Distributed control design for nonlinear output agreement in convergent systems

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1 Abstract

This work studies the problem of output agreement in homogeneous networks of nonlinear dynamical systems under time-varying disturbances using controllers placed at the nodes of the networks. For the class of contractive systems, necessary and sufficient conditions for output agreement are derived, and these conditions relate the eigenvalues of the network Laplacian and the node dynamics.

2 Motivation

This result builds upon the method of [1], in which the output agreement problem is solved for the class of incrementally passive systems using dynamic couplings.

Previous results [2] study the problem of output *regulation*, and propose a solution using similar methods. However, such approaches are in general of limited use in large-scale networks, as conditions derived depend on the entire Laplacian matrix of the graph, and the resulting control strategy needs to be computed in a centralized manner.

3 Setting

We study networks of n systems

$$\begin{aligned}\dot{w}_i &= s_i w_i\\ \dot{x}_i &= f(x_i) + u_i + p_i w_i\\ y_i &= C x_i\end{aligned}$$

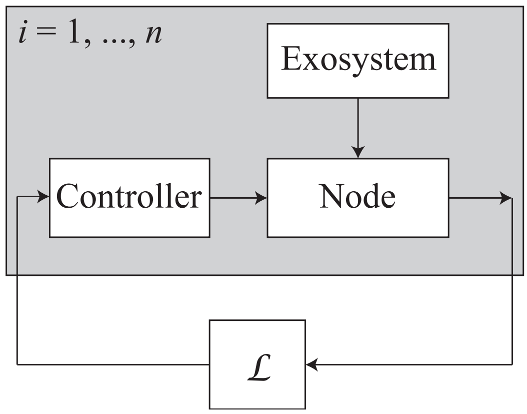
Here, s_i is assumed to be skew-symmetric. The stacked version of the interconnected system is denoted by $\bar{x}, \bar{u}, \bar{w}$ along with $\bar{f}, \bar{s}$ and $\bar{h}$.

When these are placed on the nodes of a graph $\mathscr{G}$, we denote the adjacency matrix by B and the Laplacian matrix by $\mathscr{L}$.

To achieve output agreement, we place controllers at the nodes of the form

$$\begin{aligned}\dot{\xi}_i &= p_i^T Q x_i + s_i \xi_i + H \sum_{j \in \mathscr{N}_i} (\xi_j - \xi_i),\\ u_i &= -p_i \xi_i.\end{aligned}$$

The objective is then to design H, Q and p_i such that $y_i - y_j \to 0$ as $t \to \infty$ for all pairs of nodes.



4 Approach

If the Jacobian of the entire system is quadratically stable, we achieve output agreement trivially. However, we aim to show that it's possible to achieve the same result by satisfying a simpler condition on the node systems for each eigenvalue of $\mathscr{L}$: we require instead that an expression of the form

$$f'(x) + \lambda_i H \tag{1}$$

be quadratically stable.

This result extends to dynamic controllers positioned at the nodes. In the case of scalar systems, this condition guarantees state synchronization; for the case of non-scalar systems, we have output synchronization.

References

[1] M. Bürger and C. De Persis. Dynamic coupling design for nonlinear output agreement and time-varying flow control. *Automatica*, 51:210–222, 2015.

[2] A. Pavlov, N. van de Wouw, and H. Nijmeijer. Global nonlinear output regulation: Convergence-based controller design. *Automatica*, 43(3):456–463, Mar 2007.

[3] M. Arcak. Certifying spatially uniform behavior in reaction-diffusion PDE and compartmental ODE Systems. *Automatica*, 47(6):1219–1229, Jun 2011.

[4] H. Bai and S. Y. Shafi. Output synchronization of nonlinear systems under input disturbances. *Under review*, arXiv:1312.6421, 2013.