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# Optimal remanufacturing strategies in name-your-own-price auctions with limited capacity

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ABSTRACT

We study optimal pricing and production strategies faced by a manufacturer in a remanufacturing/manufacturing system. In the reverse channel, returns are collected under a name-your-own-price (NYOP) bidding mechanism. The manufacturer has a limited capacity to produce new and remanufactured products. We characterize the optimal decisions of the consumers and the manufacturer. We find that under the NYOP mechanism, the manufacturer's optimal strategies mainly depend on the bidding cost, the cost saving of remanufacturing, the production capacity, and the market scale. In addition, when remanufacturing needs more capacity than manufacturing , the manufacturer may adopt pure manufacturing strategy without remanufacturing. We also compare this mechanism with the traditional list-price mechanism and find that the manufacturer prefers the NYOP mechanism under the conditions of a low reverse market share, a high manufacturing cost, a sufficient capacity, or a low capacity requirement of remanufacturing. Numerical studies investigate the effect of key parameters on the manufacturer's profit and some managerial insights are obtained.



## 1. Introduction

Nowadays the environment protection is becoming one of the most pressing issues all around the world. To protect the environment, legislation has been introduced into Europe, North America, Asia and South America to encourage enterprises to enhance the awareness of environmental protection (Zeng et al., 2013; Lagarinhos and Tenório, 2013). In China, the situation seems more urgent. The grievous haze diffusing the whole country in the winter of 2013 has made Chinese people reconsider the huge environmental cost of rapid economic development during the last 35 years.

Besides environmental legislation, more and more companies become interested in remanufacturing operations because of economical benefits (Kaya, 2010). Empirical evidences show that in the process of recycling and remanufacturing, labor, energy as well as materials can be saved, and production lead time is also reduced. In the practice of remanufacturing, the capacity constraint and the mechanism design are two important problems. The former means that the manufacturer may not have enough production capability, which hinders the willingness to collect returns, and the latter means that the manufacturer should design an effective network and mechanism to collect used items. Therefore, it is imperative to simultaneously address these two issues in order to achieve a sustainable remanufacturing strategy.

In the practice, we find that there are two pricing mechanisms used to acquire returns: list-price strategy and name-your-own-price (NYOP) strategy. Traditionally, the OEM offers a list price for all the consumers and then each consumer decides whether he or she likes to do the return. The NYOP mechanism has become popular since the inception of Priceline in 1998. In the NYOP auction, a given price is fixed for every item. This price is called reserve price, and a consumer wins the item only if his or her bid is no less than the reserve price. Now this mechanism has already been widely used in the reverse logistics. In China, a typical example is a famous e-commerce website built by the largest e-commerce company in China, Alibaba Group. This website is specially for collecting second-hand items such as used mobile phones. On this website, used items are listed with a price named by the owners. Some items are clearly labeled with 'no bargaining' while others are not. Even though this website was initially created for other consumers to buy cheaper items, companies such as mobile phone manufacturers can also obtain used items as remanufacturing materials from these channels. Another example is the on-line resource recycling platform named China Resources

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Recycling. On this platform, sellers can name their own prices for the returned products and wait for buyers who can accept these bids. Besides on-line auction, the NYOP strategy has also been applied by many bricks-and-mortar recycling markets in China. In January 2013, Yahoo! News reported that a successful recycling business yielded more than 600 millionaires from Hubei Province in China. All of them have worked as scrap traders. During the scrap trade process, it is a common practice that the trader asks consumers to name their own price to trade recyclables. In these above business practices, if the consumers charge different prices, the payment from manufacturers may be different even if the returns are in similar conditions. This mechanism that consumers name the price is obviously different from the traditional list price mechanism that manufacturers name an identical price. Gönsch (2014) notices consumers' heterogeneity and challenges traditional posted-price(list-price) mechanism. He studies a bargaining process where the manufacturer initially sets an identical price paid for used products and consumers with heterogeneous valuation of used products can then bargain with the manufacturer. Different from the traditional posted price mechanism, the final payment is not identical for different consumers. This important feature is very similar to the NYOP mechanism. For example, the manufacturer's price in the bargaining is very similar to reserve price under the NYOP mechanism.

In this paper, we mainly investigate two important issues: (1) What are the optimal strategies of consumers and the manufacturer under this new NYOP mechanism? (2) How does the manufacturer make a choice between the NYOP mechanism and the traditional list-price mechanism? To address the above issues, we study optimal pricing and production strategies faced by a manufacturer in a remanufacturing/manufacturing system. In the reverse channel, returns are collected under a name-your-own-price (NYOP) bidding mechanism. The manufacturer has a limited capacity to produce new and remanufactured products. We characterize the optimal decisions of the consumers and the manufacturer. Numerical studies investigate the effect of key parameters on the manufacturer's profit and some managerial insights are obtained. We believe that our findings can help the manufacturer make the optimal choice about the pricing mechanism for return collection. Specifically, our paper contributes to the existing literature in threefold. First, we creatively introduce the NYOP mechanism for return acquisition into remanufacturing system. As many scholars argued (Gönsch, 2014; He, 2015; Zhou and Yu, 2011), return acquisition from consumers is one of the most important issues in remanufacturing. Therefore, the consumers' behavior plays an important role in the reverse logistics. We model the consumers' objective function and decision variable under this new NYOP mechanism. Our work enriches the studies of consumers' behavior in the reverse logistics. Second, we compare the NYOP mechanism with the traditional list-price mechanism, and find the conditions under which the manufacturer prefers the NYOP mechanism or the list-price mechanism, which explains why both NYOP mechanism and list-price mechanism are used in practice. In addition, it helps the manufacturer make the optimal choice about pricing mechanism. Third, we also consider the capacity constraint of production which makes this paper more realistic and complex.

The remainder of this paper is organized as follows. In Section 2, we make a brief literature review and state the innovation of this paper. In Section 3, the model and assumptions are presented. We first investigate the case that remanufacturing consumes less capacity than manufacturing and then the case that remanufacturing consumes more capacity. In Section 4, the effect of parameters associated with the NYOP mechanism is investigated through numerical experiments. We also compare these two mechanisms in this section. Section 5 summarizes the main results and points out some directions for future research. All the proofs are given in the appendix.

## 2. Literature review

The most related stream of research focuses on the pricing mechanism for return collection. Under the list-price strategy, Savaskan et al. (2004) address the problem of choosing the appropriate reverse channel structure for the collection of used products from consumers. They find that among these three recycling models, the retailer is the most effective undertaker of product collection activity for the manufacturer. Sun et al. (2013) study a multi-period acquisition pricing and remanufacturing decision problem under random price-sensitive returns, and they analyze characteristics of the optimal acquisition price and derive a monotonic pricing policy depending on the starting level of the whole inventory in each period. Atamer et al. (2013) focus on pricing and production decisions in utilizing reusable containers with stochastic customer demand. In their model, the return quantity depends on both demand and the acquisition fee determined by the manufacturer. Bulmus et al. (2014b) consider acquisition prices offered for returns with different quality types and on selling prices of new and remanufactured products. He (2015) models a closed-loop supply chain (CLSC)with a manufacturer and its supply channels-recycle channel and reliable supply channel, and he finds the effect similar to double marginalization often occurred in the normal forward supply chain. To the best of our knowledge, only very few researchers investigate the NYOP mechanism in recycling. Most research about NYOP pricing mechanism which focus on the forward flow provides us some managerial insights and hints. As argued by many researchers, the NYOP channel provides a niche market where consumers are sensitive to price or psychologically prefer this kind of auction (see Segan, 2005). Terwiesch et al. (2005) provide dynamic programming models to identify the optimal bidding strategy for consumers, and their results show that a haggling model may be better than a list-price model if the consumers are rather heterogeneous. Ding et al. (2005) and Cai et al. (2009) study the case where there exist both NYOP channel and list-price channel. Furthermore, Wang et al. (2010) examine the NYOP retailer's information revelation strategy when competing with list-price channel. Their results suggest that the NYOP mechanism can increase the expected profit for supply chain participants. Mostly related to our work in reverse logistics, Gönsch (2014) and Agrawal et al. (2015) notice the consumers' heterogeneity and adopt a generalized Nash bargaining solution to model the negotiation outcome. Gönsch (2014) studies a bargaining process where the manufacturer sets an identical price paid for used products and consumers with heterogeneous valuation of used products can bargain with the manufacturer. Agrawal et al. (2015) investigate when and how an original equipment manufacturer should offer a trade-in rebate to recover used products in order to achieve better price discrimination and weaken competition from third-party remanufacturers. Slightly different from our work, they use exogenous parameters to model negotiation outcome in equilibrium, therefore, the consumers' objective function and decisions are not well investigated.

The other related research is on capacity constraint problem, Bayındır et al. (2007) investigate the effect of finite production capacity and initial inventory levels on the optimal policy as well as the effect of substitution policy on the optimal order-up-to levels and the expected profit. However, they assume that the market demand is independent of retail price and ignore the pricing decision. Georgiadis et al. (2006) study how the lifecycles and return patterns of various products affect the optimal policies regarding expansion and contraction of collection and remanufacturing capacities. Their results show that the collection and remanufacturing capacity policies are insensitive to the total product demand. Georgiadis and Athanasiou (2013) deal with long-term demand-driven capacity planning policies in the reverse channel of closed-loop supply chains (CLSCs) with remanufacturing, and the key findings propose flexible policies as improved alternatives to large-scale capacity expansions/contractions in the terms of adaptability. Bulmus et al. (2013) examine the effect of remanufacturing on capacity and production decisions and one insightful finding is that the availability of the less capital intensive remanufacturing option sometimes leads to an increased capital investment.

Different from the above literature, our main innovation is to investigate the NYOP mechanism for return collection with capacity constraint in the reverse channel and compare the NYOP mechanism with the list-price mechanism.

## 3. Model description and analysis

We consider a remanufacturing/manufacturing system where the recycling business is run by a manufacturer, as illustrated in Fig. 1 (see the notation in Table 1). In this configuration, the manufacturer first decides the reserve price for returned items and the retail price. Second, given the manufacturer's decisions, the consumers bid with the manufacturer for their used products, and finally the used items are transferred to the manufacturer.

Moreover, We assume that one returned product can only be used to produce $\gamma$ percentage of one new product because of damage and other reasons where $0 \le \gamma \le 1$. In addition, the manufacturer purchases raw materials from an outside supplier. We denote $c_m$ as the total cost of unit new product which includes procurement cost of raw materials and manufacturing cost.

In the following subsections, we first analyze the optimal decisions for consumers, and then explore the optimal decisions of the manufacturer.

### 3.1. Consumers' behavior under the NYOP bidding mechanism

We examine consumers' behavior under the NYOP bidding mechanism. We assume that a particular consumer first bids in the NYOP auction and then sells with ratio $\lambda$ directly through the list-price channel if he/she fails in the NYOP auction. Since the consumer has only one chance to win in the NYOP auction, the consumer will choose the optimal bidding price $x$ to maximize the expected profit, which can be expressed as follows:

$$\Pi_c(\theta) = \max_x \quad \{F(x \le R)(x-\theta) + \lambda B[1 - F(x \le R)]\}$$
$$\text{s.t.} \quad B \le x \le A. \qquad (1)$$

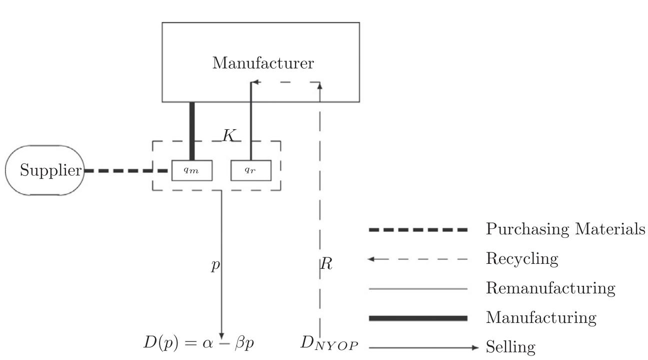


**Fig. 1.** Configuration of the remanufacturing/manufacturing system.

**Table 1**
Notation.

| | |
|---|---|
| **Decision variables** | |
| $x$ | A consumer's bid under the NYOP auction |
| $p$ | Retail price |
| $R$ | Reserve price under the NYOP auction |
| **Model parameters** | |
| $\alpha$ | Potential market size |
| $\beta$ | Price sensitive factor |
| $\lambda$ | Ratio of consumers who fails in the NYOP channel and turns to the list-price channel |
| $B$ | List price from either the other party or competitors |
| $[B, A]$ | Domain of a consumer's belief for the reserve price $R$ |
| $\theta$ | A random consumer's bidding cost |
| $[c, d]$ | Domain of $\theta$ |
| $c_m$ | Total cost of unit new product (procurement cost plus manufacturing cost) |
| $c_r$ | Unit remanufacturing cost and $c_r < c_m$ |
| $\gamma$ | Remanufacturing ratio and $\gamma \in (0, 1)$ |
| $s$ | Relative capacity requirement of remanufacturing |
| $K$ | Total production capacity |
| **Other notations** | |
| $D_{NYOP}$ | Amount of returned used items |
| $q$ | Total quantity of products and $q = \alpha - \beta p$ |
| $q_r$ | Amount of remanufactured products and $q_r = \gamma D_{NYOP}$ |
| $q_m$ | azmount of new products and $q_m = q - q_r$ |
| $\Pi_M{}^N$ | Manufacturer's profit under the NYOP mechanism |
| $\Pi_M{}^L$ | Manufacturer's profit under the list-price mechanism |
| $*$ | Denotes optimality, e.g., $q^*$ means the optimal total quantity |

Here, $F(x \le R)$ means the probability that the consumer wins in the NYOP bidding and successfully sells used items to the manufacturer. Correspondingly, $1 - F(x \le R)$ means the probability to fail in the NYOP bidding. The first term presents the consumer's expected revenue from the NYOP channel, and the second term presents his revenue from the list-price channel. Similar to Cai et al. (2009), we assume that the domain of a consumer's belief for the reserve price is given by a uniform distribution. In reality, the consumer only knows the lower and upper bounds of reserve price, and assume that the reserve price lies in the interval $[B, A]$ with the same probability. Therefore, an assumption of uniform distribution is reasonable. Based on this assumption, we obtain $F(x \le R) = \frac{A-x}{A-B}$. By optimizing the Equation (1) without the constraint, we have the optimal solution $x^* = \frac{\lambda B + A + \theta}{2}$.

**Assumption 1.** $c \ge (2-\lambda)B - A$ and $d \le A - \lambda B$.

Assumption 1 can guarantee that the optimal solution to $\Pi_c(\theta)$ is $x^* = \frac{\lambda B + A + \theta}{2}$ under the constraint that $B \le x \le A$ so that we can avoid tedious technical process and still keep the essence of the problem. The existence of $x^*$ requires $B \le x^* \le A$, which is equivalent to $(2-\lambda)B - A \le \theta \le A - \lambda B$. If a consumer's bidding cost is larger than $A - \lambda B$, then the consumer does not bid on the NYOP channel. Thus, the assumption can guarantee the bid occurs so that we can investigate the insight for remanufacturing with the NYOP mechanism.

In the NYOP channel, the consumers' willingness to sell their used products can be described by the reserve price. A consumer sells an item if his or her bid is lower than the reserve price, i.e., $x^* = \frac{\lambda B + A + \theta}{2} \le R$, which is equivalent to $\theta \le 2R - A - \lambda B$. Hence, the portion of all consumers who eventually sells used items is given by $D_{NYOP}$, where the maximal amount of $D_{NYOP}$ is normalized to 1.

Therefore, we have

$$D_{NYOP}=\begin{cases}1, & \text{if } R\geq\dfrac{A+\lambda B+d}{2};\\ \dfrac{2R-A-\lambda B-c}{d-c}, & \text{if } R\in\left[\dfrac{A+\lambda B+c}{2},\dfrac{A+\lambda B+d}{2}\right);\\ 0, & \text{if } R<\dfrac{A+\lambda B+c}{2}.\end{cases}\qquad(2)$$

### 3.2. Manufacturer's optimal strategies when $s\leq 1$

In what follows, we investigate the manufacturer's optimal strategies. Research has shown that in the capacitated remanufacturing/manufacturing system, the manufacturer's optimal strategies largely depend on whether remanufacturing needs less or more capacity than manufacturing process (Bulmus et al., 2013). In this paper, $s$ is denoted as the relative capacity requirement of remanufacturing process compared with manufacturing process. $s\leq 1$ indicates remanufacturing needs less capacity than manufacturing process while $s>1$ indicates remanufacturing needs more capacity than manufacturing process. We first investigate the case that $s\leq 1$ and then the case that $s>1$. For these two cases, we investigate two pricing mechanisms, i.e., name-your-own-price mechanism and list-price mechanism.

