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Koshelev, Alexey S.; Marto, Joao; Mazumdar, Anupam

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Schwarzschild $1/r$ singularity is not permissible in ghost-free quadratic-curvature infinite-derivative gravity

Alexey S. Koshelev,[1,2,3] João Marto,[1,2] and Anupam Mazumdar[4,5]

[1]*Departamento de Física, Universidade da Beira Interior, Rua Marquês D'Ávila e Bolama, 6201-001 Covilhã, Portugal*
[2]*Centro de Matemática e Aplicações da Universidade da Beira Interior (CMA-UBI), Rua Marquês D'Ávila e Bolama, 6201-001 Covilhã, Portugal*
[3]*Theoretische Natuurkunde, Vrije Universiteit Brussel and The International Solvay Institutes, Pleinlaan 2, B-1050 Brussels, Belgium*
[4]*Van Swinderen Institute, University of Groningen, 9747 AG, Groningen, The Netherlands*
[5]*Kapteyn Astronomical Institute, University of Groningen, 9700 AV Groningen, The Netherlands*

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In this paper, we will study the complete equations of motion for a ghost-free quadratic-curvature infinite-derivative gravity. We will argue that within the scale of nonlocality, a Schwarzschild-type singular metric solution is not *permissible*. Therefore, the Schwarzschild-type vacuum solution which is a prediction in Einstein-Hilbert gravity may *not* persist within the region of nonlocality. We will also show that just quadratic-curvature gravity, without infinite derivatives, always allows Schwarzschild-type singular metric solution.

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I. INTRODUCTION

Arguably, Einstein's theory of general relativity is one of the most successful descriptions of spacetime. It has seen numerous confirmations of observational tests at different length scales, predominantly in the infrared (IR) (far away from the source and at late time scales) [1], including the fascinating detection of gravitational waves [2]. In spite of this success, at short distances and at small time scales, i.e., in the ultraviolet (UV), the Einstein-Hilbert action leads to well-known singular solutions, in terms of black-hole solutions in the vacuum, and the cosmological singularity in a time-dependent background [3]. The nature of the latter singularity is indeed very different from the former, which brings uncertainty to the cosmological models at the level of initial conditions for inflation and the big bang cosmology. In reality, one would expect that nature would avoid any kind of classical singularities, whether they are covered by an event horizon or they are naked—a stronger version of the *cosmic censorship* hypothesis [4,5]. In this respect, it can be argued that the singularities present in the Einstein-Hilbert action are mere artifacts of the action, and there must be a way to ameliorate the singularities in nature. Indeed, removing the singularities is one of the foremost fundamental questions of gravitational physics.

Recently, Biswas, Gerwick, Koivisto, and Mazumdar (BGKM) have shown that the quadratic-curvature infinite-derivative theory of gravity in four spacetime dimensions can be made *ghost free* and avoid both cosmological and black-hole singularities at the linearized level around the Minkowski background [6],[1] while the cosmological singularity can be resolved even at the full nonlinear level [9–13]. At the linear level (around asymptotically Minkowski background), resolution of black-hole singularities has been studied both in the static case [6,14–19] and in a rotating case [20] by various groups. Furthermore, lack of formation of singularity at the linear level has also been studied in a dynamical context by Frolov and his collaborators [21,22].

In Ref. [6], the authors have shown that for *ghost-free* quadratic-curvature gravitational form factors, at short distances the gravitational metric-potential tends to be a constant, while at large distances from the source, the metric potential takes the usual form of $1/r$ behavior in the IR. Furthermore, the gravitational force quadratically vanishes towards the center in the UV. Such a system behaves very much like a compact object, but by construction there is no curvature singularity, nor there is an event horizon. The gravitational entropy calculated by the Wald's formalism [23] leads to the *area* law [24]. Since, all the interactions are *purely* derivative in nature, the gravitational *form factors* give rise to nonlocal interactions for such a spacetime [25–29]. The nonlocality is indeed *confined* within the scale M_s, which has a very interesting behavior.

[1]See previous to this work other relevant Refs. [7–9], where the authors have argued absence of singularity in infinite-derivative gravity motivated from the string theory. However, the full quadratic-curvature action including the Weyl term with two gravitational metric potentials were first presented in [6].

The aim of this short paper is to show that the full nonlinear metric solution of the BGKM gravity will not permit a$1/r$-type metric potential, i.e., Schwarzschild-type solution, for the static background. Note that what is relevant for us is indeed the $1/r$ part of the metric potential, be it in isotropic coordinates or Schwarzschild coordinates. Near the vicinity of singularity, at $r=0$, what dominates is indeed the $1/r$ part of the metric potential in Einstein's theory of gravity. Also, such a singular solution exists in quadratic-curvature gravity as well [30], see for instance [31], therefore it is a pertinent question to ask whether $1/r$-kind of metric potential would survive the infinite-derivative theory of gravity or not?

II. THE INFINITE COVARIANT DERIVATIVE ACTION

The most general quadratic-curvature action (parity invariant and free from torsion) has been derived around constant curvature backgrounds in Refs. [6,14,32], given by[2]

