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Published in:
Proceedings of the IFAC 2017 conference

DOI:
10.1016/j.ifacol.2017.08.2034

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 10009–10014

A power consensus algorithm for DC microgrids ⋆

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Abstract: A novel power consensus algorithm for DC microgrids is proposed and analyzed. DC microgrids are networks composed of DC sources, loads, and interconnecting lines. They are represented by differential-algebraic equations connected over an undirected weighted graph that models the electrical circuit. A second graph represents the communication network over which the source nodes exchange information about the instantaneous powers, which is used to adjust the injected current accordingly. This give rise to a nonlinear consensus-like system of differential-algebraic equations that is analyzed via Lyapunov functions inspired by the physics of the system. We establish convergence to the set of equilibria consisting of weighted consensus power vectors as well as preservation of the weighted geometric mean of the source voltages. The results apply to networks with constant impedance, constant current and constant power loads.

Keywords: DC microgrids, Power sharing, Distributed control, Nonlinear consensus, Lyapunov stability analysis

1. INTRODUCTION

The proliferation of renewable energy sources and storage devices that are intrinsically operating using the DC regime is stimulating interest in the design and operation of DC microgrids, which have the additional desirable feature of preventing the use of inefficient power conversions at different stages. These DC microgrids might have to be deployed in areas where an AC microgrid is already in place, creating what is called a hybrid microgrid Loh et al. [2013], for which rigorous analytical studies are still in their infancy. Furthermore, the envisioned future in which power generation is far away from the major consumption sites raises the problem of how to transmit power with low losses, a problem for which High Voltage Direct Current (HVDC) networks perform comparatively better than AC networks. Finally, also mobile grids on ships, aircrafts, and trains are based on a DC architecture.

With DC and hybrid microgrids, as well as HVDC networks, on the rise, we need to develop a deeper system-theoretic understanding of this interesting class of dynamical networks. In this paper we propose and analyse a control algorithm for a DC microgrid that enforces power sharing among the different power sources.

1.1 Literature review

The literature on DC microgrids is rapidly growing. We summarize below the contributions that share a systems and control-theoretic point of view on these networks. The work Nasirian et al. [2015] relies on a cooperative control paradigm for DC microgrids to replace the conventional secondary control by a voltage and a current regulator. In Zhao and Dörfler [2015] a voltage droop controller for DC microgrids inspired by frequency droop in AC power networks is analyzed, and a secondary consensus control strategy is added to prevent voltage drift and achieve optimal current injection. The paper Belk et al. [2016] models the DC microgrid via the Brayton-Moser equations and uses this formalism to show that with the addition of a decentralized integral controller voltage regulation to a desired reference value is achieved. Other schemes achieving desirable power sharing properties are proposed but no formal analysis is provided. In Tucci et al. [2016], a secondary consensus-based control scheme for current sharing and voltage balancing in DC microgrids is designed in a Plug-and-Play fashion to allow for the addition or removal of generation units. A distributed control method to enforce power sharing among a cluster of DC microgrids is proposed in Moayedi and Davoudi [2016]. Other work has focused on the challenges in the stability analysis of DC microgrids using consensus-like algorithms due to the interaction between the communication network and the

⋆ The work of Claudio De Persis is partially supported by NWO within the program "Uncertainty Reduction in Smart Energy Systems (URSES)" under the auspices of the project ENBARK, by the DST-NWO Indo-Dutch Cooperation on "Smart Grids" under the auspices of the project "Energy management strategies for interconnected smart microgrids" and by the STW Perspectief program "Robust Design of Cyber-physical Systems" under the auspices of the project "Energy Autonomous Smart Microgrids". The work of Florian Dörfler is supported by ETH Zürich funds.

10.1016/j.ifacol.2017.08.2034

physical one [Meng et al. 2016]. In Cucuzzella et al. [2017] a decentralized sliding mode control scheme is proposed which achieves finite-time voltage regulation in presence of network parameter uncertainty and unknown load dynamics. Finally, feasibility of the nonlinear algebraic equations in DC power circuits is studied by Barabanov et al. [2016], Simpson-Porco et al. [2015], and Lavei et al. [2011].