#### 3.2.1. NYOP mechanism when $s\leq 1$

Under the NYOP mechanism, the manufacturer sets an optimal reserve price that determines the quantity of returned items in the reverse channel. Although the conditions of used products may be uncertain, we may use the identical reserve price under the NYOP mechanism as the reserve price can be determined based on the estimation of the conditions of used products on average. Further, such an assumption is also common in literature related to the list-price mechanism, such as Savaskan et al. (2004), Savaskan and Van Wassenhove (2006). In the forward market, we assume the remanufactured products and manufactured products are homogenous with the identical price $p$. This is the case that remanufactured products are sold with the same quality and warranty as new products and consumers can not distinguish them. In practice, this holds for some products such as single-use cameras (Akcali and Cetinkaya, 2011) as well as reusable containers for beverage and food manufacturing, packaging and transportation (Atamer et al., 2013). This assumption is also widely used in other relevant literatures including Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), Bulmus et al. (2014a). With such an assumption, the market demand is given by $D=\alpha-\beta p$. A similar demand model is widely used in literature, e.g., Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), Bulmus et al. (2013). We can adopt the utility-based method to explain this linear demand model (see Debo et al., 2005; Ferguson and Toktay, 2006; Bulmus et al., 2014a).

To motivate the manufacturer to recycle and remanufacture, we have the following assumption.

**Assumption 2.** $\alpha>\beta c_m$, $c_m-c_r>\frac{A+\lambda B+c}{2\gamma}$.

The first part of the assumption is reasonable since there must exist a positive demand when the retail price is equal to the manufacturing cost $c_m$. The second part indicates that the manufacturer should pay the consumer at least $\frac{A+\lambda B+c}{2}$ if he wants to recycle the products. Note that one recycled product can only produce $\gamma$ new product and the unit remanufacturing cost is $c_r$. Hence, the average unit cost of remanufacturing including recycling process is at least $\frac{A+\lambda B+c}{2\gamma}+c_r$. If the manufacturer directly manufactures new product, the unit cost is $c_m$. When $c_m\leq\frac{A+\lambda B+c}{2\gamma}+c_r$, the manufacturer only manufactures new products without remanufacturing. Therefore, to motivate the manufacturer to remanufacture used items, we assume that $c_m-c_r>\frac{A+\lambda B+c}{2\gamma}$. We define $c_m-c_r$ as production cost saving that is the manufacturer's economic motivation to acquire returns. Note that based on this assumption, we obtain $\alpha>\beta c_r+\frac{\beta(A+\lambda B+c)}{2\gamma}$.

In the reverse channel, the manufacturer sets $R$ to recycle returns. By Equation (2), the manufacturer never sets $R>\frac{A+\lambda B+d}{2}$ since a higher reserve price than $\frac{A+\lambda B+d}{2}$ cannot yield more returns. When the manufacturer sets $R<\frac{A+\lambda B+c}{2}$, no used product is recycled. From now on, we only focus on the range for $R\in\left[\frac{A+\lambda B+c}{2},\frac{A+\lambda B+d}{2}\right]$. Then, the amount of returns is given by $D_{NYOP}=\frac{2R-A-\lambda B-c}{d-c}$, where $0\leq D_{NYOP}\leq 1$.

After recycling, the manufacturer utilizes the capacity to produce. We denote $s$ as the relative capacity requirement of remanufacturing. Then, the capacity constraint is given by $q_m+sq_r\leq K$. The manufacturer's profit function is given as follows:

$$\Pi_M^N=\max_{R,p}\quad\{p(q_m+q_r)-c_rq_r-c_mq_m-\mathrm{E}_\theta[x^*\cdot D_{NYOP}\,|\,x^*\leq R]\}$$
$$\text{s.t.}\quad q_m+sq_r\leq K\qquad(3)$$

Here, $\mathrm{E}_\theta[x^*\cdot D_{NYOP}\,|\,x^*\leq R]$ represents the manufacturer's recycling cost given by

$$\mathrm{E}_\theta[x^*\cdot D_{NYOP}\,|\,x^*\leq R]=\frac{\int_c^{2R-A-\lambda B}\frac{A+\lambda B+\theta}{2}\frac{1}{d-c}\,d\theta}{P(x^*\leq R)}\cdot\frac{2R-A-\lambda B-c}{d-c}=\frac{4R^2-(A+\lambda B+c)^2}{4(d-c)}.\qquad(4)$$

Note that $q=\alpha-\beta p$ and $q_r=\gamma D_{NYOP}=\frac{(2R-A-\lambda B-c)\gamma}{d-c}$. Therefore, the retail price and the reserve price are uniquely determined by the total quantity and the remanufactured quantity. For convenience, from now on, we adopt $q$ and $q_r$ as our new decision variables. We

**Table 2**
Potential optimal solutions under the NYOP mechanism.

| Cases | $q_r^*$ | $q^*$ |
|---|---|---|
| Case 1 | $\frac{\gamma(2\gamma c_m-2\gamma c_r-A-c-\lambda B)}{d-c}$ | $\frac{\alpha-\beta c_m}{2}$ |
| Case 2 | $\gamma$ | $\frac{\alpha-\beta c_m}{2}$ |
| Case 3 | $\frac{\gamma[(1-s)(4K-2\alpha)\gamma+\beta(A+\lambda B+c)+2\beta\gamma(-sc_m+c_r)]}{c\beta-d\beta-4(-1+s)^2\gamma^2}$ | $K+\frac{(1-s)\gamma[(1-s)(4K-2\alpha)\gamma+\beta(A+\lambda B+c)+2\beta\gamma(-sc_m+c_r)]}{c\beta-d\beta-4(-1+s)^2\gamma^2}$ |
| Case 4 | $\gamma$ | $K+\gamma-s\gamma$ |
| Case 5 | $\frac{\gamma[2\alpha\gamma-\beta(A+\lambda B+c)-2\beta\gamma c_r]}{d\beta-c\beta+4\gamma^2}$ | $\frac{\gamma[2\alpha\gamma-\beta(A+\lambda B+c)-2\beta\gamma c_r]}{d\beta-c\beta+4\gamma^2}$ |
| Case 6 | $\gamma$ | $\gamma$ |
| Case 7 | $\frac{K}{s}$ | $\frac{K}{s}$ |

rewrite the manufacturer's profit function as follows:

$$\Pi_M^N = \max_{q,q_r} \left\{ \frac{(\alpha-q)q}{\beta} - c_m q + \left(c_m - c_r - \frac{A+\lambda B+c}{2\gamma}\right) q_r - \frac{q_r^2(d-c)}{4\gamma^2} \right\}$$

$$\text{s.t.} \quad q_r \le q; \quad q-(1-s)q_r \le K; \quad q_r \le \gamma. \tag{5}$$

By Eq. (5), the marginal cost of remanufacturing with recycling is $c_r + \frac{A+\lambda B+c}{2\gamma} + \frac{q_r(d-c)}{2\gamma^2}$ and the maximal marginal cost is $c_r + \frac{A+\lambda B+d}{2\gamma}$. Note that the optimal solutions depend on whether $s \le 1$ or $s > 1$. We first consider the case of $s \le 1$ in this section and then the case of $s > 1$. The potential optimal solutions for the case of $s \le 1$ are shown in Table 2.

Proposition 1 shows the manufacturer's optimal strategies under different scenarios. Denote

$\alpha_1 = \beta c_m + 2K + \frac{[4\gamma^2(c_m-c_r) - 2\gamma(A+\lambda B+c)](1-s)}{d-c}$,

$\alpha_2 = \frac{(d-c)K\beta + \gamma s\beta(A+\lambda B+c) - 2s\beta\gamma^2(sc_m-c_r)}{2(1-s)s\gamma^2} + \frac{2K}{s}$,

$\alpha_3 = \frac{\beta(A+\lambda B+d) - 2s\beta\gamma c_m + 2\beta\gamma c_r}{2(1-s)\gamma} + 2(K+\gamma-s\gamma)$,

$\alpha_4 = \beta c_m + \frac{4\gamma^2(c_m-c_r) - 2\gamma(A+\lambda B+c)}{d-c}$,

$\alpha_5 = \frac{dK\beta + c\beta(-K+s\gamma) + \gamma[4K\gamma + s\beta(A+\lambda B)]}{2s\gamma^2} + \beta c_r$, and

$\alpha_6 = \beta c_r + 2\gamma + \frac{\beta(A+\lambda B+d)}{2\gamma}$.

**Proposition 1.** *The manufacturer's optimal strategies are given by*:

(1) *If* $d \ge 2\gamma(c_m-c_r) - A - \lambda B$ *and* $K \le \frac{2s\gamma^2(c_m-c_r) - s\gamma(A+\lambda B+c)}{d-c}$, *then when* $\alpha < \alpha_5$, *Case 5 is optimal. Otherwise, Case 7 is optimal.*
(2) *If* $d \ge 2\gamma(c_m-c_r) - A - \lambda B$ *and* $\frac{2s\gamma^2(c_m-c_r) - s\gamma(A+\lambda B+c)}{d-c} \le K \le s\gamma$, *then, when* $\alpha \le \alpha_4$, *Case 5 is optimal. When* $\alpha \in [\alpha_4, \alpha_1]$, *Case 1 is optimal. When* $\alpha \in [\alpha_1, \alpha_2]$, *Case 3 is optimal. Otherwise, Case 7 is optimal.*
(3) *If* $d \ge 2\gamma(c_m-c_r) - A - \lambda B$ *and* $K \ge s\gamma$, *then, when* $\alpha \le \alpha_4$, *Case 5 is optimal. When* $\alpha \in [\alpha_4, \alpha_1]$, *Case 1 is optimal. When* $\alpha \in [\alpha_1, \alpha_3]$, *Case 3 is optimal. Otherwise, Case 4 is optimal.*
(4) *If* $d < 2\gamma(c_m-c_r) - A - \lambda B$ *and* $K \le s\gamma$, *then when* $\alpha < \alpha_5$, *Case 5 is optimal. Otherwise, Case 7 is optimal.*
(5) *If* $d < 2\gamma(c_m-c_r) - A - \lambda B$ *and* $K \ge s\gamma$, *then, when* $\alpha < \alpha_6$, *Case 5 is optimal. When* $\alpha_6 < \alpha < \beta c_m + 2\gamma$, *Case 6 is optimal. When* $\beta c_m + 2\gamma < \alpha < \beta c_m + 2K + 2(1-s)\gamma$, *Case 2 is optimal. Otherwise, Case 4 is optimal.*

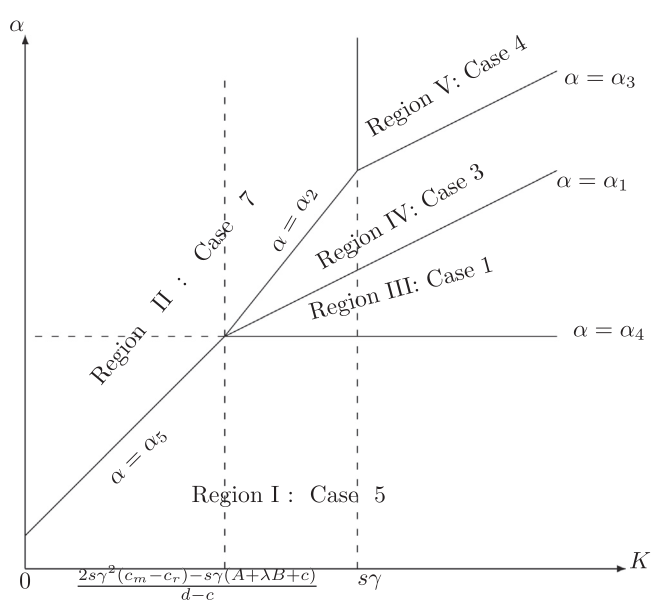


**Fig. 2.** Regions of manufacturer's optimal strategies for $d \ge 2\gamma(c_m-c_r) - A - \lambda B$.

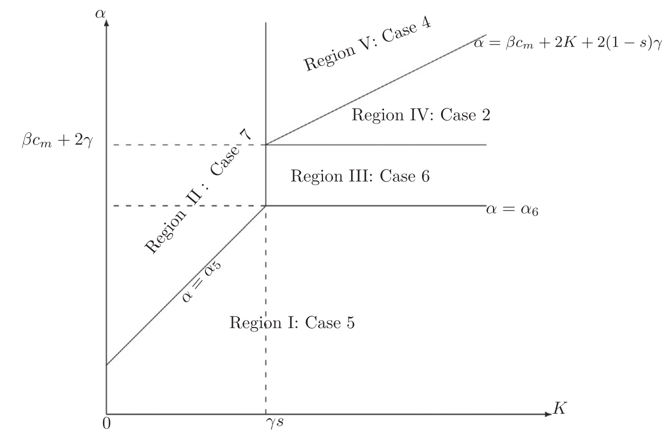


**Fig. 3.** Regions of manufacturer's optimal strategies for $d < 2\gamma(c_m-c_r) - A - \lambda B$.

**Table 3**
Manufacturer's optimal policy for $d \ge 2\gamma(c_m-c_r) - A - \lambda B$.

| Region | Optimal policy | Managerial implication for the manufacturer |
|---|---|---|
| I | $q_m = 0$, $q_r < \gamma$, $K_c < K$ | Remanufacturing, partially recycling, capacity surplus |
| II | $q_m = 0$, $q_r < \gamma$, $K_c = K$ | Remanufacturing , partially recycling, capacity used up |
| III | $q_m > 0$, $q_r < \gamma$, $K_c < K$ | Remanufacturing and manufacturing, partially recycling, capacity surplus |
| IV | $q_m > 0$, $q_r < \gamma$, $K_c = K$ | Remanufacturing and manufacturing, partially recycling, capacity used up |
| V | $q_m > 0$, $q_r = \gamma$, $K_c = K$ | Remanufacturing and manufacturing, totally recycling, capacity used up |

**Table 4**
Manufacturer's optimal policy for $d < 2\gamma(c_m-c_r) - A - \lambda B$.

| Region | Optimal policy | Managerial Implication for the manufacturer |
|---|---|---|
| I | $q_m = 0$, $q_r < \gamma$, $K_c < K$ | Remanufacturing, partially recycling, capacity surplus |
| II | $q_m = 0$, $q_r < \gamma$, $K_c = K$ | Remanufacturing, partially recycling, capacity used up |
| III | $q_m = 0$, $q_r = \gamma$, $K_c < K$ | Remanufacturing, totally recycling, capacity surplus |
| IV | $q_m > 0$, $q_r = \gamma$, $K_c < K$ | Remanufacturing & manufacturing, totally recycling, capacity surplus |
| V | $q_m > 0$, $q_r = \gamma$, $K_c = K$ | Remanufacturing & manufacturing, totally recycling, capacity used up |

By Proposition 1, we find that the manufacturer's optimal strategy mainly depends on the bidding cost $d$, the remanufacturing cost saving $c_m - c_r$, production capacity $K$, and the potential market size $\alpha$. To achieve a better illustration, Figs. 2 and 3 show the manufacturer's optimal policy in different regions in terms of $K$ and $\alpha$ for two scenarios: $d \ge 2\gamma(c_m-c_r) - A - \lambda B$ and $d < 2\gamma(c_m-c_r) - A - \lambda B$. Tables 3 and 4 provide a detailed explanation of optimal policies, where $K_c = q_m + sq_r$ represents the consumed capacity for manufacturing and/or remanufacturing. In Tables 3 and 4, $q_m$=0 or $q_m > 0$ represents the manufacturer's production mode. The former indicates a single production mode without manufacturing and the latter indicates a mixed production mode with both remanufacturing and manufacturing. $K_c < K$ means the capacity is sufficient to meet the market demand and $K_c = K$ means all the capacity is used up to meet the demand.

