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Advance Selling and Advertising: A Newsvendor Framework

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ABSTRACT

Many firms offer consumers the opportunity to place advance orders at a discount when introducing a new product to the market. Doing so has two main advantages. First, it can increase total expected sales by exploiting valuation uncertainty of the consumers at the advance ordering stage. Second, total sales can be estimated more accurately based on the observed advance orders, reducing the need for safety stock and thereby obsolescence cost. In this research, we derive new insights into trading off these benefits against the loss in revenue from selling at a discount at the advance stage. In particular, we are the first to explore whether firms should advertise the advance ordering opportunity. We obtain several structural insights into the optimal policy, which we show is driven by two dimensions: the fraction of consumers who potentially buy in advance (i.e., strategic consumers) and the size of the discount needed to make them buy in advance. If the discount is below some threshold, then firms should sell in advance and they should advertise that option if the fraction of strategic consumers is sufficiently large. If the discount is above the threshold, then firms should not advertise and only sell in advance if the fraction of strategic consumers is sufficiently small. Graphical displays based on the two dimensions provide further insights. [Submitted: September 24, 2018. Revised: October 15, 2019. Accepted: November 11, 2019.]

Subject Areas: Advance Selling, Advertising, Demand Forecast Accuracy Improvement, Newsvendor, and Strategic Consumer.

INTRODUCTION

Advance selling is a marketing strategy in which a seller offers opportunities for consumers to pre-order a product before it is available. It has two important benefits: First, advance selling can increase sales by exploiting the consumer's valuation uncertainty during the advance selling period (e.g., Xie & Shugan, 2001). Second,

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it helps the retailer reduce demand uncertainty (e.g., Tang, Rajaram, Alptekinoglu, & Ou, 2004; Li & Zhang, 2013). As orders from advance selling may correlate with the demand in the spot selling period, advance selling can be used to obtain a better demand forecast (e.g., Zhao & Stecke, 2010; Prasad, Stecke & Zhao, 2011). Advance selling has been widely used in many industries as an important marketing strategy. For example, both Amazon and JD.com sell most new products in advance; these products include smartphones, consumer electronics, fashion products, books, video games, and software.

Advance selling is also beneficial to consumers. To encourage consumers to pre-order a product, the retailer normally offers a price discount as a pre-order incentive. For example, Amazon offered a 20% pre-order discount for all new-to-be-released video games from 2016 to 2018. Furthermore, advance selling guarantees prompt delivery on release. This is particularly valuable when the product may be hard to find in the spot selling period due to its popularity.

Most extant research (e.g., Xie & Shugan, 2001; Prasad et al., 2011; Zhao, Pang & Stecke, 2016a) on advance selling has assumed that the number of consumers who will be informed about a potential advance selling opportunity is fixed. Without advertising, this group mainly consists of brand-loyal consumers who stay informed at all times. However, the discussed benefits of exploiting valuation uncertainty and improving forecasts both increase with the number of informed consumers. This research is the first to analyze whether informing consumers of the advance selling opportunity is profitable. We remark that Cheng, Li and Thorstenson (2018) do discuss the effect on demand of starting "regular" marketing already during the advanced selling period, but they do not assess the profitability of advertising the advanced sales opportunity itself.

Another important contribution of this study lies in the sophisticated modeling of the consumer's choice between buying in advance and waiting for the sales season. We consider consumers who are loss averse and reference dependent (anchoring). To the best of our knowledge, this is the first study on advance selling strategy that considers the anchoring effect. Many studies have illustrated that loss aversion and the anchoring effect are prevalent in human decision-making in a variety of fields. We suggest interested readers refer to Camerer and Loewenstein (2004), Furnham and Boo (2011), and Neumann and Böckenholt (2014) for detailed reviews of loss aversion and the anchoring effect, respectively.

This study develops a newsvendor model that incorporates all important elements mentioned above: consumer valuation uncertainty, strategic response, loss aversion, anchoring effect, retailer's ordering and pricing decisions, demand uncertainty, demand information updates, advertising, and forecasting improvements. Specifically, we provide answers to the following questions: *When a retailer should use an advance selling strategy? When should a retailer advertise the pre-order opportunity? How do the anchoring effect and loss aversion affect the retailer's advance selling and advertising strategies?*

Our research reveals many new structural results and insights. Prior research (e.g., Xie & Shugan, 2001; Zhao & Stecke, 2010; Zhao, Pang & Stecke, 2016a) on advance selling has considered doing so to strategic informed customers (brand-loyal consumers) only. The present work is the first to explore whether a retailer should advertise the advance ordering opportunity. The analytical and numerical results show that a more accurate forecast of the market size plays a vital role in making decisions on advance selling. We provide new insights for retail managers into the advance selling strategy (no advance selling, without advertising, with advertising) selection. This should be based on both consumer characteristics and the market demand structure. Specifically, even without advertising, the retailer's optimal advance selling strategy depends on both the market size of strategic consumers and the optimal advance selling price (or price discount) determined by the consumer's valuation. Furthermore, we show that whether or not a retailer should advertise the pre-order opportunity also depends on both the consumer's valuation and the market size. Specifically, if a consumer's valuation is above a certain level, then a retailer can benefit from advertising by increased sales and forecast improvement; otherwise, advertising may hurt the retailer. Moreover, by considering loss aversion and anchoring effects, we model human behavior in decision-making. Our results show that the optimal advance selling strategy can be significantly affected when the consumer's expectation or target utility changes. We also show that the overall impact of loss aversion on the retailer's profit is negative, and that a lower consumer target valuation (anchor) makes advance selling more attractive.

The remainder of this article is organized as follows. In the next section, we review the related literature. "Problem Setting" section introduces key model settings. "A Benchmark: The Classic Newsvendor" section proposes a model for the newsvendor problem without advance selling as a benchmark. "Advance Selling without Advertising" section studies the newsvendor's advance selling decisions without advertising. "Advance Selling and Advertising" section studies the newsvendor's advance selling and advertising strategies. "Numerical Study" section provides numerical studies to discuss the sensitivity of advance selling and advertising strategies to market conditions. Finally, "Conclusion" section concludes the article.

LITERATURE REVIEW

In line with the extant literature and our modeling, we will mainly review contributions on advance selling in a newsvendor framework. In "The Newsvendor Problem with Advance Selling" and "Advertising" sections, the newsvendor literature is surveyed from the two aspects that our research integrates: advance selling and advertising. Our contributions are discussed in "Contribution" section.

The Newsvendor Problem with Advance Selling

A number of authors have considered the profitability of exploiting valuation uncertainty of consumers at the advance ordering stage, where consumers have not yet seen the actual product. Based on independent consumer valuation, Shugan and Xie (2000, 2005) and Xie and Shugan (2001) show that advance selling can improve a retailer's profit. Fay and Xie (2010) compare an advance selling strategy with a probabilistic selling strategy and show that uncertainty and heterogeneity of consumer valuation can benefit the retailer under both strategies in different ways. Yu, Kapuscinski and Ahn (2015) study the impact of interdependent consumer valuations on a retailer's advance selling strategy under limited capacity, revealing that the valuation interdependence can lead to a different pricing strategy. If valuations are highly diverse (related), the retailer should use a discount (premium) advance selling strategy. Under the advance selling strategy, informed consumers can strategically choose when to buy the product. Therefore, the literature mostly considers the strategic behavior of consumers and how that affects decisions. For example, Şeref, Alptekinoğlu and Erengüç (2016) consider the advance selling problem with strategic consumers. Lee, Choi and Cheng (2015) further consider the advance selling problem when strategic consumers are loss averse. They show that consumer's loss aversion leads to an inventory reduction at the retailer. Lim and Tang (2013) scrutinize advance selling by speculators who resell rather than use products. Nasiry and Popescu (2012) study the effect of anticipated regret on consumer decisions, retailer profits, and the advance selling strategy. They show that action regret hurts the retailer, whereas inaction regret helps. Wei and Zhang (2018) further show that the pre-order contingent production strategy, as a new advance selling strategy, can effectively mitigate strategic waiting behavior and can significantly improve the retailer's profit.

Zhao, Pang and Xiao (2016b) compare three pricing strategies: dynamic pricing, price commitment, and a pre-order price guarantee. One interesting find is that a pre-order price guarantee can reduce pre-order demand uncertainty. Cachon and Feldman (2017) examine the advance selling problem in a competitive environment to show that advance selling is less beneficial and may, in fact, be harmful under competition. Ma, Li, Sethi and Zhao (2019) show that whether a manufacturer should sell in advance depends on the manufacturer's market power and consumer risk aversion. In more recent literature, Cheng et al. (2018) study the advance selling problem with marketing efforts. They model market demand as a dynamic diffusion process and marketing decisions as a deterministic Markov decision-making process. Their investigation reveals that prolonging the selling season with an advance selling season can improve sales.

Another stream of research has studied improved demand forecasting from advance selling by focusing on whether demand updates benefit the retailer. Tang et al. (2004) show that advance selling can reduce inventory risk for a retailer. McCardle, Rajaram and Tang (2004) extend their work by considering brand competition among retailers. Zhao and Stecke (2010) and Prasad et al. (2011) examine benefits from both demand updating and consumer valuation uncertainty to decide when a retailer should sell in advance to risk-averse and loss-averse consumers. Zhao et al. (2016a) further study how the advance selling option affects the interactions between a retailer and a manufacturer in a decentralized supply chain. They show that the advance selling option can hurt both the retailer's profit and the supply chain performance. To summarize, these studies discuss when and how the retailer can benefit from demand updates, but all assume a fixed number of informed consumers, whereas we will explore the profitability of informing more consumers using advertising.

Advertising

At the normal ordering stage, the retailer can advertise the new-to-be-released product by adopting a new product pre-announcement (NPP) strategy. The NPP allows consumers to speculate about a product's characteristics, resulting in valuation uncertainty. Different aspects of the NPP have been investigated: timing and content (e.g., Homburg, Bornemann & Totzek, 2009; Jung, 2011), consumer effects, for example, the effects on valuation and response (e.g., Dahlén, Thorbjørnsen & Sjödin, 2011; Thorbjørnsen, Ketelaar, van't Riet & Dahlén, 2015), and competitive effects (e.g., Zhang, Liang & Huang, 2016). However, to the best of our knowledge, advertising of the advance ordering stage has not been considered. Advertising in general has, of course, been intensively studied, and we will review those contributions that are most relevant to our study.

Vidale and Wolfe (1957) published one of the earliest advertising response models based on three parameters: a sales decay constant, saturation level, and response constant. Advertising response functions are typically assumed to be either concave or S-shaped (Jones, 1995). A concave response function indicates that increased advertising expenditure leads to increased sales but at a diminishing rate. An S-shaped response function has a critical threshold: when advertising expenditure is low, sales exhibit little response to advertising; when advertising expenditure exceeds the threshold, sales respond more to advertising.

