

University of Groningen

Heuristics for the stochastic economic lot sizing problem with remanufacturing under backordering costs

Kilic, Onur A.; Tunc, Huseyin

Published in:
European Journal of Operational Research

DOI:
[10.1016/j.ejor.2019.01.051](https://doi.org/10.1016/j.ejor.2019.01.051)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):
Kilic, O. A., & Tunc, H. (2019). Heuristics for the stochastic economic lot sizing problem with remanufacturing under backordering costs. *European Journal of Operational Research*, 276(3), 880-892. <https://doi.org/10.1016/j.ejor.2019.01.051>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.



Production, Manufacturing, Transportation and Logistics

Heuristics for the stochastic economic lot sizing problem with remanufacturing under backordering costs

Onur A. Kilic^{a,b}, Huseyin Tunc^{b,*}^a Department of Operations, University of Groningen, Groningen, the Netherlands^b Department of Policy and Strategy Studies, Hacettepe University, Ankara, Turkey

ARTICLE INFO

Article history:

Received 22 November 2017

Accepted 23 January 2019

Available online 29 January 2019

Keywords:

Inventory

Stochastic lot-sizing

Remanufacturing

Heuristic

Mixed integer programming

ABSTRACT

We consider a production system where demand can be met by manufacturing new products and remanufacturing returned products, and address the economic lot sizing problem therein. The system faces stochastic and time-varying demands and returns over a finite planning horizon. The problem is to match supply with demand, while minimizing the total expected cost which is comprised of fixed production costs and inventory (holding and backordering) costs. We introduce heuristic policies for this problem which offer different levels of flexibility with respect to production decisions. We present computational methods for these policies based on convex optimization and certainty equivalent mixed integer programming, and numerically assess their cost performance and computational efficiency by means of simulation.

© 2019 Elsevier B.V. All rights reserved.

1. Introduction

The environmental considerations have been universally recognized over the last decade. This has led governments to put policies and regulations into action toward promoting environment-friendly businesses. Besides environmental legislation, increasing consumer awareness has stimulated companies to become more environment-friendly. As a result of all these, a large variety of new concepts, such as, environmental enterprises, sustainable businesses, and green brands, have been introduced into today's business world in the quest of fostering environment-friendly business operations. The practice of "remanufacturing" falls into the context of environment-friendly production operations. It refers to a set of value-added recovery operations where used and/or returned components or products are restored to as-good-as-new condition (Van Der Laan & Teunter, 2006). It has been proven to be economical and environment-friendly and well-received in practice. The production environments where manufacturing and remanufacturing operations are carried out in concert are referred to as hybrid manufacturing and remanufacturing systems and are common in a variety of industries (see e.g. Ferguson and Toktay, 2006; Kenne, Dejax, and Gharbi, 2012, and references therein). It is difficult to manage inventories in these systems as it requires effective coordination of manufacturing

and remanufacturing operations, especially in presence of fixed production costs and uncertainty in demands and returns.

We aim to address the aforementioned challenge in the current manuscript. We consider the inventory control problem in hybrid manufacturing and remanufacturing systems with stochastic and time-varying demands and returns. The objective is to minimize the expected total cost which is comprised of production costs of manufacturing and remanufacturing and inventory costs of serviceables and returns. We refer to this problem as the stochastic economic lot-sizing problem with returns—henceforth abbreviated as SELSR.

There is a sizeable literature on hybrid manufacturing and remanufacturing systems where a variety of integrated inventory control approaches have been presented. For a recent review, we refer the reader to Guide Jr and Van Wassenhove (2009) and Chan, Li, Chung, and Saadat (2017). The studies which fall into this domain can be classified based on, among others, their assumptions regarding demands and returns. For instance, there are many studies on lot sizing (see e.g. Retel Helmrich, Jans, van den Heuvel, & Wagelmans, 2014; Schulz, 2011; Teunter, Bayindir, & Van Den Heuvel, 2006) and lot scheduling problems (see e.g. Tang & Teunter, 2006; Teunter, Kaparis, & Tang, 2008; Zanoni, Segerstedt, Tang, & Mazzoldi, 2012) in hybrid manufacturing and remanufacturing systems. These studies, however, assume deterministic demands and returns. There are also studies which address stochastic demands and returns (see e.g. Ahiska & King, 2010a; Benedito & Corominas, 2013; Panagiotidou, Nenes, Zikopoulos, & Tagaras, 2017; Van Der Laan & Teunter, 2006). However, they

* Corresponding author.

E-mail address: huseyin.tunc@hacettepe.edu.tr (H. Tunc).

assume time-invariant demand and return distributions over time. These assumptions do not hold in most production systems. It is well-known that demand processes significantly vary over time due to factors including short product life cycles and seasonal fluctuations (see e.g. Graves & Willems, 2008; Neale & Willems, 2009; Silver, 2008) and return processes are subject to time-varying uncertainty (see e.g. Fleischmann & Kuik, 2003; Guide, 2000). The studies addressing time-varying stochastic demands and returns are rather scarce. Li, Liu, Cao, and Wang (2009) and Tao and Zhou (2014) are among those. They do not, however, consider fixed manufacturing and remanufacturing costs, and, as such, ignore economies of scale in production.

There are only a few studies where time-varying stochastic demands and returns are addressed in presence of fixed manufacturing and remanufacturing costs. These are outlined in the following. Ahiska and King (2010b) consider a case where a product's life-cycle involves five stages. The demand and return distributions vary between these stages but remain stationary within each stage. They approximate the original problem by a sequence of stationary problems—one for each stage of the product life cycle, and employ a stationary heuristic policy for each of these problems. This intuitive approach is reasonable if stages of the product life cycle span large intervals of time. Naeem, Dias, Tibrewal, Chang, and Tiwari (2013) consider a hybrid manufacturing and remanufacturing system with period-specific demand and return distributions. They model and solve the problem as a stochastic dynamic program. Hilger, Sahling, and Tempelmeier (2016) and Kilic, Tunc, and Tarim (2018) consider the problem under different service level constraints. They present heuristic policies inspired by simple control rules and develop mathematical models thereof.

To the best of authors' knowledge; Naeem et al. (2013), Hilger et al. (2016), and Kilic et al. (2018) are the only studies that address variants of SELSR. These adopt different policies in approaching the problem which can be classified by Bookbinder and Tan's (1988) well-known scheme. This classification scheme differentiates between static, dynamic, and static-dynamic uncertainty strategies. The static uncertainty strategy encompasses policies where all replenishment decisions are made at the beginning of the planning horizon. For instance, Hilger et al. (2016) adopt a static-uncertainty strategy where manufacturing and remanufacturing periods, as well as corresponding quantities in each period, are determined at the beginning of the planning horizon. The dynamic uncertainty strategy, on the other hand, stands for the other extreme where replenishment decisions are dynamically made in a just-in-time fashion. Naeem et al.'s (2013) stochastic dynamic program employs a dynamic uncertainty strategy as manufacturing and remanufacturing quantities are strictly state dependent. The static-dynamic uncertainty strategy is a hybrid of the first two strategies where a replenishment schedule is fixed at the outset, but replenishment quantities are dynamically determined upon observing inventory levels at each replenishment epoch. Kilic et al.'s (2018) policies follow a static-dynamic uncertainty strategy as they are characterized by fixed manufacturing and remanufacturing schedules, while allowing flexibility in manufacturing and/or remanufacturing quantities. It is important to remark that these strategies have their advantages and disadvantages. For instance, because the cost-effectiveness of a policy improves as it effectively exploits more information in making decisions, the dynamic uncertainty strategy is the best in terms of cost performance. The static uncertainty strategy, on the other hand, offers advanced information on production quantities, and, as such, it is very suitable for systems characterized by limited flexibility. The static-dynamic uncertainty strategy eliminates the uncertainty (or the so-called nervousness) in the replenishment schedule which is known to be critical in practice (Heisig, 2001; Inderfurth, 1994), while exploiting the cost advantage of flexible production quantities.

The aim of this paper is to present heuristic policies for SELSR, based on dynamic and static-dynamic uncertainty strategies. Our contributions are outlined as follows. First, we propose a heuristic policy following the dynamic uncertainty strategy. This heuristic is aimed at alleviating the computational burden of the optimal stochastic dynamic program of SELSR, while providing cost-effective solutions by exploiting the advantages of the dynamic uncertainty strategy. It is a stochastic adaptation of Silver and Meal's (1973) heuristic tailored for hybrid manufacturing and remanufacturing systems with stochastic demands and returns. Second, we adopt the static-dynamic policies introduced by Kilic et al. (2018). The mathematical models of these policies were built on the restrictive assumption that the effect of backorders can be neglected in cost computations. This assumption is only reasonable under service level constraints that require high non-stockout probabilities and enables one to safely replace the true non-linear cost function by a linear approximation, and thereby lead to simpler mathematical models. In the current study, we relax this assumption and develop certainty equivalent MIP models of Kilic et al.'s (2018) static-dynamic policies which allow for more general measures of service quality. Finally, we conduct a numerical study and evaluate the cost performance and the computational efficiency of the proposed dynamic and static-dynamic policies, while using Hilger et al.'s (2016) static policy as a benchmark. Our results clearly demonstrate the trade-off between the cost performance and the flexibility of the policies and point out problem settings where a particular policy can be a better alternative to others.

The remainder of the paper is organized as follows. In Section 2, we provide a formal definition of the problem and set the notation. In Section 3, we introduce the dynamic heuristic and the associated algorithm. In Section 4, we introduce the static-dynamic heuristics as well as their MIP models. In Section 5, we conduct a computational study. In Section 6, we conclude.

2. Problem definition and background

SELSR can be defined as follows. We consider a hybrid manufacturing and remanufacturing system with stochastic demands and returns where remanufactured products are considered as-good-as-new. The system has two inventories: serviceable and return products. The serviceable inventory is replenished by manufacturing new products and remanufacturing returned products. The return inventory is fed by product returns. The planning horizon consists of T periods. The demands and returns over these periods are independent but not necessarily identically distributed random variables with known distribution functions. The demand and return in period n are denoted as D_n and R_n , respectively. Demands that are not satisfied from stock are backordered. There are fixed production costs which are charged in each manufacturing and remanufacturing period. These are denoted as K^m and K^r . The unit manufacturing and remanufacturing costs are ignored for convenience (see Teunter et al., 2006), yet they can easily be incorporated in the analysis. The holding costs h^s and h^r are incurred for each unit of serviceable and return inventory carried from one period to the next, and a backorder cost p is incurred for each unit of backordered demand per period. The problem is to determine a manufacturing and remanufacturing schedule, and the corresponding production quantities so as to minimize the expected total cost over the planning horizon.

