Hypothesis testing problems with the alternative restricted by a number of inequalities

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PREFACE

This thesis presents methods to derive decision procedures (tests), as solutions of clearly stated optimum problems, for certain classes of hypothesis testing problems where the alternative is defined by a number of inequalities.

A general theory is given in part I; this theory is applied to a number of actual testing problems in part II.

Besides the Neyman-Pearson formulation of a testing problem, two decision-theoretical formulations will be considered.

For testing problems in the *Neyman-Pearson formulation* the criterion "*most stringent S.M.P.(D)*" is introduced. Methods to derive most stringent S.M.P.(D) tests will be given in chapter 2.

For *decision theoretical* testing problems the similar criterion "*minimax regret S.M.R.(W)*" is formulated. Methods to derive minimax regret S.M.R.(W) decision procedures will be given in chapter 3.

In a number of cases of actual importance (*v.* chapters 7, ..., 11), the *exact* construction of the procedure, with the formulated optimum property, is practically impossible. A method to deal with such problems will be described in section 2.11. The original problem is replaced by an *approximately equivalent* problem that admits the construction of a procedure with the formulated optimum property. Next this procedure is proposed for the original problem. One might support the choice of this procedure by proving certain *limit theorems* stating that a *sequence* of such procedures has *asymptotically* the optimum property formulated for a *sequence* of original problems. However only weakened forms of such limit theorems will be considered in rather special cases (sections 2.12 and 7.2).

As to the applications in part II, special attention may be requested to the problems (*i*) to test homogeneity against *trend* (chapters 5 and 8), (*ii*) to test additivity of effects against *positive interaction* (chapter 6), (*iii*) to test independence against *positive dependence* or against *positive correlation* (chapter 9) because the results for these problems are of most actual importance. The notions "positive interaction" and "positive correlation" are believed to be new.

The list of references is not intended to give a representative review of the work done in the theory of testing against restricted alternatives.

The theory of this thesis originated in discussions with research-workers who apply statistical methods especially to the medical field. These discussions

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resulted in the problem to determine the "best weights in the test (5.14) against trend" and to give reasons for choosing a test of this form (5.14). For that purpose I introduced the criterion "most stringent S.M.P. size-α" and derived the theorems 1 of chapter 5 and the result of section 4.5. Further I proposed to use certain "asymptotically" most stringent S.M.P. size-α tests for the corresponding problems $\{(H, K_1), \sigma^2 \text{ unknown}\}$ and for the problem of section 5.6 to test homogeneity of probabilities against an upward trend (1963).

Next *Professor Dr. L. J. Smid* made an indispensable contribution. He introduced the general formulation of the problems (section 2.4) and he arrived at the results of the sections 2.6 and 2.7 by using the "straightforward" geometrical method described in section 2.5 and applied throughout this thesis (v. the collective paper [15]).

This contribution made it possible to write the sections 2.8 and 2.9 and to describe the method of section 2.11. Thus I could obtain the results of part II by applying the general results of chapter 2 (1964).

The theory of chapter 3 originated in sessions, devoted to Lehmann's "Testing Statistical Hypothesis", of the Netherlands Statistical Association (1965).

An appointment as assistent and later on as scientific officer to the Applied Mathematical Department of the Mathematical Institute of the University of Groningen enabled me to work out the theory of this thesis.

Useful suggestions concerning the final form of the manuscript have been made by Prof. Dr. L. J. Smid.

Figure 7 of section 3.4 has been computed by the TR-4 of the University of Groningen by means of a program written by Mr. L. Th. van der Weele.

Mrs. A. E. van Deemter-Loman and Miss A. J. Meijer typed the manuscript. Mr. B. Kamps of the Physical Laboratory of Groningen University drew the figures. Mr. G. Bosma read the English.

PART I

THE GENERAL THEORY

This part contains a theory to obtain procedures for certain general classes of hypothesis testing problems where the alternative is *restricted by a number of inequalities*.

The formulation of a testing problem has to depend on the nature of the problem as it arises from actual practice. Various possible formulations are discussed in chapter 1.

The *Neyman-Pearson* formulation by means of the restriction to the class of size-α procedures seems to be useful for many problems from practice. A theory based on this formulation is developed in chapter 2.

A *decision-theoretical* formulation where losses are attached to errors of several kinds may be more appropriate than the Neyman-Pearson formulation. Especially a *three-decision* formulation seems to be attractive for many actual situations including those where the Neyman-Pearson formulation is suitable. Certain decision-theoretical testing problems are treated in chapter 3.

The theory of part I will be applied in part II to a number of actual testing problems.

Finally we give some *directions for the reading* of part I.

It is not necessary to read chapter 1 before chapter 2. So the reader may confine his attention to chapter 2 if he is only interested in testing problems in the Neyman-Pearson formulation. Moreover there is no need for him to read the complete chapter 2 before one of the applications of part II (see the directions for reading at the beginning of part II).

The reader whose main interest lies in three-decision testing problems may restrict his accurate reading to the sections 1.3, 1.4, 1.7, 2.3, ..., 2.6(2.7), 2.10, 3.3, 3.4(3.5).

We remark that in all these cases reading of the sections 2.3, ..., 2.6 and 2.10 cannot be dispensed with. These sections and section 2.11 may be regarded as the most important part of this work.