There is an ongoing debate on how to best model the advertising response (Simon & Arndt, 1980; Bronnenberg, 1998). Since Hollander (1949) first reported that advertising has a carry-over effect on sales, that is, a positive effect that influences both current and future periods, several studies have suggested that advertising activities have a long-term effect on sales growth or firm value (e.g., Tull, 1965; Moriarty, 1983). Conversely, other studies have claimed that the effects of advertising are merely short-lived (Dekimpe & Hanssens, 1995; Duffy, 2001). For instance, Dekimpe and Hanssens (1995) show that the effects of advertising on financial performance disappear within a year. We refer interested readers to Danaher (2008) for summaries of the advertising response models.

Given a sales response function, key decisions include what budget to assign to advertising and how many products to keep in stock. A substantial number of authors have focused on this marketing–operations interface, many of them considering single-period newsvendor formulations. The first to do so were Gerchak and Parlar (1987). They considered a setting where market penetration can be estimated accurately, but the size of the market remains uncertain. Others later generalized this setting to demand that is stochastic and advertising-sensitive (e.g., Eliashberg & Steinberg, 1993; Hausman, Montgomery & Roth, 2002). By assuming that the mean demand is increasing and concave in advertising expenditures, Khouja and Robbins (2003) study three cases of demand variation as a function of advertising expenditure: (i) constant variance, (ii) constant coefficient of variation, and (iii) increasing coefficient of variation. They provide solutions for several demand distributions. These results were adapted by Lee and Hsu (2011) and Güler (2014) to a distribution free newsvendor setting. Their results confirm that advertising can improve a retailer's profit. Further, increasing sales through advertising leads to an increase in the optimal order quantity, and the optimal advertising expenditure increases with the profit margin. We refer interested readers to Khouja (1999) for a detailed review of the newsvendor problem with advertising.

Table 1: Notation.

p_A	Advance selling price per unit.
p_λ	Price discount in the advance selling period, that is, $p - p_A$.
p	Spot selling price per unit.
c	Cost per unit, $c < p$.
s	Salvage value per unit, $s < c$.
V	Consumer valuation for a product which has mean μ_v, standard deviation σ_v, CDF $F(\cdot)$ and PDF $f(\cdot)$ with support on $[L, U]$.
D_A	Demand (number of consumers who buy) in the advance selling period.
D_{SS}	Demand (number of consumers who buy) in the selling season with mean μ_{SS} and standard deviation σ_{SS}.
$\mathcal{U}_A$	Expected utilities for buying in the advance selling period.
$\mathcal{U}_\mathcal{W}$	Expected utilities for waiting until the spot selling period.
N_s	Number of strategic consumers, a normally distributed random variable, that is, $N_s \sim N(\mu_s, \sigma_s^2)$.
$N_{si}(N_{su})$	Number of informed (uninformed) strategic consumers, that is, $N_{si} = \eta_0 N_s$ and $N_{su} = (1 - \eta_0)N_s$ where $\eta_0 \in (0, 1]$ is a portion of informed strategic consumers.
N_m	Number of myopic consumers, a normally distributed random variable, that is, $N_m \sim N(\mu_m, \sigma_m^2)$.
B	Expenditure on advertising.
$\eta(B)$	A consumers-to-advertising response function, $\eta'(B) \geq 0$, $\eta''(B) \leq 0$.
$N_{si}(B)$	Number of informed strategic consumers after advertising under a given budget B.

Contribution

This study differs from the extant literature on the advance selling newsvendor problem in three ways. First, our study is the first to explore whether retailers should advertise the advance ordering opportunity or limit advance sales to brand fans who always stay informed. Second, although some studies (e.g., Zhao & Stecke, 2010) have considered advance selling under a loss-averse utility, they implicitly assume a zero anchor. We include the anchoring effect, which leads to new and more complete insights. Third, although the extant literature on advance selling has considered the effects of demand updating (e.g., Zhao & Stecke, 2010; Prasad et al., 2011), we explicitly model the forecast improvement from advance selling. As our analysis will show, this provides important new insights on the profitability of advanced selling in relation to the market size.

PROBLEM SETTING

Our general setting is that of a retailer who faces a single-period newsvendor problem. The retailer can allow consumers to place advance orders and, if so, can advertise this opportunity. Table 1 lists the notation used in this article. Please note that all proofs are in the online appendix.

Retailer Settings

In the advance selling period, which ends before the start of the spot selling period, the retailer allows consumers to make purchases at a price p_A per item and commits to fulfilling advance purchase orders in the spot selling period, guaranteeing delivery to these consumers (i.e., avoiding a potential stock out). The retailer uses a price commitment pricing strategy, that is, announces the advance price and the spot selling price p simultaneously at the beginning of the advance selling period. As the spot selling market is more competitive and the demand is more sensitive to the spot selling price, in our main analysis, the spot selling price p is assumed to be exogenous and its value is determined by using the reference pricing strategy, that is, the selling price equals that of its competitors. This also keeps the analysis tractable. Note that in "Extension: Endogenousness of Spot Selling Price" section, we study numerically how an endogenous spot price affects the advance selling strategy of the retailer.

At the beginning of the spot selling period, the retailer decides on the order quantity. This quantity can be split as $Q + d_A$, where d_A products are used to fulfill orders from the advance selling period. Note that the observed demand in the advance selling period can be used to update the demand forecast for the spot selling season.

Consumer Settings

Consumer valuation

Consumer valuation is the maximum value a consumer is willing to pay. Consumers are uncertain about their own valuation during the advance selling period. Therefore, we assume that consumer valuation V is a random variable which has mean μ_v, standard deviation σ_v, a PDF $f(\cdot)$ and a CDF $F(\cdot)$, with a finite support on $[L, U]$ where $L > s > 0$, and s is the salvage value per unit. The realized valuation in the spot selling period is denoted by v. Note that the valuation uncertainty can be affected by many factors and valuations; it can also vary across consumers. To keep the problem tractable, we assume that all consumers have the same valuation distribution. However, as we will explain next, we distinguish between strategic and myopic consumers.

Consumer surplus, behavior, classification, and decision-making

Consumer surplus is the difference between the consumer valuation and the actual price the consumer pays, that is, $V - p$. If consumers buy in advance at price p_A, the uncertain surplus is $V - p_A$. Buying in the spot selling period implies a certain consumer surplus of $v - p$.

Although all consumers are assumed to have the same valuation distribution, we introduce consumer heterogeneity by grouping consumers into two types: myopic (or nonstrategic) and strategic. Strategic consumers potentially (when informed) consider buying in advance, but myopic consumers do not (because they either are not informed or do not act upon it). Myopic consumers never purchase in the advance selling period, which can be for a number of reasons, such as they are conservative and never buy an inexperienced product, their value for an inexperienced product is very low, they do not take any valuation risk at all, their buying and consumption behaviors are not separated, they are unwilling to return to the retailer in the spot selling period, or they are simply short-sighted. Consistent with most of the literature (e.g., Cachon & Swinney, 2009a, 2009b), myopic consumers make a buy-now-or-leave-forever purchase decision only in the spot selling period. Thus, let N_m and N_s denote the number of myopic and strategic consumers, respectively. We assume that N_m and N_s are independent and normally distributed with means μ_m and μ_s, and standard deviations σ_m and σ_s, respectively. That is, $N_m \sim N(\mu_m, \sigma_m^2)$ and $N_s \sim N(\mu_s, \sigma_s^2)$.

As in Varian (1980), depending on whether strategic consumers are aware of advance purchases offerings, we further introduce consumer heterogeneity by grouping strategic consumers into two types: informed and uninformed, with group sizes denoted by N_{si} and N_{su}, respectively. As part of the strategic consumers are informed, we let $N_{si} = \eta_0 N_s$, where $\eta_0 \in (0, 1]$ represents the portion of informed strategic consumers. Correspondingly, $N_{su} = (1 - \eta_0)N_s$.

A myopic or uninformed strategic consumer buys a product if his or her realized surplus $v - p$ is nonnegative. Note that myopic (uninformed strategic) consumers never face uncertainty because the consumer valuation is realized and certain. In general, myopic (uninformed strategic) consumers are risk neutral. However, consumers who buy in advance are uncertain about their valuation because they cannot observe or (fully) experience the product in advance. Based on the information received, they may form a target valuation (reference point/anchor, consumer's expectation). Due to the valuation uncertainty, informed strategic consumers who have to choose between buying in advance or in the regular season (or not at all) face a more difficult decision problem, and take the risk that their realized surplus is lower than expected. In other words, they may suffer a loss from buying in advance, where the target valuation determines whether a realized surplus is perceived as a loss or a gain. As a result, informed strategic consumers are assumed to be loss averse with anchoring.

Note that we do not consider the stockout risk for strategic consumers who decided not to buy in advance. This seems justified for the following two reasons: First, we do not consider a clearance sale in our model. Therefore, for informed strategic consumers who choose to buy in the spot selling period, it is rational to buy the product at the start of the spot selling period rather than wait until the end. Hence, the availability risk would be minimal. Second, the retailer's order quantity is private information, making it difficult for consumers to estimate the stockout risk. Therefore, we do not consider a stockout risk in our main analysis. However, in "Advance Selling and Advertising" section, we will address the effects of including such a risk on our results.

To reflect both reference/target dependency and loss aversion, we adopt a piecewise loss-averse utility to describe both loss aversion and reference dependency for informed strategic consumers. More specifically, the utility (valuation) is defined as:

$$U(V) = V - \lambda(V_0 - V)^+,$$

where $\lambda \geq 0$ is a degree of loss aversion and V_0 is the consumer's target valuation (expectation). Note that, if $\lambda = 0$, then the loss-averse utility reduces to the risk-neutral utility. This piecewise-linear form of the loss-averse utility function is a special case of a prospect theory function, which has been widely used in the literature on economics and operations management (e.g., Long & Nasiry, 2015; Wu, Bai & Zhu, 2018). Furthermore, to make the problem nontrivial, we assume that the reference target valuation V_0 is greater than the spot selling price p, that is, $V_0 \geq p$; otherwise, any loss-averse utility reduces to risk neutral utility. To keep the problem tractable, we assume that all consumers have the same loss-averse utility function with the same loss-averse degree and the same target valuation. Note that the consumer's target valuation is usually driven by perceived performance. Such a target valuation (expectation) would be affected by a number of factors, such as product valuation, performance, feature, and a comparison with substitute products. However, we do not model these issues and restrict our focus to pricing and ordering. Hence, the target valuation V_0 is assumed to be given exogenously. Then, the expected surplus of buying early is:

$$\begin{aligned}\mathcal{U}_A &= E(U(V)) - p_A \\ &= E(V - \lambda(V_0 - V)^+) - p_A \\ &= \mu_v - p_A - \lambda \int_L^{V_0} F(x)dx. \end{aligned} \tag{1}$$

If consumers do not buy early, there are several possibilities. If the realized consumer valuation v is smaller than a critical utility $\frac{\lambda V_0 + p}{\lambda+1}$, then the utility valuation is negative. Hence, informed strategic consumers do not buy and receive a zero surplus. If $v > \frac{\lambda V_0 + p}{\lambda+1}$, then the consumer makes a purchase and obtains a surplus of $v - p$ when $v > V_0 (\geq p)$ and a surplus of $(\lambda + 1)v - \lambda V_0 - p$ when $V_0 \geq v > \frac{\lambda V_0 + p}{\lambda+1}$. In summary, the loss-averse consumer's surplus for strategic consumers who do not buy in advance is:

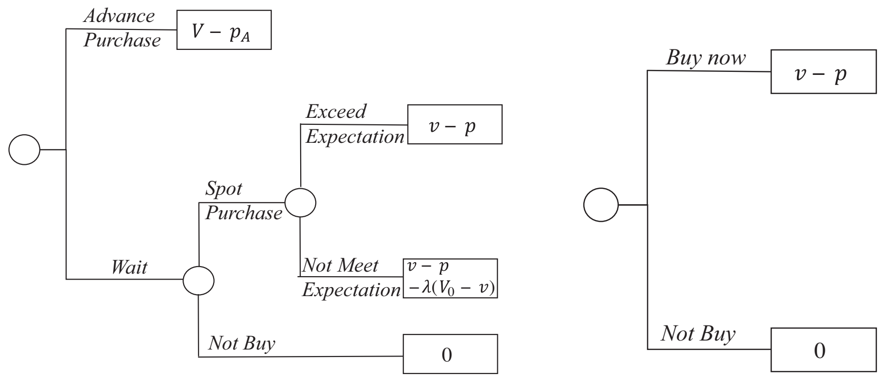
$$U(V) - p = \begin{cases} v - p, & \text{if } v > V_0, \\ v - p - \lambda(V_0 - v), & \text{if } V_0 \geq v > \frac{\lambda V_0 + p}{\lambda+1}, \\ 0, & \text{if } v \leq \frac{\lambda V_0 + p}{\lambda+1}. \end{cases}$$

Hence, the expected utility of waiting is:

$$\begin{aligned}\mathcal{U}_W &= E(U(V) - p) \\ &= \int_{V_0}^{U} (x - p)dF(x) + \int_{\frac{\lambda V_0 + p}{\lambda+1}}^{V_0} ((\lambda + 1)x - \lambda V_0 - p)dF(x) \\ &= \mu_v - p + (\lambda + 1) \int_L^{\frac{\lambda V_0 + p}{\lambda+1}} F(x)dx - \lambda \int_L^{V_0} F(x)dx. \end{aligned} \tag{2}$$

An informed strategic consumer buys early if $\mathcal{U}_A \geq 0$ and $\mathcal{U}_A \geq \mathcal{U}_W$; otherwise, a strategic consumer waits until the spot selling season and buys if his or her realized consumer surplus is nonnegative, that is, $v - p \geq 0$, or the consumer does not buy at all. Note that a nonnegative expected utility of waiting $\mathcal{U}_W \geq 0$ at the advance selling stage does not imply that a strategic consumer always buys in the spot selling period. This is because a strategic consumer reassesses the purchase decision after the valuation for the product is realized. More specifically, due to the valuation uncertainty, consumers are loss averse when deciding whether to buy in the advance selling period. If they choose to wait, then strategic consumers behave in the same way as myopic consumers do in deciding whether to buy in

Figure 1: Consumer surplus for different decisions.



(a) Advance selling period (b) Spot selling periodt

the spot period, because there is no uncertainty (risk) at all. In other words, for the consumers who have not pre-purchased, whether to buy in the spot selling periods is unaffected by either loss aversion or anchoring. Figure 1 summarizes the consumer surplus with advance selling for different decisions.

We incorporate loss aversion and anchoring into modeling the strategic buying behavior because, first, consumers who buy in advance may suffer a utility loss. Therefore, it is necessary to consider consumers' attitude toward loss in order to better model their pre-buying behavior. Second, advance selling separates consumer behavior into purchasing behavior (for the advance selling period) and consumption behavior (for the spot selling period). Comparing pre-buying with spot buying, our model clearly shows how valuation realization reassesses the decision of buying. We remark that if the anchor is not included in the model, that is, $V_0 = p$, then waiting (not buying in advance) is identical to spot buying, though these two behaviors are obviously different. More specifically, informed consumers usually form an expectation about a product, whereas consumers who spot buy a product may not (or the expectation is less relevant), because the decision can be made after they fully experience the product.

Advertising Settings

Not all strategic consumers are informed of the advance ordering opportunity. Indeed, without advertising, they may form a relatively small group of brand-loyal fans (aficionado) and/or technology enthusiasts. Advertising can thus help stimulate advance ordering.

Let B denote the advertising budget. The market size in the advance selling period N_{si} can be enlarged to $N_{si}(B)$ by advertising. More specifically, we assume that $N_{si}(B)$ is given by

$$N_{si}(B) = \eta(B)N_s + \epsilon = (\eta_0 + \omega B)N_s + \epsilon, \tag{3}$$

where $\eta(B)$ is a linear advertising response function, $\eta_0 \geq 0$ is the initial fraction of informed strategic consumers, $\omega \geq 0$ is the effectiveness of advertising, and ϵ is a normally distributed random variable with mean zero and standard deviation σ_ϵ, that is, $\epsilon \sim N(0, \sigma_\epsilon^2)$, which is independent of both N_s and N_m. Without the error term ϵ, the response of advertising is certain, and the number of strategic consumers is known exactly after advance demand is realized, and the problem becomes trivial. Moreover, we assume for tractability that $\sigma_\epsilon \leq \eta_0 \sigma_s$. That is, we assume that the uncertainty in the number of strategic informed consumers is mainly related to uncertainty in the number of strategic consumers (i.e., the strategic market size) and, to a lesser degree, related to estimating the effectiveness of advertising. This is reasonable because we are dealing with a new product for which the potential market size is especially difficult to judge. However, past experiences with other products should help estimate the advertisement response function accurately. It can be easily verified that this advertising response model has a constant coefficient of variation. For other types of response functions, we refer interested readers to Khouja and Robbins (2003) and Lilien, Rangaswamy and De Bruyn (2007). Note that in "Extension: Concavity of Advertising Response" section, we study numerically how a concave advertising response affects the advance selling strategy of the retailer.

Using $N_s = N_{si}(B) + N_{su}(B)$, we find that the number of uninformed strategic consumers is given by

$$N_{su}(B) = (1 - \eta(B))N_s - \epsilon. \tag{4}$$

Hence, the uncertainty in the number of uninformed strategic consumers decreases with the fraction of informed consumers. This is a desirable property, as (i) more informed strategic consumers implies fewer "remaining" uninformed strategic consumers, and (ii) higher advance sales should statistically lead to a more accurate forecast of the remaining number of strategic consumers. It is important to remark that our setting is different from that of Prasad et al. (2011) in this respect. They assume that the fraction of informed consumers does not affect the benefit of reduced risk in the spot selling period. The retailer's decisions are summarized and depicted in Figure 2.

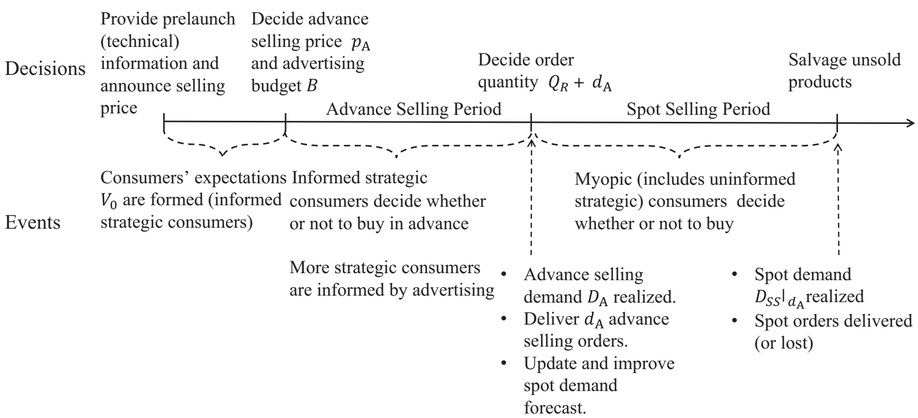
A BENCHMARK: THE CLASSIC NEWSVENDOR

We first determine the optimal order quantity and associated profit without advance selling, which serves as a benchmark for assessing the profitability of introducing the advance ordering opportunity and of advertising it.

Under this scenario, demand in the advance selling period is obviously zero, that is, $D_A = 0$. During the spot selling period, consumer valuations are realized. Consumer i makes a purchase at price p if and only if his or her consumer surplus is nonnegative, that is, $v_i - p \geq 0$. As all consumers have the same valuation distribution, the fraction of all consumers who buy at price p is $E(1(v_i \geq p)) = \bar{F}(p)$. Therefore, the spot selling demand is given by

$$D_{SS} = \sum_{i=1}^{N_s+N_m} E(1(v_i \geq p)) = (N_s + N_m)\bar{F}(p),$$

Figure 2: Timeline of decisions and events under the advance selling strategy.



where $1(k)$ is an indicator function, that is, $1(k) = 1$ if k is true; otherwise, $1(k) = 0$. As the numbers of myopic and strategic consumers are normally distributed and independent, the spot selling demand is normally distributed with mean and variance as follows:

$$\mu_{SS} = E(D_{SS}) = (\mu_s + \mu_m)\bar{F}(p),$$
$$\sigma_{SS}^2 = \mathrm{Var}(D_{SS}) = (\sigma_s^2 + \sigma_m^2)\bar{F}^2(p).$$

The retailer's expected profit is:

$$\pi_{NA}(Q) = E_{D_{SS}}\{p \min\{Q, D_{SS}\} + s(Q - D_{SS})^+ - cQ\}.$$

Deciding on the optimal order quantity is a classic newsvendor problem with normally distributed demand. Thus, we have the following result.

Lemma 1: For a newsvendor problem without advance selling and advertising, the optimal order quantity and the optimal expected profit are as follows:

$$Q_{NA}^* = (\mu_s + \mu_m)\bar{F}(p) + k\sqrt{\sigma_s^2 + \sigma_m^2}\,\bar{F}(p),$$

$$\pi_{NA}^* = (p - c)(\mu_s + \mu_m)\bar{F}(p) - (p - s)\phi(k)\bar{F}(p)\sqrt{\sigma_s^2 + \sigma_m^2},$$

where $k = \Phi^{-1}((p - c)/(p - s))$; $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and the PDF of the standard normal distribution, respectively.

ADVANCE SELLING WITHOUT ADVERTISING

We next consider advance selling, but without advertising. Compared with the case with advertising that will be considered in the next section, this is easier to analyze as it does not involve deciding on the advertising budget. Furthermore, it allows a direct comparison with results in the literature.

Advance selling only makes sense if the advance selling price is attractive enough for informed strategic consumers to indeed buy in advance. As discussed in "Consumer Surplus, Behavior, Classification, and Decision-Making" section, this implies that $\mathcal{U}_A \geq \mathcal{U}_W$ must hold. As the retailer aims to maximize his or her profit, he or she selects the advance selling price for which sets $\mathcal{U}_A = \mathcal{U}_W$. From (1) and (2), we obtain

$$\mu_v - p_A - \lambda \int_L^{V_0} F(x)dx \geq \mu_v - p + (\lambda+1) \int_L^{\frac{\lambda V_0 + p}{\lambda+1}} F(x)dx - \lambda \int_L^{V_0} F(x)dx,$$

which implies

$$p_A^* = p - (\lambda+1) \int_L^{\frac{\lambda V_0 + p}{\lambda+1}} F(x)dx, \tag{5}$$

where the latter term on the right-hand side is the discount offered to advance buyers.