The manufacturer's production policy is briefly explained as follows. From Fig. 2, in Region I where the market scale is rather small, the manufacturer remanufactures with the unconstrained optimal quantity, i.e., $q_r^* = \frac{\gamma[2\alpha\gamma - \beta(A+\lambda B+c) - 2\beta\gamma c_r]}{d\beta - c\beta + 4\gamma^2}$, and there is no need to manufacture. In Region II, for the limited capacity, the market scale is large enough and the optimal strategy is to utilize all capacity to produce as many as possible. Therefore, the manufacturer allocates all capacity for remanufacturing since it consumes less capacity than manufacturing. In Region III where the market slightly expands, both remanufacturing and manufacturing are necessary. Similarly, in Region IV, the manufacturer utilizes all capacity to remanufacture and manufacture. In Region V where the market scale is extremely large, it is profitable to produce as many as possible. That is why Case 4 is optimal in this region. In Fig. 3, when $d<2\gamma(c_m-c_r)-A-\lambda B$, manufacturing is dominated by remanufacturing and the manufacturer's policy is rather simple. By Proposition 1, it can be briefly summarized as follows: $q_r^*=\min\left\{\frac{\gamma[2\alpha\gamma-\beta(A+\lambda B+c)-2\beta\gamma c_r]}{d\beta-c\beta+4\gamma^2},\frac{K}{s},\gamma\right\}$, and $q_m^*=\min\{K-sq_r^*,\ [\frac{\alpha-\beta c_m}{2}-q_r^*]^+\}$. The managerial insight is to do remanufacturing with the returns available, and further try to utilize the remaining capacity to manufacture until the total quantity reaches the optimal quantity, i.e., $\frac{\alpha-\beta c_m}{2}$.

We also investigate the impacts of the parameters on the manufacturer's optimal strategies. By Proposition 1, we obtain the following corollary.

**Corollary 1.** (1) *Under the NYOP mechanism, the remanufactured quantity $q_r^*$ is nondecreasing with $\gamma$, $c_m$, $\alpha$ and nonincreasing with $A$, $B$, $c$, $d$, $\lambda$, $\beta$ and $c_r$.*

(2) *Under the NYOP mechanism, the total production quantity $q^*$ is nondecreasing with $\gamma$, $K$, $\alpha$ and nonincreasing with $A$, $B$, $c$, $d$, $\lambda$, $\beta$, $c_r$ and $s$.*

Since we assume $q=\alpha-\beta p$, we can also obtain the effect of key parameters on the manufacturer's optimal pricing strategy $p^*$ from Corollary 1. Obviously, $p^*$ is nonincreasing with $\gamma$, $K$ and nondecreasing with $A$, $B$, $c$, $d$, $\lambda$, $c_r$ and $s$ due to the negative, linear relationship between $p^*$ and $q^*$. In addition, with some complex mathematical process, we can prove that $p^*$ is nondecreasing with $\alpha$ and nonincreasing with $\beta$. For conciseness, the proof about $p^*$ is omitted.

*3.2.2. List-price mechanism when $s\le 1$*

In this section, we investigate the manufacturer's optimal strategies under the list-price mechanism for return collection. We assume that the competition in the reverse market is so fierce that no agent can gain more returns with a higher payment. In other words, due to perfect competition, the manufacturer chooses the same payment as his competitors do. In consistent with Eq. (1), we denote the list price as $B$.

With such a list price, the potential return quantity depends on the manufacturer's market influence and consumers' preference. We use $\tau$ to represent the manufacturer's share in the reverse market which is also the maximal return quantity. Since one returned product can only be used to produce $\gamma$ percentage of one new product, if the remanufactured quantity is $q_r$, the amount of return should be $q_r/\gamma$. Further, we require that $c_m>c_r+B/\gamma$. Otherwise, the manufacturer has no incentive to do recycling and remanufacturing.

The manufacturer makes the pricing and production decisions to maximize his profit as follows:

$$\Pi_M^L=\max_{B,p}\left\{(q_r+q_m)p-c_rq_r-c_mq_m-\frac{Bq_r}{\gamma}\right\}$$
$$\text{s.t.}\quad q_m+sq_r\le K,\quad q_r\le\gamma\tau,\quad q_m\ge 0, q_r\ge 0. \tag{6}$$

Since $q_r+q_m=q=\alpha-\beta p$, we rewrite the manufacturer's profit function under the list-price mechanism as follows:

$$\Pi_M^L=\max_{\{q_r,q\}}\left\{\frac{(\alpha-q)q}{\beta}+\left(c_m-c_r-\frac{B}{\gamma}\right)q_r-c_mq\right\}$$
$$\text{s.t.}\quad q-(1-s)q_r\le K,\quad q_r\le\gamma\tau,\quad q_r\le q. \tag{7}$$

Based on this profit function, the manufacturer's optimal remanufactured/manufactured decisions are obtained. Denote $\hat{q}_r^*$ as the optimal quantity of remanufactured products and $\hat{q}^*$ as the optimal total quantity of products. We obtain all potential optimal solutions for Eq. (7) in Table 5. As shown by the proof in Appendix B, there does not exist a feasible solution for Case 1. Thus, Table 5 starts with Case 2.

**Table 5**
Potential optimal solutions under the list-price mechanism.

| Cases | $\hat{q}_r^*$ | $\hat{q}^*$ |
|---|---|---|
| Case 2 | $\frac{\alpha\gamma-B\beta-\beta\gamma c_r}{2\gamma}$ | $\frac{\alpha\gamma-B\beta-\beta\gamma c_r}{2\gamma}$ |
| Case 3 | $\tau\gamma$ | $\frac{\alpha-\beta c_m}{2}$ |
| Case 4 | $\tau\gamma$ | $\tau\gamma$ |
| Case 5 | $\frac{2K-\alpha+\beta c_m}{2(s-1)}+\frac{\beta[\gamma(c_m-c_r)-B]}{2\gamma(s-1)^2}$ | $\frac{\alpha-\beta c_m}{2}-\frac{\beta[\gamma(c_m-c_r)-B]}{2\gamma(s-1)}$ |
| Case 6 | $\frac{K}{s}$ | $\frac{K}{s}$ |
| Case 7 | $\tau\gamma$ | $K+(1-s)\gamma\tau$ |

Denote $\hat{\alpha}_1=\frac{Bs\beta+2K\gamma+s\beta\gamma c_r}{s\gamma}$ and $\hat{\alpha}_2=\beta c_m+2K+2(1-s)\tau\gamma$, and we obtain:

**Proposition 2.** *The manufacturer's optimal strategies under the list-price mechanism are given by*

(1) *If the capacity is low, i.e., $K\le\tau\gamma s$, then when $\alpha<\hat{\alpha}_1$, Case 2 is optimal. Otherwise, Case 6 is optimal*;
(2) *If the capacity is high, i.e., $K>\tau\gamma s$, then when $\alpha<\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}$, Case 2 is optimal. When $\alpha\in[\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma},\beta c_m+2\tau\gamma]$, Case 4 is optimal. When $\alpha\in[\beta c_m+2\tau\gamma,\hat{\alpha}_2]$, Case 3 is optimal. Otherwise, Case 7 is optimal.*

The results of Proposition 2 are presented in Fig. 4. By Proposition 2, we find that the manufacturer's optimal decisions are rather simpler under the list-price mechanism than under the NYOP mechanism. An interesting observation is that $\hat{q}_r^*$ is independent of $c_m$. It indicates that when the manufacturer recycles returns, the manufacturer only considers the market scale and the capacity which facilitates the manufacturer to make optimal decisions. In fact, by Proposition 2, we can rewrite $\hat{q}_r^*=\min\left\{\tau\gamma,\frac{K}{s},\frac{\alpha\gamma-\beta c_r\gamma-\beta B}{2\gamma}\right\}$, where $\tau\gamma$ is the maximal remanufactured quantity due to potential returns, $\frac{K}{s}$ is the maximal remanufactured quantity due to capacity constraint, $\frac{\alpha\gamma-\beta c_r\gamma-\beta B}{2\gamma}$ represents the unconstrained optimal remanufacturing quantity. In short, the manufacturer's optimal strategies are as follows: the manufacturer decides the optimal remanufactured quantity as $\hat{q}_r^*=\min\left\{\tau\gamma,\frac{K}{s},\frac{\alpha\gamma-\beta c_r\gamma-\beta B}{2\gamma}\right\}$, and then he further decides the optimal manufactured quantity as $\hat{q}_m^*=\min\left\{\left[\frac{\alpha-\beta c_m}{2}-\hat{q}_r^*\right]^+,K-s\hat{q}_r^*\right\}$. If the optimal remanufacturing quantity has already exceeded $\frac{\alpha-\beta c_m}{2}$, no new product will be manufactured, i.e., $\hat{q}_m^*=0$. Otherwise, the manufacturer utilizes the rest of capacity $K-s\hat{q}_r^*$ until the total quantity reaches $\frac{\alpha-\beta c_m}{2}$, thus $\hat{q}_m^*=\min\left\{\left[\frac{\alpha-\beta c_m}{2}-\ \hat{q}_r^*\right]^+,K-s\hat{q}_r^*\right\}$.

Similarly, we also investigate the impact of parameters on the manufacturer's optimal strategies. Since the expression of optimal decisions are simple, we can easily obtain the following corollary.

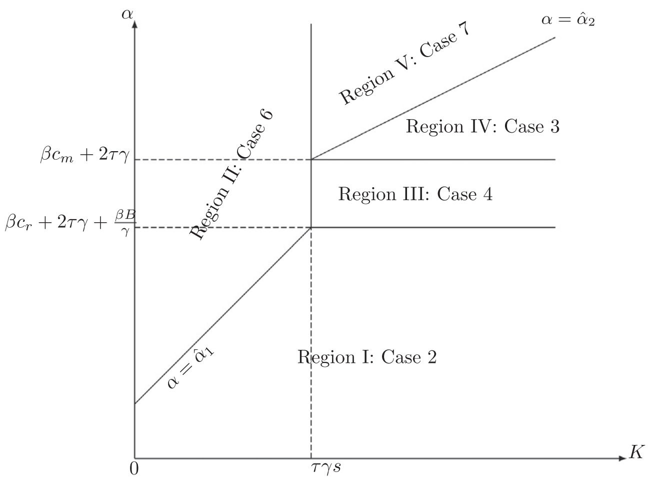


**Fig. 4.** Regions for manufacturer's optimal list-price strategies.

**Corollary 2.** (1) *Under the list-price mechanism, $q_r^*$ is nondecreasing with $\alpha$, $K$, $\gamma$, $\tau$, nonincreasing with $s$, $\beta$, $B$, $c_r$ and independent with $c_m$.*

(2) *Under the list-price mechanism, $q^*$ is nondecreasing with $\alpha$, $K$, $\gamma$, $\tau$, and nonincreasing with $s$, $\beta$, $B$, $c_r$, $c_m$.*

We omit the proof of Corollary 2 since it is straightforward by Proposition 2. As for the manufacturer's optimal pricing strategy, $p^*$ is nonincreasing with $K$, $\gamma$, $\tau$ and nondecreasing with $s$, $B$, $c_r$, $c_m$ due to the linear relationship between $p^*$ and $q^*$. Similarly, we can also prove that $p^*$ is nondecreasing with $\alpha$ and nonincreasing with $\beta$.

### 3.3. Manufacturer's optimal strategies with $s > 1$

In this section, we investigate the manufacturer's strategy under the scenario that remanufacturing is more capacity intensive than manufacturing, i.e., $s > 1$. We first analyze the NYOP mechanism and then the list-price mechanism.

#### 3.3.1. NYOP mechanism with $s > 1$

Similar to the case of $s \le 1$, the optimal strategy is derived from Equation (5). The local optimal solutions are the same to the case of $s \le 1$ (see Table 2). However, the optimality conditions are different due to $s$. Denote $\alpha_7 = \beta c_m + 2K + \frac{2\beta\gamma(c_m - c_r) - \beta(A + \lambda B + c)}{2(s-1)\gamma}$. Here, we directly give the optimal strategies in the following proposition.

**Proposition 3.** *When $s > 1$, the manufacturer's optimal strategies under the NYOP mechanism are as follows.*

(1) *If $d \ge 2\gamma(c_m - c_r) - A - \lambda B$ and $K \le \frac{2s\gamma^2(c_m - c_r) - s\gamma(A + \lambda B + c)}{d - c}$, or if $d < 2\gamma(c_m - c_r) - A - \lambda B$ and $K \le s\gamma$, then when $\alpha < \alpha_5$, Case 5 is optimal. When $\alpha \in [\alpha_5, \alpha_2]$, Case 7 is optimal. When $\alpha \in [\alpha_2, \alpha_7]$, Case 3 is optimal. Otherwise, $q^* = K$ and $q_r^* = 0$.*
(2) *If $d \ge 2\gamma(c_m - c_r) - A - \lambda B$ and $K \ge \frac{2s\gamma^2(c_m - c_r) - s\gamma(A + \lambda B + c)}{d - c}$, then when $\alpha < \alpha_4$, Case 5 is optimal. When $\alpha \in [\alpha_4, \alpha_1]$, Case 1 is optimal. When $\alpha \in [\alpha_1, \alpha_7]$, Case 3 is optimal. Otherwise, $q^* = K$ and $q_r^* = 0$.*
(3) *If $d < 2\gamma(c_m - c_r) - A - \lambda B$ and $K \ge s\gamma$, then when $\alpha < \alpha_6$, Case 5 is optimal. When $\alpha \in [\alpha_6, \beta c_m + 2\gamma]$, Case 6 is optimal. When $\alpha \in [\beta c_m + 2\gamma, \beta c_m + 2K + 2(1 - s)\gamma]$, Case 2 is optimal. When $\alpha \in [\beta c_m + 2K + 2(1 - s)\gamma, \alpha_3]$, Case 4 is optimal. When $\alpha \in [\alpha_3, \alpha_7]$, Case 3 is optimal. Otherwise, $q^* = K$ and $q_r^* = 0$.*

For a clear view of the manufacturer's optimal strategies, Figs. 5 and 6 show different regions in terms of $K$ and $\alpha$. Similar to the case of $s \le 1$, there are also two subcases that $d < 2\gamma(c_m - c_r) - A - \lambda B$ and $d \ge 2\gamma(c_m - c_r) - A - \lambda B$. Note that, when $s > 1$, $\alpha_2$ may

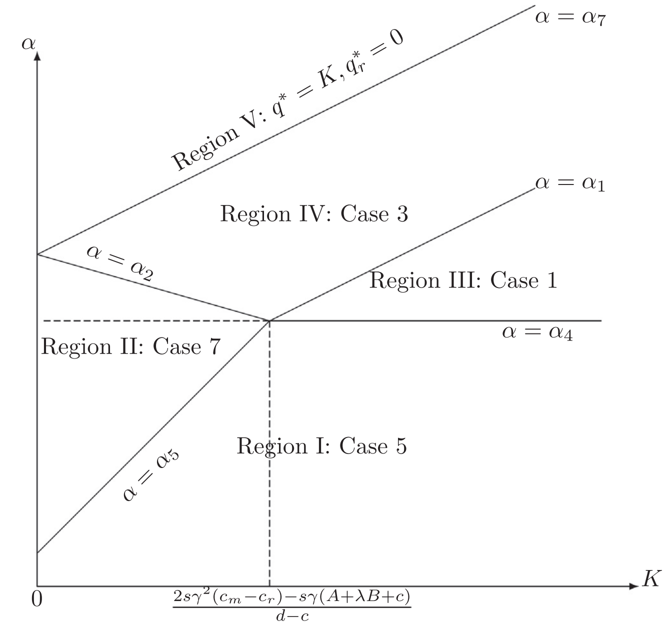


**Fig. 5.** Regions of optimal NYOP strategies when $s > 1$ and $d \ge 2\gamma(c_m - c_r) - A - \lambda B$.

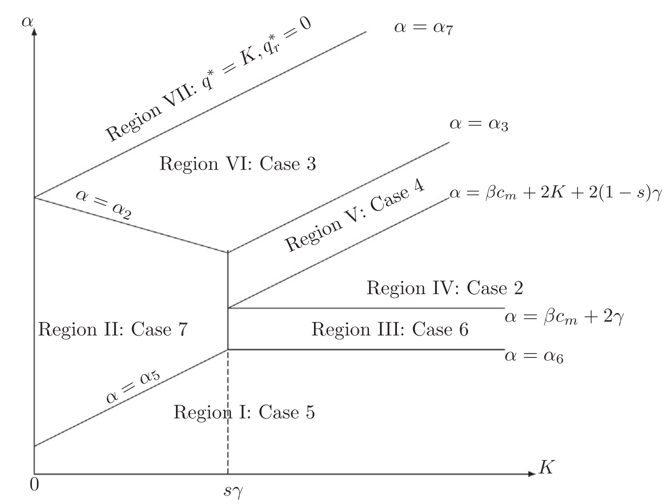


**Fig. 6.** Regions of optimal NYOP strategies when $s > 1$ and $d < 2\gamma(c_m - c_r) - A - \lambda B$.

not always increase with $K$ and therefore the lines of $\alpha = \alpha_2$ in Figs. 5 and 6 are only representative. Specifically, when $s > \frac{4\gamma^2 + \beta(d - c)}{4\gamma^2}$, $\alpha_2$ increases with $K$, otherwise, $\alpha_2$ decreases with $K$.