A closely related research area is that of multi-terminal HVDC transmission systems. In Andreasson et al. [2014] distributed controllers that keep the voltages close to a nominal value and guarantee a fair power sharing are considered, whereas passivity-based decentralized PI control for the global asymptotic stabilisation of multi-terminal high-voltage is studied in Zonetti et al. [2015]. The paper Zonetti et al. [2016] studies feasibility and power sharing under decentralized droop control. We refer to [Zonetti 2016, Chapter 4] for an annotated bibliography of HVDC transmission systems.

1.2 Main contribution

This paper focuses on a new control algorithm to stabilize a DC microgrid under different load characteristics while achieving power sharing among the sources. Our controller is enabled by communicating the instantaneous source power measurements among neighboring source nodes, averaging these measurements and setting the voltage at the source terminals accordingly. An additional feature of the algorithm is that a weigthed geometric average of the source voltages is preserved.

The system dynamics present interesting features. By averaging the power measurements that the sources communicate amongst each other, the system dynamics becomes an intriguing combination of the physical network (the weighted Laplacian of the electrical circuit appearing in the power measurements) and the communication network (over which the information about the power measurements is exchanged). "ZIP" (constant impedance, constant current and constant power) loads introduce algebraic equations in the system's dynamics, adding additional complexity and nonlinearities.

To analyze this system of nonlinear differential-algebraic equations without going through a linearization of the dynamics, Lyapunov-based arguments become very convenient. The Lyapunov functions in this case are constructed starting from the power dissipated in the network that is further shaped to take into account the specifics of the dynamics. The presence of the loads, which shift the equilibrium of interest, is taken into account by the so-called Bregman function [De Persis and Monshizadeh 2016]. The level sets of the Lyapunov functions are used to estimate the excursion of the state response of these systems and therefore, combined with the preservation of the geometric average of the source voltages, can be used to obtain an estimate of the voltage at steady state.

Reactive power sharing algorithms have been first suggested by Schiffer et al. [2016] for network-reduced AC microgrids whose voltage dynamics show similar features as in DC grids. In this paper we show that a similar idea can be adopted also for network preserved DC microgrids. The novelties of this contribution with respect to Schiffer et al. [2016] are the different dynamics of the system under study, the explicit consideration of algebraic equations in the model and the use of Lyapunov arguments to prove the main results.

1.3 Paper organization

The model of the DC microgrid is described in Section 2. The power consensus algorithm is introduced in Section 3. The analysis of the closed-loop system is carried out in Section 4 for the general case of ZIP loads. Numerical tests of the algorithm are provided in Section 5. Conclusions are drawn in Section 6. All the proofs are omitted due to space constraints and can be found in De Persis et al. [2016].

1.4 Notation

Given a vector v, the symbol $[v]$ represents the diagonal matrix whose diagonal entries are the components of v. The notation $\mathrm{col}(v_1, v_2, \ldots, v_n)$, with v_i scalars, represents the vector $[v_1\ v_2\ \ldots\ v_n]^T$. If v_i are matrices having the same number of columns, then $\mathrm{col}(v_1, v_2, \ldots, v_n)$ denotes the matrix $[v_1^T\ v_2^T\ \ldots\ v_n^T]^T$. The symbol $\mathbb{1}_n$ represents the n-dimensional vector of all 1's, whereas $\mathbb{0}_{m\times n}$ is the $m \times n$ matrix of all zeros. When the size of the matrix is clear from the context the index is omitted. The $n \times n$ identity matrix is represented as $\mathbb{I}_n$. Given a vector $v \in \mathbb{R}^n$, the symbol $\mathbf{ln}(v)$ denotes the element-wise logarithm, i.e., the vector $[\ln(v_1) \ldots \ln(v_n)]^T$.

2. DC RESISTIVE MICROGRID

The DC microgrid is modeled as an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} := \{1, 2, \ldots, n\}$ the set of nodes (or buses) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ the set of edges. The edges represent the interconnecting lines of the microgrid, which we assume here to be resistive. Associated to each edge is a weight modeling the conductance (or reciprocal resistance) $1/r_k > 0$, with $k \in \mathcal{E}$. The set of nodes is partitioned into the two subsets of n_s DC sources $\mathcal{V}_s$ and n_l loads $\mathcal{V}_l$, with $n_s + n_l = n$.