We next determine the optimal ordering quantity (and corresponding expected profit), which depends on the distributions of the numbers of strategic informed, strategic uninformed, and myopic buyers. For the case without advertising, we obtain the following from (4):

$$N_{si} := \eta_0 N_s + \epsilon,$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ is independent of both N_s and N_m. Therefore, we obtain $N_{si} \sim N(\mu_{si}, \sigma_{si}^2) = N(\eta_0 \mu_s, \eta_0^2 \sigma_s^2 + \sigma_\epsilon^2)$. Correspondingly, the number of uninformed strategic consumers $N_{su} = (1-\eta_0)N_s - \epsilon$ also follows a normal distribution, that is, $N_{su} \sim N(\mu_{su}, \sigma_{su}^2) = N((1-\eta_0)\mu_s, (1-\eta_0)^2\sigma_s^2 + \sigma_\epsilon^2)$. We easily obtain the covariance and the coefficient of variation between N_{si} and N_{su} as:

$$\mathrm{Cov}(N_{si}, N_{su}) = \eta_0(1-\eta_0)\sigma_s^2 - \sigma_\epsilon^2$$

and

$$\rho_0 = \frac{\mathrm{Cov}(N_{si}, N_{su})}{\sqrt{\mathrm{Var}(N_{si})\mathrm{Var}(N_{su})}} = \frac{\eta_0(1-\eta_0)\sigma_s^2 - \sigma_\epsilon^2}{\sqrt{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} \cdot \sqrt{(1-\eta_0)^2\sigma_s^2 + \sigma_\epsilon^2}},$$

respectively.

We can now use the following general statistical result to obtain the conditional mean and variance of N_{su}: For two bivariate normal random variables $(X, Y) \sim N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, the conditional mean and the conditional variance of Y given $X = x$ are $E(Y|X=x) = \mu_Y + \rho(x - \mu_X)\frac{\sigma_Y}{\sigma_X}$ and $\mathrm{Var}(Y|X = x) = \sigma_Y^2(1-\rho^2)$.

Application of this general result to our case gives:

$$\begin{aligned}\mu_{su|n_{si}} &= \mu_{su} + \rho_0(n_{si} - \mu_{si})\frac{\sigma_{su}}{\sigma_{si}} \\ &= (1-\eta_0)\mu_s + (n_{si} - \mu_{si})\frac{\eta_0(1-\eta_0)\sigma_s^2 - \sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2}\end{aligned}$$

and

$$\sigma^2_{su|n_{si}} = \sigma^2_{su}(1-\rho_0^2) = (1-\eta_0)^2\sigma_s^2 + \sigma_\epsilon^2 - \frac{(\eta_0(1-\eta_0)\sigma_s^2 - \sigma_\epsilon^2)^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2}$$
$$= \frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2}.$$

Hence, for a realized advance selling demand $d_A = n_{si}$, the total demand (strategic uninformed and myopic) during the spot selling season D_{SS} has an updated mean and a standard deviation as follows:

$$\mu_{SS|d_A} = (\mu_{su|d_A} + \mu_m)\bar{F}(p),$$
$$\sigma^2_{SS|d_A} = (\sigma^2_{su|d_A} + \sigma^2_m)\bar{F}^2(p) = \left(\frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2\right)\bar{F}^2(p).$$

Let π_{AS} denote the retailer's total expected profit with advance selling. Then, we obtain

$$\pi_{AS} = E_{D_A}\{(p_A - c)D_A + E_{D_{SS}|D_A=d_A}\{p\min\{Q, D_{SS}\} + s(Q-D_{SS})^+ - cQ\}\}$$
$$= E_{D_A}\{(p_A - c)D_A\} + E_{D_A}\{E_{D_{SS}|D_A=d_A}\{p\min\{Q, D_{SS}\}$$
$$+ s(Q-D_{SS})^+ - cQ\}\}.$$

The retailer must decide the order quantity Q to maximize the total expected profit and decide on the advance selling price p_A to make all informed strategic consumers buy in the advance selling season. That is:

$$\begin{aligned} \max_{p_A}\quad & E_{D_A}\{(p_A - c)D_A + \max_Q E_{D_{SS}|D_A=d_A}\{p\min\{Q, D_{SS}\} \\ & + s(Q-D_{SS})^+ - cQ\}\} \\ \text{s.t.}\quad & \mathcal{U}_A \geq \mathcal{U}_W. \end{aligned} \tag{6}$$

Then, we obtain the following result.

Proposition 1: For the advance selling newsvendor problem without advertising, the optimal order quantity and total expected profit are:

$$Q^*_{AS}|_{d_A} = \mu_{SS|d_A} + k\sigma_{SS|d_A}$$
$$= \left((1-\eta_0)\mu_s + \mu_m + (d_A - \eta_0\mu_s)\frac{\eta_0(1-\eta_0)\sigma_s^2 - \sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2}\right)\bar{F}(p)$$
$$+ k\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2},$$
$$\pi^*_{AS} = (p_A^* - c)\eta_0\mu_s + (p-c)((1-\eta_0)\mu_s + \mu_m)\bar{F}(p)$$
$$- (p-s)\phi(k)\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2}.$$

The difference between the optimal profit of the advance selling newsvendor and the classic newsvendor problem is:

$$\begin{aligned}\Delta\pi^*_{AS} &= \pi^*_{AS} - \pi^*_{NA} \\ &= (p^*_A - c)\eta_0\mu_s - (p-c)\eta_0\mu_s\bar{F}(p) \\ &\quad + (p-s)\phi(k)\bar{F}(p)\left(\sqrt{\sigma_s^2+\sigma_m^2} - \sqrt{\frac{\sigma_s^2}{\eta_0^2\sigma_s^2/\sigma_\epsilon^2+1}+\sigma_m^2}\right) \\ &= (p^*_A - p_0)\eta_0\mu_s + (p-s)\phi(k)\bar{F}(p)\Delta\sigma_0, \end{aligned} \tag{7}$$

where $p_0 := p\bar{F}(p) + cF(p)$ is a critical price determined by the consumer's valuation and $\Delta\sigma_0 := \sqrt{\sigma_s^2+\sigma_m^2} - \sqrt{\frac{\sigma_s^2}{\eta_0^2\sigma_s^2/\sigma_\epsilon^2+1}+\sigma_m^2}$. As $\sigma_\epsilon \leq \eta_0\sigma_s$, $\Delta\sigma_0 \geq 0$ always holds.

Note that the profit difference $\Delta\pi^*_{AS}$ can be further rewritten as

$$\Delta\pi^*_{AS} = (p^*_A - c)\eta_0\mu_s F(p) - (p - p^*_A)\eta_0\mu_s\bar{F}(p) + (p-s)\phi(k)\bar{F}(p)\Delta\sigma_0.$$

The three terms of the above profit difference expressions have clear interpretations. The first term represents the added profit from selling $\eta_0\mu_s F(p)$ at price p^*_A. The second term is the loss from selling at a discount to informed strategic consumers who would otherwise have bought the item at the spot selling price. The final term is the safety stock reduction that results from better forecasting of the spot selling demand.

The profit difference and its interpretation also lead to the identification of parameter settings where advance selling is profitable. If the pre-order discount $p - p^*_A$ is sufficiently small, then the profit from additional sales $(p^*_A - c)\eta_0\mu_s F(p)$ is larger than the discount loss $(p - p^*_A)\eta_0\mu_s\bar{F}(p)$ for all values of μ_s because both terms are linear in μ_s. For a large pre-order discount, the combined effect of terms $(p^*_A - c)\eta_0\mu_s F(p)$ and $(p - p^*_A)\eta_0\mu_s\bar{F}(p)$ on the profit is negative, and the corresponding profit deduction is increasing with μ_s. The safety stock reduction $(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0$ is not affected by μ_s and, therefore, only outweighs the discount-related profit reduction if μ_s is sufficiently small. This is formalized in the following theorem. That is, all the proofs can be found online in the Supporting Information Section.

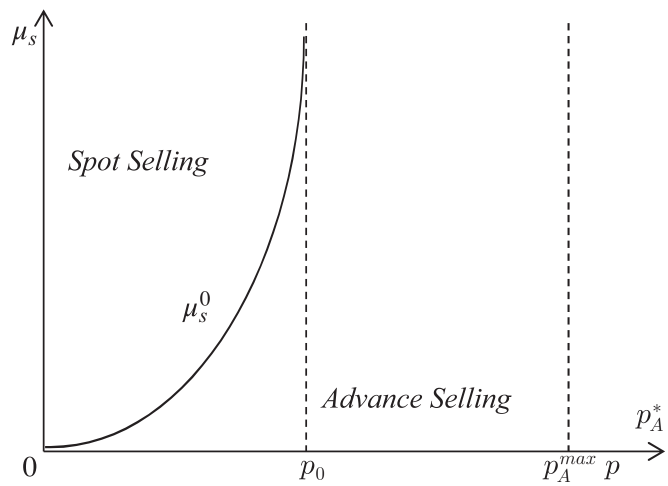
Theorem 1: (Advance Selling vs. Spot Selling)

- If the optimal advance selling price is higher than a threshold, that is, $p^*_A \geq p_0$, then a retailer should always sell in advance.
- Otherwise, the retailer should only sell in advance if the average number of strategic consumers is less than a threshold, that is, $\mu_s \leq \mu_s^0 := \frac{(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0}{\eta_0(p_0 - p^*_A)}$.

The interpretation of this result is that selling in advance can only be profitable if either the discount is not too large, or the discount is large but the number of strategic informed consumers who obtain the discount is limited. Figure 3 provides further graphical insight into when advance selling is profitable. It becomes clear that, as the advance selling price drops below p_0, the maximum relative size of the strategic consumers at which advance selling is profitable rapidly decreases.

Note that the fraction of informed consumers to all strategic consumers η_0 affects strategy selection. More specifically, as the profit from the safety stock reduction $(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0$ is always increasing in η_0, if $p^*_A \geq p_0$, then the

Figure 3: Optimal policy without advertising.



retailer should always sell in advance. The retailer will earn more when more strategic consumers are informed, that is, $\frac{d\Delta\pi_{AS}^*}{d\eta_0} \geq 0$. However, if $p_A^* < p_0$, then the critical threshold of the strategic market size μ_s^0 may be increasing or decreasing in η_0. In other words, whether selling in advance is optimal depends on the trade-off between the benefit from the safety stock reduction $(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0$ and the loss from offering a discount to all informed strategic consumers $(p_A^* - p_0)\eta_0\mu_s$.

