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## Inventory Control for Multi-location Rental Systems

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# Chapter 1

## Introduction

Rental products are a common phenomenon in our society. Renting fulfills demand for products that are typically expensive to purchase and required for a limited period of time. There is a large variety of products that can be rented, such as cars, bikes, movies, books, tools, clothing, and jewelry. Customers can retrieve these products from a local facility of a rental company or have them delivered at their home address. For most products, a per-rental cost has to be paid which varies with the choice of product and the duration of the rental period. Other products, such as books and movies, are typically rented out on a subscription basis. By paying a monthly fee, customers have the right to select any title from an available collection.

While traditionally items are rented from a rental company, nowadays consumers also rent items from each other. Consumers seem to become less interested in physically owning products. Via online channels, consumers start networks in which product usage is shared. Prime examples are sharing of apartments, cars, and tools. Sharing allows the available resources to be utilized better, the environmental impact to be lower, and can also provide opportunities for social networking. The cost for customers of renting through these new channels is also lower than through traditional channels. With sharing initiatives becoming increasingly prevalent in practice, it is safe to say that renting is as popular as ever.

Rental companies often face complex inventory problems, especially if comprising of multiple rental locations. For example, decisions have to be made how to deal with customers' demand if all products are currently rented out at a certain location.

Possible options are letting the customer wait until another customer returns the product, or having the product shipped from another location which has it on stock. For the shipment option, a logistical system to transport the products has to be in place.

In the light of the societal importance of rental products and the involved challenges in inventory management, the goal of this thesis is to develop new methods for controlling inventory and organizing logistical processes in multi-location rental systems. In specific, we consider rental systems which allow shipments of rental items between locations in response to demand. We analyze various types of rental systems and focus on optimizing the stock levels and logistical operations for these different types of systems.

This introduction is organized as follows. We characterize the inventory control problems of multi-location rental systems in §1.1. In §1.2 we discuss an important practical motivation for writing this thesis: the public library system in the Netherlands. In §1.3 we review academic literature on rental systems. Finally, in §1.4 we provide an outline for the chapters in this thesis and discuss the academic and practical contribution of each chapter.

## 1.1 Inventory Control for Rental Systems

In this thesis we deal with the inventory control of multi-location rental systems, i.e., rental systems that have multiple local facilities from which customers can rent items. From an inventory control perspective, it is useful to characterize rental systems by the possibilities for stock transfer between locations. If no stock can be transferred between locations, then each location has its own inventory, so the inventory control of a location can be considered in isolation. Customers enter the location, rent a certain product, and return it at the same location after the rental period. Typical challenges arising in this setting are determining the amount of inventory to hold for each product type, and rationing rental items in case some customer types have priority over others.

If stock transfer is possible, it typically occurs in the following two ways. The first way is stock transfer by customers. For example, customers in a car rental system may rent a car from one location and return it to another location after the rental period. Some locations end up with a surplus while others have a shortage. Such inventory

imbalance is typically resolved by periodically repositioning rental inventory between the locations. The second way is stock transfer by carrying out shipments. For specialized expensive tools or small rental products such as books, unfulfilled demand may be met by carrying out shipments from locations which still have available items. These shipments can also lead to inventory imbalance, so in this setting it may also be necessary to reposition inventory. However, one of the advantages of a system with shipments is the possibility to choose shipping and receiving locations, which may be employed to reduce the imbalance.

We focus on this second type of stock transfer with shipments. These shipments lead to logistical challenges on strategic, tactical, and operational levels. On a strategic level, the main challenge is the design of the network for shipments. Possible options include shipping directly from rental location to rental location or shipping from one or more dedicated storage depots. Given the choice for a network design, on a tactical level the challenge is to determine the amount of rental stock in the system as a whole and at each location individually. On an operational level the challenge is to make cost-efficient day-to-day decisions for fulfilling demands by shipments. These decisions concern the determination of shipping and receiving locations, as well as the repositioning of returning rented items. Various trade-offs have to be made. For example, a choice for high rental stock levels at each location may lead to relatively low costs for shipments and unmet demand at the expense of significant holding costs. A policy in which stock is always balanced perfectly over the locations may lead to low costs for unmet demand at the expense of high transportation costs. In this thesis we address several of the challenges mentioned above. For different network designs, we investigate the relevant trade-offs on tactical and operational levels.