The current-potential relation in a resistive network is given by the identity $I = B\Gamma B^T V$, with $B \in \mathbb{R}^{n\times|\mathcal{E}|}$ being the incidence matrix of $\mathcal{G}$ and $\Gamma = \mathrm{diag}\{r_1^{-1}, \ldots, r_{|\mathcal{E}|}^{-1}\}$ the diagonal matrix of conductances. Considering the partition of the nodes in sources and loads, the relation can be rewritten as

$$\begin{bmatrix} I_s \\ I_l \end{bmatrix} = \begin{bmatrix} B_s\Gamma B_s^T & B_s\Gamma B_l^T \\ B_l\Gamma B_s^T & B_l\Gamma B_l^T \end{bmatrix} \begin{bmatrix} V_s \\ V_l \end{bmatrix} =: \begin{bmatrix} Y_{ss} & Y_{sl} \\ Y_{ls} & Y_{ll} \end{bmatrix} \begin{bmatrix} V_s \\ V_l \end{bmatrix}, \quad (1)$$

where $I_s = \mathrm{col}(I_1, \ldots, I_{n_s})$, $I_l = \mathrm{col}(I_{n_s+1}, \ldots, I_n)$, $V_s = \mathrm{col}(V_1, \ldots, V_{n_s})$, $V_l = \mathrm{col}(V_{n_s+1}, \ldots, V_n)$ and $B = \mathrm{col}(B_s, B_l)$.

Observe that both Y_{ss} and Y_{ll} are positive definite since they are principal submatrices of a Laplacian of a connected undirected graph. This allows us to eliminate the load voltages as $V_l = Y_{ll}^{-1} I_l - Y_{ll}^{-1} Y_{ls} V_s$ and reduce the network to the source nodes $\mathcal{V}_s$ with balance equations

$$I_s - Y_{sl} Y_{ll}^{-1} I_l = Y_{red} V_s\,, \quad (2)$$

where $Y_{red} = Y_{ss} - Y_{sl} Y_{ll}^{-1} Y_{ls}$ is known as the Kron-reduced conductance matrix [Dörfler and Bullo 2013] and

$-Y_{sl}Y_{ll}^{-1}I_l$ is the mapping of the load current injections to the sources.

3. POWER CONSENSUS CONTROLLERS

We propose controllers that force the different sources to share the total power injection in prescribed ratios [Schiffer et al. 2016]. For this purpose, a communication network is deployed to connect the source nodes, through which the controllers exchange information about the instantaneous injected powers. This communication network is modelled as an undirected unweighted graph $(\mathcal{V}_c, \mathcal{E}_c)$, where $\mathcal{V}_c = \mathcal{V}_s$. Associated with the communication graph is the $n_s \times n_s$ Laplacian matrix $L_c = D_c - A_c$, where D_c is the degree matrix and A_c is the adjacency matrix of the communication graph. Note that the nodes of the communication network (but not necessarily the edges) coincide with the source nodes of the microgrid. For each node $i \in \mathcal{V}_s$, the set $\mathcal{N}_{c,i} = \{j \in \mathcal{V}_s : \{i,j\} \in \mathcal{E}_c\}$ represents the neighbors connected to node i via the communication graph.

Controllers. The proposed controllers are of the form

$$\mathcal{C}_i(V_i)\dot{V}_i = -I_i + u_i, \quad i \in \mathcal{V}_s, \tag{3}$$

where

$$\mathcal{C}_i(V_i) = V_i^{-2}D_{ci}^{-1}C_i^2, \quad i \in \mathcal{V}_s \tag{4}$$

can be interpreted as a nonlinear capacitance, $C_i > 0$ is a positive parameter of suitable units such that $\mathcal{C}_i(V_i)$ actually has the units of a capacitance, I_i is the injected current at node $i \in \mathcal{V}_s$ as defined in (1), and the term

$$u_i = V_i^{-1}D_{ci}^{-1}C_i \sum_{j\in\mathcal{N}_{c,i}} C_j^{-1}P_j, \quad i \in \mathcal{V}_s \tag{5}$$

represents an ideal current source that is controlled as a function of the local voltage V_i and the injected power $P_j = V_jI_j$ at the neighboring node sources $j \in \mathcal{N}_{c,i}$.