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R+\alpha_c[R\mathcal{F}_1(\Box_s)R + R^{\mu\nu}\mathcal{F}_2(\Box_s)R_{\mu\nu}+W^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box_s)W_{\mu\nu\lambda\sigma}]), \quad (2)$$

where $G=1/M_p^2$ is the Newton's gravitational constant, $\alpha_c\sim 1/M_s^2$ is a dimensionful coupling, $\Box_s\equiv\Box/M_s^2$, where M_s signifies the scale of nonlocality at which new gravitational interaction becomes important. In the limit $M_s\to\infty$, the action reduces to the Einstein-Hilbert term. The d'Alembertian term is $\Box=g^{\mu\nu}\nabla_\mu\nabla_\nu$, where $\mu,\nu=0$, 1, 2, 3, and we work with a metric convention which is mostly positive $(-,+,+,+)$. The $\mathcal{F}_i$'s are three gravitational *form-factors*,

$$\mathcal{F}_i(\Box_s)=\sum_n f_{i,n}\Box_s^n, \quad (3)$$

reminiscence to any massless theory possessing *only* derivative interactions. In this theory, the graviton remains massless with transverse and traceless degrees of freedom (d.o.f.). However, the gravitational interactions are nonlocal due to the presence of the form factors $\mathcal{F}_i$'s, see [6]. These form factors contain infinite covariant derivatives, which shows that the interaction vertex in this class of theory becomes nonlocal. In fact, the gravitational interaction in this class of theory leads to smearing out the point source by modifying the gravitational potential, as shown in [6]. The nonlocal gravitational interactions are also helpful to ameliorate the quantum aspects of the theory, which is believed to be UV finite [25–28]. The scale of nonlocality is governed by M_s^{-1}.

Around the Minkowski background the three *form factors* obey a constraint equation, in order to maintain *only* the transverse-and traceless graviton d.o.f., i.e., the perturbative tree-level unitarity [6,14][3]

$$6\mathcal{F}_1(\Box_s)+3\mathcal{F}_2(\Box_s)+2\mathcal{F}_3(\Box_s)=0. \quad (4)$$

Let us first discuss very briefly the linear properties of this theory around an asymptotically Minkowski background before addressing the nonlinear equations of motion. The linear solutions are indeed insightful and provides a lot of understanding of the solutions within BGKM gravity. Even though, we will not discuss explicitly nonlinear solution, but any nonlinear solution should have a limit in the linear regime. Note that the mass of the source is the relevant parameter, which plays a crucial role in determining linear and nonlinear solutions. In Refs. [6,15,16], it was shown that for $a(\Box_s)=e^{\gamma(\Box_s)}$, where γ is an *entire function*, the central singularity is avoided, while recovering the correct $1/r$ dependence in the metric potential in the IR. For a specific choice of $a(\Box_s)=e^{\Box_s}$, and assuming the Dirac-delta mass distribution, $m\delta^3(r)$ at the center, the gravitational metric potential, i.e., the Newtonian potential remains linear, as long as:

$$mM_s\leq M_p^2, \quad (5)$$

with the gravitational metric potential in static and isotropic coordinates is given by [6]:

$$\phi(r)=-\frac{Gm}{r}\mathrm{Erf}\left(\frac{rM_s}{2}\right), \quad (6)$$

[2]The original action was first written in terms of the Riemann, but it is useful to write the action in terms of the Weyl term which is related to the Riemann as:

$$W^\mu_{\alpha\nu\beta}=R^\mu_{\alpha\nu\beta}-\frac{1}{2}(\delta^\mu_\nu R_{\alpha\beta}-\delta^\mu_\beta R_{\alpha\nu}+R^\mu_\nu g_{\alpha\beta}-R^\mu_\beta g_{\alpha\nu}) + \frac{R}{6}(\delta^\mu_\nu g_{\alpha\beta}-\delta^\mu_\beta g_{\alpha\nu}) \quad (1)$$

[3]In order to make sure that the full action Eq. (2) contains the same original dynamical d.o.f. as that of the massless graviton in four dimensions. This is to make sure that the action is *ghost free*, there are no other dynamical d.o.f. in spite of the fact that there are infinite derivatives. The graviton propagator for the above action gives rise to

$$\Pi(k^2)=\frac{1}{a(k^2)}\Pi(k^2)_{GR}=\frac{1}{a(k^2)}\left[\frac{P^{(2)}}{k^2}-\frac{P^{(0)}}{2k^2}\right],$$

where $P^{(2)}$ and $P^{(0)}$ are spin-2 and 0 projection operators, and $a(k^2)=e^{\gamma(k^2/M_s^2)}$, is exponential of an *entire function*—γ, which does not contain any poles in the complex plane, therefore no new d.o.f. other than the transverse and traceless graviton, see for details [6,33]. The gravitational form factors $\mathcal{F}_i(\Box_s)$ cannot be determined simultaneously in terms of $a(\Box_s)$, if we switch one of the $\mathcal{F}_i=0$, then we can express the other form factors in terms of $a(\Box_s)$, for instance for $\mathcal{F}_2=0$ yields, $\mathcal{F}_1=-[(a(\Box_s)-1)/12\Box_s]$ and $\mathcal{F}_3=[(a(\Box_s)-1)/4\Box_s]$, see [14].

approaches to be constant with a magnitude less than 1 for $r < 2/M_s$. Since the error function goes linearly in r for $r < 2/M_s$, the metric potential becomes finite in this ultraviolet region. For $r > 2/M_s$, the metric potential follows as $\sim Gm/r$, in the infrared region. However, in our case, the typical scale of nonlocality is actually larger than the Schwarzschild radius as shown in [18,19]

$$r_{NL} \sim \frac{2}{M_s} \geq r_{\rm sch} = \frac{2m}{M_p^2}, \tag{7}$$

thus avoiding the event horizon as well.[4] Now, for the rest of the discussion, let us focus on the full nonlinear equations for the above action Eq. (2).