We assume the following sequence of events in each period. First, returns are realized and fed into the return inventory. Then, production operations take place and serviceable and return inventory levels are updated. If a remanufacturing operation takes place, then remanufactured products are added to serviceable inventory and the same amount of returns are depleted from the return inventory. If a manufacturing operation takes place, then

manufactured products are added into the serviceable inventory. Next, demand is realized and deducted from the serviceable inventory. Finally, excess or shortage inventories and returns are carried over to the next period.

The following notational convention is used throughout the paper. We let $[u, v]$ denote the set of consecutive integers $\{u, u + 1, \dots, v\}$. For a sequence of random variables ξ_1, ξ_2, \dots , we let $\xi_{[u,v]} := \sum_{n \in [u,v]} \xi_n$. We define $[x]^+ = \max\{x, 0\}$ and $[x]^- = \max\{-x, 0\}$. We also define $\lceil x \rceil$ as the least integer greater than or equal to x . We let E be the expectation operator and $\mathbb{1}\{\cdot\}$ be the indicator function. Finally, we denote the cardinality of a set A as $|A|$.

SELSR can be modeled as a stochastic dynamic program (c.f. Naeem et al., 2013). For the sake of completeness, we begin our analysis by presenting such a program. Let us define the single-period inventory cost function

$$G_n(y^s, y^r) = h^s E[y^s - D_n]^+ + p E[y^s - D_n]^- + h^r y^r$$

which provides the expected holding costs of serviceable and returns, and backorder costs in period n , provided that post-production serviceable and return inventory levels are respectively y^s and y^r .

The stochastic dynamic program can be built upon a recursive function $C_n(x^s, x^r)$ which expresses the expected total cost of following optimal policy from period n onward, if the initial serviceable and return inventory levels in this period are x^s and x^r , respectively. This function can be written as

$$C_n(x^s, x^r) = \min_{0 \leq q^m, 0 \leq q^r \leq x^r} \{ K^m \mathbb{1}\{q^m > 0\} + K^r \mathbb{1}\{q^r > 0\} + G_n(x^s + q^m + q^r, x^r - q^r) + E C_{n+1}(x^s + q^m + q^r - D_n, x^r - q^r + R_{n+1}) \}$$

where the minimization is done over q^m and q^r which respectively stand for manufacturing and remanufacturing quantities. The terminal cost function reads $C_{T+1}(x^s, x^r) = 0$ for all x^s and x^r . It is easy to see that the first line of the expression stands for the production costs and expected inventory costs in the immediate period whereas the second line provides the optimal expected cost-to-go from next period onwards.

The dynamic program above can be solved for discrete demands and returns by standard methods, but it poses a significant computational challenge even for small-scale problems. It is therefore not suitable to be used in most real-life applications, at least without exploiting structural properties of its optimal solution which to date are not available. The heuristic policies we present in the following sections bypass this issue and offer practical means of approaching SELSR.

3. Dynamic heuristic

In this section, we present a computationally efficient heuristic for SELSR. The heuristic follows a dynamic uncertainty strategy and determines production quantities in each period upon observing the initial serviceable and return inventories. It is inspired by Silver and Meal's (1973) well-known myopic heuristic. Silver and Meal's heuristic has been adapted for many other lot-sizing problems in the literature, including Askin's (1981) extension which captures stochastic demands and Teunter et al.'s (2006) extension which accounts for product returns. For the purposes of the current paper, we build on and blend these two approaches. The proposed heuristic is based on the idea of determining production quantities in connection with a virtual replenishment cycle which starts in the immediate period. The length of this cycle is chosen such that the average expected cost per period is minimum. The production quantities in the immediate period are then myopically set as the

optimal production quantities minimizing the total expected cost of the chosen replenishment cycle.

In what follows, we first introduce the concept of replenishment cycles and associated cost minimization problems. Then, we explain how replenishment decisions are made.

Let us consider an arbitrary period n and suppose that post-production serviceable and return inventory levels are y^s and y^r , respectively. Furthermore, assume that no production operation takes place up-to (but not including) period $n + a$ ($a \geq 1$). That is, we have a replenishment cycle that spans over periods $[n, n + a - 1]$. The length of this cycle is a periods. The expected total inventory holding and backorder costs over the replenishment cycle can be expressed by means of the following function:

$$G_{na}(y^s, y^r) = \sum_{u=n}^{n+a-1} (h^s E[y^s - D_{[n,u]}]^+ + p E[y^s - D_{[n,u]}]^- + h^r E[y^r + R_{[n+1,u]}])$$

It is easy to see that $G_{na}(y^s, y^r)$ is separable and jointly convex on y^s and y^r . Its partial derivatives are $\partial G_{na} / \partial y^s = (h^s + p) \sum_{u=n}^{n+a-1} F_{nu}(y^s) - pa$ and $\partial G_{na} / \partial y^r = h^r a$ where F_{nu} is the distribution function of the random variable $D_{[n,u]}$.

Next, we analyze the problem of finding the optimal production quantities. For initial serviceable and return inventories x^s and x^r , the minimum expected total cost of the replenishment cycle can be written as

$$V_{na}(x^s, x^r) = \min_{0 \leq q^m, 0 \leq q^r \leq x^r} \{ K^m \mathbb{1}\{q^m > 0\} + K^r \mathbb{1}\{q^r > 0\} + G_{na}(x^s + q^m + q^r, x^r - q^r) \}$$

and computing it is essential in the context of the proposed heuristic. We now present an efficient approach to compute $V_{na}(x^s, x^r)$. Our approach decomposes the problem by breaking down the domain of production quantities q^m and q^r into four different parts such that each sub-problem stands for a different production policy; namely, manufacture-only ($q^m \geq 0$ and $q^r = 0$), remanufacture-only ($q^m = 0$ and $q^r \geq 0$), manufacture-and-remanufacture ($q^m \geq 0$ and $q^r \geq 0$), and do-nothing ($q^m = 0$ and $q^r = 0$). In all these cases, the fixed production costs are constant and independent of the production quantities; and because $G_{na}(y^s, y^r)$ is jointly convex on its arguments, so are the cost expressions to be minimized. This enables us to optimize the overall problem by solving a series of convex optimization problems and choosing the case with the minimum cost solution.

We now elaborate on how each case can be optimized independently. Let us first concentrate on manufacture-only problem. Here, the optimization problem reduces to finding a q^m ($q^m \geq 0$) such that $K^m + G_{na}(x^s + q^m, x^r)$ is minimized. Because $G_{na}(y^s, y^r)$ is convex in y^s , the optimal policy is a manufacture-up-to policy. That is, there exists a manufacture-up-to level α_{na} for the manufacture-only problem associated with the replenishment cycle which spans over periods $[n, n + a - 1]$ such that it is optimal to manufacture $\alpha_{na} - x^s$ units if $x^s < \alpha_{na}$, and not to manufacture otherwise. This manufacture-up-to level satisfies

$$\frac{1}{a} \sum_{u=n}^{n+a-1} F_{nu}(\alpha_{na}) = \frac{p}{h^s + p} \tag{1}$$

which is obtained by setting $\partial G_{na} / \partial y^s = 0$. Thus, we have that $q^m_* = \max\{0, \alpha_{na} - x^s\}$ is the optimal manufacturing quantity and $K^m + G_{na}(x^s + q^m_*, x^r)$ is the associated cost of the manufacturing-only problem.

We can analyze the remanufacturing-only problem in a similar vein. Here, the optimization problem is to find a q^r ($x^r \geq q^r \geq 0$) such that $K^r + G_{na}(x^s + q^r, x^r - q^r)$ is minimized. We know that $G_{na}(y^s, y^r)$ is jointly convex in y^s and y^r . Thus, the optimal policy is a remanufacture-up-to policy. That is, there exists a

Table 1

The cost expressions and optimal production quantities for manufacture-only (**m**), remanufacture-only (**r**), manufacture-and-remanufacture (**mr**), and do-nothing (**0**) problems.

Sub-problem	Cost expression	q_*^m	q_*^r
m	$K^m + G_{na}(x^s + q_*^m, x^r)$	$\max\{0, \alpha_{na} - x^s\}$	0
r	$K^r + G_{na}(x^s + q_*^r, x^r - q_*^r)$	0	$\min\{x^r, \max\{\beta_{na} - x^s, 0\}\}$
mr	$K^m + K^r + G_{na}(x^s + q_*^m + q_*^r, x^r - q_*^r)$	$\max\{0, \alpha_{na} - x^s - q_*^r\}$	$\min\{x^r, \max\{\beta_{na} - x^s, 0\}\}$
0	$G_{na}(x^s, x^r)$	0	0

remanufacture-up-to level β_{na} for the manufacture-only problem associated with the replenishment cycle which spans over periods $[n, n + a - 1]$ such that it is optimal to remanufacture $\beta_{na} - x^s$ units if $x^s < \beta_{na}$, and not to remanufacture otherwise. The remanufacture-up-to level can be characterized by

$$\frac{1}{a} \sum_{u=n}^{n+a-1} F_{nu}(\beta_{na}) = \frac{h^r + p}{h^s + p} \tag{2}$$

which is obtained by setting $\partial G_{na} / \partial y^s - \partial G_{na} / \partial y^r = 0$. For the cases where the available return inventory x^r is not sufficient to remanufacture $\beta_{na} - x^s$ units, the convexity suggests that it is optimal to remanufacture as much as possible, or, put in other words, remanufacture all return inventory. Therefore, we can conclude that $q_*^r = \min\{x^r, \max\{\beta_{na} - x^s, 0\}\}$ is the optimal remanufacturing quantity and $K^r + G_{na}(x^s + q_*^r, x^r - q_*^r)$ is the optimal cost of the remanufacturing-only problem.

We note that the results presented above immediately apply to systems where demands and returns are discrete. The manufacture- and remanufacture-up-to levels should then be defined as the smallest values such that the left-hand-sides of the expressions in (1) and (2) exceed the right-hand-sides.

Before proceeding with the manufacture-and-remanufacture problem, we present the following results which facilitate our analysis.