In Fig. 5, when $d \ge 2\gamma(c_m - c_r) - A - \lambda B$, it is never optimal to recycle and remanufacture all returns. That is why Cases 2, 4, 6 do not show up. The optimal strategies in Regions I, II, III are the same to those regions in Fig. 2. In Region IV, since the market scale is rather large with respect to the capacity, the optimal strategy is to utilize all capacity to produce. Since $s > 1$, manufacturing is preferred. If the market further enlarges as in Region V, the manufacturer takes the strategy of pure manufacturing to yield the most products. In Fig. 6, the optimal strategies in Regions I, II, III, IV, V are the same to those regions in Fig. 3. In addition, the explanation of Regions VI, VII is similar to that in Fig. 5 as stated above.

Similarly to the case of $s \le 1$, we obtain the effect of key parameters on optimal decisions as follows.

**Corollary 3.** (1) *when $s > 1$, $q_r^*$ is nondecreasing with $K$, $c_m$, $\gamma$ and nonincreasing with $A$, $B$, $c$, $d$, $c_r$, $\lambda$, $s$.*

(2) *when $s > 1$, $q^*$ is nondecreasing with $K$, $\alpha$, and nonincreasing with $c_m$, $\beta$.*

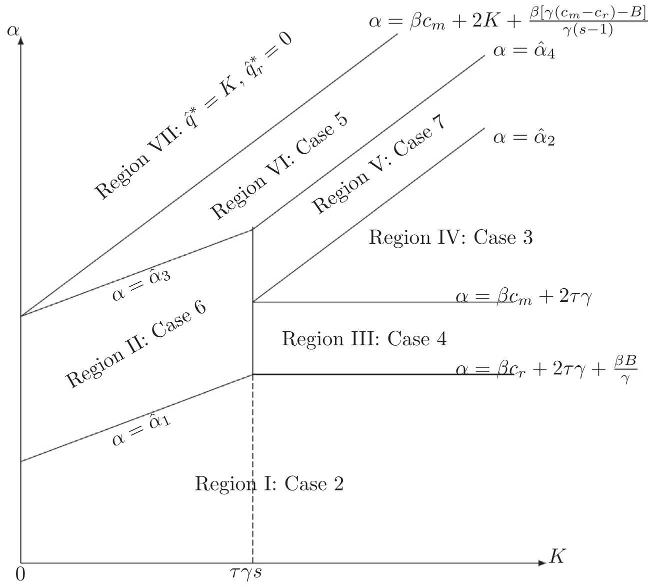


**Fig. 7.** Regions of manufacturer's optimal list-price strategies when $s > 1$.

From Corollary 3, we conclude that the optimal price $p^*$ is nonincreasing with $K$ and nondecreasing with $c_m$. In addition, we prove that $p^*$ is nondecreasing with $\alpha$ and nonincreasing with $\beta$.

### 3.3.2. List-price mechanism with $s > 1$

In the section, we investigate the optimal strategies under the list-price mechanism when remanufacturing consumes more capacity than manufacturing. We obtain the same potential solutions to the case of $s \le 1$ (see Table 5) and then the optimality conditions. Denote $\hat{\alpha}_3 = \beta c_m + \frac{2K}{s} + \frac{\beta[\gamma(c_m - c_r) - B]}{(s-1)\gamma}$ and $\hat{\alpha}_4 = \beta c_m + 2K + 2(1-s)\tau\gamma + \frac{\beta[\gamma(c_m - c_r) - B]}{(s-1)\gamma}$, and we obtain:

**Proposition 4.** *When $s > 1$, the manufacturer's optimal strategies under the list-price mechanism are given as follows:*

(1) *If the capacity is low, i.e., $K \le \tau\gamma s$, then when $\alpha < \hat{\alpha}_1$, Case 2 is optimal. When $\alpha \in [\hat{\alpha}_1, \hat{\alpha}_3]$, Case 6 is optimal. When $\alpha \in [\hat{\alpha}_3, \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}]$, Case 5 is optimal. Otherwise, $\hat{q}^* = K$, and $\hat{q}_r^* = 0$.*
(2) *If the capacity is high, i.e., $K > \tau\gamma s$, then when $\alpha \le \beta c_r + 2\tau\gamma + \frac{\beta B}{\gamma}$, Case 2 is optimal. When $\alpha \in [\beta c_r + 2\tau\gamma + \frac{\beta B}{\gamma}, \beta c_m + 2\tau\gamma]$, Case 4 is optimal. When $\alpha \in [\beta c_m + 2\tau\gamma, \hat{\alpha}_2]$, Case 3 is optimal. When $\alpha \in [\hat{\alpha}_2, \hat{\alpha}_4]$, Case 7 is optimal. When $\alpha \in [\hat{\alpha}_4, \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}]$, Case 5 is optimal. Otherwise, $\hat{q}^* = K$, and $\hat{q}_r^* = 0$.*

The results are illustrated in Fig. 7 for a better understanding. In Region I, because the market is rather small, the manufacturer only remanufactures some available returns with some surplus capacity. In Region II, although the market expands, due to the limited capacity, it is optimal to do remanufacturing with the whole capacity. In Region III, as the capacity enlarges, the manufacturer has a sufficient capacity to recycle all available returns. It is optimal to recycle all returns for remanufacturing but not to manufacture new products since the market scale is not that large. In Region IV, as the market booms, besides remanufacturing, it is necessary to produce a certain amount of new products. In this region, there is still some remaining capacity. In Region V, as the market further booms, all available returns are recycled and all capacity is utilized to manufacture and remanufacture. In Region VI, as the market further expands, more products are needed. As a result, it is optimal to utilize all capacity for manufacturing and remanufacturing. In addition, Region VI can be divided into two subregions that $K < \tau\gamma s$ and $K \ge \tau\gamma s$. In the subregion that $K \ge \tau\gamma s$, it is interesting that the manufacturer only remanufactures some available returns even though the market is large and he has a sufficient capacity to remanufacture all available returns due to the capacity-intensive remanufacturing. In Region VII, if the market scale is extremely large, the manufacturer abandons remanufacturing and take the strategy of pure manufacturing. This strategy yields the maximal profit.

We also investigate the effect of parameters on optimal strategies under the list-price mechanism as follows:

**Corollary 4.** (1) *When $s > 1$, $\hat{q}_r^*$ is nondecreasing with $K$, $\gamma$, $\tau$, $c_m$ and nonincreasing with $s$, $B$, $c_r$.*

(2) *When $s > 1$, $\hat{q}^*$ is nondecreasing $\alpha$, $K$, and nonincreasing with $\beta$ and $c_m$.*

From Corollary 4, under list-price mechanism and when $s > 1$, $p^*$ is obviously nonincreasing with $K$ and nondecreasing with $c_m$. In addition, we can also prove that $p^*$ is nondecreasing with $\alpha$ and nonincreasing with $\beta$.

### 3.4. Comparison of $s \le 1$ and $s > 1$

In this section, we compare the manufacturer's strategies under different cases of $s \le 1$ and $s > 1$. Based on Propositions 1–4, we obtain these following results.

**Observation 1.** *When $s \le 1$, the manufacturer utilizes remanufacturing in priority. However, when $s > 1$, the manufacturer may not utilize remanufacturing. Specifically, when $s > 1$, under the NYOP mechanism, the manufacturer remanufactures nothing if $\alpha \ge \beta c_m + 2K + \frac{2\beta\gamma(c_m - c_r) - \beta(A + \lambda B + c)}{2(s-1)\gamma}$, and under the list-price mechanism, the manufacturer remanufactures nothing if $\alpha \ge \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}$.*

Under the NYOP mechanism, remanufacturing process has the advantage of saving cost at the beginning since $c_m - c_r > \frac{A + \lambda B + c}{2\gamma}$. Similarly, under the list-price mechanism, remanufacturing process also has the advantage of saving cost since $c_m - c_r > \frac{B}{\gamma}$. Therefore, when $s \le 1$, manufacturing process is dominated by remanufacturing process and remanufacturing process certainly occurs. However, when $s > 1$, the remanufacturing process has the disadvantage of consuming more capacity. Due to the trade-off between the cost saving and capacity limit, remanfuacturing is not always preferred compared with manufacturing. Under some conditions where saving capacity brings about more profit than saving production cost, remanufacturing is not preferred and only manufacturing process occurs. Observation 1 gives these exact conditions and shows a higher $s$, a larger $\alpha$ and a smaller $K$ may motive the manufacturer to adopt the strategy of pure manufacturing.

Based on Observation 1, an interesting finding is that if the manufacturer adopts the strategy of pure manufacturing under the list-price mechanism, then he will also adopt the same strategy under the NYOP mechanism. Because of $c \ge (2-\lambda)B - A$ in Assumption 1, we obtain $\frac{2\beta\gamma(c_m - c_r) - \beta(A + \lambda B + c)}{2(s-1)\gamma} \le \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}$. Therefore, when $\alpha \ge \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}$, we obtain $\alpha \ge \beta c_m + 2K + \frac{2\beta\gamma(c_m - c_r) - \beta(A + \lambda B + c)}{2(s-1)\gamma}$.

In addition, we also observe that the effect of some key parameters are different under the case of $s \le 1$ and $s > 1$.

**Observation 2.** *Under the NYOP mechanism, some parameters have opposite effect on optimal decisions with $s$ converting from $s \le 1$ to $s > 1$. When $s \le 1$, $A$, $B$, $c$, $d$ have negative effect on total quantity in case 3 while the effect is positive when $s > 1$.*

Obviously, higher $A$, $B$, $c$, $d$ bring about obstacle for recycling which is independent with the fact whether $s \le 1$ or $s > 1$. In Case

3 where $q=K+(1-s)q_r$, $q$ increases with $q_r$ if $s\le 1$ while decreases with $q_r$ if $s>1$. Therefore, the effect of $A$, $B$, $c$, $d$ on $q$ is opposite for the case of $s\le 1$ and $s>1$. Based on this observation, we find that when $s>1$, higher $A$, $B$, $c$, $d$ may be beneficial to consumers since they can purchase cheaper products although the manufacturer's profit may be reduced.

## 4. Numerical study

In this section, we first perform sensitivity analysis with respect to several key parameters and perform a comparison of the NYOP mechanism and the list-price mechanism. To guarantee the assumptions, the basic parameters are set as $\alpha = 3.0$, $\beta = 0.2$, $c_m=3.5$, $c_r=2.0$, $\gamma = 0.8$, $\lambda = 0.8$, $A=1.5$, $B=0.5$, $c=0$, $d=1.0$, $K=1.2$. $s=0.9$ represents the case that remanufacturing needs less capacity than manufacturing and $s=1.5$ represents the case that remanufacturing needs more capacity. In each numerical experiment, some parameters are changed with a clear statement; otherwise, they are the same to the basic values.

### 4.1. Sensitivity analysis under the NYOP mechanism

We first investigate the effect of NYOP bidding cost ($c$ and $d$) on the manufacturer's profit and return rate. In the numerical experiments, to comply with those assumptions stated in Section 3, $c$ is varied between 0 and 0.4 and $d$ is set to be 0.6, 1.0.

Fig. 8 shows that the profit decreases with $c$ and $d$. As $c$ or $d$ increases, the average bidding cost $\frac{d+c}{2}$ increases. Consequently, a relatively high cost occurs to consumers for returning used items under the NYOP mechanism. In order to attract them to participate in this auction, the manufacturer has to make compensation for such costs , which results in the decrease of its own profit. We also find this decrease is milder as $s$ increases from 0.9 to 1.5.

Second, we investigate the effect of the bounds of reserve price ($B$ and $A$). As stated in Table 1, $[B,A]$ represents the bounds of the reserve price. In the experiments, $B$ is set to be 0.1, 0.5, and $A$ is varied in the interval $[1.5, 1.9]$.

Fig. 9 shows that both the profit and return rate decrease with $A$ and $B$. Since the optimal bid from a consumer is given by $\frac{A+\lambda B+\theta}{2}$, it is easy to see that the bid increases with $A$ or $B$. Since the recycling activities become more expensive, the manufacturer has less motivation to recycle and thus the return rate is low. To gain more profit, the manufacturer should reveal certain information about the reserve price to consumers.

Third, we investigate the effect of the ratio of consumers switching to the list-price channel ($\lambda$) on optimal decisions. Since the consumer's optimal bid equals $\frac{A+\lambda B+\theta}{2}$ which increases with $\lambda$, the manufacturer's recycling cost obviously increases with $\lambda$. Further, $D_{NYOP}=\frac{2R-A-\lambda B-c}{d-c}$ indicates that a higher $\lambda$ yields a lower return rate. Fig. 10 shows how $\lambda$ influences the manufacturer's profit and return rate in equilibrium. A higher $\lambda$ reduces the manufacturer's profit and return rate, which indicates that the manufacturer should put more efforts in attracting customers, such as advertisement.

Fourth, we investigate the effect of the relative capacity requirement of remanufacturing ($s$). Here, $K=0.8$ and $s$ is varying between 0.1 and 1.9. In Fig. 11, we find that the manufacturer's profit concavely decreases with $s$, which indicates that capacity-intensive remanufacturing hurts the manufacturer's profit. In Fig. 11, when $s=0.7$ or $s=0.8$, the manufacturer recycles the most returns. The numerical experiments demonstrate that when $s$ is rather small, $q_r^*$ increases with $s$. On the contrary, when $s$ is rather large, $q_r^*$ decreases. Further, when $s$ is larger than 1, no returns are collected under such an experimental setting.

### 4.2. Comparison of the NYOP mechanism and the list-price mechanism

Under different settings of key parameters, including $s$, $c_m$, $K$ and $\tau$, we first compare the performance difference between the NYOP mechanism and the list-price mechanism, and investigate under what conditions the NYOP mechanism is more beneficial than the list-price mechanism. For the experiments, $s$ is set to be 0.9, 1.5, $c_m$ is set to be 3.5, 5.5, $K$ is set to be 0.3, 0.8, 1.3, and $\tau$ is set to be 0.2, 0.4, 0.6, 0.8. Note that the superscripts $N$ and $L$ denote the notation for the NYOP mechanism and the list-price mechanism, respectively.