III. TOWARDS THE IMPOSSIBILITY OF THE SCHWARZSCHILD METRIC SOLUTION

The complete equations of motion have been derived from action Eq. (2), and they are given by [14],

$$\begin{aligned}
P^{\alpha\beta} = &-\frac{G^{\alpha\beta}}{8\pi G} + \frac{\alpha_c}{8\pi G}(4G^{\alpha\beta}\mathcal{F}_1(\Box_s)R + g^{\alpha\beta}R\mathcal{F}_1(\Box_s)R - 4(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box_s)\mathcal{F}_1(\Box_s)R \\
&- 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^\sigma + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\Box_s)R^{\mu\beta} \\
&- g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\Box_s)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box_s)R^{\mu\alpha}) + 2\Box_s(\mathcal{F}_2(\Box_s)R^{\alpha\beta}) \\
&+ 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box_s)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^\sigma + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\
&- g^{\alpha\beta}W^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box_s)W_{\mu\nu\lambda\sigma} + 4W_{\mu\nu\sigma}^\alpha\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\sigma} \\
&- 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^\gamma + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta}) \\
= &-T^{\alpha\beta},
\end{aligned} \tag{8}$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components, and we have defined the following symmetric tensors, for the detailed derivation, see [14]:

$$\begin{aligned}
\Omega_1^{\alpha\beta} &= \sum_{n=1}^\infty f_{1_n}\sum_{l=0}^{n-1}\nabla^\alpha R^{(l)}\nabla^\beta R^{(n-l-1)}, \\
\bar{\Omega}_1 &= \sum_{n=1}^\infty f_{1_n}\sum_{l=0}^{n-1}R^{(l)}R^{(n-l)},
\end{aligned} \tag{9}$$

$$\begin{aligned}
\Omega_2^{\alpha\beta} &= \sum_{n=1}^\infty f_{2_n}\sum_{l=0}^{n-1}R_\nu^{\mu;\alpha(l)}R_\mu^{\nu;\beta(n-l-1)}, \\
\bar{\Omega}_2 &= \sum_{n=1}^\infty f_{2_n}\sum_{l=0}^{n-1}R_\nu^{\mu(l)}R_\mu^{\nu(n-l)},
\end{aligned} \tag{10}$$

$$\Delta_2^{\alpha\beta} = \sum_{n=1}^\infty f_{2_n}\sum_{l=0}^{n-1}[R_\sigma^{\nu(l)}R^{(\beta\sigma;\alpha)(n-l-1)} - R_\sigma^{\nu;\alpha(l)}R^{\beta\sigma(n-l-1)}]_{;\nu}, \tag{11}$$

$$\begin{aligned}
\Omega_3^{\alpha\beta} &= \sum_{n=1}^\infty f_{3_n}\sum_{l=0}^{n-1}W^{\mu;\alpha(l)}_{\ \ \nu\lambda\sigma}W_\mu^{\ \nu\lambda\sigma;\beta(n-l-1)}, \\
\bar{\Omega}_3 &= \sum_{n=1}^\infty f_{3_n}\sum_{l=0}^{n-1}W^{\mu(l)}_{\ \ \nu\lambda\sigma}W_\mu^{\ \nu\lambda\sigma(n-l)},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\Delta_3^{\alpha\beta} = \sum_{n=1}^\infty f_{3_n}\sum_{l=0}^{n-1}[&W^{\lambda\nu(l)}_{\ \ \sigma\mu}W_\lambda^{\ \beta\sigma\mu;\alpha(n-l-1)} \\
&- W^{\lambda\nu\ \ ;\alpha(l)}_{\ \ \sigma\mu}W_\lambda^{\ \beta\sigma\mu(n-l-1)}]_{;\nu}.
\end{aligned} \tag{13}$$

The notation $\mathcal{R}^{(l)} \equiv \Box^l\mathcal{R}$ has been used for the curvature tensors and their covariant derivatives. The trace equation is much more simple, and just for the purpose of illustration, we write it below [14]:

$$\begin{aligned}
P = &\frac{R}{8\pi G} + \frac{\alpha_c}{8\pi G}(12\Box_s\mathcal{F}_1(\Box_s)R + 2\Box_s(\mathcal{F}_2(\Box_s)R) \\
&+ 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box_s)R^{\mu\nu}) + 2(\Omega_{1\sigma}{}^\sigma + 2\bar{\Omega}_1) \\
&+ 2(\Omega_{2\sigma}{}^\sigma + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}{}^\sigma + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}{}^\sigma - 8\Delta_{3\sigma}{}^\sigma) \\
= &-T \equiv -g_{\alpha\beta}T^{\alpha\beta}.
\end{aligned} \tag{14}$$

The Bianchi identity has been verified explicitly in Ref. [14]. Here we briefly sketch the Weyl part, since this will be the most important part of our discussion. To accomplish this, note that the computations are simplified if one uses the following tricks by rewriting the equations of motion with one upper and one lower index, express Ricci tensor through

[4] This could potentially resolve the information-loss paradox, since there is no event horizon and the graviton interactions for $r_{NL} \sim 2/M_s$ becomes nonlocal; therefore, for interacting gravitons, the spacetime ceases to hold any meaning in the Minkowski sense.

the Einstein tensor (who's divergence is zero due to the Bianchi identities), and recalling the fact that the divergence of the Weyl tensor is the third rank Cotton tensor, which can be expressed through the Schouten tensor:

$$\nabla^{\gamma} W_{\alpha\mu\nu\gamma} = -\nabla_{\alpha} S_{\mu\nu} + \nabla_{\mu} S_{\alpha\nu},$$

where the Schouten tensor in four dimensions is given by

$$S_{\mu\nu} = \frac{1}{2}\left(R_{\mu\nu} - \frac{1}{6} g_{\mu\nu} R\right).$$

With this in mind, the rest of the computations amount to careful accounting of the symmetry properties of the Weyl tensor (which are identical to those of the Riemann tensor). We should also note that the Bianchi identities should hold regardless of the precise form of functions $\mathcal{F}_i$, and independently for each and every coefficient $f_{i,n}$, because these are mere numerical coefficients, which are required to make the theory *ghost free* [14].[5] Technically, this means that we should not bother about the summation over n, but rather concentrating on the inner summation over l in Eqs. (9)–(13). Finally, the symmetry with respect to $\alpha \leftrightarrow \beta$ permutation in the equations of motion can be accounted by rearranging the summation over l in the inverse order from $n-1$ to 0. With all the above precautions in mind, we can perform a direct substitution and check term by term that all the contributions vanish upon computing the divergence of the equations of motion. As stated above, it is not a surprise that the Bianchi identities hold. However, it is a very good check for the equations of motion, mostly for the mutual coefficients in front of the different terms, details can be found in Ref. [14].