Property 1. *The optimal production quantities q_*^m and q_*^r of the remanufacture-and-manufacture problem satisfies $q_*^m(x^r - q_*^r) = 0$. That is, the optimal manufacturing quantity can only be strictly positive if it is also optimal to remanufacture all return inventory.*

Proof. Proof by contradiction. Suppose q_*^m and q_*^r are the optimal manufacturing and remanufacturing quantities such that $q_*^m > 0$ and $x^r > q_*^r \geq 0$. Let us consider an alternative solution where manufacturing and remanufacturing quantities are $q^m = q_*^m - (x^r - q_*^r)$ and $q^r = x^r$. The costs of these solutions are $K^m + K^r + G_{na}(x^s + q_*^m + q_*^r, x^r - q_*^r)$ and $K^m + K^r + G_{na}(x^s + q_*^m + q_*^r, 0)$, respectively. Because the former is the optimal, we must have $G_{na}(x^s + q_*^m + q_*^r, x^r - q_*^r) < G_{na}(x^s + q_*^m + q_*^r, 0)$. However, this contradicts with the fact that $G_{na}(y^s, y^r)$ is increasing in y^r . □

Property 2. *The manufacture-up-to and remanufacture-up-to levels satisfy $\alpha_{na} \leq \beta_{na}$.*

Proof. The proof immediately follows from (1) and (2). □

The properties above clearly demonstrate that in the manufacture-and-remanufacture problem it is always better to remanufacture rather than to manufacture (provided there is available return inventory) when approaching a specific target serviceable inventory level. The consequence of this is that we can approach the manufacture-and-remanufacture problem by sequentially deploying the optimal policies of the remanufacturing-only and manufacturing-only problems. To that end, we first set the optimal remanufacturing quantity as $q_*^r = \min\{x^r, \max\{\beta_{na} - x^s, 0\}\}$. Then, based on the post-remanufacturing serviceable inventory level $x^s + q_*^r$, we set the optimal manufacturing quantity as $q_*^m = \max\{0, \alpha_{na} - x^s - q_*^r\}$. The

optimal cost of the remanufacturing-only problem is therefore $K^m + K^r + G_{na}(x^s + q_*^m + q_*^r, x^r - q_*^r)$.

This concludes our analysis on optimal production quantities, as the do-nothing option does not entail an optimization problem. We can summarize the optimization procedure as follows. First, we compute the manufacture-up-to and remanufacture-up-to levels. These can easily be obtained by standard root-finding techniques. The initial serviceable and return inventories along with the manufacture-up-to and remanufacture-up-to levels immediately provide the optimal manufacturing and remanufacturing quantities for all four sub-problems. We then compute the cost expressions associated with each sub-problem based on the optimal production quantities. Finally, we choose the sub-problem with the minimum cost and establish production quantities accordingly. For convenience, we provide the cost expressions and optimal production quantities for all each sub-problem in Table 1.

We have thus far presented an efficient method to compute the minimum expected total cost of a replenishment cycle. We now turn our attention to how replenishment decisions are made in each period.

The heuristic determines manufacturing and remanufacturing quantities in each period upon observing the initial serviceable and return inventories. The production quantities are established in connection with a virtual replenishment cycle which starts in the immediate period. We use the term “virtual” because the replenishment cycle is merely used as a proxy to determine production quantities, it is not implemented. That is, as opposed to the definition of a replenishment cycle which suggests that no production operation takes place over the replenishment cycle, the heuristic initiates production operations on a period-by-period basis.

The replenishment cycle that will constitute a basis for production quantities is chosen such that its average expected cost per period is as small as possible. Let us consider an arbitrary period n and suppose that the initial serviceable and return inventory levels are x^s and x^r . The average expected cost per period of a replenishment cycle that starts in period n and spans periods $[n, n + a - 1]$ is written as

$$\frac{V_{na}(x^s, x^r)}{a}.$$

We thus aim at finding a replenishment cycle for which this expression is minimized. To that end, we use a simple forward search routine which increments the cycle length a as long as the average expected cost per period is decreasing. That is, we compute a sequence of expressions $\frac{V_{n1}(x^s, x^r)}{1}, \frac{V_{n2}(x^s, x^r)}{2}, \dots$ and stop when $\frac{V_{na}(x^s, x^r)}{a} \leq \frac{V_{n(a+1)}(x^s, x^r)}{a+1}$ for some cycle length a . Then, we put the optimal production quantities associated with the replenishment cycle that spans over the next a periods into action and the system transitions from period n to $n + 1$. This procedure is deployed in each and every period over the planning horizon upon observing initial serviceable and return inventories.

4. Static-dynamic heuristics

In this section, we present two heuristic policies for SELSR: All-or-Nothing Policy (AoN) and Threshold Policy (THR). These are

both static-dynamic policies, i.e., they fix the periods in which manufacturing and remanufacturing operations take place at the very beginning of the planning horizon, yet they significantly differ from each other as to how manufacturing and remanufacturing quantities are determined. AoN and THR are introduced by Kilic et al. (2018). Because Kilic et al. (2018) addressed SELSR under a service level constraint that limits the probability of stock-outs, their mathematical models are built on the restrictive assumption that non-stockout probabilities are sufficiently high to justify neglecting backorders in cost computations. We shall relax this assumption and devise certainty equivalent MIP models of AoN and THR which can account for backordering costs.

In what follows, we first present some preliminary analysis of static-dynamic policies. Then, we elaborate on AoN and THR policies and their mathematical models. For the sake of brevity, we keep the discussion on policy descriptions brief and refer the reader to Kilic et al. (2018) for details.

4.1. Preliminaries

Any static-dynamic policy is characterized by a set of manufacturing and remanufacturing periods as well as a set of rules that define how production quantities are determined in these periods. We define the set of manufacturing and remanufacturing periods as $\Pi^m \subseteq [1, T]$ and $\Pi^r \subseteq [1, T]$, respectively. The decision rules may vastly change from one policy to another, but, in essence, they characterize manufacturing and remanufacturing quantities, and thereby serviceable and return inventories. Let us define the following for each period n over the planning horizon; Q_n^m and Q_n^r are the respective manufacturing and remanufacturing quantities, and X_n^s and X_n^r are the respective serviceable and return inventory levels at the end of the period. For any given period n , the policy inputs initial serviceable and return inventory levels X_{n-1}^s and $X_{n-1}^r + R_n$, respectively, and yields Q_n^m ($Q_n^m \geq 0$) and Q_n^r ($X_{n-1}^r + R_n \geq Q_n^r$ and $Q_n^r \geq 0$), where it should be clear that we can only have $Q_n^m > 0$ for $n \in \Pi^m$ and $Q_n^r > 0$ for $n \in \Pi^r$. It is important to note that the actual values of Q_n^m and Q_n^r may or may not be known at the outset based on the structure of the decision rules.

The dynamics of the system is governed by the following equations which dictate the flow conservation for serviceable and return inventories

$$X_n^s = X_{n-1}^s + Q_n^m + Q_n^r - D_n$$

$$X_n^r = X_{n-1}^r + R_n - Q_n^r$$

where X_0^s and X_0^r are the serviceable and return inventories at the beginning of the planning horizon, respectively.

The expected total cost of the policy over the planning horizon can be written as

$$K^m |\Pi^m| + K^r |\Pi^r| + \sum_{n=1}^T (\mathbb{E}\{h^s [X_n^s]^+ + p [X_n^s]^- + h^r X_n^r\})$$

or alternatively

$$K^m |\Pi^m| + K^r |\Pi^r| + \sum_{n=1}^T (h^s \mathbb{E}X_n^s + h^r \mathbb{E}X_n^r + (h^s + p) \mathbb{E}[X_n^s]^-). \quad (3)$$

The main challenge here is to compute the expected total cost of a given policy. This requires obtaining the expected values $\mathbb{E}X_n^s$, $\mathbb{E}X_n^r$, and $\mathbb{E}[X_n^s]^-$. These respectively stand for the expected serviceable and return inventories, and backorders in period n . In what follows, we address this challenge for AoN and THR policies.

4.2. AoN

AoN is a static-dynamic policy with fixed manufacturing and remanufacturing periods; where manufacturing quantities

are determined at the outset as constants and remanufacturing quantities are dynamically determined following an all-or-nothing rule, i.e., all return inventory is remanufactured in remanufacturing periods. Thus, in addition to the set of manufacturing and remanufacturing periods Π^m and Π^r , the parameters of AoN involve manufacturing quantities for manufacturing periods, denoted by $q = \{q_n : n \in \Pi^m\}$. Following the policy parameters, we have $Q_n^m = q_n$ if period $n \in \Pi^m$ and $Q_n^m = 0$ otherwise, and, similarly, $Q_n^r = X_{n-1}^r + R_n$ if period $n \in \Pi^r$ and 0 otherwise. Then, the expected manufacturing and remanufacturing quantities can be expressed as $\mathbb{E}Q_n^m = q_n$ and $\mathbb{E}Q_n^r = \mathbb{E}X_{n-1}^r + \mathbb{E}R_n$, respectively.

We now concentrate on serviceable and return inventories in an arbitrary period n . For the serviceable inventory, we rely on the simple observation that the cumulative demand $D_{[1,n]}$ is satisfied by cumulative manufacturing and remanufacturing over the interval $[1, n]$. The former can be derived from the policy parameters as $\sum_{u \in \Pi^m: u \leq n} q_u$. The latter, on the other hand, can be expressed in connection with the last remanufacturing period prior to period n . Let $k = \max\{u \in \Pi^r \cup \{0\} : u \leq n\}$ denote this period where $\{0\}$ suggests there may be no remanufacturing period before period n . It should be obvious that k is dependent on n , yet it is suppressed in the notation for simplicity. The all-or-nothing property then suggests that cumulative remanufacturing over the interval $[1, n]$ can be expressed as $R_{[1,k]}$. Hence, we can write the serviceable inventory level in period n as $X_n^s = \sum_{u \in \Pi^m: u \leq n} q_u - (D_{[1,n]} - R_{[1,k]})$. For the return inventory, we use the observation that there is no remanufacturing period in the interval $[k+1, n]$. This implies that the return inventory in period n equals to the cumulative return over interval $[k+1, n]$. That is $X_n^r = R_{[k+1,n]}$.

The expressions above demonstrate that we can derive the expected values of serviceable and return inventories in an arbitrary period n from the policy parameters as $\mathbb{E}X_n^s = \sum_{u \in \Pi^m: u \leq n} q_u - (\mathbb{E}D_{[1,n]} - \mathbb{E}R_{[1,k]})$ and $\mathbb{E}X_n^r = \mathbb{E}R_{[k+1,n]}$. Further, they enable us to characterize the expected backorders in period n as

$$\mathbb{E}[X_n^s]^- = \mathbb{E} \left[\sum_{u \in \Pi^m: u \leq n} q_u - (D_{[1,n]} - R_{[1,k]}) \right]^-$$

$$= L_{kn} \left(\sum_{u \in \Pi^m: u \leq n} q_u \right)$$

$$= L_{kn} (\mathbb{E}X_n^s + (\mathbb{E}D_{[1,n]} - \mathbb{E}R_{[1,k]})) \quad (4)$$

where $L_{kn}(\cdot)$ is the first-order loss function of the random variable $D_{[1,n]} - R_{[1,k]}$.

We shall make use of the results presented here on the expected values $\mathbb{E}X_n^s$, $\mathbb{E}X_n^r$, and $\mathbb{E}[X_n^s]^-$ in building a certainty equivalent MIP model of AoN.