The dynamic controllers (3)–(5) are initialised at positive values of the voltage, that is $V_i(0) > 0$ for all $i \in \mathcal{V}_s$. It will be made evident in later sections that these controllers render the positive orthant $\mathbb{R}^{n_s}_{>0}$ positively invariant, thus showing that the positivity of the initial source voltages yields positivity of these variables for all $t \geq 0$.

Remark 1. **(Circuit interpretation)** The control algorithm has the circuit interpretation given in Fig. 1. Com-

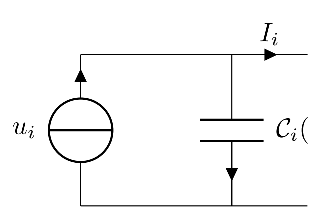

Fig. 1. A circuit interpretation of the controller (3).

paring with [Belk et al. 2016, (4)], the ideal current source u_i can be generated also by a voltage source with value v_i in series with a resistance r_i provided that $v_i = r_iu_i + V_i$. Finally, the dynamic droop controller in Zhao and Dörfler [2015] corresponds in our notation to a constant capacitance $\mathcal{C}_i$ and current source u_i.

Multiplying both sides of (3) by $V_i^2D_{c,i}C_i^{-1}$, one arrives at the closed-loop system

$$\begin{aligned} C_i\dot{V}_i &= -V_iD_{ci}C_i^{-1}P_i + V_i\sum_{j\in\mathcal{N}_{c,i}} C_j^{-1}P_j \\ &= V_i\sum_{j\in\mathcal{N}_{c,i}} (C_j^{-1}P_j - C_i^{-1}P_i), \quad i \in \mathcal{V}_s, \end{aligned} \tag{6}$$

that is, the voltage at the source terminal is updated according to a weighted power consensus algorithm scaled by the voltage. Provided that $V_i \neq 0$ (a property that will be established in the next sections), equation (6) shows that at steady state the algorithm achieves *proportional power sharing* according to the C_i ratios, namely

$$\frac{P_j}{C_j} = \frac{P_i}{C_i}, \quad \forall i,j \in \mathcal{V}_s. \tag{7}$$

A detailed characterisation of the steady-state power signals is given in the next section (Lemma 1).

Loads. Depending on the particular load models, the term I_l in (1) takes different expression and will henceforth be denoted as $I_l(V_l)$ to stress the functional dependence on the load voltages. Prototypical load models that are of interest include the following:

(i) constant current loads: $I_l(V_l) = I_l^* \in \mathbb{R}^{n_l}_{<0}$,
(ii) constant impedance: $I_l(V_l) = -Y_l^*V_l$, with $Y_l^* > 0$ a diagonal matrix of load conductances, and $V_l = \mathrm{col}(V_{n_s+1}, \ldots, V_{n_s+n_l})$, and
(iii) constant power: $I_l(V_l) = [V_l]^{-1}P_l^*$, with $P_l^* \in \mathbb{R}^{n_l}_{<0}$.

To refer to the three load cases above, we will use the indices "I", "Z" and "P" respectively. The analysis of this paper will focus on the more general case of a parallel combination of the three loads, thus on the case of "ZIP" loads. Moreover, additional and stronger statements results on the "ZI" case can be obtained.

Bearing in mind (1), (6), and vectorizing the expressions to avoid cluttered formulas, the closed-loop system is

$$\begin{bmatrix} C_s\dot{V}_s \\ -I_l(V_l) \end{bmatrix} = -\begin{bmatrix} [V_s]L_cC_s^{-1}P_s \\ B_l\Gamma B^TV \end{bmatrix}, \tag{8}$$

where $V = \mathrm{col}(V_s, V_l)$, $C_s = \mathrm{diag}(C_1, \ldots, C_{n_s})$, $P_s = \mathrm{col}(P_1, \ldots, P_{n_s})$ given by

$$P_s = [V_s]I_s = [V_s](Y_{ss}V_s + Y_{sl}V_l) \tag{9}$$

are source power injections and

$$I_l(V_l) = I_l^* - Y_l^*V_l + [V_l]^{-1}P_l^* \tag{10}$$

are the load currents. The interconnected closed-loop DC microgrid is then entirely described by equations (8), (9), (10).