Let us note that in GR, we have a vacuum solution, around an *asymptotically Minkowski* background for a static case,

$$R = 0, \qquad R_{\mu\nu} = 0. \tag{15}$$

In this case the energy momentum tensor vanishes in all the region except at $r = 0$, where the source $m\delta^3(r)$ is localized. One of the properties of such a vacuum solution is the presence of $1/r$- static and spherically symmetric metric solution, similar to the Schwarzschild metric, given by

$$ds^2 = -b(r)dt^2 + b^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2), \tag{16}$$

where $b(r) = 1–2Gm/r$ with the presence of a central singularity at $r = 0$, and also the presence of an event horizon. As we have already discussed, for $r < 2Gm$, what dominates is the $1/r$ part of $b(r)$, which dictates the rise in the gravitational potential all the way to $r = 0$. Note that, although the vacuum solution permits $R = 0$, $R_{\mu\nu} = 0$, the Weyl-tensor is nonvanishing in the case of a Schwarzschild metric, where

$$W_{\mu\nu\lambda\sigma} W^{\mu\nu\lambda\sigma} \to \infty,$$

as $r \to 0$. Now in our case, indeed the full equations of motion are quite complicated, nevertheless, we might be able to test this hypothesis of setting $R = 0$, $R_{\mu\nu} = 0$, and study whether the Schwarzschild metric, or $1/r$-type metric potential is a viable metric solution of our theory of gravity or not?

Let us then demand that the above action, Eq. (2), along with the equations of motion Eq. (8), permits a solution which is Schwarzschild metric with $P_{\alpha\beta} = 0$, and $R = 0$ and $R_{\mu\nu} = 0$. In fact, in the region of nonlocality where higher derivative terms in the action are dominant, it suffices to demand that $R = \text{const}$ and $R_{\mu\nu} = \text{const}$. Let us now concentrate on the full equations of motion (8) with the Weyl part of the full equations of motion:

$$\begin{aligned} P^{\alpha\beta} = 0 = P_3^{\alpha\beta} = \frac{\alpha_c}{8\pi G}(&-g^{\alpha\beta} W^{\mu\nu\lambda\sigma} \mathcal{F}_3(\Box_s) W_{\mu\nu\lambda\sigma} \\ &+ 4 W^{\alpha}{}_{\mu\nu\sigma} \mathcal{F}_3(\Box_s) W^{\beta\mu\nu\sigma} \\ &- 4(R_{\mu\nu} + 2\nabla_{\mu}\nabla_{\nu})(\mathcal{F}_3(\Box_s) W^{\beta\mu\nu\alpha}) \\ &- 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}{}^{\gamma} + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta}). \end{aligned} \tag{17}$$

Indeed, we would expect that in order to fulfill the necessary condition (but not sufficient) for the Schwarzschild metric to be a solution of Eq. (8), we would have both the left and the right-hand side of the above equation vanishes identically. The failure of this test will imply that the Schwarzschild metric *cannot* be the permissible solution of the equation of motion for Eq. (8).

There are a couple of important observations to note, which we summarize below:

(1) $\mathcal{F}_i(\Box_s)$ contain an infinite series of $\Box_s$.
(2) The Bianchi identity holds for each and every order in $\Box_s$, as we have already discussed.
(3) The right-hand side of Eq. (17) should vanish at each and every order in $\Box_s$. This is due to the fact that when we compare the terms, assigned to coefficients $f_{i,n}$ (where the box operator has been applied n times, i.e., $\Box_s^n$) with terms where the box operator has been applied $n+1$ times ($\Box_s^{n+1}$, assigned to coefficient $f_{i,n+1}$), then the $1/r^n$ dependence would at least be changed to $1/r^{n+2}$ in this process. Note that the box operator has roughly two covariant derivatives in r. Therefore, if we are not seeking any miraculous cancellation, between different orders in $\Box_s$, it is paramount that each and every order in $\Box_s$, the right-hand side must vanish to yield the Schwarzschild-like metric solution.

[5]We have checked that the Bianchi identity holds true at each and every order of $\Box_s$.

(4) In fact, we could repeat the same argument for higher-order singular metric *Ansätze*, such as $1/r^\alpha$, for $\alpha > 0$ at short distances, near the ultraviolet.

In order to obtain some insight into this problem, let us first consider the right-hand side of $P_3^{\alpha\beta}$ with one $\Box_s$ *only*, such that

$$\mathcal{F}_3(\Box_s) = (f_{30} + f_{31}\Box_s).$$

Therefore, Eq. (17) becomes

$$\begin{aligned} P_3^{\alpha\beta} = \frac{\alpha_c}{8\pi G}(&-g^{\alpha\beta}W^{\mu\nu\lambda\sigma}(f_{30}+f_{31}\Box_s)W_{\mu\nu\lambda\sigma} \\ &+ 4W^{\alpha}{}_{\mu\nu\sigma}(f_{30}+f_{31}\Box_s)W^{\beta\mu\nu\sigma} \\ &- 4(R_{\mu\nu}+2\nabla_\mu\nabla_\nu)((f_{30}+f_{31}\Box_s)W^{\beta\mu\nu\alpha}) \\ &- 2f_{31}\nabla^\alpha W^{\mu\nu\rho\gamma}\nabla^\beta W_{\mu\nu\rho\gamma} \\ &+ g^{\alpha\beta}f_{31}(\nabla^\alpha W^{\mu\nu\rho\gamma}\nabla^\beta W_{\mu\nu\rho\gamma} + W^{\mu\nu\rho\gamma}\Box_s W_{\mu\nu\rho\gamma}) \\ &- 8f_{31}(W^{\gamma\nu}{}_{\rho\mu}\nabla^\alpha W_\gamma{}^{\beta\rho\mu} - W_\gamma{}^{\beta\rho\mu}\nabla^\alpha W^{\gamma\nu}{}_{\rho\mu})_{;\nu}). \end{aligned} \tag{18}$$

