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Bifurcations in Hamiltonian systems

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Bifurcations in Hamiltonian systems

Computing singularities by Gröbner bases

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Computing singularities by Gröbner bases

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