## 1.2 The Public Library System in the Netherlands

An important motivation for writing this thesis is the public library system in the Netherlands. This is a rental system of tremendous size, with over a thousand locations. First we provide background about this public library system and its societal and financial challenges. After that, we discuss the logistical processes of public libraries and the various network designs that are used in practice.

### 1.2.1 Historical and Current Trends

We now give a short historical background about the public libraries in the Netherlands and discuss various statistics and trends that are of importance to understand the current situation, see also *Bibliotheekmonitor* (2015) and *Leesmonitor* (2015).

In the 19th century, people visited public reading rooms to read and study. It is believed that by the end of the 19th century there were close to thousand of such rooms. The first public library that allowed people to borrow books was founded in Dordrecht in 1899. Starting in 1921, the national government and municipalities started with a temporary program to subsidize public libraries. Since then, several laws have been passed in the parliament that established and strengthened the role of public libraries as an institution for the provision of reading material and information. Over time, this has supported the increase in the number of public libraries from 31 in 1920 to a peak of 1165 in 1990. This number decreased to 1070 in 2000, and decreased further to 810 in 2013. However, some of the libraries that were closed in the last decade have been replaced by so-called service points with small collections and limited opening hours. As of 2013 there are 225 service points. In the period 2010-2012, 8% of the library organizations closed down one or more library facilities. It has to be noted that the closing down of libraries may lead to significant cancellation of subscriptions (Baardman et al., 2012). Cancellation rates are particularly high for the elderly who often do not have the mobility and the means to travel to a distant library.

In terms of membership numbers, there was a peak of 4.59 million members in 1994. In the period from 2000 to 2013 this number has steadily declined from 4.31 to 3.86 million. Due to efforts to promote reading for children at schools, the number of children with a membership has increased in this period from 2.05 to 2.23 million. Hence, the decrease can be attributed completely to a decline in the number of adult members, which in this same period has decreased with 29% from 2.26 million to 1.62 million. In addition, the yearly number of loaned items has almost halved from 155 million to 84 million.

This clear decrease in the number of public libraries, members, and loaned items in the last two decades can be attributed to various trends. First and foremost, the availability of digital sources has significantly reduced the demand for books. Information that previously required consultation of books can now readily be found on the internet. In addition, many books are available electronically (Van der Velde

& Ernst, 2009). In 2013, the relative share of e-books in the total book sales was 23% in the United States, 18% in the United Kingdom, and 5% in Germany (Robehmed, 2015). Evidently, these numbers are expected to only increase further. Companies are setting up new initiatives for e-books similar to online video and music streaming services such as Netflix and Spotify. Customers are then able to read a selection of e-books for a fixed monthly fee. In addition to the above trends, reading has become less popular as leisure activity, because computers, smartphones, and tablets provide alternative sources of entertainment that compete with reading.

A second development is the economic situation. The municipalities provide the majority of the funding for public libraries. In 2013, the percentage funded by the municipalities was 76.8%. In response to the global crisis of 2008, the national government has gradually reduced the funding to municipalities and in turn the municipalities have been forced to cut budgets for public libraries. In the period 2000 to 2010, the income of public libraries increased with approximately 20 million Euros per year. This contrasts with the period 2010 to 2012, in which the income reduced from 610 million to 606 million Euros. Many budget cuts have taken place since 2012; however, no general statistics have been published for this period, yet. For budget cuts on an international level, see Rowlands & Nicholas (2010). Most of the library expenditures are due to costs for personnel (47.9%), buildings (23.9%) and media rights (13.1%). The remaining 15.1% include costs for administration and other expenditures, including costs for transportation. Similar figures are found for public libraries in the United States (Davis, 2010).