The comparison of the profit is presented in Table 6. Table 6 shows that the profit under both mechanisms is nonincreasing with $c_m$ and nondecreasing with $K$. Since a higher $c_m$ increases manufacturing cost and the cost of capacity expansion is ignored in this model, this result is intuitive. Moveover, the NYOP mechanism shows its advantage compared with the list-price

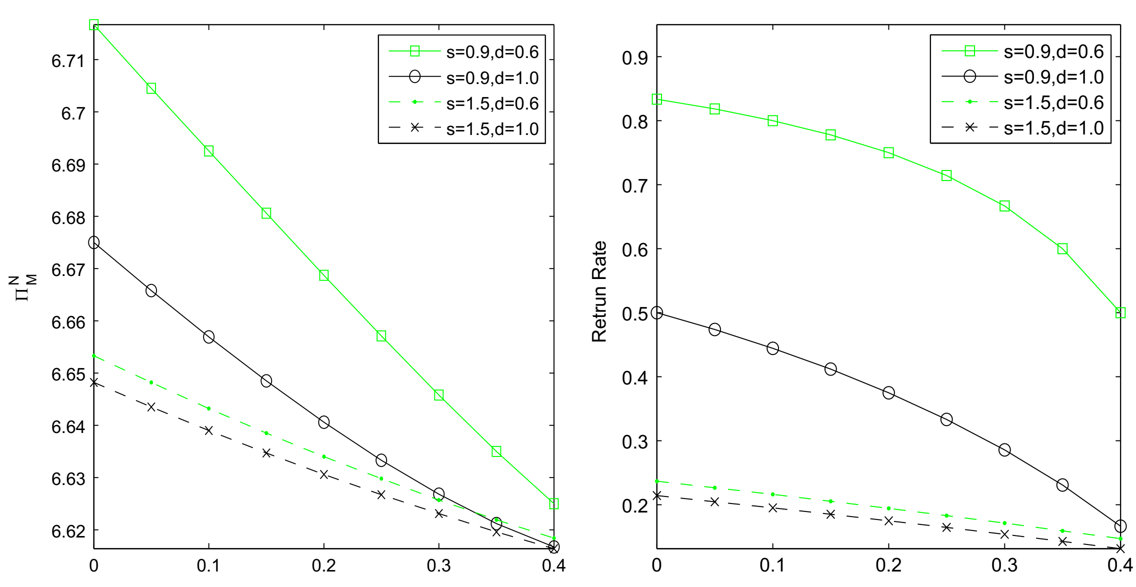


**Fig. 8.** The effect of bidding cost on the profit and return rate.

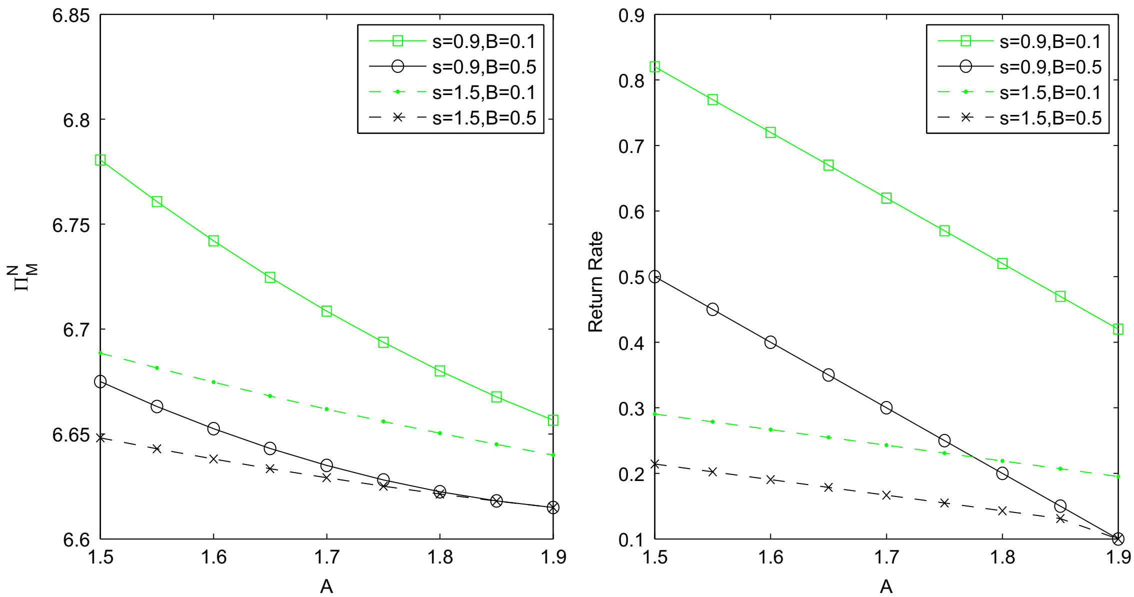


**Fig. 9.** The effect of reserve price on the profit and return rate.

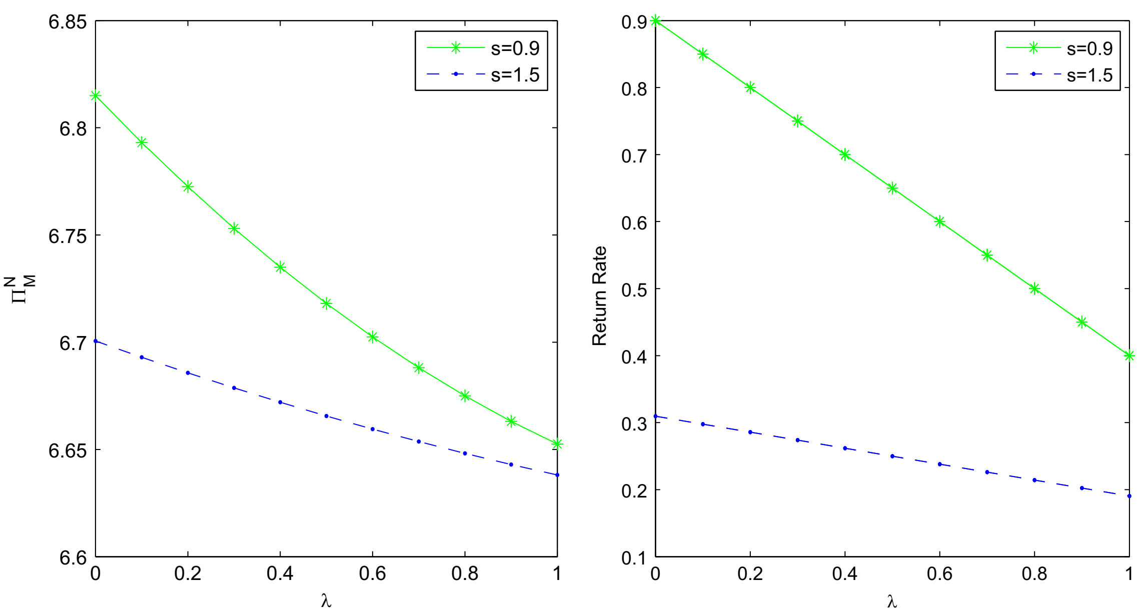


**Fig. 10.** The effect of $\lambda$ on the profit and return rate.

mechanism under the following four conditions. First, when the market share of the manufacturer in the reverse market ($\tau$) is small, we find that for $\tau = 0.2$, the NYOP mechanism gains a higher profit among most of cases. In fact, when $\tau$ is small, under the list-price mechanism, the return resources are limited which increases production cost. It indicates that if the manufacturer has a small market share in the reverse market, the NYOP mechanism is a better choice. Second, when the manufacturing cost ($c_m$) is high, Table 6 shows that for $c_m$=5.5, the NYOP mechanism brings the manufacturer a higher profit under eleven out of twenty-four cases. When manufacturing is more expensive than remanufacturing, the more the manufacturer recycles and remanufactures, the more he saves production cost. Third, a high capacity $K$ is beneficial to the NYOP mechanism. The reason is that a sufficient capacity guarantees that the manufacturer can achieve the maximal cost saving of remanufacturing, which motivates the manufacturer to collect more returns by using the NYOP mechanism. Fourth, when the capacity requirement of remanufacturing is relatively low, i.e., when $s$ is small. Table 6 shows that for $s > 1$ there are less cases under which the NYOP mechanism results in a higher profit than the list-price mechanism. The reason is that as $s$ becomes larger, the cost saving of remanufacturing is offsetted by the intensive capacity requirement.

Next, we compare the return rate under the NYOP and the list-price mechanism shown in Table 7. An observation is that when $s \le 1$, for most cases, the return rate is equal to the market share which indicates the manufacturer would like to recycle as many as possible. Another interesting finding is the effect of $K$ on return rate under the NYOP mechanism. We observe that a rather low capacity may hinder the manufacturer to recycle many and thus $r^N|K = 0.3$ is lower than $r^N|K = 0.8$ for $s$=0.9. Intuitively, a high capacity results in a high return rate. However, by Table 7, we find

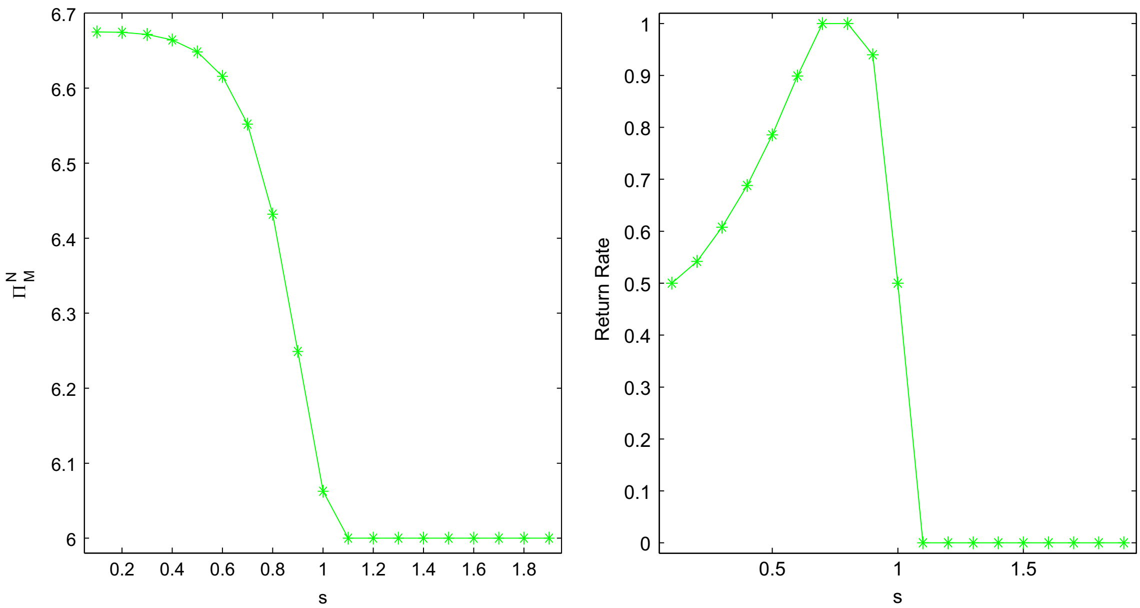


**Fig. 11.** The effect of $s$ on the profit and return rate.

**Table 6**
Manufacturer's profit under the NYOP mechanism and the list-price mechanism.

| $s$ | $c_m$ | $K$ | Profit | $\tau$ | | | |
|---|---|---|---|---|---|---|---|
| | | | | 0.2 | 0.4 | 0.6 | 0.8 |
| 0.9 | 3.5 | 0.3 | $\Pi_M^N/\Pi_M^L$ | **3.339** / 3.275 | 3.339/ 3.547 | 3.339 / 3.569 | 3.339 / 3.569 |
| 0.9 | 3.5 | 0.8 | $\Pi_M^N/\Pi_M^L$ | **6.249** / 6.195 | 6.249 / 6.387 | 6.249/ 6.577 | 6.249 / 6.764 |
| 0.9 | 3.5 | 1.3 | $\Pi_M^N/\Pi_M^L$ | 6.675 / 6.753 | 6.675 / 6.893 | 6.675 / 7.033 | 6.675 / 7.173 |
| 0.9 | 5.5 | 0.3 | $\Pi_M^N/\Pi_M^L$ | **3.339** / 2.963 | 3.339 / 3.523 | 3.339 / 3.569 | 3.339 /3.569 |
| 0.9 | 5.5 | 0.8 | $\Pi_M^N/\Pi_M^L$ | **6.088** /4.883 | **6.088** / 5.363 | **6.088** / 5.841 | 6.088 / 6.316 |
| 0.9 | 5.5 | 1.3 | $\Pi_M^N/\Pi_M^L$ | **6.113** / 4.973 | **6.113** / 5.433 | **6.113** / 5.893 | 6.113 / 6.353 |
| 1.5 | 3.5 | 0.3 | $\Pi_M^N/\Pi_M^L$ | 3.000 / 3.000 | 3.000 / 3.000 | 3.000 / 3.0000 | 3.000 / 3.000 |
| 1.5 | 3.5 | 0.8 | $\Pi_M^N/\Pi_M^L$ | 6.000 / 6.000 | 6.000 / 6.000 | 6.000 / 6.000 | 6.000 / 6.000 |
| 1.5 | 3.5 | 1.3 | $\Pi_M^N/\Pi_M^L$ | 6.672 / 6.753 | 6.672 / 6.892 | 6.672 / 6.992 | 6.672 / 7.028 |
| 1.5 | 5.5 | 0.3 | $\Pi_M^N/\Pi_M^L$ | 2.400 / 2.400 | 2.400 / 2.400 | 2.400 / 2.400 | 2.400 / 2.400 |
| 1.5 | 5.5 | 0.8 | $\Pi_M^N/\Pi_M^L$ | **4.772** / 4.708 | 4.772 / 4.952 | 4.772 / 5.132 | 4.772 / 5.178 |
| 1.5 | 5.5 | 1.3 | $\Pi_M^N/\Pi_M^L$ | **6.100** / 4.973 | **6.100** / 5.433 | **6.100**/ 5.893 | 6.100 / 6.353 |

In the cases that the profit is higher under the NYOP mechanism, the numbers are in bold.

that when $c_m$=3.5, $r^N|K=1.3$ is lower than $r^N|K=0.8$. Thus, we conclude that on one hand, a rather low capacity hinders the manufacturer's enthusiasm to recycle. On the other hand, a rather high capacity also reduces the return rate due to the concern of cannibalization of new products. Further, we compare the return rates under these two different mechanisms and find that $r^N$ tends to be higher than $r^L$ with a low reverse market share, a high manufacturing cost. In addition, we observe a lower return rate under both mechanisms as $s$ increases to 1.5.

Next, we compare the retail price under the NYOP and the list-price mechanism shown in Table 8. We find that when $s \le 1$, the NYOP mechanism yields a lower retail price in more cases than $s > 1$. Combining with Table 6, we find that in some cases with a low market share, a low capacity requirement, and a high manufacturing cost ($\tau=0.2$, $s$=0.9, $c_m$=5.5), the manufacturer gains more profit under the NYOP mechanism. An interesting finding is that, for $s$=0.9 and $K$=0.8, as $c_m$ increases to 5.5, $p^N$ decreases which indicates the manufacturer may produce more products as cost increases. Since the total quantity is nondecreasing with $K$, the retail price is therefore nonincreasing with $K$ under both the NYOP mechanism and the list-price mechanism.