As the advance selling price relates directly to the degree of loss aversion and the anchor of strategic consumers, a similar result can be obtained on whether advance selling is profitable in terms of loss aversion and the anchoring effect. By using $p_A^* = p - (\lambda+1)\int_L^{\frac{\lambda V_0 + p}{\lambda+1}} F(x)dx$, we can rewrite $\Delta\pi_{AS}^*$ as

$$\Delta\pi_{AS}^* = ((p-c)F(p) - (\lambda+1)\int_L^{\frac{\lambda V_0+p}{\lambda+1}} F(x)dx)\eta_0\mu_s + (p-s)\phi(k)\bar{F}(p)\Delta\sigma_0.$$

As $\Delta\pi_{AS}^*$ is decreasing with respect to both λ and V_0, if $\Delta\pi_{AS}^*|_{\lambda=0} \geq 0$ or $\Delta\pi_{AS}^*|_{V_0=p} \geq 0$, then there exist critical values $\hat{\lambda}$ and $\hat{V}_0$ that satisfy the following equations, respectively:

$$(\hat{\lambda}+1)\int_L^{\frac{\hat{\lambda} V_0+p}{\hat{\lambda}+1}} F(x)dx = (p-c)F(p), \tag{8}$$

$$(\lambda+1)\int_L^{\frac{\lambda \hat{V}_0+p}{\lambda+1}} F(x)dx = (p-c)F(p). \tag{9}$$

Then, we obtain the following result.

Proposition 2: If the critical target utility $\hat{V}_0$ (or the critical loss aversion degree $\hat{\lambda}$) does not exist, that is, $\Delta\pi_{AS}^*|_{V_0=p} \leq 0$ (or $\Delta\pi_{AS}^*|_{\lambda=0} \leq 0$), then the spot selling only strategy is always optimal; otherwise, the advance selling strategy is optimal in the following cases:

- If the target utility (or the loss aversion degree) is lower than a threshold, that is, $V_0 \leq \hat{V}_0$ (or $\lambda \leq \hat{\lambda}$), then a retailer should always sell in advance.
- Otherwise, the retailer should only sell in advance if the average number of all strategic consumers is less than a threshold, that is, $\mu_s \leq \mu_s^0$ where

$$\mu_s^0 = \frac{(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0}{\eta_0((\lambda+1)\int_L^{\frac{\lambda V_0+p}{\lambda+1}} F(x)dx - (p-c)F(p))}.$$

It is clear that the anchoring effect can significantly affect the advance selling strategy even if the consumers' valuation and their attitude toward loss are unchanged. Furthermore, the retailer can benefit from advance selling by setting realistic consumer expectations (narrowing the consumer's expectation gap). From a marketing perspective, this provides the following insights: On the one hand, if a new product has few comparative advantages (e.g., cost effectiveness, better appearance, product uniqueness, and good quality) compared with existing substitute products, then a retailer usually does not benefit from advance selling. On the other hand, a good and unknown product should be sold in advance and the retailer should accurately state the features of the product. Our findings stress the importance of the anchoring effect for decision-making on advance selling.

Note that these structural insights are different and, arguably, a refined version of those in Prasad et al. (2011). Recall from "Advertising Settings" section that, different from their model, we consider a setting where the fraction of informed consumers and, thereby, the number of advanced sales affects the benefit of reduced risk in the spot selling periods. This moderates the effect of the number of strategic consumers on the profitability of allowing advance orders.

Without an anchoring effect and loss aversion, Theorem 1 reduces to the following result by setting $\lambda = 0$ and $V_0 = p$.

Corollary 1: If all consumers are risk neutral, then the optimality conditions for the different selling strategies are as follows:

- If the cost is lower than a threshold, that is, $c \leq c_0$, then a retailer should always sell in advance.
- Otherwise, the retailer should only sell in advance if the average number of strategic consumers is less than a threshold, that is, $\mu_s \leq \tilde{\mu}_s^0$,

where $c_0 = p - \int_L^p \frac{F(x)}{F(p)}dx$ and $\tilde{\mu}_s^0 := \frac{(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0}{\eta_0(c-c_0)F(p)}$.

Under the assumption of constant market sizes, Xie and Shugan (2001) find that retailers should sell in advance when the marginal costs are below some lower threshold of consumer valuation. If market sizes are uncertain, then Prasad et al. (2011) find that no retailer should sell in advance if the difference the between consumer expected valuation and consumer expected surplus of waiting is above some threshold. Our results show that, besides the marginal valuation/cost, the market size of strategic consumers affects the optimal strategy on advance selling.

ADVANCE SELLING AND ADVERTISING

The advance selling price is unchanged from the case without advertising because advertising makes more consumers aware of the advance selling opportunity but does not alter their valuation distribution. The analysis of the optimal order quantity decision, given the advertising budget, is more tedious, but similar to that without advertising (i.e., with a zero budget) in the previous section. This leads to the following proposition.

Proposition 3: Given advertising budget B, the optimal order quantity and total expected profit are:

$$\begin{aligned} &Q^*_{AS}(B)|_{d_A} \\ &= \left((1-\eta(B))\mu_s + \mu_m + (d_A - \eta(B)\mu_s)\frac{\eta(B)(1-\eta(B))\sigma_s^2 - \sigma_\epsilon^2}{\eta^2(B)\sigma_s^2 + \sigma_\epsilon^2}\right)\bar{F}(p) \\ &\quad + k\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta^2(B)\sigma_s^2+\sigma_\epsilon^2} + \sigma_m^2}, \\ &\pi^*_{AS}(B) = (p^*_A - c)\eta(B)\mu_s - B + (p-c)((1-\eta(B))\mu_s + \mu_m)\bar{F}(p) \\ &\quad - (p-s)\phi(k)\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta^2(B)\sigma_s^2+\sigma_\epsilon^2} + \sigma_m^2}. \end{aligned}$$

We now find the optimal advertising budget. Differentiating $\pi^*_{AS}(B)$ with respect to B gives us:

$$\frac{d\pi^*_{AS}(B)}{dB} = (p^*_A - p_0)\eta'(B)\mu_s - 1 + \frac{(p-s)\phi(k)\bar{F}(p)}{\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta^2(B)\sigma_s^2+\sigma_\epsilon^2} + \sigma_m^2}} \cdot \frac{\eta(B)\eta'(B)\sigma_\epsilon^2\sigma_s^4}{(\eta^2(B)\sigma_s^2+\sigma_\epsilon^2)^2}. \quad (10)$$

As shown in the online appendix, the second order derivative is nonpositive under the condition that $p^*_A \geq p_0 := p\bar{F}(p) + cF(p)$. Hence, by setting $\frac{d\pi^*_{AS}}{dB} = 0$, we find that the optimal advertising expenditure B^* satisfies the following equation:

$$\eta'(B^*)\left((p^*_A - p_0)\mu_s + \frac{(p-s)\phi(k)\bar{F}(p)}{\sqrt{\sigma_s^2\sigma_\epsilon^2 + \sigma_m^2(\eta^2(B^*)\sigma_s^2+\sigma_\epsilon^2)}}\frac{\eta(B^*)\sigma_\epsilon^2\sigma_s^4}{(\eta^2(B^*)\sigma_s^2+\sigma_\epsilon^2)^{3/2}}\right) = 1.$$

The optimal budget B^* is positive if $\frac{d\pi^*_{AS}}{dB}|_{B=0} > 0$ and zero otherwise. This condition can be rewritten as

$$p^*_A \geq p_0 + \hat{p}_0 := p_0 + \frac{Z}{\mu_s},$$

where

$$Z = \left(\frac{1}{\eta'(B)|_{B=0}} - \frac{(p-s)\phi(k)\bar{F}(p)}{\sqrt{\sigma_s^2\sigma_\epsilon^2 + \sigma_m^2(\eta_0^2\sigma_s^2+\sigma_\epsilon^2)}}\frac{\eta_0\sigma_\epsilon^2\sigma_s^4}{(\eta_0^2\sigma_s^2+\sigma_\epsilon^2)^{3/2}}\right) \geq 0.$$

This leads to the following result.

Theorem 2: If the optimal advance selling price is higher than a certain value, that is, $p_A^* \geq p_0 + \hat{p}_0$ where $\hat{p}_0 = \frac{Z}{\mu_s} \geq 0$, then the optimal advertising expenditure is positive and satisfies the following equation:

$$\eta'(B^*)\left((p_A^* - p_0)\mu_s + \frac{(p-s)\phi(k)\bar{F}(p)}{\sqrt{\sigma_s^2\sigma_\epsilon^2 + \sigma_m^2(\eta^2(B^*)\sigma_s^2 + \sigma_\epsilon^2)}} \frac{\eta(B^*)\sigma_\epsilon^2\sigma_s^4}{(\eta^2(B^*)\sigma_s^2 + \sigma_\epsilon^2)^{3/2}}\right) = 1. \tag{11}$$

An alternative formulation is that, if the average number of strategic consumers is higher than a threshold, that is, $\mu_s \geq \bar{\mu}_s^0 := \frac{Z}{p_A^* - p_0}$, then the retailer should advertise products and sell them in advance; otherwise, the retailer should not advertise. Combined with Theorem 1, we summarize the optimality conditions for the different selling strategies as follows.

Theorem 3:

- (Advance Selling and Advertising) If the optimal advance selling price and the average number of strategic consumers are higher than certain thresholds, that is, $p_A^* \geq p_0$ and $\mu_s \geq \bar{\mu}_s^0$, then a retailer should sell in advance and advertise this opportunity.
- (Spot Selling) If the optimal advance selling price is lower than a certain threshold and the average number of strategic consumers is higher than a certain threshold, that is, $p_A^* \leq p_0$ and $\mu_s \geq \mu_s^0$, then a retailer should only sell in the spot selling period.
- (Advance Selling) Otherwise, the retailer should sell in advance but without advertising.

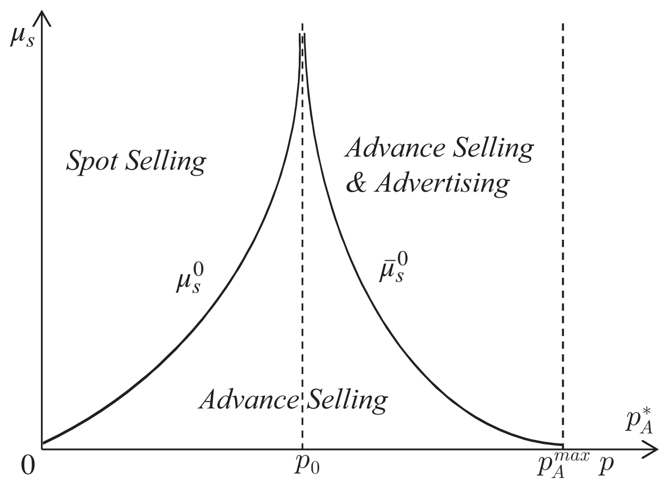
This result is displayed graphically in Figure 4. Advertising is profitable when the advance selling price is relatively high and the strategic market size is relatively large. As the advance selling price drops, the maximum relative size of the strategic consumers at which advertising is profitable rapidly increases. If the advance selling price drops below p_0, the retailer should not sell in advance unless the strategic market size is relatively small.

A similar result can again be obtained in terms of loss aversion and anchoring effect. As $\Delta\pi_{AS}^*$ is decreasing with respect to both λ and V_0, if $\Delta\pi_{AS}^*|_{\lambda=0} \geq 0$ or $\Delta\pi_{AS}^*|_{V_0=p} \geq 0$, then there exist critical values $\hat{\lambda}$ and $\hat{V}_0$ which satisfy (8) and (9), respectively. This gives the following result.