In the static limit, after some computations, we can infer the following:

(1) All the terms combining f_{30} terms cancel each other from the above expression in Eq. (18). This is indeed reminiscence, and agrees to the earlier computations performed in this regard in Ref. [31], where the action corresponds to just the quadratic in curvature, but with local quadratic-curvature action:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R + \alpha_c[R^2 + R^{\mu\nu}R_{\mu\nu} + W^{\mu\nu\lambda\sigma}W_{\mu\nu\lambda\sigma}]). \tag{19}$$

Such an action indeed provides *singular* solutions with metric coefficients $b(r) \sim 1/r$ for $r \ll r_{\rm sch}$, as the leading order contribution, in spite of the fact that the above action has been shown to be renormalizable, but with an unstable vacuum, due to spin-2 ghost [30]. The BGKM action indeed attempts to address the ghost problem of quadratic-curvature gravity.

(2) The first nontrivial result comes from the fact that the *only* terms that *do not cancel*, and survive from the right-hand side of Eq. (18), are those proportional to f_{31}, and one can show explicitly that they go as

$$1/r^8,$$

in the UV ($r \ll 1/M_s$); for details, see the Appendix. This means that, indeed, $1/r$ as a metric solution does not pass through our test, since the right-hand side of the above equation of motion is nonvanishing, but the left-hand side ought to vanish in lieu of the vacuum condition, $P^{\alpha\beta} = 0$.

(3) In fact, we may be able to generalize our results to any orders in $\Box_s$ by noting that the higher orders beyond one box would contribute at least two more covariant derivatives in r in going from $\Box_s^n$ to $\Box_s^{n+1}$ terms (assuming that $\Box_s \sim \frac{1}{M_s^2}\partial_r^2$).[6] This means that the full computation for the right-hand side of Eq. (18) would yield

$$P_3^{\alpha\beta} \backsim g^{\alpha\beta}\left(f_{31}\mathcal{O}\left(\frac{1}{r^8}\right) + f_{32}\mathcal{O}\left(\frac{1}{r^{10}}\right) + \cdots + f_{3n}\mathcal{O}\left(\frac{1}{r^{6+2n}}\right) + \cdots\right), \tag{20}$$

($g^{\alpha\beta}$ is defined from the metric (16), see the exact definition of $P_3^{\alpha\beta}$ in the Appendix) which would require too much fine-tuning to cancel each and every term, while keeping in mind that f_{3n} are mere constant coefficients. Barring such unjustified cancellation, it is fair to say that indeed $1/r$ for $r \ll 1/M_s$ as a metric potential for the BGKM gravity is not a valid solution, if $\mathcal{F}_3$ has a nontrivial dependence on $\Box_s$.

Similar conclusions have already been drawn in Ref. [19], with a complementary arguments. In Ref. [19], the argument was based on taking a smooth limit from the nonlinear solution of Eq. (2) to the linear solution. For any physical solution to be valid, the nonlinear solution must pave the way smoothly to the linear solution.

At the linear level (where the metric potential is bounded below 1), it was shown that the Weyl term vanishes quadratically in r [19], for a nonsingular metric solution given by a metric potential Eq. (6). Therefore, at the full nonlinear level, the $1/r$-type metric potential cannot be promoted as a full solution for the nonlinear equations of motion for the BGKM action, since there is no way it can be made to vanish quadratically at the linear level. Similar conclusions can be made for any metric potential which goes as $1/r^\alpha$ for $\alpha > 1$.

Indeed, this intriguing and potentially very powerful conclusion leads to the fact that the BGKM action with quadratic-curvature, infinite–covariant derivative gravitational action not only ameliorates the curvature singularity at $r = 0$, but also gives rise to a metric potential which is bounded below one in the entire spacetime regime. The notion about the physical mechanism which avoids forming a trapped surface, also yields a static metric solution of gravity, which has no horizon, see [34]. The only viable solution of Eq. (2) remains that of the linear solution, around the Minkowski background, already described by

[6]For the Schwarzschild metric, this takes the form $\Box_s = \frac{1}{M_s^2}g^{\nu\mu}\nabla_\nu\nabla_\mu = \frac{1}{M_s^2}[(1-\frac{2m}{r})\partial_r^2 - 2(-\frac{m}{r^2} + (1-\frac{2m}{r})\frac{1}{r})\partial_r]$.

Eq. (6). Indeed, this last step has to be shown more rigorously, which we leave for future investigation.

Another important conclusion arises due to the nonlocal interactions in the gravitational sector, which yields a nonvacuum solution, such that $R \neq 0$ and $R_{\mu\nu} \neq 0$, within length scale $\sim 2/M_s$ [19]. This is due to the fact that the BGKM gravity smears out the Dirac-delta source, and therefore a vacuum solution does not exist any more like in the case of the Einstein's gravity, or any $f(R)$ gravity, or even in the context of local quadratic-curvature gravity; see Eq. (19).