## 5. Concluding remarks

This paper characterizes the optimal remanufacturing strategies in a remanufacturing/manufacturing system under different recycling pricing mechanism, including the NYOP mechanism and the list-price mechanisms. These pricing and production decisions are made with a limited capacity. We study two different cases that the remanufacturing process needs less or more capacity than manufacturing. The main conclusions are as follows. First, we find the manufacturer's optimal NYOP strategy depends on the bidding cost, the cost saving, the capacity, and the market scale. Second, under the NYOP mechanism, when remanufacturing needs less capacity, the remanufactured quantity may decrease. Similarly, a larger capacity may yield a lower return rate. In addition, the total quantity may increase with manufacturing cost. Third, when remanufacturing process needs less capacity than manufacturing, remanufacturing always occurs and bidding cost has a negative effect on the total quantity. However, when remanufacturing process needs more capacity than manufacturing, the manufacturer may adopt the strategy of pure manufacturing and bidding cost may have a positive effect on total quantity. Fourth, we

**Table 7**
Return rate under the NYOP Mechanism and the List-Price Mechanism.

| $s$ | $c_m$ | $K$ | Return rate | $\tau$ | | | |
|---|---|---|---|---|---|---|---|
| | | | | 0.2 | 0.4 | 0.6 | 0.8 |
| 0.9 | 3.5 | 0.3 | $r^N/r^L$ | **0.417** / 0.200 | **0.417** / 0.400 | **0.417** / 0.400 | **0.417**/ 0.400 |
| 0.9 | 3.5 | 0.8 | $r^N/r^L$ | **0.940** / 0.200 | **0.940** / 0.400 | **0.940** / 0.600 | **0.940** / 0.800 |
| 0.9 | 3.5 | 1.3 | $r^N/r^L$ | **0.500** / 0.200 | **0.500** / 0.400 | 0.500 / 0.600 | 0.500 / 0.800 |
| 0.9 | 5.5 | 0.3 | $r^N/r^L$ | **0.417** / 0.200 | **0.417** / 0.400 | 0.417 / 0.417 | 0.417 / 0.417 |
| 0.9 | 5.5 | 0.8 | $r^N/r^L$ | **1.000** / 0.200 | **1.000** / 0.400 | **1.000** / 0.600 | **1.000** / 0.800 |
| 0.9 | 5.5 | 1.3 | $r^N/r^L$ | **1.000** / 0.200 | **1.000** / 0.400 | **1.000** / 0.600 | **1.000** / 0.800 |
| 1.5 | 3.5 | 0.3 | $r^N/r^L$ | 0.000 / 0.000 | 0.000 / 0.000 | 0.000 / 0.000 | 0.000 / 0.000 |
| 1.5 | 3.5 | 0.8 | $r^N/r^L$ | 0.000 / 0.000 | 0.000 / 0.000 | 0.000 / 0.000 | 0.000 / 0.000 |
| 1.5 | 3.5 | 1.3 | $r^N/r^L$ | **0.405** / 0.200 | **0.405** / 0.400 | 0.405 / 0.600 | 0.405 / 0.800 |
| 1.5 | 5.5 | 0.3 | $r^N/r^L$ | 0.000 / 0.063 | 0.000 / 0.063 | 0.000 / 0.063 | 0.000 / 0.063 |
| 1.5 | 5.5 | 0.8 | $r^N/r^L$ | **0.595** / 0.200 | **0.595** / 0.400 | 0.595 / 0.600 | 0.595 / 0.667 |
| 1.5 | 5.5 | 1.3 | $r^N/r^L$ | **1.000** / 0.200 | **1.000** / 0.400 | **1.000** / 0.600 | **1.000** / 0.800 |

In the cases that the return rate is higher under the NYOP mechanism, the numbers are in bold.

**Table 8**
Retail Price under the NYOP Mechanism and the list-price mechanism.

| $s$ | $c_m$ | $K$ | Retail price | $\tau$ | | | |
|---|---|---|---|---|---|---|---|
| | | | | 0.2 | 0.4 | 0.6 | 0.8 |
| 0.9 | 3.5 | 0.3 | $p^N/p^L$ | 13.333 / 13.420 | 13.333 / 13.340 | 13.333 / 13.333 | 13.333 / 13.333 |
| 0.9 | 3.5 | 0.8 | $p^N/p^L$ | 10.624 / 10.920 | 10.624 / 10.840 | 10.624 / 10.760 | 10.624 / 10.680 |
| 0.9 | 3.5 | 1.3 | $p^N/p^L$ | 9.250 / 9.250 | 9.250 / 9.250 | 9.250 / 9.250 | 9.250 / 9.250 |
| 0.9 | 5.5 | 0.3 | $p^N/p^L$ | 13.333 /13.420 | 13.333 / 13.340 | 13.333 / 13.333 | 13.333 / 13.333 |
| 0.9 | 5.5 | 0.8 | $p^N/p^L$ | 10.600 / 10.920 | 10.600 / 10.840 | 10.600 / 10.760 | 10.600 / 10.680 |
| 0.9 | 5.5 | 1.3 | $p^N/p^L$ | 10.250 / 10.250 | 10.250 / 10.250 | 10.250 / 10.250 | 10.250 / 10.250 |
| 1.5 | 3.5 | 0.3 | $p^N/p^L$ | 13.500 / 13.500 | 13.500 / 13.500 | 13.500 / 13.500 | 13.500 / 13.500 |
| 1.5 | 3.5 | 0.8 | $p^N/p^L$ | 11.000 / 11.000 | 11.000 / 11.000 | 11.000 / 11.000 | 11.000 / 11.000 |
| 1.5 | 3.5 | 1.3 | $p^N/p^L$ | **9.310** / 9.250 | **9.310** / 9.300 | 9.310 / 9.700 | 9.310 / 10.100 |
| 1.5 | 5.5 | 0.3 | $p^N/p^L$ | 13.500 / 13.500 | 13.500 / 13.500 | 13.500 / 13.500 | 13.500 / 13.500 |
| 1.5 | 5.5 | 0.8 | $p^N/p^L$ | **12.191** / 11.400 | **12.191** / 11.800 | 12.191 / 12.200 | 12.191 / 12.333 |
| 11.5 | 5.5 | 1.3 | $p^N/p^L$ | **10.500** / 10.250 | **10.500** / 10.250 | **10.500** / 10.250 | **10.500** / 10.250 |

In the cases that the retail price is higher under the NYOP mechanism, the numbers are in bold.

make a comparison between the NYOP mechanism and the list-price mechanism through numerical experiments. We find that under certain conditions, such as a low reverse market share, a high manufacturing cost, a high capacity, and a low capacity requirement of remanufacturing, the manufacturer prefers the NYOP mechanism.

The current study has certain limitations. The interesting direction may be to consider that new products and remanufactured products are heterogeneous in the forward market. That is, they are sold with different retail prices. Another direction is to address demand uncertainty and investigate whether our findings still hold under a stochastic demand setting.

## Acknowledgments

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## Appendix A. The manufacturer's optimal strategies under the NYOP mechanism

**Proof of Proposition 1.** The manufacturer decides $q$ and $q_r$ to optimize his profit function in Eq. (5). Since the profit function is joint concave in $q$ and $q_r$ and these constraints are all linear with $q$ and $q_r$, we use the KKT method to solve this problem. The Lagrangian function is as follows:

$$L(q,q_r,\lambda_1,\lambda_2)=\frac{(\alpha-q)q}{\beta}-c_m q+\left(c_m-c_r-\frac{A+\lambda B+c}{2\gamma}\right)q_r -\frac{q_r^2(d-c)}{4\gamma^2}+\lambda_1(q-q_r) +\lambda_2[K+(1-s)q_r-q]+\lambda_3(\gamma-q_r) \tag{A.1}$$

$$s.t \quad \frac{\partial L(q_r,q_r,\lambda_1,\lambda_2)}{\partial q}=\frac{\alpha-2q}{\beta}-c_m+\lambda_1-\lambda_2=0 \tag{A.2}$$

$$\frac{\partial L(q,q_r,\lambda_1,\lambda_2)}{\partial q_r}=c_m-c_r-\frac{A+\lambda B+c}{2\gamma}-\frac{q_r(d-c)}{2\gamma^2} -\lambda_1+\lambda_2(1-s)-\lambda_3=0 \tag{A.3}$$

$$\lambda_1(q-q_r)=0 \tag{A.4}$$

$$\lambda_2[K+(1-s)q_r-q]=0 \tag{A.5}$$

$$\lambda_3(\gamma - q_r) = 0 \tag{A.6}$$

$$q_r \le q \tag{A.7}$$

$$q - (1-s)q_r \le K \tag{A.8}$$

$$q_r \le \gamma \tag{A.9}$$

From (A.2)–(A.6), we can obtain these feasible solutions as follows.

*Case* 1: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, q_r = \frac{\gamma(2\gamma c_m - 2\gamma c_r - A - \lambda B - c)}{d-c}, q = \frac{\alpha - \beta c_m}{2}$

By (A.7), we obtain $\alpha > \frac{4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)}{d-c} + \beta c_m$. By (A.9), we obtain $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$. By (A.8), we obtain $\alpha \le 2K + \beta c_m + \frac{2(1-s)\gamma(2\gamma c_m - 2\gamma c_r - A - \lambda B - c)}{d-c}$.

To guarantee this case, $2K + \beta c_m + \frac{2(1-s)\gamma(2\gamma c_m - 2\gamma c_r - A - \lambda B - c)}{d-c} > \frac{4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)}{d-c} + \beta c_m$ must hold. It is equivalent to $K \ge \frac{2s\gamma^2(c_m - c_r) - s\gamma(A + \lambda B + c)}{d-c}$. The integral conditions for this case to be globally optimal are as follows: $K \ge \frac{2s\gamma^2(c_m - c_r) - s\gamma(A + \lambda B + c)}{d-c}, c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$ and $\frac{4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)}{d-c} + \beta c_m \le \alpha \le 2K + \beta c_m + \frac{2(1-s)\gamma(2\gamma c_m - 2\gamma c_r - A - \lambda B - c)}{d-c}$.

*Case* 2: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = \frac{2\gamma c_m - 2\gamma c_r - A - \lambda B - d}{2\gamma}, q_r = \gamma, q = \frac{\alpha - \beta c_m}{2}$

By $\lambda_3 > 0$, we obtain $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$. By (A.7), we obtain $\alpha \ge \beta c_m + 2\gamma$. By (A.8), we obtain $\alpha \le \beta c_m + 2K + 2(1-s)\gamma$.

To guarantee this case optimal, these conditions must hold: $K \ge s\gamma, c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$ and $\beta c_m + 2\gamma \le \alpha \le \beta c_m + 2K + 2(1-s)\gamma$.

*Case* 3: $\lambda_1 = 0$,

$\lambda_2 = \frac{(d-c)(2K - \alpha + \beta c_m) + 2(-1+s)\gamma(A + \lambda B + c) + 4(1-s)\gamma^2(c_m - c_r)}{c\beta - d\beta - 4(-1+s)^2\gamma^2}$, $\lambda_3 = 0$,

$$q_r = \frac{\gamma[(1-s)(4K - 2\alpha)\gamma + \beta(A + \lambda B + c) + 2\beta\gamma(-sc_m + c_r)]}{c\beta - d\beta - 4(-1+s)^2\gamma^2},$$

$$q = K + \frac{(1-s)\gamma[(1-s)(4K - 2\alpha)\gamma + \beta(A + \lambda B + c) + 2\beta\gamma(-sc_m + c_r)]}{c\beta - d\beta - 4(-1+s)^2\gamma^2}.$$

By $\lambda_2 > 0$, we obtain $\alpha \ge \beta c_m + 2K + \frac{[4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)](1-s)}{d-c}$. By (A.7), we obtain $\alpha \le \frac{(d-c)K\beta + \gamma s\beta(A + \lambda B + c) - 2s\beta\gamma^2(sc_m - c_r)}{2(1-s)s\gamma^2} + \frac{2K}{s}$. By (A.9), we obtain $\alpha \le \frac{\beta(A + \lambda B + d) - 2s\beta\gamma c_m + 2\beta\gamma c_r}{2(1-s)\gamma} + 2(K + \gamma - s\gamma)$.

Define $\alpha_1 = \beta c_m + 2K + \frac{[4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)](1-s)}{d-c}$, $\alpha_2 = \frac{(d-c)K\beta + \gamma s\beta(A + \lambda B + c) - 2s\beta\gamma^2(sc_m - c_r)}{2(1-s)s\gamma^2} + \frac{2K}{s}$, and $\alpha_3 = \frac{\beta(A + \lambda B + d) - 2s\beta\gamma c_m + 2\beta\gamma c_r}{2(1-s)\gamma} + 2(K + \gamma - s\gamma)$. Since

$$\alpha_1 - \alpha_2 = \frac{[c\beta - d\beta - 4(-1+s)^2\gamma^2][dK + c(-K + s\gamma) + s\gamma(A + \lambda B) + 2s\gamma^2(-c_m + c_r)]}{2(c-d)(-1+s)s\gamma^2}. \tag{A.10}$$

$$\alpha_1 - \alpha_3 = \frac{[c\beta - d\beta - 4(-1+s)^2\gamma^2](A + \lambda B + d - 2\gamma c_m + 2\gamma c_r)}{2(c-d)(-1+s)\gamma}. \tag{A.11}$$

$$\alpha_2 - \alpha_3 = \frac{(K - s\gamma)[c\beta - d\beta - 4(-1+s)^2\gamma^2]}{2(-1+s)s\gamma^2}. \tag{A.12}$$

To guarantee (A.10) $\langle 0, K \rangle$ $\frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - \lambda B - c)}{d-c}$ holds. To guarantee (A.11) $< 0$, $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$ holds. Observe (A.12), to make this case optimal, either of the following conditions must hold: (1) $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $\frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c} \le K \le s\gamma$, $\alpha_1 \le \alpha \le \alpha_2$. (2)$c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $K > s\gamma$, $\alpha_1 \le \alpha \le \alpha_3$.

*Case* 4: $\lambda_1 = 0$, $\lambda_2 = \frac{-2K + \alpha + 2(-1+s)\gamma - \beta c_m}{\beta}$,

$\lambda_3 = \frac{2(1-s)\gamma[-2K + \alpha + 2(-1+s)\gamma] - \beta(A + \lambda B + d)}{2\beta\gamma} + sc_m - c_r$, $q_r = \gamma$, $q = K + \gamma - s\gamma$.

By $\lambda_2 > 0$, we obtain $\alpha > \beta c_m + 2K + 2(1-s)\gamma$. By $\lambda_3 > 0$, we obtain $\alpha > 2(K + \gamma - s\gamma) + \frac{\beta(A + \lambda B + d) - 2\beta\gamma(sc_m - c_r)}{2(1-s)\gamma} = \alpha_3$. By (A.7), we obtain $K \ge s\gamma$. Since

$$\beta c_m + 2K + 2(1-s)\gamma - \alpha_3 = \frac{\beta(A + \lambda B + d - 2\gamma c_m + 2\gamma c_r)}{(-1+s)\gamma}.$$

The integral conditions for this case are as follows: (1) $K > s\gamma$, $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, and $\alpha > \alpha_3$, or (2) $K > s\gamma$, $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$, and $\alpha > \beta c_m + 2K + 2(1-s)\gamma$.

*Case* 5: $\lambda_1 = \frac{(d-c)\alpha + 2\gamma(A + \lambda B + c) + (\beta c - \beta d - 4\gamma^2)c_m + 4\gamma^2 c_r}{c\beta - d\beta - 4\gamma^2}$, $\lambda_2 = 0, \lambda_3 = 0$,

$$q_r = \frac{\gamma(-2\alpha\gamma + \beta(A + \lambda B + c) + 2\beta\gamma c_r)}{c\beta - d\beta - 4\gamma^2},\ q = \frac{\gamma(-2\alpha\gamma + \beta(A + \lambda B + c) + 2\beta\gamma c_r)}{c\beta - d\beta - 4\gamma^2}.$$

By $\lambda_1 > 0$, we obtain $\alpha < \beta c_m + \frac{4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)}{d-c}$. By (A.8), we obtain $\alpha \le \frac{(d-c)K\beta + \gamma s\beta(A + \lambda B + c)}{2s\gamma^2} + \beta c_r + \frac{2K}{s}$. By (A.9), we obtain $\alpha < \beta c_r + 2\gamma + \frac{\beta(A + \lambda B + d)}{2\gamma}$.

Similarly, we define $\alpha_4 = \beta c_m + \frac{4\gamma^2(c_m - c_r) - 2\gamma(A + \lambda B + c)}{d-c}$, $\alpha_5 = \frac{(d-c)K\beta + \gamma s\beta(A + \lambda B + c)}{2s\gamma^2} + \beta c_r + \frac{2K}{s}$, and $\alpha_6 = \beta c_r + 2\gamma + \frac{\beta(A + \lambda B + d)}{2\gamma}$. We have:

$$\alpha_4 - \alpha_5 = \frac{(c\beta - d\beta - 4\gamma^2)[(d-c)K + s\gamma(A + \lambda B + c) + 2s\gamma^2(-c_m + c_r)]}{2(d-c)s\gamma^2}. \tag{A.13}$$

$$\alpha_4 - \alpha_6 = \frac{(c\beta - d\beta - 4\gamma^2)(A + \lambda B + d - 2\gamma c_m + 2\gamma c_r)}{2(d-c)\gamma}. \tag{A.14}$$

$$\alpha_5 - \alpha_6 = \frac{(K - s\gamma)(-c\beta + d\beta + 4\gamma^2)}{2s\gamma^2}. \tag{A.15}$$

Observe (A.13)–(A.15), and we can get the integral conditions which make Case 5 optimal: (1) $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $K \le \frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c}$, $\alpha < \alpha_5$. (2) $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $K \ge \frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c}$, $\alpha < \alpha_4$. (3) $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$, $K \le s\gamma$, $\alpha < \alpha_5$. (4) $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$, $K \ge s\gamma$, $\alpha < \alpha_6$. One of these conditions must hold.