Remark 2. (**Nonlinear consensus algorithms**) To compare the algorithm (6) with related nonlinear consensus algorithms proposed in the literature, we neglect the algebraic constraints and the differentiation between sources and loads. This allows us to rewrite (6) as

$$C\dot{V} = -[V]L_cC^{-1}[V]B\Gamma B^TV.$$

The weighted power mean consensus algorithms of Bauso et al. [2006], Cortes [2008], on the other hand, can be written as $[W]\dot{V} = [V]^{1-r}B\Gamma B^TV$, where W is vector of weights satisfying $\mathbb{1}^TW = 0$ and $r \in \mathbb{R}$. In the special case $r = 0$, we get

$$[W]\dot{V} = [V]B\Gamma B^TV,$$

which is known to converge to the consensus value $V_1^{w_1} \dots V_n^{w_n}$. The analysis is based on the Lyapunov function $\sum_{i=1}^n w_i V_i - \prod_{i=1}^n V_i^{w_i}$.
The nonlinear power consensus algorithm is different in that it uses another layer of averaging in addition to the averaging induced by the physical network. This, and the algebraic constraints, requires a different analysis based on physically inspired Lyapunov functions.

4. POWER CONSENSUS ALGORITHM WITH ZIP LOADS

In this section we analyze the closed-loop system (8), (9), (10). We start by studying its equilibria, namely the set of points $V \in \mathbb{R}^n_{>0}$ that satisfy (9), (10), and

$$\begin{bmatrix} \mathbb{0} \\ -I_l(V_l) \end{bmatrix} = -\begin{bmatrix} [V_s] L_c C_s^{-1} P_s \\ B_l \Gamma B^T V \,. \end{bmatrix} \tag{11}$$

4.1 Steady-state characterization

In the following, we show that the equilibria are fully characterized by power balance equations at the sources and current balance equations at the loads, respectively.

Lemma 1. **(System equilibria)** The equilibria of the system (8), (9), (10) are equivalently characterized by

$$\mathcal{E}_{ZIP} = \{V \in \mathbb{R}^n_{>0} : \mathcal{I}_{ZIP}(V) = \mathbb{0}\,,\, \mathcal{P}_{ZIP}(V) = \mathbb{0}\},$$

where $\mathcal{I}_{ZIP}(V) = \mathbb{0}$ is the current balance at the loads

$$\mathcal{I}_{ZIP}(V) = I_l(V_l) - Y_{ll} V_l - Y_{ls} V_s \,,$$

$\mathcal{P}_{ZIP}(V) = 0$ depicts the power balance at the sources

$$\mathcal{P}_{ZIP}(V) = \underbrace{[V_s] Y_{red} V_s}_{\substack{\text{network} \\ \text{dissipation}}} + \underbrace{[V_s] Y_{sl} Y_{ll}^{-1} I_l(V_l)}_{\substack{\text{load} \\ \text{demands}}} - \underbrace{P_s}_{\substack{\text{source} \\ \text{injections}}} \,,$$

Y_{red} is the Kron-reduced conductance matrix, $Y_{ll}^{-1} Y_{sl} I_l(V_l)$ is the mapping of the ZIP loads $I_l(V_l)$ to the source buses in the Kron-reduced network as in (2), and P_s is vector of power injections by the sources written for $V \in \mathcal{E}_{ZIP}$ as

$$P_s = -C_s \mathbb{1} \frac{\mathbb{1}^T I_l(V_l)}{\mathbb{1}^T [V_s]^{-1} C_s \mathbb{1}} =: C_s \mathbb{1} p_s^*. \tag{12}$$

Observe that the steady-state injections (12) achieve indeed power sharing, and the asymptotic power value p_s^* to which the source power injections converge (in a proportional fashion according to the coefficients C_i, $i \in \mathcal{V}_s$) is the total current demand divided by the weighted sum of the steady-state source voltages. The latter values and those of the load voltages are interestingly entangled by the power balance at the sources $\mathcal{P}_{ZIP}(V) = \mathbb{0}$ and the current balance equations at the loads $\mathcal{I}_{ZIP}(V) = \mathbb{0}$.

We make the standing assumption that equilibria exist:

Assumption 2. $\mathcal{E}_{ZIP} \neq \emptyset$.

