PWSDI with SPI-12

District	Hazzan	Weibull	Tukey	Laplace	Gringorten
Gupis	0.992872	0.88894	0.992839	0.888472	0.992859
Drosh	0.922051	0.91516	0.922008	0.914922	0.922021
Pasni	0.969786	0.838459	0.96966	0.838411	0.969738
Jiwani	0.969963	0.814967	0.970121	0.814202	0.970042
Quetta	0.925676	0.864437	0.9253	0.863944	0.925537
Nawab Shah	0.992911	0.820631	0.993003	0.823298	0.992952
Karachi	0.982931	0.929966	0.983426	0.931505	0.983142
Sialkot	0.968072	0.96251	0.969405	0.963736	0.968614
Jehlam	0.971161	0.974303	0.971965	0.974896	0.971492
Sargodha	0.950611	0.916986	0.951046	0.917329	0.950796

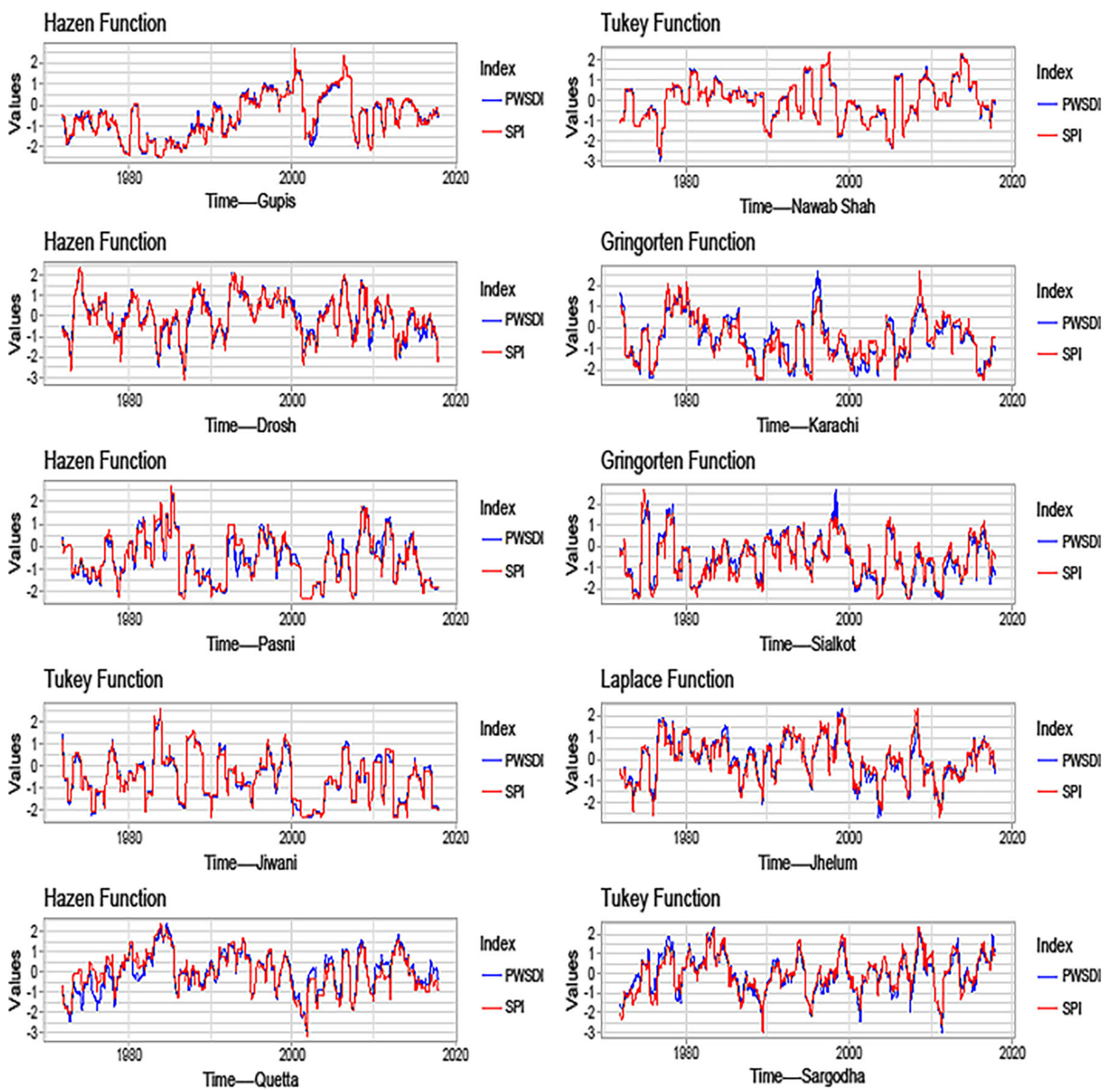

Fig. 6 Temporal behaviour of PWSDI and SPI-12 under probability plotting position functions

candidates for measuring hydrological drought in more representative ways.