### 1.2.2 Challenges and Undertaken Efforts

The trends discussed above may in the long run threaten the existence of the public libraries as an institution. In order to counter this, various digitalization efforts have been undertaken over the years to modernize the public libraries. The online presence is increased to better fit the demands of the client-base. For example, most library organizations allow clients access an online catalog that shows the collection of items and their availability. Items in the catalog can often be ordered online. Since 2013, a nationwide catalog known as the ‘Nationale Bibliotheek Catalogus’ is under development that shows availability of books in all member libraries and which is slowly being adopted by more library organizations (Bibliotheekblad, 2015). These developments make it easier for clients to access books from other libraries, and

increase the variety of the offered collection.

Despite these efforts, the decrease in demand for printed books in a digitalized world is expected to continue; hence in the future libraries are required less and less for physical storage of books. Sullivan (2003) claims that the most important feature of public libraries will be to serve as a meeting place. However, because especially elderly people prefer printed books over e-books, in the coming one or two decades it is expected there will still be considerable demand for printed books. For this transitional period, the structure of the library network is gradually being adapted. The network is moving in a direction with a focus on central libraries, with the smaller satellite libraries being closed down. In order to keep serving clients in remote areas, cost-efficient strategies have to be developed to fulfill demand. Another important aspect is determining how to adapt the collection to demand that decreases over time. In this thesis we focus on cost-efficient demand fulfillment strategies for various network designs and consider the effect of decreasing demand on the choice of collection.

In the short run, the budget cuts demand a more efficient use of resources. Some of this efficiency is reached through increased cooperation. In the last decade, various library organizations have merged. Other efforts include collaboratively building book collections within provinces of the Netherlands so that fewer items are needed in total and they are divided better between the existing libraries.

### **1.2.3 Interlibrary Loans**

An important service that public libraries offer to their clients is the possibility of interlibrary loans (ILL). That is, clients can request items to be shipped from another library. The process is often as follows. Clients are able to place reservations for items that are unavailable at their regular library. If the library possesses the item, but it is currently borrowed by another client, the item will be reserved for the client. Once the item has returned, the client receives a message that the item is ready for pick-up. Typically, the number of reservations per item per library is limited to one. If the library does not possess the item, the librarian can issue an ILL request to have it shipped from another library. In most cases the ILL request is shipped from a library within the same municipality. If the item is unavailable within the municipality, it is requested from a library elsewhere, preferably in the province. Within the municipality, clients can often use the ILL service for free, but between

municipalities the client may have to pay a fee ranging between 0 and 5 euros. The goal of the fee is typically not to fully cover the costs of ILL, but to encourage a limited use of the ILL option. Another practical alternative is to provide clients with an e-book, however this is not often used at the moment.

Though not available in the same detail as membership numbers, some numbers are available about the usage of ILL in the Netherlands. The organization of so-called Plusbibliotheken, having 15 facilities with specialized research collections, has reported their ILL numbers for 2012 (Plusbibliotheken, 2015). The organization had 250,000 ILL requests from various university and public libraries. 210,000 of these requests were solved within the province, whereas 40,000 items were shipped between provinces. The shared catalog, as discussed in §1.2.2, may lead to an increase in ILL usage. As a consequence, costs for transportation may increase.

#### 1.2.4 Network Designs

Currently, library organizations use several network designs for shipping and repositioning requested items. The design typically depends on the level at which the ILL request takes place, i.e., on the municipal, regional, or national level. Cooperation is typically very high at the municipal level because of small distances and shared funding. We can roughly distinguish between the following network designs.

- Standard single-echelon system with fixed ownership: all libraries are equipped for shipping ILL requests and are located in the same echelon. After the requested item is returned, the item is shipped back to its original library. This is the most commonly used system, applied in many municipalities, but also between the various university libraries in the Netherlands.
- Standard single-echelon system with floating: all libraries are equipped for shipping ILL requests and are located in the same echelon. Returned items remain at the library of the request. Hence, items ‘float’ through the system depending on the demand. This system is common for organizations with several libraries in large cities, for instance Utrecht.
- System with shipments from a support depot: there is a support depot dedicated to shipment of ILL requests. Requested items are shipped from the support depot to libraries on a certain route, while at the same time returned items



are picked up. The shipments may also be outsourced to a third party logistics company. This type of system exists within municipalities and provinces. On a national level, public libraries can place requests for rare books at the central facility of the Royal Library.