Remark 3. **(Existence of the equilibria $\mathcal{E}_{ZIP}$)** The analytical investigation of the existence of the equilibria $\mathcal{E}_{ZIP}$ is deferred to a future research. This is a topic of interest on its own and similar problems have been dealt with in recent work about the solvability of reactive power flow equations [Bolognani and Zampieri 2016, Barabanov et al. 2016, Simpson-Porco et al. 2015, 2016]. For instance, the problem in Simpson-Porco et al. [2016] boils down to the solution of quadratic algebraic equations of the form $[V_l] Y_{ll} V_l - [V_l] Y_{ll} V_l^* + Q_l = 0$, where Q_l is the vector of constant power load demands and V_l^* is the so called vector of open circuit voltages (again constant). Although similarities between these equations and the equations $\mathcal{P}_{ZIP}(V_s) = \mathbb{0} = [V_s] Y_{red} V_s + [V_s] Y_{sl} Y_{ll}^{-1} I_l(V_l) + P_s$ could be useful to investigate the nature of the set $\mathcal{E}_{ZIP}$, the non-quadratic nature of $\mathcal{P}_{ZIP}(V_s) = \mathbb{0}$, as well as the presence of the additional equations $Y_{ll}^{-1} I_l(V_l) - V_l = Y_{ll}^{-1} Y_{ls} V_s$ pose additional challenges. Extra insights could come from the convex relaxation of the DC power flow equations in the context of optimal DC power flow dispatch Lavei et al. [2011].

4.2 A Lyapunov function and hidden gradient form

We pursue a Lyapunov-based analysis of the stability of the closed-loop system (8), (9), (10). Inspired by the Lyapunov analysis of the reactive power consensus algorithm in De Persis and Monshizadeh [2016], we consider the total power dissipated through the network resistors, $\frac{1}{2} V^T B \Gamma B^T V$, as the first natural Lyapunov candidate for our analysis, to which we add the power dissipated through the impedance loads, to obtain the power losses at passive devices as

$$J(V) = \frac{1}{2} V^T \left(B \Gamma B^T + \begin{bmatrix} \mathbb{0} & \mathbb{0} \\ \mathbb{0} & Y_l^* \end{bmatrix} \right) V. \tag{13}$$

Let $\overline{V} \in \mathcal{E}_{ZIP}$, and define $\overline{P}_s = [\overline{V}_s] B_s \Gamma B^T \overline{V}$ the source power injection corresponding to the equilibrium source voltage $\overline{V}$ (see (12)). To cope with the asymmetry in the dynamics of the sources and loads we add to J the terms

$$H(V) = -\overline{P}_s^T \mathbf{ln}(V_s),$$

and

$$K(V) = -P_l^{*T} \mathbf{ln}(V_l),$$

which is the way classical power systems transient stability analysis absorbs constant power injections [Chiang 2011] into a so-called *energy function* defined here as

$$\begin{aligned} M(V) &:= J(V) + H(V) + K(V) \\ &= \tfrac{1}{2} V^T (B \Gamma B^T + \begin{bmatrix} 0 & 0 \\ 0 & Y_l^* \end{bmatrix}) V - \overline{P}_s^T \mathbf{ln}(V_s) - P_l^{*T} \mathbf{ln}(V_l). \end{aligned} \tag{14}$$

The natural "energy function" (14) has its minimum at the trivial zero voltage level. To center the function M with respect to a non-trivial equilibrium $\overline{V} \in \mathcal{E}_{ZIP}$, we use the following Bregman function [De Persis and Monshizadeh 2016]

$$\mathcal{M}(V) = M(V) - M(\overline{V}) - \left.\frac{\partial M}{\partial V}\right|_{V=\overline{V}}^T (V - \overline{V}). \tag{15}$$

The next result shows a (perhaps surprising) gradient relation between the dynamics of system (8), (9), (10) and the Bregman function (15) above:

Lemma 3. **(Gradient dynamics)** The following holds

$$\begin{bmatrix} L_c C_s^{-1} P_s \\ B_l \Gamma B^T V - I_l(V_l) \end{bmatrix} = \begin{bmatrix} L_c [V_s] C_s^{-1} & \mathbb{0} \\ \mathbb{0} & \mathbb{I}_{n_l} \end{bmatrix} \frac{\partial \mathcal{M}(V)}{\partial V} \tag{16}$$

for all $V \in \mathbb{R}^n_{>0}$. Hence the system (8), (9), (10) can be rewritten as a weighted gradient flow

$$\begin{bmatrix} C_s \dot{V}_s \\ \mathbb{0} \end{bmatrix} = -\begin{bmatrix} [V_s] L_c [V_s] C_s^{-1} & \mathbb{0} \\ \mathbb{0} & \mathbb{I}_{n_l} \end{bmatrix} \frac{\partial \mathcal{M}(V)}{\partial V}. \tag{17}$$

4.3 Convergence of solutions

The particular form of the dynamics (8), (9), (10) elucidated in Lemma 3 permits a straightforward analysis of the convergence properties of the solutions.