*Case* 6:

$\lambda_1 = -\frac{\alpha - 2\gamma}{\beta} + c_m, \lambda_2 = 0, \lambda_3 = \frac{2(\alpha - 2\gamma)\gamma - \beta(A + \lambda B + d) - 2\beta\gamma c_r}{2\beta\gamma}, q_r = \gamma, q = \gamma.$

By $\lambda_1 > 0$, we obtain $\alpha \le \beta c_m + 2\gamma$. By $\lambda_3 > 0$, we obtain $\alpha > \frac{4\gamma^2 + \beta(A + \lambda B + d) + 2\beta\gamma c_r}{2\gamma} = \alpha_6$. By (A.8), we obtain $K \ge s\gamma$.

Similarly, we easily obtain conditions that make Case 6 optimal: $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$, $K > s\gamma$ and $\alpha_6 \le \alpha \le \beta c_m + 2\gamma$.

*Case* 7:

$\lambda_1 = \frac{K[c\beta - d\beta + 4(-1+s)\gamma^2] - s\gamma[2(-1+s)(\alpha - \beta c_r)\gamma + \beta(A + \lambda B + c)]}{2s^2\beta\gamma^2} + c_m - c_r$,

$\lambda_2 = -\frac{(d-c)K\beta + \beta s\gamma(A + \lambda B + c) + 4K\gamma^2 - 2s\alpha\gamma^2 + 2s\beta\gamma^2 c_r}{2s^2\beta\gamma^2}$, $\lambda_3 = 0$, $q_r = \frac{K}{s}, q = \frac{K}{s}$.

By $\lambda_1 > 0$, we obtain $\alpha > \frac{-dK\beta + c\beta(K - s\gamma) + \gamma[-As\beta + 4K(-1+s)\gamma - Bs\beta\lambda] + 2s\beta\gamma^2(sc_m - c_r)}{2(-1+s)s\gamma^2} = \alpha_2$. By $\lambda_2 > 0$, we obtain $\alpha \ge \frac{(d-c)K\beta + \gamma[4K\gamma + s\beta(A + \lambda B + c)]}{2s\gamma^2} + \beta c_r = \alpha_5$. By (A.9), we obtain $K \le s\gamma$.

Similarly, we can obtain the conditions that make Case 7 optimal: (1) $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $\frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c} \le K \le s\gamma$ and $\alpha > \alpha_2$; (2) $c_m - c_r \le \frac{A + \lambda B + d}{2\gamma}$, $K \le \frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c}$ and $\alpha \ge \alpha_5$; (3) $c_m - c_r \ge \frac{A + \lambda B + d}{2\gamma}$, $K \le s\gamma$ and $\alpha \ge \alpha_5$.

*Case* 8: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, $q_r = \gamma$, $q = \gamma$.

By (A.5), we obtain $K = s\gamma$. If so, the solutions of Case 4 and Case 7 are also as $q_r = \gamma$, $q = \gamma$. Thus, we can regard Case 8 as a special case of Case 4 and Case 7.

We see that these potential optimal solutions are distinguished by $c_m - c_r$, $K$, and $\alpha$. Therefore, we make an overall comparison among these cases.

(1) If $c_m - c_r \le \frac{A+\lambda B+d}{2\gamma}$ and $K \le \frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c}$, then Case 5 and Case 7 successionally become the optimal solutions, and the threshold of $\alpha$ is $\alpha_5$.
(2) If $c_m - c_r \le \frac{A+\lambda B+d}{2\gamma}$ and $\frac{s\gamma(2\gamma c_m - 2\gamma c_r - A - c - \lambda B)}{d-c} \le K \le s\gamma$, then Case 5, Case 1, Case 3 and Case 7 successionally become the optimal solutions, and the threshold of $\alpha$ are $\alpha_4$, $\alpha_1$ and $\alpha_2$.
(3) If $c_m - c_r \le \frac{A+\lambda B+d}{2\gamma}$ and $K \ge s\gamma$, then Case 5, Case 1, Case 3 and Case 4 successionally become the optimal solutions, and the threshold of $\alpha$ are $\alpha_4$, $\alpha_1$ and $\alpha_3$.
(4) If $c_m - c_r \ge \frac{A+\lambda B+d}{2\gamma}$ and $K \le s\gamma$, then Case 5 and Case 7 successionally become the optimal solutions, and the threshold of $\alpha$ is $\alpha_5$.
(5) If $c_m - c_r \ge \frac{A+\lambda B+d}{2\gamma}$ and $K \ge s\gamma$, then Case 5, Case 6, Case 2 and Case 4 successionally become the optimal solutions, and the threshold of $\alpha$ are $\alpha_6$, $\beta c_m + 2\gamma$, and $\beta c_m + 2K + 2(1-s)\gamma$.

Note that $c_m - c_r \le \frac{A+\lambda B+d}{2\gamma}$ is equivalent to $d \ge 2\gamma(c_m - c_r) - A - \lambda B$, and we complete the proof of Proposition 1.

**Proof of Corollary 1.** There are seven potential optimal solutions of $q_r^*$ and $q^*$ whose optimality conditions are shown in Figs. 2 and 3. Observing these seven solutions, we find that the solutions of Cases 1, 2, 4, 6, 7 are simple. Therefore we first investigate Case 3 as well as Case 5 and then the other cases.

**Case 3:** $q_r^* = \frac{\gamma[(1-s)(4K-2\alpha)\gamma + \beta(A+\lambda B+c) + 2\beta\gamma(-sc_m + c_r)]}{c\beta - d\beta - 4(-1+s)^2\gamma^2}$,

$$q^* = K + \frac{(1-s)\gamma[(1-s)(4K-2\alpha)\gamma + \beta(A+\lambda B+c) + 2\beta\gamma(-sc_m + c_r)]}{c\beta - d\beta - 4(-1+s)^2\gamma^2}$$

(3.1)
$$\frac{\partial q^*}{\partial c} = \frac{(-1+s)\beta\gamma\{2(-1+s)\gamma[-2K+\alpha-2(1-s)\gamma] + \beta(A+\lambda B+d) + 2\beta\gamma(-sc_m + c_r)\}}{[d\beta - c\beta + 4(1-s)^2\gamma^2]^2}.$$

We denote $f_1(\alpha) = (-1+s)\beta\gamma\{2(-1+s)\gamma[-2K+\alpha-2(1-s)\gamma] + \beta(A+\lambda B+d) + 2\beta\gamma(-sc_m + c_r)\}$, and $\frac{df_1(\alpha)}{d\alpha} = 2(1-s)^2\beta\gamma^2 > 0$. One optimality region of Case 3 is $\frac{2s\gamma^2(c_m - c_r) - s\gamma(A+\lambda B+c)}{d-c} \le K \le s\gamma$ with $\alpha \in [\alpha_1, \alpha_2]$ where $f_1(\alpha_2) = \frac{(-1+s)\beta(K-s\gamma)[c\beta - d\beta - 4(1-s)^2\gamma^2]}{s} < 0$. Another optimality region of Case 3 is $K \ge s\gamma$, and $\alpha \in [\alpha_1, \alpha_3]$ where $f_1(\alpha_3) = 0$. Therefore, for Case 3, $\frac{\partial q^*}{\partial c} < 0$.

(3.2)
$$\frac{\partial q^*}{\partial d} = \frac{(1-s)\beta\gamma[2(1-s)(2K-\alpha)\gamma + \beta(A+\lambda B+c) + 2\beta\gamma(-sc_m + c_r)]}{[-c\beta + d\beta + 4(1-s)^2\gamma^2]^2}.$$

We denote $f_2(\alpha) = (1-s)\beta\gamma[2(1-s)(2K-\alpha)\gamma + \beta(A+\lambda B+c) + 2\beta\gamma(-sc_m + c_r)$ and $\frac{df_2(\alpha)}{d\alpha} = -2\beta(1-s)^2\gamma^2 < 0$. Since $\alpha > \alpha_1$,

then $f(\alpha) \le f(\alpha_1) = \frac{(1-s)\beta\gamma[-c\beta + d\beta + 4(1-s)^2\gamma^2](A+\lambda B+c-2\gamma c_m + 2\gamma c_r)}{d-c} < 0$. Therefore, for Case 3, $\frac{\partial q^*}{\partial d} < 0$.

(3.3)
$$\frac{\partial q^*}{\partial \beta} = \frac{2(1-s)^2\gamma^2[(d-c)(2K-\alpha) + 2(-1+s)\gamma(A+\lambda B+c) + 4(-1+s)\gamma^2(-sc_m + c_r)]}{[-c\beta + d\beta + 4(1-s)^2\gamma^2]^2}.$$

We denote the numerator as $f_3(\alpha)$ and $\frac{df_3(\alpha)}{d\alpha} = 2(c-d)(1-s)^2\gamma^2 < 0$. Since $\alpha > \alpha_1$, then $f_3(\alpha) \le f_3(\alpha_1) = -2(1-s)^2\gamma^2[-c\beta + d\beta + 4(1-s)^2\gamma^2]c_m < 0$. Therefore, $\frac{\partial q^*}{\partial \beta} < 0$.

(3.4)
$$\frac{\partial q^*}{\partial s} = \frac{\beta\gamma\{(A+\lambda B+c+2\gamma c_r - 2\gamma c_m)[(d-c)\beta - 4(1-s)^2\gamma^2] + 4(d-c)(1-s)(2K-\alpha+\beta c_m)\gamma\}}{[-c\beta + d\beta + 4(1-s)^2\gamma^2]^2}.$$

We denote the numerator as $f_4(\alpha)$ and $\frac{df_4(\alpha)}{d\alpha} = 4(d-c)(s-1)\beta\gamma^2 < 0$. Since $\alpha > \alpha_1$, then $f_4(\alpha) < f_4(\alpha_1) = \beta\gamma[\beta(d-c) + 4(1-s)^2\gamma^2](A+\lambda B + c - 2\gamma c_m + 2\gamma c_r) < 0$, therefore $\frac{\partial q^*}{\partial s} < 0$.

(3.5)
$$\frac{\partial q^*}{\partial \gamma} = \frac{(1-s)\beta((A+\lambda B+c)[(c-d)\beta + 4(1-s)^2\gamma^2] + 4(d-c)\gamma[(-1+s)(2K-\alpha) + \beta s c_m - \beta c_r])}{[-c\beta + d\beta + 4(-1+s)^2\gamma^2]^2}.$$

We denote the numerator as $f_5(\alpha)$ and $\frac{df_5(\alpha)}{d\alpha} = 4(d-c)(1-s)^2\beta\gamma > 0$. We obtain $f_5(\alpha) \ge f_5(\alpha_1) = (1-s)\beta[c\beta - d\beta - 4(1-s)^2\gamma^2](A+\lambda B + c - 4\gamma c_m + 4\gamma c_r) > 0$ and therefore $\frac{\partial q^*}{\partial \gamma} > 0$.

For other parameters, we obtain the results as:

$$\frac{\partial q^*}{\partial A} = \frac{-(1-s)\beta\gamma}{-c\beta + d\beta + 4(1-s)^2\gamma^2} < 0 \quad \frac{\partial q^*}{\partial \lambda} = \frac{B(-1+s)\beta\gamma}{-c\beta + d\beta + 4(-1+s)^2\gamma^2} < 0$$

$$\frac{\partial q^*}{\partial K} = \frac{(d-c)\beta}{4(-1+s)^2\gamma^2 + (d-c)\beta} > 0 \quad \frac{\partial q^*}{\partial \alpha} = \frac{2(1-s)^2\gamma^2}{-c\beta + d\beta + 4(1-s)^2\gamma^2} > 0$$

$$\frac{\partial q^*}{\partial c_m} = \frac{2(-1+s)s\beta\gamma^2}{c\beta - d\beta - 4(-1+s)^2\gamma^2} > 0 \quad \frac{\partial q^*}{\partial c_r} = \frac{2(-1+s)\beta\gamma^2}{-c\beta + d\beta + 4(-1+s)^2\gamma^2} < 0$$

$$\frac{\partial q^*}{\partial B} = \frac{(s-1)\beta\gamma\lambda}{-c\beta + d\beta + 4(1-s)^2\gamma^2} < 0$$

We notice that in Case 3, $q_r^* = (q-K)/(1-s)$, therefore expect $K$ and $s$, other parameters have the same effect on $q_r$ and $q$ as stated above. In addition, it is easy to prove $\frac{\partial q_r^*}{\partial K} = \frac{4(-1+s)\gamma^2}{-c\beta + d\beta + 4(1-s)^2\gamma^2} < 0$.

*Case* 5: $q_r^* = q^* = \frac{\gamma[2\alpha\gamma - \beta(A+\lambda B+c) - 2\beta\gamma c_r]}{4\gamma^2 + \beta(d-c)}$.

(5.1)
$\frac{\partial q^*}{\partial c} = \frac{\beta\gamma[2(\alpha - 2\gamma)\gamma - \beta(A+\lambda B+d) - 2\beta\gamma c_r]}{(d\beta - c\beta + 4\gamma^2)^2}$. Since for Case 5, $\alpha \le \beta c_r + 2\gamma + \frac{\beta(A+\lambda B+d)}{2\gamma}$ always holds, see Figs. 2 and 3. In Fig. 2, $\alpha \le \beta c_m + \frac{4\gamma^2(c_m - c_r) - 2\gamma(A+\lambda B+c)}{d-c}$ with $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, we have $\alpha \le \beta c_r + 2\gamma + \frac{\beta(A+\lambda B+d)}{2\gamma}$. Therefore, we obtain $\frac{\partial q^*}{\partial c} < 0$.

(5.2)
$\frac{\partial q^*}{\partial \gamma} = \frac{\beta[4\gamma(d-c)(\alpha - \beta c_r) + (4\gamma^2 + c\beta - d\beta)(A+\lambda B+c)]}{[-c\beta + d\beta + 4\gamma^2]^2}$. We denote $f_6(\alpha) = \beta[4\gamma(d-c)(\alpha - \beta c_r) + (4\gamma^2 + c\beta - d\beta)(A+\lambda B+c)]$, since $\frac{df_6(\alpha)}{d\alpha} = 4(d-c)\beta\gamma > 0$ and $\alpha \ge \beta c_r + \frac{\beta(A+\lambda B+c)}{2\gamma}$, we obtain $f_6(\alpha) \ge f_6(\beta c_r + \frac{\beta(A+\lambda B+c)}{2\gamma}) = \beta(4\gamma^2 + \beta d - \beta c)(A+\lambda B+c) > 0$. Therefore $\frac{\partial q^*}{\partial \gamma} > 0$.

As for other parameters, we can easily obtain their effect as follows:

$$\frac{\partial q^*}{\partial d} = \frac{-\beta\gamma[2\alpha\gamma - \beta(A+\lambda B+c) - 2\beta\gamma c_r]}{(-c\beta + d\beta + 4\gamma^2)^2} < 0 \quad \frac{\partial q^*}{\partial A} = \frac{\beta\gamma}{c\beta - d\beta - 4\gamma^2} < 0$$

$$\frac{\partial q^*}{\partial B} = \frac{\beta\gamma\lambda}{c\beta - d\beta - 4\gamma^2} < 0 \quad \frac{\partial q^*}{\partial \alpha} = \frac{2\gamma^2}{-c\beta + d\beta + 4\gamma^2} > 0$$

$$\frac{\partial q^*}{\partial c_r} = \frac{2\beta\gamma^2}{c\beta - d\beta - 4\gamma^2} < 0 \quad \frac{\partial q^*}{\partial c_m} = 0$$

$$\frac{\partial q^*}{\partial \lambda} = \frac{B\beta\gamma}{c\beta - d\beta - 4\gamma^2} < 0 \quad \frac{\partial q^*}{\partial s} = 0$$

**Table 9**
The effect of parameters in different cases under the NYOP mechanism when $s \le 1$.

| Decisions | Case | $A$ | $B$ | $c$ | $d$ | $\alpha$ | $\beta$ | $K$ | $c_m$ | $c_r$ | $\gamma$ | $s$ | $\lambda$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q^*$ | Case 1 | 0 | 0 | 0 | 0 | + | – | 0 | – | 0 | 0 | 0 | 0 |
| | Case 2 | 0 | 0 | 0 | 0 | + | – | 0 | – | 0 | 0 | 0 | 0 |
| | Case 3 | – | – | – | – | + | – | + | + | – | + | – | – |
| | Case 4 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | + | – | 0 |
| | Case 5 | – | – | – | – | + | – | 0 | 0 | – | + | 0 | – |
| | Case 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 7 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | 0 | – | 0 |
| $q_r^*$ | Case 1 | – | – | – | – | 0 | 0 | 0 | + | – | + | 0 | – |
| | Case 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 3 | – | – | – | – | + | – | – | + | – | + | () | – |
| | Case 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 5 | – | – | – | – | + | – | 0 | 0 | – | + | 0 | – |
| | Case 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 7 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | 0 | – | 0 |

$$\frac{\partial q^*}{\partial \beta}=\frac{2\gamma^2[(c-d)\alpha-2\gamma(A+\lambda B+c)-4\gamma^2 c_r]}{(d\beta-c\beta+4\gamma^2)^2}<0 \quad \frac{\partial q^*}{\partial K}=0$$

Since the expressions of optimal solutions are simple in Cases 1, 2, 4, 6 and 7 where they are independent with most parameters, we thus drop detailed analysis here. In all seven cases, the effect of parameters are shown in Table 9.