- Cross-docking system: there is a facility dedicated to cross-docking of ILL requests. Requested items are first shipped to a cross-dock, sorted, and then shipped to the requesting libraries. This system is used within the province of Drenthe.

The above categorization of network designs forms a simplification of reality, since libraries may use several of these systems simultaneously. Variations are also possible with multiple centralized facilities and alternative shipment policies. For example, in the province of Friesland an alternative cross-docking policy is applied, with items picked up early in the route being sorted on the truck and delivered to libraries later in the route.

Clearly, each choice of network design requires different policies for shipments and repositioning. As there is a significant number of shipment requests, we believe there is a benefit in carrying out these operations as efficiently as possible. Similar to the public libraries, rental companies may use the above network designs to deal with shipment requests. One of the important questions that we try to answer in this thesis is therefore how to efficiently coordinate the shipments and repositioning of items in various network designs.

### 1.3 Literature on Rental Systems

Even though rental businesses are common in practice, the available research on this topic is comparatively limited (Jain et al., 2015). This may come as a surprise since rental systems feature various relevant and complex problems which include, but are not limited to, optimizing the number of rental items, the repositioning of stock after returns, and the replacement of broken items. We start by giving a short overview of problems considered in settings with a single location, after which we discuss contributions in settings with multiple locations.

### 1.3.1 Single-location Rental Systems

The main branch of literature on rental systems considers single location systems with a generic type of rental product. An early contribution is due to Tainiter (1964), who formulated a stochastic model to optimize the number of rental items for a variety of profit functions. Whisler (1967) considers a dynamic program in which the number of rental items can be adapted at several points in time to minimize expected costs. Both these authors observed the close relation between rental products and servers in queueing models. This direction with queueing has been followed by various authors. One particular stream concerns rental businesses with different customer classes. Savin et al. (2005) consider two customer classes and formulate an approximation based on fluid queues to optimize the size of the rental fleet. The authors show how the use of allocation schemes may lead to increased profit margins. Fluid queues also appear in Bassamboo et al. (2009) and Jain et al. (2015). Jain et al. (2015) consider two customer classes in a setting with decreasing demand, and show how the optimal prioritization of one customer class over the other changes as demand decreases.

The following authors have dealt with initial stocking decisions for new rental items. Pasternack & Drezner (1999) determine the rental stock for a video rental company which incorporates cinema visitor data. They consider that demand decreases over time and formulate variations of newsvendor models to determine rental stock levels in different regimes of the demand height. In a similar fashion, Baron et al. (2011) consider the purchasing of initial stock and the allocation of this stock over several rental locations, in presence of different types of revenue-sharing contracts. Inspired by a dress rental company, use-based loss is considered by Slauch et al. (2015). They show the importance of accounting for use-based loss in initial stocking decisions and investigate the optimal recirculation policy, i.e., the policy that specifies which rental item to rent out to a new customer. The collection for a public library with limited capacity is determined by Bijvank (2009), who solves an assortment problem using data of library loans.

Inventory control models of single stock points with base stock control are also relevant for rental systems. The process of receiving a returned rental product after a stochastic rental period is similar to receiving an order from a supplier after a stochastic lead time.

### 1.3.2 Multi-location Rental Systems

Single location models are a powerful tool for analyzing practical problems of rental businesses, but they do not capture the dynamics of multi-location rental systems in which rental stock may transfer between locations by customers or by use of shipments. Such shipments are also known as lateral transshipments, i.e., shipments between locations in the same echelon. These provide possibilities for pooling stock, similar as in inventory models. Paterson et al. (2011) categorize literature about inventory models depending on whether reactive or proactive transshipment is considered. Reactive transshipment is carried out in response to demand, whereas proactive transshipment is carried out in anticipation of demand. In these inventory models products are consumed on demand, whereas in rental systems all products return after some time. Hoadley & Heyman (1977) and Ching et al. (2003) do consider lateral transshipment in a setting with returning products, however, this return process is exogenously given.