Proposition 4: If the critical target utility $\hat{V}_0$ (or the critical loss aversion degree $\hat{\lambda}$) does not exist, then the spot selling only strategy is always optimal; otherwise, the optimal strategy is as follows:

- (Advance Selling and Advertising) If the target utility (or the loss aversion degree) is lower than a threshold and the average number of strategic consumers is greater than certain thresholds, that is, $V_0 \leq \hat{V}_0$ (or $\lambda \leq \hat{\lambda}$) and $\mu_s \geq \bar{\mu}_s^0$, then a retailer should always sell in advance and advertise this opportunity.
- (Spot Selling) If the target utility (or the loss aversion degree) and the average number of strategic consumers are higher than certain thresholds, that is, $V_0 \geq \hat{V}_0$

Figure 4: Optimal policy with advertising.



(or $\lambda \geq \hat{\lambda}$) and $\mu_s \geq \mu_s^0$, then a retailer should only sell in the spot selling period.
- (Advance Selling) Otherwise, the retailer should always sell in advance but without advertising.

In the following, we examine the effect of a stockout probability, which was ignored in the main analysis, on purchasing decisions, especially for strategic consumers. Following the assumption of the exogenous perceived stocking out probability ξ (e.g., Prasad et al., 2011), the expected utility of waiting with a stockout risk $\mathcal{U}_W(\xi)$ satisfies $\mathcal{U}_W(\xi) = (1 - \xi)\,\mathcal{U}_W$, where $\mathcal{U}_W$ defined in (2) is the expected utility of waiting with a zero stocking out probability. As discussed in "Advance Selling without Advertising" section, the retailer selects the advance selling price $p_A^*(\xi)$ for which $\mathcal{U}_A = \mathcal{U}_W(\xi)$. This gives $p_A^*(\xi) = p_A^* + \xi\mathcal{U}_W$, where p_A^* is the optimal advance selling price with a zero stocking out probability. By replacing p_A^* with $p_A^*(\xi)$ in Theorem 3, the optimal selling strategy with a stockout risk is characterized as follows.

Corollary 2: With a perceived stocking out probability ξ, the optimal selling strategy is as follows:

- (Advance Selling and Advertising) If the optimal advance selling price and the average number of strategic consumers are higher than certain thresholds, that is, $p_A^*(\xi) \geq p_0$ and $\mu_s \geq \bar{\mu}_s^0(\xi)$, then a retailer should sell in advance and advertise this opportunity.
- (Spot Selling) If the optimal advance selling price is lower than a certain threshold and the average number of strategic consumers is higher than a certain threshold, that is, $p_A^*(\xi) \leq p_0$ and $\mu_s \geq \mu_s^0(\xi)$, then a retailer should only sell in the spot selling period.

- (Advance Selling) Otherwise, the retailer should sell in advance but without advertising,

where $p_A^*(\xi) = p_A^* + \xi\mathcal{U}_W$, $\bar{\mu}_s^0(\xi) = \frac{Z}{p_A^*(\xi)-p_0}$ *and* $\mu_s^0(\xi) = \frac{(p-s)\phi(k)\bar{F}(p)\Delta\sigma_0}{\eta_0(p_0-p_A^*(\xi))}$.

If the expected utility of waiting without a stockout risk $\mathcal{U}_W$ is nonnegative, then considering a stockout risk leads to an increase in the advance selling price $p_A^*(\xi)$. Further, the critical strategic market sizes $\bar{\mu}_s^0(\xi)$ and $\mu_s^0(\xi)$ are smaller and larger, respectively, due to the stockout risk. These changes imply that the advance selling (with advertising) strategy is more likely to be optimal. We remark that $p_A^*(\xi)$ has an upper bound of p if the retailer uses a price commitment pricing strategy. These discussions show that considering stockout risk does alter the location of the "boundaries" where one strategy outperforms another, but that the key insights of where what policy is optimal, as graphically presented in Figure 4, carries over.

NUMERICAL STUDY

In this section, we examine numerically how market parameters (e.g., unit cost, standard deviation of myopic/strategic consumers et al.) affect the retailer's advance selling strategy. Moreover, we calculate the retailer's profit gain from advance selling and advertising. As an extension of our model, we also discuss how an endogenous spot selling price affects the retailer's advance selling strategy.

Impact of Varying Market Conditions on Optimal Policy

Some parameters have fixed values: $p = 5$, $s = 1$, $\eta(B) = 0.1 + 0.05B$, $V \sim N(5.2, 2.5^2)$, $N_m \sim N(50, 30^2)$, and $N_\epsilon \sim N(0, 1^2)$. We will examine the effects of varying the overage costs (unit cost) and of varying the standard deviation of strategic market size.

We start by varying the unit overage cost $c - s \in \{0.1, 1.4\}$ and fixing σ_s at 20. Figure 5 presents the critical thresholds of the advance selling price and the strategic market sizes, that is, p_0, μ_s^0 and $\bar{\mu}_s^0$. Note that the maximum advance selling price $p_A^{\max} := p - \int_L^p F(x)dx$ is unaffected by the overage cost. Therefore, as the unit overage cost increases, the critical advance selling price p_0 approaches $p_A^{\max}$, indicating that the retailer is less likely to benefit from advance selling (and advertising). Instead, for low-margin products, the retailer is better off with the spot selling strategy unless the strategic market size is relatively small or strategic consumers have a high valuation (or low expectation).

We next vary the standard deviations of strategic consumers $\sigma_s \in \{10, 20, 30\}$, fixing c at 2. Note that the critical advance selling price p_0 is independent with σ_s. Figure 6 presents the critical thresholds of the strategic market size, that is, μ_s^0 and $\bar{\mu}_s^0$. Note from this figure (and also Figure 4) that more uncertainty in the strategic market size increases the area where having advance selling is beneficial, and also increases the area where this opportunity should be advertised. A more uncertain strategic market size implies a higher inventory risk and, correspondingly, a larger benefit from reducing that risk by selling in advance.

Figure 5: Optimal policy with varying unit overage costs.

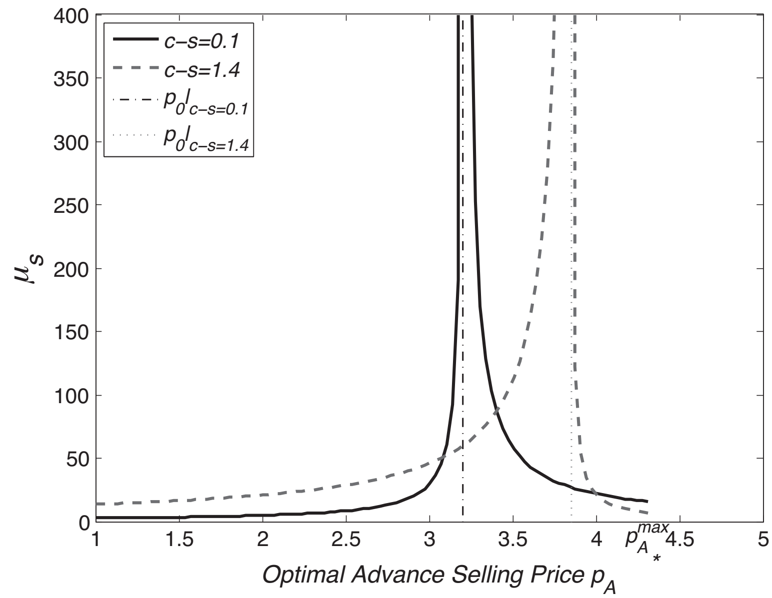


Figure 6: Optimal policy with varying standard deviations of strategic consumers.

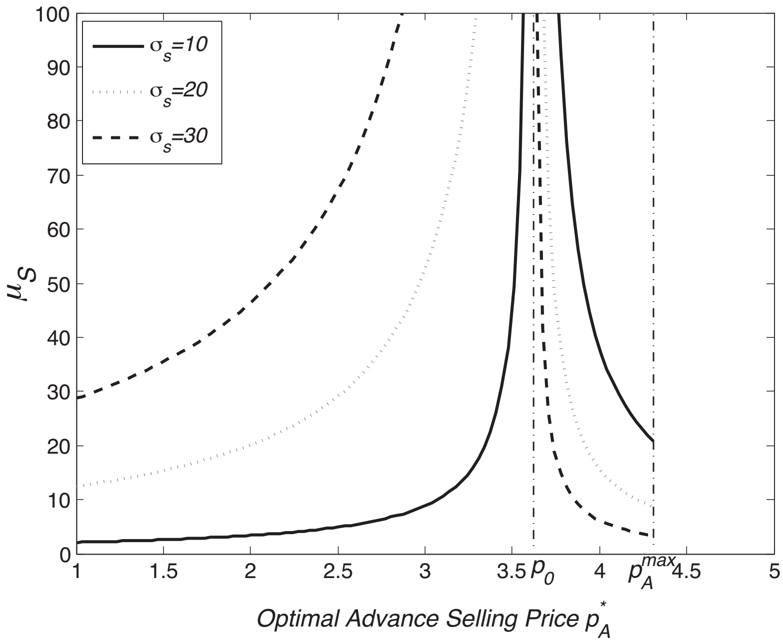
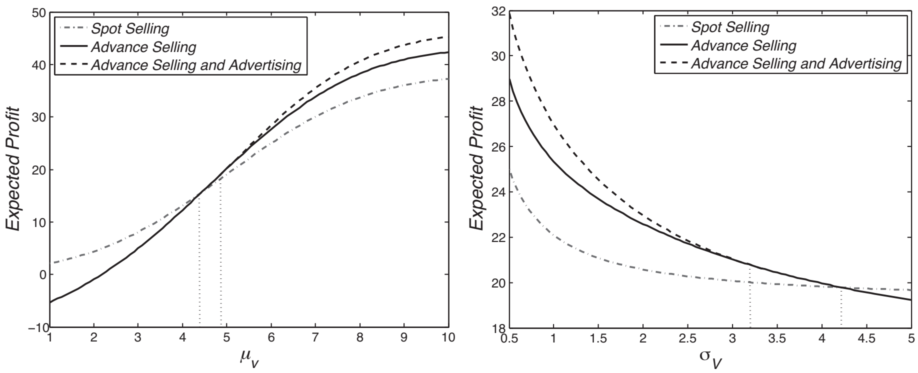


Figure 7: The expected profits under three policies with varying consumer's valuation.



(a) Expected valuation μ_v (b) Valuation uncertainty σ_v

Retailer's Profit Gain from Advance Selling and Advertising

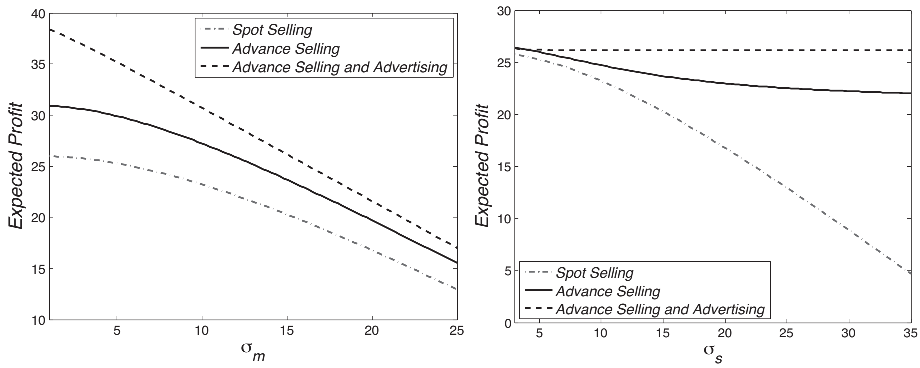
In this section, the parameters used are as follows: $p = 5$, $c = 3.5$, $s = 0$, $V_0 = 5.5$ $\lambda = 0.4$ $\eta(B) = 0.07 + 0.1B$, $V \sim N(5.2, 2.5^2)$, $N_s \sim N(30, 15^2)$, $N_m \sim N(20, 15^2)$, and $N_\epsilon \sim N(0, 1^2)$. We examine the retailer's profit gain from two aspects: consumer valuation and market size (demand) uncertainty.