Theorem 4. **(Main result)** Assume that there exists $\overline{V} \in \mathcal{E}_{ZIP}$ such that

$$Y_{ll}+Y_l^*+[\overline{V}_l]^{-2}[P_l^*]-Y_{ls}(Y_{ss}+[\overline{V}_s]^{-2}[\overline{P}_s])^{-1}Y_{sl} > 0 \quad (18)$$

Then there exists a compact sublevel set Λ_{ZIP} of $\mathcal{M}$ contained in $\mathbb{R}^n_{>0}$ such that any solution to (8), (9), (10) that originates from initial conditions $V(0)$ belonging to Λ_{ZIP} exists, always remain in Λ_{ZIP} with strictly positive voltages for all times, and asymptotically converges to the set $\mathcal{E}_{ZIP}\cap\Lambda_{ZIP}\cap\mathcal{V}_{ZIP}$, where $\mathcal{V}_{ZIP}$ specifies the preserved weighted geometric mean of the source voltages

$$\begin{aligned}\mathcal{V}_{ZIP} := \{&(V_s,V_l)\in\Lambda_{ZIP} : \mathcal{I}_{ZIP}(V)=\mathbb{0}\,,\\ &V_1^{C_1}\cdot\ldots\cdot V_{n_s}^{C_{n_s}} = V_1^{C_1}(0)\cdot\ldots\cdot V_{n_s}^{C_{n_s}}(0)\}.\end{aligned}$$

Remark 4. **(Interpretation of the main condition)** The main condition (18) guarantees regularity of the algebraic equations and stability of the solutions. Its role is revealed when converting the constant power loads and the asymptotically constant power injections at the sources to the equivalent impedances $[\overline{V}_l]^{-2}[P_l^*]$ and $[\overline{V}_s]^{-2}[\overline{P}_s]$. In this case, the equivalent conductance matrix in the steady-state current-balance equations (1) read as

$$Y_{eq} = \begin{bmatrix} Y_{ss} & Y_{sl}\\ Y_{ls} & Y_{ll}\end{bmatrix} + \begin{bmatrix}[\overline{V}_s]^{-2}[\overline{P}_s] & \mathbb{0}\\ \mathbb{0} & [\overline{V}_l]^{-2}[P_l^*]+Y_l^*\end{bmatrix}. \quad (19)$$

By a Schur complement argument, observe that Y_{eq} is a well-defined (i.e., positive definite) conductance matrix if and only if the main condition (18) holds.

Remark 5. **(Capacitors at the loads)** If loads are interconnected to the network via capacitors, the load equations are modified as

$$C_l\dot{V}_l = -I_l(V_l) + B_l\Gamma B^T V.$$

Notice that the equilibria of the system remain the same. Bearing in mind (16), the load dynamics read as

$$C_l\dot{V}_l = -\frac{\partial\mathcal{M}}{\partial V_l}.$$

It follows that

$$\dot{\mathcal{M}} = -\frac{\partial\mathcal{M}}{\partial V_s}^T C_s^{-1}[V_s]L_c[V_s]C_s^{-1}\frac{\partial\mathcal{M}}{\partial V_s} - \frac{\partial\mathcal{M}}{\partial V_l}^T C_l^{-1}\frac{\partial\mathcal{M}}{\partial V_l},$$

and one can infer convergence to the set $\mathcal{E}_{ZIP}\cap\Lambda_{ZIP}\cap\mathcal{V}_{ZIP}$ similarly as for the differential-algebraic model.

In the case of ZI loads the previous results can be strengthened [De Persis et al. 2016]. First, the set of equilibria can be more easily characterized. Second, the convergence result can be established without any extra condition on the equivalent conductance matrix in (19). Finally, the convergence is to a point rather than to a set.