We find it useful to distinguish between fixed-return systems and free-return rental systems. In fixed-return rental systems products are typically rented from and returned to a specific rental location. Prime examples are books and DVDs. Reactive transshipments can be carried out in response to stock-outs. Depending on the application, proactive transshipments may also be possible. These rental products are usually not needed immediately and can therefore be backordered. In free-return systems products may be rented from one location and returned at another location after the rental period. Typically these systems are for mobile products such as cars and bikes. Reactive transshipments are rarely used in free-return systems, since customers are less inclined to wait. A typical policy is to offer a substitute product on a stock-out, so that demand will not be lost. Proactive transshipment (or: repositioning of stock) is extremely common in free-return systems. These operations are carried out overnight to meet the stochastic demand of the next day.

Literature has paid attention almost exclusively to free-return rental systems. The aspects considered here are the sizing of the rental fleet and the optimal repositioning policy. (Beaujon & Turnquist, 1991) optimize these aspects simultaneously by proposing a network approximation. George & Xia (2011) formulate a closed queuing network where items rented from one location return at another location with a certain probability. The availability of rental items at each location is analyzed in steady-state and the results are used to optimize the total size of the fleet. Reposi-

tioning is not considered explicitly in the model, but based on the results the authors provide several suggestions for repositioning. Loss models are also applied by Fricker & Gast (2014) for a bike-sharing system with limited capacity stations. They evaluate how optimal stock levels change when stock is repositioned and when incentives are introduced for customers to return items to low-stock locations. Papier & Thonemann (2008) too base availability of rental stock at a location on a loss model, and analyze how having a combined rental fleet can reduce the expected lost demands.

An ILP for optimal routing and repositioning of stock in a bike-sharing system is formulated by Chemla et al. (2013). The operations are carried out overnight and should lead to a certain target number of bikes at each location under minimal cost. This can be regarded as a static repositioning problem. Nair & Miller-Hooks (2011) consider a dynamic repositioning model without such fixed target numbers. They solve a stochastic mixed integer program, but do not consider routing aspects. The branch of literature on vehicle and bike-sharing systems is rapidly expanding. We refer to DeMaio (2009) and its forward references. An important difference between bike-sharing models and rental models is that all stock is assumed to be available at the end of the day, whereas in rental models the stock may still be rented out.

Given their prevalence in practice, it is notable that we found almost no literature about fixed-return rental systems. Vliegen (2009) considers parts and service tools that can be shipped to other warehouses in response to demand. The parts are consumed, whereas the service tools are returned to their original warehouse after use. Optimal repositioning of these service tools is not considered. The special fixed-return rental system with a support depot and several local facilities has significant overlap with repairable inventory systems. Under specific assumptions for the backordering policy and the shipping policy for returned items at local facilities, such rental systems can be equal to the METRIC model from the seminal paper by Sherbrooke (1968).

## 1.4 Contribution and Outline

In the preceding sections, we discussed the logistical challenges of multi-location rental systems, in particular those of public libraries. In academic literature there seems to be relatively limited attention for fixed-return rental systems, i.e., rental systems with reactive transshipments. We contribute to the field by analyzing and optimizing these shipment decisions for various network designs. We focus on three designs commonly

used in practice as discussed in §1.2.4: the standard single-echelon system, the system with shipments from a support depot, and the system with shipments from and cross-docking via a support depot. These systems are displayed schematically in Figure 1.1. For these systems we develop methods to minimize operational costs by carrying out shipments, repositioning stock, and setting appropriate stock levels.

