Some aspects of the theory of elastic scattering of spin 0 and spin 1/2 particles by spin 0 targets

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S U M M A R Y

In this thesis some aspects of the process of elastic scattering have been investigated. All the considerations are connected with the question whether and to what extent the interaction between the bombarded and the bombarding particle can be derived from experimental data. This problem is divided here (as usual) into two steps by the introduction of the so-called phase shifts. In chapter 1 both the projectile and the target have a spin o. In chapters 2, 3 and 4 one of the spins is o and the other $\frac{1}{2}$. The research is restricted to experimental data obtained at only one value of the energy.

In chapter 1 we have first given a brief survey of the formalism of the scattering theory and the first Born approximation for spin o - spin o scattering. Next we have shown that in a phase shift analysis of the differential effective cross section each of the phase shifts can be determined only except for an integer multiple of π. A second ambiguity is a consequence of the invariance of the scattering cross section for the simultaneous reversal of the sign of each phase shift. Then the possibility of a unique phase shift analysis (apart from the above modulo π and sign ambiguities) has been investigated. A method for the performance of the phase shift analysis is indicated, which can, under certain conditions, be used in practice, and which is usually unique in that case. In most cases, however, the uniqueness of the analysis is uncertain, and no method is known with the help of which the analysis can be executed in practice. Finally a few remarks are made in this chapter about the problem of finding the interaction from a set of scattering phase shifts. In an appendix a few formulae are added which may be useful for the actual execution of the phase shift analysis.

Chapter 2 contains a number of aids to the considerations in chapters 3 and 4. First the formalism for spin $\frac{1}{2}$ - spin o scattering is indicated, and the first Born approximation for this case is briefly sketched. As experimentally measurable quantities we have here not only the ordinary effective cross section (for unpolarized particles), but also the cross section for initially completely or partially polarized particles; besides the polarization of the particles after the scattering can be measured as a function of the initial polarization. These data can be taken into account in a clear way by making use, for the spin states of the spin $\frac{1}{2}$ particles, of the density operator.

The spin $\frac{1}{2}$ particles are described with the help of the nonrelativistic Pauli theory. It is proved, however, that the scattering process can be formally described in the same way if the

spin $\frac{1}{2}$ particles obey the relativistic one-particle Dirac equation. But a further discussion shows that this is presumably of little practical use in most cases.

Then methods are discussed for producing polarized beams of particles and for measuring the polarization of a beam. We have investigated to what extent and in what way the amplitude operator can be calculated from the experimental data. It turns out that the amplitude operator can in principle be determined completely apart from a phase factor from the results of single, double, triple and quadruple scattering experiments.

In this chapter also a coordinate transformation, which is called a compass rotation, is introduced. It appears that it enables us to give a more symmetrical description of the scattering process. With a view to possible other applications this transformation has been formulated more generally than would have been necessary for our purpose. Finally the amplitude operator is cast in a different form, which is useful for some of our considerations.

In chapter 3 we begin by applying the compass rotation to spin $\frac{1}{2}$ - spin 0 scattering. It appears in a simple way that from an arbitrary set of phase shifts, belonging to some scattering process, three closely related sets of phase shifts can be derived. The fact is that the four sets of phase shifts turn out to give rise to exactly the same differential scattering cross section if the polarizations of the incoming beams of particles in the four corresponding scattering processes have equal but oppositely directed longitudinal and/or transversal components. The polarizations of the particles scattered in an arbitrary direction then turn out to exhibit the same symmetry relations with respect to each other. These symmetry properties are treated in two different ways. They prove to have far-reaching consequences for the interpretation of measurements. In order to decide which of the four sets of phase shifts describes a certain scattering process we must perform experiments with an incoming beam of particles with non-vanishing polarization, and the final polarization in that case has to be measured. Either the initial polarization must have a known non-vanishing longitudinal component, or the longitudinal component of the final polarization must be measured. This means that at least quadruple scattering experiments must be performed, which meets with large difficulties for the intensity. A number of cases in which the four sets of phase shifts are still more closely related, is discussed. Then a few practical cases are briefly explained.

Finally we have paid some attention in this chapter to the connection between scattering phase shifts and interaction. In particular we have dwelt on the question whether, given an interaction corresponding to a certain set of phase shifts, other interactions can be found corresponding to the three sets of related phase shifts mentioned above.

In an appendix some considerations are devoted to a time-dependent formulation of the scattering problem in connection with invariance properties with respect to reversal of the direction of motion, rotations and reflections.

Chapter 4 treats of a number of properties which facilitate a (generally not unique) phase shift analysis of the effective cross section for initially unpolarized spin $\frac{1}{2}$ particles. Moreover different kinds of ambiguities in the phase shift analysis are discussed. Here it turns out that usually a denumerably infinite number of sets of phase shifts will be exactly equally consistent with the cross section. In a few degenerate cases, however, a continuously infinite number of sets of phase shifts can be equally well used.

The thesis ends with a number of general conclusions.