5. SIMULATIONS

In this section, we present simulation results comparing the proposed control strategy to an averaging-based control method. We use an example network obtained from Belk et al. [2016]. The network topology is sketched in Fig. 4, and the physical parameters are given in Table 1. It can be checked that condition (18) is satisfied for this network. As in the reference experiment, there are seven constant power loads, five of which are initially turned off and are turned on gradually between 9.5 and 10.5 ms. This means that there is a gradual increase of the total power load from 70 W to 245 W. We simulate the control strategy (5). The power measured at the source nodes is shown in Fig. 3. As predicted by the analysis, at steady state proportional power sharing is achieved by the power sources in conformity with (7). The voltage evolution both at the sources and at the loads is depicted in Fig. 2.

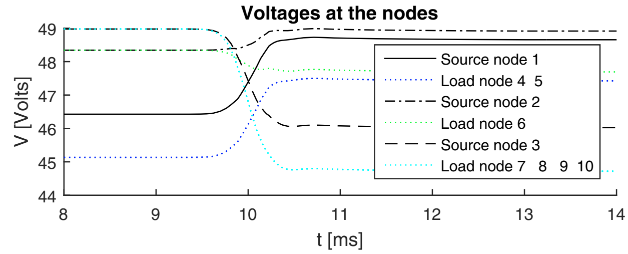

Fig. 2. Voltage plots of the simulation

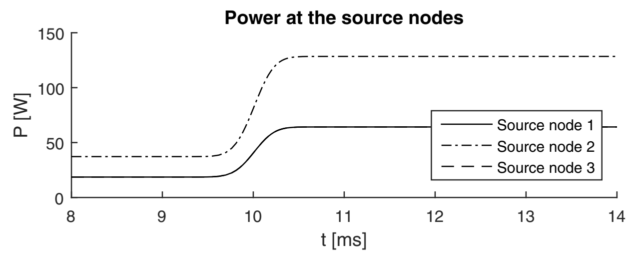

Fig. 3. Power plots of the simulation

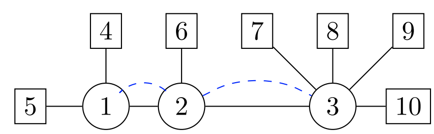

Fig. 4. The node network used for the simulations. Sources are depicted as circles, loads as rectangles. Solid lines denote the interconnecting lines, while dashed blue lines represent the communication graph used by the controllers.

Parameter	Value
Transmission line weights Γ_i	$6\times10^{-1}\,\Omega$
Capacitance weight C_i, $i=1,3$	$4\times10^{-2}\sqrt{\text{kgm}}/\text{s}$
$i=2$	$8\times10^{-2}\sqrt{\text{kgm}}/\text{s}$
Nominal voltage V^*	48 V
Load values $-P_l^*$	35 W

Table 1. Simulation parameter values.

6. CONCLUSIONS

We have proposed controllers for DC microgrids that average power measurement at the sources. The results apply to network preserved model (systems of DAE) of the microgrid in the presence of ZIP loads. Capacitors at the terminals of the grid that model either Π-models of lines or power converter components can be included by means of passivity-based analysis.

Many interesting new research directions can be taken. The first one is to consider more complex scenarios such as the inclusion of dynamical (inductive) lines and loads. Another one is the extensions of the controllers to network preserved AC microgrids. Moreover, although the preservation of the geometric mean of the voltages allows for an estimate of the voltage excursion, no active voltage regulation is present in the proposed scheme. An addition of voltage controllers to the power consensus algorithm is an interesting open problem. The power consensus algorithms lead to a new set of power flow equations, whose solvability still needs to be investigated, e.g., starting from recent advances concerning power flow feasibility and approximations; see Bolognani and Zampieri [2016], Barabanov et al. [2016], Simpson-Porco et al. [2016] and references therein. Other distributed averaging integral controllers achieving power consensus have been proposed in De Persis et al. [2016] that enjoy the nice feature of not requiring power measurements and could be an enthralling algorithm to investigate further. Finally, the power consensus algorithms preserves the weighted geometric mean of the voltages and is thus a compelling application for nonlinear consensus schemes [Bauso et al. 2006, Cortes 2008]. We believe this connection deserves a deeper investigation.

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