4.3. THR

THR is a static-dynamic policy with fixed manufacturing and remanufacturing periods; where manufacturing and remanufacturing quantities are dynamically determined following threshold values. That is, manufacturing quantities are set so as to increase the serviceable inventory to a pre-specified manufacturing-up-to level and remanufacturing quantities are set so as to decrease the return inventory to a pre-specified remanufacturing-down-to level. If both manufacturing and remanufacturing take place in the same period, then remanufacturing precedes manufacturing and the same decision rules apply. AoN and THR significantly differ in how manufacturing and remanufacturing quantities are determined. The manufacturing quantities are dynamically determined by means of manufacture-up-to levels in THR, whereas they are fixed quantities in AoN. The remanufacturing quantities, on the other hand, are set by means of the remanufacture-down-to levels in both AoN and THR. The former is a special

case where the down-to level is zero due to the all-or-nothing rule. The latter is more flexible as any threshold level can be used. The parameters of THR include the set of manufacturing and remanufacturing periods Π^m and Π^r as well as manufacture-up-to levels $w^m = \{w_n^m : n \in \Pi^m\}$ and remanufacture-down-to levels $w^r = \{w_n^r : n \in \Pi^r\}$. Following the policy parameters, we have $Q_n^m = w_n^m - Q_n^r - X_{n-1}^s$ if period $n \in \Pi^m$ and $Q_n^m = 0$ otherwise, and, similarly, $Q_n^r = X_{n-1}^s + R_n - w_n^r$ if period $n \in \Pi^r$ and $Q_n^r = 0$ otherwise. We note that production quantities mentioned above can be negative following realized demands and returns. We neglect such cases as they rarely occur if reasonable policy parameters are in place, which should call for sizeable production quantities in presence of fixed production costs.

We now focus on serviceable and return inventories. It is possible to derive expressions of serviceable and return inventories in an arbitrary period n based on particular production periods scheduled for earlier periods. Let $j \in [0, n]$ be the last manufacturing period before n . That is, $j = \max\{u \in \Pi^m \cup \{0\} : u \leq n\}$. Also, let $i \in [0, j]$ and $k \in [0, n]$ be the last remanufacturing periods before j and n , respectively. That is, $i = \max\{u \in \Pi^r \cup \{0\} : u \leq j\}$ and $k = \max\{u \in \Pi^r \cup \{0\} : u \leq n\}$. For any period n ; periods i, j , and k are readily available once the policy parameters are known. These periods are all dependent on n , but we suppress this in the notation for brevity. For the serviceable inventory, we use the that observation that the post-manufacturing serviceable inventory level w_j^m in period j and the cumulative remanufacturing quantity over interval $[j, n]$ are used to satisfy the cumulative demand $D_{[j,n]}$. The former is already a policy parameter. The latter can be expressed in connection with the production periods introduced above. The total remanufacturing quantity over the interval $[j, n]$ is equal to the total remanufacturing quantity over the interval $[i + 1, k]$ as there is no remanufacturing period over the intervals $[i + 1, j]$ and $[k + 1, n]$. The post-production return inventories in periods i and k are w_i^r and w_k^r , respectively. These suggest that the total remanufacturing quantity over the interval $[j, n]$ is $w_i^r - w_k^r + R_{[i+1,k]}$. Then, we can write the serviceable inventory level in period n as $X_n^s = w_j^m + w_i^r - w_k^r - (D_{[j,n]} - R_{[i+1,k]})$. For the return inventory, we use the observation that the only remanufacturing period over the interval $[k, n]$ is k , and post-production return inventory in this period is w_k^r . Therefore, all returns received over the interval $[k + 1, n]$ accumulate up to period n . Then, we can write the return inventory level in period n as $X_n^r = w_k^r + R_{[k+1,n]}$.

The expressions provided above show that the expected values of serviceable and return inventories can be obtained from the policy parameters for an arbitrary period n as $EX_n^s = w_j^m + w_i^r - w_k^r - (ED_{[j,n]} - ER_{[i+1,k]})$ and $EX_n^r = w_k^r + ER_{[k+1,n]}$. They also allow us to write the expected backorders in period n as

$$\begin{aligned} E[X_n^s]^- &= E[w_j^m + w_i^r - w_k^r - (D_{[j,n]} - R_{[i+1,k]})]^- \\ &= L_{ijkn}(w_j^m + w_i^r - w_k^r) \\ &= L_{ijkn}(EX_n^s + (ED_{[j,n]} - ER_{[i+1,k]})) \end{aligned} \tag{5}$$

where $L_{ijkn}(\cdot)$ is the first-order loss function of the random variable $D_{[j,n]} - R_{[i+1,k]}$.

The results presented here on the expected values EX_n^s , EX_n^r , and $E[X_n^s]^-$ will be functional in devising a MIP model of THR.

4.4. Approximation of the expected backorder level

The non-linearity induced by the loss functions in (4) and (5) precludes us from directly embedding expected backorder levels into a MIP model. We overcome this issue by approximating the loss function by a piecewise linear function. The first-order loss function L of a random variable ξ is non-negative, and it is convex and decreasing on x . It approaches $-E\xi$ as $x \rightarrow -\infty$ and 0 as $x \rightarrow \infty$. It is well-known that point-wise maximum of a given set

of linear functions is also convex. Therefore, any loss function $L(\cdot)$ can be well approximated as $L(x) \approx \max_{(a,b) \in W} \{a + bx\}$. Here, W defines a finite set of linear functions by means of their intercept and slope pairs as $W = \{(a_1, b_1), (a_2, b_2), \dots, (a_m, b_m)\}$. Because L is non-negative and decreasing and due to its limiting behavior, it only makes sense to have an intercept such that $a \geq 0$ and a slope such that $0 \geq b \geq -1$.

The aforementioned idea can immediately be employed in the context of AoN and THR policies, as provided the replenishment schedule, we know the loss function of which random variable characterizes the backorder level.

For AoN, the backorder level in period n is characterized by the loss function of the random variable $D_{[1,n]} - R_{[1,k]}$ as derived in (4), and it is obtained by evaluating this function at $EX_n^s + (ED_{[1,n]} - ER_{[1,k]})$. Hence, it can be approximated as

$$\max_{(a,b) \in W_{kn}} \{a + b(EX_n^s + (ED_{[1,n]} - ER_{[1,k]}))\} \tag{6}$$

where W_{kn} is the set of linear functions used for random variable $D_{[1,n]} - R_{[1,k]}$.

For THR, on the other hand, the backorder level in period n is characterized by the loss function of the random variable $D_{[j,n]} - R_{[i+1,k]}$ as derived in (5), and it is obtained by evaluating this function at $EX_n^s + (ED_{[j,n]} - ER_{[i+1,k]})$. Therefore, it can be approximated as

$$\max_{(a,b) \in W_{ijkn}} \{a + b(EX_n^s + (ED_{[j,n]} - ER_{[i+1,k]}))\} \tag{7}$$

where W_{ijkn} denotes the set of linear functions used for random variable $D_{[j,n]} - R_{[i+1,k]}$.

We assume here that an efficient set of linear functions are readily available in (6) and (7). The reader is referred to the literature on the piecewise linear approximation of convex functions for the details of obtaining such efficient sets of functions (see e.g. Frenzen, Sasao, & Butler, 2010; Gavrilovic, 1975; Rossi, Tarim, Prestwich, & Hnich, 2014).

4.5. Models

We now develop certainty equivalent MIP models to compute the parameters of AoN and THR. Because these policies have comparable mathematical backgrounds, we present their MIP models in a unified fashion.

The decision variables used in MIP models are outlined in Table 2. It is important to note that the decision variables do not represent policy parameters explicitly. This is to provide notational consistency in AoN and THR models. They can yet be derived immediately from the decision variables. For both AoN and THR, policy parameters involve manufacturing and remanufacturing periods. These are captured by binary variables and written as $\Pi^m = \{n \in [1, T] : \delta_n^m = 1\}$ and $\Pi^r = \{n \in [1, T] : \delta_n^r = 1\}$. The parameters of AoN also include manufacturing quantities q_n for manufacturing periods. These are expressed by $q^m = \{\tilde{Q}_n^m : \delta_n^m = 1\}$. The parameters of THR further include manufacturing-up-to levels w_n^m and remanufacturing down-to levels w_n^r for manufacturing and remanufacturing periods, respectively. These are given by $w^m = \{\tilde{X}_{n-1}^m + \tilde{Q}_n^m + \tilde{Q}_n^r : \delta_n^r = 1\}$ and $w^r = \{\tilde{X}_n^r : \delta_n^r = 1\}$.

The objective function is the same for AoN and THR and minimizes the sum of fixed production costs and expected inventory costs. It is written as

$$\min \sum_{n=1}^T (K^m \delta_n^m + K^r \delta_n^r + h^s \bar{X}_n^s + h^r \bar{X}_n^r + (h^s + p) H_n). \tag{8}$$

Notice that (8) is the same as the expression (3) except the expected backorder level in period n is replaced by H_n . We will make sure that this indeed captures the expected backorder level later on.

Table 2
The domains and definitions of decision variables used in MIP models.

Variable	Domain	Definition
\bar{Q}_n^m	\mathbb{R}^+	expected manufacturing quantity in period n
\bar{Q}_n^r	\mathbb{R}^+	expected remanufacturing quantity in period n
\bar{X}_n^s	\mathbb{R}	expected serviceable inventory level at the end of period n
\bar{X}_n^r	\mathbb{R}^+	expected return inventory level at the end of period n
H_n	\mathbb{R}^+	approximate expected backorder level in period n
δ_n^m	$\{0, 1\}$	binary variable that takes value of 1 if period n is a manufacturing period and 0 otherwise
δ_n^r	$\{0, 1\}$	binary variable that takes value of 1 if period n is a remanufacturing period 0 otherwise
Δ_{kn}	$\{0, 1\}$	binary variable indicating that the last remanufacturing period prior to period n is period k (used only for AoN)
Δ_{ijkn}	$\{0, 1\}$	binary variable indicating that the last manufacturing period prior to period n is period j , and the last remanufacturing periods prior to periods j and n are periods i and k , respectively (used only for THR)

We first present constraints that apply to both AoN and THR. The inventory conservation constraints which regulate expected values of production quantities and inventory levels in subsequent periods are expressed as

$$\bar{X}_n^s = \bar{X}_{n-1}^s + \bar{Q}_n^m + \bar{Q}_n^r - ED_n \quad \forall n \in [1, T] \tag{9}$$

$$\bar{X}_n^r = \bar{X}_{n-1}^r + ER_n - \bar{Q}_n^r \quad \forall n \in [1, T]. \tag{10}$$

The production schedule is governed by indicator variables. Therefore expected production quantities should be in line with these variables. This can be expressed for manufacturing and remanufacturing periods as

$$\bar{Q}_n^m \leq M\delta_n^m \quad \forall n \in [1, T] \tag{11}$$

$$\bar{Q}_n^r \leq M\delta_n^r \quad \forall n \in [1, T] \tag{12}$$

where M stands for a sufficiently large constant.