To observe the effect of the consumer's valuation, we vary the expected valuation $\mu_v \in [1, 10]$ with a fixed $\sigma_v = 2.5$ and the standard deviation of valuation $\sigma_v \in [0.5, 5]$ with a fixed $\mu_v = 5.2$, respectively. Figure 7 represents the retailer's profits with varying consumer valuations under the three possible policies.

For a very low expected valuation, the retailer should offer a very large discount for advance sales, which may hurt the retailer. Thus, Figure 7(a) suggests that spot selling is optimal. As the expected valuation increases, the discount decreases. Correspondingly, the absolute profit gap between spot selling and advance selling first decreases and then increases. After a certain value of expected valuation, the retailer can obtain more profits by selling in advance. As the expected valuation increases further, the retailer can benefit from advertising. Moreover, the profit gap between advance selling and spot selling and that between advance selling with advertising and advance selling only are increasing, but at a diminishing rate that heavily depends on the initial fraction of informed strategic consumers η_0.

Figure 7(b) shows how the retailer's profit changes as valuation uncertainty increases. A smaller valuation uncertainty leads to a higher advance selling price. Therefore, the retailer should always sell in advance and advertise this opportunity. As the valuation uncertainty increases, the advance selling price decreases and profit gaps among the three policies diminish. Above a critical value of σ_v, the retailer cannot benefit from advertising. Moreover, further decreasing the valuation uncertainty leads to a situation where the retailer should not sell in advance at all.

Figure 8: The expected profits under three policies as market size uncertainty increases.



(a) Myopic market (b) Strategic market

To showcase the effect of uncertain market size, we vary the standard deviation of strategic market size $\sigma_s \in [5, 35]$ with a fixed $\sigma_m = 15$ and vary the standard deviation of the myopic market size $\sigma_m \in [0, 25]$ with a fixed $\sigma_s = 15$, respectively. Figure 8 presents the retailer's profit with increasing market size uncertainty under three policies.

As the uncertainty of the myopic market size increases, the inventory risk in the spot selling period enlarges, which leads to a dramatic profit drop, as shown in Figure 8(a). However, this does not hold for the strategic market. As the retailer can benefit from reducing the inventory risk by advance selling and from improved forecasting by advertising, both advance selling and advertising can limit the profit drops, as shown in Figure 8(b).

Extension: Endogenousness of Spot Selling Price

In this section, we study how an endogenous spot selling price affects the retailer's advance selling strategy. This is done by numerically solving the first order conditions of π_{NA}, π^*_{AS} and $\pi^*_{AS}(B^*)$. We use the parameter settings in "Retailer's Profit Gain from Advance Selling and Advertising" section. In Table 2, we show the optimal selling prices and their corresponding optimal profits under the three different strategies (spot selling, advance selling without advertising, and advance selling with advertising).

From Table 2, it is clear that the spot selling (advance selling without advertising) strategy is optimal only if the retailer has a relatively high cost or offers a big advance selling discount. Advance selling with advertising is the best strategy if the retailer has a low purchasing cost or offers a small advance selling discount. For a relatively high cost ($c \geq 5$), advertising is unprofitable. Recall that the same effects were observed analytically for a fixed spot price comparison in Figure 4. The numerical study further confirms that, for a high-margin product, the retailer should sell in advance and advertise the pre-order opportunity; for a low-margin

Table 2: Optimal pricing and maximal profit.

	Spot Selling		AS without Advertising			AS with Advertising		
Cost c	Spot Price p^*_{NA}	Profit π^*_{NA}	Spot Price p^*_{AS}	AS Price Ratio p_A^{AS}/p^*_{AS}	Profit π^*_{AS}	Spot Price p^*_{AD}	AS Price Ratio p_A^{AD}/p^*_{AD}	Profit $\pi^*_{AS}(B^*)$
6.5	8.497	**4.162**	8.561	56.2%	0.933	–	–	–
6	8.113	**6.071**	8.167	58.0%	3.829	–	–	–
5.5	7.741	**8.647**	7.787	59.9%	7.369	–	–	–
5	7.379	**12.039**	7.421	61.7%	11.693	–	–	–
4.73	7.189	14.268	7.229	62.6%	**14.406**	11.730	43.2%	1.841
4.5	7.030	16.412	7.068	63.4%	**16.949**	7.684	60.4%	15.985
4	6.692	21.936	6.729	65.1%	23.290	6.749	65.0%	**23.425**
3.5	6.367	28.792	6.403	66.7%	30.876	6.464	66.4%	**31.414**
3	6.052	37.171	6.090	68.2%	39.870	6.214	67.6%	**40.393**
2.5	5.749	47.277	5.788	69.7%	50.449	6.049	68.4%	**53.053**
2	5.456	59.347	5.497	71.1%	62.810	5.521	70.9%	**65.994**
1.5	5.170	73.674	5.215	72.4%	77.201	5.063	73.1%	**80.442**
1	4.889	90.680	4.938	73.6%	93.973	4.647	74.9%	**96.162**
0.5	4.607	111.120	4.662	74.8%	113.762	4.288	76.4%	**114.941**

Note: Maximum profits in different strategies are in bold and underlined.

product, no advance selling is the best strategy; for a medium-margin product, the retailer should sell in advance, but not advertise the pre-order opportunity.

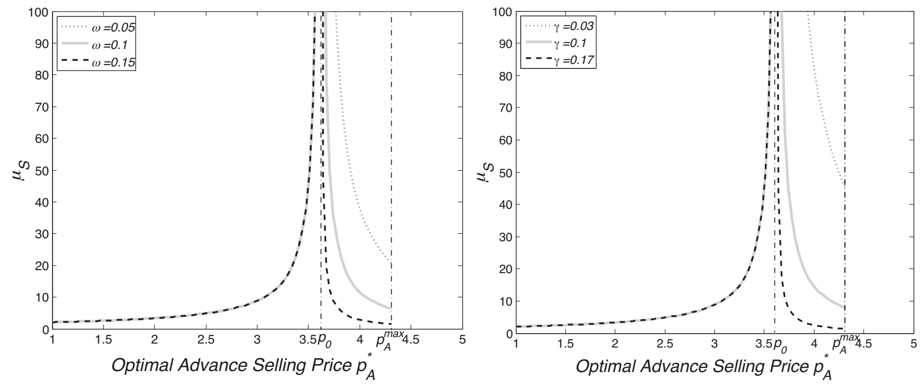
However, some changes can be noticed that relate to the endogenousness of the spot selling price. Table 2 clarifies that selling in advance always leads to an increase in the spot price, that is, $p^*_{NA} \leq p^*_{AS}$. This is because the discount for the advanced selling period is relative to that spot price. Hence, a higher spot price improves profitability for the advance selling period. However, whether advertising the pre-order opportunity can further increase the spot price depends on the purchasing cost. More specifically, if the retailer has a relatively high cost, then it is optimal to further increase the spot price because advertising can offset the demand reduction caused by an increased price, that is, $p^*_{AS} \leq p^*_{AD}$. However, if the purchasing cost is relatively low, then it is optimal to lower the spot price, that is, $p^*_{AS} \geq p^*_{AD}$. This is because the increase in the total (myopic and strategic) market size offsets the lower profit margin. Similarly, we also find that the advertising strategy does not always offer the biggest discount.

Extension: Concavity of Advertising Response

We have assumed a linear advertising response curve in our main analysis. In real-life situations, there may be diminishing effects from advertising which a nonlinear concave response function can reflect. In this section, we consider such a response function.

The fraction of informed strategic consumers is at most 1 and so this provides an upper bound for the response function $\eta(B)$. Based on the review and summary

Figure 9: Optimal policy when varying the effectiveness of advertising.



(a) Linear response model (b) Concave response model

of advertising response functions by Lilien et al. (2007), we consider the following modified exponential advertising response model:

$$\eta(B) = \eta_0 + \alpha(1 - e^{-\gamma B}),$$

where $\gamma > 0$ indicates the effectiveness of advertising and η_0 is the initial fraction of informed strategic consumers. Note that this function has an upper bound or horizontal asymptote at $\eta_0 + \alpha$, and a lower of bound of η_0. In our model, α is defined on $[0, 1 - \eta_0]$. If $\alpha = 1 - \eta_0$, then all consumers can be informed when advertising budget is sufficiently large; if $\alpha = 0$, then there is no response at all.

We use the parameter setting in "Retailer's Profit Gain from Advance Selling and Advertising" section. Let $\alpha = 1 - \eta_0 = 0.9$, $c = 2$, and $\sigma_s = 10$. Let $\gamma \in \{0.03, 0.1, 0.17\}$ and $\omega \in \{0.1, 0.05, 0.3\}$ vary. As the effectiveness and concavity of advertising do not affect the choice of spot selling, the critical threshold μ_s^0 is unchanged. Figure 9 presents the critical thresholds of the strategic market size, that is, μ_s^0 and $\bar{\mu}_s^0$. Note from Figure 9(a) that, for the linear advertising response model, higher effectiveness of advertising increases the area where this advance selling opportunity should be advertised. Figure 9(b) confirms that this effect and the structural results carry over if the advertising response function is concave. The same is observed for other parameter settings.

Extension: Heterogeneous Consumer Target Valuations

As consumer target valuations differ, we divide informed strategic consumers into two segments: high-target-valuation V_0^H and low-target-valuation V_0^L, where $V_0^L, V_0^H \geq p$. Further, we let $\beta \in [0, 1]$ be the market share of informed strategic consumers with low target valuations in the total market.

By replacing V_0 with V_0^L or V_0^H in (5), the optimal advance selling prices for all consumers with low target valuations and high target valuations are given as follows:

$$p_A^H := p_A^*(V_0^L) = p - (\lambda + 1)\int_L^{\frac{\lambda V_0^L + p}{\lambda+1}} F(x)dx,$$

$$p_A^L := p_A^*(V_0^H) = p - (\lambda + 1)\int_L^{\frac{\lambda V_0^H + p}{\lambda+1}} F(x)dx.$$

As the optimal selling price is decreasing in the target valuation, we obtain $p_A^*(V_0^L) \geq p_A^*(V_0^H)$, that is, $p_A^H \geq p_A^L$.