IV. CONCLUSION

The conclusion of this paper is very powerful. We have argued that the Schwarzschild metric or $1/r$-type metric potentials cannot be the solution of the full BGKM action given by Eq. (2), and the full nonlinear equations of motion (8). By the $1/r$-type metric potential, we mean the nonlinear part of the Schwarzschild metric, for $r < 2Gm$, where m is the Dirac delta source. The presence or absence of singularity is judged by the Weyl contribution. In the pure Einstein-Hilbert action, indeed the Weyl term in the Schwarzschild metric is nonvanishing, and contributes towards the Kretschmann singularity at $r = 0$. In the case of infinite derivatives in four dimensions, we have shown here that this is not the case, and the infinite-derivative Weyl contribution contradicts with $1/r$ being the metric solution for a vacuum configuration, for which the energy momentum tensor vanishes, for a static and spherically symmetric solution for the BGKM action. By itself the result does not prove or disprove a nonsingular metric potential, but it provides a strong hint that the full equations of motion cannot support the Schwarzschild-type of $1/r$-type metric potential. We have also argued that on a similar basis even $1/r^\alpha$ for $\alpha > 0$ will not serve as a full solution to the BGKM gravity. It would be very interesting to explore that if the BGKM gravity may allow other static/nonstatic singular metric solutions or not.

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APPENDIX: NONVANISHING CONTRIBUTIONS FROM THE WEYL TERM

Here we show the relevant terms, present in Eq. (18), assuming $b(r) = 1\text{–}2Gm/r$ in the metric (16). The explicit enumeration of each term is important to understand how the coefficient f_{30} and f_{31} appear and how they might cancel. Let us define

$$P_3^{\alpha\beta} = \frac{\alpha_c}{8\pi G}\sum_{i=1}^{6} F_i^{\alpha\beta}$$

(1) For the first term, $F_1^{\alpha\beta} = -g^{\alpha\beta}W^{\mu\nu\lambda\sigma}(f_{30} + f_{31}\Box_s)W_{\mu\nu\lambda\sigma}$, the calculation yields

$$F_1^{\alpha\beta} = g^{\alpha\beta}\frac{48G^2}{r^8}\frac{m^2}{M_s^2}\begin{pmatrix} -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 & 0 & 0 \\ 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 & 0 \\ 0 & 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 \\ 0 & 0 & 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} \end{pmatrix}. \quad \text{(A1)}$$

(2) The second term, $F_2^{\alpha\beta} = +4W^{\alpha}{}_{\mu\nu\sigma}(f_{30} + f_{31}\Box_s)W^{\beta\mu\nu\sigma}$, is given by

$$F_2^{\alpha\beta} = -g^{\alpha\beta}\frac{48G^2}{r^8}\frac{m^2}{M_s^2}\begin{pmatrix} -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 & 0 & 0 \\ 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 & 0 \\ 0 & 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} & 0 \\ 0 & 0 & 0 & -\frac{(f_{30}M_s^2r^3-6f_{31}Gm)}{r} \end{pmatrix}. \quad \text{(A2)}$$

We can verify, at this point, that the first two terms cancel each other.

(3) The third term, $F_3^{\alpha\beta} = -4(2R_{\mu\nu} + \nabla_\mu \nabla_\nu)(f_{30} + f_{31}\Box_s)W^{\beta\mu\nu\alpha}$, is given by

$$F_3^{\alpha\beta} = g^{\alpha\beta}\frac{288G^2}{r^8}\frac{m^2}{M_s^2}f_{31}\begin{pmatrix} -\frac{(5r-11Gm)}{r} & 0 & 0 & 0 \\ 0 & -\frac{(r-3Gm)}{r} & 0 & 0 \\ 0 & 0 & \frac{(3r-7Gm)}{r} & 0 \\ 0 & 0 & 0 & \frac{(3r-7Gm)}{r} \end{pmatrix}, \tag{A3}$$

which only depends on the f_{31} coefficient.

(4) The fourth term, $F_4^{\alpha\beta} = -2f_{31}\nabla^\alpha W^\lambda{}_{\mu\nu\sigma}\nabla^\beta W_\lambda{}^{\mu\nu\sigma}$, is given by

$$F_4^{\alpha\beta} = g^{\alpha\beta}\frac{288G^2}{r^8}\frac{m^2}{M_s^2}f_{31}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{3(r-2Gm)}{r} & 0 & 0 \\ 0 & 0 & -\frac{(r-2Gm)}{r} & 0 \\ 0 & 0 & 0 & -\frac{(r-2Gm)}{r} \end{pmatrix}. \tag{A4}$$

(5) The fifth term, $F_5^{\alpha\beta} = +g^{\alpha\beta}f_{31}(\nabla^\alpha W^{\mu\nu\rho\gamma}\nabla^\beta W_{\mu\nu\rho\gamma} + W^{\mu\nu\rho\gamma}\Box_s W_{\mu\nu\rho\gamma})$, is given by

$$F_5^{\alpha\beta} = g^{\alpha\beta}\frac{144G^2}{r^8}\frac{m^2}{M_s^2}f_{31}\begin{pmatrix} \frac{(5r-12Gm)}{r} & 0 & 0 & 0 \\ 0 & \frac{(5r-12Gm)}{r} & 0 & 0 \\ 0 & 0 & \frac{(5r-12Gm)}{r} & 0 \\ 0 & 0 & 0 & \frac{(5r-12Gm)}{r} \end{pmatrix}. \tag{A5}$$