Next, we turn our attention to constraints that are policy-specific. AoN is characterized by an all-or-nothing rule according to which all the return inventory is consumed by remanufacturing in remanufacturing periods. This can be embedded into the formulation as

$$\bar{X}_n^r \leq M(1 - \delta_n^r) \quad \forall n \in [1, T]. \tag{13}$$

We now focus on the expected backorder levels. For AoN, the expected backorder level in period n can be determined if it is known that the last remanufacturing period prior to period n is period k . The following constraints make sure that the binary variable Δ_{kn} takes the value 1 if this condition holds

$$\Delta_{kn} \geq \delta_k^r - \sum_{t \in [k+1, n]} \delta_t^r \quad \forall k \in [0, n], n \in [1, T] \tag{14}$$

$$\sum_{k \in [0, n]} \Delta_{kn} = 1 \quad \forall n \in [1, T] \tag{15}$$

where the dummy remanufacturing period 0 is used to ensure that there is always a most recent remanufacturing period prior to any period.

For THR, the expected backorder level can be determined if it is known that the last manufacturing period prior to period n is period j , and the last remanufacturing periods prior to periods j and n are periods i and k , respectively. The following constraints

make sure that the binary variable Δ_{ijkn} takes the value 1 if these conditions indeed hold

$$\begin{aligned} \Delta_{ijkn} \geq & \delta_j^m - \sum_{t \in [j+1, n]} \delta_t^m + \delta_i^r - \sum_{t \in [i+1, j]} \delta_t^r \\ & + \delta_k^r - \sum_{t \in [k+1, n]} \delta_t^r - 2 \\ & \forall i \in [0, j], j \in [0, n], k \in i \cup [j + 1, n], n \in [1, T] \end{aligned} \tag{16}$$

$$\sum_{i \in [0, j]} \sum_{j \in [0, n]} \sum_{k \in i \cup [j+1, n]} \Delta_{ijkn} = 1 \quad \forall n \in [1, T] \tag{17}$$

where dummy manufacturing and remanufacturing periods 0 are used to ensure that there is always a most recent manufacturing and remanufacturing period prior to any period.

Finally, we write the constraints which make sure that H_n captures the expected backorder level in period n by means of the piecewise linearization of the associated loss function. For AoN, the constraint provided below guarantees that H_n is at least as large as the approximation in (6) as

$$\begin{aligned} H_n \geq & a \Delta_{kn} + b(\bar{X}_n^s + (ED_{[1, n]} - ER_{[1, k]}))\Delta_{kn} \\ & \forall k \in [0, n], n \in [1, T], (a, b) \in W_{kn}. \end{aligned} \tag{18}$$

For THR, we can write a similar constraint that bounds H_n from below, but this time in accordance with (7) as

$$\begin{aligned} H_n \geq & a \Delta_{ijkn} + b(\bar{X}_n^s + (ED_{[j, n]} - ER_{[i+1, k]}))\Delta_{ijkn} \\ & \forall i \in [0, j], j \in [0, n], k \in i \cup [j + 1, n], n \in [1, T], (a, b) \in W_{ijkn}. \end{aligned} \tag{19}$$

It is easy to observe that (18) and (19) condition on the respective indicator variables Δ_{kn} and Δ_{ijkn} , and bound H_n from below by linear segments of the associated approximation. These constraints are equivalent to (6) and (7) if the indicator variables are active. Otherwise, they yield the inequality $H_n \geq b\bar{X}_n^s$. Let us recall that for any sensible piecewise linear approximation, b should lie in between 0 and -1. Hence, if $\bar{X}_n^s \geq 0$ then the inequality is dominated by $H_n \geq 0$, and if $\bar{X}_n^s \leq 0$ then it is dominated by $H_n \geq -\bar{X}_n^s$. These cases hold by definition, as we have $E\xi^- \geq 0$ and $E\xi^+ \geq -E\xi^-$ for any random variable ξ .

These constraints finalize the MIP models. For convenience, below we provide complete MIP models for AoN and THR:

(AoN) min (8)
 subject to (9), (10), (11), (12), (13), (14), (15), (18)
 $\bar{X}_0^s = 0, \bar{X}_0^r = 0, \delta_0^m = 1, \delta_0^r = 1$
 $\bar{X}_n^s \in \mathbb{R} \quad \forall n \in [1, T]; \bar{X}_n^r, \bar{Q}_n^m, \bar{Q}_n^r, H_n \in \mathbb{R}_+ \quad \forall n \in [1, T]$
 $\delta_n^m, \delta_n^r \in \{0, 1\} \quad \forall n \in [1, T]$
 $\Delta_{kn} \in \{0, 1\} \quad \forall k \in [0, n], n \in [1, T]$

(THR) min (8)
 subject to (9), (10), (11), (12), (16), (17), (19)
 $\bar{X}_0^s = 0, \bar{X}_0^r = 0, \delta_0^m = 1, \delta_0^r = 1$
 $\bar{X}_n^s \in \mathbb{R} \quad \forall n \in [1, T]; \bar{X}_n^r, \bar{Q}_n^m, \bar{Q}_n^r, H_n \in \mathbb{R}_+ \quad \forall n \in [1, T]$
 $\delta_n^m, \delta_n^r \in \{0, 1\} \quad \forall n \in [1, T]$
 $\Delta_{ijkn} \in \{0, 1\} \quad \forall i \in [0, j], j \in [0, n],$
 $k \in i \cup [j + 1, n], n \in [1, T]$

5. Numerical study

In this section, we numerically assess the cost performance of the dynamic heuristic—referred to as SM—and the static-dynamic

Table 3
Results on Set-S.

ρ	SM			THR			AoN			STA		
	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
0.1	3.74	2.51	5.42	2.77	0.00	5.04	2.86	0.00	5.76	7.23	6.09	8.24
0.2	3.42	2.41	4.32	3.88	2.04	5.15	5.04	2.04	7.48	11.75	10.71	13.01
0.3	3.55	3.11	4.77	7.43	6.90	8.25	9.28	6.90	11.34	17.18	14.92	20.58
0.4	3.78	2.97	5.23	10.64	8.95	13.93	13.24	12.26	14.43	20.90	18.40	25.86
0.5	3.37	2.24	4.81	13.78	12.27	16.48	18.08	17.27	18.78	24.66	20.95	29.93
0.6	4.14	2.99	5.60	17.48	15.72	21.38	24.06	22.78	24.83	28.79	25.35	34.65
0.7	4.46	3.01	5.54	20.40	19.07	22.73	28.48	26.54	29.64	32.06	28.48	38.71
0.8	4.89	3.30	6.24	24.87	22.21	26.88	34.11	32.13	36.42	36.64	32.89	44.01
0.9	5.12	3.05	6.75	26.99	24.50	29.62	38.06	35.67	41.94	40.66	36.34	48.21
1.0	5.84	3.36	7.19	28.90	26.09	32.49	41.45	37.72	46.65	43.29	38.92	51.94

Table 4
Results on Set-L for STAT.

Parameter	Value	SM			THR			AoN			STA		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
T	6	2.79	1.02	5.35	0.07	0.00	0.16	0.02	0.00	0.07	0.06	0.00	0.21
	9	2.97	1.64	4.83	0.02	0.00	0.05	0.07	0.00	0.19	2.26	2.03	2.44
	12	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	15	0.48	0.16	0.82	0.00	0.00	0.00	2.68	2.58	2.79	7.32	6.92	7.52
	18	0.11	0.00	0.55	0.34	0.00	0.62	3.69	3.15	4.26	7.57	6.85	8.22
φ	0.1	1.77	0.05	4.25	0.00	0.00	0.00	3.41	2.99	3.62	3.37	2.96	3.54
	0.3	2.93	2.55	3.28	0.00	0.00	0.00	2.40	2.28	2.50	3.98	3.86	4.09
	0.5	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	0.7	1.34	0.66	2.14	0.01	0.00	0.07	0.06	0.00	0.11	7.32	7.17	7.58
	0.9	0.03	0.00	0.11	0.28	0.00	0.68	0.28	0.02	0.68	6.48	6.05	7.11
ρ	0.10	7.70	6.62	8.05	0.00	0.00	0.00	1.15	1.09	1.17	2.65	2.52	2.78
	0.15	5.38	5.04	5.61	0.00	0.00	0.00	1.72	1.65	1.78	3.31	3.12	3.42
	0.20	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	0.25	0.30	0.17	0.41	0.00	0.00	0.00	2.82	2.67	3.07	5.26	5.09	5.35
	0.30	0.00	0.00	0.00	1.87	1.56	2.29	5.61	5.21	6.14	8.41	7.92	8.99
K^m	200	2.90	1.58	3.83	0.00	0.00	0.00	9.02	8.35	9.66	11.24	10.56	11.98
	500	1.63	0.95	2.56	0.00	0.00	0.00	4.58	4.44	4.82	6.58	6.32	6.78
	1000	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	1250	1.28	0.00	3.24	0.21	0.00	0.42	2.33	2.05	2.50	4.09	3.99	4.21
	2000	4.04	3.43	4.35	0.02	0.00	0.09	0.16	0.00	0.32	3.55	2.75	4.10
K^r	200	4.58	4.26	5.00	0.00	0.00	0.00	4.76	4.56	4.86	11.14	10.87	11.55
	500	2.69	2.38	3.06	0.00	0.00	0.00	3.06	2.91	3.17	7.44	7.31	7.73
	1000	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	1250	2.35	2.12	2.73	0.00	0.00	0.00	2.29	2.14	2.44	4.08	3.96	4.17
	2000	0.27	0.14	0.49	0.00	0.00	0.00	2.69	2.47	2.79	2.68	2.42	2.84
h^r	0.1	2.08	0.00	5.76	0.22	0.00	0.53	2.57	2.35	2.83	2.20	1.75	2.61
	0.3	0.87	0.78	1.00	0.00	0.00	0.00	2.19	2.11	2.32	2.61	2.52	2.76
	0.5	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	0.7	2.67	2.30	3.16	0.00	0.00	0.00	2.24	2.17	2.39	5.64	5.38	5.86
	0.9	3.25	2.83	3.46	0.00	0.00	0.00	2.41	2.27	2.52	7.83	6.79	8.27
p	2	4.45	3.24	6.28	0.00	0.00	0.00	0.43	0.35	0.53	5.26	5.07	5.42
	5	2.64	2.35	3.20	0.00	0.00	0.00	1.67	1.58	1.79	3.95	3.83	4.05
	10	2.74	2.60	2.92	0.00	0.00	0.00	2.32	2.21	2.50	4.33	4.20	4.56
	15	2.31	2.04	2.56	0.00	0.00	0.00	2.61	2.42	2.88	4.41	4.18	4.58
	20	2.13	2.00	2.38	0.00	0.00	0.00	3.05	2.84	3.30	4.59	4.44	4.73

heuristics AoN and THR, while using Hilger et al.'s (2016) static heuristic—referred to as STA—as a benchmark. STA adopts a static-uncertainty strategy where production schedule and production quantities are determined at the outset. It is therefore a logical benchmark for dynamic and static-dynamic heuristics.