For heterogeneous informed strategic consumers, the retailer has two pricing strategies for advance selling: low pricing versus high pricing. Intuitively, a high-pricing strategy will prevent consumers with high target valuations from pre-buying. However, selling at a low price will attract both types of consumers, but may result in lost profits due to retailers offering too much of a discount for high-target-valuation consumers. Based on an analysis that is similar to the one for homogeneous consumers in “A Benchmark: The Classic Newsvendor” section, we find that the maximal profits for the low-pricing and high-pricing strategies are:

$$\begin{aligned}\pi_{AS}^*(p_A^L) &= \left(p_A^L - c\right)\eta_0\mu_s + (p-c)((1-\eta_0)\mu_s + \mu_m)\bar{F}(p) \\ &\quad -(p-s)\phi(k)\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2}, \\ \pi_{AS}^*(p_A^H) &= \left(p_A^H - c\right)\beta\eta_0\mu_s + (p-c)((1-\beta\eta_0)\mu_s + \mu_m)\bar{F}(p) \\ &\quad -(p-s)\phi(k)\bar{F}(p)\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\beta^2\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2}.\end{aligned}$$

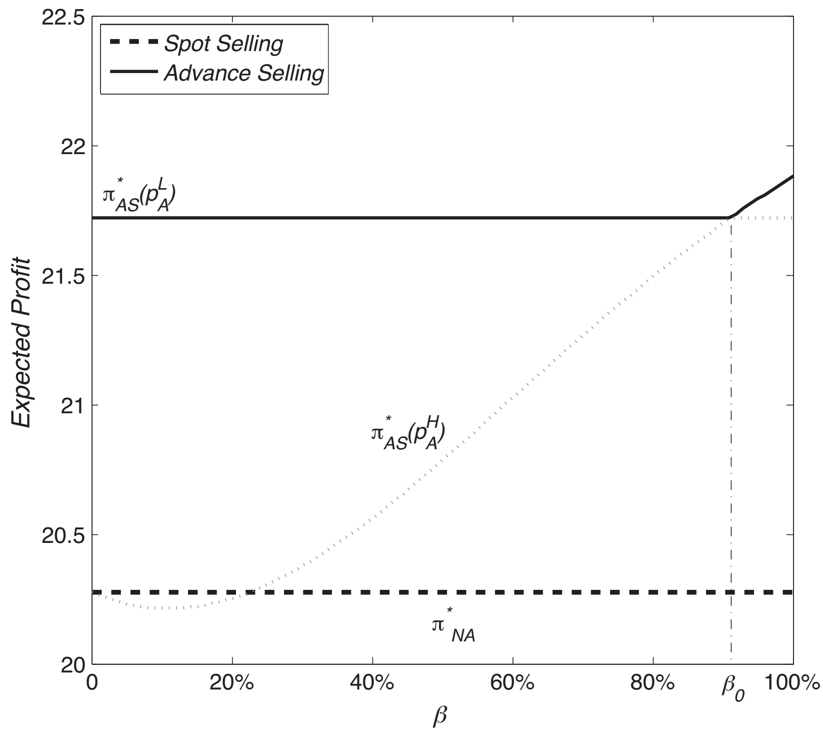
The profit difference between the two strategies is:

$$\begin{aligned}\Delta\pi_{AS}^{V_0} &= \pi_{AS}^*\left(p_A^H\right) - \pi_{AS}^*(p_A^L) \\ &= \left(p_A^H - p_A^L\right)\beta\eta_0\mu_s - \left(p_A^L - c\right)(1-\beta)\eta_0\mu_s + (1-\beta)\eta_0(p-c)\mu_s\bar{F}(p) \\ &\quad -(p-s)\phi(k)\bar{F}(p)\left(\sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\beta^2\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2} - \sqrt{\frac{\sigma_s^2\sigma_\epsilon^2}{\eta_0^2\sigma_s^2 + \sigma_\epsilon^2} + \sigma_m^2}\right).\end{aligned}$$

As $\Delta\pi_{AS}^{V_0}$ may be not monotone or convex in β, we use numerical examples to further analyze the profit difference between the two strategies. We use the parameter setting in “Retailer’s Profit Gain from Advance Selling and Advertising” section and set $V_0^H = V_0 = 5.5$, $V_0^L = 5.1$, and $\beta \in [0, 1]$. Then, we obtain $p_A^L = p_A^* = 3.667$, $p_A^H = 3.744$, $\pi_{AS}^*(p_A^L) = 21.719$, $\pi_{NA}^* = 20.276$.

Note from Figure 7(b) that advance selling with homogeneous target valuations is more profitable than spot selling under these parameter settings ($\sigma_v = 2.5$). Figure 10 shows that this result for heterogeneous target valuations carries over, that is, $\hat{\pi}_{AS}^* \geq \pi_{NA}^*$, where $\hat{\pi}_{AS}^* = \max\{\pi_{AS}^*(p_A^L), \pi_{AS}^*(p_A^H)\}$ is the optimal profit of advance selling with heterogeneous target valuation. This can be explained as follows:

Figure 10: The expected profits with heterogeneous target valuations.



Pricing low stimulates all informed strategic consumers to buy in advance. Therefore, the market share of low-target-valuation consumers has no impact on the optimal profits of the advance selling strategy. That is, $\pi^*_{AS}(p^L_A) > \pi^*_{NA}$ is independent of β. Different from the low pricing strategy, the expected profit of the high pricing strategy heavily relies on the market share of low-target-valuation consumers. If that market share is higher than a threshold ($\beta \geq \beta_0 = 92\%$), then high pricing strategy outperforms the low pricing strategy, that is, $\pi^*_{AS}(p^H_A) \geq \pi^*_{AS}(p^L_A) > \pi^*_{NA}$.

CONCLUSION

This article studies advance selling and advertising strategies for a newsvendor retailer. The retailer sells a new product with a short life cycle in a market with uncertain size. Consumers are heterogeneous (strategic and myopic) and have unknown product valuation before the start of the selling season. Informed consumers, who are eager to obtain the product, benefit from advance selling through a discount and guaranteed prompt delivery on release. Advance selling is also beneficial to the retailers because it exploits valuation uncertainty, reduces the inventory risk, and improves the spot selling demand forecast.

In this study, we develop a modeling framework to study advance selling and advertising strategies and, thus, analyze the demand forecast accuracy improvement. The retailer has three options: spot selling only, advance selling without advertising, and advance selling with advertising. To the best of our knowledge, our modeling framework is the first to simultaneously incorporate the following important elements: consumer behavior that includes strategic response, loss aversion, and the anchoring effect; demand information updating; increased advance sales from advertising; and forecast accuracy improvement from advance selling.

Our results on whether to sell in advance build on the studies of Xie and Shugan (2001) and Prasad et al. (2011). When market sizes are assumed to be deterministic, Xie and Shugan (2001) find that retailers should sell in advance when marginal costs are below a lower threshold of consumer valuation. If market sizes are uncertain, then Prasad et al. (2011) find that selling in advance never benefits retailers if the difference between the consumer's expected valuation and the consumer's expected surplus of waiting is higher than a threshold. Moreover, Zhao and Stecke (2010) further find that if a portion of consumers are loss averse, then whether a small or a deep discount for advanced sales is optimal depends on the profit margin of the product and the consumer's expected valuation. Our results confirm that advance selling is not always an optimal strategy for a retailer. However, besides the optimal advance selling price determined by the consumer's marginal valuation, the optimal strategy on advance selling heavily depends on the market size of potential (informed strategic) consumers.

Our contribution to the literature is threefold. First, as discussed above, most advance-selling literature (e.g., Xie & Shugan, 2001; Prasad et al., 2011) relates the decision of whether to sell in advance solely on the profit margin of advanced sales. Our results show that besides the profit margin, the market size of informed strategic consumers in the advance selling period plays a crucial role in determining whether selling in advance is optimal. Moreover, advanced sales lead to a more accurate demand forecast and we quantify the inventory cost savings.

Second, our work is the first to explore whether a retailer should advertise the advance ordering opportunity. We provide new insights for retail managers into the selection of the advance selling strategy (i.e., no advance selling, without advertising, and with advertising). The selection should be based on both consumer characteristics and the market demand structure. More specifically, if the advance selling price is higher than a threshold, then retailers should sell in advance and they should advertise that option if the strategic market size is relatively large. If the advance selling price is under the threshold, then retailers should not advertise and only sell in advance if the size of the strategic market is relatively small. Otherwise, the retailers should sell only in the spot period.

Finally, we also contribute to a more realistic modeling of the consumer's choice between buying in advance and waiting for the start of the sales season. We consider consumers who are loss averse and reference dependent (anchoring). By considering the consumers' preferences on loss and his or her expected/target utility based on the consumer valuation uncertainty, we obtain insights into the impact of loss aversion and anchoring on the retailer's optimal advance selling and advertising strategies. When consumers are loss averse, the risk from the negative utility gap between the expectation and reality motivates them to buy later, while a large price discount is needed to motivate them to buy early. Our results show that the overall impact of loss aversion on the retailer's profit is negative. A higher expectation (anchor) or a larger expectation gap can further diminish the retailer's profit. However, interestingly, lowering consumer expectations or narrowing the expectation gap can help counter the negative effect caused by loss aversion. This would increase the appeal of advance selling for retailers. For spot selling a new product, it is a common belief that a little positive exaggeration favorably influences a consumer's judgment. However, overstatement or understatement may not be an effective strategy for selling in advance, because consumers may suffer a utility loss from pre-buying. Note that consumers' expectations are affected by pre-launch information. Our results indicate that the retailer should carefully choose what information to provide.

A numerical investigation revealed further insights under multiple market and consumer settings. We find that, as the profit margin decreases, spot selling is more likely to be the optimal strategy. However, as the strategic market size becomes more uncertain, the retailer should sell in advance. If the optimal advance selling price is relatively high, the retailer should advertise the pre-order opportunity. Moreover, the retailer can benefit from advance selling and advertising if consumers have a relatively high average valuation or a relatively low valuation uncertainty.

We also numerically explained how an endogenous spot selling price affects the advance selling strategy. We find that the structural results for a fixed spot price carry over. Further, both selling in advance and advertising the pre-order opportunity can increase the spot price. Therefore, the advance selling with advertising (no advance selling) strategy has the highest (lowest) spot price. However, advertising can help lower the advance selling discount if the purchasing cost is relatively low.

Some limitations of our work and corresponding future research opportunities should be noted. In line with previous research, our model assumes that all consumers are independent and have the same product valuation and expectation. In practice, consumers' valuations and expectations are different, and consumers' decisions may be dependent (i.e., through group buying). It is worthwhile to extend our model by including heterogeneous and/or dependent consumers. Moreover, our model assumes that the number of informed strategic consumers is independent of the myopic market size. As advance demand also can be used to forecast the myopic market size under certain scenarios, correlation between strategic market size and myopic market size deserves to be considered and analyzed.

Another future line of research is to consider product returns. In most countries, consumers have a statutory right to return a nonfaulty product and receive a full refund. By allowing consumers to return unsatisfying products, the retailer can no longer benefit from consumer valuation uncertainty. However, selling in advance still reduces the inventory risk through increased forecast accuracy; the return option also reduces the discount needed to make consumers buy in advance. Therefore, it would be interesting to investigate the impact of product returns on the advance selling strategy.

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

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