(6) The sixth term, $F_6^{\alpha\beta} = -8f_{31}(W^{\gamma\nu}{}_{\rho\mu}\nabla^\alpha W_\gamma{}^{\beta\rho\mu} - W_\gamma{}^{\beta\rho\mu}\nabla^\alpha W^{\gamma\nu}{}_{\rho\mu})_{;\nu}$, is given by

$$F_6^{\alpha\beta} = g^{\alpha\beta}\frac{576G^2}{r^8}\frac{m^2}{M_s^2}f_{31}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{(r-2Gm)}{r} & 0 & 0 \\ 0 & 0 & \frac{(3r-7Gm)}{r} & 0 \\ 0 & 0 & 0 & \frac{(3r-7Gm)}{r} \end{pmatrix}. \tag{A6}$$

Having computed each term of $P_3^{\alpha\beta}$, we can conclude that the stress energy momentum tensor dependence on the f_{30} coefficient is vanishing, and *only* the one box, $\Box_s$, contributions survive. Finally, we have the nonvanishing contribution,

$$P_3^{\alpha\beta} = g^{\alpha\beta}\frac{144G}{8\pi r^8}\frac{m^2}{M_s^4}f_{31}\begin{pmatrix} -\frac{5(r-2Gm)}{r} & 0 & 0 & 0 \\ 0 & -\frac{7(r-2Gm)}{r} & 0 & 0 \\ 0 & 0 & \frac{(21r-50Gm)}{r} & 0 \\ 0 & 0 & 0 & \frac{(21r-50Gm)}{r} \end{pmatrix}. \tag{A7}$$

Second order in $\Box_s$ contributions: In order to strengthen our arguments, we present below the additional contribution for the second order in box, i.e., $\Box_s^2$:

$$P_3^{\alpha\beta}(\Box_s^2) = -g^{\alpha\beta}\frac{576G}{8\pi r^{10}}\frac{m^2}{M_s^6}f_{32}\begin{pmatrix} a_{00} & 0 & 0 & 0 \\ 0 & a_{11} & 0 & 0 \\ 0 & 0 & a_{22} & 0 \\ 0 & 0 & 0 & a_{33} \end{pmatrix}. \tag{A8}$$

with the dimensionless matrix elements, defined as

$$a_{00} = \frac{(939G^2m^2 - 744Gmr + 140r^2)}{r^2},$$
$$a_{11} = \frac{(195G^2m^2 - 132Gmr + 20r^2)}{r^2},$$
$$a_{22} = -\frac{(789G^2m^2 - 534Gmr + 80r^2)}{r^2},$$
$$a_{33} = -\frac{(789G^2m^2 - 534Gmr + 80r^2)}{r^2}.$$

Let us now consider, e.g., the P_3^{22} element at $\Box_s^2$, namely:

$$P_3^{22} = \frac{\alpha_c}{8\pi G}\Bigg[f_{31}\left(\frac{3024G^2m^2}{r^{10}M_s^2} - \frac{7200G^3m^3}{r^{11}M_s^2}\right) + f_{32}\left(\frac{46080G^2m^2}{r^{12}M_s^4} - \frac{307584G^3m^3}{r^{13}M_s^4} + \frac{454464G^4m^4}{r^{14}M_s^4}\right) + \cdots\Bigg]. \tag{A9}$$

Demanding that $P_3^{22} = 0$, implies that $f_{31} = f_{32} = 0$. We can now ask what would happen for higher orders in $\Box_s$. Since $\Box_s \sim \frac{1}{M_s^2}\partial_r^2$, we have at the lowest third order contribution in box, in powers of r, is proportional to

$$f_{33}\frac{G^2m^2}{r^{14}M_s^6}. \tag{A10}$$

Therefore, since we already concluded that $f_{31} = f_{32} = 0$, we now have to demand that the contribution of $f_{33}\frac{G^2m^2}{r^{14}M_s^6}$ vanishes identically. The lowest fourth order contribution, in powers of r, will go as

$$f_{34}\frac{G^2m^2}{r^{16}M_s^8}, \tag{A11}$$

we are left with the option that $f_{33} = f_{34} = 0$. Obviously, we do not claim that this is a rigorous mathematical demonstration; however, we can hint, by dimensional analysis, that the lowest nth order contribution will be always proportional to

$$f_{3n}\frac{G^2m^2}{r^{8+2n}M_s^{2n}}. \tag{A12}$$

Indeed, the above analysis suggests that for a nonvanishing coefficient f_{3n} it is hard to imagine how we could make the contribution from P_3^{22} vanish.

[1] C. M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17, 4 (2014).
[2] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. **116**, 061102 (2016).
[3] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge Univeristy Press, Cambridge, England, 1975).
[4] R. Penrose, Gravitational collapse: The role of general relativity, Riv. Nuovo Cim. **1**, 252 (1969) [Gen. Relativ. Gravit. **34**, 1141 (2002)].
[5] R. Penrose, Gravitational Collapse and Space-time Singularities, Phys. Rev. Lett. **14**, 57 (1965).
[6] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Towards Singularity and Ghost Free Theories of Gravity, Phys. Rev. Lett. **108**, 031101 (2012).
[7] A. A. Tseytlin, On singularities of spherically symmetric backgrounds in string theory, Phys. Lett. B **363**, 223 (1995).
[8] W. Siegel, Stringy gravity at short distances, arXiv:hep-th/0309093.
[9] T. Biswas, A. Mazumdar, and W. Siegel, Bouncing universes in string-inspired gravity, J. Cosmol. Astropart. Phys. 03 (2006) 009.
[10] T. Biswas, R. Brandenberger, A. Mazumdar, and W. Siegel, Non-perturbative gravity, the Hagedorn bounce and the cosmic microwave background, J. Cosmol. Astropart. Phys. 12 (2007) 011.
[11] T. Biswas, T. Koivisto, and A. Mazumdar, Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity, J. Cosmol. Astropart. Phys. 11 (2010) 008.
[12] A. S. Koshelev and S. Y. Vernov, On bouncing solutions in non-local gravity, Phys. Part. Nucl. **43**, 666 (2012); T. Biswas, A. S. Koshelev, A. Mazumdar, and S. Y. Vernov, Stable bounce and inflation in non-local higher derivative cosmology, J. Cosmol. Astropart. Phys. 08 (2012) 024.
[13] A. Conroy, A. S. Koshelev, and A. Mazumdar, Geodesic completeness and homogeneity condition for cosmic inflation, Phys. Rev. D **90**, 123525 (2014); Defocusing of null rays in infinite derivative gravity, J. Cosmol. Astropart. Phys. 01 (2017) 017.