We organize our numerical study in two parts in each of which we employ a different set of problem instances, namely Set-S and Set-L. In Set-S, we assess the performance of heuristic approaches against the optimal policy—henceforth referred to as OPT. This test set involves a modest number of instances due to the extensive computational time required to obtain the optimal policy. But it provides critical insights into the optimality gaps of heuristics. In Set-L, we compare heuristics against each other. This test set includes a large number of instances which enable us to explore how the heuristic performance is affected by a variety of factors,

involving, progression of demands and returns over time, the length of the planning horizon, the return-demand coverage, demand and return uncertainty, and cost parameters. We shall provide further details of these sets of instances in the following sub-sections.

To evaluate the performance of heuristics, we proceed as follows. We compute the expected cost of OPT by solving the stochastic dynamic program provided in Section 2. SM is an online heuristic whose expected cost cannot be obtained by solving a monolithic model. Thus, we compute its cost by means of simulation. In each simulation run, production decisions over periods are made following the lines provided in Section 3. We compute the parameters of AoN and THR by solving the MIP models provided in Section 4, and STA by the MIP model of Hilger et al. (2016). The models of AoN and THR do not provide exact expected cost

Table 5
Results on Set-L for INC.

Parameter	Value	SM			THR			AoN			STA		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
T	6	0.77	0.52	1.56	0.04	0.00	0.09	0.02	0.00	0.06	1.21	0.01	2.38
	9	3.93	2.61	6.01	0.00	0.00	0.00	0.11	0.00	0.28	4.42	3.97	4.68
	12	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	15	3.18	2.59	3.97	0.00	0.00	0.00	1.68	1.60	1.76	5.18	4.59	5.50
	18	3.19	2.61	3.62	0.00	0.00	0.00	2.49	2.29	2.78	5.69	5.55	5.94
φ	0.1	0.31	0.00	0.75	0.02	0.00	0.10	2.26	2.09	2.38	2.27	2.16	2.40
	0.3	2.13	1.91	2.42	0.00	0.00	0.00	0.95	0.76	1.11	3.22	2.84	3.58
	0.5	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	0.7	1.90	1.30	2.99	0.00	0.00	0.00	1.34	0.78	1.83	3.89	3.15	4.67
	0.9	1.80	1.02	2.87	0.10	0.00	0.22	0.02	0.00	0.07	7.18	6.80	7.62
ρ	0.10	5.29	4.02	6.56	0.00	0.00	0.00	0.84	0.80	0.87	1.37	0.82	1.82
	0.15	3.29	2.50	4.01	0.00	0.00	0.00	0.63	0.34	0.94	1.35	0.98	1.67
	0.20	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	0.25	0.22	0.00	0.59	0.05	0.00	0.16	1.21	1.02	1.43	2.81	2.50	3.18
	0.30	0.00	0.00	0.00	1.21	0.95	1.57	3.00	2.89	3.17	5.04	4.66	5.68
K^m	200	0.05	0.00	0.19	0.09	0.00	0.18	7.27	7.02	7.44	7.47	7.11	7.87
	500	3.81	3.68	3.93	0.00	0.00	0.00	3.03	2.69	3.43	6.49	6.11	6.89
	1000	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	1250	1.86	0.80	2.70	0.00	0.00	0.00	0.78	0.74	0.91	1.83	1.35	2.33
	2000	5.93	3.27	9.76	0.02	0.00	0.12	0.06	0.00	0.12	4.95	3.79	5.91
K^r	200	7.11	5.12	9.30	0.00	0.00	0.00	2.76	2.54	3.07	8.23	7.61	8.96
	500	2.41	0.97	3.48	0.00	0.00	0.00	2.38	2.31	2.47	5.43	5.29	5.49
	1000	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	1250	1.38	1.17	1.66	0.00	0.00	0.00	0.80	0.70	0.92	1.86	1.27	2.27
	2000	2.35	0.10	3.92	0.00	0.00	0.00	1.80	1.69	1.88	1.81	1.70	1.93
h^r	0.1	1.91	0.39	3.21	0.00	0.00	0.00	2.21	1.89	2.47	1.16	1.03	1.33
	0.3	1.99	1.46	2.56	0.00	0.00	0.00	1.24	0.94	1.59	1.23	1.19	1.27
	0.5	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	0.7	1.21	0.72	1.48	0.00	0.00	0.00	1.33	1.06	1.63	3.67	2.90	4.38
	0.9	1.58	1.21	1.91	0.00	0.00	0.00	1.38	1.19	1.69	6.04	5.37	6.36
p	2	6.07	5.32	6.63	0.00	0.00	0.00	0.20	0.09	0.38	4.54	4.45	4.65
	5	2.06	1.60	2.62	0.00	0.00	0.00	1.03	0.84	1.14	3.49	3.31	3.67
	10	1.67	1.32	1.98	0.00	0.00	0.00	0.85	0.61	0.96	1.96	1.38	2.42
	15	1.27	0.46	2.38	0.00	0.00	0.00	1.28	1.12	1.48	1.79	1.60	1.98
	20	1.38	0.89	2.30	0.00	0.00	0.00	1.54	1.37	1.73	1.83	1.72	1.98

expressions for these heuristics. This is also the case for STA. Therefore, we also assess the expected costs of these heuristics via simulation. We conduct 10^4 simulation runs (with common random numbers) for all heuristics. This turned out to be more than sufficient as 95% confidence interval was less than ± 0.1 percent of the average cost for all heuristics and each test instance. We perform all numerical experiments on an Intel Core i7-3770 CPU with 16 gigabyte RAM and use Gurobi v7.01 as a MIP solver. We do not impose a time limit and solve all MIP models to optimality.

In what follows, we present our computational study on Set-S and Set-L. Then, we discuss the managerial implications of our findings.

5.1. Set-S

In this part of the numerical study, we assess the performance of heuristic approaches against OPT. Because AoN, THR, and STA differ as to how they approach uncertainty, we initially focus on better understanding how the extent of uncertainty affect the heuristic performance. To that end, Set-S is designed as follows. We use the same cost parameters in all numerical instances; $K^m = 1000$, $K^r = 1000$, $h^s = 10$, $h^r = 5$, and $p = 100$. We consider problem instances with 10 periods. The expected demands ED_n over these periods are drawn from a discrete uniform distribution on $[5,15]$ and expected returns are set as $ER_n = \lceil \frac{1}{2}ED_n \rceil$. We randomly generate five series of expected demands and returns. To capture the effects of uncertainty in a unified fashion, we assume that the coefficient of variation of demands and returns is (almost)

the same over the planning horizon. We denote the coefficient of variation by ρ , and consider ten different values $\rho \in \{0.1, 0.2, \dots, 1.0\}$. We assume that demands and returns follow a three-point distribution, i.e., a discrete distribution concentrated at three points $v = (v_1, v_2, v_3)$ with probabilities $\vartheta = (\vartheta_1, \vartheta_2, \vartheta_3)$. To make sure that the support is integral—required to compute the optimal policy—and the mean and coefficient of variation are in line with the specified values, we characterize these distributions as follows. Let μ and ρ be the mean and the coefficient of variation, respectively. Then, we establish the distribution as $v = (0, \mu, \lceil \mu(2\rho^2 + 1) \rceil)$ and $\vartheta = (\frac{1}{2} - \frac{1}{2} \frac{\mu}{\lceil \mu(2\rho^2 + 1) \rceil}, \frac{1}{2}, \frac{1}{2} \frac{\mu}{\lceil \mu(2\rho^2 + 1) \rceil})$. This scheme makes sure that the distribution exactly matches the specified mean, while it may slightly overshoot the coefficient of variation.

Set-S includes 50 test instances in total, i.e., five random instances for each of ten different values of coefficient of variation. For each of these instances, we compute the expected costs of OPT, SM, AoN, THR, and STA. The lower bound approximation of the loss functions employed in the MIP models of AoN, THR, and STA are established numerically with a maximum error of 1 cost unit. We note that the MIP models of AoN, THR and STA are solved to optimality in the order of seconds, whereas the time required to solve the stochastic dynamic program of OPT is not less than several hours for each test instance.

For each heuristic, we measure the (percentage) optimality gap as $(\mathcal{C}(H)/\mathcal{C}(OPT) - 1) \times 100$ where $\mathcal{C}(H)$ and $\mathcal{C}(OPT)$ are the heuristic cost and the optimal cost, respectively.

Table 3 presents our results on Set-S. It reports the average, minimum, and maximum optimality gaps of all heuristics for

Table 6
Results on Set-L for DEC.

Parameter	Value	SM			THR			AoN			STA		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
<i>T</i>	6	3.65	2.32	5.55	0.03	0.00	0.06	0.02	0.00	0.04	0.26	0.06	0.54
	9	2.88	2.25	3.72	0.07	0.00	0.20	0.01	0.00	0.04	5.25	5.00	5.55
	12	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	15	1.95	1.52	2.37	0.00	0.00	0.00	4.78	4.63	4.99	9.55	9.40	9.78
	18	0.00	0.00	0.00	1.38	1.24	1.47	7.20	6.97	7.31	13.11	12.70	13.34
φ	0.1	1.70	1.62	1.83	0.00	0.00	0.00	4.29	3.36	5.03	4.24	3.25	5.03
	0.3	4.03	3.62	4.38	0.00	0.00	0.00	3.39	3.20	3.64	5.74	5.47	5.96
	0.5	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	0.7	0.00	0.00	0.00	2.74	2.14	3.18	2.73	2.24	3.11	9.43	9.05	9.79
	0.9	1.44	0.00	3.64	1.98	0.19	3.48	1.90	0.00	3.28	6.91	5.07	8.25
ρ	0.10	0.52	0.41	0.63	0.00	0.00	0.00	1.49	1.42	1.59	3.34	3.21	3.58
	0.15	0.00	0.00	0.00	0.09	0.00	0.20	2.70	2.62	2.82	4.55	4.33	4.78
	0.20	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	0.25	0.00	0.00	0.00	3.22	2.69	3.48	8.14	7.65	8.61	11.06	10.71	11.37
	0.30	0.00	0.00	0.00	4.93	4.45	5.41	11.04	10.54	11.47	14.72	13.92	15.39
K^m	200	7.45	6.64	8.25	0.00	0.00	0.00	10.77	10.29	11.21	14.52	14.17	14.92
	500	0.00	0.00	0.00	0.13	0.01	0.32	6.18	5.98	6.49	8.54	8.47	8.62
	1000	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	1250	0.00	0.00	0.00	1.09	0.59	2.60	2.52	2.11	3.56	6.28	5.74	7.06
	2000	0.00	0.00	0.00	2.49	2.26	2.72	2.59	2.44	2.71	6.33	5.63	6.90
K^r	200	0.00	0.00	0.00	0.73	0.29	1.47	6.31	5.54	6.79	13.25	12.95	13.94
	500	0.02	0.00	0.08	0.07	0.00	0.21	3.77	2.79	4.63	9.69	9.59	9.96
	1000	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	1250	0.74	0.00	1.88	0.14	0.00	0.36	3.59	3.40	3.91	5.72	5.45	5.99
	2000	2.12	1.59	2.68	0.00	0.00	0.00	3.20	3.12	3.35	4.41	3.10	5.21
h^r	0.1	2.06	1.64	2.32	0.00	0.00	0.00	3.49	3.25	4.04	2.90	2.71	3.21
	0.3	0.00	0.00	0.00	0.33	0.06	0.60	4.16	3.93	4.47	4.72	4.33	5.05
	0.5	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	0.7	0.00	0.00	0.00	1.26	0.89	1.66	4.72	4.42	4.96	8.92	8.68	9.11
	0.9	1.94	0.00	3.31	0.00	0.00	0.02	3.45	3.34	3.67	10.70	10.34	11.13
<i>p</i>	2	4.80	4.34	5.82	0.00	0.00	0.00	0.44	0.36	0.56	5.70	5.60	5.76
	5	0.25	0.00	0.66	0.02	0.00	0.10	2.62	2.51	2.69	5.54	5.43	5.64
	10	0.00	0.00	0.00	1.49	1.43	1.59	5.23	5.15	5.29	7.49	7.36	7.70
	15	0.00	0.00	0.00	1.81	1.63	2.00	6.15	6.04	6.39	8.28	7.97	8.59
	20	0.00	0.00	0.00	2.51	2.38	2.65	7.04	6.88	7.15	9.13	8.67	9.36