However, besides careful selection of probability model, experimental findings show that extreme values and outliers are beyond the coverage of the optimal probability functions. In the previous study, it proved that uncertainty always exists in probability functions. Inferences related to this kind of uncertainty are reported in (Hirsch and Stedinger 1987; Yahaya et al. 2012; Hao et al. 2018; Buyukada and Aydogmus 2018). Here, in our case, Fig. 3 shows that the exponential distribution having the lowest BIC value does not capture the uncertainty at the significant parts in both records at Pasni station. Moreover, substantial deviations are observed in gamma and normal density plots at Jehlam stations. Similar results are found for all other distributions. These analyses advocate non-parametric propagation of uncertainty (Hao and AghaKouchak 2014). Therefore, to check and evaluate the improved estimate with simple precipitation records under a non-parametric way, the study integrates five PP formulas (see Table 1).

After numerical integration of PP formulas, the resulting probability vectors are further used in SDI procedure for obtaining standardized values. Here, standardized values based on these PP formulas are further used to explore the temporal behaviour of PWSDI and SPI-12. Table 6 provides the correlation between SPI and PWSDI in all PP formulas. PWSDI and SPI have a maximum correlation under the Hazen PP formula for these five stations (Gupis, Drosh, Pasni, Jiwani and Quetta). Under Tukey function, the maximum correlations are observed at Jiwani, Nawab Shah and Sargodha Station. Karachi and Sialkot have a maximum correlation under Gringorten formula, while Laplace has maximum correlation at Jhelum. However, due to limitations of the page, the study includes those temporal plots in which probability plotting formula has a maximum correlation.

Figure 6 shows the temporal behaviour of the quantitative values of PWSDI and SPI. Although there is a strong correlation between PWSDI and SPI-12, significant variations are observed in their temporal plots at Sialkot and Pasni. In the mid of the twentieth century, relative to SPI, quantitative values of PWSDI are large. Equivalently, some deviations are observed in Quetta and Sargodha. However, for the rest of the stations, both the series behave in similar manners. Overall, the standardized vectors of PWSDI and SPI-12 under non-parametric methods are consistent and stable. That is, in all probability plotting functions, the correlation of PSWDI with SPI is significantly higher in all stations.

5 Conclusion

This research strengthens the standard module of SDI-type drought indices (Erhardt and Czado 2018) by giving a new procedure of data improvement under auxiliary information. Consequently, this research suggests a new procedure drought index to assess and characterize drought using improved time series data of rainfall estimates. Here, the improved estimates of precipitation are based on regression estimator of two auxiliary information.

To check the efficiency and consistency of the proposed method, the paper included an application on real data of various meteorological station of Pakistan (see Fig. 1). In this research, a comparative analysis of PWSDI is made with SPI-12. We have concluded the following points:

1. More regional resolution and representativeness precipitation estimates: It is observed that there are substantial deviations between the original and the proposed time series (see Fig. 2, Table 3). Although we have used varying distribution-based standardizations, the quantification of SDI in PWSDI is very close to SPI-12 (see Figs. 4 and 5). This equilibrium between PWSDI and SPI also proves the findings of Stagge et al. (2015).
2. Consistency: In both the standardization approaches (parametric and non-parametric), our analysis shows that there is a strong correlation between PWSDI and SPI. Hence, the proposed method is consistent with the existing one (see Table 5 and Table 6).
3. Addressing extreme values: Non-parametric approaches corresponding with correlation analysis between PWSDI and SPI-12 also supports the use of improved precipitation estimates under extreme values.
4. Multivariate standardized drought characterization: Inclusion of multi-auxiliary information (minimum and maximum temperature) before the standardization makes rationale to declare PWSDI index as a multivariate type drought index.

In summation, the ability of the PWSDI of drought characterization and monitoring over existing indices is used for calibrating time series data of precipitation. However, PWSDI cannot be generalized in multi-scale temporal settings (Edwards 1997).

Acknowledgements Authors are very grateful to the Deanship of Scientific Research at King Khalid University, Kingdom of Saudi Arabia for their funding this work through research groups program under the project number RGP-1/103/40.

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