[14] T. Biswas, A. Conroy, A. S. Koshelev, and A. Mazumdar, Generalized ghost-free quadratic curvature gravity, Classical Quantum Gravity **31**, 015022 (2014); Corrigendum, Classical Quantum Gravity **31**, 159501 (2014).

[15] V. P. Frolov and A. Zelnikov, Head-on collision of ultrarelativistic particles in ghost-free theories of gravity, Phys. Rev. D **93**, 064048 (2016).

[16] J. Edholm, A. S. Koshelev, and A. Mazumdar, Behavior of the Newtonian potential for ghost-free gravity and singularity-free gravity, Phys. Rev. D **94**, 104033 (2016).

[17] V. P. Frolov, Mass Gap for Black-Hole Formation in Higher-Derivative and Ghost-Free Gravity, Phys. Rev. Lett. **115**, 051102 (2015).

[18] A. S. Koshelev and A. Mazumdar, Do massive compact objects without event horizon exist in infinite derivative gravity?, Phys. Rev. D **96**, 084069 (2017).

[19] L. Buoninfante, A. S. Koshelev, G. Lambiase, and A. Mazumdar, Classical properties of non-local, ghost- and singularity-free gravity, arXiv:1802.00399.

[20] A. S. Cornell, G. Harmsen, G. Lambiase, and A. Mazumdar, Rotating metric in Non-Singular Infinite Derivative Theories of Gravity, Phys. Rev. D **97**, 104006 (2018).

[21] V. P. Frolov, A. Zelnikov, and T. de Paula Netto, Spherical collapse of small masses in the ghost-free gravity, J. High Energy Phys. 06 (2015) 107.

[22] V. P. Frolov and A. Zelnikov, Head-on collision of ultrarelativistic particles in ghost-free theories of gravity, Phys. Rev. D **93**, 064048 (2016).

[23] R. M. Wald, Black hole entropy is the Noether charge, Phys. Rev. D **48**, R3427 (1993); V. Iyer and R. M. Wald, Some properties of the Noether charge and a proposal for dynamical black hole entropy, Phys. Rev. D **50**, 846 (1994); T. Jacobson, G. Kang, and R. C. Myers, On black hole entropy, Phys. Rev. D **49**, 6587 (1994).

[24] A. Conroy, A. Mazumdar, and A. Teimouri, Wald Entropy for Ghost-Free, Infinite Derivative Theories of Gravity, Phys. Rev. Lett. **114**, 201101 (2015); Erratum, Phys. Rev. Lett. **120**, 039901(E) (2018); A. Conroy, A. Mazumdar, S. Talaganis, and A. Teimouri, Nonlocal gravity in D dimensions: Propagators, entropy, and a bouncing cosmology, Phys. Rev. D **92**, 124051 (2015).

[25] E. Tomboulis, Renormalizability and asymptotic freedom in quantum gravity, Phys. Lett. B **97**, 77 (1980). Renormalization and asymptotic freedom in quantum gravity, in *Quantum Theory Of Gravity*, edited by S. M. Christensen (North Carolina University, Chapel Hill, 1984), p. 251; Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146.

[26] E. T. Tomboulis, Nonlocal and quasilocal field theories, Phys. Rev. D **92**, 125037 (2015).

[27] L. Modesto, Super-renormalizable quantum gravity, Phys. Rev. D **86**, 044005 (2012).

[28] S. Talaganis, T. Biswas, and A. Mazumdar, Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity, Classical Quantum Gravity **32**, 215017 (2015); S. Talaganis and A. Mazumdar, High-energy scatterings in infinite-derivative field theory and ghost-free gravity, Classical Quantum Gravity **33**, 145005 (2016).

[29] T. Biswas and N. Okada, Towards LHC physics with nonlocal Standard Model, Nucl. Phys. **B898**, 113 (2015); A. Ghoshal, A. Mazumdar, N. Okada, and D. Villalba, On the stability of infinite derivative Abelian Higgs, Phys. Rev. D **97**, 076011 (2018).

[30] K. S. Stelle, Renormalization of higher-derivative quantum gravity, Phys. Rev. D **16**, 953 (1977).

[31] H. Lu, A. Perkins, C. N. Pope, and K. S. Stelle, Black Holes in Higher Derivative Gravity, Phys. Rev. Lett. **114**, 171601 (2015); Spherically symmetric solutions in higher-derivative gravity, Phys. Rev. D **92**, 124019 (2015).

[32] T. Biswas, A. S. Koshelev, and A. Mazumdar, Gravitational theories with stable (anti-)de Sitter backgrounds, Fundam. Theor. Phys. **183**, 97 (2016). Consistent higher derivative gravitational theories with stable de Sitter and anti–de Sitter backgrounds, Phys. Rev. D **95**, 043533 (2017).

[33] T. Biswas, T. Koivisto, and A. Mazumdar, Nonlocal theories of gravity: the flat space propagator, arXiv:1302.0532; L. Buoninfante, Ghost and singularity free theories of gravity, arXiv:1610.08744.

[34] V. P. Frolov and A. Zelnikov, *Introduction to Black Hole Physics* (Oxford University Press, New York, 2011).