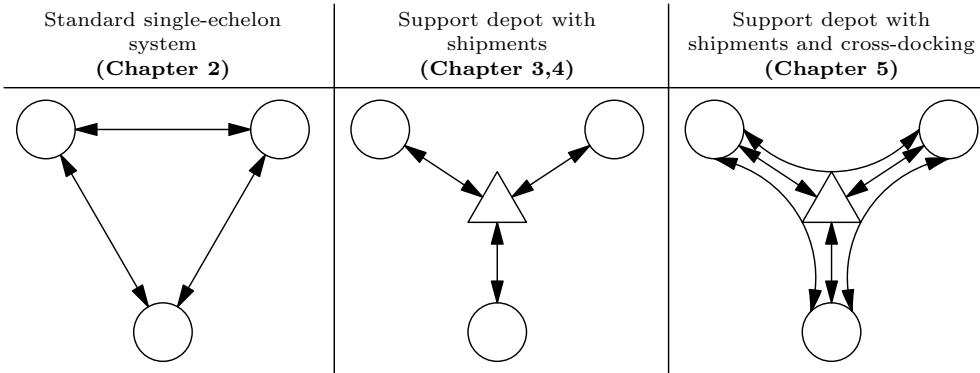


Figure 1.1: Types of network designs

For each of the considered network designs we have two common goals. The first goal is to gain intuition into optimal shipment and repositioning policies. Typically, the approach involves obtaining analytical results for small size problems and using numerical methods such as dynamic programming and Markov decision theory to investigate optimal policies. The second goal is to use this intuition to propose heuristics that lead to good solutions. As rental systems in practice may have a substantial number of locations, these heuristics need to scale well in order to effectively obtain solutions for large size problems.

Now we summarize the contribution of each chapter in the thesis. In Chapter 2 we investigate a single-echelon rental system with lateral transshipments, displayed in the left of Figure 1.1, and determine the optimal policies for transshipment and repositioning. In practice, it is customary to return transshipped items to the location that owns the item after the rental period. Clearly, any transshipped item will have to be shipped twice in such a regime: once to fulfill the demand at another location and once to return it to the owner location after the rental period. We evaluate how this fixed ownership influences the operational costs of the system. We compare the optimal repositioning policy, as obtained by a stochastic dynamic program, with the

fixed ownership policy as well as with a floating policy where returned items remain at the location they have been shipped to. Comparison of these policies provides various directions for efficiently organizing the process of returning transshipped items in single-echelon rental systems.

In Chapters 3 and 4 we consider a system with a support depot dedicated to storage and shipment of rental products, displayed in the middle of Figure 1.1. In Chapter 3 we focus on the tactical decision of setting optimal stock levels in such a system, taking the operational policy for shipments and repositioning as given. For settings without backordering, an easy-to-calculate approximation based on Erlang blocking models can be used to determine costs of given stock levels. We extend this approximation with partial backordering, as this is a common feature of rental systems. By using single-depot single-location decompositions we obtain bounds on the optimal stock levels and we propose a new efficient method for determining the optimal stock levels. In a sensitivity analysis, we determine how stock levels have to be adapted in the common situation that demand for rental items decreases over time.

In Chapter 4 we optimize the operational policy for the same system as in Chapter 3. Unfulfilled demands are met by shipments from the depot. However, this depletes the inventory of the depot, so (returned) stock at the locations must be used to replenish inventory at the depot. This is referred to as a take-back operation. In order to gain insight in how to coordinate these operations, we study the optimal shipment and take-back policies by solving a Markov decision process and analyzing its optimal policy. This serves as a basis for formulating a heuristic, which is compared with the optimal policy in small instances and with alternative heuristics in large instances.

In Chapter 5 we consider a support depot with shipments and cross-docking, displayed in the right of Figure 1.1. The model from Chapter 4 is extended with an option for cross-docking between locations as well as nonidentical cost parameters at the locations. Structure results are provided for the optimal take-back policy in a setting with a single location and a single depot, and these results are extended to special cases of the two- and multi-location setting. We formulate a heuristic for this new situation with cross-docking and we show in which situations it can be important to enable such cross-docking operations. Finally, since in all chapters we assume memoryless rental time distributions, we test the validity of this assumption here.

By solving these problems we extend academic literature with an analysis of various network designs that have not been considered before in inventory control models for rental systems. In addition, we develop methods which are easy to implement in software packages and which apply to common network designs used by rental companies in practice.