instances characterized by the same coefficient of variation. The results clearly demonstrate that the extent of demand and return uncertainty is a major driver of the heuristic performance. We observe that the cost performance of all heuristics diminish with increasing demand and return uncertainty. AoN, THR, and STA are yet more sensitive to the level of uncertainty as compared to SM. We see that SM clearly overperforms all other heuristics when the coefficient of variation is higher than 0.3. This is not surprising as fixed replenishment schedules and replenishment quantities leave less room for dynamic adjustments in response to realized demands and returns. THR and AoN are competitive when the coefficient of variation is below 0.3. They also significantly outperform STA.

Based on these results, it is fair to conclude that employing static-dynamic heuristics THR and AoN, and the static heuristic STA could be very expensive when the extent of uncertainty is high. In the following part of the numerical study, we shall thus concentrate on instances characterized by moderate levels of demand and return uncertainty and provide detailed insights into the performance of the proposed heuristics.

5.2. Set-L

In the second part of the numerical study, we conduct a larger numerical experiment and explore how heuristic performance is affected by a variety of factors. These involve the following: (1) progression of demands and returns over time (2) length of the planning horizon, (3) return-demand coverage, (4) demand and

return uncertainty, (5) manufacturing and remanufacturing setup costs, and (6) serviceable and return holding costs.

Set-L is designed as follows. We use a variety of cost parameters involving five manufacturing and remanufacturing setup costs $K^m, K^r = \{200, 500, 1000, 1250, 2000\}$, five return inventory holding costs $h^r = \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and five backorder costs $p = \{2, 5, 10, 15, 20\}$. We employ the same serviceable holding cost in all experiments $h^s = 1$. We use five different planning horizon lengths $T = \{6, 9, 12, 15, 18\}$. We consider four different demand and return progressions over time, namely, stationary, increasing, decreasing, and random. We abbreviate these as STAT, INC, DEC, and RND, respectively. The demand and return progressions are reflected on expected demands and returns over the planning horizon. For RND, the expected demand for each period is randomly drawn from a discrete uniform distribution on interval $[0, 200]$. For STAT, INC, and DEC, the expected demand in any period n is established following the expression $ED_n = 100 - \lambda(\frac{1}{2}(T - 1) - (n - 1)) + \epsilon$ where λ is the trend parameter and ϵ is a random noise. We set λ to 0 for STAT, 6 for INC, and -6 for DEC. We draw ϵ from a discrete uniform distribution on interval $[-5, 5]$. The expected return is set as $ER_n = \lceil \varphi ED_n \rceil$ where φ is the parameter reflecting on return-demand coverage. We use five expected return to demand ratios $\varphi = \{0.1, 0.3, 0.5, 0.7, 0.9\}$. For any combination of planning horizon length, demand and return progression, and return-demand coverage, we randomly generate five series of expected demands. We assume that period demands and returns are normally distributed with a fixed coefficient of variation over the planning horizon. We consider five different coefficient of variation values $\rho = \{0.1, 0.15, 0.2, 0.25, 0.3\}$.

Table 7
Results on Set-L for RND.

Parameter	Value	SM			THR			AoN			STA		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
T	6	1.78	0.00	3.52	0.37	0.00	1.64	0.45	0.00	1.55	1.35	0.06	2.35
	9	1.14	0.00	3.75	0.56	0.00	1.76	1.37	0.00	2.03	5.24	2.79	7.98
	12	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	15	0.40	0.00	0.92	0.54	0.00	2.71	3.33	1.56	5.65	7.07	5.01	8.86
	18	2.13	0.00	5.23	0.48	0.00	1.74	5.12	3.42	7.71	9.98	7.89	11.83
φ	0.1	1.55	0.00	5.16	0.27	0.00	1.09	4.20	2.13	5.32	4.18	2.08	5.27
	0.3	1.06	0.00	2.72	0.45	0.00	1.13	2.54	1.65	3.24	5.98	4.61	6.94
	0.5	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	0.7	0.00	0.00	0.00	2.17	0.93	3.83	3.07	0.89	6.54	10.28	6.92	14.32
	0.9	0.21	0.00	1.05	1.54	0.00	3.93	1.57	0.53	3.80	6.07	2.96	13.93
ρ	0.10	2.51	0.37	4.84	0.00	0.00	0.00	1.45	0.94	2.18	2.82	1.02	3.77
	0.15	1.56	0.00	3.76	0.11	0.00	0.57	2.45	1.36	3.27	3.59	2.14	4.07
	0.20	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	0.25	0.11	0.00	0.55	1.67	0.00	3.06	5.41	2.14	8.33	7.41	6.53	9.04
	0.30	0.00	0.00	0.00	3.39	1.78	4.88	7.60	4.27	10.76	10.42	9.17	12.48
K^m	200	4.42	0.00	12.25	0.33	0.00	1.66	11.76	9.75	13.64	14.51	9.86	18.41
	500	2.03	0.00	3.51	0.02	0.00	0.11	5.85	2.74	8.71	7.57	5.02	9.74
	1000	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	1250	0.70	0.00	2.25	0.66	0.00	1.67	2.87	1.60	3.95	4.53	3.82	5.35
	2000	3.42	0.00	6.61	0.66	0.00	2.23	0.74	0.00	2.49	2.92	1.42	4.57
K^r	200	1.30	0.00	2.46	0.49	0.00	2.44	5.01	2.99	9.46	11.82	9.55	15.33
	500	0.00	0.00	0.00	1.77	0.05	3.44	5.24	2.79	7.89	10.02	6.41	14.29
	1000	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	1250	2.36	0.00	5.19	0.27	0.00	1.33	3.18	1.61	4.27	4.50	3.69	5.37
	2000	2.37	0.00	6.42	0.05	0.00	0.25	2.92	1.69	4.00	3.11	1.74	4.59
h^r	0.1	1.01	0.00	4.74	0.72	0.00	1.73	4.35	1.92	6.55	3.33	2.11	4.89
	0.3	0.76	0.00	3.80	0.71	0.00	1.54	3.98	2.04	5.14	4.16	3.33	5.05
	0.5	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	0.7	0.65	0.00	1.83	0.58	0.00	1.48	3.31	1.69	4.29	6.39	5.41	7.20
	0.9	1.53	0.00	3.47	0.09	0.00	0.45	2.81	1.57	4.69	8.37	6.50	10.48
p	2	8.54	2.73	17.93	0.00	0.00	0.00	0.87	0.41	1.70	5.09	3.57	6.41
	5	2.73	0.00	5.82	0.20	0.00	0.99	2.10	1.04	3.32	4.30	3.25	5.14
	10	0.74	0.00	2.56	0.66	0.00	1.64	3.75	1.85	5.26	5.04	4.16	5.50
	15	0.32	0.00	1.12	1.09	0.00	2.21	4.56	2.17	6.46	5.72	5.05	6.79
	20	0.03	0.00	0.15	1.49	0.00	2.64	5.25	2.27	7.66	6.31	5.42	8.01

For each of the four demand and return progression, the instance characterized by the following parameters is considered as the base instance; $T = 12$, $\varphi = 0.5$, $\rho = 0.2$, $K^m = 1000$, $K^r = 1000$, $h^s = 1$, $h^r = 0.5$, and $p = 10$. Then, variations of this instance are populated by changing the value of one parameter at a time. This leads to 580 problem instances in total, i.e. for each demand and return progression, we have five random instances for the base case and its 28 different variations. For each of these test instances, we compute the expected costs of SM, AoN, THR, and STA. We use Rossi et al.'s (2014) 11-piece lower bound approximation of the standard normal distribution loss function for piecewise linearization of the loss functions in all MIP models. The computational times of the MIP models are mainly associated with the length of the planning horizon, as it is the parameter defining model size (i.e., the number of variables and constraints). Therefore, the most challenging instances with respect to computational time are 18-period instances. The MIP models of STA and AoN are solved to optimality within a few seconds for these instances, while that of THR are solved within a few minutes.

Because it is not viable to compute the optimal policy for a large number of instances, in Set-L we opt for assessing heuristics by comparing them against each other. To that end, we measure the performance of a heuristic by means of its (percentage) cost gap from the heuristic with the minimum expected cost as $(\mathcal{C}(H)/\mathcal{C}(H^*) - 1) \times 100$ where $\mathcal{C}(H)$ and $\mathcal{C}(H^*)$ are the heuristic cost and the cost of the best heuristic, respectively.

Tables 4–7 summarize our results on Set-L. The results presented in each of these tables reflect on a particular demand and

return progression and report the average, minimum, and maximum cost gaps of all heuristics for instances characterized by the same pivot parameter.