In Table 9, $+$ or $-$ mean the parameters have positive or negative effect on optimal solutions, respectively. 0 means the optimal solutions are independent with parameters. () means the effect of the parameters are not certain.

## Appendix B. The manufacturer's optimal strategies under the list-price mechanism

Under the list-price mechanism, the manufacturer also decides $q_r$ and $q$ to optimize his profit function in Eq. (7). Similarly, since the profit function is joint concave with $q$ and $q_r$ and these constraints are all linear with $q$ and $q_r$, we use KKT method to solve this problem. The Lagrangian function is as follows:

$$L(q,q_r,\lambda_1,\lambda_2,\lambda_3)=\frac{(\alpha-q)q}{\beta}+\left(c_m-c_r-\frac{B}{\gamma}\right)q_r-c_m q +\lambda_1[K-q+(1-s)q_r]+\lambda_2(\gamma\tau-q_r)+\lambda_3(q-q_r) \tag{B.1}$$

$$\text{s.t.}\quad \frac{\partial L}{\partial q_r}=\left(c_m-c_r-\frac{B}{\gamma}\right)+(1-s)\lambda_1-\lambda_2-\lambda_3=0 \tag{B.2}$$

$$\frac{\partial L}{\partial q}=\frac{\alpha-2q}{\beta}-c_m-\lambda_1+\lambda_3=0 \tag{B.3}$$

$$\lambda_1[K-q+(1-s)q_r]=0 \tag{B.4}$$

$$\lambda_2(\gamma\tau-q_r)=0 \tag{B.5}$$

$$\lambda_3(q-q_r)=0 \tag{B.6}$$

$$q-(1-s)q_r\le K \tag{B.7}$$

$$q_r\le\gamma\tau \tag{B.8}$$

$$q_r\le q \tag{B.9}$$

From (B.2)–(B.6), we obtain all solutions as follows.

*Case* 1: $\lambda_1=0,\lambda_2=0,\lambda_3=0$.

By (B.2), we obtain $c_m-c_r-\frac{B}{\gamma}=0$. However, based on our assumptions that $c_m-c_r>\frac{A+\lambda B+c}{2\gamma}$ and $c\ge(2-\lambda)B-A$, it is easy to obtain $c_m-c_r>\frac{B}{\gamma}$. So this case can not hold.

*Case* 2: $\lambda_1=0,\lambda_2=0,\lambda_3=c_m-c_r-\frac{B}{\gamma}, q=\frac{\alpha\gamma-B\beta-\beta\gamma c_r}{2\gamma}$, $q_r=\frac{\alpha\gamma-B\beta-\beta\gamma c_r}{2\gamma}$.

By (B.7), we obtain $\alpha<\frac{Bs\beta+2K\gamma+s\beta\gamma c_r}{s\gamma}$. By (B.8), we obtain $\alpha\le\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}$. So the integral conditions for the case are: (1) $K<\tau\gamma s$ and $\alpha<\frac{Bs\beta+2K\gamma+s\beta\gamma c_r}{s\gamma}$. (2) $K\ge\tau\gamma s$ and $\alpha\le\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}$.

*Case* 3: $\lambda_1=0,\lambda_2=c_m-c_r-\frac{B}{\gamma},\lambda_3=0, q=\frac{\alpha-\beta c_m}{2}, q_r=\tau\gamma$.

By (B.9), we obtain $\alpha>\beta c_m+2\gamma\tau$. By (B.7), we obtain $\alpha<\beta c_m+2K+2(1-s)\tau\gamma$. So the conditions for this case are $K>\tau\gamma s$ and $\beta c_m+2\gamma\tau\le\alpha\le\beta c_m+2K+2(1-s)\tau\gamma$.

*Case* 4: $\lambda_1=0,\lambda_2=-\frac{B}{\gamma}+\frac{\alpha-2\gamma\tau}{\beta}-c_r,\lambda_3=-\frac{\alpha-2\gamma\tau}{\beta}+c_m$, $q=\gamma\tau, q_r=\gamma\tau$.

By $\lambda_2>0$, we obtain $\alpha>\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}$. By $\lambda_3>0$, we obtain $\alpha<\beta c_m+2\tau\gamma$. By (B.7), we obtain $K\ge\tau\gamma s$. Thus the conditions for this case are $K\ge\tau\gamma s$ and $\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}\le\alpha<\beta c_m+2\tau\gamma$.

*Case* 5: $\lambda_1=\frac{\gamma(c_m-c_r)-B}{\gamma(s-1)},\lambda_2=0,\lambda_3=0, \quad q=\frac{\alpha-\beta c_m}{2}-\frac{\beta[\gamma(c_m-c_r)-B]}{2\gamma(s-1)}$, $q_r=\frac{2K-\alpha+\beta c_m}{2(s-1)}+\frac{\beta[\gamma(c_m-c_r)-B]}{2\gamma(s-1)^2}$.

By $\lambda_1>0$, we obtain $s>1$, so this case is not optimal when $s\le1$ but potentially optimal when $s>1$.

*Case* 6: $\lambda_1=\frac{-Bs\beta-2K\gamma+s\alpha\gamma-s\beta\gamma c_r}{s^2\beta\gamma},\lambda_2=0$, $\lambda_3=\frac{-Bs\beta+(-1+s)(2K-s\alpha)\gamma+s\beta\gamma(sc_m-c_r)}{s^2\beta\gamma}$, $q=\frac{K}{s}, q_r=\frac{K}{s}$.

By $\lambda_1>0$, we obtain $\alpha>\frac{2K}{s}+\frac{B\beta}{\gamma}+\beta c_r$. By $\lambda_3>0$, we obtain $\alpha>\frac{Bs\beta+2K(1-s)\gamma-s\beta\gamma(sc_m-c_r)}{(1-s)s\gamma}$. By (B.8), we obtain $K\le\tau\gamma s$. Since $\frac{2K}{s}+\frac{B\beta}{\gamma}+\beta c_r-\frac{-Bs\beta+2K(-1+s)\gamma+s\beta\gamma(sc_m-c_r)}{(-1+s)s\gamma}=\frac{s\beta(\gamma c_m-\gamma c_r-B)}{(1-s)\gamma}>0$, the conditions of this case are $K\le\tau\gamma s$ and $\alpha>\frac{2K}{s}+\frac{B\beta}{\gamma}+\beta c_r$.

*Case* 7: $\lambda_1=\frac{-2K+\alpha+2(-1+s)\gamma\tau-\beta c_m}{\beta}$, $\lambda_2=\frac{-B\beta+(1-s)\gamma[\alpha-2K+2(-1+s)\gamma\tau]+\beta\gamma(sc_m-c_r)}{\beta\gamma}$, $\lambda_3=0$, $q=K-(-1+s)\gamma\tau$, $q_r=\gamma\tau$.

By $\lambda_1>0$, we obtain $\alpha>2K+2(1-s)\gamma\tau+\beta c_m$. By $\lambda_2>0$, we obtain $\alpha>\frac{B\beta+2(1-s)\gamma\left(K+\gamma\tau-s\gamma\tau\right)-s\beta\gamma c_m+\beta\gamma c_r}{(1-s)\gamma}$. By (B.9), we obtain $K>\tau\gamma s$.

Since $B\beta+2(1-s)\gamma(K+\frac{\gamma\tau-s\gamma\tau)-s\beta\gamma c_m+\beta\gamma c_r}{(1-s)\gamma-2K-2(1-s)\gamma\tau-\beta c_m=\frac{\beta(B-\gamma c_m+\gamma c_r)}{\gamma-s\gamma}<0}$, the integral conditions for this case are $\alpha>2K+2(1-s)\gamma\tau+\beta c_m$ and $K>\tau\gamma s$.

*Case* 8: $\lambda_1>0$, $\lambda_2>0$, $\lambda_3>0$, $q=\tau\gamma$, $q_r=\tau\gamma$.

By (B.4), we obtain $K=\tau s\gamma$. If so, the solutions of Cases 4, 6, and 7 are all as $q=\tau\gamma$, $q_r=\tau\gamma$. Thus we can regard Case 8 as a special case of Cases 4, 6 and 7.

All these cases are distinguished by $K$ and $\alpha$ so that we can compare them under different conditions of $K$ and $\alpha$ as follows:

(1) If $K<\tau\gamma s$, when $\alpha\le\frac{2K}{s}+\frac{B\beta}{\gamma}+\beta c_r$, Case 2 is optimal, and when $\alpha>\frac{2K}{s}+\frac{B\beta}{\gamma}+\beta c_r$, Case 6 is optimal;

(2) If $K\ge\tau\gamma s$, when $\alpha\le\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}$, Case 2 is optimal. When $\beta c_r+2\tau\gamma+\frac{\beta B}{\gamma}<\alpha<\beta c_m+2\tau\gamma$, Case 4 is optimal. When $\beta c_m+2\gamma\tau\le\alpha\le\beta c_m+2K+2(1-s)\tau\gamma$, Case 3 is optimal. When $\alpha>\beta c_m+2K+2(1-s)\tau\gamma$, Case 7 is optimal.

These above results are presented in Proposition 2.

## Appendix C. The case that remanufacturing is more capacity intensive, i.e., $s>1$

### C.1. *The manufacturer's optimal strategies under the NYOP mechanism when* $s>1$

When $s>1$, the model is the same to the case when $s\le1$. Specifically, we obtain the optimal decisions by (A.1)–(A.9). Note that Case 1 to Case 7 in Appendix A are also applicative here. However, the optimality conditions may be different. In addition, when $s>1$, the solution of $q_r$ in Case 3 may be negative if $\alpha>\alpha_7=\beta c_m+2K+\frac{2\beta\gamma(c_m-c_r)-\beta(A+\lambda B+c)}{2(s-1)\gamma}$. Obviously, if this condition holds, then $q_r^*=0$, $q^*=K$. In total, when $s>1$, there are eight possible optimal strategies including this additional case of $q_r^*=0$, $q^*=K$.

Since the method and the process to obtain optimality conditions of each case are presented in Appendix A, we ignore the detailed and tedious mathematical process. Here, we directly show the optimality conditions of Case 1 to Case 7 as well as the case of $q_r^*=0$ and $q^*=K$ in Table 10. These results are summarized by Proposition 3.

Based on these optimality conditions, we investigate the effect of parameters on the optimal decisions for each case. Since the method is very similar to the case of $s\le1$, we omit the detailed process and directly give the results in Table 11. Further, we can obtain the results of Corollary 3.

**Table 10**
Cases and optimality conditions under the NYOP mechanism when $s > 1$.

| Cases | Optimality conditions of each case | | |
|---|---|---|---|
| Case 1 | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K \geq \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha_4 \leq \alpha \leq \alpha_1$ |
| Case 2 | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \geq s\gamma$, | $\alpha \in [\beta c_m + 2\gamma, \beta c_m + 2K + 2(1-s)\gamma]$ |
| Case 3 | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K \leq \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha \in [\alpha_2, \alpha_7]$ |
| | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K \geq \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha \in [\alpha_1, \alpha_7]$ |
| | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \leq s\gamma$, | $\alpha \in [\alpha_2, \alpha_7]$ |
| | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \geq s\gamma$, | $\alpha \in [\alpha_3, \alpha_7]$ |
| Case 4 | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \geq s\gamma$, | $\alpha \in [\beta c_m + 2K + 2(1-s)\gamma, \alpha_3]$ |
| Case 5 | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K < \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha \leq \alpha_5$ |
| | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K \geq \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha \leq \alpha_4$ |
| | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \leq s\gamma$, | $\alpha \leq \alpha_5$ |
| | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \geq s\gamma$, | $\alpha \leq \alpha_6$ |
| Case 6 | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \geq s\gamma$, | $\alpha \in [\alpha_6, \beta c_m + 2\gamma]$ |
| Case 7 | $c_m - c_r < \frac{A+\lambda B+d}{2\gamma}$, | $K \leq \frac{2s\gamma^2(c_m-c_r)-s\gamma(A+\lambda B+c)}{d-c}$, | $\alpha \in [\alpha_5, \alpha_2]$ |
| | $c_m - c_r \geq \frac{A+\lambda B+d}{2\gamma}$, | $K \leq s\gamma$, | $\alpha \in [\alpha_5, \alpha_2]$ |
| $q_r = 0$, $q = K$ | | | $\alpha > \alpha_7$ |

**Table 11**
The effect of parameters in different cases under the NYOP mechanism when $s > 1$.

| Decisions | Case | $A$ | $B$ | $c$ | $d$ | $\alpha$ | $\beta$ | $K$ | $c_m$ | $c_r$ | $\gamma$ | $s$ | $\lambda$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q^*$ | Case 1 | 0 | 0 | 0 | 0 | + | − | 0 | − | 0 | 0 | 0 | 0 |
| | Case 2 | 0 | 0 | 0 | 0 | + | − | 0 | − | 0 | 0 | 0 | 0 |
| | Case 3 | + | + | + | + | + | − | + | − | + | − | () | + |
| | Case 4 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | − | − | 0 |
| | Case 5 | − | − | − | − | + | − | 0 | 0 | − | + | 0 | − |
| | Case 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 7 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | 0 | − | 0 |
| $q_r^*$ | Case 1 | − | − | − | − | 0 | 0 | 0 | + | − | + | 0 | − |
| | Case 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 3 | − | − | − | − | − | + | + | + | − | + | − | − |
| | Case 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 5 | − | − | − | − | + | − | 0 | 0 | − | + | 0 | − |
| | Case 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 |
| | Case 7 | 0 | 0 | 0 | 0 | 0 | 0 | + | 0 | 0 | 0 | − | 0 |

### *C.2. The manufacturer's optimal strategies under the list-price mechanism when $s > 1$*

With the method similar to that in Appendix B, we obtain the manufacturer's optimal strategies under the list-price mechanism when $s > 1$. Besides Case 1 to Case 7 in Appendix B, now Case 5 can also be potentially optimal. From the constraints that $q_r < q$, $q_r < \tau\gamma$ and $q_r > 0$, the corresponding optimality conditions for Case 5 is $\alpha \in [\beta c_m + \frac{2K}{s} + \frac{\beta[\gamma(c_m - c_r) - B]}{(s-1)\gamma}, \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}]$ for $K \leq \tau\gamma s$, and $\alpha \in [\beta c_m + 2K + 2(1-s)\gamma\tau + \frac{\beta\gamma(c_m - c_r) - \beta B}{(s-1)\gamma}, \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}]$ for $K > \tau\gamma s$. In addition, to guarantee non-negativity of the optimal solution, if $\alpha > \beta c_m + 2K + \frac{\beta[\gamma(c_m - c_r) - B]}{\gamma(s-1)}$, then $\hat{q}^* = K$ and $\hat{q}_r^* = 0$.

For other cases, the optimality conditions are easy to obtain. Therefore, we omit the detailed mathematical process and directly give the final results in Proposition 4. Since the effect of parameters are straightforward, we directly summarize the results in Corollary 4.

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