The results on Set-L provide a large variety of insights, yet they do not demonstrate a consistent trend on the heuristic with respect to most of the problem parameters. The most consistent trend is on the level of uncertainty, i.e. coefficient of variation. That is, SM performs better while THR, AoN, and STA perform worse with increasing levels of uncertainty. SM outperforms all other heuristics for instances when the coefficient of variation reaches up to 0.3, while it is outperformed by other heuristics for instances with lower coefficient of variation values. THR is the most competitive heuristic for instances with a coefficient of variation below 0.3. SM and THR are thus in complete contrast with each other regarding the trade-off between the cost performance and the extent of uncertainty. This is in line with our earlier results in Set-S. There are several other consistent trends. AoN and STA, perform much worse for instances with longer planning horizon. Besides, AoN performs better for instances with higher manufacturing setup costs, while STA performs better for instances with higher remanufacturing setup costs. There are also apparent tendencies. For instance, SM tends to perform better when backorder cost is higher, whereas THR, AoN, and STA performs better when it is lower. It is also possible to see that THR tends to perform relatively better when return-demand coverage is low. As for the demand and return progressions, we observe that THR, AoN, and STA perform relatively better for instances with stationary and increasing demand patterns. The performance of SM is quite stable over different progressions.

In summary, SM and THR arise as the most competitive heuristics, while AoN and STA perform relatively good when demand and return uncertainty is low and the planning horizon is short. The cost performance of STA and AoN are mostly in line with each other, but AoN consistently outperforms STA in all problem instances.

5.3. Discussion

In what follows, we provide a discussion on the managerial implications of our numerical findings. Because we considered heuristics that follow different strategies in managing uncertainty, we organize our discussion on the basis of uncertainty strategies.

SM follows a dynamic uncertainty strategy. This provides SM a significant advantage over other heuristics as it can take immediate recourse actions in response to realized demands. This is apparent in our numerical results especially in those instances where demand and return uncertainty is high. That SM is not competitive for the rest of the instances is due to its myopic nature. SM's decision rules take only a few upcoming periods' demand and return information into account and neglects that of future periods. This has a more severe effect on its cost performance when the extent of uncertainty is low. All in all, SM appears to be a good heuristic for highly uncertain environments where future demand and return information is fairly limited. It is also very appealing due to its efficiency with respect to computational time.

AoN and THR both follow a static-dynamic uncertainty strategy. Because they fix replenishment periods in advance, they lack flexibility in timing of replenishment orders. But they offer some extent of flexibility in replenishment quantities. They are far sighted in nature. That is, they take all future demand and return information into account when setting policy parameters. We observe that this gives AoN and THR a competitive advantage especially when demand and return uncertainty is low. THR is very competitive when demands and returns have a moderate level of uncertainty. It is important to recall that AoN is more conservative as compared to THR because it fixes manufacturing quantities in advance and remanufacture all return inventories in remanufacturing periods. This takes a heavy toll on AoN's cost performance, especially when demands and returns are highly uncertain. We can conclude that AoN and THR are viable heuristics for environments where demand and return uncertainty is limited and a rigid replenishment schedule is of importance. THR appears to be the preferred option as it is more cost-effective. However, its computational performance could be a drawback. AoN could be a viable alternative for systems characterized with short planning horizons. Besides, as it offers fixed production quantities for manufacturing and a simple all-or-nothing decision rule for manufacturing, it may have a particular appeal for practitioners.

STA follows a static uncertainty strategy where the replenishment schedule as well as production quantities are fixed at the outset for manufacturing and remanufacturing. We observed that STA is consistently outperformed by all other heuristic policies with respect to cost performance under all parameter settings. Therefore, it is a viable option only if a fixed production plan is absolutely necessary. This is the case, for instance, in multi-item production environments with limited capacities where revising production schedules and/or quantities is either very costly or not even possible due to practical concerns.

6. Conclusions and Further Research

We considered SELSR—a prominent inventory control problem which appears in hybrid manufacturing and remanufacturing systems. We proposed three heuristic policies for this problem: SM,

AoN, and THR. These policies provide different levels of flexibility with respect to production schedules and quantities. Therefore, they can be appealing in different environments, based on the extent of instability the production system can accommodate over the planning horizon. We presented mathematical models to compute the cost-minimizing parameters of the proposed policies, and illustrated their advantages and disadvantages with respect to cost effectiveness and computational efficiency by means of a numerical study.

There are many interesting directions for further research. We mention a few in the following. We showed that SM is very competitive with respect to cost performance, especially for instances characterized by high demand and return uncertainty. This is despite the fact that SM is based on simple myopic decision rules—which in turn enables us to compute its parameters by a very efficient algorithm. Therefore, it must be possible to improve on SM's cost performance by extending it with more elaborate decision rules, probably in (a limited) expense of computational efficiency. AoN and THR are sensible static-dynamic policies, but we do not yet know how they compare against the optimal static-dynamic policy. It is thus of importance to characterize the best and/or explore better static-dynamic policies. Finally, there is a variety of relevant aspects concerning hybrid manufacturing and remanufacturing systems, by which the work presented in the current study can be extended. These involve, among others, lead times for manufacturing and remanufacturing, quality considerations and disposal of returned products, distinct demand processes for manufactured and remanufactured products, and substitution.

Acknowledgments

This research is supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant no. MAG-114M389.

References

- Ahiska, S. S., & King, R. E. (2010a). Inventory optimization in a one product recoverable manufacturing system. *International Journal of Production Economics*, 124(1), 11–19.
- Ahiska, S. S., & King, R. E. (2010b). Life cycle inventory policy characterizations for a single-product recoverable system. *International Journal of Production Economics*, 124(1), 51–61.
- Askin, R. G. (1981). A procedure for production lot sizing with probabilistic dynamic demand. *AIIE Transactions*, 13(2), 132–137.
- Benedito, E., & Corominas, A. (2013). Optimal manufacturing policy in a reverse logistic system with dependent stochastic returns and limited capacities. *International Journal of Production Research*, 51(1), 189–201.
- Bookbinder, J. H., & Tan, J. Y. (1988). Strategies for the probabilistic lot-sizing problem with service-level constraints. *Management Science*, 34(9), 1096–1108.
- Chan, F. T. S., Li, N., Chung, S. H., & Saadat, M. (2017). Management of sustainable manufacturing systems—a review on mathematical problems. *International Journal of Production Research*, 55(4), 1210–1225.
- Ferguson, M. E., & Toktay, L. B. (2006). The effect of competition on recovery strategies. *Production and Operations Management*, 15(3), 351.
- Fleischmann, M., & Kuik, R. (2003). On optimal inventory control with independent stochastic item returns. *European Journal of Operational Research*, 151(1), 25–37.
- Frenzen, C., Sasao, T., & Butler, J. T. (2010). On the number of segments needed in a piecewise linear approximation. *Journal of Computational and Applied Mathematics*, 234(2), 437–446.
- Gavrilovic, M. M. (1975). Optimal approximation of convex curves by functions which are piecewise linear. *Journal of Mathematical Analysis and Applications*, 52(2), 260–282.
- Graves, S. C., & Willems, S. P. (2008). Strategic inventory placement in supply chains: Nonstationary demand. *Manufacturing & Service Operations Management*, 10(2), 278–287.
- Guide, V. D. R. (2000). Production planning and control for remanufacturing: industry practice and research needs. *Journal of Operations Management*, 18(4), 467–483.
- Guide Jr, V. D. R., & Van Wassenhove, L. N. (2009). OR FORUM - The evolution of closed-loop supply chain research. *Operations Research*, 57(1), 10–18.
- Heisig, G. (2001). Comparison of (s, s) and (s, nq) inventory control rules with respect to planning stability. *International Journal of Production Economics*, 73(1), 59–82.

- Hilger, T., Sahling, F., & Tempelmeier, H. (2016). Capacitated dynamic production and remanufacturing planning under demand and return uncertainty. *OR Spectrum*, 38(4), 849–876.
- Inderfurth, K. (1994). Nervousness in inventory control: Analytical results. *OR Spectrum*, 16(2), 113–123.
- Kenne, J.-P., Dejax, P., & Gharbi, A. (2012). Production planning of a hybrid manufacturing–remanufacturing system under uncertainty within a closed-loop supply chain. *International Journal of Production Economics*, 135(1), 81–93.
- Kilic, O. A., Tunc, H., & Tarim, S. A. (2018). Heuristic policies for the stochastic economic lot sizing problem with remanufacturing under service level constraints. *European Journal of Operational Research*, 267(3), 1102–1109. doi:10.1016/j.ejor.2017.12.041.
- Li, C., Liu, F., Cao, H., & Wang, Q. (2009). A stochastic dynamic programming based model for uncertain production planning of re-manufacturing system. *International Journal of Production Research*, 47(13), 3657–3668.
- Naeem, M. A., Dias, D. J., Tibrewal, R., Chang, P.-C., & Tiwari, M. K. (2013). Production planning optimization for manufacturing and remanufacturing system in stochastic environment. *Journal of Intelligent Manufacturing*, 24(4), 717–728.
- Neale, J. J., & Willems, S. P. (2009). Managing inventory in supply chains with non-stationary demand. *Interfaces*, 39(5), 388–399.
- Panagiotidou, S., Nenes, G., Zikopoulos, C., & Tagaras, G. (2017). Joint optimization of manufacturing/remanufacturing lot sizes under imperfect information on returns quality. *European Journal of Operational Research*, 258(2), 537–551.
- Retel Helmrich, M. J., Jans, R., van den Heuvel, W., & Wagelmans, A. P. (2014). Economic lot-sizing with remanufacturing: complexity and efficient formulations. *IIE Transactions*, 46(1), 67–86.
- Rossi, R., Tarim, S. A., Prestwich, S., & Hnich, B. (2014). Piecewise linear lower and upper bounds for the standard normal first order loss function. *Applied Mathematics and Computation*, 231, 489–502.
- Schulz, T. (2011). A new Silver–Meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing. *International Journal of Production Research*, 49(9), 2519–2533.
- Silver, E. A. (2008). Inventory management: An overview, Canadian publications, practical applications and suggestions for future research. *Infor*, 46(1), 15.
- Silver, E. A., & Meal, H. C. (1973). A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. *Production and Inventory Management*, 14(2), 64–74.
- Tang, O., & Teunter, R. (2006). Economic lot scheduling problem with returns. *Production and Operations Management*, 15(4), 488–497.
- Tao, Z., & Zhou, S. X. (2014). Approximation balancing policies for inventory systems with remanufacturing. *Mathematics of Operations Research*, 39(4), 1179–1197.
- Teunter, R., Kaparis, K., & Tang, O. (2008). Multi-product economic lot scheduling problem with separate production lines for manufacturing and remanufacturing. *European Journal of Operational Research*, 191(3), 1241–1253.
- Teunter, R. H., Bayindir, Z. P., & Van Den Heuvel, W. (2006). Dynamic lot sizing with product returns and remanufacturing. *International Journal of Production Research*, 44(20), 4377–4400.
- Van Der Laan, E. A., & Teunter, R. H. (2006). Simple heuristics for push and pull remanufacturing policies. *European Journal of Operational Research*, 175(2), 1084–1102.
- Zanoni, S., Segerstedt, A., Tang, O., & Mazzoldi, L. (2012). Multi-product economic lot scheduling problem with manufacturing and remanufacturing using a basic period policy. *Computers & Industrial Engineering*, 62(4), 1025–1033.